Fundamental Axions

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1. **No Global Symmetries in Quantum Gravity**

2. The Role of Axions in Quantum Gravity

3. Axion Strings and the Weak Gravity Conjecture

4. Axion Potentials from Magnetic Monopoles

No Global Symmetries

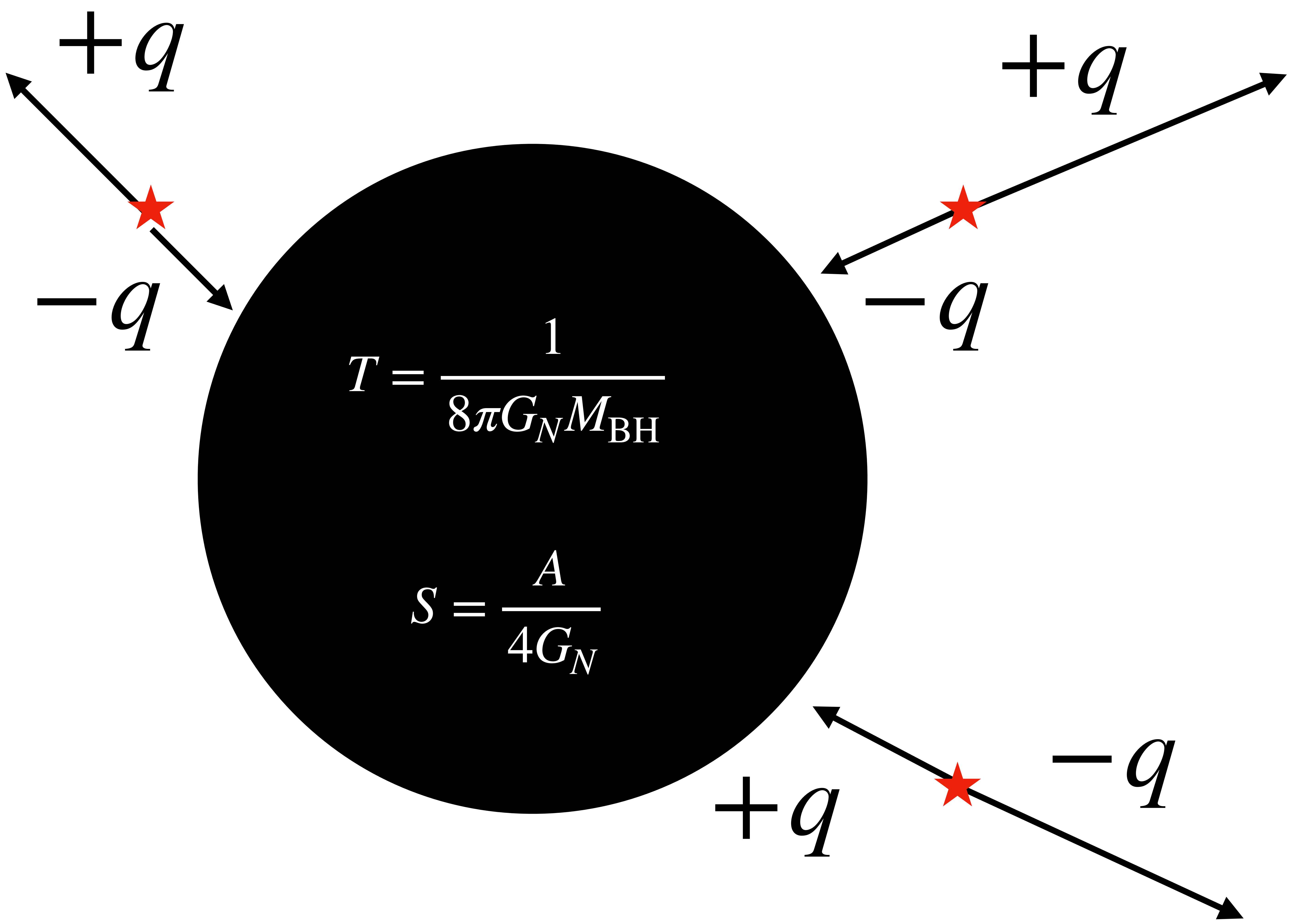
(Wheeler; Hawking; Zeldovich; Banks, Dixon; Banks, Seiberg; Harlow, Ooguri; rapidly growing list of others….)

Expectation: consistent theories of quantum gravity have **no global symmetries.** At the UV cutoff scale, not even ***approximate*** global symmetries.

