Higgs enhancement and bound state formation in coannhilation scenarios of dark matter

Julia Harz

in collaboration with

Kalliopi Petraki

based on

JHEP 1904 (2019) 130, [arXiv:1901.10030]
JHEP 1902 (2019) 186, [arXiv:1811.05478]
Phys. Rev. D97 (2018) no.7, 075041, [arXiv:1711.03552]

Quarkonia meet Dark Matter
17.06.2021

Technische Universität München

Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

Why Coannihilation?

What is the status of the WIMP?

Weakly **i**nteracting **m**assive **p**article – **WIMP** – a failed miracle?

Berkeley News   Research ▾   People ▾   Campus & community

RESEARCH, SCIENCE & ENVIRONMENT

MACHOs are dead. WIMPs are a no-show. Say hello to SIMPs.

By Robert Sanders, Media relations | DECEMBER 4, 2017

nature

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NEWS · 02 OCTOBER 2020

Last chance for WIMPs: physicists launch all-out hunt for dark-matter candidate

Researchers have spent decades searching for the elusive particles – a final generation of detectors should leave them no place to hide.

Forbes

Feb 22, 2019, 02:00am EST | 57.866 views

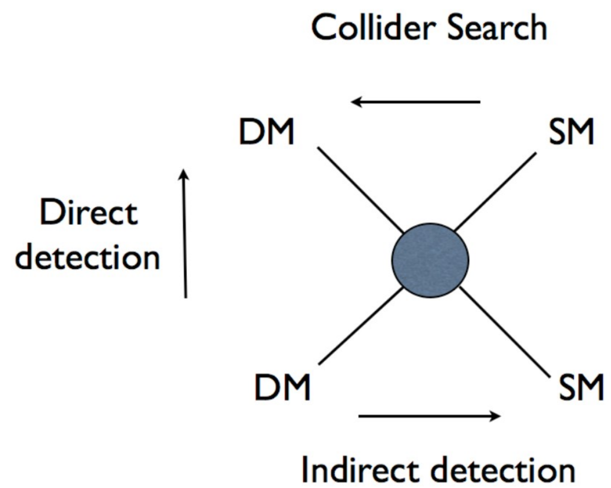
The 'WIMP Miracle' Hope For Dark Matter Is Dead

Ethan Siegel Senior Contributor
Starts With A Bang Contributor Group ⓘ
Science
The Universe is out there, waiting for you to discover it.

WIMP miracle?

$$\Omega h^2 \sim \frac{10^{-10}\mathrm{GeV}^{-2}}{\langle \sigma v \rangle} \sim 0.1$$

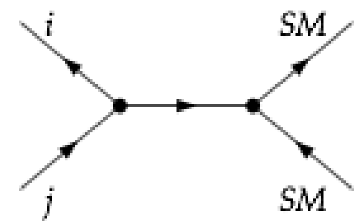
$$\Rightarrow \quad \langle \sigma v \rangle \sim \frac{g^4}{m_\chi^2} \sim 10^{-9}\mathrm{GeV}^{-2}$$



- no observations at the LHC, direct or indirect detection so far that supports the *minimal* WIMP model

- **one** reason could be the realisation of a more complex WIMP model, *e.g. non-trivial dark sector including co-annihilation scenarios etc.*

Dark Matter Freeze-out

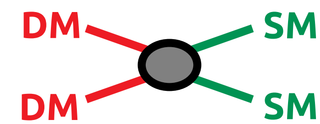


Thermal equilibrium regime (T >> m)

annihilation and production of DM in thermal equilibrium $Y \approx \text{const.}$

(2) Annihilation regime (T ~ m/10)

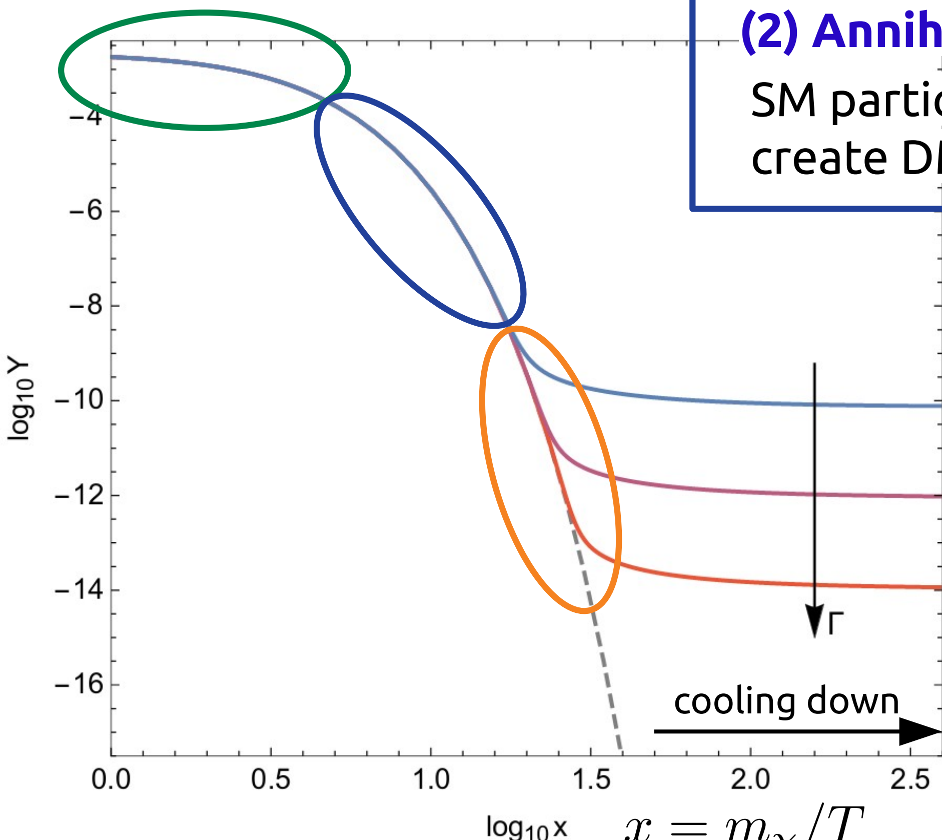
SM particles not energetic enough to create DM particles $Y \approx \exp(-m_{DM}/T)$



(3) Freeze-out (T ~ m/30)

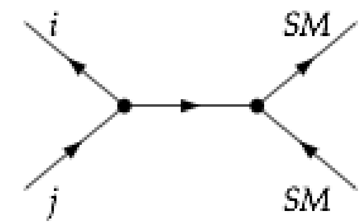
Annihilation rate falls behind expansion rate

$$\frac{\Gamma}{H} < 1$$



$x = m_\chi / T$

Dark Matter Freeze-out



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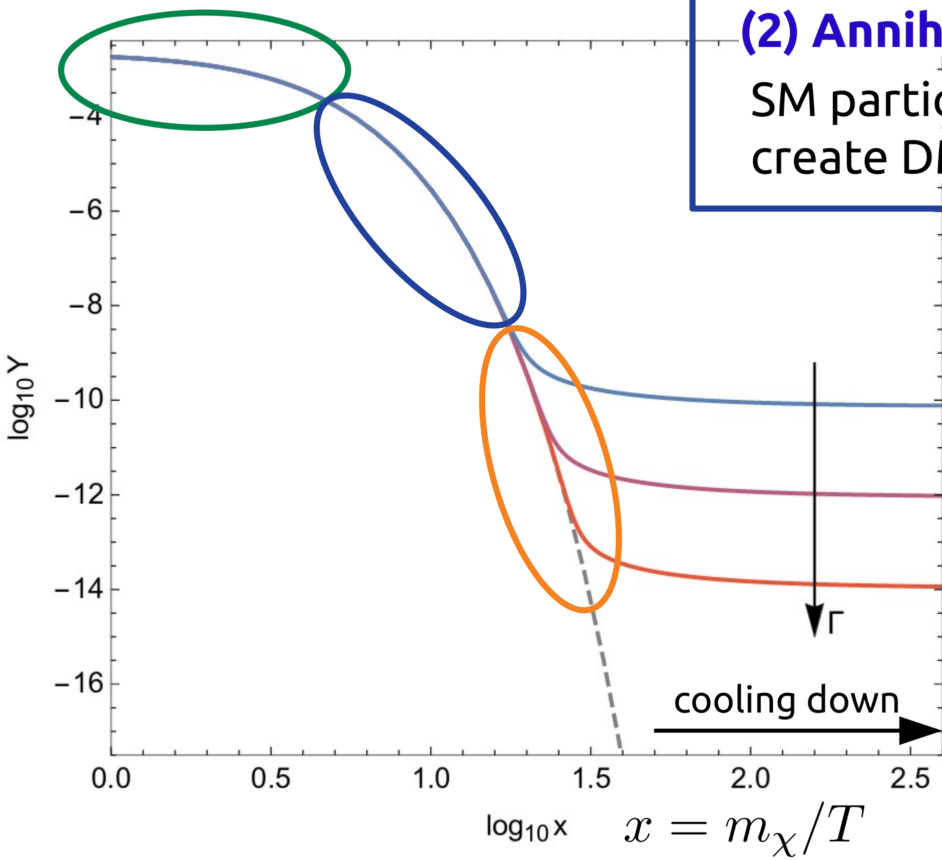

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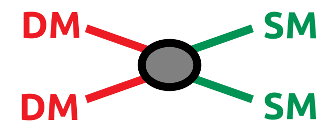

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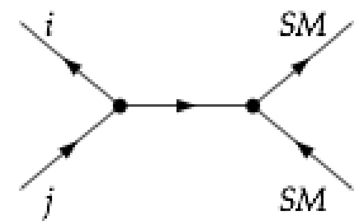
$$\dot{n} + 3Hn = -\langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2)$$

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\text{eff}} v \rangle}$$

Dark Matter Freeze-out



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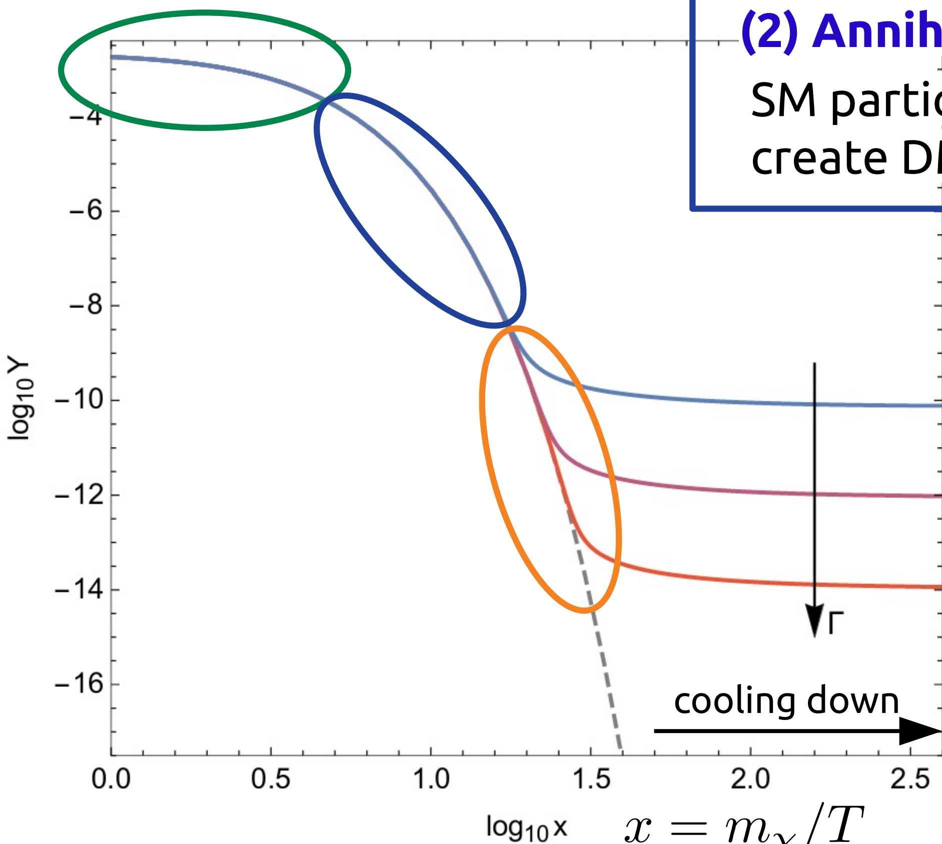

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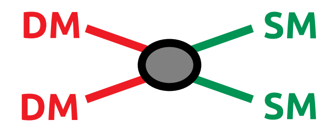

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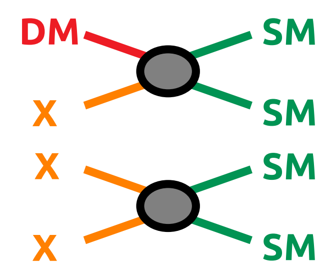
$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\text{eff}} v \rangle}$$

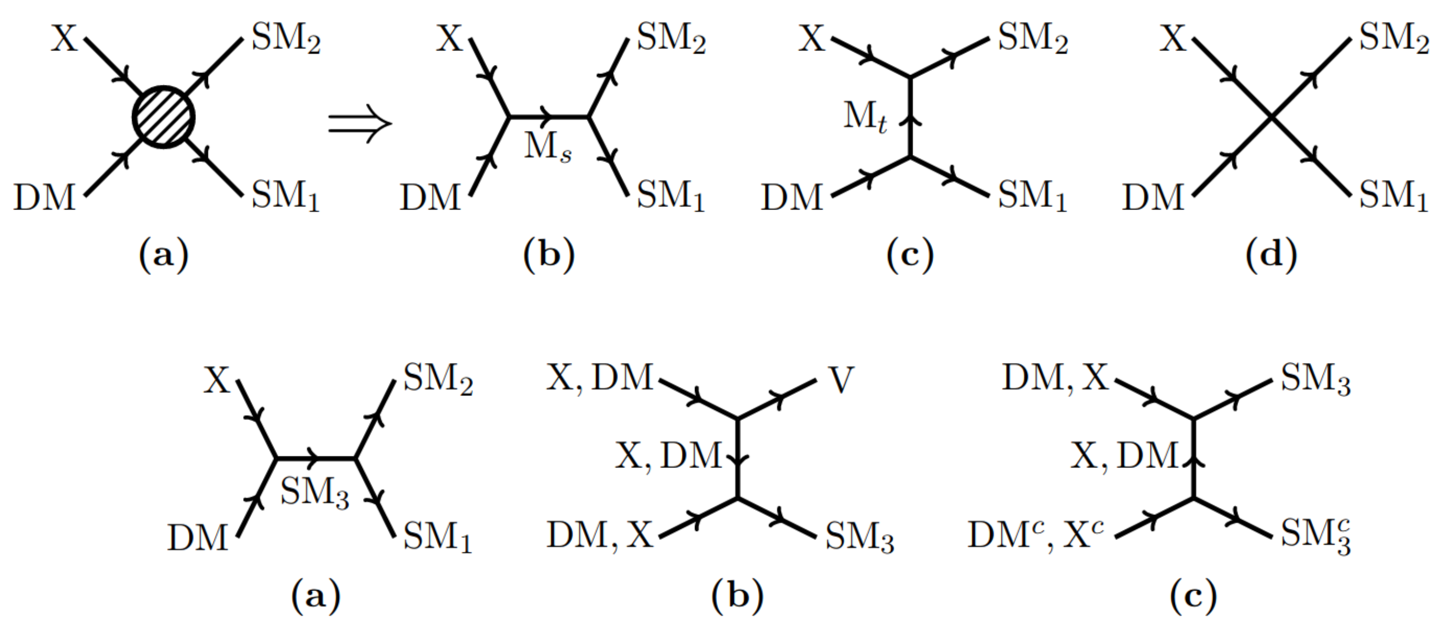
$$\langle \sigma_{\text{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq}}{n^{eq}} \frac{n_j^{eq}}{n^{eq}}$$



$$\frac{n_i^{eq}}{n^{eq}} \propto \exp^{\frac{-(m_i - m_\chi)}{T}}$$

co-annihilation

Recent specific focus on (colored) Coannihilation



Coloured coannihilations: Dark matter phenomenology meets non-relativistic EFTs, Biondini et al (2018)

Cornering Colored Coannihilation, El Hedri et al (2018)

Stop Coannihilation in the CMSSM and SubGUT Models, Ellis et al (2018)

Simplified Phenomenology for Colored Dark Sector, El Hedri et al (2017)

The Coannihilation Codex, Baker et al (2016)

Anatomy of Coannihilation with a Scalar Top Partner, Ibarra et al (2015)

To name only few examples…

Exceptions in the calculation of the relic density

1990

Famous three exceptions:

(1) particles degenerate in mass to DM (*coannihilation*)

(2) annihilation in heavier states (*forbidden channels*)

(3) resonant enhancement

October 1990

CfPA-TH-90-001
BA-90-79

UNIVERSITY OF CALIFORNIA, BERKELEY

CENTER FOR PARTICLE ASTROPHYSICS

Three Exceptions in the Calculation of Relic Abundances

KIM GRIEST

Center for Particle Astrophysics and Astronomy Department, University of California, Berkeley, CA 94720

and

DAVID SECKEL

Bartol Research Institute, University of Delaware, Newark, DE 19716

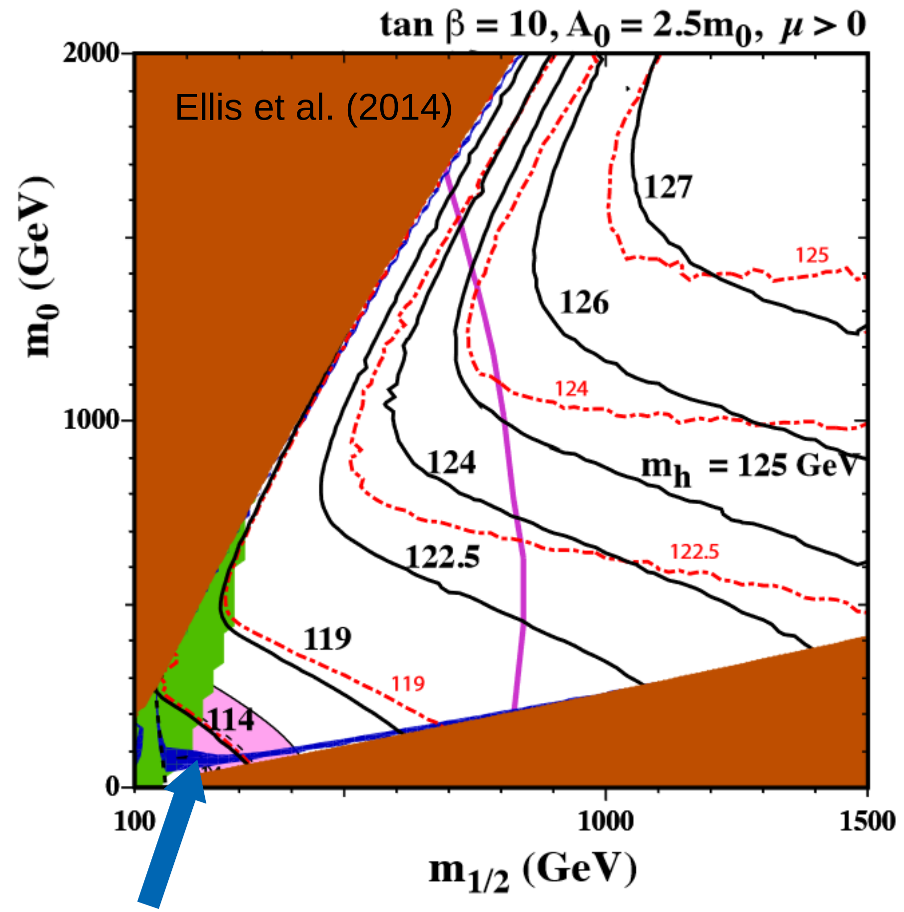
Test your favorite model

Software tools are publicly available to constrain the parameter space, e.g.:

MicrOMEGAs, Belanger, Boudjema, Pukhov, Semenov et al. [2002-]

DarkSUSY, Bringmann, Edsjo, Gondolo, Ullio, Bergstrom [2002-]

MadDM, Ambrogio, Arina, Backovic, Heisig, Maltoni, Mantani, Mattelaer, Mohlabeng [2014-]



$$\Omega_{\rm CDM}h^2 = 0.120 \pm 0.001$$

Everything settled and straightforward?

Exceptions in the calculation of the relic density

2021

modified Hubble expansion

$$\boxed{\dot{n} + 3Hn = -\langle \sigma_{\rm eff} v \rangle (n^2 - n_{\rm eq}^2)}$$

Break down of assumptions

modification of particle physics cross section

- NLO
- theoretical uncertainties
- Sommerfeld enhancement
- Boundstate formation

finite temperature effects

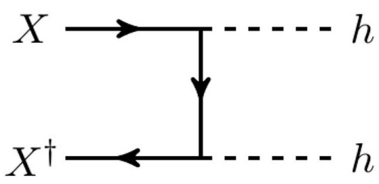
Effects impacting the particle cross section

Effects impacting the particle cross section

$$\dot{n} + 3Hn = -\langle \sigma_{\mathrm{eff}} v \rangle (n^2 - n_{\mathrm{eq}}^2)$$

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\mathrm{eff}} v \rangle}$$

Born level annihilation



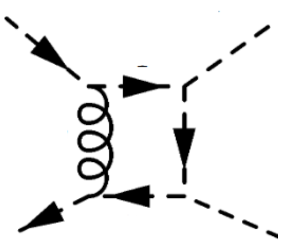
$$\sigma_{\mathrm{eff}} v_{\mathrm{rel}} = \sigma^{\mathrm{tree}} v_{\mathrm{rel}}$$

usually DM codes include *only* born level calculation

Effects impacting the particle cross section

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\text{eff}} v \rangle}$$

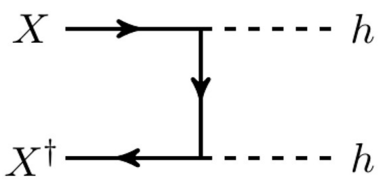
Higher order corrections



$$\sigma_{\text{eff}} v_{\text{rel}} = \sigma^{\text{NLO}} v_{\text{rel}}$$

can lead to sizeable corrections to the DM abundance

Born level annihilation



$$\sigma_{\text{eff}} v_{\text{rel}} = \sigma^{\text{tree}} v_{\text{rel}}$$

usually DM codes include *only* born level calculation

Impact of higher order corrections

non-exhaustive list!

