The Research Council of Norway

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Strongly interacting systems at finite T: results from the lattice

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University of Stavanger

For a recent review on in-medium quarkonium:
"Heavy quarkonium in extreme conditions",
A.R., Phys.Rept. 858 (2020)

Norwegian Particle, Astroparticle & Cosmology Theory network

Physics motivation

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Physics challenges at this workshop

(Thermal) bound states

idealized heavy-quarkonium
T. Matsui and H. Satz, Phys.Lett.B 178 (1986)

idealized WIMPonium
Shepherd et. al. Phys.Rev.D 79 (2009) 055022

Kinetic equilibration

heavy-quarkonium in HIC
Stavanger/Osaka: PRD 101 (2020) 3, 034011
Kent State: JHEP 03 (2021) 235
Munich: JHEP 05 (2021) 136
Saclay/Jyvaskyla: PRD 98 (2018) 7, 074007

dark matter decoupling
Binder et. al. PRD 96 (2017) 11, 115010

Chemical equilibration

heavy quarks at the FCC
Bodecker, Laine JHEP 07 (2012) 130

dark-matter (co)annihilation
Binder, Covi, Mukaida
PRD 98 (2018) 11, 115023
Biondini, Kim, Laine JCAP 10 (2019) 078

select recent Lattice insights on T>0 quarkonium

T>0 static potential

Euclidean quantum
Y. Burnier, O. Kaczmarek, A.R.
JHEP 12 (2015) 101
Y. Burnier, A.R. PRD 95 (2017) 5, 054511
P. Petreczky, A.R., J. Weber NPA 982 (2019)

classical statistical
A. Lehmann, A.R. arXiv:2012.10089
K. Boguslavski, B. Kasmai, M. Strickland
arXiv:2102.12587

T>0 quarkonium spectra

relativistic formulation
Y. Burnier et. al. JHEP 11 (2017) 206

using lattice EFT (NRQCD)
S. Kim, P. Petreczky, A.R. JHEP 11 (2018) 088
FASTSUM PoS LATTICE2019 (2019) 076
R. Larsen et. al. Phys.Lett.B 800 (2020) 135119

Transport coefficients

heavy quark diffusion
N. Brambilla et.al. PRD 100 (2019) 5, 054025
N. Brambilla et.al. PRD 102 (2020) 7, 074503
L. Altenkort et.al. PRD 103 (2021) 1, 014511

Sommerfeld enhancement
Kim Laine JHEP 07 (2016) 143,
Biondini, Kim, Laine JCAP 10 (2019) 078

The two faces of lattice QCD

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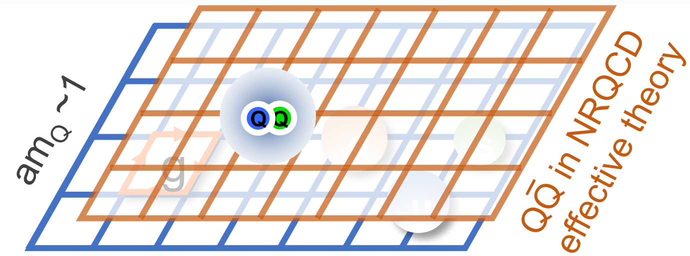
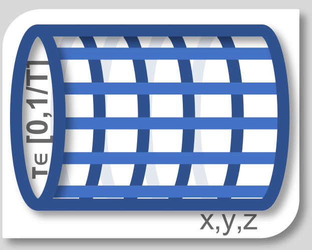


Lattice discretization

- Gauge fields as links: $U_\mu(x)=\exp[\, i\, g\, a_\mu A_\mu(x)\,]$
- Discretized N_f flavors of light fermions on the nodes
- Heavy quarks as relativistic fermion fields OR non-relativistic EFT fields (“without loops”)

Euclidean quantum



Formulated in compact **imaginary time** for MC

Gattringer, Lang, QCD on a lattice 10.1007/978-3-642-01850-3

$$\langle O(\tau)\rangle = \int \mathcal{D}U O(U) e^{-S_E^{QCD}[U]}$$

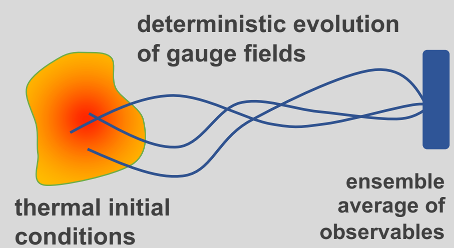
ab-initio sim. of a quantum path integral

