On the quantum origin of a small positive cosmological constant

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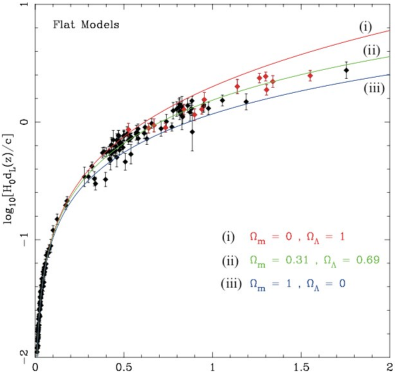
Overview

1. Dark Matter and Dark Energy problem
2. Friedmann equation $\leftarrow$ Raychaudhuri equation
3. Quantum Friedmann equation $\leftarrow$ Quantum Raychaudhuri equation
4. Dark Matter and Λ from a Bose-Einstein Condensate
5. Potential origin of a small positive Cosmological constant

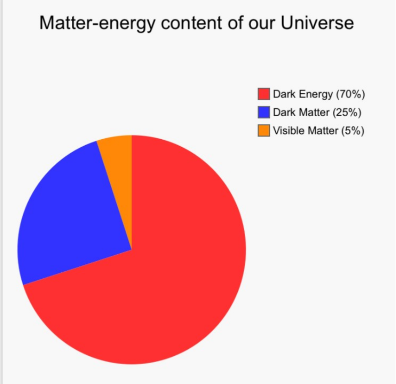
Dark Matter, Dark Energy

Luminosity distance $d_L(\Omega_\Lambda, \Omega_M, z) = \frac{(1+z)}{H_0} \int_0^z \frac{dz}{\sqrt{\underbrace{\Omega_\Lambda}_{0.7} + \underbrace{\Omega_M}_{0.3}(1+z)^3}}$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{crit}} \ , \quad \Omega_M = \frac{\rho_M}{\rho_{crit}}$$



(Pic: Copeland et al, hep-th/0603057)



Questions

- What constitutes Dark Matter?
- What constitutes Dark Energy/Λ?
- Why is Λ positive?
- Why is Λ tiny, about $10^{-123}\ell_{Pl}^{-2}$ where ℓ_{Pl} is the Planck length?
  $(\rho = \int_0^{k_{max}} dk\, k^2 \sqrt{k^2 + m^2} \approx k_{max}^4 > 10^{50}\, \rho_\Lambda)$
- Currently $\rho_{DM} \approx \underbrace{\rho_\Lambda}_{\frac{\Lambda c^2}{4\pi G}} \approx \underbrace{\rho_{crit}}_{\frac{3H_0^2}{8\pi G}} \approx 10^{-26}\, kg\, m^{-3}$

  Why? The *'coincidence problem'*

Friedmann equations

Spatially flat FLRW Universe

$$ds^2 = -dt^2 + a(t)^2 \left[dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right] \quad [a = \textit{scale factor} = 1 \textit{ Now}]$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} \;\rightarrow\; d_L = (1+z)\int_0^z \frac{dz'}{H(z')}$$

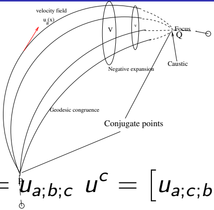
$$\dot{\rho} + 3H(p+\rho) = 0$$

Raychaudhuri $\rightarrow$ Friedmann Equation [$\theta =$ Expansion]

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - R_{cd}u^c u^d \;< 0$$

$$\theta = 3\,\frac{\dot{a}}{a}\;, \quad R_{cd}u^c u^d \rightarrow \frac{4\pi G}{3}(\rho + 3p) \text{ (Einstein eqns.)}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$



Velocity field $u_a = u_a(x) \Rightarrow \frac{du_{a;b}}{d\tau} = u_{a;b;c}\ u^c = \left[u_{a;c;b} + R_{cba}{}^d u_a\right] u^c$

$$= \left(\underbrace{u_{a;c}u^c}_{=0\,\text{(geod.eqn.)}}\right)_{;b} - u^c{}_{;b}u_{a;c} + R_{cba}{}^d u^c u_d = -u^c{}_{;b}u_{a;c} + R_{cbad}\ u^c u^d$$

