The Ubiquitous Axion in Quantum Gravity

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1. **No Global Symmetries in Quantum Gravity**
2. Moduli and Axions in String Theory
3. Axion Potentials from Magnetic Monopoles

No Global Symmetries

(Wheeler; Hawking; Zeldovich; Banks, Dixon; Banks, Seiberg; Harlow, Ooguri; rapidly growing list of others….)

One big idea behind multiple things that I will discuss in this talk is that consistent theories of quantum gravity have **no global symmetries.** At the UV cutoff scale, not even ***approximate*** global symmetries.

Surprisingly wide range of applications! **e.g.:** (time for only a subset today)

- Existence of particles in all representations of gauge group
- Weak Gravity Conjecture
- Chern-Simons terms and axions
- Existence of “twist” strings (Z_N strings, Alice strings, …)

Example: Symmetries in Free U(1) Gauge Theory

In free Maxwell theory, we have no electric or magnetic sources, so

$$\mathrm{d}(\star F) = 0$$

Closed (d–2)-form current
$\Longrightarrow$ **Global 1-form symmetry**

$$\mathrm{d}F = 0$$

Closed 2-form current
$\Longrightarrow$ **Global (d-3)—form symmetry**

The quantization of fluxes means that these are both U(1) symmetries.
In 4d, they are both *1-form* global symmetries.

- **Electric symmetry, current $\star$ *F*, charged objects are *Wilson loops*.**
- **Magnetic symmetry, current *F*, charged objects are *'t Hooft loops*.**

The symmetries basically *count* Wilson or 't Hooft loops.

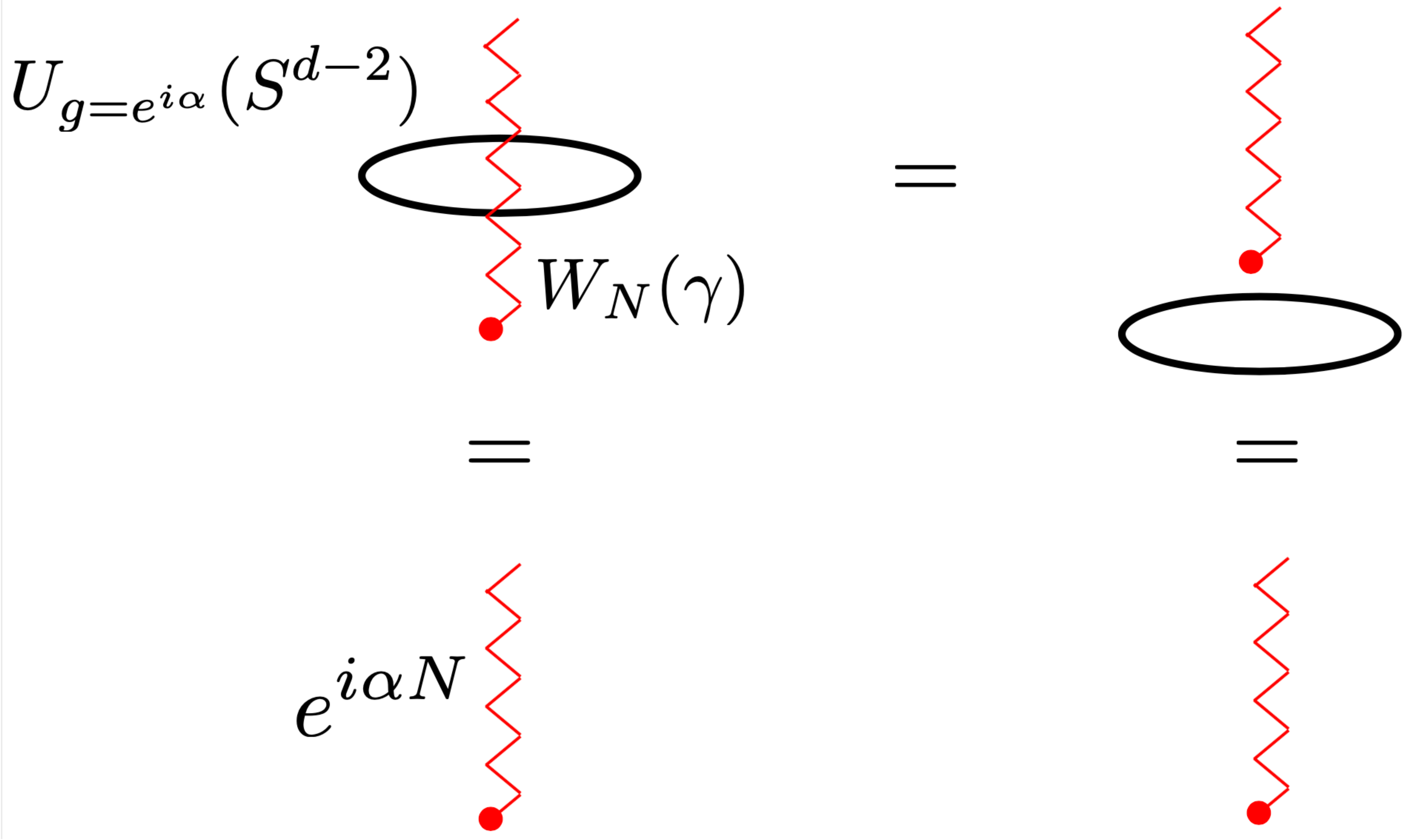
Complete spectrum of charged particles $\Longleftrightarrow$ absence of global symmetries

$$\mathrm{d}(\star F) = J$$

Charged particles break the *1-form symmetry*'s conservation law
(while gauging a *0-form symmetry* with current *J*)

Wilson operators can end on local operators that create charged particles.

No longer a topologically invariant flux.



Wilson lines can *end* $\Longleftrightarrow$ 1-form electric symmetry is *explicitly broken*.