$$P[U] \propto e^{-S_E[U,\psi,\bar\psi]}$$

$$\langle O\rangle = \frac{1}{N} \lim_{N\to\infty} \sum_{k=1}^{N} O(U^k)$$

return to real-time very costly

Real-time classical statistical



Formulated in **Minkowski time** directly

V. Kasper et. al. PRD 90, 025016 (2014)

$$\langle O(t)\rangle = \int dE_0 dU_0\, P[U_0,E_0] O(U(t),E(t))$$

valid at highly occupancy: glasma
or deep in the IR: sphaleron transitions

$$P[U_i,E_i]|_{t=0} \sim e^{-H/T}$$

$$\partial_\mu F^a_{\mu\nu}[U,E] = j^a_\nu[\psi]$$

continuum limit intricate & no confinement

Spectral functions on Euclidean lattices

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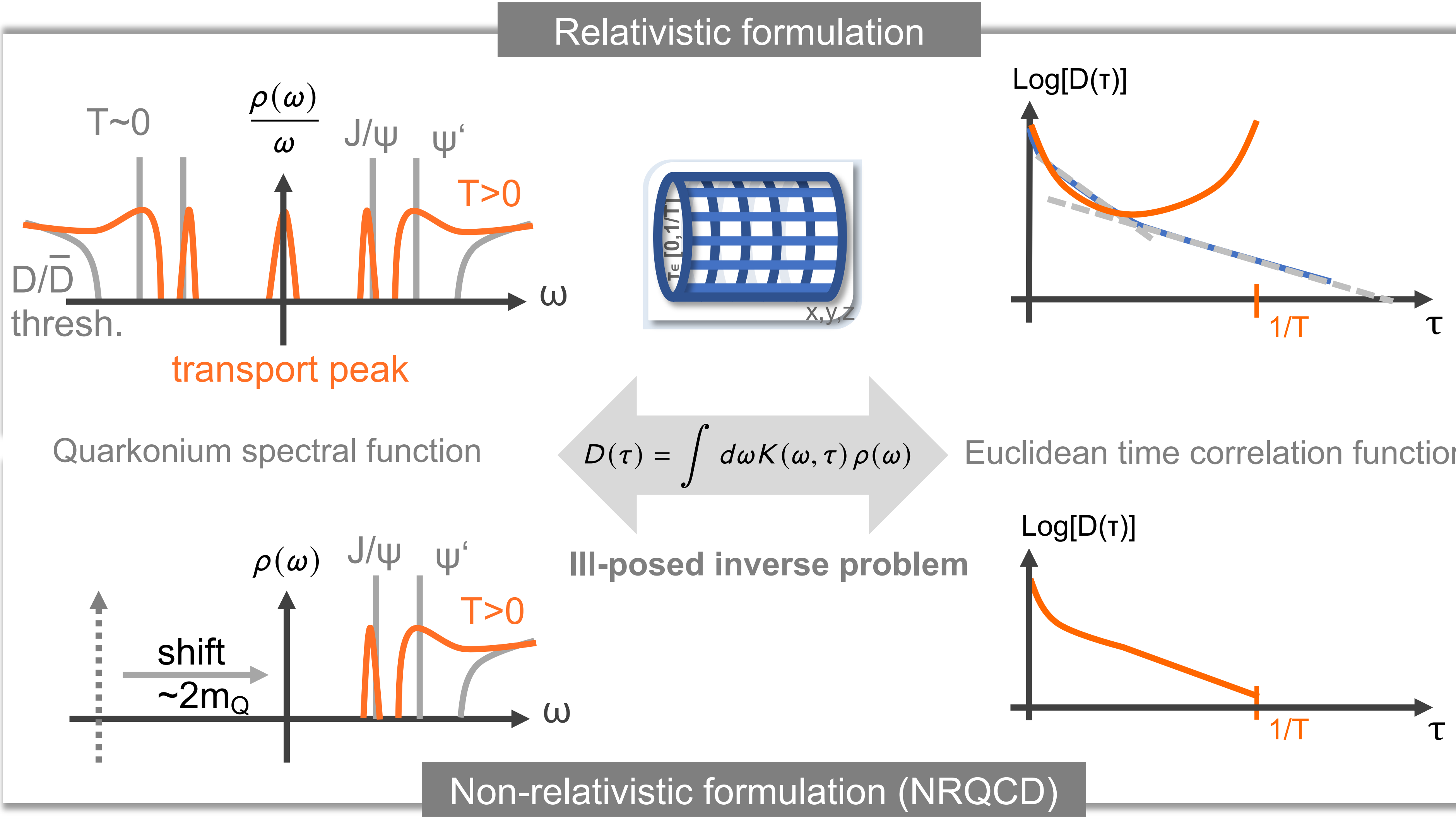
- **Euclidean lattice QCD** simulations are similar to a (very) **imperfect detector**



