Scalar mediators, the Higgs, and dark matter bound states

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Quarkonia meet dark matter
18 June 2021

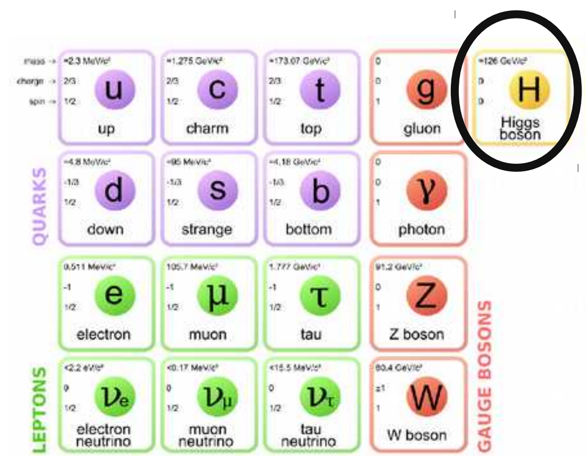
We talked about gauge theories
U(1), SU(2), SU(3).

What about light scalar mediators?

We talked about gauge theories
U(1), SU(2), SU(3).

What about light scalar mediators?
And why do we care?

Hidden sector models,
e.g. self-interacting DM



What do force mediators do?

- Generate potential → Sommerfeld effect, bound states.
- Provide channel for transitions via on/off-shell emission. Spin of the emitted mediator determines selection rules.

In many realistic models, including WIMPs:
different mediators present

Bound-state formation via emission of a neutral scalar

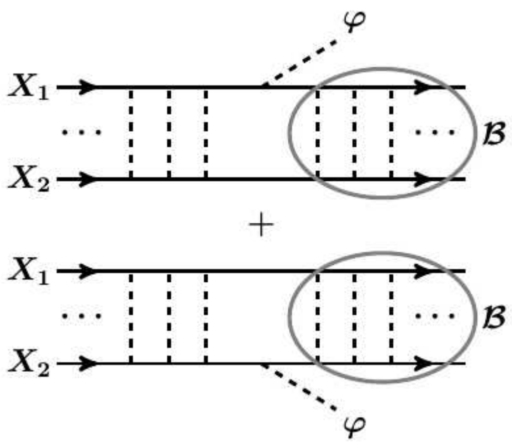
Neutral scalar mediator

Lagrangian

$$\mathcal{L} = \bar{X}_1 i\partial\!\!\!/ X_1 + \bar{X}_2 i\partial\!\!\!/ X_2 + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - m_1\bar{X}_1X_1 - m_2\bar{X}_2X_2 - \frac{m_\varphi}{2}\varphi^2 - y_1\varphi\bar{X}_1X_1 - y_2\varphi\bar{X}_2X_2$$

$$V_{12} = -\alpha\frac{e^{-m_\varphi r}}{r} \quad \text{with} \quad \alpha = \frac{y_1y_2}{4\pi}$$

$$\text{for} \quad \mu\alpha > m_\varphi \quad \text{where} \quad \mu = \frac{m_1m_2}{m_1+m_2} \quad \Rightarrow \quad \text{bound states exist}$$



Bound-state formation amplitude

$$\mathcal{M}_{\rm BSF} \sim \int d^3p\, \tilde{\psi}^*_{n\ell m}(p)\left[y_1\,\tilde{\phi}_k\left(p + \frac{m_2}{m_1+m_2}P_\varphi\right) + y_2\,\tilde{\phi}_k\left(p - \frac{m_1}{m_1+m_2}P_\varphi\right)\right]$$

$$\sim \int d^3r\, \psi^*_{n\ell m}(r)\left[y_1\,\phi_k(r)\; e^{-\frac{m_2}{m_1+m_2}P_\varphi\cdot r} + y_2\,\phi_k(r)\; e^{+\frac{m_1}{m_1+m_2}P_\varphi\cdot r}\right]$$

Energy scales

$$\left.\begin{aligned} k &\sim \mu v_{\rm rel} \\ p &\sim \mu\alpha \end{aligned}\right\} \quad \Leftrightarrow \quad r \sim \frac{1}{\sqrt{k^2+p^2}} \sim \frac{1}{\mu\sqrt{\alpha^2+v_{\rm rel}^2}}$$

$$P_\varphi \sim \mu(\alpha^2 + v_{\rm rel}^2)/2$$

$$P_\varphi\cdot r \sim \sqrt{\alpha^2+v_{\rm rel}^2} \ll 1$$

Neutral scalar mediator

Lagrangian

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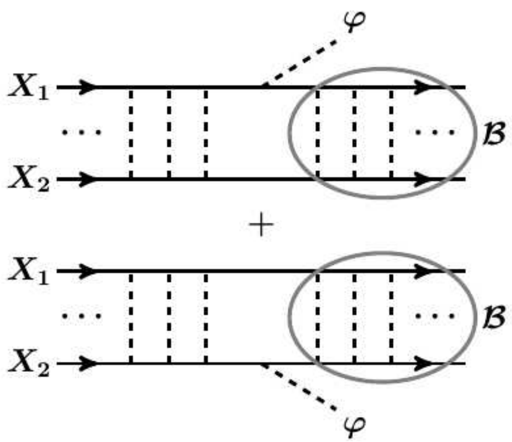

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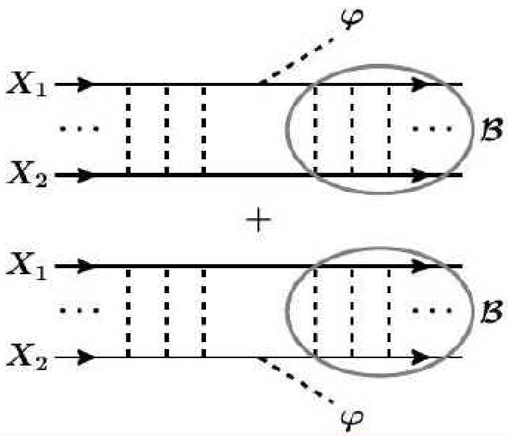
$$P_\varphi\cdot r \sim \sqrt{\alpha^2+v_{\rm rel}^2} \ll 1$$

expand

Neutral scalar mediator



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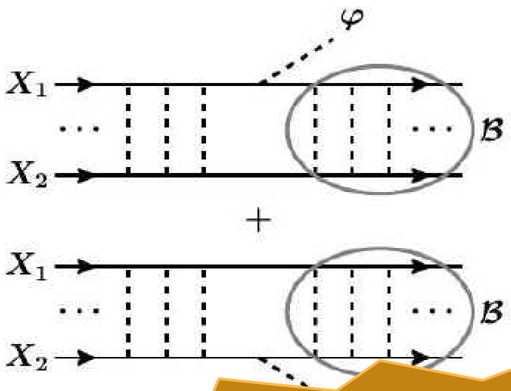
$$P_\varphi \cdot r \sim \sqrt{\alpha^2+v_{\rm rel}^2} \ll 1$$

monopole $(\Delta\ell = 0)$: $\int d^3p\, \psi^*_{n\ell m}(r)\, \phi_k(r)\, (y_1+y_2)$ — cancels due to orthogonality of wavefunctions

dipole $(\Delta\ell = 1)$: $\int d^3p\, \psi^*_{n\ell m}(r)\, \phi_k(r) (P_\varphi \cdot r) \left(-\frac{y_1 m_2}{m_1+m_2} + \frac{y_2 m_1}{m_1+m_2}\right)$ — cancels for $\frac{y_1}{m_1} = \frac{y_2}{m_2}$, suppressed by $\sqrt{\alpha^2+v_{\rm rel}^2}$

quadrapole $(\Delta\ell = 2)$: $\int d^3p\, \psi^*_{n\ell m}(r)\, \phi_k(r) (P_\varphi \cdot r)^2\, \frac{1}{2}\left[\left(\frac{y_1 m_2}{m_1+m_2}\right)^2 + \left(\frac{y_2 m_1}{m_1+m_2}\right)^2\right]$ — suppressed by $\alpha^2 + v_{\rm rel}^2$