Symmetric part: $\sigma_{ab} = u_{(a;b)} - \frac{1}{3}h_{ab}\theta$

Anti-symmetric part and trace: $\omega_{ab} = u_{[a;b]};\quad \theta = h^{ab}u_{a;b};\ ,\ h_{ab} = g_{ab} - u_a u_b$

Decomposition: $u_{a;b} = \frac{1}{3}\theta h_{ab} + \sigma_{ab} + \omega_{ab}$

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - R_{cd}u^c u^d \ < 0$$ *[Raychaudhuri Equation]*

If $\theta_0 = \theta(0) < 0$ (initially converging)

Focus/caustic for $\tau \leq \frac{3}{|\theta_0|}$ *Geodesics end in finite time! → Spacetimes are singular!*

A. K. Raychaudhuri (1955), L. D. Landau, E. M. Lifshitz (c.1959), R. Penrose (1965), S. W. Hawking and R. Penrose (1970)

Quantum Raychaudhuri Equation - 1

But: Raychaudhuri Equation/Friedmann Equation are classical

So: Compute quantum corrections and study consequences

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - R_{cd}u^c u^d + \underbrace{Tr[(u_{a;c}\, u^c)_{;b}]}_{=0}$$

How does this (geodesic equation) change on quantization?

Classical fluid $(u_a) \to$ Quantum fluid (Ψ)

Geodesic equation $\to$ Klein-Gordon equation

$$\left[\Box + \frac{m^2 c^2}{\hbar^2}\right] \Psi = 0$$

$$\Psi(x) = \mathcal{R}(x)\, e^{iS(x)} \;,\; \mathcal{R}, S \in \mathbb{R} \;,$$

$$k_a = \partial_a S \;, \quad u_a = c\,\frac{dx_a}{d\tau} = \frac{\hbar k_a}{m} \;, \quad \vec{v} = \frac{d\vec{x}}{dt} = -c^2 \frac{\vec{\nabla} S}{\partial^0 S}$$

Imaginary part of the KG equation: $\partial^a \left(\mathcal{R}^2 \partial_a S\right) = 0$

Real part of the KG equation: $k^2 = \frac{(mc)^2}{\hbar^2} + \frac{\Box \mathcal{R}}{\mathcal{R}}$

$$u^b_{;a} u^a = \frac{\hbar^2}{m^2} \left(\frac{\Box \mathcal{R}}{\mathcal{R}}\right)^{;b} \neq 0 \quad \left(\text{i.e. geodesic equation} + \text{Quantum potential } V_Q = \frac{\hbar^2}{m^2} \frac{\Box \mathcal{R}}{\mathcal{R}}\right)$$

Quantal trajectories are not geodesics!

Quantum Raychaudhuri equation

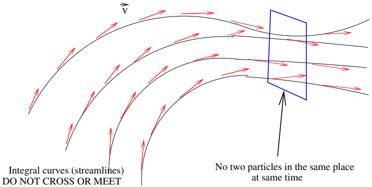
$$\frac{d\theta}{d\tau} = -\frac{1}{3}\,\theta^2 - \sigma_{ab}\sigma^{ab} - R_{cd} u^c u^d + \frac{\hbar^2}{m^2} h^{ab} \left(\frac{\Box \mathcal{R}}{\mathcal{R}}\right)_{;a;b} \leftarrow \text{Quantum Correction } \mathcal{O}(\hbar^2)$$

S. Das, Phys. Rev. **D89** (2014) 084068 [arXiv/1311.6539]

Quantum Raychaudhuri Equation and no Singularity Theorems

$$\boxed{\text{No-crossing of quantal trajectories}}$$

$$\vec{v} = \frac{d\vec{x}}{dt} = -c^2 \frac{\vec{\nabla} S}{\partial^0 S}$$



- No focusing, no conjugate points, geodesics go on forever
- No singularities! (all because of $\hbar$)

<u>FLRW Universe:</u> $ds^2 = -dt^2 + a(t)^2 \left[dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$ [a = *scale factor*]