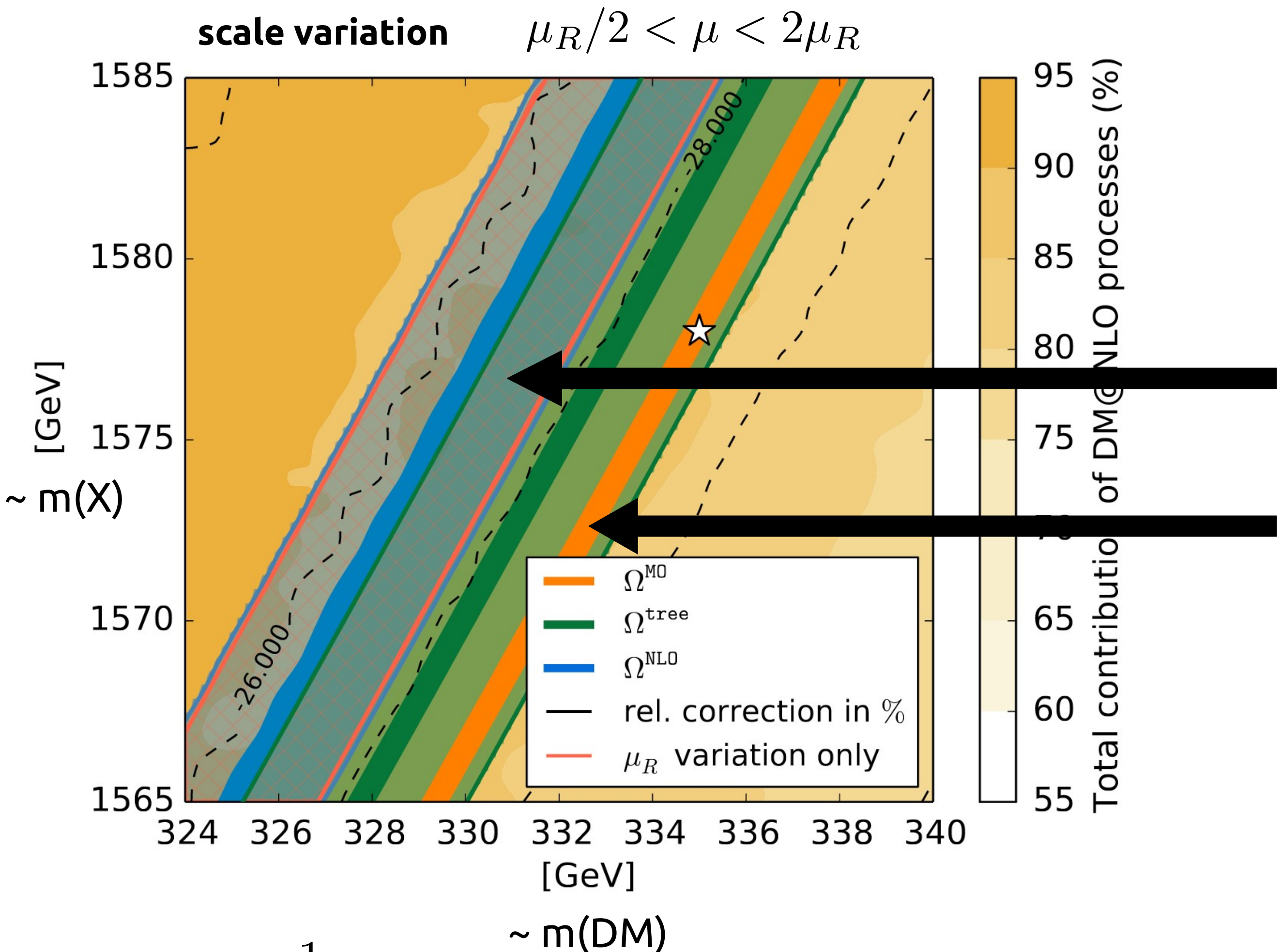
QED:

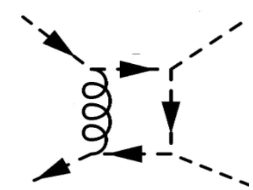
- **Relic density calculations beyond tree-level, exact calculations versus effective couplings: the ZZ final state,** Boudjema, Drieu La Rochelle, Mariano (2014)
- **Radiative Corrections to the Neutralino Dark Matter Relic Density - an Effective Coupling Approach,** Chatterjee, Drees, Kulkarni (2012)
- **One-loop corrections, uncertainties and approximations in neutralino annihilations: Examples,** Boudjema, Drieu La Rochelle, Kulkarni (2011)
- **Relic density at one-loop with gauge boson pair production,** Baro, Boudjema, Chalons, Hao (2010)
- **Full one-loop corrections to the relic density in the MSSM: A Few examples,** Baro, Boudjema, Semenov (2008)
- **SUSY dark matter: Loops and precision from particle physics,** Boudjema, Semenov , Temes (2006)

QCD:

- **SUSY-QCD corrected and Sommerfeld enhanced stau annihilation into heavy quarks with scheme and scale uncertainties,** Branahl, JH, Herrmann, Klasen, Kovařík, Schmiemann (2019)
- **Squark-pair annihilation into quarks at next-to-leading order,** Schmiemann, JH, Herrmann, Klasen, Kovařík (2019)
- **Theoretical uncertainty of the supersymmetric dark matter relic density from scheme and scale variations,** JH, Herrmann, Klasen, Kovařík, Steppeler (2016)
- **SUSY-QCD corrections to stop annihilation into electroweak final states including Coulomb enhancement effects,** JH, Herrmann, Klasen, Kovařík, Meinecke (2015)
- **One-loop corrections to neutralino-stop coannihilation revisited,** JH, Herrmann, Klasen, Kovařík (2015)
- **One-loop corrections to gaugino (co)annihilation into quarks in the MSSM,**
- Herrmann, Klasen, Kovařík, Meinecke, Steppeler (2014)
- **Neutralino-stop coannihilation into electroweak gauge and Higgs bosons at one loop,** JH, Herrmann, Klasen, Kovařík, Le Boulc'h (2013)
- **SUSY-QCD effects on neutralino dark matter annihilation beyond scalar or gaugino mass unification,**
- Herrmann, Klasen, Kovařík (2009)
- **Neutralino Annihilation into Massive Quarks with SUSY-QCD Corrections**, Herrmann, Klasen, Kovařík (2009)

Impact of higher order corrections





$$\sigma_{\text{eff}} v_{\text{rel}} = \sigma^{\text{NLO}} v_{\text{rel}}$$

Relic density based on cross section at NLO

Usual DM tool at LO without any theoretical error estimate

→ first estimate of theoretical error on relic abundance calculation

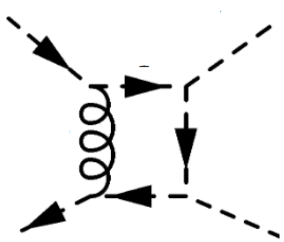
$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\text{eff}} v \rangle}$$

JH et al., Phys. Rev. D 99 (2019)
JH et al., Phys. Rev. D 93 (2016)
JH et al., Phys. Rev. D 91(2015b)
JH et al., Phys. Rev. D 91(2015a)
JH et al., Phys. Rev. D 87 (2013)

Effects impacting the particle cross section

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\mathrm{eff}} v \rangle}$$

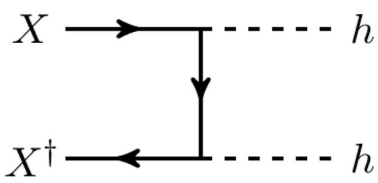
Higher order corrections



$$\sigma_{\mathrm{eff}} v_{\mathrm{rel}} = \sigma^{\mathrm{NLO}} v_{\mathrm{rel}}$$

can lead to sizeable corrections to the DM abundance

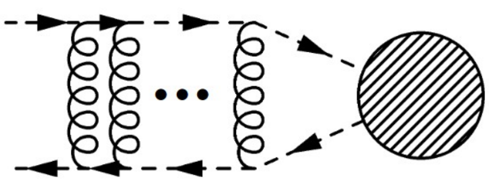
Born level annihilation



$$\sigma_{\mathrm{eff}} v_{\mathrm{rel}} = \sigma^{\mathrm{tree}} v_{\mathrm{rel}}$$

usually DM codes include *only* born level calculation

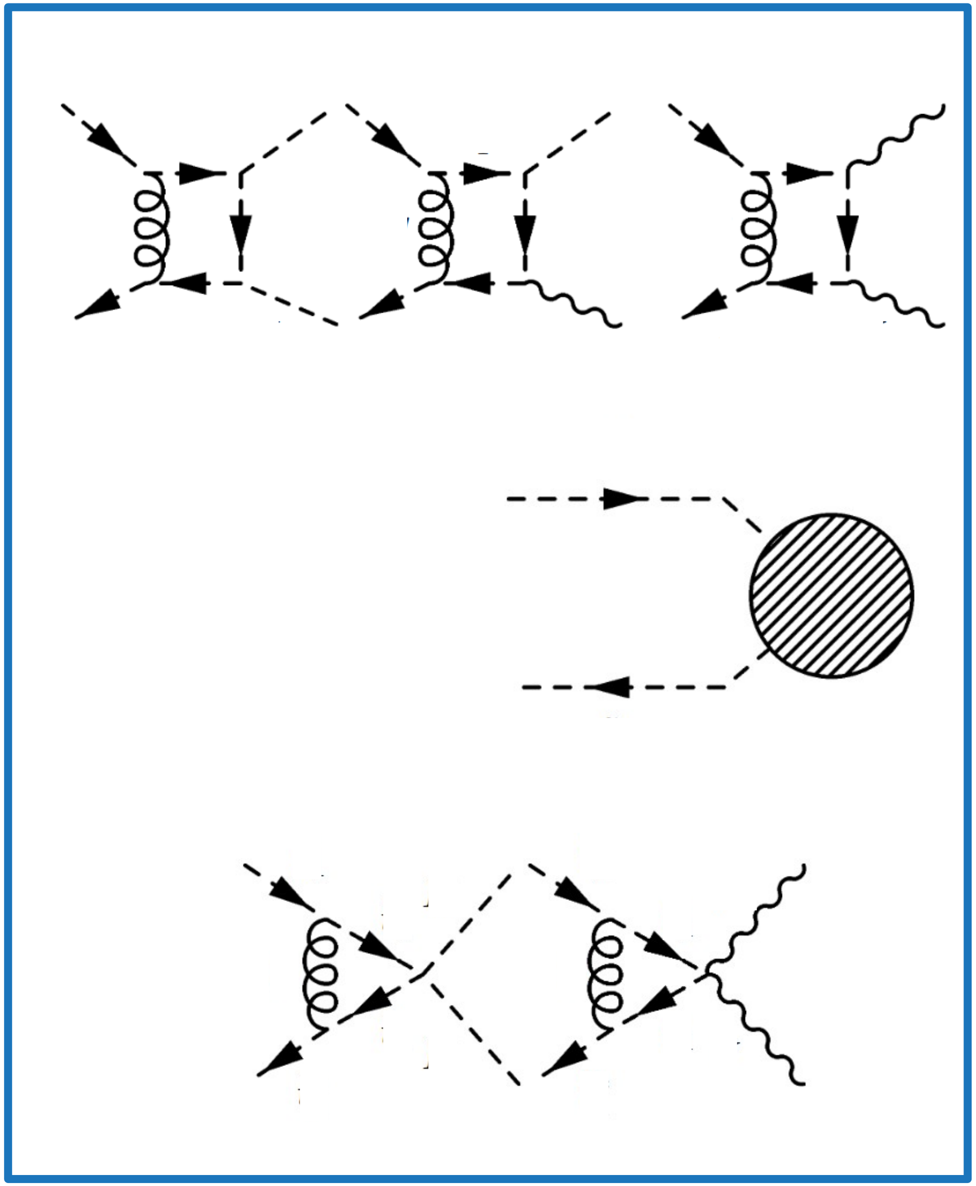
Sommerfeld enhancement



$$\left(\frac{\alpha}{v_{\mathrm{rel}}}\right)^n \sim 1$$

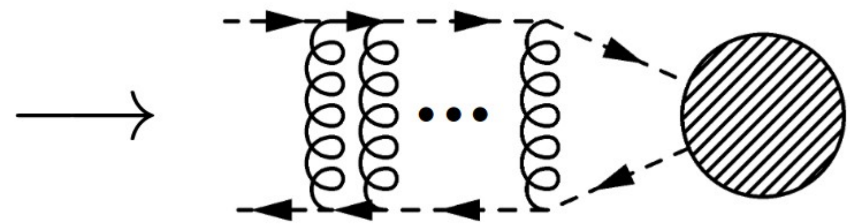
$$\sigma_{\mathrm{eff}} v_{\mathrm{rel}} = \sigma^{\mathrm{tree}} v_{\mathrm{rel}} \times S_0$$

Sommerfeld effect



Important in the regime:

$$\alpha \sim v_{\mathrm{rel}}$$



Exchange of n gluons contains a correction proportional to

$$\left(\frac{\alpha}{v_{\mathrm{rel}}}\right)^n \sim 1$$

Sommerfeld enhancement for dark matter

MSSM:

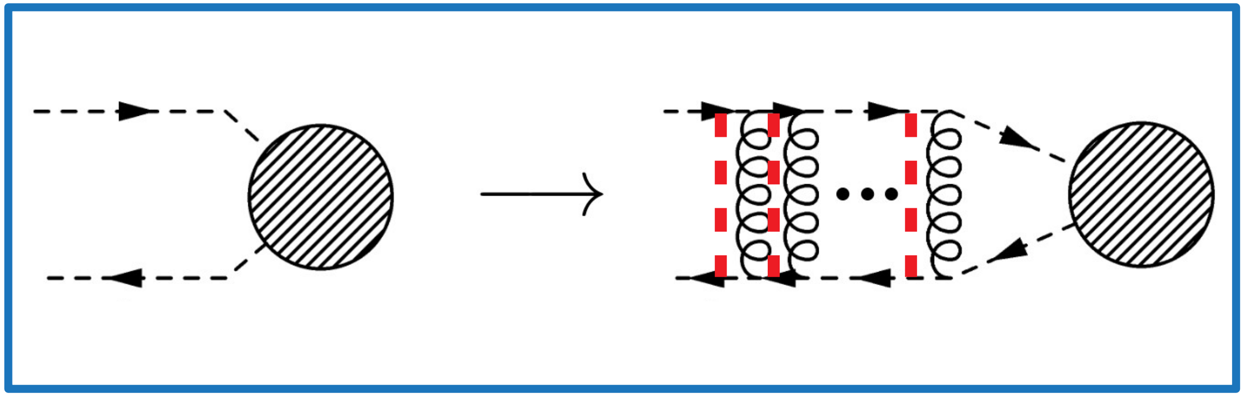
non-exhaustive list!

- **SUSY-QCD corrected and Sommerfeld enhanced stau annihilation into heavy quarks with scheme and scale uncertainties,** Branahl, JH, Herrmann, Klasen, Kovařík, Schmiemann (2019)
- **Squark-pair annihilation into quarks at next-to-leading order,** Schmiemann, JH, Herrmann, Klasen, Kovařík (2019)
- **Relic density of wino-like dark matter in the MSSM**, Beneke, Bharucha, Dighera, Hellmann, Hryczuk, Recksiegel, Ruiz-Femenia (2016)
- **Heavy neutralino relic abundance with Sommerfeld enhancements - a study of pMSSM scenarios**, Beneke, Hellmann, Ruiz-Femenia (2015)
- **SUSY-QCD corrections to stop annihilation into electroweak final states including Coulomb enhancement effects,** JH, Herrmann, Klasen, Kovařík, Meinecke (2015)
- **Non-relativistic pair annihilation of nearly mass degenerate neutralinos and charginos III. Computation of the Sommerfeld enhancements**, Beneke, Hellmann, Ruiz-Femenia (2015)
- **Non-relativistic pair annihilation of nearly mass degenerate neutralinos and charginos II. P-wave and next-to-next-to-leading order S-wave coefficients**, Hellmann, Ruiz-Femenía (2013)
- **Non-relativistic pair annihilation of nearly mass degenerate neutralinos and charginos I. General framework and S-wave annihilation,** Beneke, Hellmann, Ruiz-Femenia (2013)
- **Enhanced One-Loop Corrections to WIMP Annihilation and their Thermal Relic Density in the Coannihilation Region**, Drees, Gu (2013)

Other Models:

- **Higgs Enhancement for the Dark Matter Relic Density**, JH, Petraki, (2018)
- **A Sommerfeld Toolbox for Colored Dark Sectors**, El Hedri, Kaminska, Vries (2017)
- **Asymmetric thermal-relic dark matter: Sommerfeld-enhanced freeze-out, annihilation signals and unitarity bounds**, Baldes, Petraki (2017)
- **The Sommerfeld Enhancement in the Scotogenic Model with Large Electroweak Scalar Multiplets,** Chowdhury, Nasri (2017)
- **Self-consistent Calculation of the Sommerfeld Enhancement**, Blum, Sato, Slatyer (2016)

Higgs enhancement in co-annihilation scenarios?



What is the effect of a multiple exchange of Higgs boson?

Sommerfeld enhancement via massive scalar exchange

non-exhaustive list!

First studies before the Higgs discovery:

- **The Sommerfeld enhancement for scalar particles and application to sfermion co-annihilation regions**, Hryczuk (2011)
- **Sommerfeld Enhancements for Thermal Relic Dark Matter**, Feng, Kapling, Yu (2010)
- **Potentially Large One-loop Corrections to WIMP Annihilation**, Drees, Kim, Nagao (2009)

More specific studies after the Higgs discovery:

- **Relic density of wino-like dark matter in the MSSM**, Beneke, Bharucha, Dighera, Hellmann, Hryczuk, Recksiegel, Ruiz-Femenia (2016)
- **Non-relativistic pair annihilation of nearly mass degenerate neutralinos and charginos III. Computation of the Sommerfeld enhancements**, Beneke, Hellmann, Ruiz-Femenia (2015)
- **Heavy neutralino relic abundance with Sommerfeld enhancements - a study of pMSSM scenarios**, Beneke, Hellmann, Ruiz-Femenia (2015)
- **Higgs portal, fermionic dark matter, and a Standard Model like Higgs at 125 GeV**, Lopez-Honorez, Schwetz, Zupan (2012)

However, Higgs boson exchange had been neglected in recent studies of colored coannihilation scenarios!

Higgs enhancement

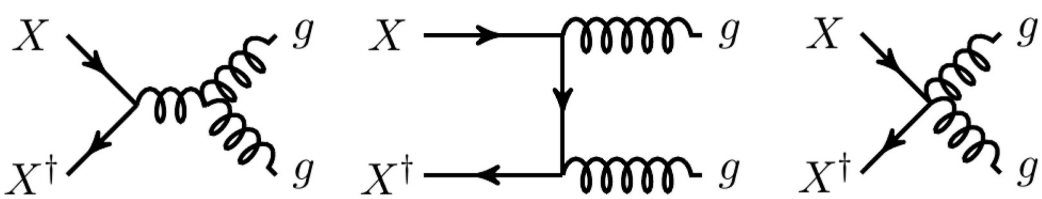
Simplified model

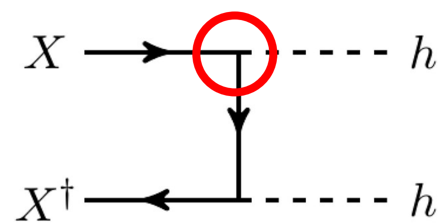
DM Majorana fermion χ, co-annihilating with complex scalar X charged under SU(3)$_c$

$$\delta\mathcal{L} = (D_{\mu,ij}X_j)^\dagger\,(D^{\mu}_{ij'}X_{j'}) - m_X^2\, X_j^\dagger X_j + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{1}{2}m_h^2 h^2 - g_h m_X\; h\, X_j^\dagger X_j$$

Annihilation processes:

$$\alpha_h = \frac{g_h^2}{16\pi}$$





$$(\sigma v_{\rm rel})^{\rm pert}_{XX^\dagger\to gg} = \frac{14}{27}\frac{\pi\alpha_s^2}{m_X^2}$$

$$(\sigma v_{\rm rel})^{\rm pert}_{XX^\dagger\to hh} = \frac{4\pi\alpha_h^2}{3m_X^2}\frac{(1-m_h^2/m_X^2)^{1/2}}{[1-m_h^2/(2m_X^2)]^2}$$

we neglect p-wave suppressed contributions $X\bar{X} \to q\bar{q},\, X\bar{X} \to gh$

Effective Boltzmann description

$$\boxed{\frac{d\tilde{Y}}{dx} = -\sqrt{\frac{\pi}{45}}\,\frac{M_{\rm Pl} m_X\, g_{*,\rm eff}^{1/2}}{x^2}\,\langle\sigma_{\rm eff}\, v_{\rm rel}\rangle\,(\tilde{Y}^2 - \tilde{Y}_{\rm eq}^2)}$$

With the effective dark matter yield

$$\tilde{Y} \equiv Y_\chi + Y_X + Y_{X^\dagger} = Y_\chi + 2Y_X$$

$$Y_\chi^{\rm eq} = \frac{90}{(2\pi)^{7/2}}\,\frac{g_\chi}{g_{*,S}}\,x^{3/2}\,e^{-x}$$

$$Y_X^{\rm eq} = Y_{X^\dagger}^{\rm eq} = \frac{90}{(2\pi)^{7/2}}\,\frac{g_\chi}{g_{*,S}}\,[(1+\varDelta)x]^{3/2}\,e^{-(1+\varDelta)x}$$

Assumptions:

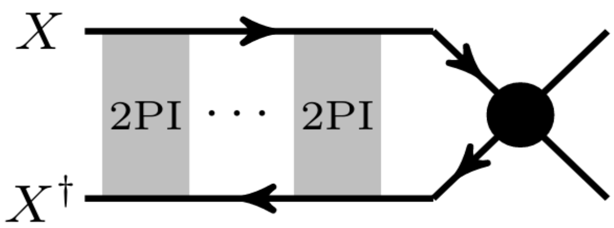
- Coannihilating particle will later decay in DM
- Coannihilating particle in thermal equilibrium with DM particle

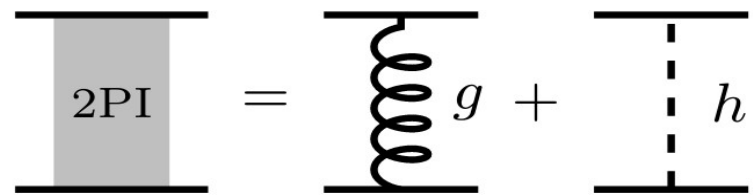
$$\varGamma(X + \mathrm{SM} \leftrightarrow \chi + \mathrm{SM}) \gg H$$

$$\sigma(X + \mathrm{SM} \leftrightarrow \chi + \mathrm{SM}) \gg \frac{17\; x_{\rm dec}}{m_\chi M_{\rm Pl}} \sim 6\times 10^{-11}\;\mathrm{pb}\left(\frac{\mathrm{TeV}}{m_\chi}\right)$$

$$x_{\rm dec} = m_\chi / T_{\rm dec}$$

Higgs as mediator of long-range interactions





$$\left[-\frac{\nabla^2}{2\mu} + V_{\rm scatt}(\mathbf{r})\right]\phi_{\mathbf{k}}(\mathbf{r}) = \mathcal{E}_{\mathbf{k}}\,\phi_{\mathbf{k}}(\mathbf{r})$$ with $$\mathrm{V}_{\rm scatt}(r) = -\frac{\alpha_g^S}{r} - \frac{\alpha_h}{r}\,e^{-m_h r}$$