Neutral scalar mediator



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expand

For particle-antiparticle pairs or pairs of identical particles,

BSF via neutral scalar emission is rather suppressed!

$$r \sim \frac{1}{\sqrt{k^2+p^2}} \sim \frac{1}{\mu\sqrt{\alpha^2+v_{\rm rel}^2}}$$

$${}_{\rm rel}^2)/2$$

$$P_\varphi \cdot r \sim \sqrt{\alpha^2+v_{\rm rel}^2} \ll 1$$

monopole $(\Delta\ell = 0)$: $\int d^3p\, \psi^*_{n\ell m}(r)\, \phi_k(r)\, (y_1+y_2)$ — cancels due to orthogonality of wavefunctions

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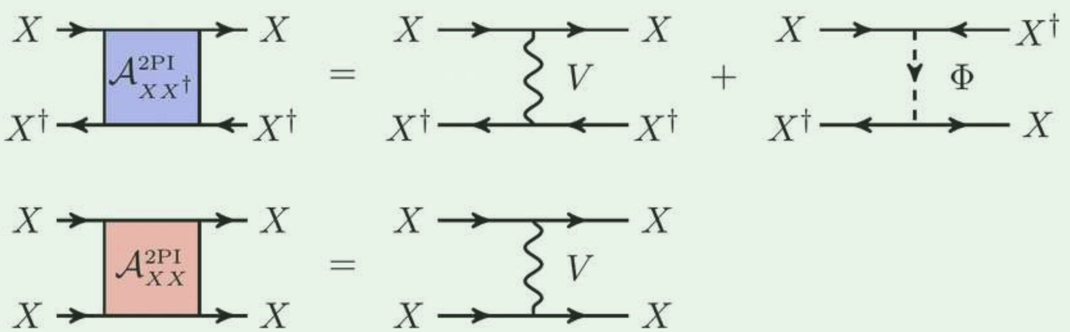
Bound-state formation via emission of a charged scalar

Scalar DM *X,X†* coupled to doubly charged light scalar mediator *Φ*

$$\mathcal{L} \supset -igX^\dagger V^\mu(\partial_\mu X) - i2g\Phi^\dagger V^\mu(\partial_\mu \Phi) - \frac{y m_X}{2} X X \Phi^\dagger + h.c.$$

$$m_X \gg m_\Phi$$



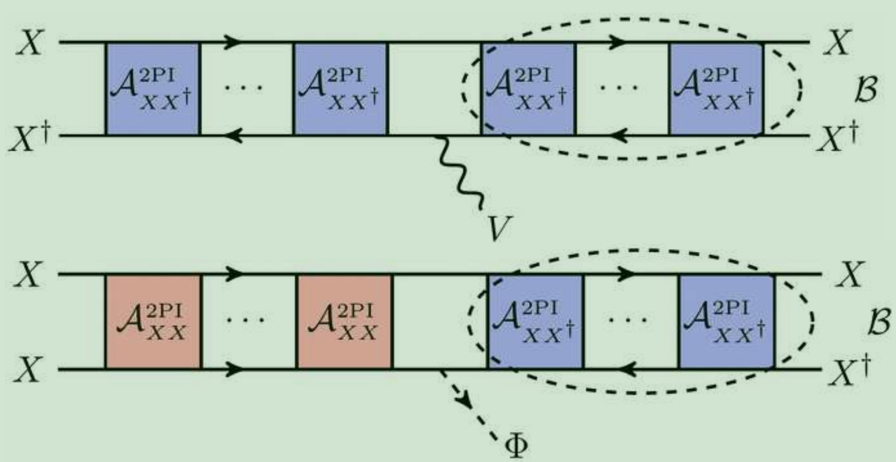
$$U_{XX^\dagger}(r) = -\frac{\alpha_V}{r} - (-1)^\ell \frac{\alpha_\Phi}{r} e^{-m_\Phi r}$$

$$U_{XX}(r) = +\frac{\alpha_V}{r}$$

Potential differs even in the global symmetry limit $\alpha_V \to 0$

BSF$_V$

BSF$_\Phi$



dipole transition

$$\mathcal{M} \sim 2g \int d^3p\; \psi^*_{n\ell m}(r) \nabla \phi_k(r) \sim 2g \int d^3p\; \psi^*_{n\ell m}(r) r \phi_k(r)$$

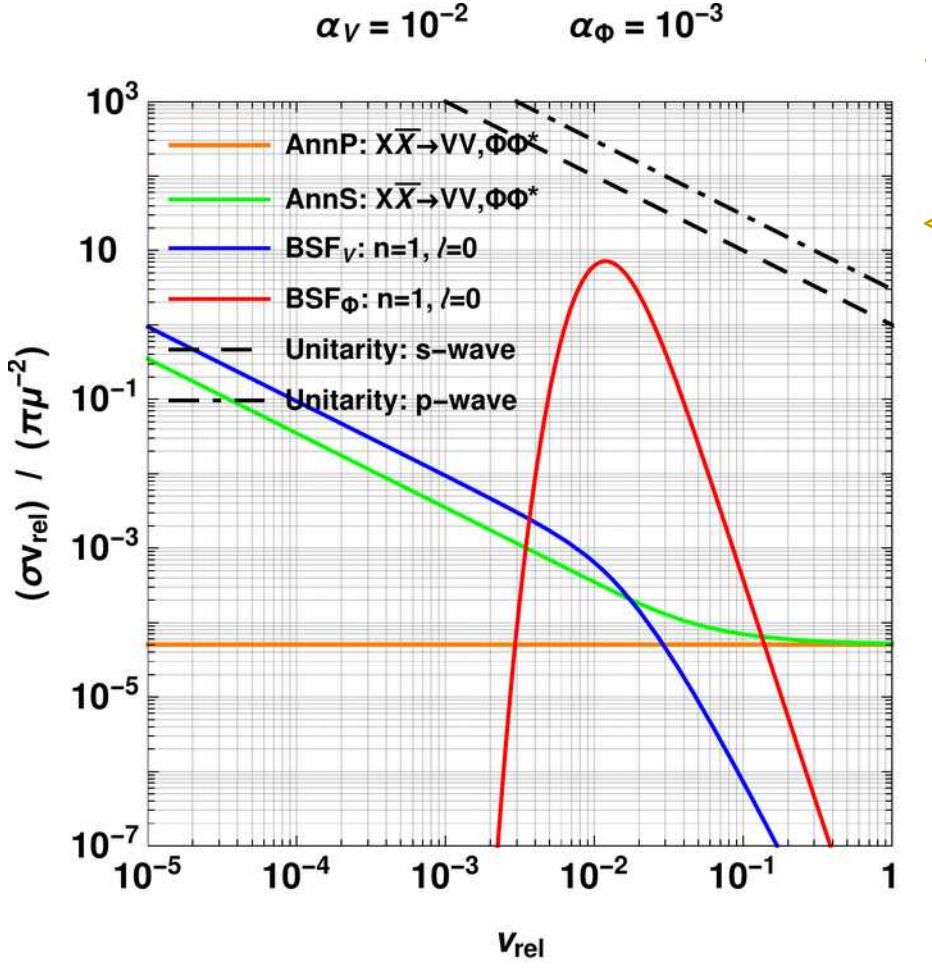
Change in effective Hamiltonian
⇒ monopole transition

$$\mathcal{M} \sim 2y \int d^3p\; \psi^*_{n\ell m}(r)\; \phi_k(r)$$

extremely fast!