Generalization to **all representations** of **any compact gauge group**: Rudelius, Shao '20; Heidenreich, McNamara, Montero, MR, Rudelius, Valenzuela '21

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Moduli and Axions for Gauge Couplings

In string theory, the gauge kinetic function is often a *dynamical field*:

$$\frac{1}{16\pi \mathrm{i}} \int d^2\theta\, \tau(x) \mathscr{W}^\alpha(x) \mathscr{W}_\alpha(x)$$

$$\tau(x) = \frac{1}{2\pi}\theta(x) + 4\pi \mathrm{i} S(x), \quad \langle S \rangle = \frac{1}{g^2}$$

axion

"saxion" or scalar modulus

Note: I am *not* assuming TeV-scale SUSY! Just compactification-scale SUSY.

Aspects of Moduli Fields

The limit where $g \to 0$, i.e., $S \to \infty$, lies at infinite distance.
No global symmetries: cannot send gauge couplings to zero.

(cf. Ooguri/Vafa "Swampland Distance Conjecture"; Arkani-Hamed/Motl/Nicolis/Vafa "Weak Gravity Conjecture")

Have in mind Lagrangians like:

$$\mathscr{L} \supset M_*^2 \partial_\mu (\log S) \partial^\mu (\log S) + \frac{M_*^2}{S^2} \partial_\mu \theta \partial^\mu \theta$$

(can be more complicated in multi-field cases).