Characteristic parameters:

$$\zeta_{g,h} \equiv \frac{\mu\alpha_{g,h}}{\mu v_{\rm rel}} = \frac{\alpha_{g,h}}{v_{\rm rel}}$$

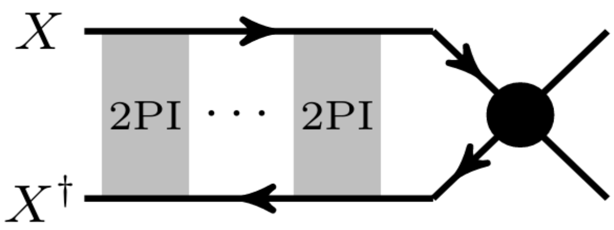
$$\frac{\text{Bohr momentum}}{\text{momentum exchange of unbound particles}} > 1$$

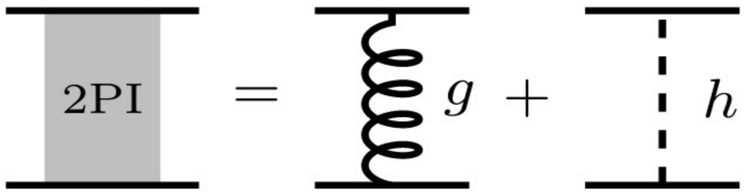
$$\mathrm{d}_h \equiv \frac{\mu\alpha_h}{m_h}$$

$$\frac{\text{Interaction range}}{\text{Bohr radius}} > 1$$

$$\left\{\nabla_{\mathbf{z}}^2 + 1 + \frac{2}{z}\left[\zeta_g + \zeta_h \exp\left(-\frac{\zeta_h z}{d_h}\right)\right]\right\}\phi_{\mathbf{k}} = 0 \quad \Rightarrow \quad \boxed{S_0(\zeta_g, \zeta_h, d_h) \equiv |\phi_{\mathbf{k}}(0)|^2}$$

Higgs as mediator of long-range interactions





$$\left[-\frac{\nabla^2}{2\mu} + V_{\text{scatt}}(\mathbf{r})\right]\phi_{\mathbf{k}}(\mathbf{r}) = \mathcal{E}_{\mathbf{k}}\,\phi_{\mathbf{k}}(\mathbf{r}) \quad \text{with} \quad \mathrm{V}_{\text{scatt}}(r) = -\frac{\alpha_g^S}{r} - \frac{\alpha_h}{r}\,e^{-m_h r}$$

Color decomposition: $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$

$$\Rightarrow \quad \alpha_g^S \equiv \alpha_s^S \times \begin{cases} C_{\mathbf{1}} = C_F = 4/3 \\ C_{\mathbf{8}} = C_F - C_A/2 = -1/6 \end{cases}$$

$$\boxed{\begin{aligned} (\sigma v_{\text{rel}})_{XX^\dagger \to gg} &= (\sigma v_{\text{rel}})^{\text{pert}}_{XX^\dagger \to gg} \times \left(\frac{2}{7} S_0^{[\mathbf{1}]} + \frac{5}{7} S_0^{[\mathbf{8}]}\right) & \qquad S_0^{[\mathbf{1}]} &= S_0[\zeta_g^{[\mathbf{1}]},\, \zeta_h,\, d_h] \\ (\sigma v_{\text{rel}})_{XX^\dagger \to hh} &= (\sigma v_{\text{rel}})^{\text{pert}}_{XX^\dagger \to hh} \times S_0^{[\mathbf{1}]} & \qquad S_0^{[\mathbf{8}]} &= S_0[\zeta_g^{[\mathbf{8}]},\, \zeta_h,\, d_h] \end{aligned}}$$

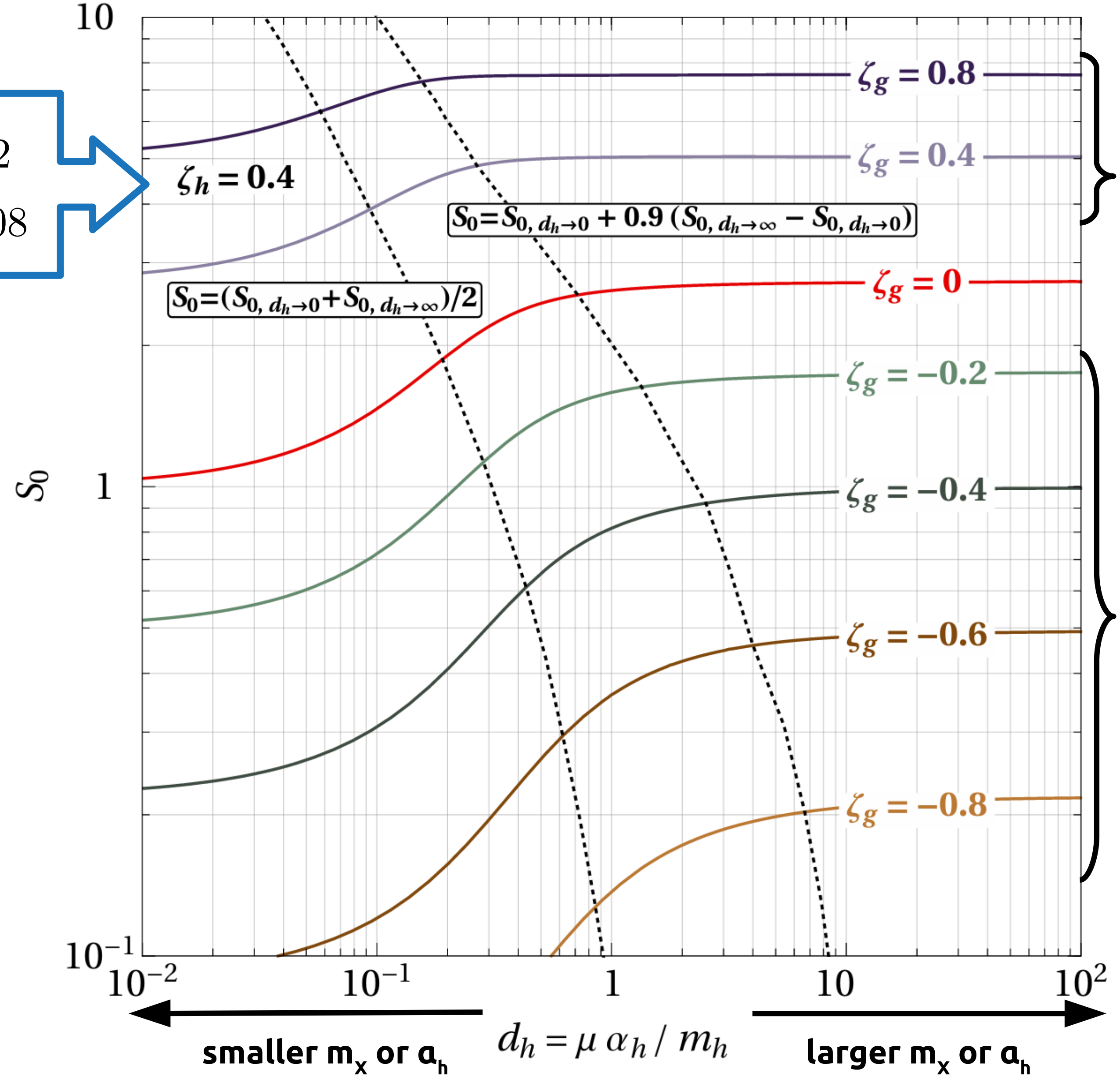
Strength of Higgs enhancement

$$\zeta_{g,h} \equiv \frac{\mu \alpha_{g,h}}{\mu v_{\rm rel}} = \frac{\alpha_{g,h}}{v_{\rm rel}},$$

$$d_h \equiv \frac{\mu \alpha_h}{m_h}$$

$v_{\rm rel} \approx 0.2$

$\alpha_h \approx 0.08$



attractive gluon potential (singlet)

repulsive gluon potential (octet)

JH, Petraki, (2018)

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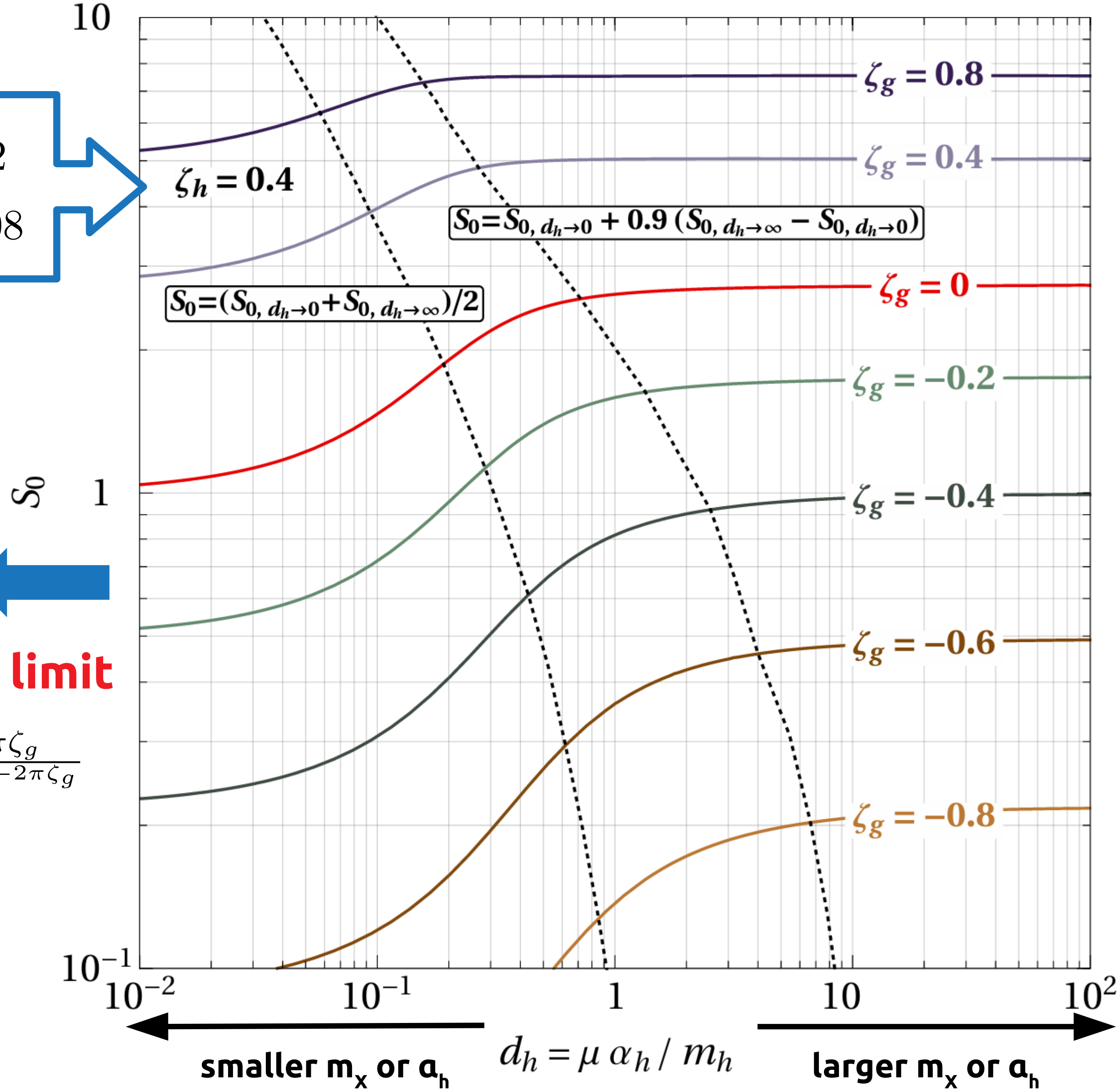

Coulomb limit

$$S_0 \simeq \frac{2\pi\zeta_g}{1-e^{-2\pi\zeta_g}}$$

Coulomb limit

$$S_0 \simeq \frac{2\pi(\zeta_g+\zeta_h)}{1-e^{-2\pi(\zeta_g+\zeta_h)}}$$

JH, Petraki, (2018)

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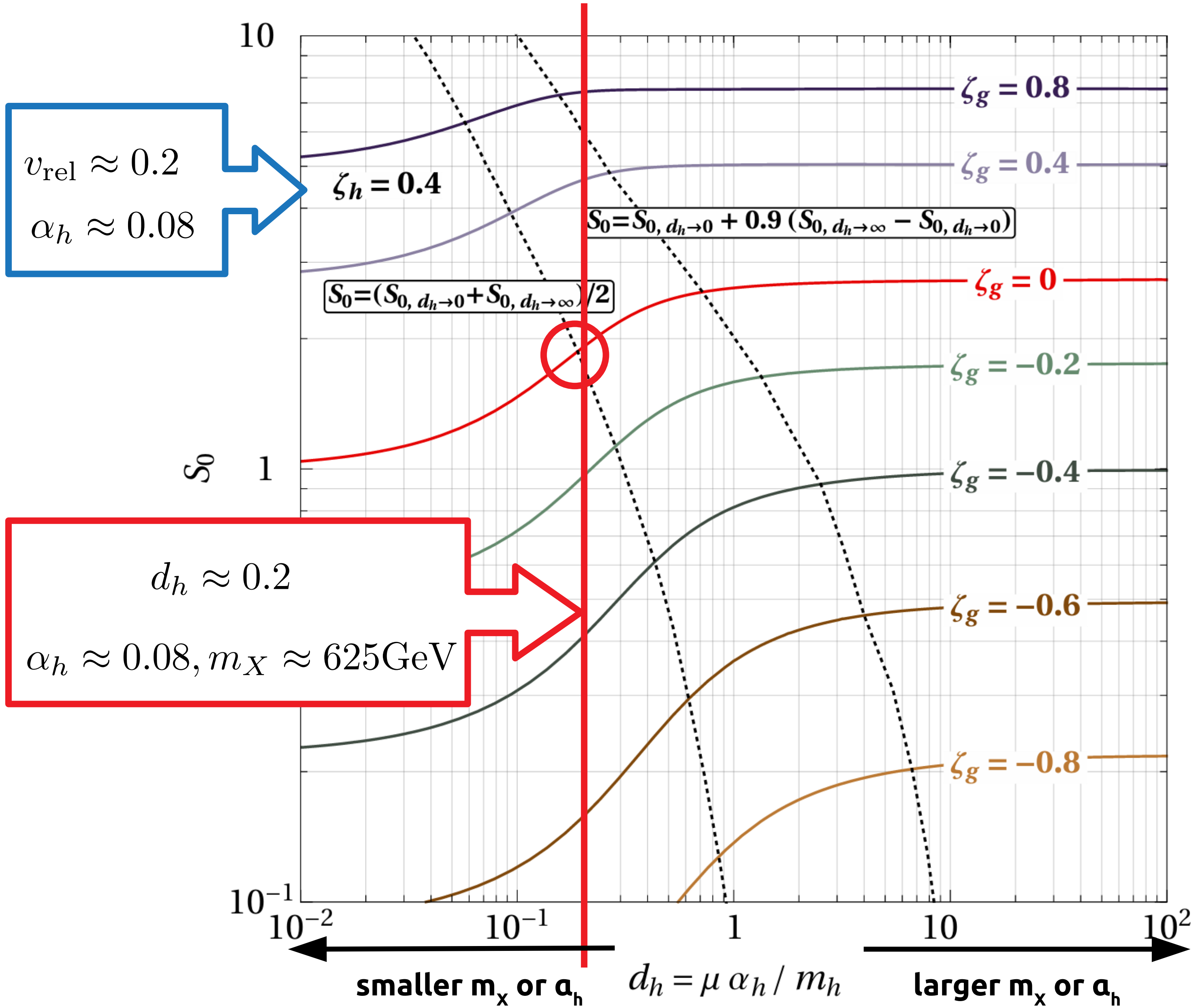
$$d_h \equiv \frac{\mu\alpha_h}{m_h}$$



d_h << 1 has already a significant impact!

JH, Petraki, (2018)

Strength of Higgs enhancement

$$\zeta_{g,h} \equiv \frac{\mu\alpha_{g,h}}{\mu v_{\rm rel}} = \frac{\alpha_{g,h}}{v_{\rm rel}}$$

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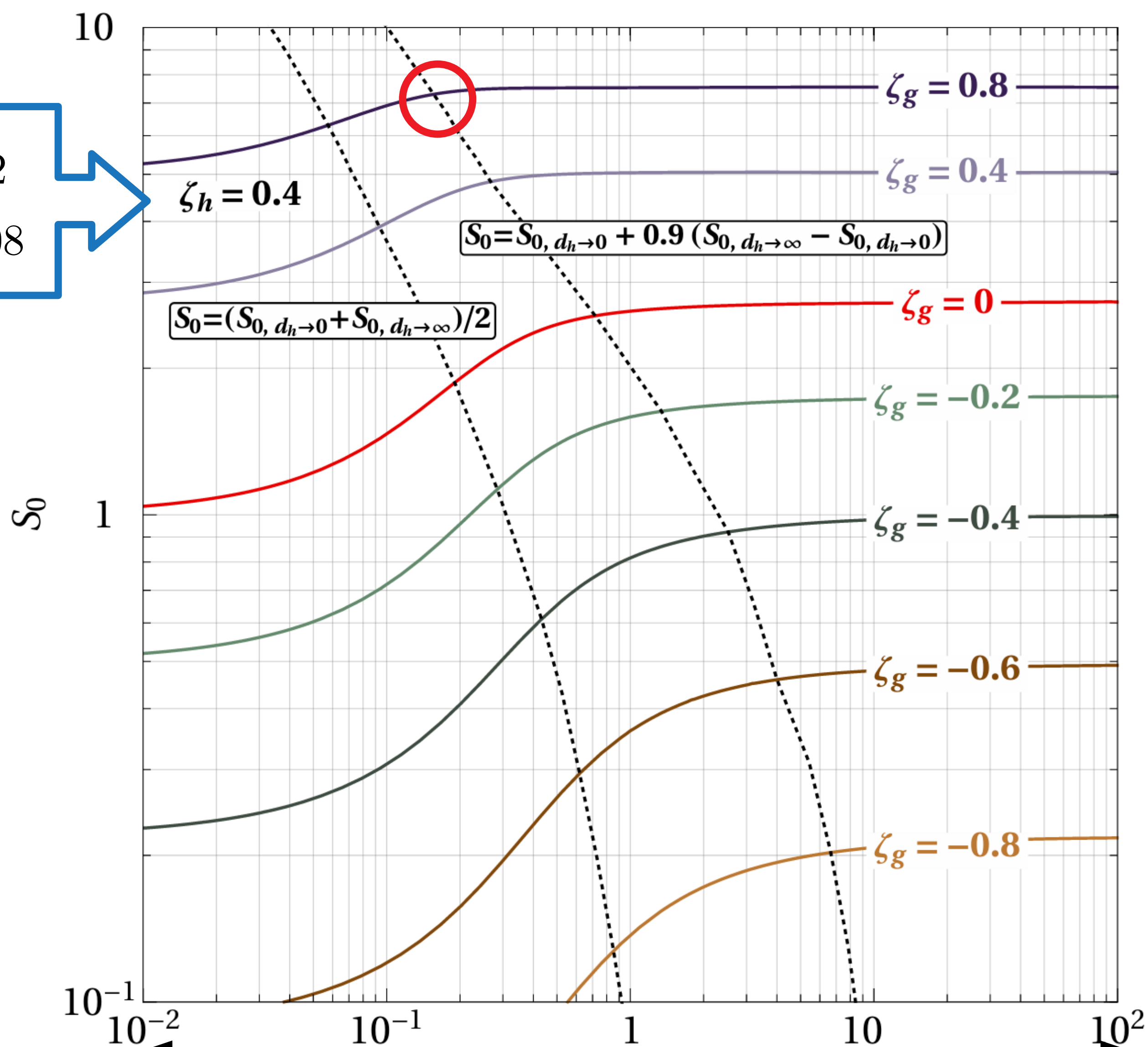

smaller m_X or α_h

larger m_X or α_h

Higgs enhances the attraction of the singlet state

JH, Petraki, (2018)

Strength of Higgs enhancement

$$\zeta_{g,h} \equiv \frac{\mu\alpha_{g,h}}{\mu v_{\text{rel}}} = \frac{\alpha_{g,h}}{v_{\text{rel}}},$$

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$v_{\text{rel}} \approx 0.2$

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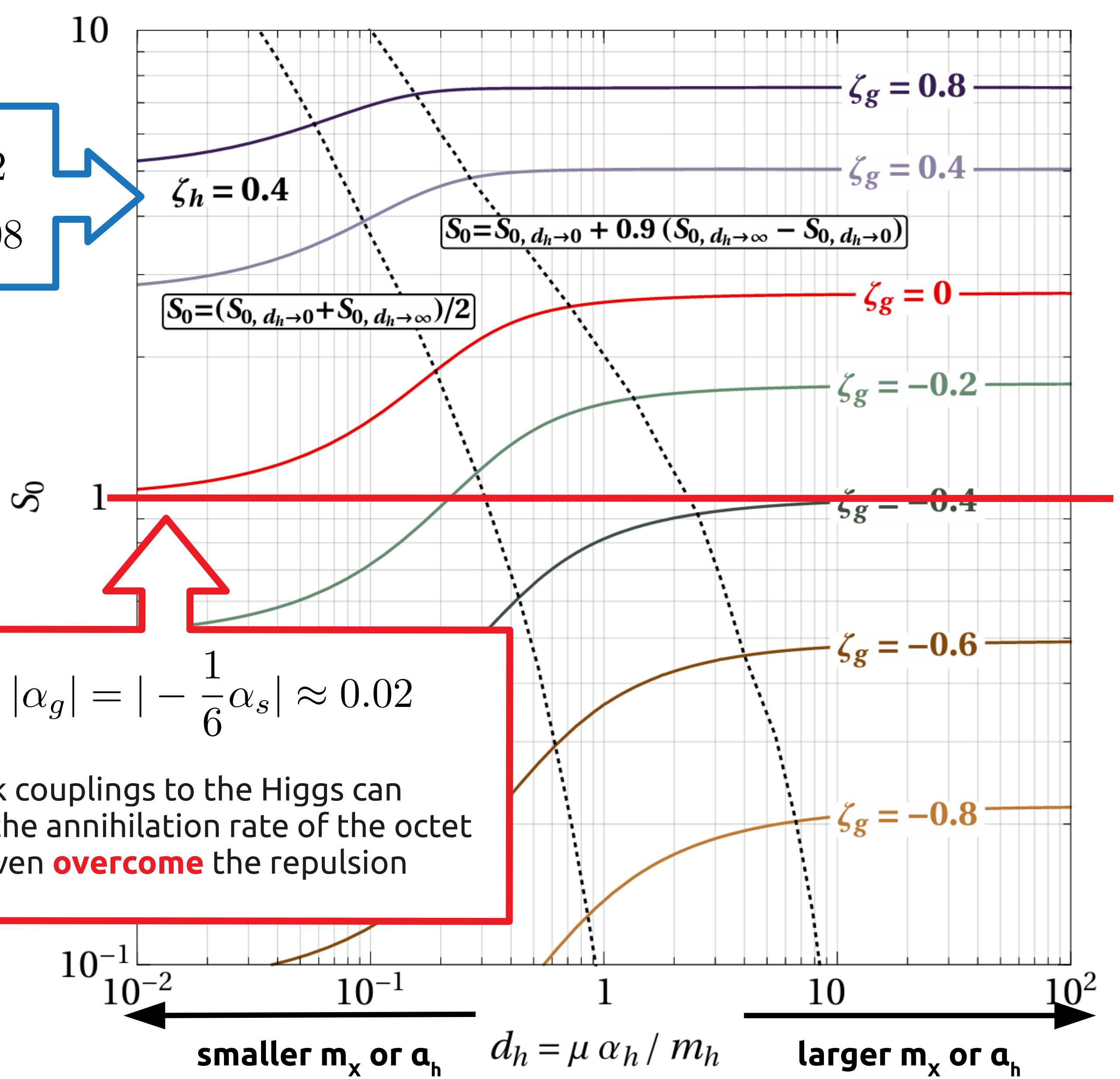