Towards spectral functions

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Bayesian Spectral reconstruction

Supply prior information to regularize the inverse problem

Maximum Entropy Method
(positivity +
do not introduce correlations where there are none in the data)
M. Asakawa, T. Hatsuda, Y. Nakahara Prog.Part.Nucl.Phys. 46 (2001) 459

BR method
(positivity + 2 x differentiability + result independent of units used)
Y. Burnier, A.R., PRL 111 (2013) 182003

Smooth BR method
(BR method + smoothness via penalty on arc length)
C. Fischer et.al., PRD 98, 014009 (2018)

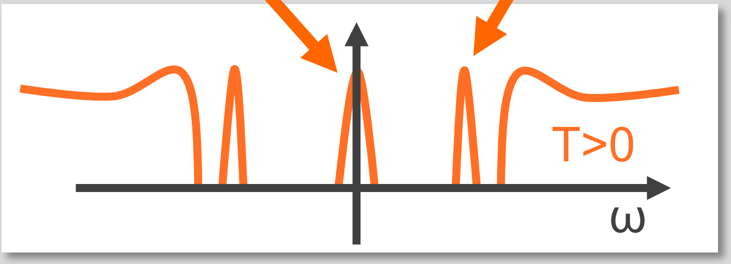
careful analysis of **regularization artifacts** necessary, often combined with mock-data analyses

Model fits

Realistic spectral forms **with few parameters from continuum comp.**

transport peak form (IR effective theory)
S. C.-Huot et.al. JHEP 04 (2009) 053
P. Petreczky et.al. PRD73 (2006) 014508

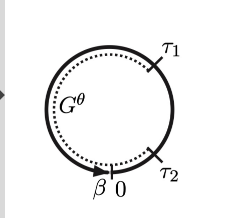
in-medium peak shape (pNRQCD)
Y. Burnier et.al. JHEP 01 (2008) 043



Reformualtion strategies

relate transport peak physics to a Euclidean correlators

$$\Omega_{\rm chem} \equiv \lim_{\omega \ll T} 2T\omega\rho_\Delta(\omega)$$



S. Kim, M. Laine, JHEP 1607 (2016) 143
A. Eller et.al. PRD 99, 094042 (2019)

Part I

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select recent Lattice insights

T>0 static potential

Euclidean quantum

Y. Burnier, O. Kaczmarek, A.R. JHEP 12 (2015) 101
Y. Burnier, A.R. PRD 95 (2017) 5, 054511
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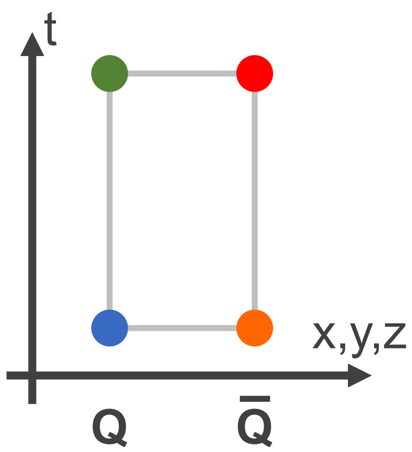
Sommerfeld enhancement

Kim Laine JHEP 07 (2016) 143,
Biondini, Kim, Laine JCAP 10 (2019) 078

Static quark potential at T>0

- Simplest model system: **infinitely heavy** color sources

$$\langle (\bar{Q}Q)(\bar{Q}Q)^\dagger \rangle \overset{m_Q\to\infty}{=} W_\square(r,t) = exp\left[ig\int_\square dz^\mu A_\mu\right]$$



Central question: can its real-time evolution be understood via a **Schrödinger equation** i.e. by a **potential**?

$$i\partial_t W_\square(r,t) \overset{?}{=} V(r) W_\square(r,t)$$

In general, we know that this is not true (pNRQCD non-potential effects). But with time coarse graining (late times) we may have a chance.