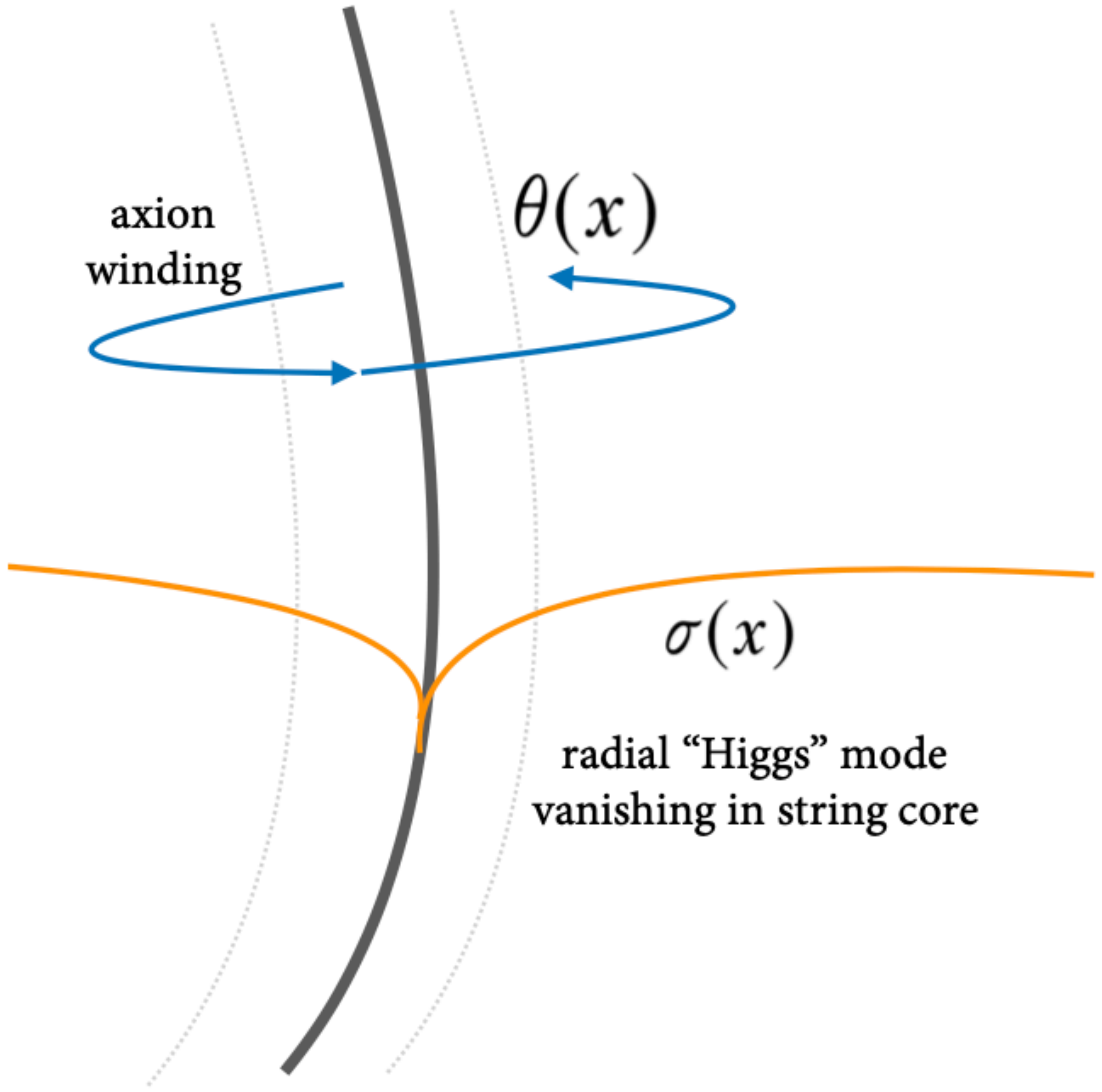
Aspects of Axion Fields

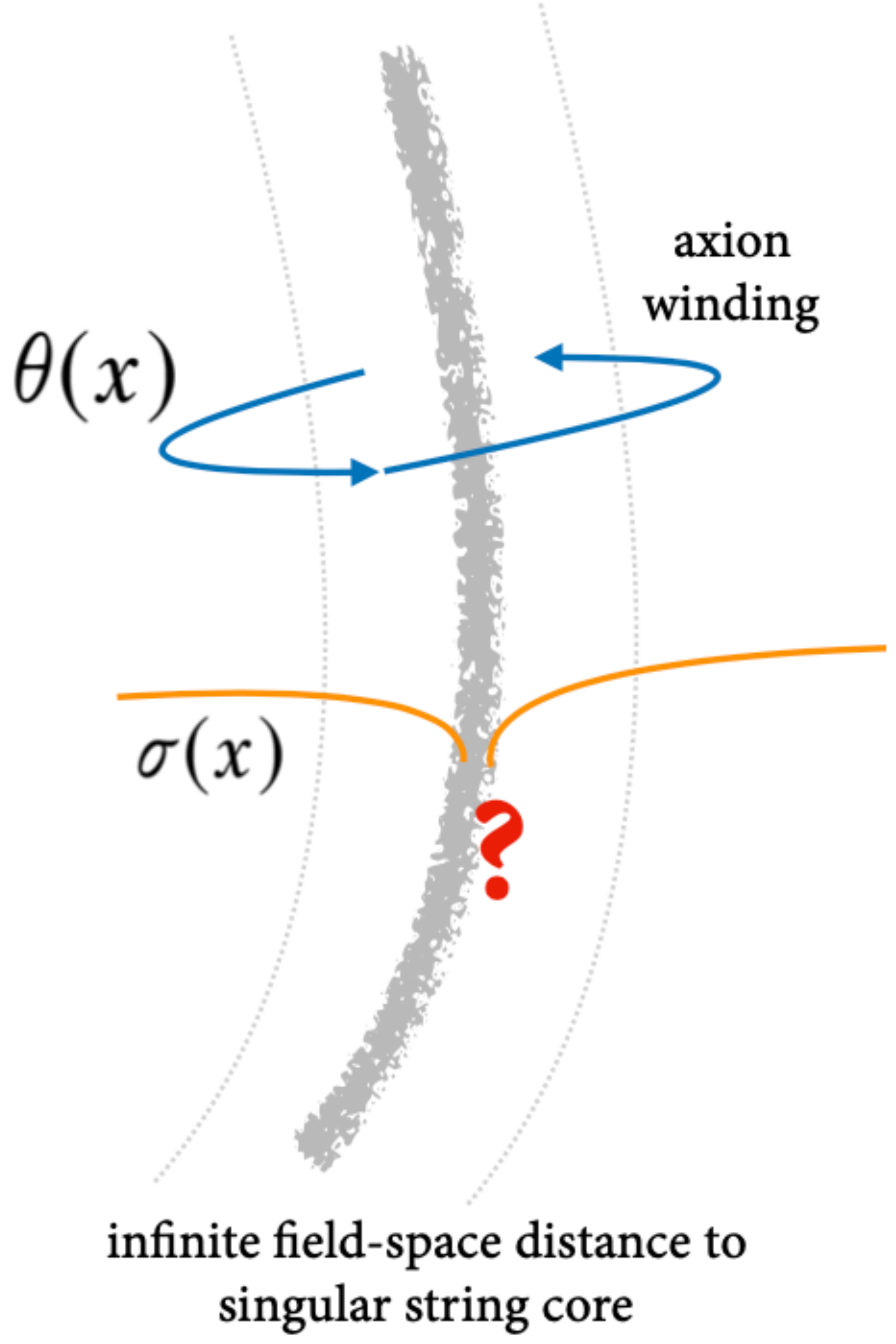
decay constant f^2

$$\mathscr{L} \supset M_*^2 \partial_\mu (\log S) \partial^\mu (\log S) + \frac{M_*^2}{S^2} \partial_\mu \theta \partial^\mu \theta$$

Axion decay constant is S-dependent, and never zero at finite distance.

“Fundamental axion”: PQ symmetry is never restored.





Axion strings are fundamental objects (e.g., F-string, wrapped D-brane).

Expectations for Scales

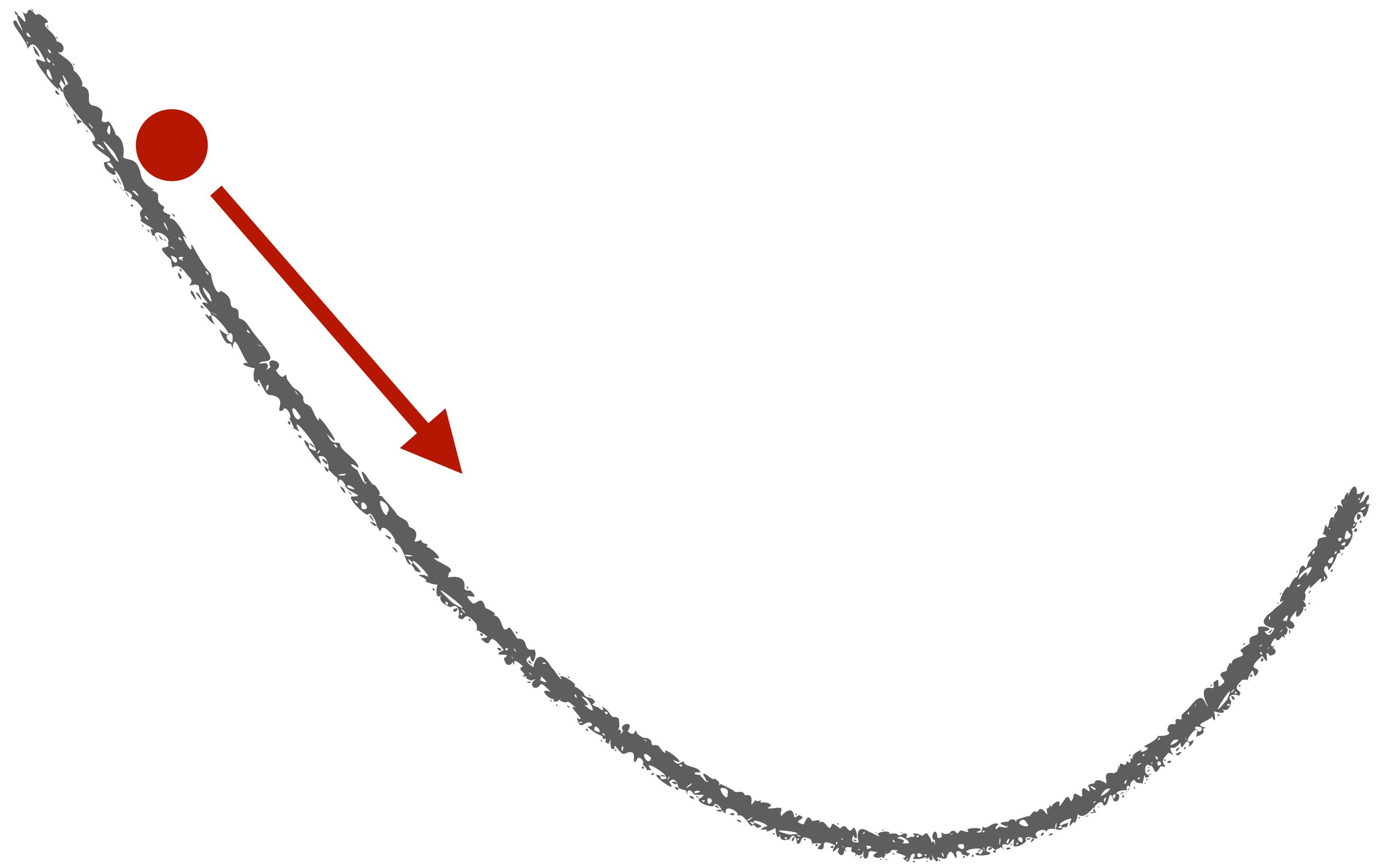
Moduli often have **Planck-suppressed** interaction strengths. Always true of the overall volume modulus.