Oncala, KP: 1911.02605

Scalar DM $X, X^\dagger$ coupled to doubly charged light scalar mediator Φ



BSF$_\Phi$ very large, even for small values of α_Φ, α_V !

Oncala, KP: 1911.02605

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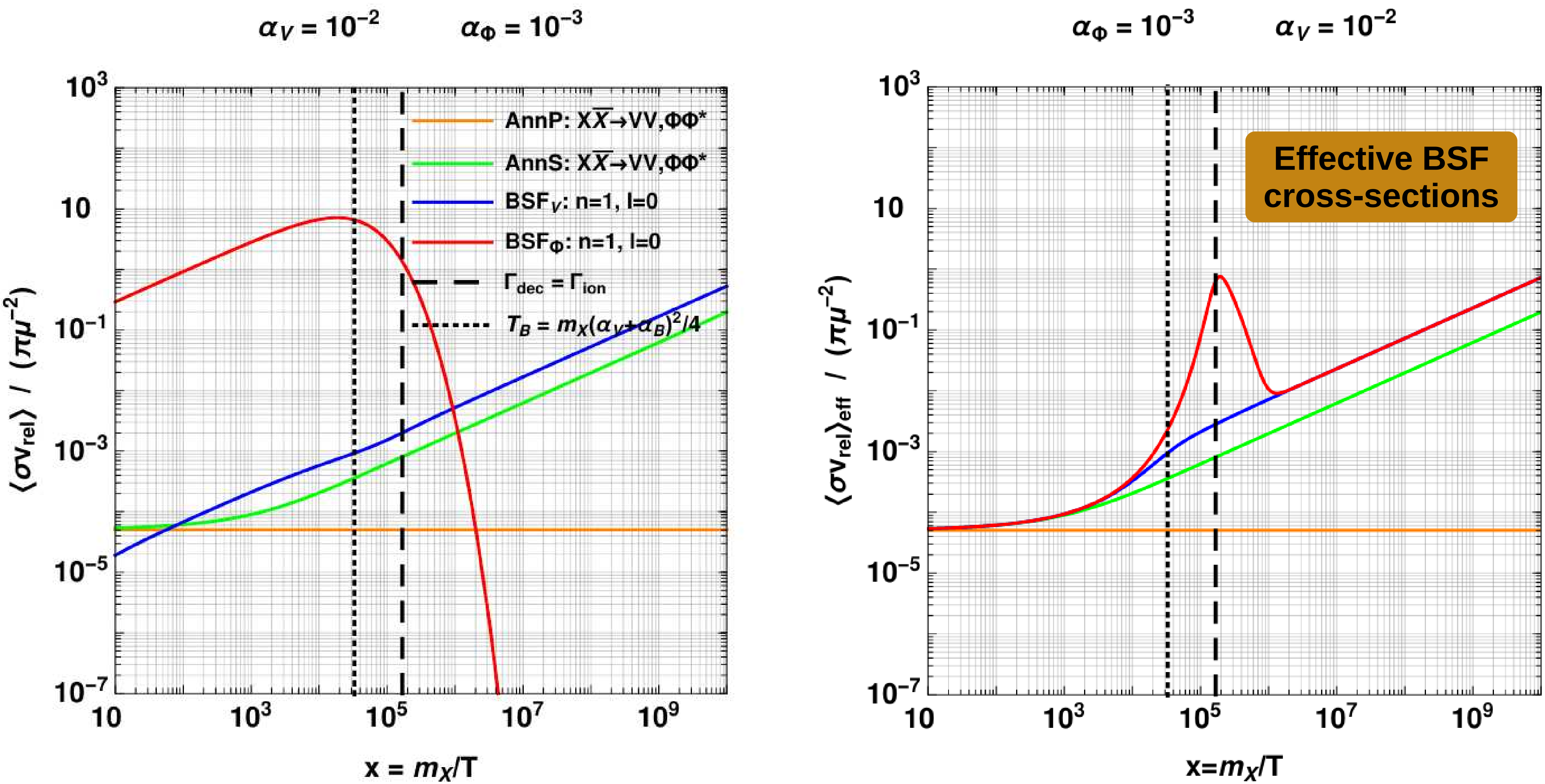