For weak coupling results see
M. Laine et.al. JHEP 03 (2007) 054,
N. Brambilla et.al. PRD 78 (2008) 014017

- If V(r) exists, how to extract it from Euclidean lattice QCD: **spectral functions**

A.R., T. Hatsuda, S. Sasaki PRL 108 (2012) 162001, Y. Burnier, A.R. PRD 86 (2012) 051503

$$V(r) = \lim_{t\to\infty} \frac{i\partial_t W_\square(r,t)}{W_\square(r,t)} = \lim_{t\to\infty} \frac{\int d\omega\omega\rho(r,\omega)e^{-i\omega t}}{\int d\omega\rho(r,\omega)e^{-i\omega t}}$$

Lowest lying spectral peak dominates late real-time behavior Re[V]=ω_0 Im[V]=Γ_0

$$W_\square(r,t) = \int d\omega\rho(r,\omega)e^{-i\omega t}$$

$$W_\square(r,\tau) = \int d\omega\rho(r,\omega)e^{-\omega\tau}$$

Static quark potential (Euclidean)

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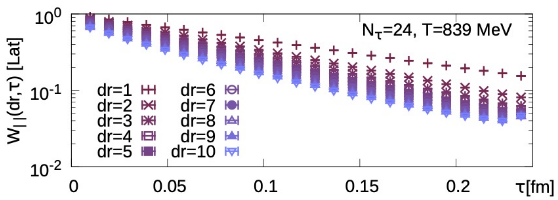
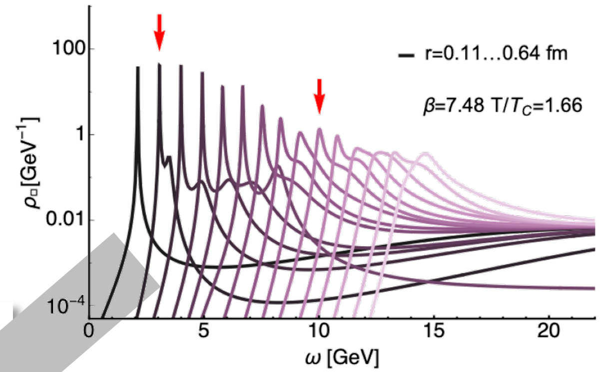
- **Bayesian reconstruction** of Wilson correlator spectral functions from Euclidean lattices with **heavier than physical** medium d.o.f. (N_f=2+1)

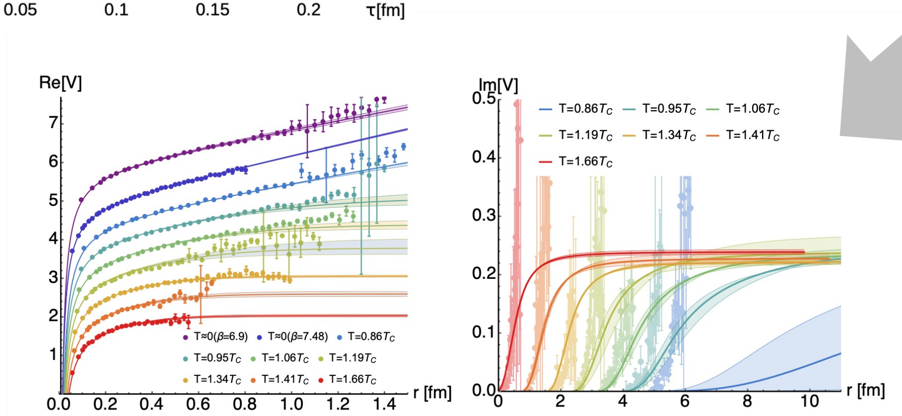
Y. Burnier, O. Kaczmarek, A.R. JHEP 12 (2015) 101

Wilson line correlator
in Coulomb gauge to avoid cusp divergencies



Identification of a potential peak
in spectra
150MeV
< T<
300MeV





Stay tuned for updates: analysis with realistic medium lattices ongoing, but results so far ambiguous

P. Petreczky et.al. Nucl.Phys.A 982 (2019) 735,
TUMQCD Nucl.Phys.A 967 (2017) 592

- Extracted values of Re[V] show gradual screening, Im[V]>0 in QGP phase
- Re[V] and Im[V] relevant input to the Osaka/Stavanger OQS approach

see T. Miura et.al. PRD 101 (2020) 3, 034011

Static quark potential (class. stat.)

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- **Fourier transform** of real-time Wilson loop from classical statistical simulations with a purely **gluonic medium**

M. Laine, M. Tassler JHEP 09 (2007) 066
A. Lehmann, A.R. arXiv:2012.10089
K. Boguslavski, B. Kasmei, M. Strickland arXiv:2102.12587

In axial gauge A_0=0 quarks only **modify Gauss' law &** affect sampling of physical **init. cond.**