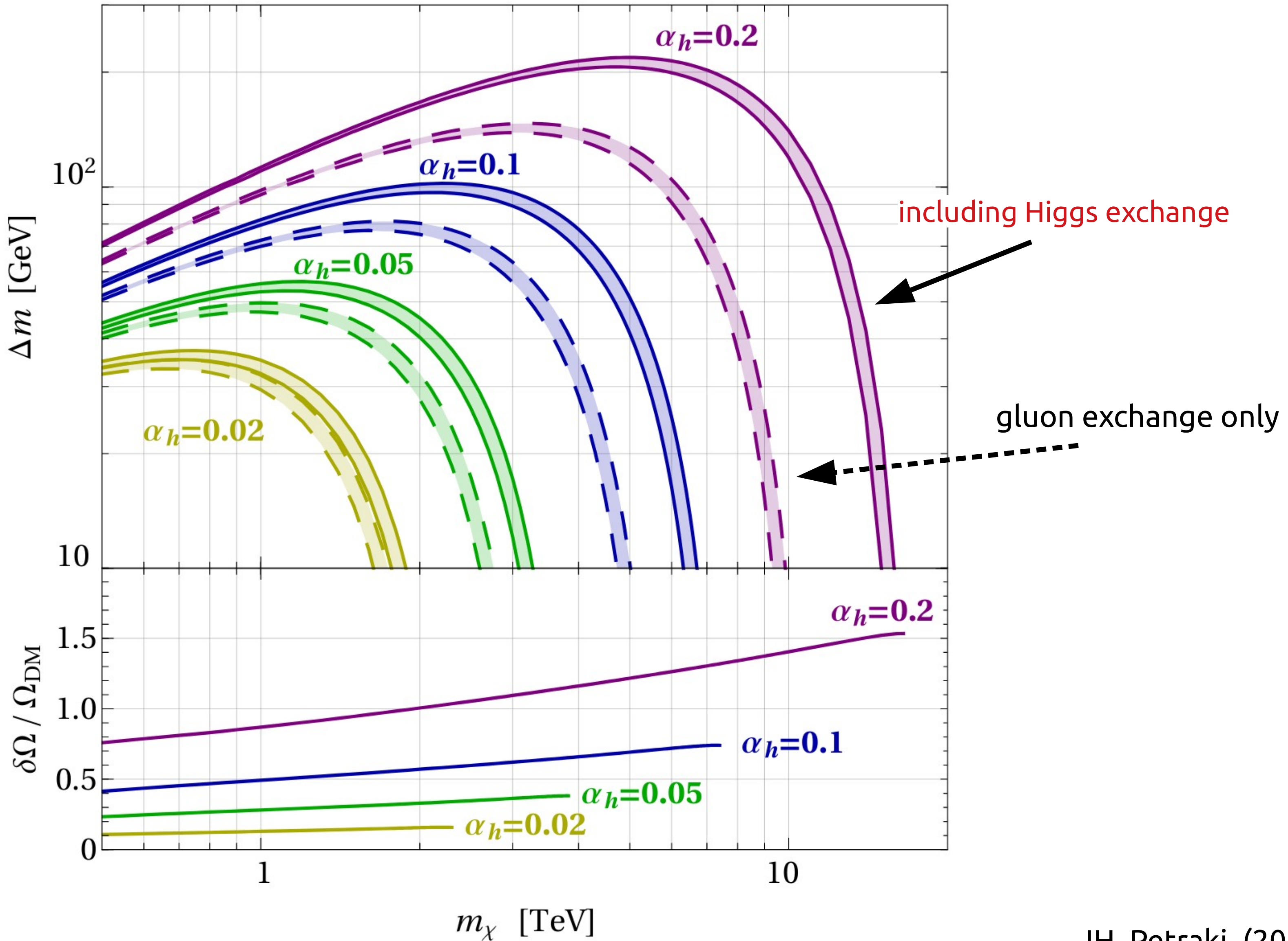
$$\alpha_h \approx |\alpha_g| = |-\frac{1}{6}\alpha_s| \approx 0.02$$

Even weak couplings to the Higgs can **enhance** the annihilation rate of the octet state or even **overcome** the repulsion

Higgs reduces the repulsion of the octet state

JH, Petraki, (2018)

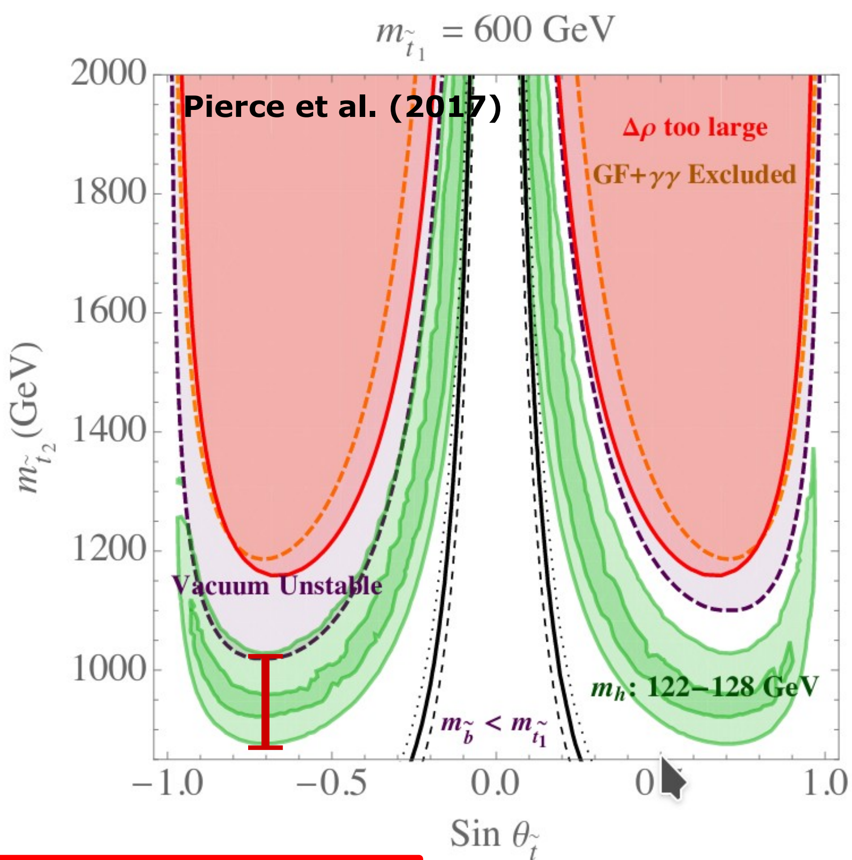
Impact on the relic density

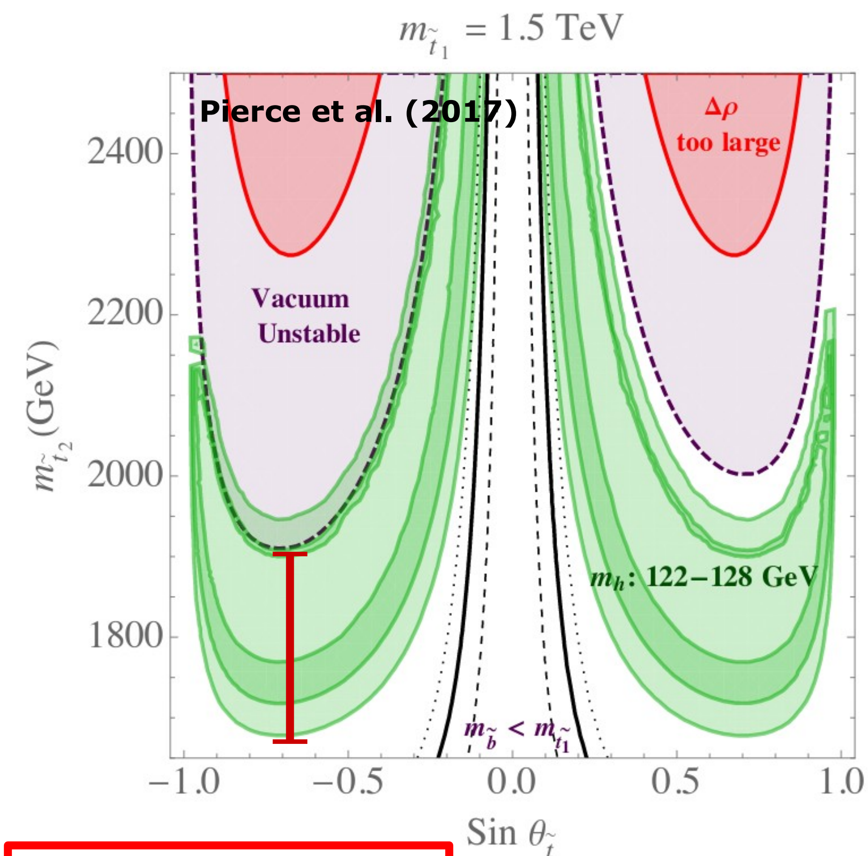


JH, Petraki, (2018)

Vacuum stability?

Example: realization within the MSSM:





$\alpha_h \approx (0.02 - 0.07)$

$\alpha_h \approx (0.01 - 0.05)$

We checked also explicitly an MSSM example scenario with *vevacious*

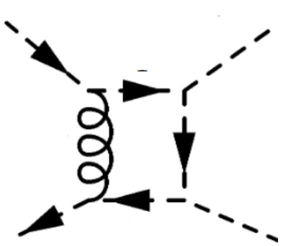
$$\alpha_h \approx 0.15$$

$$m_{\tilde{\chi}_1^0} = 982.5 \text{ GeV}$$
$$m_{\tilde{t}_1} = 1066.1 \text{ GeV}$$

Effects impacting the particle cross section

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\rm eff} v \rangle}$$

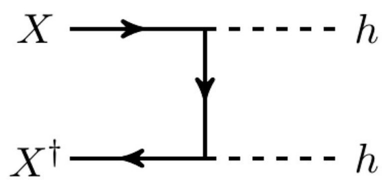
Higher order corrections



$$\sigma_{\rm eff} v_{\rm rel} = \sigma^{\rm NLO} v_{\rm rel}$$

can lead to sizeable corrections to the DM abundance

Born level annihilation

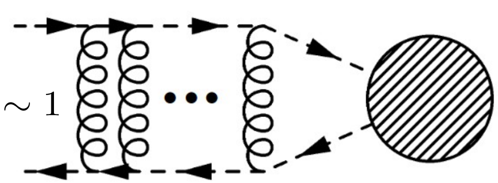


$$\sigma_{\rm eff} v_{\rm rel} = \sigma^{\rm tree} v_{\rm rel}$$

usually DM codes include *only* born level calculation

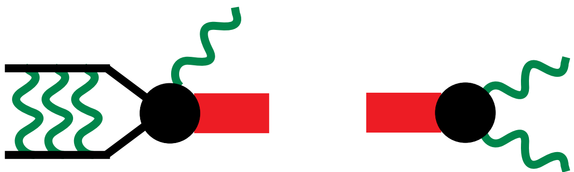
Sommerfeld enhancement

$$\left(\frac{\alpha}{v_{\rm rel}}\right)^n \sim 1$$



$$\sigma_{\rm eff} v_{\rm rel} = \sigma^{\rm tree} v_{\rm rel} \times S_0$$

Bound state formation

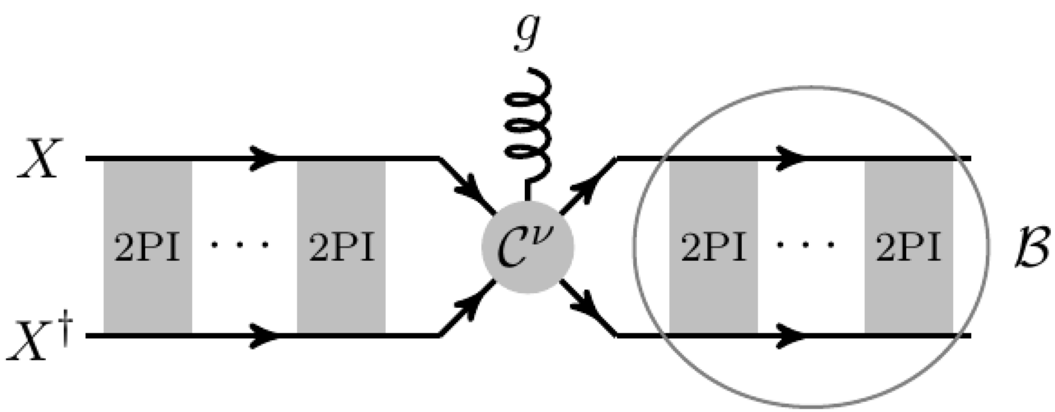


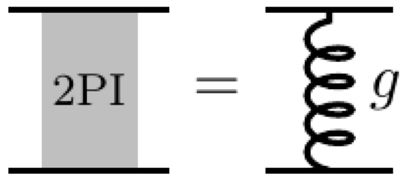
$$\langle \sigma_{\rm eff} v_{\rm rel} \rangle = \langle \sigma_{\rm ann} v_{\rm rel} \rangle + \langle \sigma_{\rm BSF} v_{\rm rel} \rangle_{\rm eff}$$

bound state formation and subsequent decay open up a new effective DM annihilation channel

Bound state formation

Bound states with gluon exchange





$$(X + X^\dagger)_{[\mathbf{8}]} \to \mathcal{B}(XX^\dagger)_{[\mathbf{1}]} + g_{[\mathbf{8}]}$$ **bound state formation**

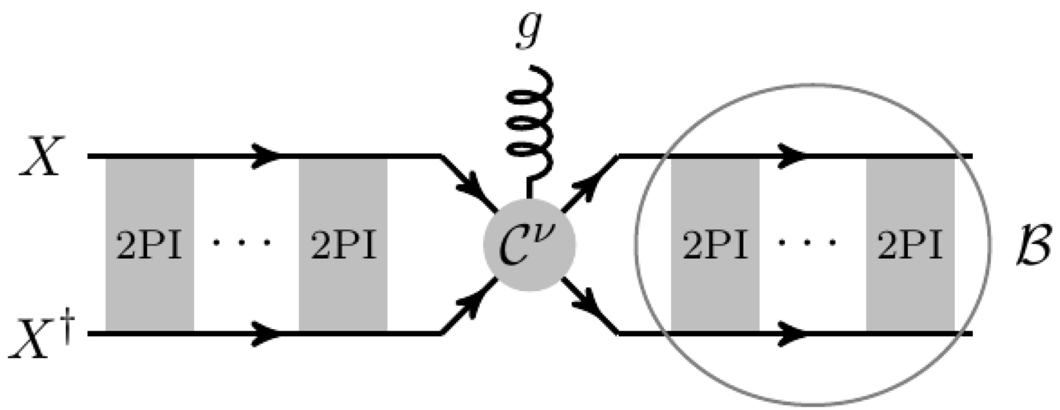
$$(\mathrm{XX}^\dagger)_{[\mathbf{1}]} + g_{[\mathbf{8}]} \to (X + X^\dagger)_{[\mathbf{8}]}$$ **bound state ionisation**

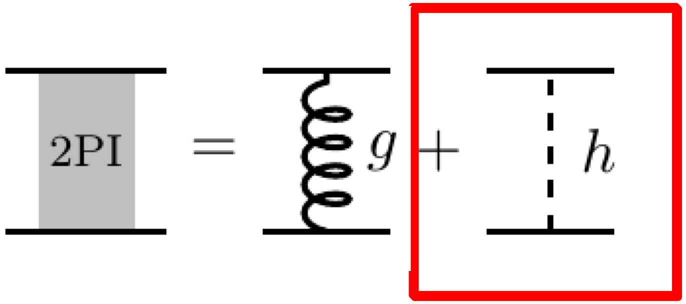
$$\mathcal{B}(XX^\dagger)_{[\mathbf{1}]} \to g_{[\mathbf{8}]}\ g_{[\mathbf{8}]}$$ **bound state decay**

$$\boxed{\langle \sigma_{\mathrm{BSF}} v_{\mathrm{rel}} \rangle_{\mathrm{eff}} = \langle \sigma_{\mathrm{BSF}} v_{\mathrm{rel}} \rangle \times \left(\frac{\Gamma_{\mathrm{dec}}}{\Gamma_{\mathrm{dec}} + \Gamma_{\mathrm{ion}}} \right)}$$

→ **additional "annihilation" channel alters the relic density prediction**

Bound states with gluon and Higgs exchange





$$(X + X^\dagger)_{[\mathbf{8}]} \to \mathcal{B}(XX^\dagger)_{[\mathbf{1}]} + g_{[\mathbf{8}]}$$

$$(X + X^\dagger)_{[\mathbf{1}]} \to \{\mathcal{B}(XX^\dagger)_{[\mathbf{8}]} + g_{[\mathbf{8}]}\}_{\mathbf{1_S}}$$

$$(X + X^\dagger)_{[\mathbf{8}]} \to \{\mathcal{B}(XX^\dagger)_{[\mathbf{8}]} + g_{[\mathbf{8}]}\}_{\mathbf{8_S}\ \mathrm{or}\ \mathbf{8_A}}$$

$$(X + X^\dagger)_{[\mathbf{1}]} \to \mathcal{B}(XX^\dagger)_{[\mathbf{1}]} + h$$

$$(X + X^\dagger)_{[\mathbf{8}]} \to \mathcal{B}(XX^\dagger)_{[\mathbf{8}]} + h$$

bound state formation

Higgs may allow

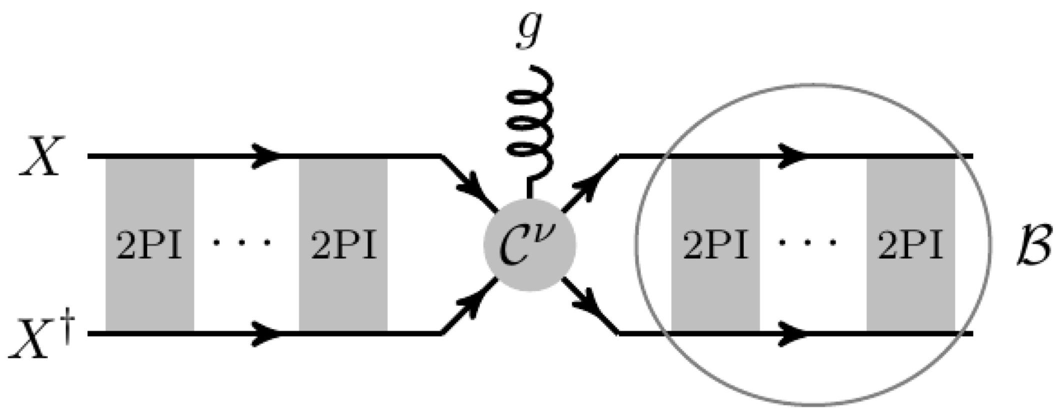
(1) to form tighter bound states

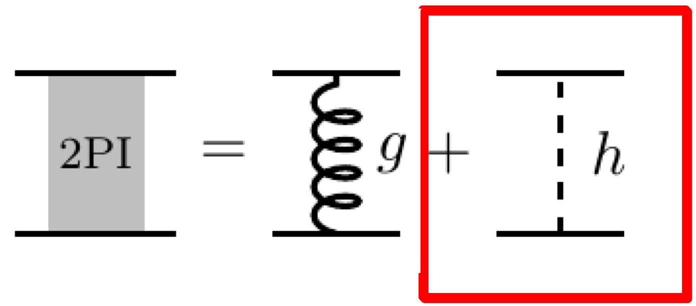
(2) to form color octet bound states

(3) to form bound states via emission of a Higgs

$$\boxed{\langle \sigma_{\mathrm{BSF}} v_{\mathrm{rel}} \rangle_{\mathrm{eff}} = \langle \sigma_{\mathrm{BSF}} v_{\mathrm{rel}} \rangle \times \left(\frac{\Gamma_{\mathrm{dec}}}{\Gamma_{\mathrm{dec}} + \Gamma_{\mathrm{ion}}} \right)}$$

Bound states with gluon and Higgs exchange





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bound state formation

Higgs may allow

(1) to form tighter bound states

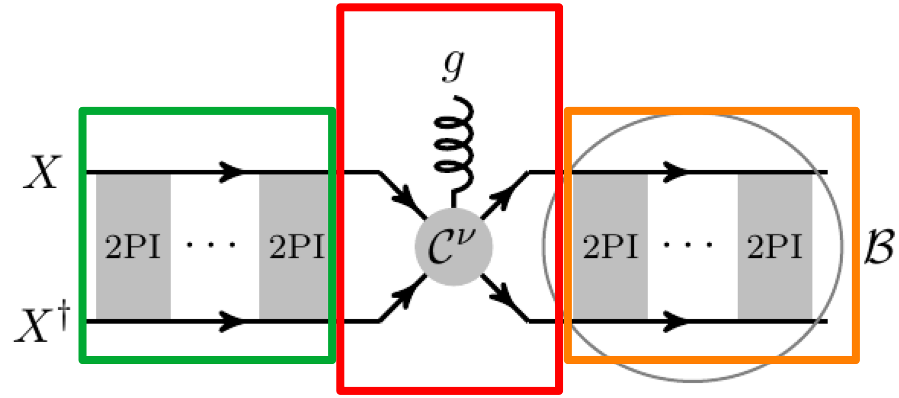
(2) to form color octet bound states

(3) to form bound states via emission of a Higgs

→ **Kallia Petraki's talk on Friday**

$$\boxed{\langle \sigma_{\mathrm{BSF}} v_{\mathrm{rel}} \rangle_{\mathrm{eff}} = \langle \sigma_{\mathrm{BSF}} v_{\mathrm{rel}} \rangle \times \left(\frac{\Gamma_{\mathrm{dec}}}{\Gamma_{\mathrm{dec}} + \Gamma_{\mathrm{ion}}} \right)}$$