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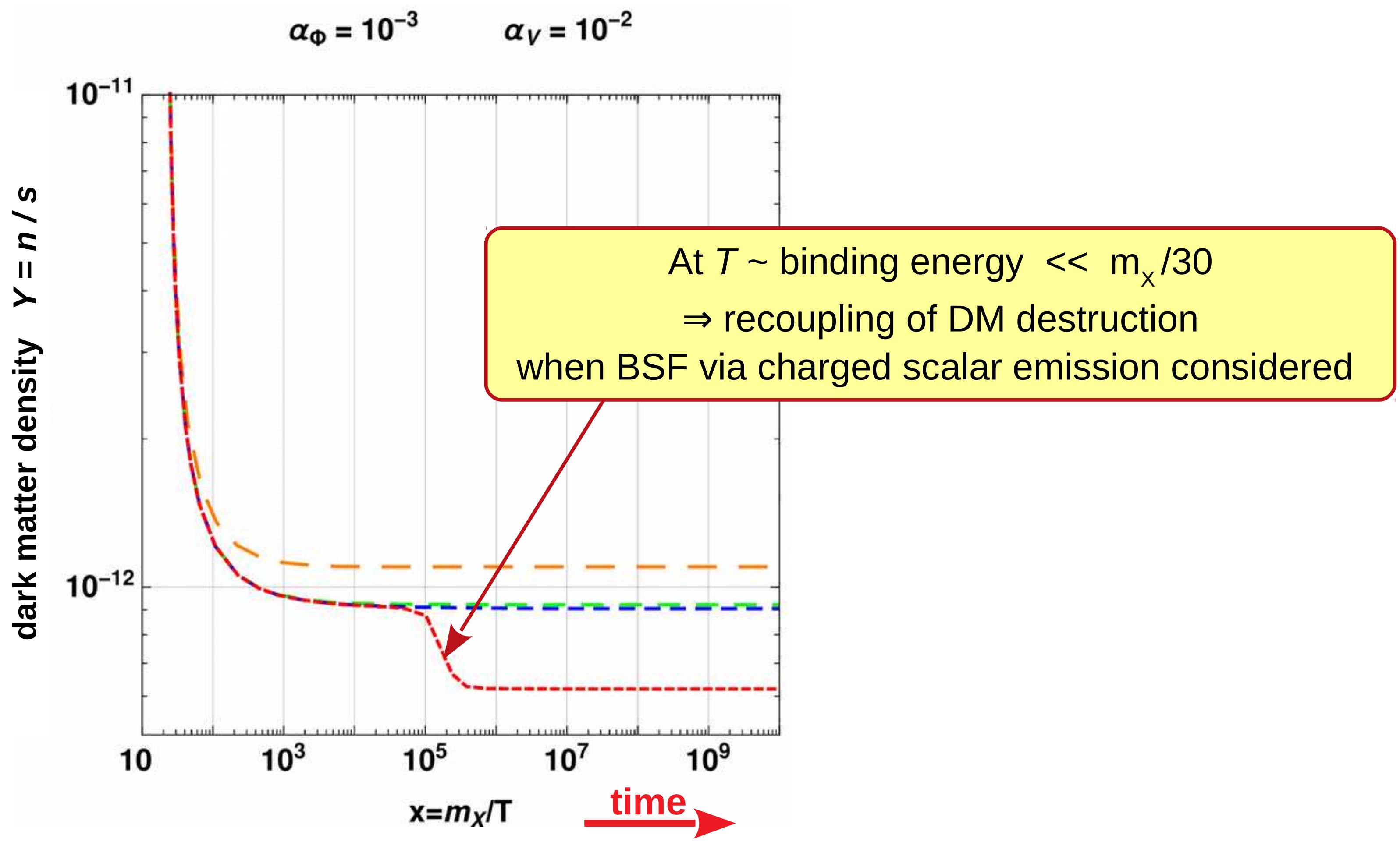
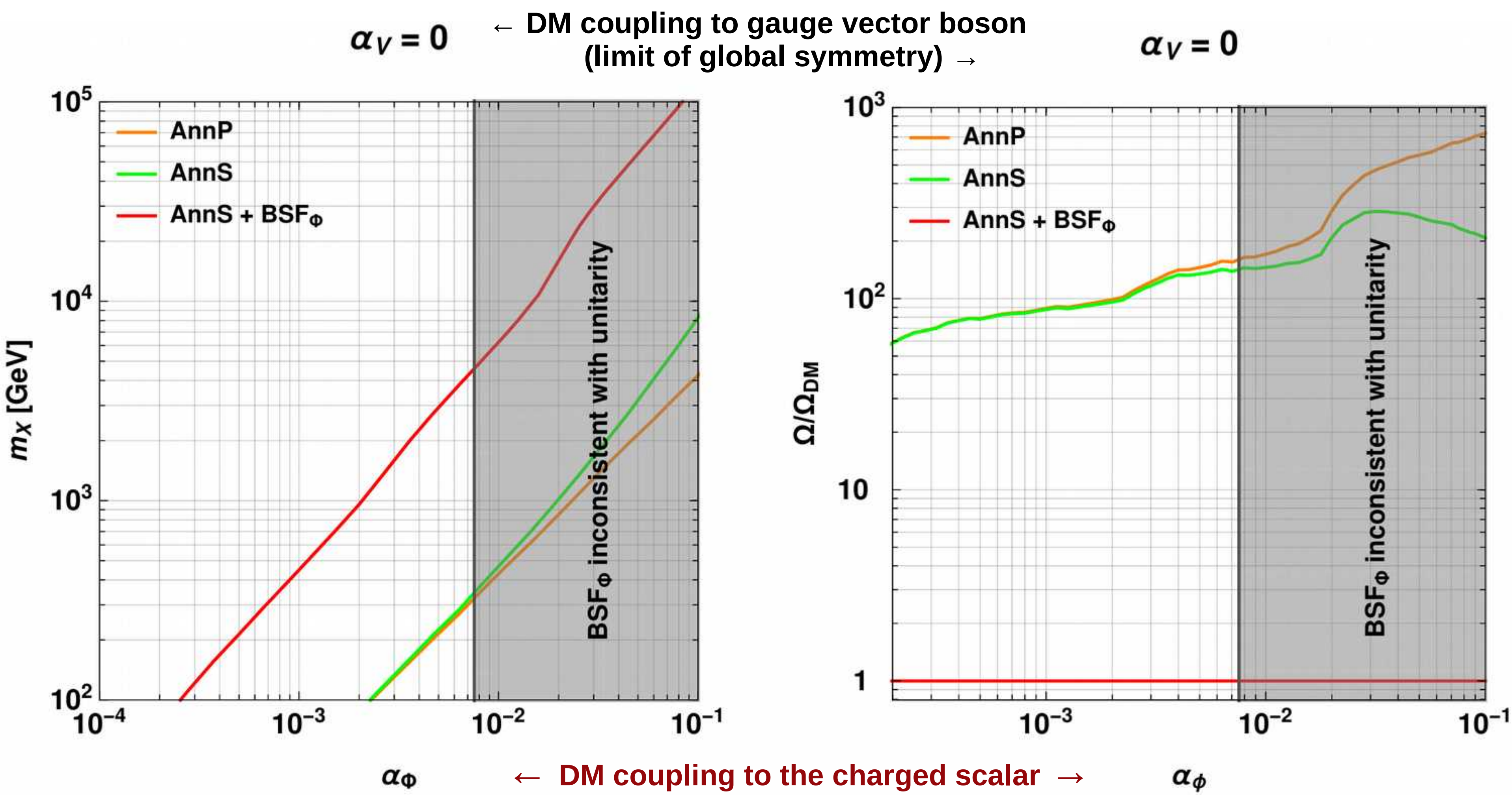


Oncala, KP: 1911.02605
(see also Ko,Matsui,Tang:1910:04311)

Scalar DM $X,X^\dagger$ coupled to doubly charged light scalar mediator Φ



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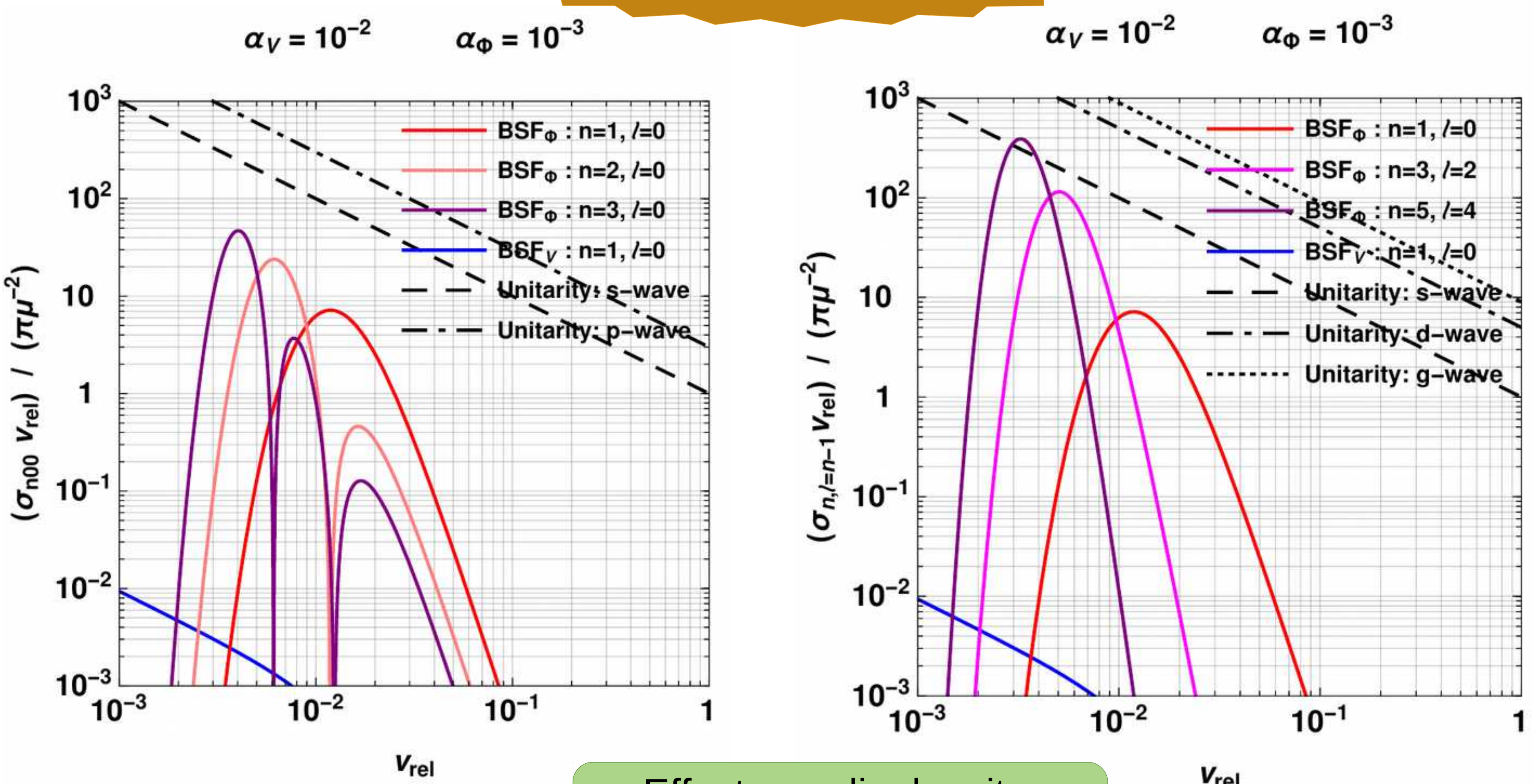
Scalar DM $X,X^\dagger$ coupled to doubly charged light scalar mediator Φ