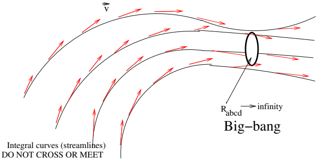
$$\theta = 3\,\frac{\dot a}{a}\ , \quad R_{cd}u^c u^d \to \frac{4\pi G}{3}(\rho + 3p)$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \underbrace{\frac{\hbar^2}{3m^2} h^{ab}\left(\frac{\Box \mathcal{R}}{\mathcal{R}}\right)_{;a;b}}_{\frac{\Lambda_Q}{3}}$$

$$\Lambda_Q = \frac{\hbar^2}{m^2} h^{ab} \left(\frac{\Box\mathcal{R}}{\mathcal{R}}\right)_{;a;b} = \text{Wavefunction-dependent Quantum correction}$$

$$\boxed{\text{Consequences}}$$

- No crossing (e.g. at the Big bang)



S. Das, IJMPD **23**, No. 12, 1442017 (2014) [arXiv/1405.4011]

Bose-Einstein Condensate (BEC) as Dark Matter - 1

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3p\right) + \underbrace{\frac{\hbar^2}{3m^2} h^{ab} \left(\frac{\Box \mathcal{R}}{\mathcal{R}}\right)_{;a;b}}_{\frac{\Lambda_Q}{3}}$$

$$\Psi(x) = \mathcal{R}(x)\, e^{iS(x)} = \text{BEC wavefunction?}$$

Pros:

- Cold
- Dark
- Light bosons as DM $\Rightarrow$ no small scale structure
- Macroscopic *Quantum state*
- BEC $\Rightarrow$ DE ($\approx$ DM/Λ) via its (repulsive) Quantum Potential
- Few assumptions and free parameters

Bose-Einstein Condensate (BEC) as Dark Matter - 2

Is the critical temperature (below which a BEC forms) high enough?

Critical temperature $= T_c$
Universe temperature $= T(a)$
Boson mass $= m \; eV/c^2$
$\rho_{DM} = 0.25\, \rho_{crit}/a^3$
No. density $= \frac{N}{V} = 0.25\, \frac{\rho_{crit}}{m\, a^3}$

$$T_c(a) = \frac{\hbar c}{k_B} \left(\frac{(N/V)\, \pi^2}{\eta \zeta(3)} \right)^{1/3} = \frac{\hbar c}{k_B} \left(\frac{(0.25\, \rho_{crit}/ma^3)\, \pi^2}{\eta \zeta(3)} \right)^{1/3} = \frac{4.9}{m^{1/3}\; a}\; K$$

$$T(a) = \frac{3.7}{a}\; K \;,\;\; a = \text{scale factor}$$

$$T(a) < T_c(a)\; \forall a \;\; \rightarrow \quad m < 6 \; eV/c^2 \;\; \Rightarrow \text{BEC forms in the early universe}$$

.

BEC density = DM density

Quantum Potential of BEC

$BEC \Rightarrow \Psi \ \Rightarrow$ *Quantum potential!*

Quantize

$\boxed{\text{Macroscopic BEC wavefunction}}$ $\quad \Psi = \frac{R_0}{a^{3/2}}\, e^{-r^2/\sigma^2} = \mathcal{R}(x)$

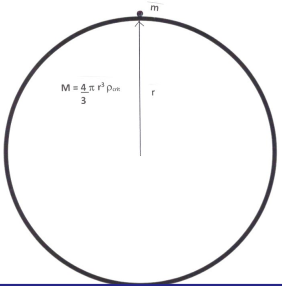
$$\rho_{DM} = |\Psi|^2 \propto \frac{1}{a^3} \ , \ \int dV |\Psi|^2 = N$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G \rho_{crit}}{3} + \frac{\Lambda_Q}{3}$$

$$\Lambda_Q = \frac{\hbar^2}{m^2 c^2}\, h^{ab} \left(\frac{\Box \mathcal{R}}{\mathcal{R}}\right)_{;a;b} = 24 \left(\frac{\hbar}{mc}\right)^2 \frac{1}{\sigma^4} = \text{constant!}$$

S. Das, R. K. Bhaduri, Class. Quant. Grav. **32** 105003 (2015) [arXiv:1411.0753]
S. Das, R. K. Bhaduri, Phys. News (special S. N. Bose anniversary issue) arXiv:1808.10505

BEC Wavefunction Ψ - 1 (from the Newtonian limit)