Bound state formation



scattering state

radiative transition

bound state

$$\mathrm{V}_{S,B}(r) = -\frac{\alpha_g^{S,B}}{r} - \frac{\alpha_h}{r}\, e^{-m_h r}$$

$$\alpha_{g,[\mathbf{1}]}^{S,B} = \frac{4\alpha_s^{S,B}}{3}$$

$$\alpha_{g,[\mathbf{8}]}^{S,B} = -\frac{\alpha_s^{S,B}}{6}$$

continuous spectrum

$$\left[-\frac{\nabla^2}{2\mu} + V_{\rm scatt}(\mathbf{r})\right]\phi_{\mathbf{k}}(\mathbf{r}) = \mathcal{E}_{\mathbf{k}}\,\phi_{\mathbf{k}}(\mathbf{r})$$

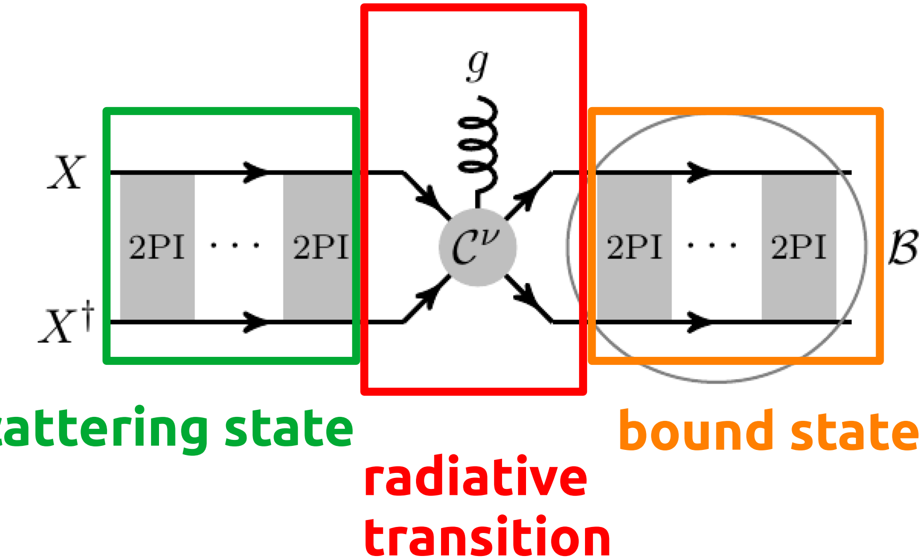
$$\mathcal{E}_{\mathbf{k}} \equiv \frac{\mathbf{k}^2}{2\mu} = \frac{\mu v_{\rm rel}^2}{2} > 0$$

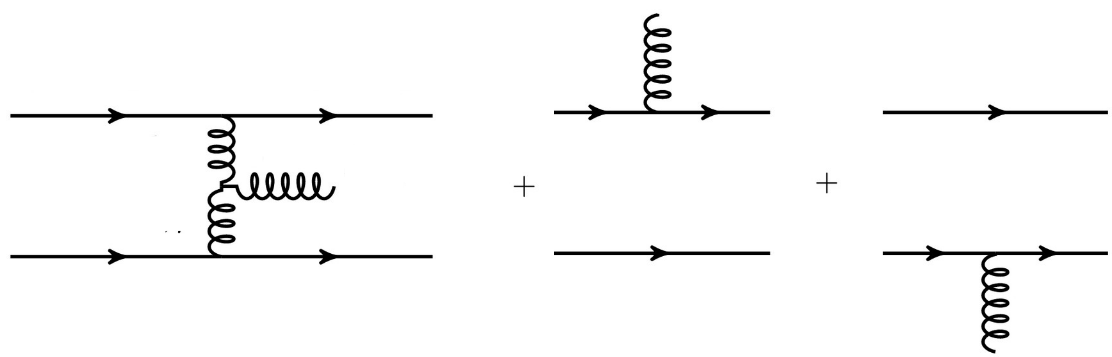
discrete spectrum

$$\left[-\frac{\nabla^2}{2\mu} + V_{\rm bound}(\mathbf{r})\right]\psi_{n\ell m}(\mathbf{r}) = \mathcal{E}_{n\ell}\,\psi_{n\ell m}(\mathbf{r})$$

$$\mathcal{E}_{n\ell} \equiv -\gamma_{n\ell}^2 \times \frac{\kappa^2}{2\mu} = -\gamma_{n\ell}^2\frac{1}{2}\mu\left(\alpha_g^B + \alpha_h\right)^2 < 0$$

Bound state formation

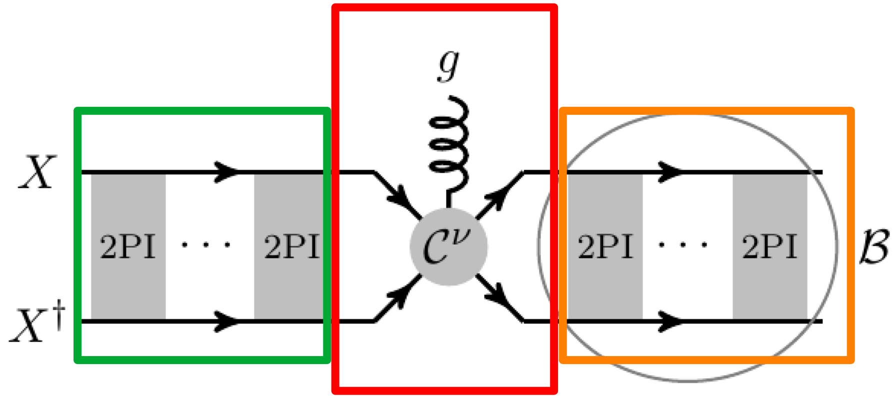




Derivation from Feynman diagrammatic approach, see
DM bound states from Feynman diagrams, Petraki, Postma, Wiechers (2018)

$$[\mathcal{M}^\nu_{\mathbf{k}\to\{n\ell m\}}]^a_{ii',jj'} = \frac{1}{\sqrt{2\mu}} \int \frac{d^3q}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} \, \tilde{\psi}^*_{n\ell m}(\mathbf{p}) \, \tilde{\phi}_{\mathbf{k}}(\mathbf{q}) \, [\mathcal{M}^\nu_{\text{trans}}(\mathbf{q},\mathbf{p})]^a_{ii',jj'}$$

Bound state formation



with: $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$

$$\frac{1}{9} \left| \mathcal{M}^{[\mathbf{8}]\to[\mathbf{1}]}_{\mathbf{k}\to 100} \right|^2 = \left(\frac{2^5 \pi \alpha_s^{\mathrm{BSF}} M^2}{\mu} \right) \times \frac{4}{27} \left[1 + \frac{3}{2} \left(\frac{\alpha_s^B}{\alpha_h + \alpha_g^B} \right) \right]^2 \left| \mathcal{J}^{[\mathbf{8},\mathbf{1}]}_{\mathbf{k},100} \right|^2$$

$$\text{for} \quad \alpha_h \to 0 : \quad \to \left[1 + \frac{9}{8} \right]^2$$

Comparison with Quarkonium literature:

Perturbative heavy quark - anti-quark systems, M. Beneke, hep-ph/9911490

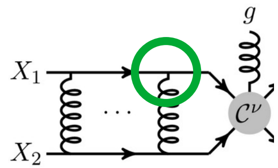
Running of the heavy quark production current and 1 / v potential in QCD, A. V. Manohar and I. W. Stewart, Phys. Rev. D63 (2001) 054004

*Renormalization group analysis of the QCD quark potential to order v**2*, A. V. Manohar and I. W. Stewart, Phys. Rev. D62 (2000) 014033

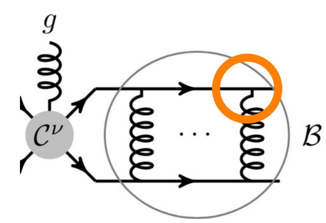
Thermal width and gluo-dissociation of quarkonium in pNRQCD, N. Brambilla, et al, JHEP 12 (2011) 116

Running of the strong coupling

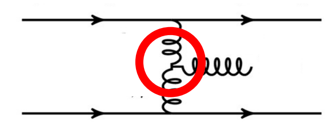
Vertices	α_s	α_g	Average momentum transfer Q
Scattering-state wavefunction (ladder diagrams)	α_s^S	Colour-singlet $\alpha_{g,[\mathbf{1}]}^S = \frac{4\alpha_s^S}{3}$ Colour-octet $\alpha_{g,[\mathbf{8}]}^S = -\frac{\alpha_s^S}{6}$	$\frac{m_X\, v_{\rm rel}}{2}$
Colour-singlet bound-state wavefunction (ladder diagrams)	$\alpha_{s,[\mathbf{1}]}^B$	$\alpha_{g,[\mathbf{1}]}^B = \frac{4\alpha_{s,[\mathbf{1}]}^B}{3}$	$\kappa_{[\mathbf{1}]}\ \gamma_{n\ell}\left(\lambda_{[\mathbf{1}]}, d_h\right) = \frac{m_X}{2}\left(\alpha_h + \frac{4\alpha_{s,[\mathbf{1}]}^B}{3}\right) \times \gamma_{n\ell}\left(\frac{4\alpha_{s,[\mathbf{1}]}^B}{3\alpha_h},\ d_h\right)$
Colour-octet bound state wavefunction (ladder diagrams)	$\alpha_{s,[\mathbf{8}]}^B$	$\alpha_{g,[\mathbf{8}]}^B = -\frac{\alpha_{s,[\mathbf{8}]}^B}{6}$	$\kappa_{[\mathbf{8}]}\ \gamma_{n\ell}\left(\lambda_{[\mathbf{8}]}, d_h\right) = \frac{m_X}{2}\left(\alpha_h - \frac{\alpha_{s,[\mathbf{8}]}^B}{6}\right) \times \gamma_{n\ell}\left(-\frac{\alpha_{s,[\mathbf{8}]}^B}{6\alpha_h},\ d_h\right)$
Formation of colour-singlet bound states: gluon emission	$\alpha_{s,[\mathbf{1}]}^{\rm BSF}$		$\frac{m_X}{4}\left[v_{\rm rel}^2 + \left(\alpha_h + \frac{4\alpha_{s,[\mathbf{1}]}^B}{3}\right)^2 \times \gamma_{n\ell}^2\left(\frac{4\alpha_{s,[\mathbf{1}]}^B}{3\alpha_h},\ d_h\right)\right]$
Formation of colour-octet bound states: gluon emission	$\alpha_{s,[\mathbf{8}]}^{\rm BSF}$		$\frac{m_X}{4}\left[v_{\rm rel}^2 + \left(\alpha_h - \frac{\alpha_{s,[\mathbf{8}]}^B}{6}\right)^2 \times \gamma_{n\ell}^2\left(-\frac{\alpha_{s,[\mathbf{8}]}^B}{6\alpha_h},\ d_h\right)\right]$



$\mu v_{\rm rel}$

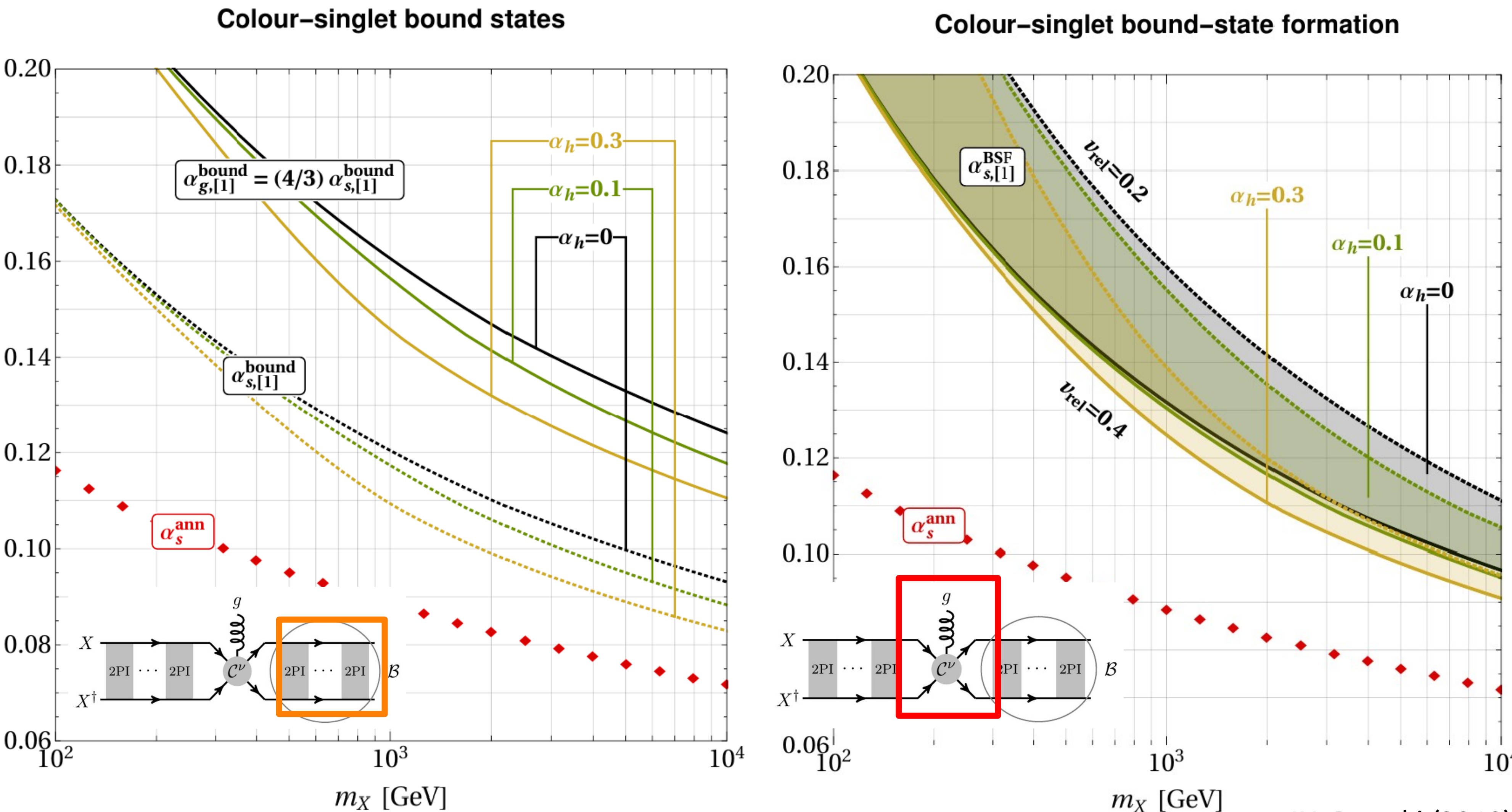


$\mu \alpha_g^B$



$|\,\mathbf{P_g}\,| = \mathcal{E}_\mathbf{k} - \mathcal{E}_{\mathbf{n}\ell}$

Running of the strong coupling – bound state



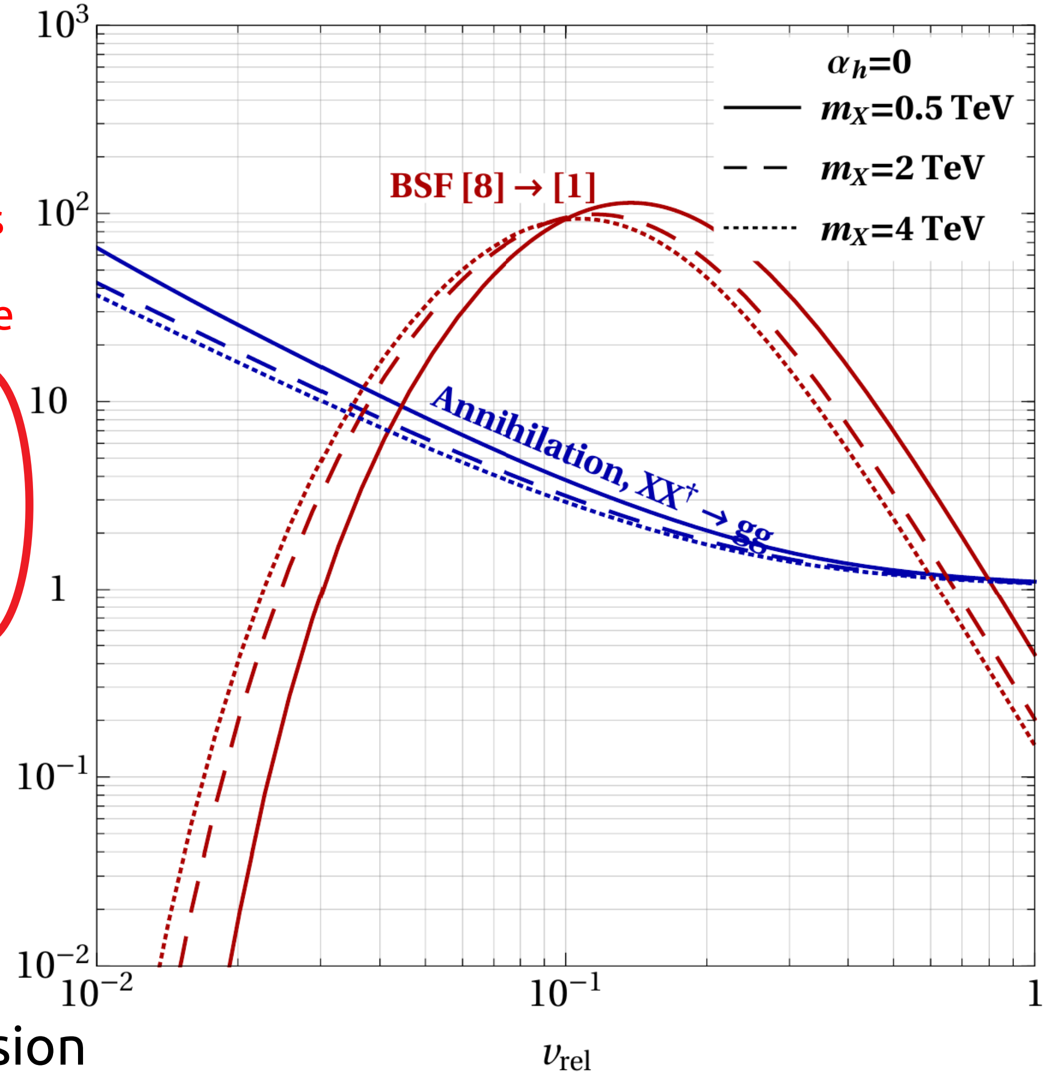
JH, Petraki (2019)
JH, Petraki (2018)

→ **Higgs interaction decreases** α_g **considerably**

Annihilation vs. BSF cross section

direct mass dependence cancels

→ only indirect scale dependence



gluon exchange only

$\alpha_s / v_{\rm rel} >> 1$

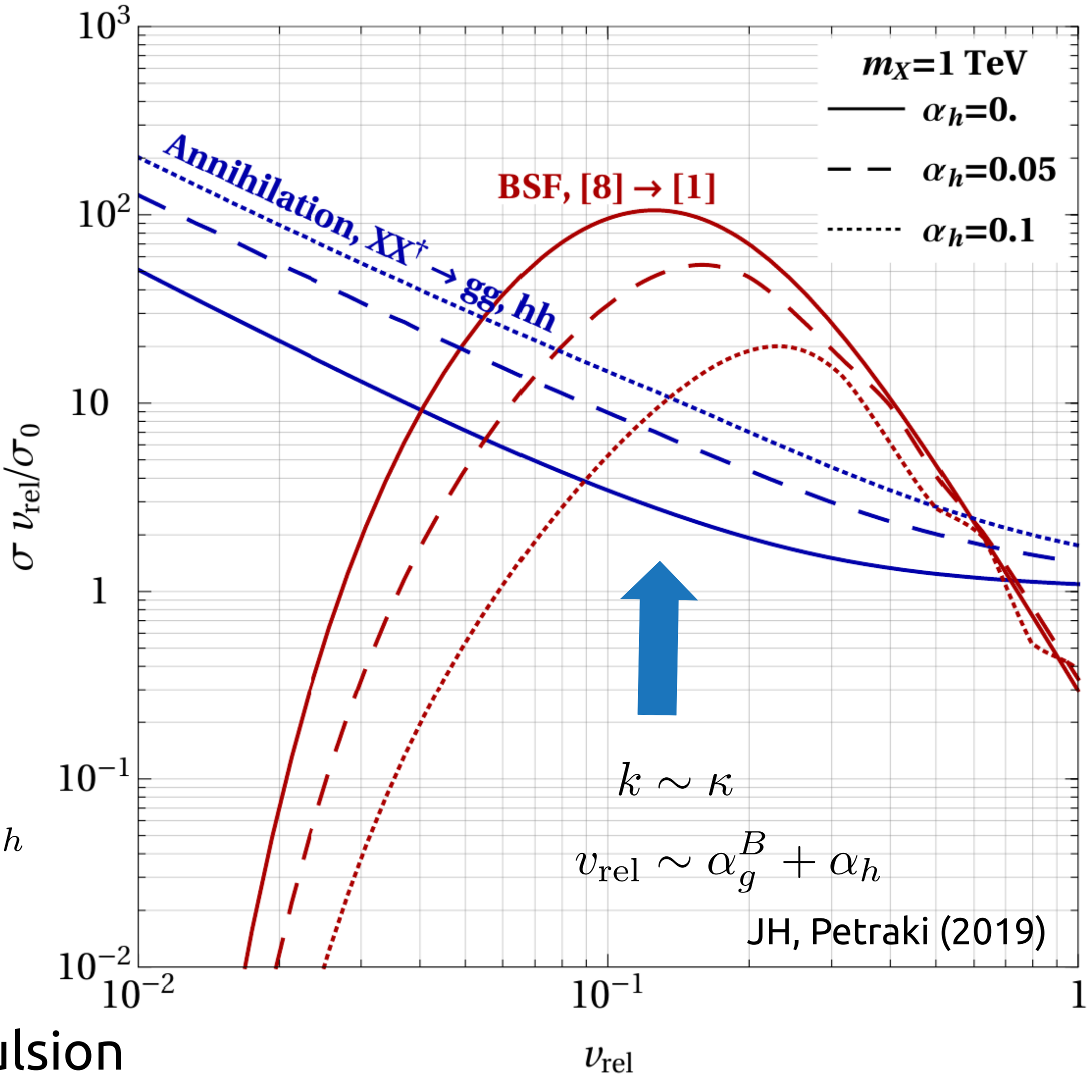
Coulomb repulsion in scattering state

$\alpha_s / v_{\rm rel} << 1$

$\sigma_{\rm BSF}^{[8]\to[1]} \propto (\alpha_s / v_{\rm rel})^4$

→ **scale dependence has impact on BSF and annihilation cross section!**

Annihilation vs. BSF cross section



with Higgs exchange

$k \sim \kappa$

$v_{\rm rel} \sim \alpha_g^B + \alpha_h$

JH, Petraki (2019)