Moduli masses generically set by **SUSY breaking.**

Axion decay constant often at **Kaluza-Klein scale** (for overall volume modulus) or **string scale** (for more generic moduli).

Axion masses are potentially **exponentially small** (when corresponding saxion has Kähler stabilization).

Moduli/Axion Cosmology



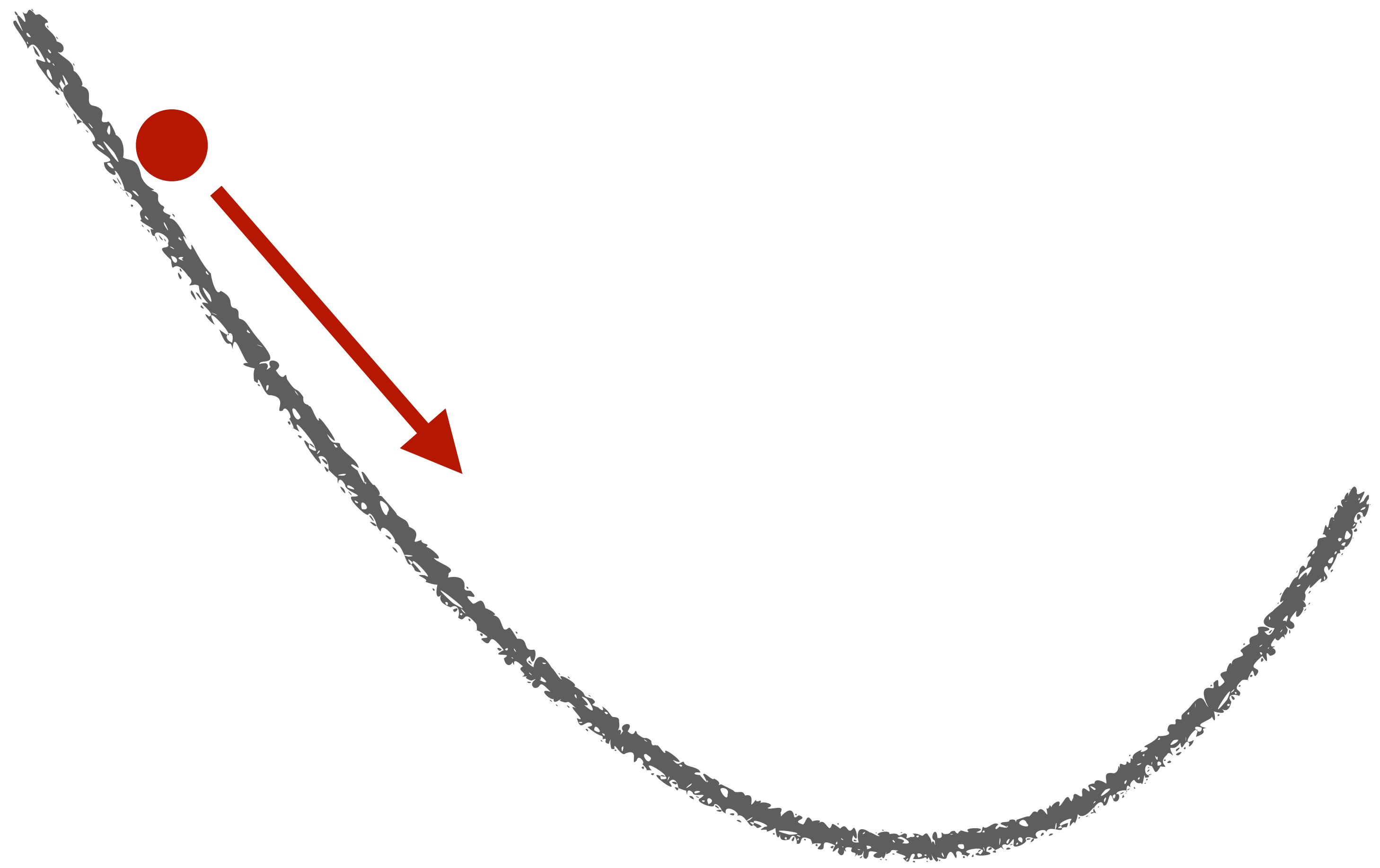
Misalignment mechanism: in the early universe, displaced from minimum.

Coherent oscillation about minimum: **matter-dominated phase**.

Moduli can dominate the universe for a long time, due to their very weak interactions.

$$\Gamma_S \sim \frac{m_S^3}{8\pi M_{\rm Pl}^2} \quad \Rightarrow \quad T_{\rm rh} \sim T_{\rm BBN} \quad \text{if} \quad m_S \sim 30 \text{ TeV}$$

Moduli Alter Cosmology



If moduli masses are **below ~10^7 GeV,** their decays reheat the universe **below** the electroweak phase transition.