Capture into excited states important!



Effect on relic density: work in progress with Anastasiia Filimonova

Oncala, KP: 1911.02605

Bound-state formation
via Higgs doublet emission

Renormalisable WIMP models with coupling to the Higgs

In some of the prototypical WIMP DM models,
DM is the lightest linear combination of the neutral component of SU(2) multiplets that couple to the Higgs

$$\boxed{\delta\mathcal{L} \supset -y\bar{X}_n H X_{n+1} + \mathrm{h.c.}}$$

Trilinear Higgs doublet-DM-DM, independent of Higgs VEV

Includes many supersymmetric scenarios,
e.g. Wino-Higgsino, coloured coannihition

If m > 5 TeV, DM freeze-out begins before electroweak phase transition.

Doublet-Singlet coupled to the Higgs

$$\delta\mathcal{L} = \frac{1}{2}\bar{S}(i\partial\!\!\!/ - m_S)S + \bar{D}(i D\!\!\!\!/ - m_D)D - (y_L\bar{D}_L H S + y_R\bar{D}_R H S) + \text{ h.c.}$$

$$\text{for simplicity, set:} \qquad y_L = y_R \equiv y, \qquad m_S = m_D \equiv m$$

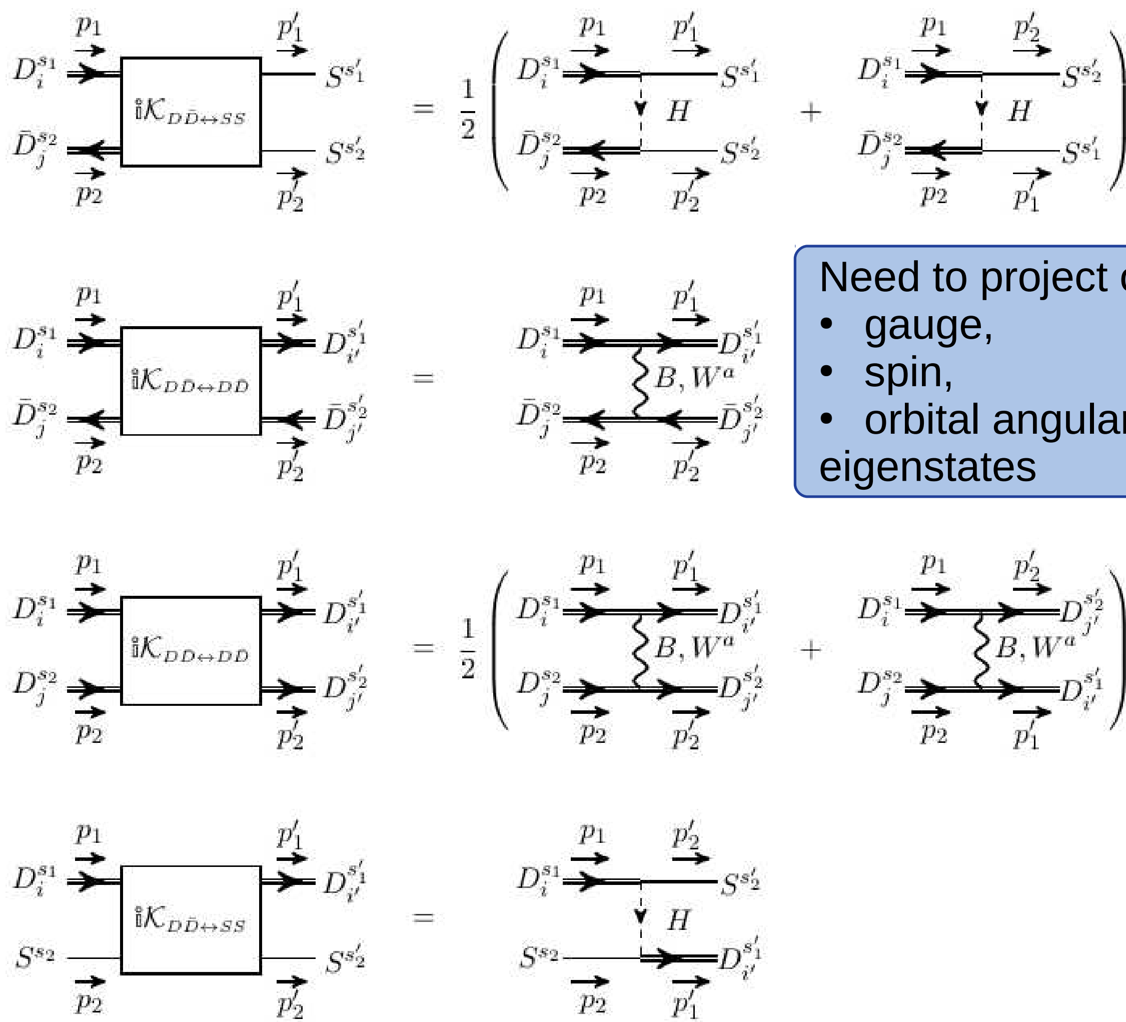
field	$SU_L(2)$	$U_Y(1)$	$\mathbb{Z}_2$
S	1	0	-1
D	2	$1/2$	-1
H	2	$1/2$	$+1$

Calculate all cross-sections and freeze-out in the symmetric electroweak phase.