$v_{\rm rel} \lesssim \alpha_g^S + \alpha_h$

Coulomb repulsion in scattering state

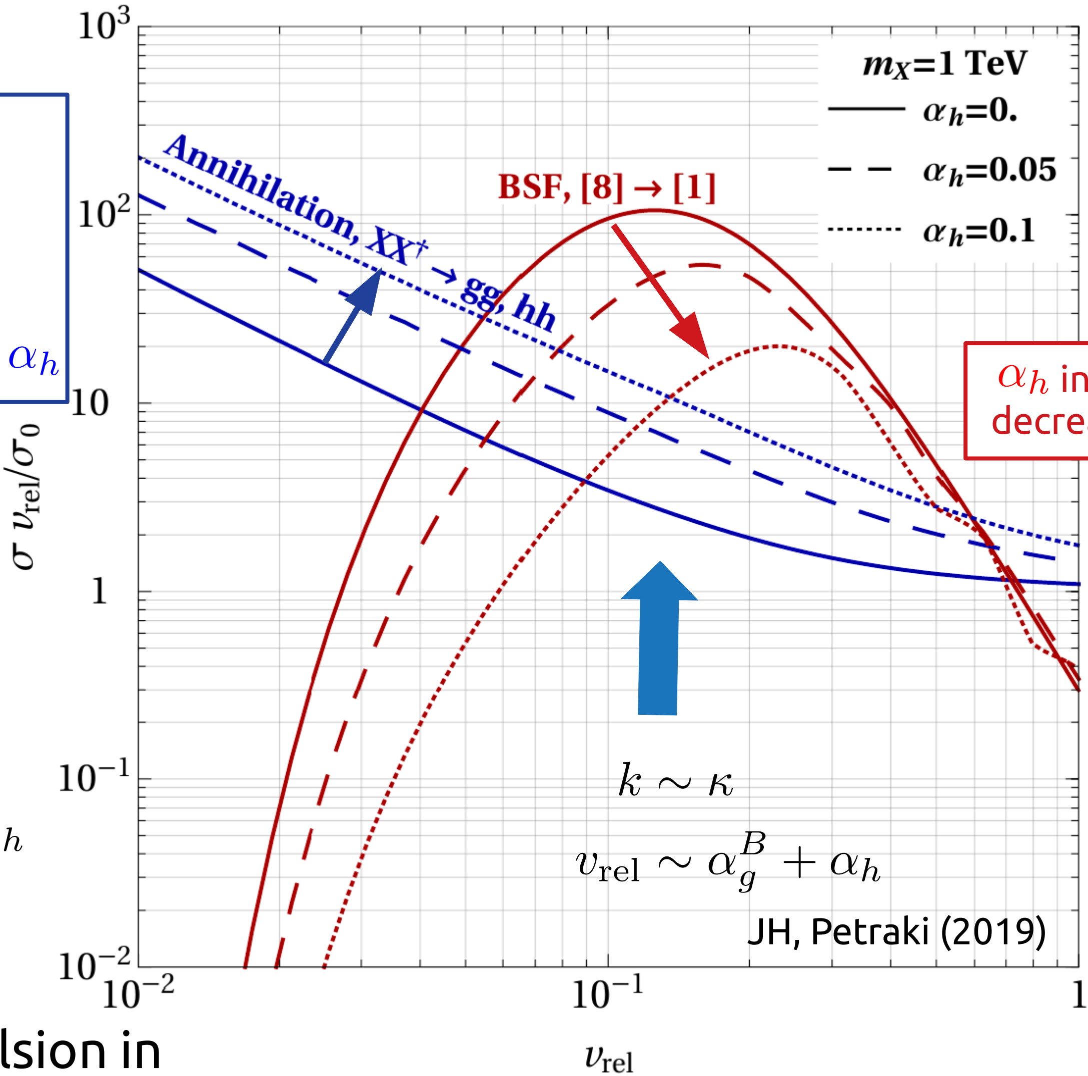
$$\sigma_{\rm BSF} v_{\rm rel} \propto (\kappa/k)^4 \approx [(\alpha_g^B + \alpha_h)/v_{\rm rel}]^4$$

Annihilation vs. BSF cross section

with Higgs exchange

new annihilation channel

pert. annihilation + Sommerfeld effect increases with larger α_h



α_h increases scale and decreases α_g

$v_{\rm rel} \lesssim \alpha_g^S + \alpha_h$

Coulomb repulsion in the scattering state

$$\sigma_{\rm BSF} v_{\rm rel} \propto (\kappa/k)^4 \approx [(\alpha_g^B + \alpha_h)/v_{\rm rel}]^4$$

→ **relative strength of BSF seems to diminish, BSF peaks at larger $v_{\rm rel}$!**

Effective BSF cross section

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\rm eff} v \rangle}$$

$$\langle \sigma_{\rm eff} v_{\rm rel}\rangle = \langle \sigma_{\rm ann}\, v_{\rm rel}\rangle + \langle \sigma_{\rm BSF} v_{\rm rel}\rangle_{\rm eff}$$

$$\langle \sigma_{\rm BSF} v_{\rm rel}\rangle_{\rm eff} = \langle \sigma_{\rm BSF} v_{\rm rel}\rangle \times \left(\frac{\Gamma_{\rm dec}}{\Gamma_{\rm dec} + \Gamma_{\rm ion}}\right)$$

bound state ionisation / dissociation

$$({\rm XX}^\dagger)_{[\mathbf{1}]} + g_{[\mathbf{8}]} \to (X + X^\dagger)_{[\mathbf{8}]}$$

$$\Gamma_{\rm dec} = (\sigma^{s-\rm wave}_{{\rm ann},[\mathbf{1},\mathbf{8}]} v_{\rm rel})\; |\psi^{[\mathbf{1},\mathbf{8}]}_{n\ell m}(0)|^2$$

$$|\psi^{[\mathbf{1},\mathbf{8}]}_{1,0,0}(0)|^2 = \frac{\mu^3(\alpha_h + \alpha^B_{g\;,[\mathbf{1},\mathbf{8}]})^3}{\pi}$$

bound state decay

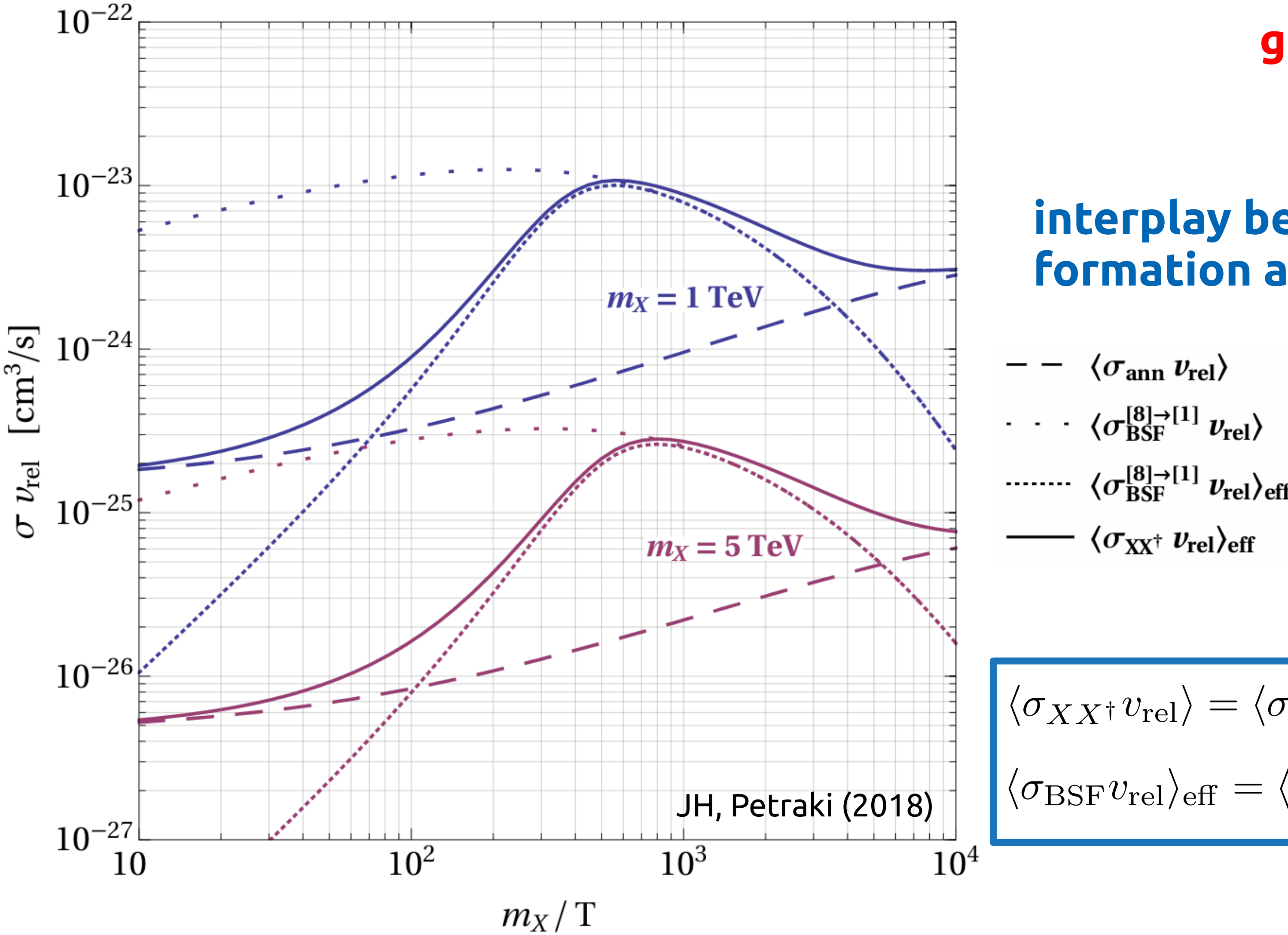
$$\mathcal{B}(XX^\dagger)_{[\mathbf{1}]} \to g_{[\mathbf{8}]}\; g_{[\mathbf{8}]}$$

$$\Gamma_{\rm ion} = g_g \int_{\omega_{\rm min}}^\infty \frac{d\omega}{2\pi^2} \frac{\omega^2}{e^{\omega/T} - 1}\, \sigma_{ion}$$

$$\sigma_{ion} = \frac{g_X^2}{g_g g_{\mathcal{B}}} \frac{\mu^2 v_{rel}^2}{\omega^2}\, \sigma_{\rm BSF}$$

Milne relation

Annihilation vs. effective BSF cross section



gluon exchange only

interplay between bound state formation and ionisation

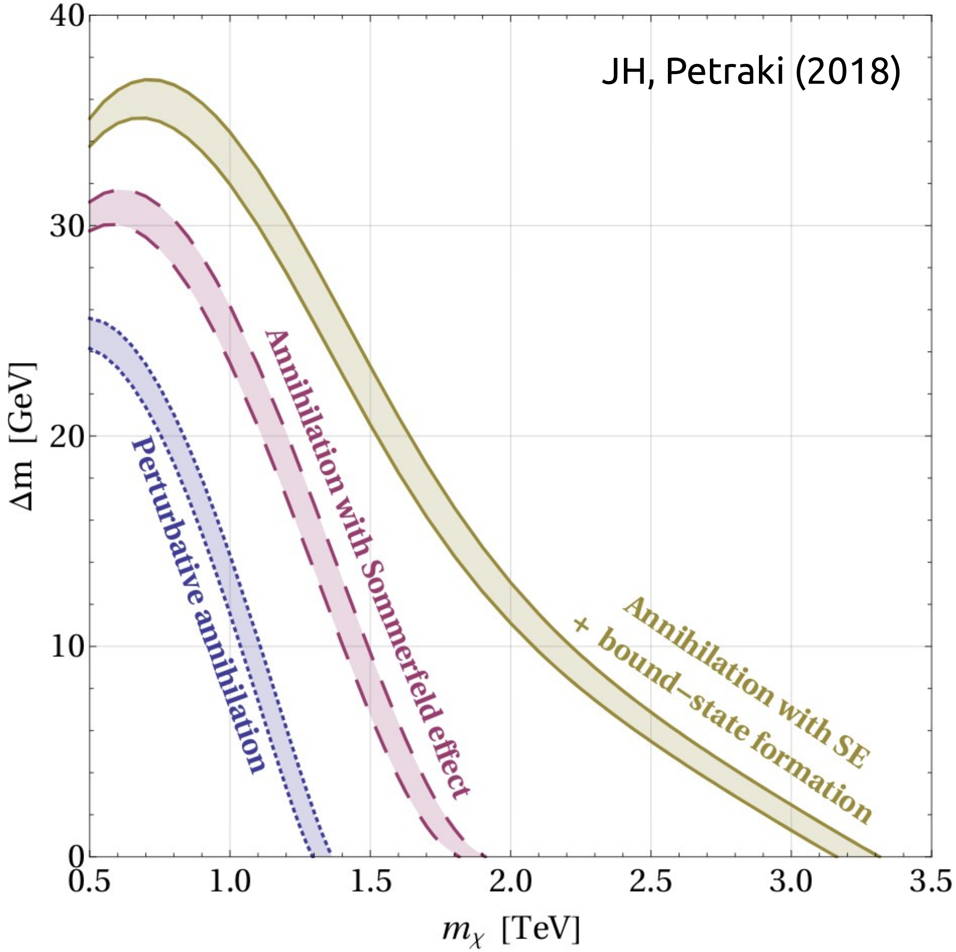
$$\langle \sigma_{XX^\dagger} v_{rel} \rangle = \langle \sigma_{ann}\, v_{rel} \rangle + \langle \sigma_{BSF} v_{rel} \rangle_{eff}$$

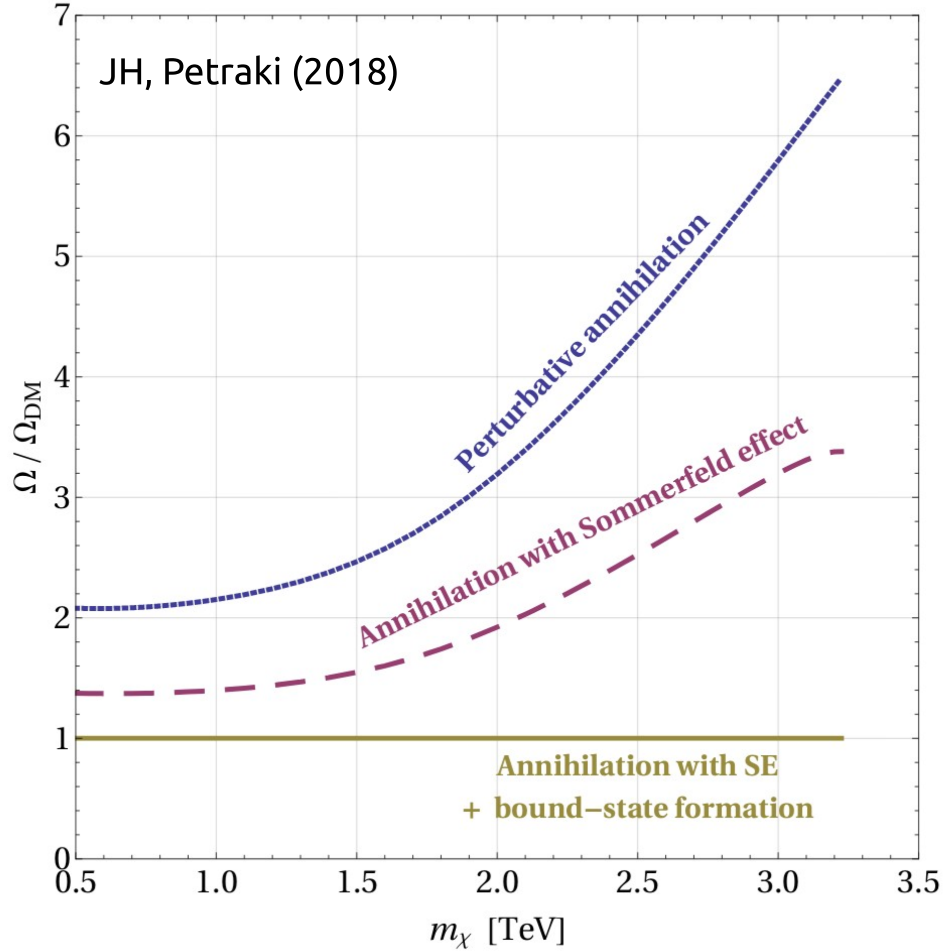
$$\langle \sigma_{BSF} v_{rel} \rangle_{eff} = \langle \sigma_{BSF} v_{rel} \rangle \times \left(\frac{\Gamma_{dec}}{\Gamma_{dec} + \Gamma_{ion}} \right)$$

→ BSF becomes more important than direct annihilation at z > 70

Impact on the relic density

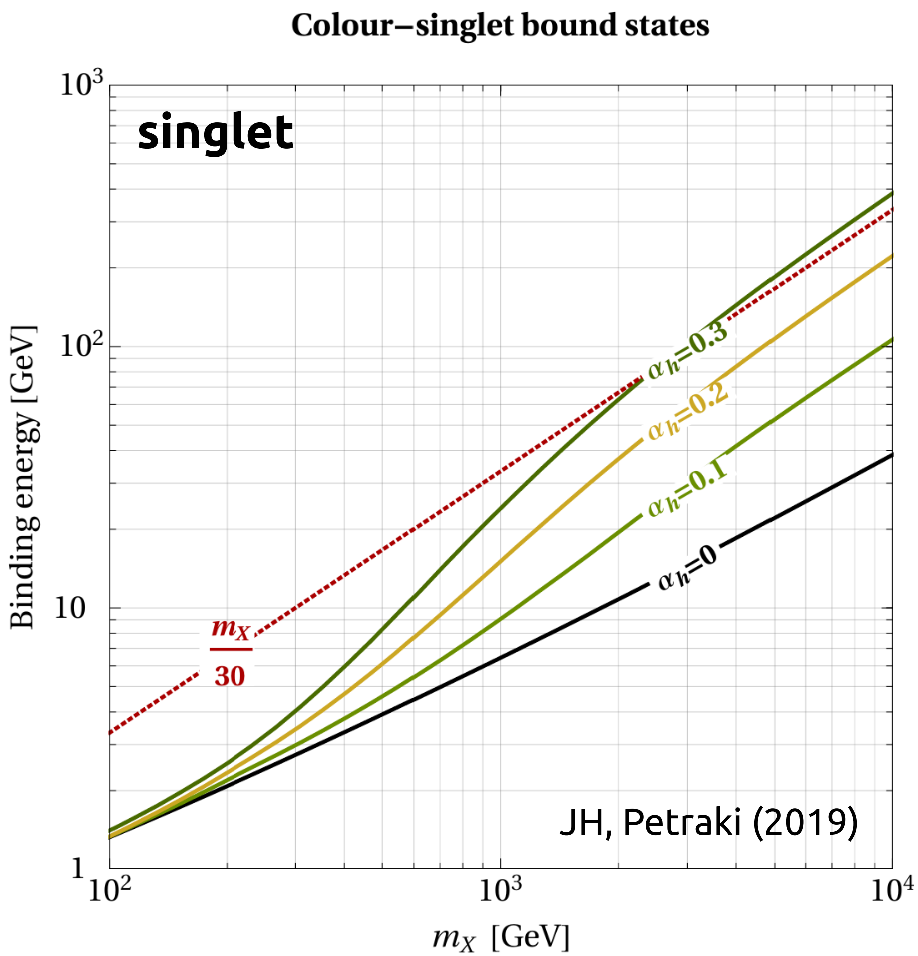
gluon exchange only

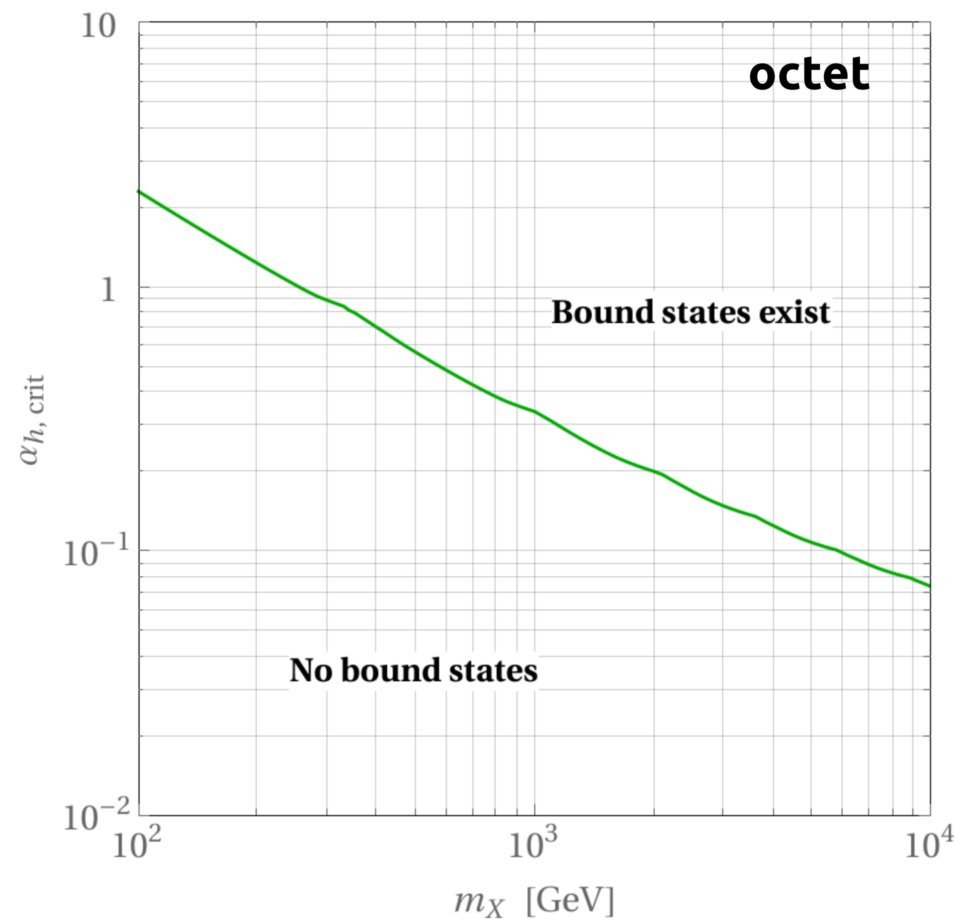




→ neglecting BSF and Sommerfeld effect would lead to a wrong relic density prediction by a factor 2 to 7