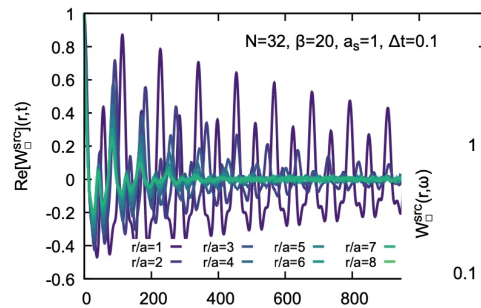
A. Lehmann, A.R. arXiv:2012.10089

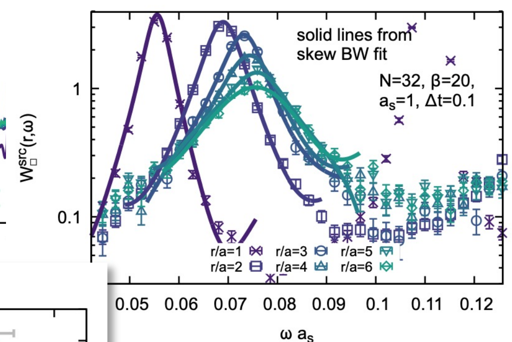
$$\partial_\mu F^a_{\mu\nu}[U,E] = j^a_\nu[\psi]$$

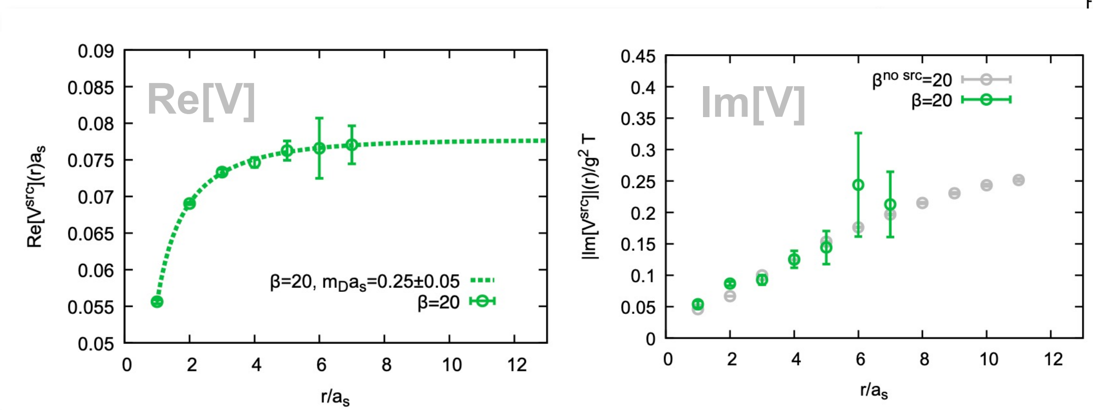
$$P[U_i,E_i]|_{t=0} \sim e^{-H/T}$$

$$D_i E_i = M(\delta^{(3)}(x-x_0) - \delta^{(3)}(x-x_1))$$

Wilson loop w/ **damped osc. behavior**: spectral peaks at ω_0>0 and width Γ_0>0







- Re[V] shows screening, Im[V]>0 present (as known from prior studies)

Part II

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select recent Lattice insights

static quark potential

Euclidean quantum

Y. Burnier, O. Kaczmarek, A.R. JHEP 12 (2015) 101
Y. Burnier, A.R. PRD 95 (2017) 5, 054511
P. Petreczky, A.R., J. Weber NPA 982 (2019)

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Y. Burnier et. al. JHEP 11 (2017) 206

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FASTSUM PoS LATTICE2019 (2019) 076
R. Larsen et. al. Phys.Lett.B 800 (2020) 135119

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heavy quark transport

N. Brambilla et.al. PRD 100 (2019) 5, 054025
N. Brambilla et.al. PRD 102 (2020) 7, 074503
L. Altenkort et.al. PRD 103 (2021) 1, 014511

Sommerfeld enhancement

Kim Laine JHEP 07 (2016) 143,
Biondini, Kim, Laine JCAP 10 (2019) 078

In-medium bound states (relativistic)

University of Stavanger

- **Modelling** of (η_c,η_b) spectral function via fit to **continuum extrapolated** Euclidean correlators on **high resolution** lattices of a **gluonic medium**