Don't expect **thermal relic WIMP DM.**

Axion DM can begin oscillating *during* the moduli-dominated epoch, then get diluted. Higher decay constants possible!

Lamppost or Principle?

(Heidenreich, McNamara, Montero, MR, Rudelius, Valenzuela ’20)

Moduli and axions are ubiquitous in string theory compactifications. But is this an accident, or are they there for a reason?

$$\frac{1}{2} f^2 (\partial\theta)^2 + \frac{\theta}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} \quad \Rightarrow \quad \partial^\mu (f^2 \partial_\mu \theta) = \frac{1}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

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instanton number density

$$\frac{1}{2} f^2 (\partial\theta)^2 + \frac{\theta}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} \quad \Rightarrow \quad \partial^\mu (f^2 \partial_\mu \theta) = \frac{1}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

The axion causes a would-be conserved quantity (instanton number) to vanish: integral of a total derivative.

Chern-Weil symmetry

In an abelian gauge theory, if $\mathrm{d}F = 0$ (no magnetic monopoles), then

$$\mathrm{d}(F \wedge F) = \mathrm{d}F \wedge F + F \wedge \mathrm{d}F = 0,$$

so $F \wedge F$ is a conserved 4-form current, and generates a $(d-5)$-form symmetry. It is broken if magnetic monopoles exist (but a modified current with localized addition, $F \wedge F + \mathrm{d}\sigma \wedge \delta_M$, can exist).

A generalization is true in nonabelian gauge theories:

$$\begin{aligned} \mathrm{d}\,\mathrm{tr}(F \wedge F) &= \mathrm{tr}(\mathrm{d}F \wedge F + F \wedge \mathrm{d}F) \\ &= \mathrm{tr}\Big((\mathrm{d}F + [A, F]) \wedge F + F \wedge (\mathrm{d}F + [A, F])\Big) \\ &= \mathrm{tr}(\mathrm{d}_A F \wedge F + F \wedge \mathrm{d}_A F) = 0 \end{aligned}$$

We call this **"Chern-Weil symmetry."**
Instanton number is an invariant charge associated with a field configuration!

Axions as Gauge Fields

(Heidenreich, McNamara, Montero, MR, Rudelius, Valenzuela '20)

The job of the axion in quantum gravity is to *eliminate* a generalized ("(-1)-form Chern-Weil") global symmetry by *gauging* it.

Indeed, axions in string theory often just *are* zero modes of higher dimensional gauge fields.

$$\tau(x) = \frac{1}{2\pi}\theta(x) + 4\pi \mathrm{i} S(x), \quad \theta = \int_{\Sigma_p} C_p, \quad S \sim \mathrm{Vol}(\Sigma_p)$$

Chern-Simons: $\theta F^{\mu\nu}\tilde{F}_{\mu\nu}$ from $\int C_p \wedge F \wedge F$

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New Origin of Axion Potential

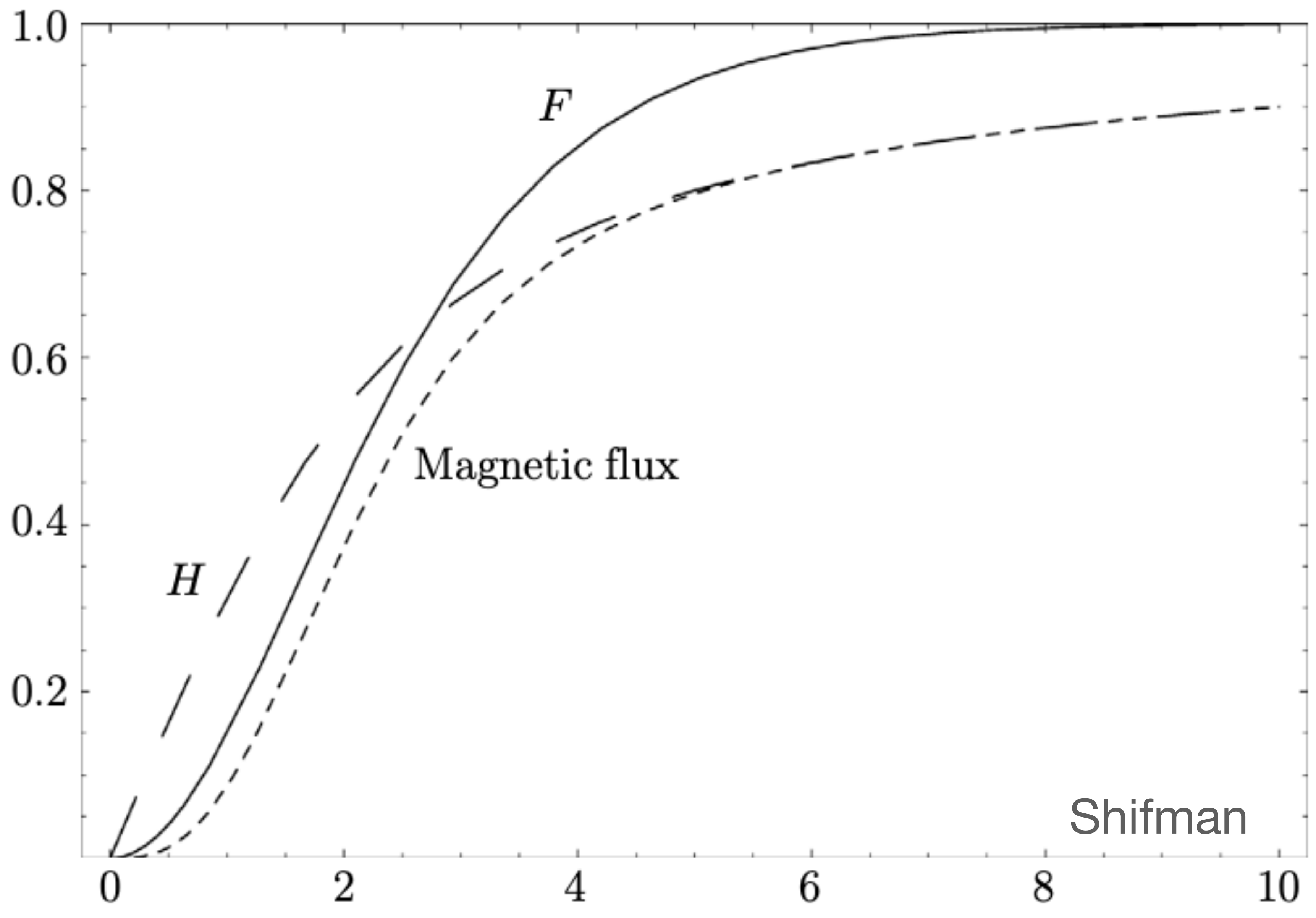
It is well known that for axion coupling to non-Abelian gauge group, instantons generate a potential for the axion.