Oncala, KP: 2101.08666/7

Doublet-Singlet coupled to the Higgs

Non-relativistic potentials



Need to project on
- gauge,
- spin,
- orbital angular momentum

eigenstates

Neglect (T-dependent) H mass in the potential

but consider it in phase-space suppression for BSF via H emission (→ analytical cross-sections)

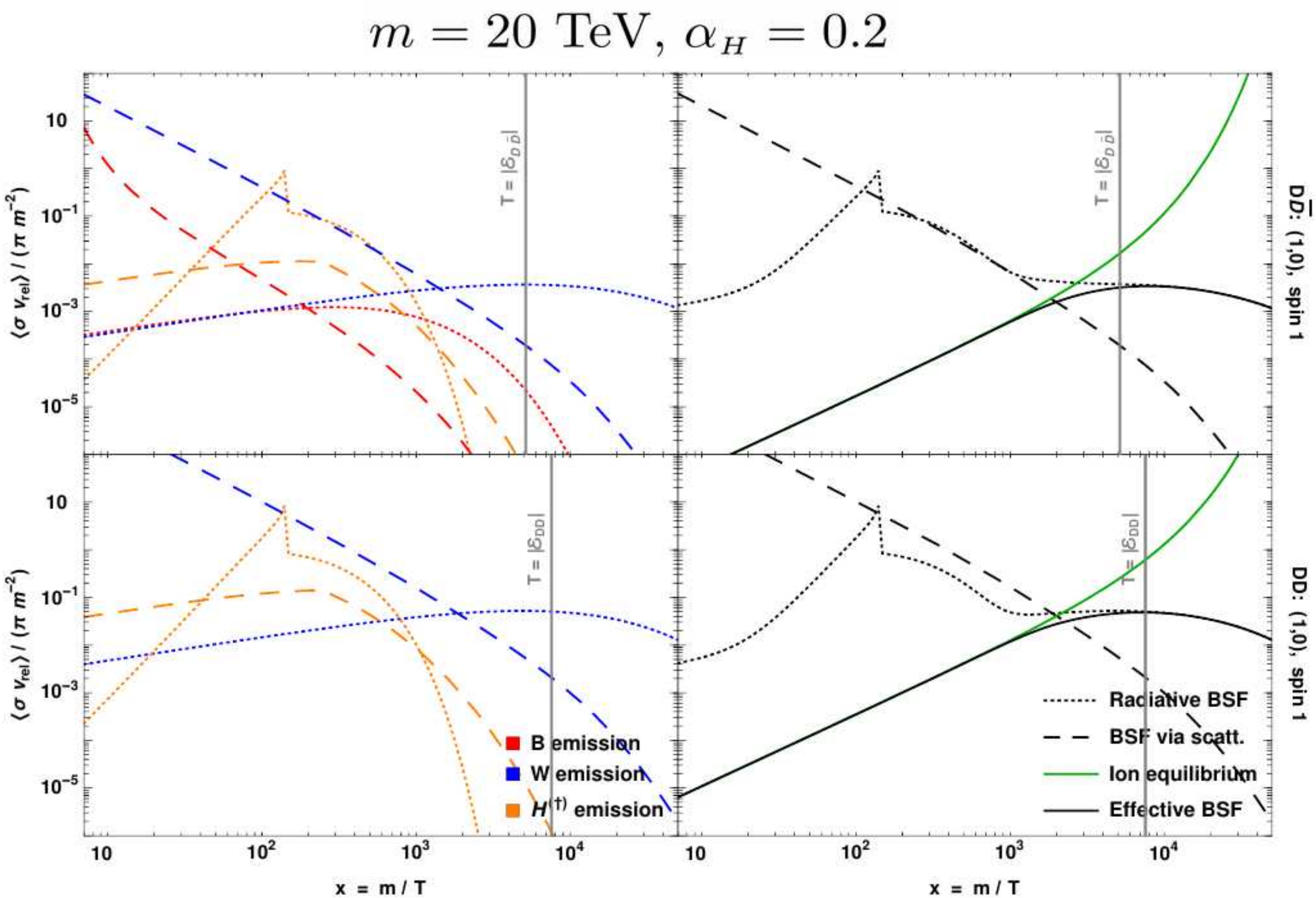
Oncala, KP: 2101.08666/7

Doublet-Singlet coupled to the Higgs

Bound-state species (n=0, ℓ=0)

Bound state ($\mathcal{B}$)	$U_Y(1)$	$SU_L(2)$	Spin	dof ($g_{\mathcal{B}}$)	Bohr momentum ($\kappa_{\mathcal{B}}$)	Decay rate ($\Gamma_{\mathcal{B}}$)
$SS/D\bar{D}$	0	$\mathbb{1}$	0	1	$\frac{m\alpha_A}{2}$	$\frac{m\alpha_A^3(\alpha_1^2+3\alpha_2^2)}{16}\left(\frac{\alpha_A}{\alpha_A+\alpha_R}\right)$
$D\bar{D}$	0	$\mathbb{1}$	1	3	$\frac{m(\alpha_1+3\alpha_2)}{8}$	$\frac{m(\alpha_1+3\alpha_2)^3[(\alpha_1+2\alpha_H)^2+40\alpha_1^2]}{2^{11}\cdot 3}$
DD	1	$\mathbb{1}$	1	3	$\frac{m(-\alpha_1+3\alpha_2)}{8}$	$\frac{m(-\alpha_1+3\alpha_2)^3\alpha_H^2}{2^7\cdot 3}$
DS	1/2	$\mathbb{2}$	0	2	$\frac{m\alpha_H}{2}$	Transition to $\mathcal{B}(SS/D\bar{D})$:

Doublet-Singlet coupled to the Higgs
Bound-state formation cross-sections

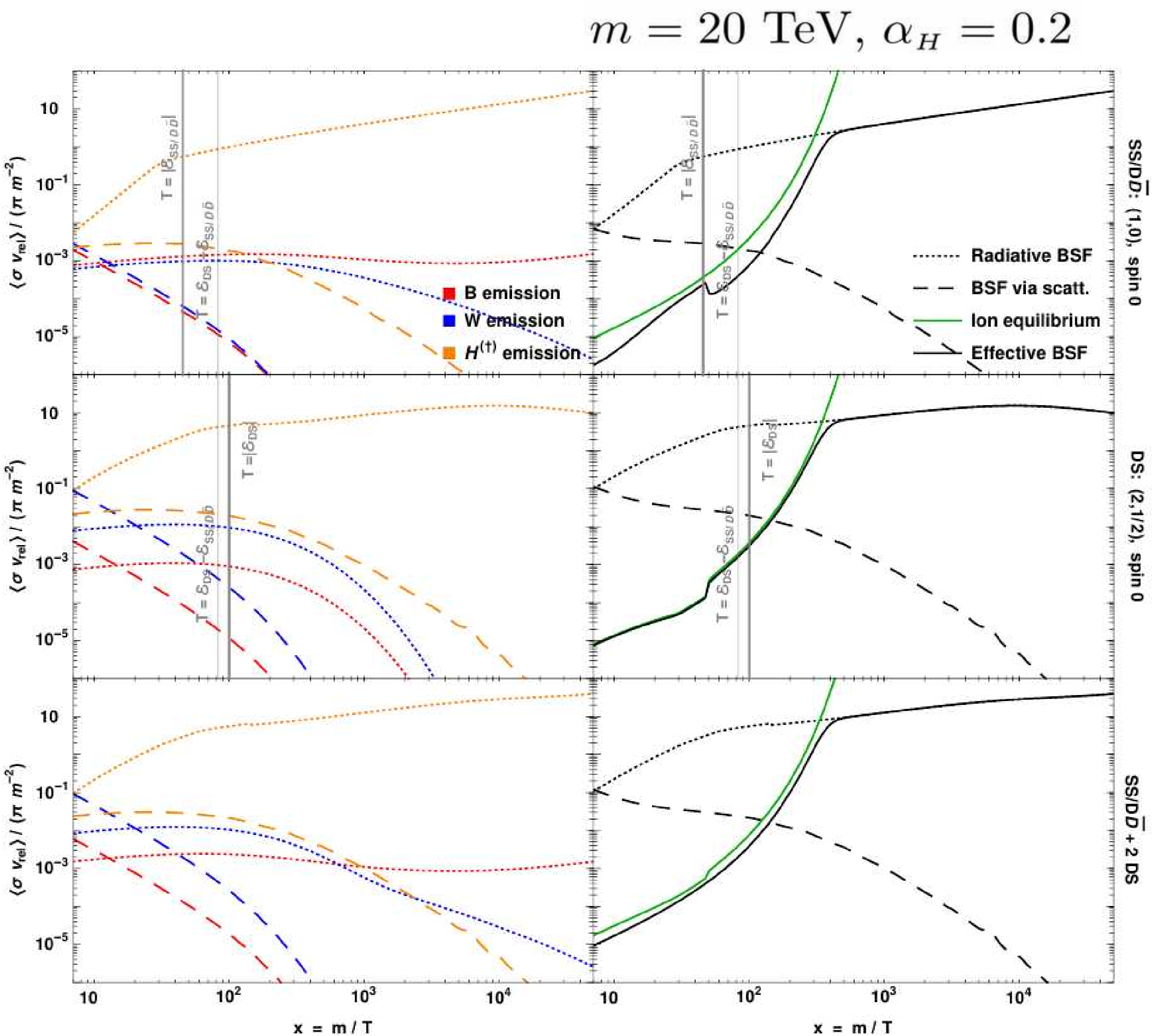


- BSF via scattering on plasma has no effect
- Departure from ionisation equilibrium at T < binding energy