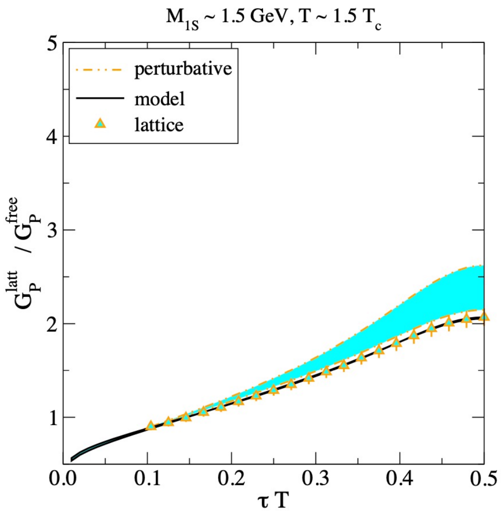
Y. Burnier et.al. JHEP 11 (2017) 206

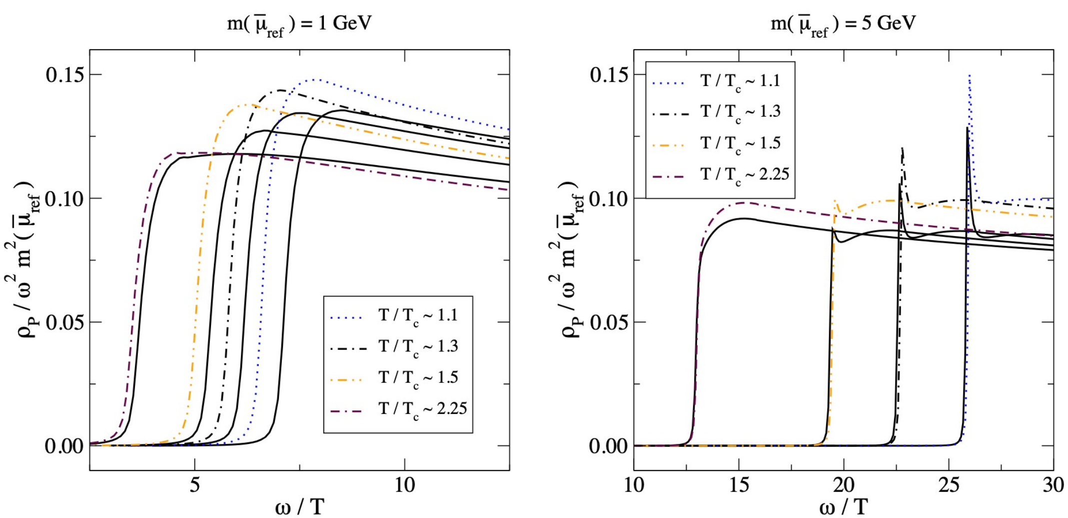
UV part of spectral function
high order perturbative computation
careful choice of renorm. scheme

Threshold part of spectral function
weak-coupling potential
(vacuum at small r, thermal at interm. r

No transport peak in this channel

$$\rho^{model}(\omega) = A\rho^{pert}(\omega - B)$$





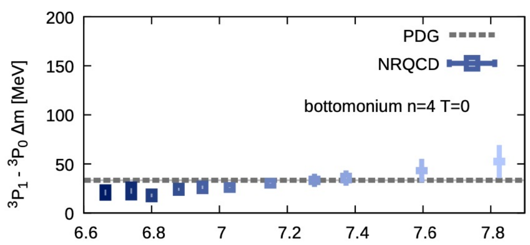
- Excellent agreement with lattice data & relatively rapid melting

In-medium bound states (non-rel.)

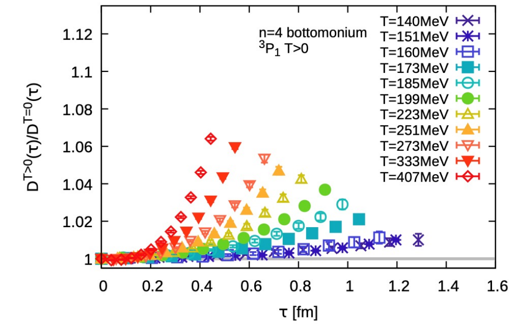
University of Stavanger

- **Bayesian reconstruction** of spectral functions from Euclidean **NRQCD correlators** on lattices with **realistic medium** d.o.f. (N_f=2+1) S.Kim, P. Petreczky, A.R. JHEP 11 (2018) 088

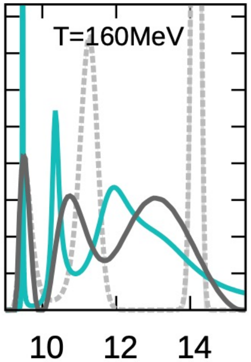
T=0 mass splitting:
establish that NRQCD approximation works well at these lattice spacings

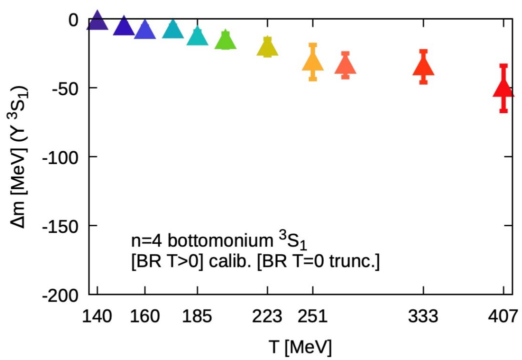


Test via spectral function models
ratio of T=0 and T>0 correlator: ground state peak moves to smaller ω



Careful study of reconstruction artifacts: Bayesian T>0 and T=0 results can only be compared at same Euclidean time extent.