Yet for axion coupling to ***Abelian*** gauge fields, the axion could still acquire a potential through ***loops of magnetic monopoles***.
(Fan, Fraser, MR, Stout 2021, just published in Phys.Rev.Lett.)

Existence of magnetic monopoles: "***completeness hypothesis***"
Polchinski 2003

Monopole Refresher: ’t Hooft-Polyakov

$SU(2) \to U(1)$ symmetry broken by an adjoint vev: classical solution of ’t Hooft-Polyakov (’t H-P) monopole.



$$\phi^a = v\hat{r}^a H(r), \quad A_i^a = \epsilon^{aij}\frac{1}{r}\hat{r}^j F(r)$$

$$r \to \infty : \quad H(r) \to 1,\ F(r) \to 1$$

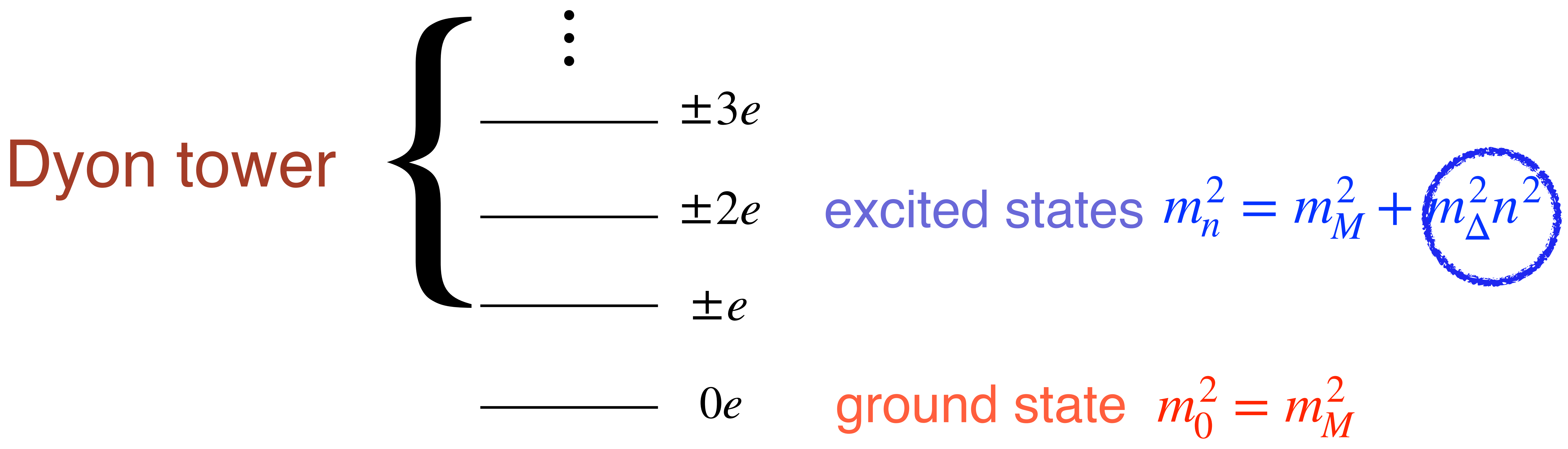
$$r \to 0 : \quad H(r) \to 0,\ F(r) \to 0$$

The solution has **4 zero modes** (collective coordinates): 3 translations, 1 U(1) (large gauge transformation, not vanishing at infinity).

review: Shifman, *Advanced Topics in Quantum Field Theory,* Chapter 4

Possible charged states: not only magnetic monopoles, but also ***dyons*** (particles with both magnetic and electric charges).

E.g., in ’t H-P case, a residual unbroken global $U(1)$ rotation could be realized by a compact real scalar. In 4d, this is described by QM of a particle living on a circle, $\sigma \cong \sigma + 2\pi$ (dyonic collective coordinate). This has a spectrum labelled by integers. The ground state is the magnetic monopole (with no electric charge) and the excited states are dyons.



Witten Effect

Given $\frac{e^2\theta}{8\pi^2} F \wedge F = \frac{e^2\theta}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$ ($\theta = a/f_a$ and e: unit of electric charge)

and a point magnetic monopole (no electric charge when $\theta = 0$) at the origin, the Maxwell equations are modified:

Magnetic Gauss' law: $\nabla \cdot \mathbf{B} = \frac{g_m}{4\pi}\delta(\mathbf{r})$, g_m: unit of magnetic charge; $eg_m = 2\pi$

due to Dirac quantization condition;

Electric Gauss' law: $\nabla \cdot \mathbf{E} + \frac{e^2}{4\pi^2}\theta(\nabla \cdot \mathbf{B}) = 0 \Rightarrow \boxed{\frac{Q_E}{e} = -\frac{\theta}{2\pi}}$

A monopole obtains an effective electric charge in the presence of an axion background!