Impact of the Higgs on the formation of bound states

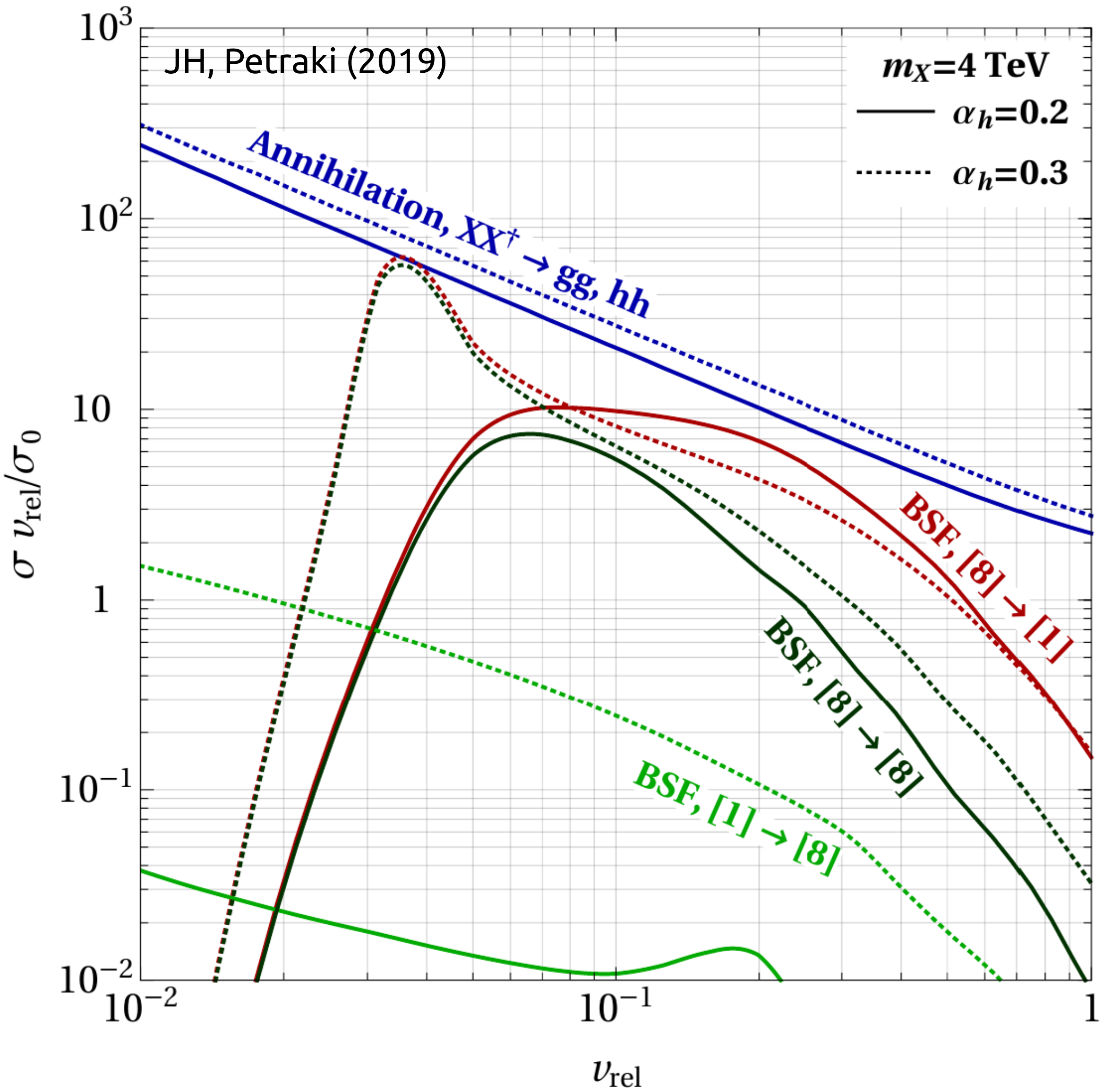




tighter bound states

additional bound states

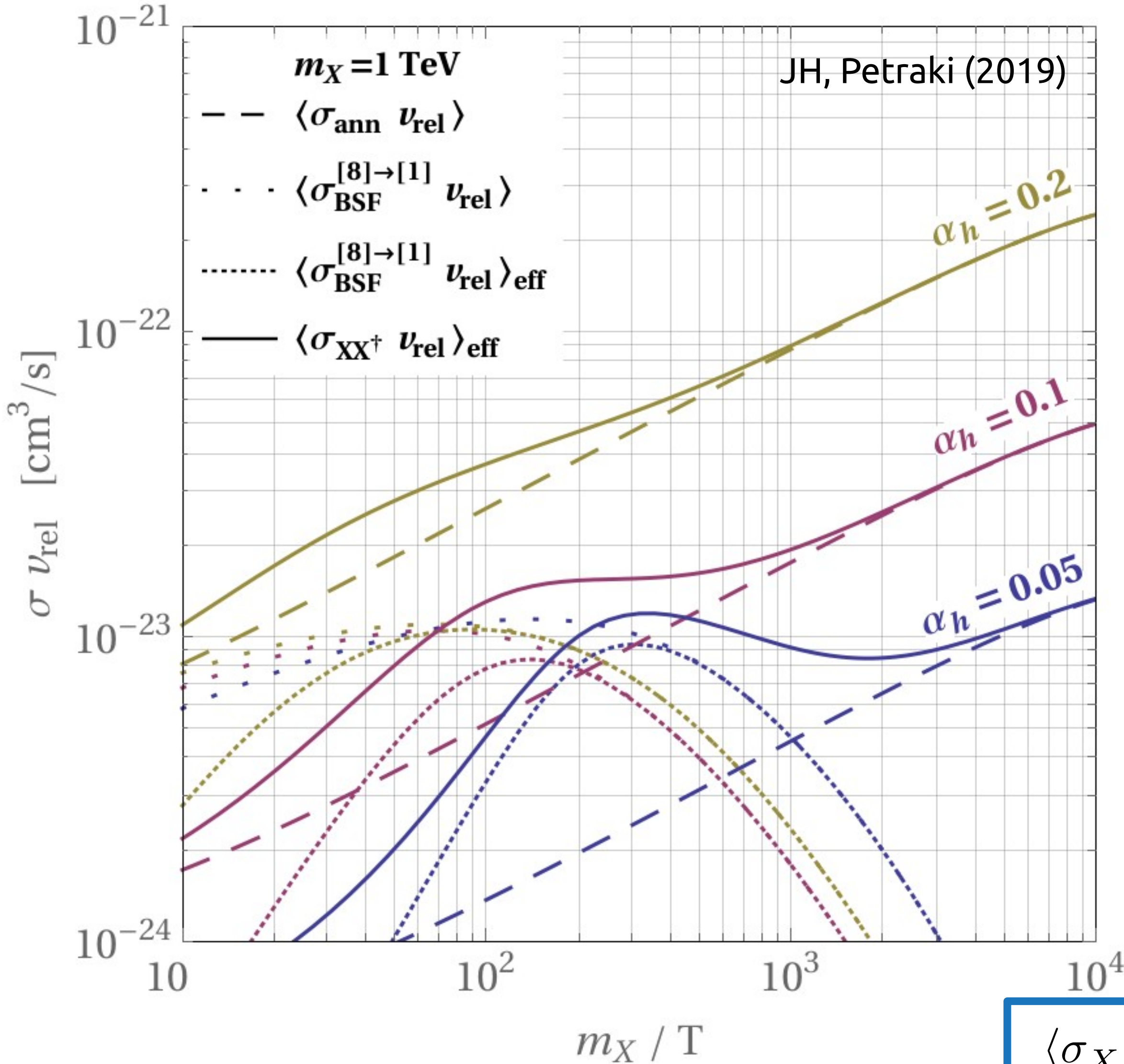
Impact of the Higgs on the BSF cross section



with Higgs exchange

at a certain mass and coupling, the formation of octets is possible

Impact of the Higgs on the effective BSF cross section



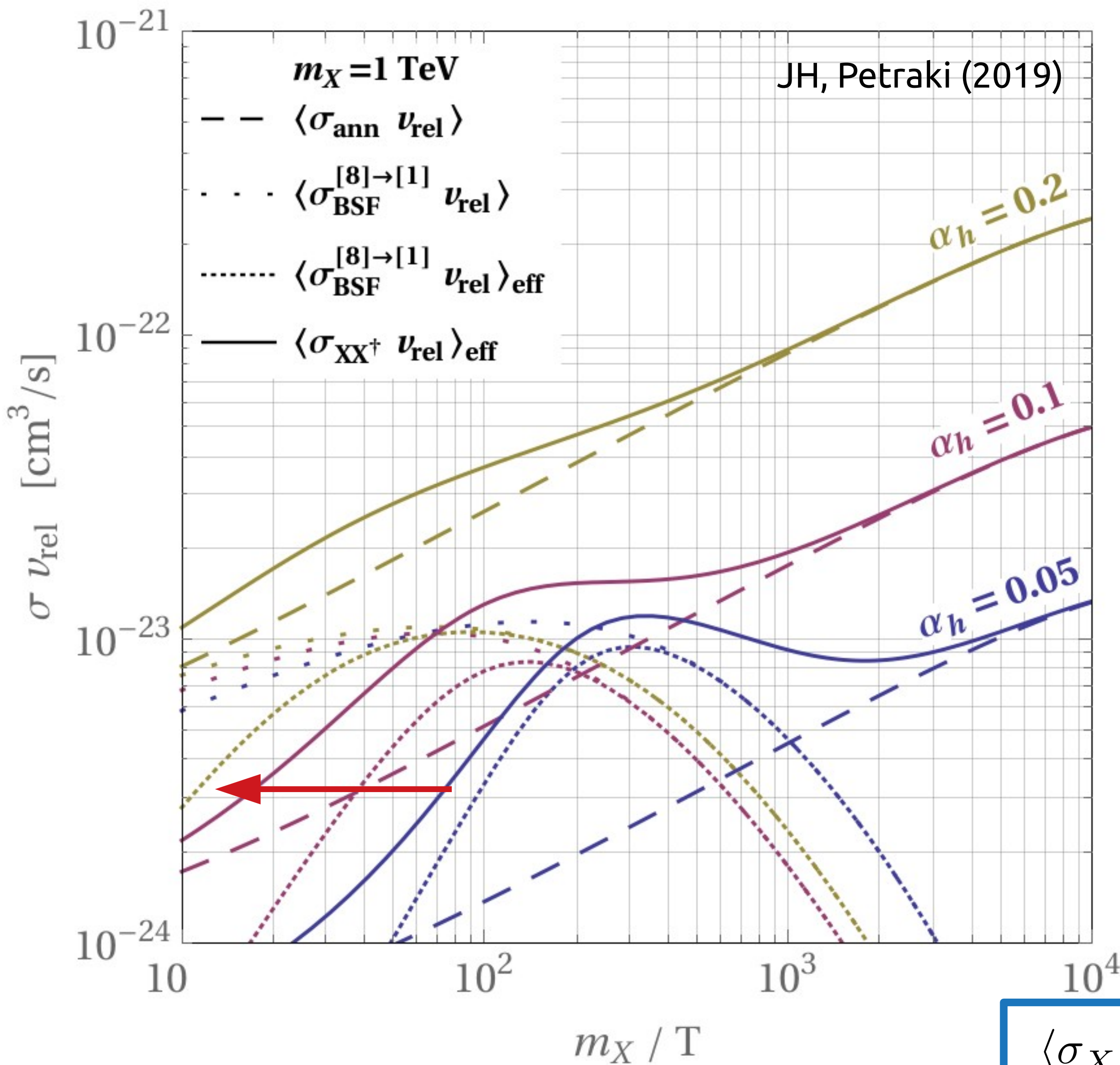
with Higgs exchange

- **Higgs coupling increases the binding energy**
- **a larger binding energy renders bound-state dissociation inefficient earlier, when the DM density is larger**
- **this enhances the efficiency to deplete DM**

$$\langle \sigma_{XX^\dagger} v_{\rm rel} \rangle = \langle \sigma_{\rm ann}\, v_{\rm rel} \rangle + \langle \sigma_{\rm BSF} v_{\rm rel} \rangle_{\rm eff}$$

$$\langle \sigma_{\rm BSF} v_{\rm rel} \rangle_{\rm eff} = \langle \sigma_{\rm BSF} v_{\rm rel} \rangle \times \left(\frac{\Gamma_{\rm dec}}{\Gamma_{\rm dec} + \Gamma_{\rm ion}} \right)$$

Impact of the Higgs on the effective BSF cross section



with Higgs exchange

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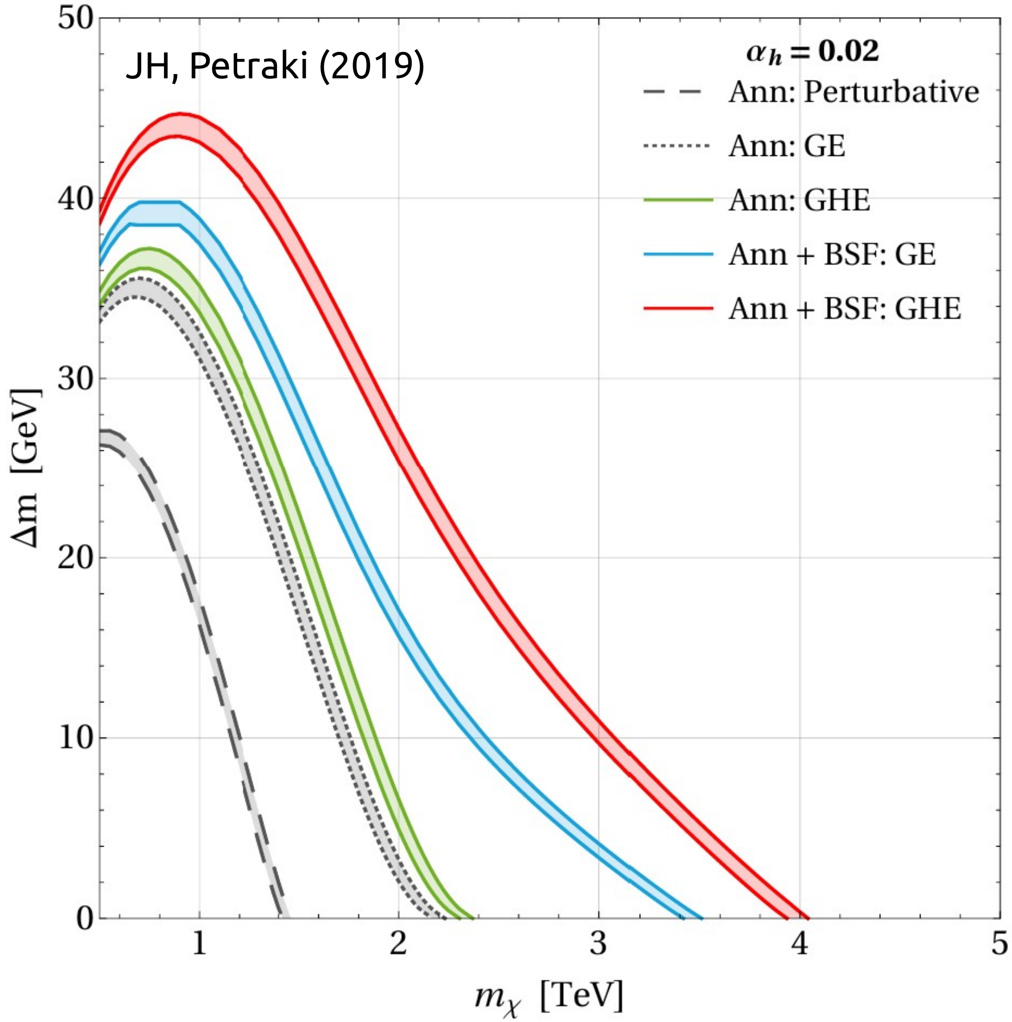
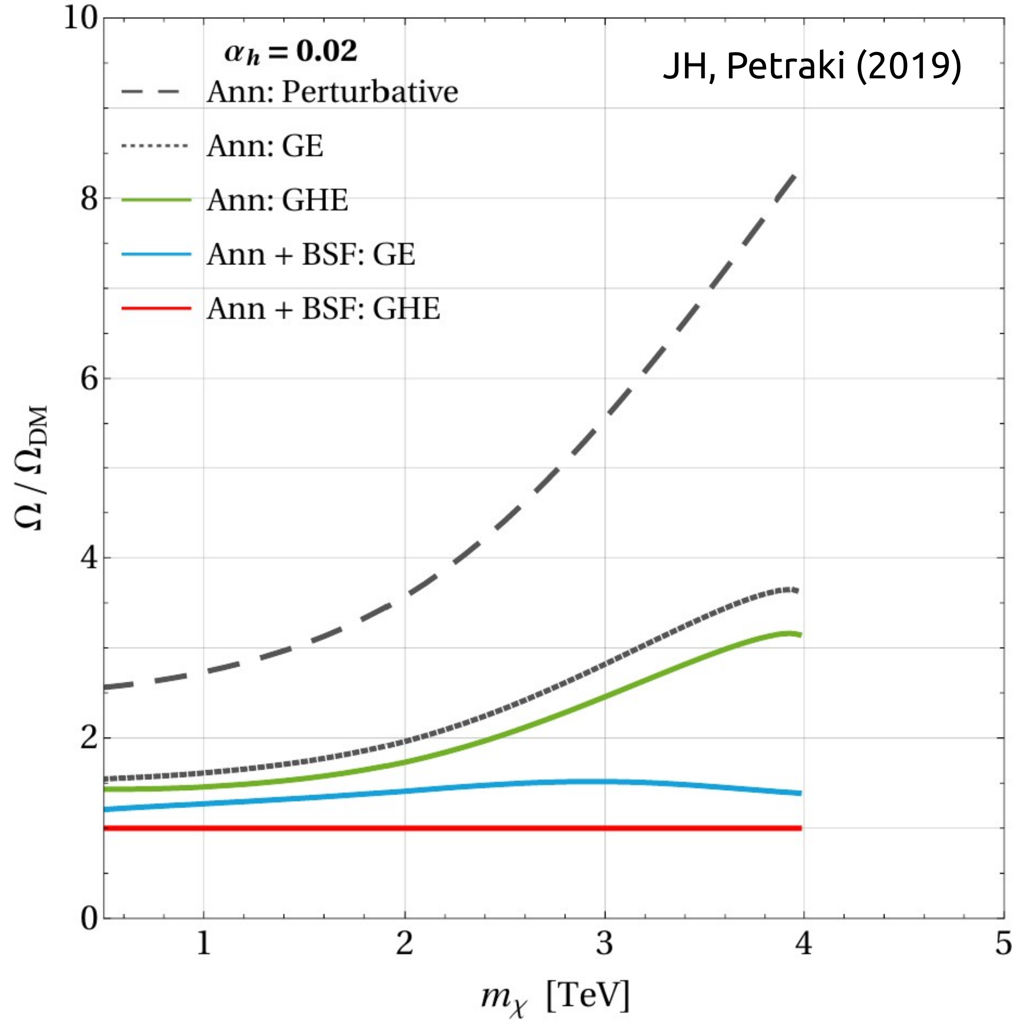
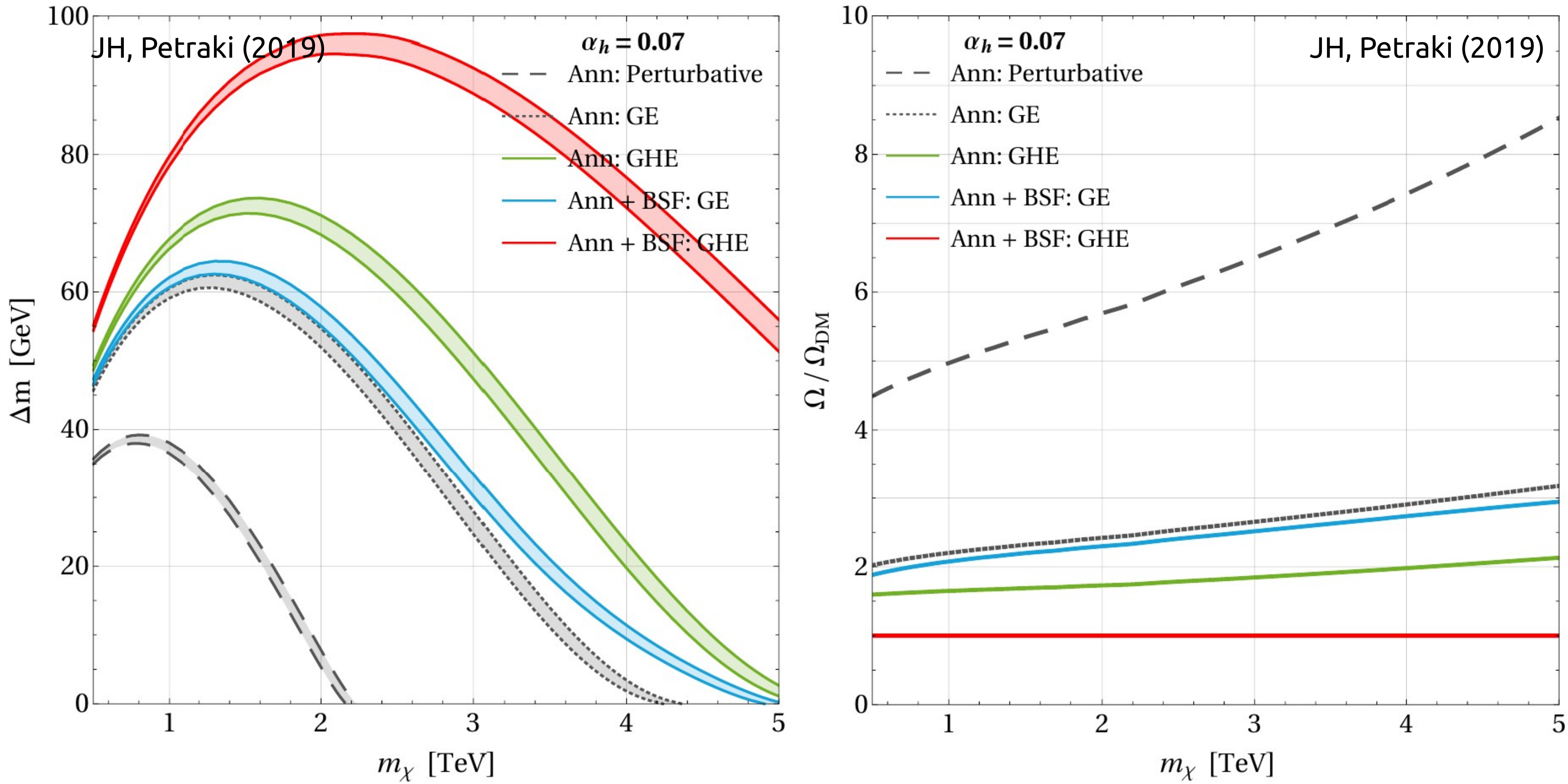
Impact on the relic density (with Higgs exchange)