- Disappearance of peaks consistent with FASTSUM & negative mass shift, no access to T>0 excited states from first principles c.f. FASTSUM JHEP 07 (2014) 097

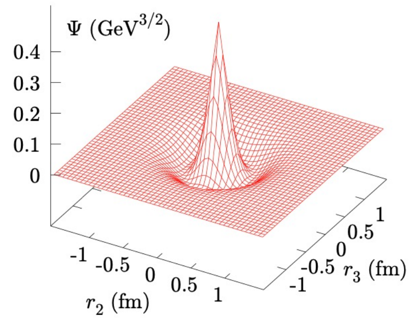
In-medium bound states (non-rel.) II

University of Stavanger

- **Modelling** of spectral function via fit to **NRQCD** Euclidean correlators w/ **pNRQCD model input** on lattices with **realistic medium** d.o.f. (N_f=2+1)

R. Larsen et.al. PLB 800 (2020) 135119

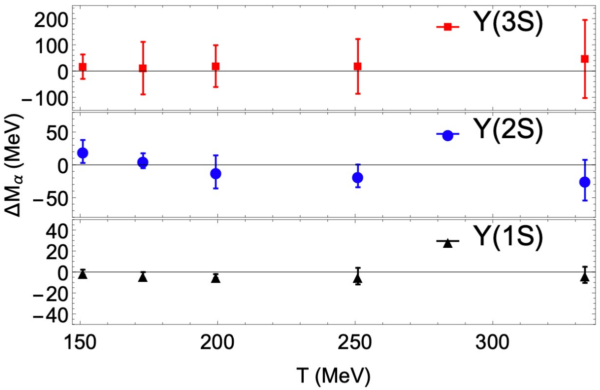
Potential model wavefunctions
attempt to project out ground or excited state contribution in T>0 Euclidean correlators



Subtract unknown UV contrib
Assuming high ω spectral function is T independent, subtract on level of correlators

Gaussian model spectral function
with extra delta peak to capture Lorentzian tail.

$$\rho_\alpha^{\rm med}(\omega,T) = A_\alpha^{\rm cut}(T)\,\delta\left(\omega-\omega_\alpha^{\rm cut}(T)\right) + A_\alpha(T)\exp\left(-\frac{[\omega-M_\alpha(T)]^2}{2\Gamma_\alpha^2(T)}\right)$$



- No change in the in-medium masses within estimated error budget. How far does T=0 model data influence the result?

Part III

University of Stavanger

select recent Lattice insights

static quark potential

Euclidean quantum

Y. Burnier, O. Kaczmarek, A.R.
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Sommerfeld enhancement

Kim Laine JHEP 07 (2016) 143,
Biondini, Kim, Laine JCAP 10 (2019) 078

Heavy quark diffusion coefficient

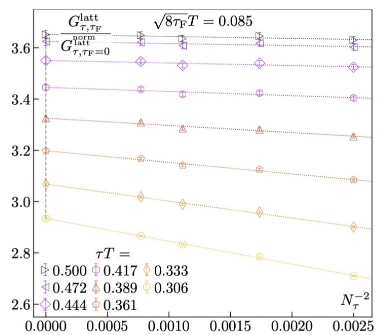
University of Stavanger

- **Modelling** of spectral function via fit to **continuum extrapolated effective** Euclidean correlators on **gluonic medium** lattices L. Altenkort et.al. PRD 103 (2021) 1, 014511

Transport peak in standard $Q\bar{Q}$ spectra **too sharp and small:** use **effective operator** with same transport physics
S. Caron-Huot, M. Laine, G. Moore JHEP 04 (2009) 053

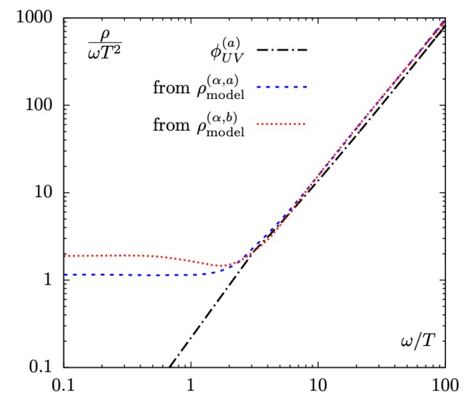
$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\frac{\Big\langle \mathrm{Re\,Tr}\Big[U(\beta,\tau)\,gE_i(\tau,\mathbf{0})\,U(\tau,0)\,gE_i(0,\mathbf{0})\Big]\Big\rangle}{\Big\langle \mathrm{Re\,Tr}\,[U(\beta,0)]\Big\rangle}$$