Witten, 1979

In general, the dyon electric charge is shifted to be

$$\frac{Q_E}{e} = n - \frac{\theta}{2\pi}, \quad n = 0, \pm 1, \pm 2, \cdots$$

The corresponding energy spectrum will be modified as well!

$$L = \frac{1}{2}\dot{\sigma}^2 + \frac{\theta}{2\pi}\dot{\sigma}$$

σ: dyonic collective coordinate

Conjugate momentum: $\Pi_\sigma = \dot{\sigma} + \frac{\theta}{2\pi}$

Hamiltonian:

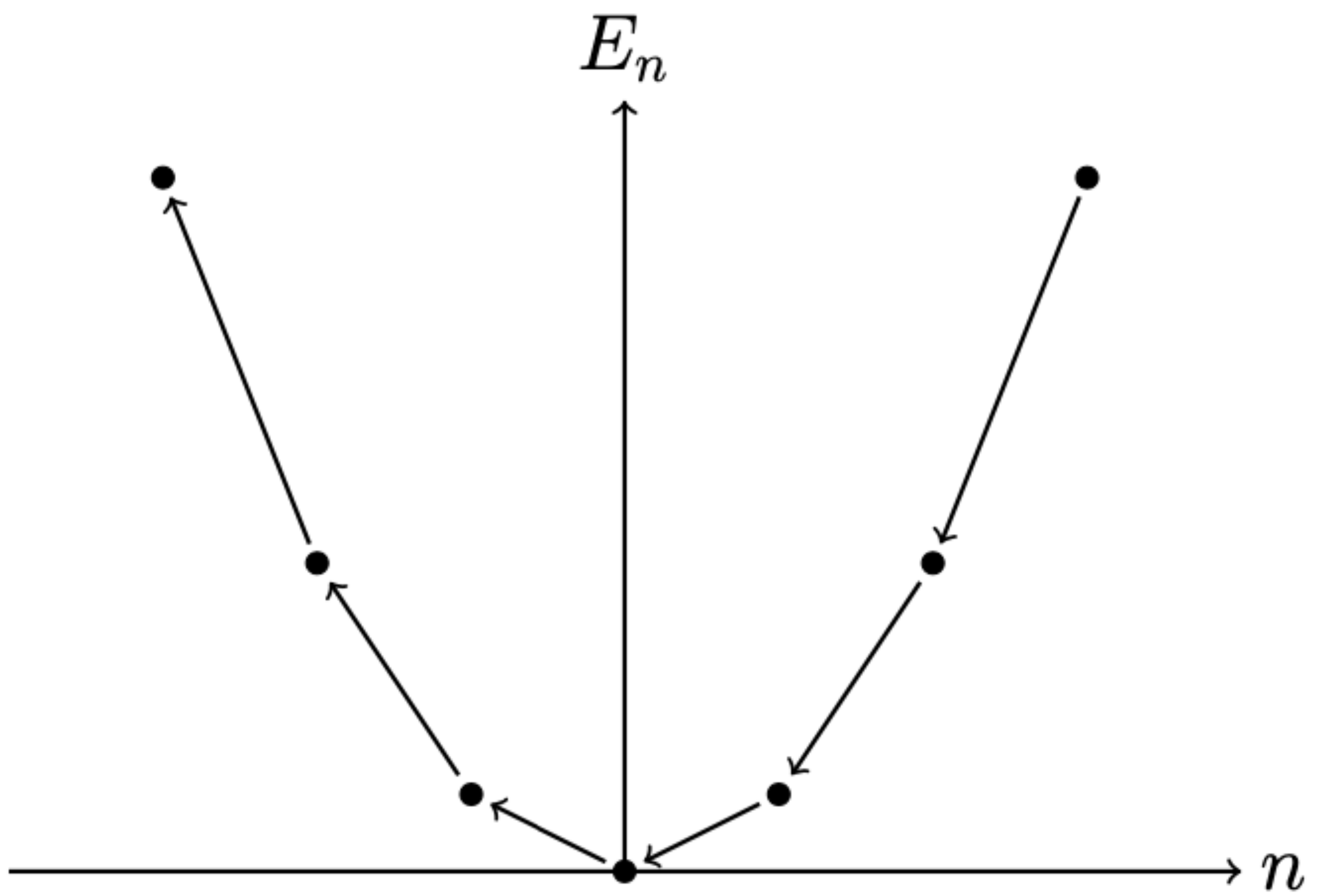
$$H = \frac{1}{2}\left(\Pi_\sigma - \frac{\theta}{2\pi}\right)^2 \quad \Rightarrow \quad E_n = \frac{1}{2}\left(n - \frac{\theta}{2\pi}\right)^2$$

$$\frac{1}{2}\left(-i\partial_\sigma - \frac{\theta}{2\pi}\right)^2 \psi_n = E_n \psi_n$$

The corresponding energy spectrum

$$m_n^2 - m_M^2 = m_\Delta^2 \left(n - \frac{\theta}{2\pi}\right)^2$$

ground state monopole mass at $\theta = 0$



periodicity through "monodromy" or rearrangement of the eigenstates:

$$n \to n+1, \quad \theta \to \theta + 2\pi$$

Integrating out these states $\Rightarrow$ vacuum potential for the axion θ!

Note: ***different*** from the axion potential generated by ***monopole and anti-monopole plasma***! Fischler, Preskill 1983; Kawasaki, Takahashi, Yamada 2015; Nomura, Rajendran, Sanches 2015; …

A plasma of monopoles and anti-monopoles could be generated through the Kibble-Zurek mechanism in the early Universe.

Here we talk about the axion potential from the ***virtual*** effects of monopole (dyon) loops.

Our calculation can be carried out from two viewpoints:

1. Integrate out the dyons to get a Coleman-Weinberg potential for axion.
2. Do the path integral over all monopole loops.