Oncala, KP: 2101.08666/7

Doublet-Singlet coupled to the Higgs

Bound-state formation cross-sections



- **BSF via scattering on plasma has no effect**

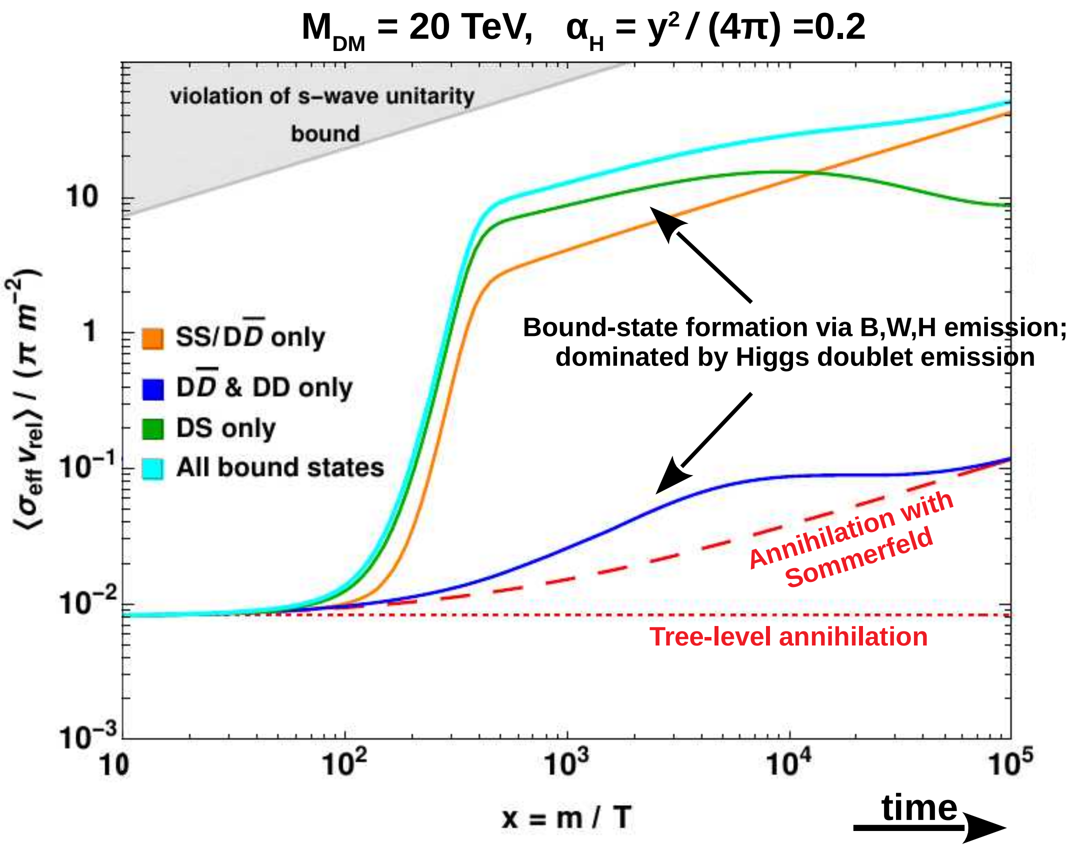
  (scattering kind of important for bound-to-bound transitions)

- **Departure from ionisation equilibrium at T >> binding energy**

  (because of largeness of monopole BSF cross-sections)

Doublet-Singlet coupled to the Higgs

Effective cross-sections



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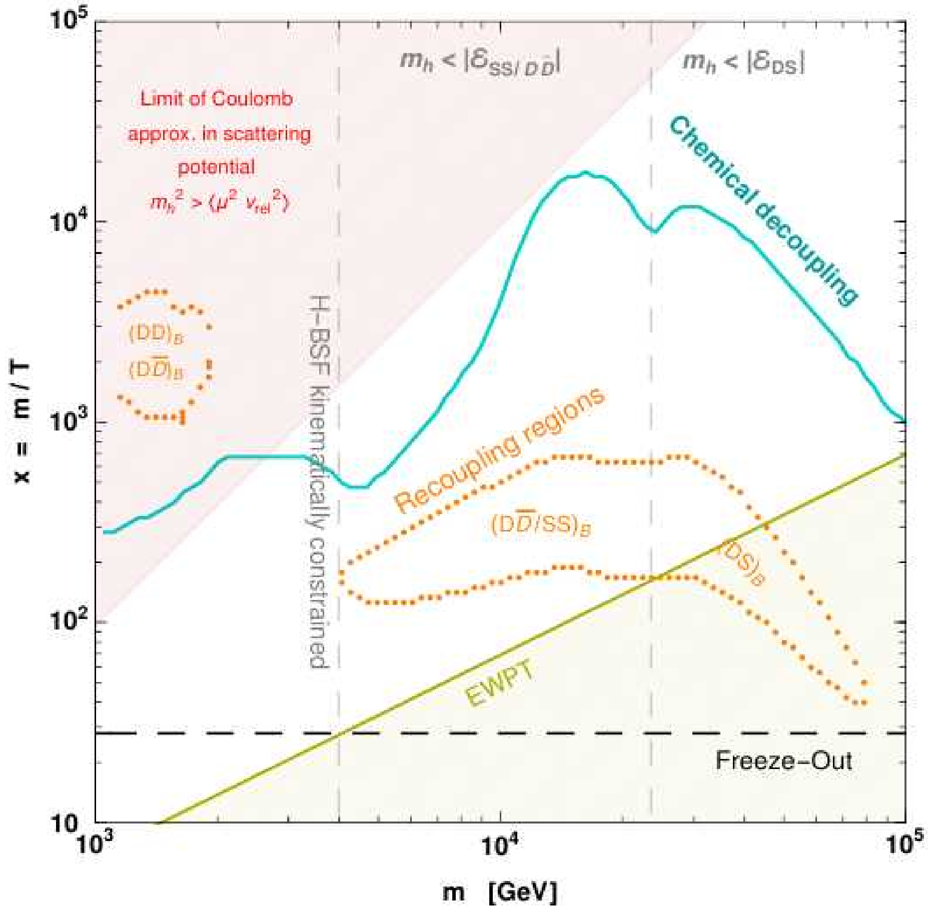
Doublet-Singlet coupled to the Higgs