Continuum extrapolation using high precision data from **Gradient Flow** or **Multilevel algorithm**



UV spectra from perturbation theory
IR part from diffusion process, weighted for fit to Euclidean correlator

$$\rho^{(UV)}(\omega) = \frac{g^2 C_F \omega^3}{6\pi} \qquad \rho^{(IR)}(\omega) = \frac{\kappa\omega}{2T}$$



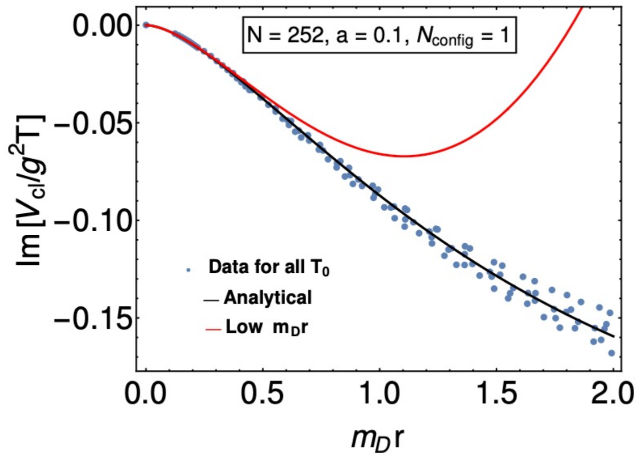
- Using different model ansätze and fitting ranges: $\kappa/T^3 = 2.31\ldots3.70$
- Recent proposal that κ can be directly estimated from Euclidean correlator
A. Eller et.al. PRD 99, 094042 (2019)

Heavy quark transport II

- Exploit relation of transport coefficients to other quantities that are accessible on the lattice

Small distance behavior of Im[V] can be related to κ in HTL perturbation theory. (here classical statistical sims.)

K. Boguslavski, B. Kasmei, M. Strickland arXiv:2102.12587

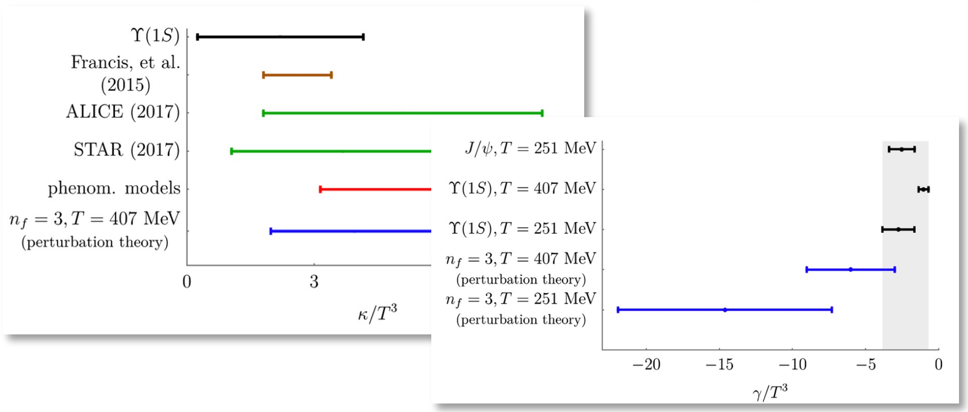


$$\kappa_V^{\rm latt} \approx C_F\, g^2 T m_{D,\rm HCL}^2 \left(0.173 \log\left(\frac{T}{m_{D,\rm HCL}}\right) - 0.023\right)$$

Masss shifts and decay widths can be related to κ and γ in EFT for Coulombic quarkonium states

N. Brambilla et.al. PRD 100 (2019) 5, 054025

$$\Gamma(1S) = 3a_0^2\kappa, \quad \delta M(1S) = \frac{3}{2}a_0^2\gamma$$



- New constraints to well known & recently identified transport coefficients

Conclusion

- Lattice QCD provides non-perturbative and first principles insight into (heavy) fermions under SU(N) interactions (quantum / classical statistical)
- Phenomenologically relevant quarkonium physics (in-medium properties, transport) requires access to real-time dynamics: spectral functions
- Progress via a combination of high precision Euclidean data, spectral functions modelling and direct spectral reconstruction

Thank you for your attention

ご清聴ありがとうございました