Related by Poisson resummation

invariant length

transition amplitude $\langle x | x \rangle_\tau$

$$V_{\rm eff} = -\int_0^\infty \frac{d\tau}{2\tau} \frac{1}{2(2\pi\tau)^2} \exp\left(-\frac{m^2\tau}{2}\right)$$

$$m_n^2 = m_{\rm M}^2 + m_\Delta^2 \left(n - \frac{\theta}{2\pi}\right)^2$$

$$-\sum_{n\in\mathbb{Z}} \int_0^\infty \frac{d\tau}{4\tau\,(2\pi\tau)^2} \exp\left(-\frac{m_{\rm M}^2\tau}{2} - \frac{m_\Delta^2\tau}{2}\left(n - \frac{\theta}{2\pi}\right)^2\right).$$

Poisson resum

$$\sum_{n\in\mathbb{Z}} e^{-\frac{1}{2}m_\Delta^2\tau\left(n-\frac{\theta}{2\pi}\right)^2} = \sum_{\ell\in\mathbb{Z}} \sqrt{\frac{2\pi}{m_\Delta^2\tau}} \exp\left(-\frac{2\pi^2\ell^2}{m_\Delta^2\tau} + i\ell\theta\right).$$

$$V_{\text{eff}}(\theta) = -\sum_{\ell=1}^{\infty} \frac{m_\Delta^2 m_{\text{M}}^2}{32\pi^4 \ell^3} \mathrm{e}^{-2\pi\ell m_{\text{M}}/m_\Delta} \cos(\ell\theta) \times \left(1 + \frac{3m_\Delta}{2\pi\ell m_{\text{M}}} + \frac{3m_\Delta^2}{(2\pi\ell m_{\text{M}})^2}\right),$$

winding number in the σ direction

Fan, Fraser, MR, Stout, 2021

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winding number in the σ direction

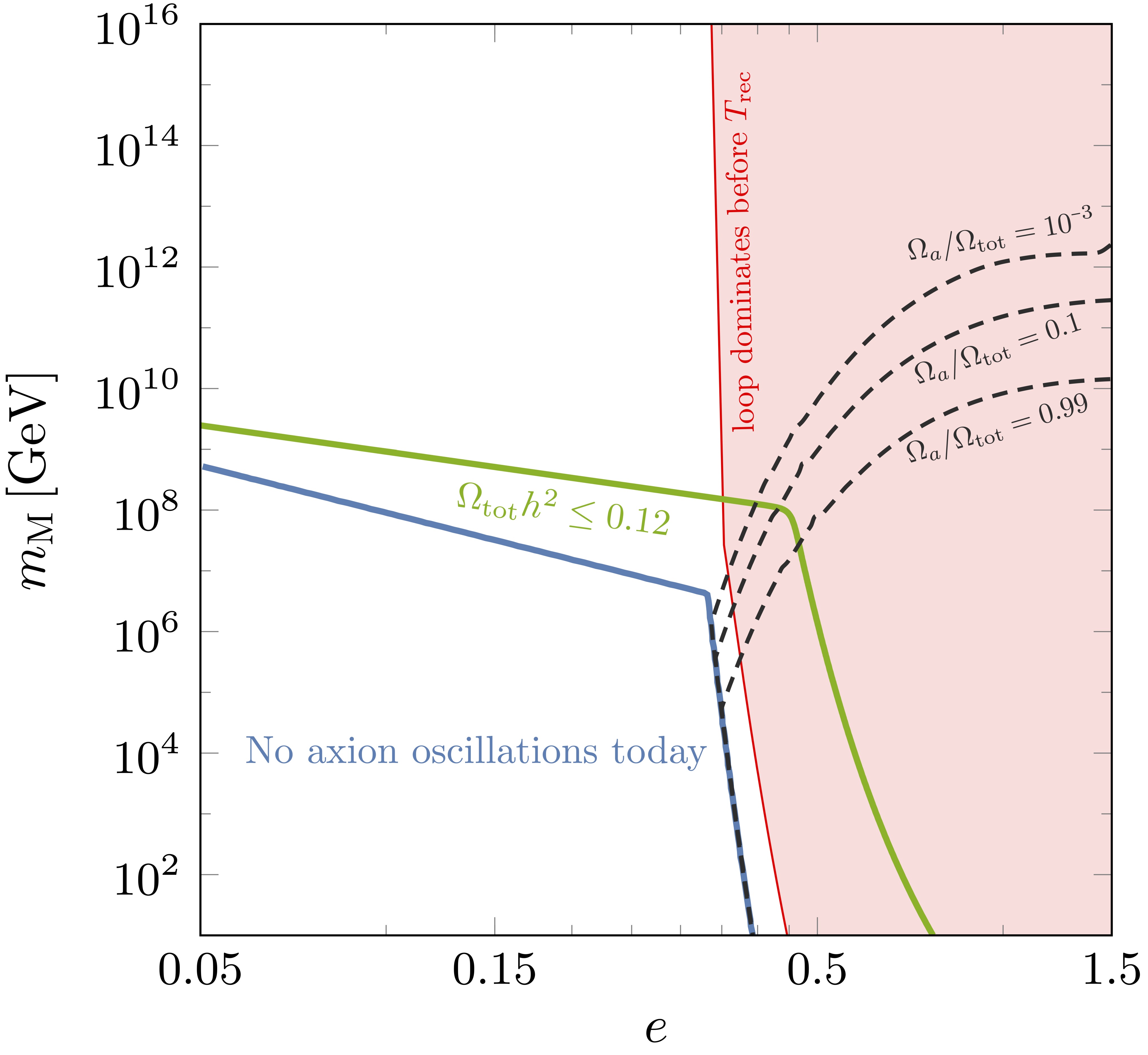
$e^{-S_{\text{inst}}} \sim e^{-8\pi^2/g^2}$ in ’t H-P model: same instanton action as in YM theory!

Fan, Fraser, MR, Stout, 2021

In a hidden gauged $U(1)$ sector with an axion and monopoles: both axion and monopole contribute to DM

$$m_a(T) = m_a^{\text{loop}} + m_a^{\text{plasma}}(T)$$

monopole mass



dark gauge coupling

Conclusions

Quantum gravity theories have ubiquitous (s)axion fields coupled to gauge fields.

Moduli and axions can lead to extended, early matter domination before BBN. Moduli dominance and decay alter any dark matter relic density calculation.

Axions have a job to do in quantum gravity: eliminating a global Chern-Weil (*instanton number*) symmetry by gauging it.

Fundamental axions need not be ordinary pseudo-Nambu-Goldstone bosons: no point in field space where Peccei-Quinn is restored.

The localized worldline fields on magnetic monopoles lead to axion potentials.

Minimum mass for axion coupled to photons? Depends on subtleties about fermion mass dependence. Work in progress (w/ Fan, Fraser, Stout, Telem)

Thank You!