Timeline

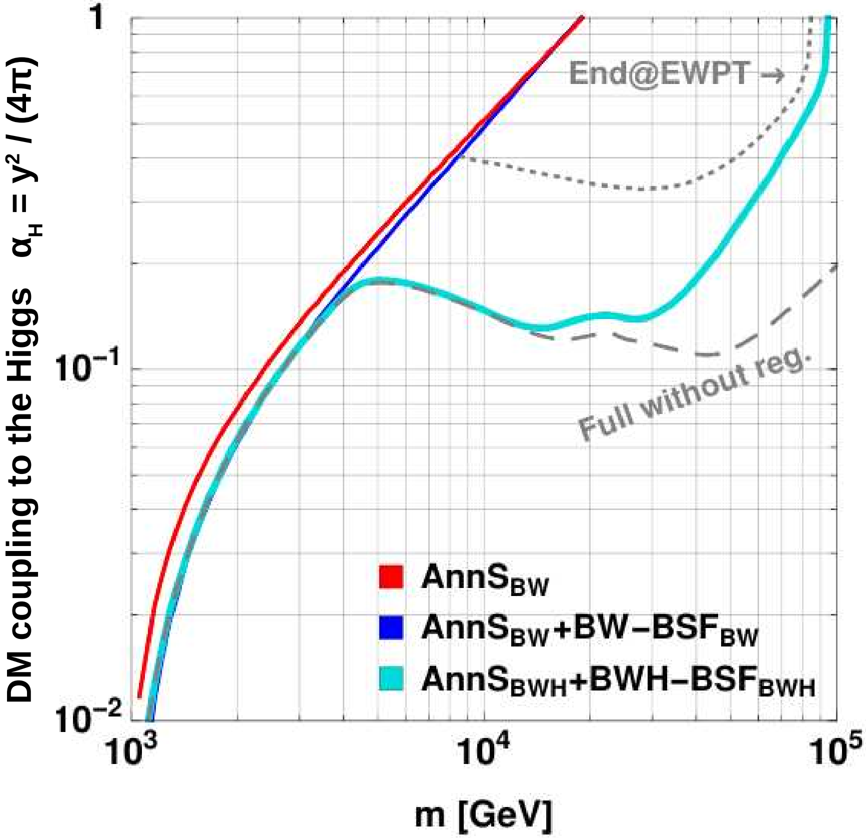


BSF via Higgs doublet emission
~ BSF via h^0, longitudinal $W^\pm$ & Z below EWPT :
Goldstone Boson equivalence theorem
Ko,Matsui,Tang:1910:04311

Oncala, KP: 2101.08666/7

Doublet-Singlet coupled to the Higgs

Relic density



Oncala, KP: 2101.08666/7

Doublet-Singlet coupled to the Higgs

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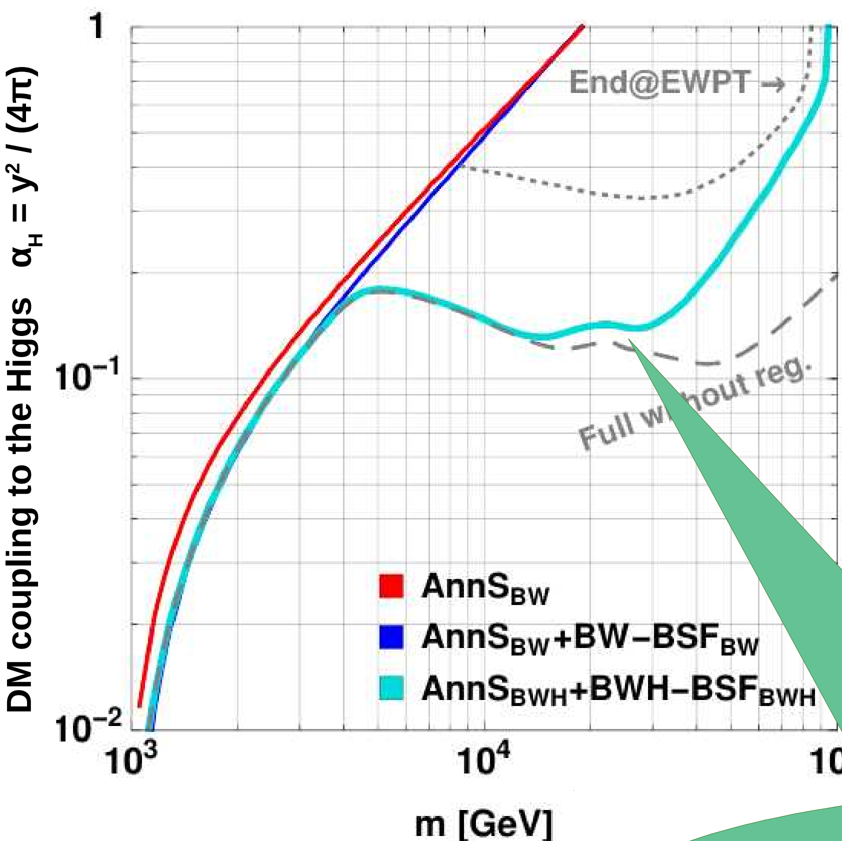

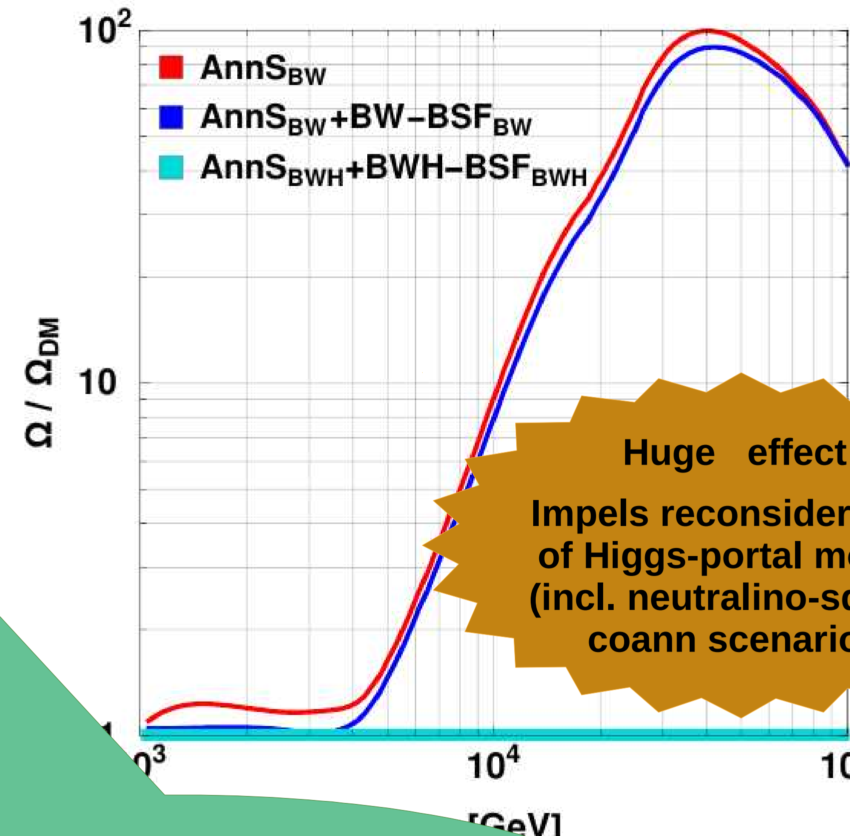

Huge effect!

Impels reconsideration of Higgs-portal models (incl. neutralino-squark coann scenarios)

Effect only at large α_H, due to phase-space suppression in Higgs emission.

For higher multiplets, important effect expected also at lower α_H.

Oncala, KP: 2101.08666/7

Conclusion

As Martin Beneke said:

The electroweak sector is very interesting!