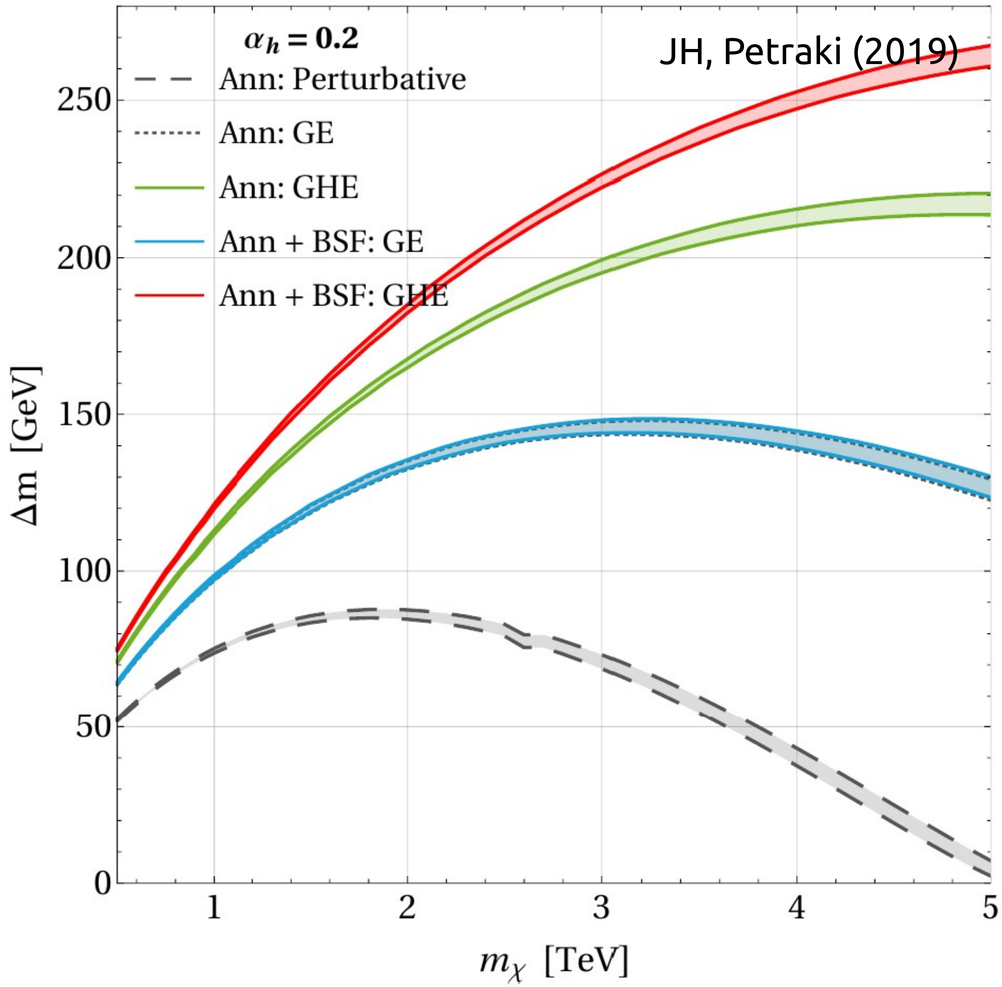
→ **impact of gluon dominant for small Higgs couplings**

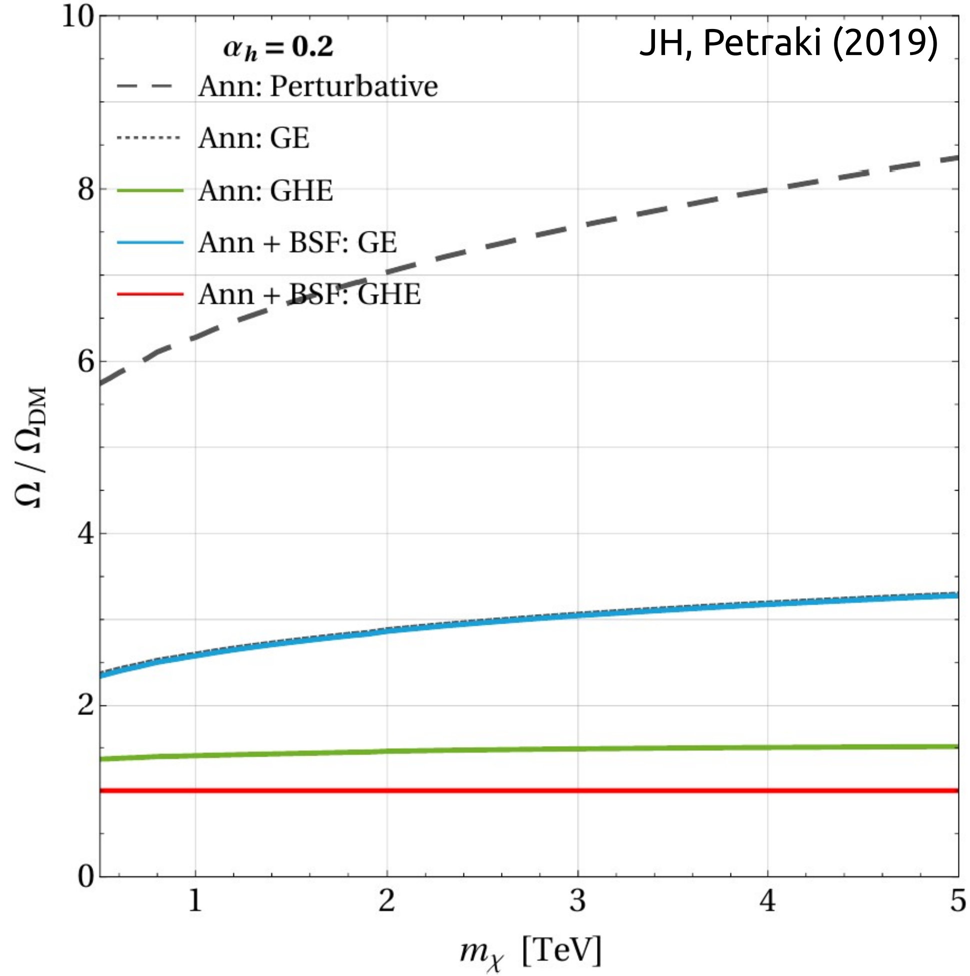
Impact on the relic density (with Higgs exchange)



→ **effect of gluon relatively less prominent**
→ **main impact from Higgs enhancement and BSF**

Impact on the relic density (with Higgs exchange)

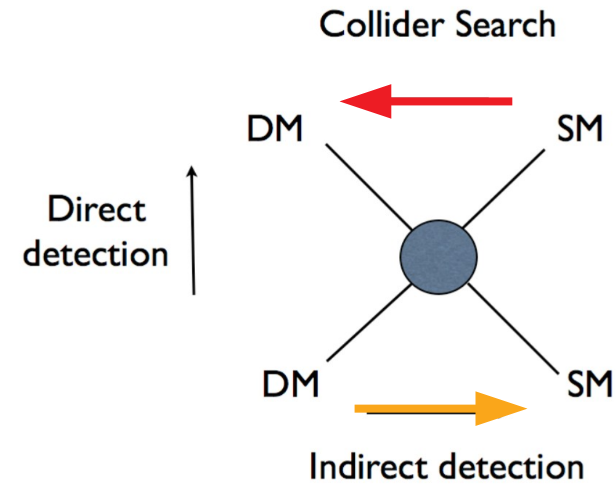


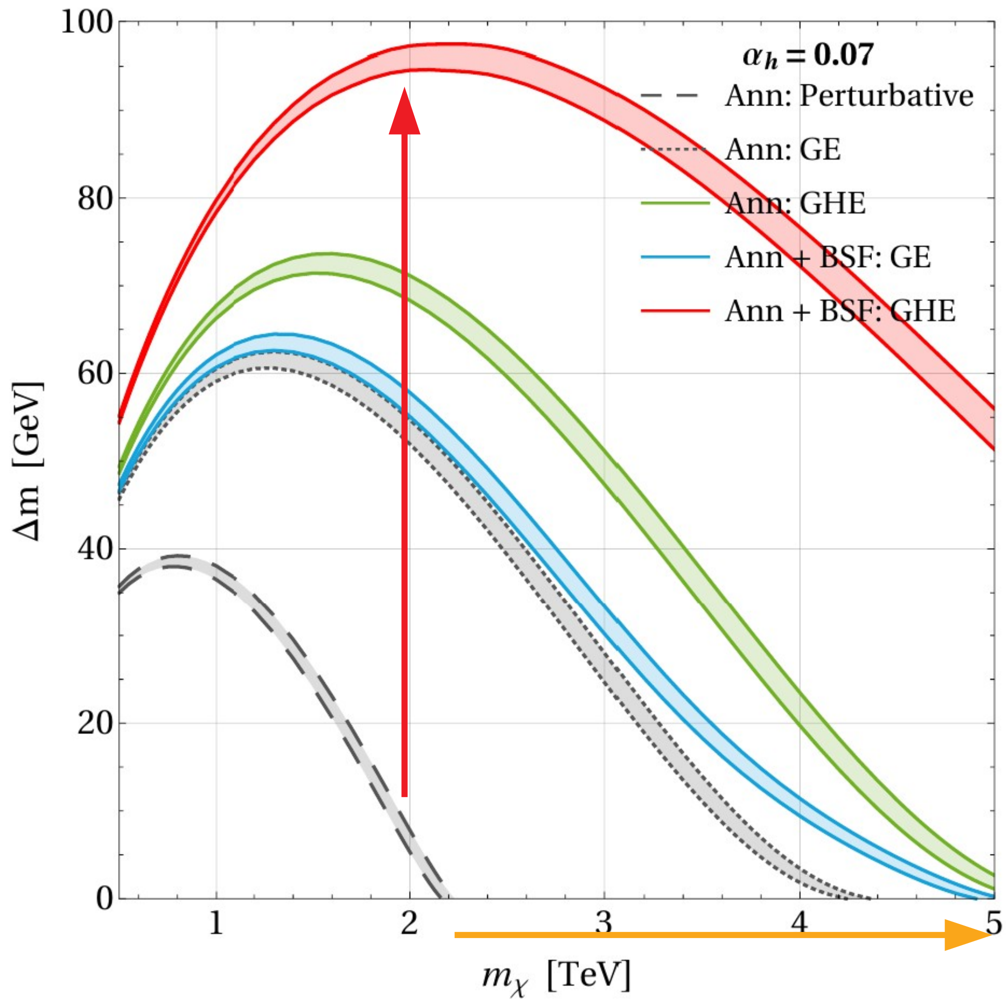


→ **Higgs enhancement most prominent**

→ **Higgs mediated BSF still sizable**

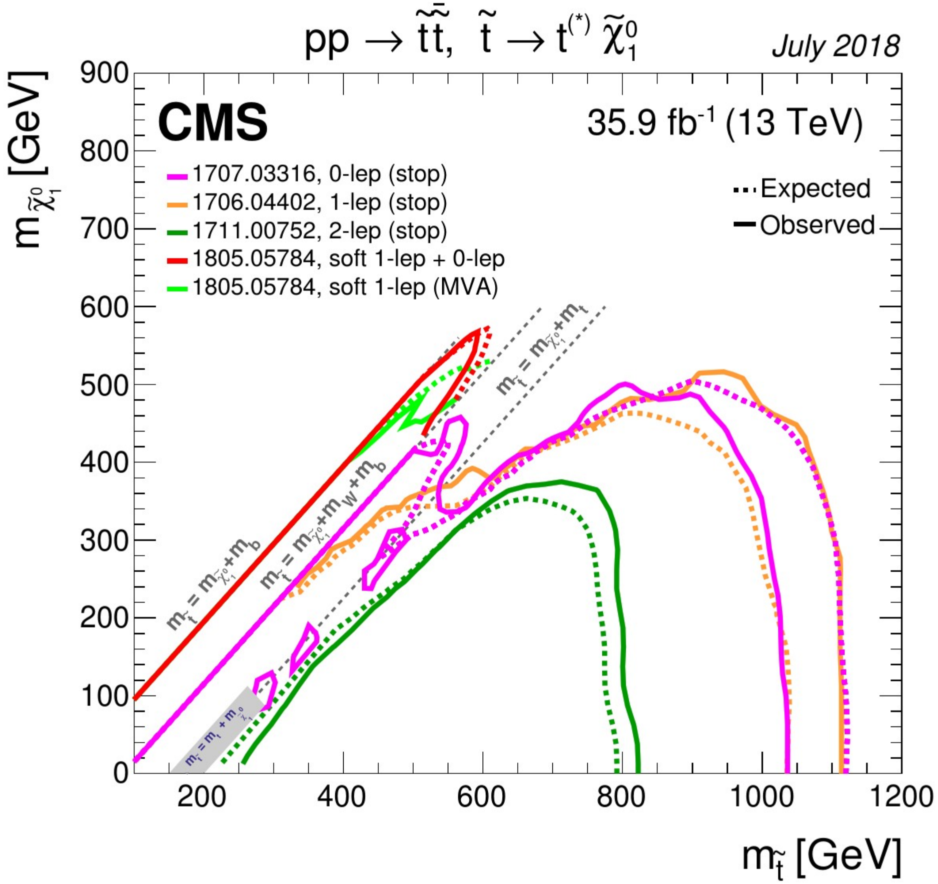
Why relevant?

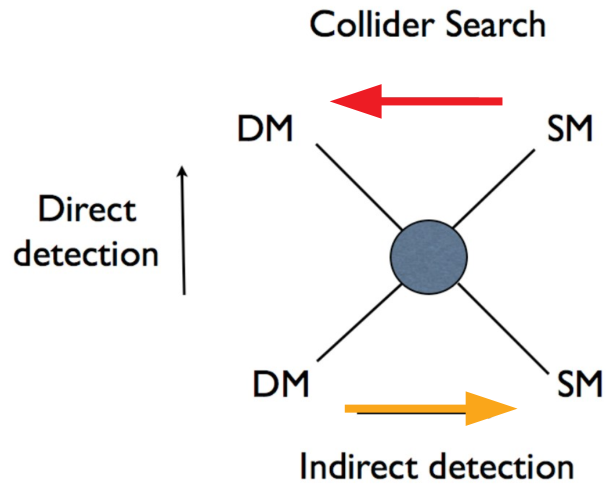




- More precise theoretical predictions
- Increased predicted mass splitting → **improved detection prospects with respect to multi-/mono-jet searches**
- DM can be heavier than anticipated → **interesting multi-TeV regime to be probed with indirect detection**

Why relevant?





- More precise theoretical predictions

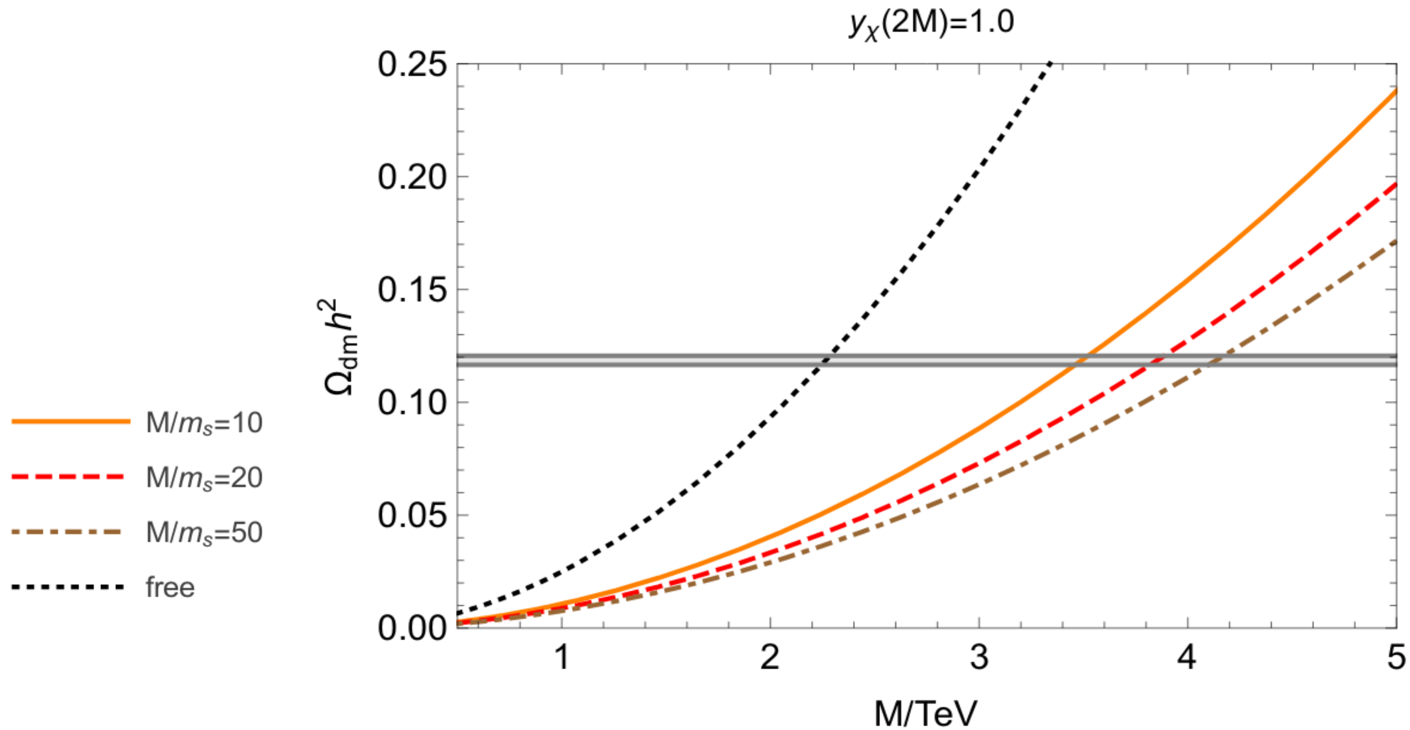
- Increased predicted mass splitting
  → **improved detection prospects with respect to multi-/mono-jet searches**

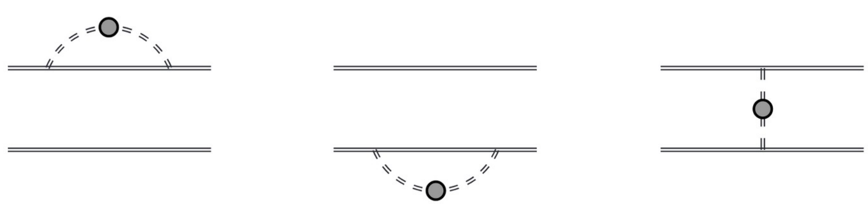
- DM can be heavier than anticipated
  → **interesting multi-TeV regime to be probed with indirect detection**

Finite temperature effects

Finite temperature effects

Dark Higgs as long-range force mediator considered (different approach)





→ **earlier talk by Simone Biondini**

Biondini, Laine (2018), Biondini (2018), Biondini, Vogl (2018)

Conclusions

- Recent interest in colored *coannihilation scenarios*
- *theoretical uncertainties* shall be included
- *Sommerfeld enhancement* and *bound state formation* can lead to huge corrective factors on the relic abundance
- *Higgs boson* can act as *long-range mediator* and impact previously studied scenarios
- *consistent running* of the couplings are crucial
- important *implications on experimental studies* (collider, indirect detection)

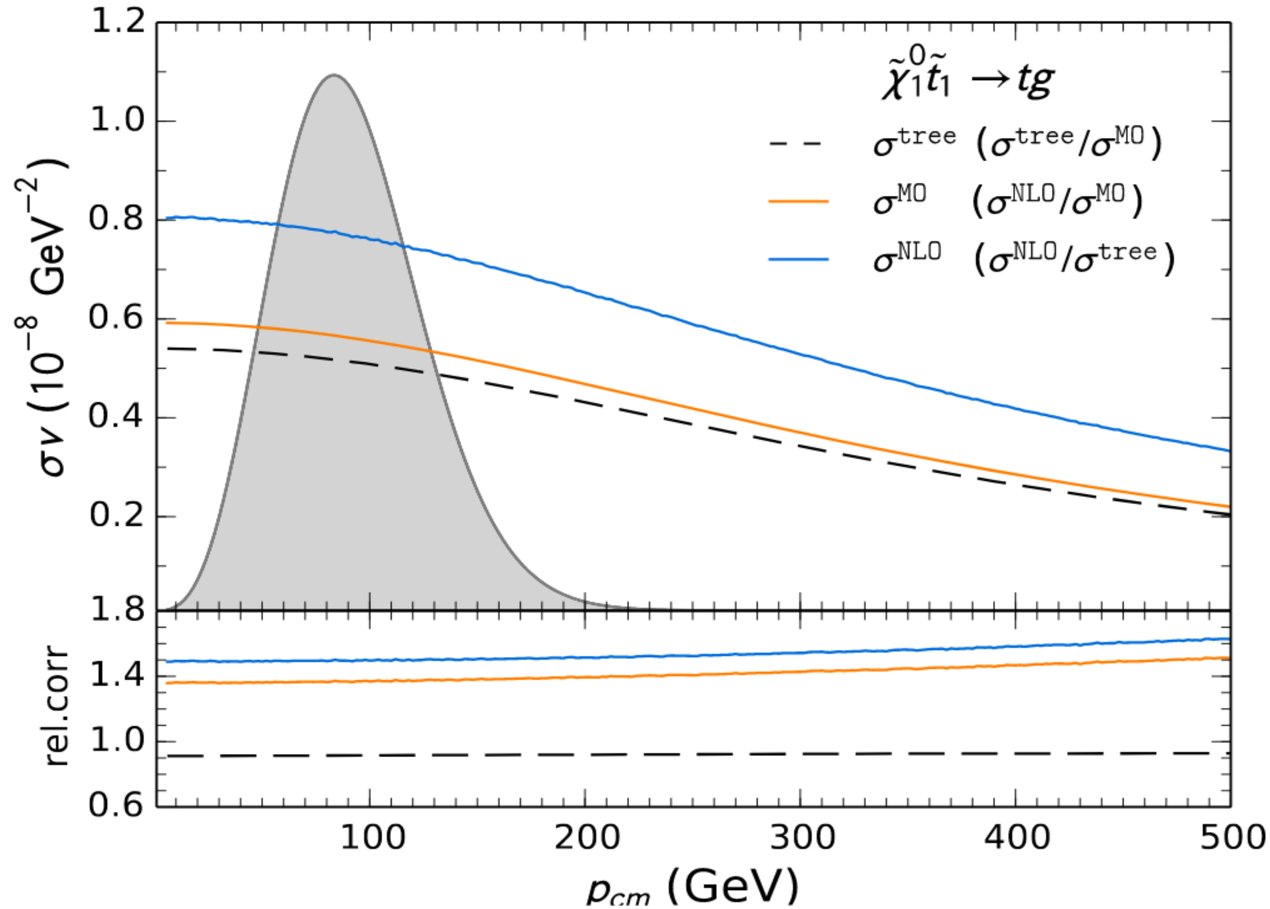
“Three exceptions” in 1990, but ongoing until today!

Thank you for your attention!

Impact on the cross section

$$\Omega_\chi h^2 = \frac{n_\chi m_\chi}{\rho_{\rm crit}} \propto \frac{1}{\langle \sigma_{\rm eff} v_{\rm rel} \rangle}$$

Coannihilation scenario on cross section level



→ **Corrections of around 40% to the MicrOMEGAs cross section**

JH, B. Herrmann, M. Klasen, K. Kovařík and Q. Le Boulc'h, Phys. Rev. D 87, 054031 (2013)
JH, B. Herrmann, M. Klasen, and K. Kovařík, Phys. Rev. D 91, 034028 (2015)