Newtonian limit, $R_{space} = 0, R_{spacetime} = 10^{-123}\,\ell_{Pl}^{-2}$

$$m\ddot{r} = -\frac{GMm}{r^2} = -\frac{G(\frac{4}{3}\pi r^3\,\epsilon\,\rho_{crit})m}{r^2}\ , \quad r = r_0\,a(t)\ ,\ \epsilon \approx 0.25$$

$$\frac{\ddot{a}}{a} \;=\; -\frac{4\pi G}{3}\,\epsilon\,\rho_{crit} = -\omega^2$$ Raychaudhuri Equation

BEC in a harmonic trap for $t \ll H_0^{-1}$ (14 Gyr)

Quantize $\rightarrow$ $$\Psi = R(a)\,e^{-\frac{m\omega r^2}{2\hbar}} = R(a)\,e^{-\frac{m(4\pi G\,\epsilon\,\rho_{crit}/3)^{1/2}r^2}{2\hbar}} = \frac{R_0}{a^{3/2}}\,e^{-\frac{r^2}{\sigma^2}}$$

$$\sigma^2 = \frac{2\hbar}{m(4\pi G\epsilon\rho_{crit}/3)^{1/2}}$$

BEC Wavefunction Ψ - 2

$$a(t) = a_0 + a_1(t) = \text{constant} + \text{slowly varying}$$

$$\Psi = \Psi_0 + \Psi_1 = \text{time-indep.} + \text{slowly varying}$$

$$\mathcal{R} = \frac{R_0}{a^{3/2}}\, e^{-(r^2/\sigma)^2} = \frac{R_0}{a_0^{3/2}} - \left(\frac{3R_0}{2\, a_0^{5/2}}\right) a_1\, e^{-(r/\sigma)^2}$$
$$= \text{time-indep.} + \text{slowly varying}$$

$$\Lambda_Q = \Lambda_Q^{(0)} + \Lambda_Q^{(1)} = \text{constant} + \text{slowly varying}$$

<u>How slow is slow?</u> $(\frac{a_1}{a_0}|_{t_1} \ll 1)$

$a(t) \propto (t - t_0)^{\frac{2}{3(1+w)}}$ *(Matter/radiation. $p = w\rho,\ w = 0, \frac{1}{3}$)*

$a(t) = a_0 e^{H_0\, t}$ *(de Sitter. $p = -\rho,\ w = -1$)*

$\Delta t \equiv t - t_1 \ll t_1 - t_0$ *(Matter/radiation)*

current time - ref.time $\ll$ ref.time - BB

$\Delta t \equiv t - t_1 \ll H_0^{-1} \simeq 16\ Gyr$ *(de Sitter)*

Don't go too far in the past!

Λ_Q from quantum potential

$$\Psi = R(a)\, e^{-\frac{m(4\pi G\, \epsilon\, \rho_{crit}/3)^{1/2} r^2}{2\hbar}} = \frac{R_0}{a^{3/2}}\, e^{-\frac{r^2}{\sigma^2}} = \mathcal{R}$$

$$\Lambda_Q = \frac{\hbar^2}{m^2} h^{ab} \left(\frac{\Box\mathcal{R}}{\mathcal{R}}\right)_{;a;b} = 8\,\pi\, G \epsilon\, \rho_{crit} \quad \textit{(independent of m!)}$$

$$\rho_\Lambda = \frac{\Lambda}{4\pi G} = 2\,\epsilon\,\rho_{crit}$$

$$\rho_{DM} = \epsilon \rho_{crit}$$

$$\frac{\rho_\Lambda}{\rho_{DM}} = 2$$

$$\rho_\Lambda \approx \rho_{DM}$$

Summary

- What constitutes DM? *BEC*
- What constitutes DE? *Quantum potential of the BEC*
- Why is Λ positive?
  Because negative gravitational potential $\Rightarrow$ positive Quantum Potential
- Why is $\rho_{DM} \approx \rho_\Lambda \approx \rho_{crit}$?
  Because $|$Quantum potential$|$ = $|$classical potential$|$ for stationary states

Remarks

- We get $\rho_\Lambda = \cancel{3}\ \rho_{DM}$, because (i) $\rho_{DM} \propto 1/a^3, \rho_\Lambda \propto$ constant (ii) all bosons not in the ground state
- Prediction: ultralight bosons of $m < 6eV/c^2$. gravitons? axions?
- Prediction: Λ has changed in the far past and will change in the future
- Interacting DM and DE model and estimate m from data *(M. Sharma, S. Sur, SD, arXiv:2102.03032)*
- Full quantization of gravity/spacetime?