***Surprisingly wide range of applications!* e.g.:**

- Weak Gravity Conjecture
- Chern-Simons terms and axions
- Existence of “twist” strings (Z_N strings, Alice strings, …)

Black Holes Destroy Global Charges

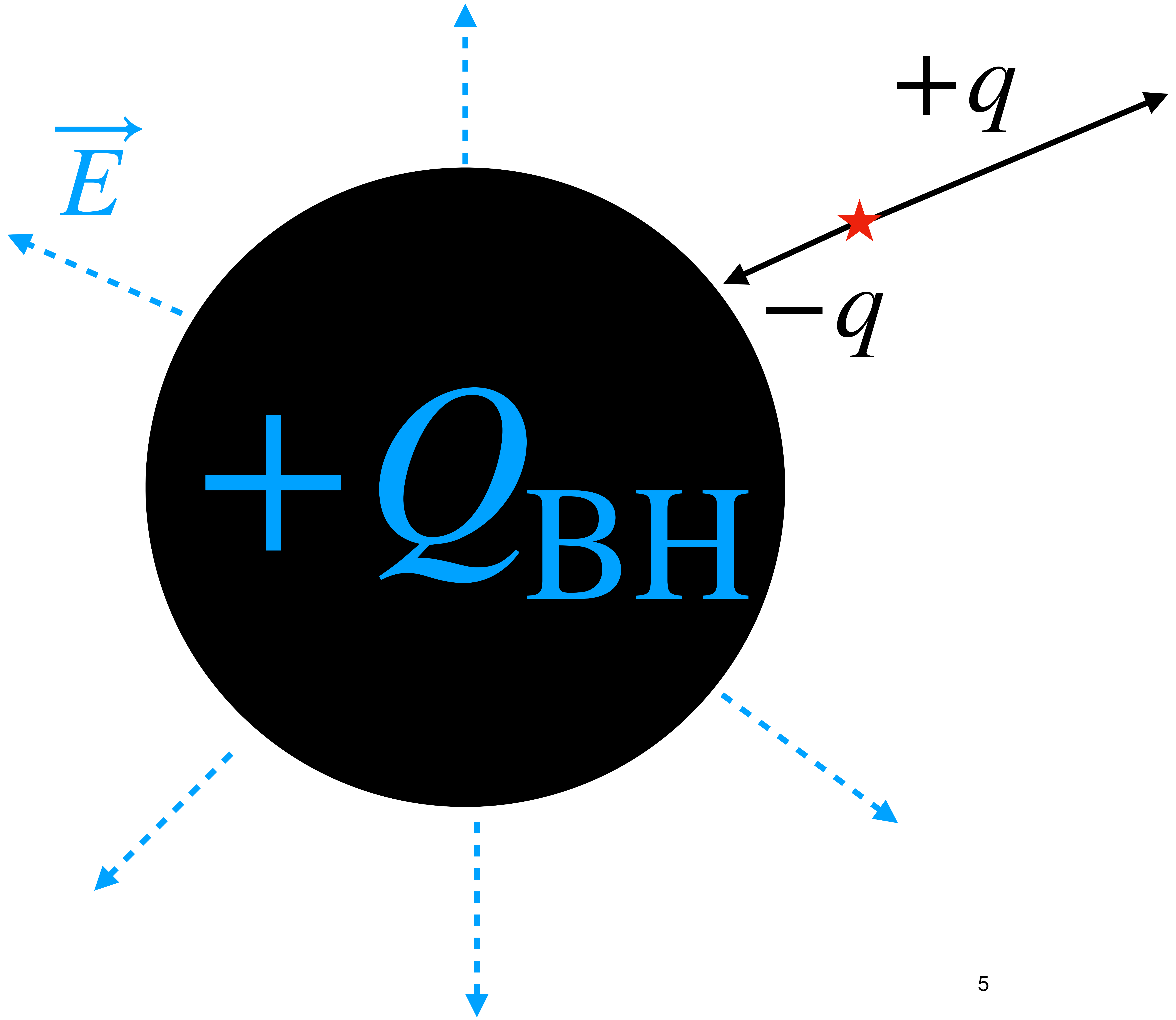


Hawking radiation:

Random thermal emission of global charge.

Modern argument: Banks, Seiberg 2010

Black Holes and Gauge Charge



Measurable $\vec{E}$ field outside BH: **preferential discharge**, if light charged particles exist.

$$\mu \propto Q_{\rm BH}$$

$\vec{E}$ field contributes to BH energy: **extremality bound**

$$M_{\rm BH} \geq \sqrt{2} e Q_{\rm BH} M_{\rm Pl}$$

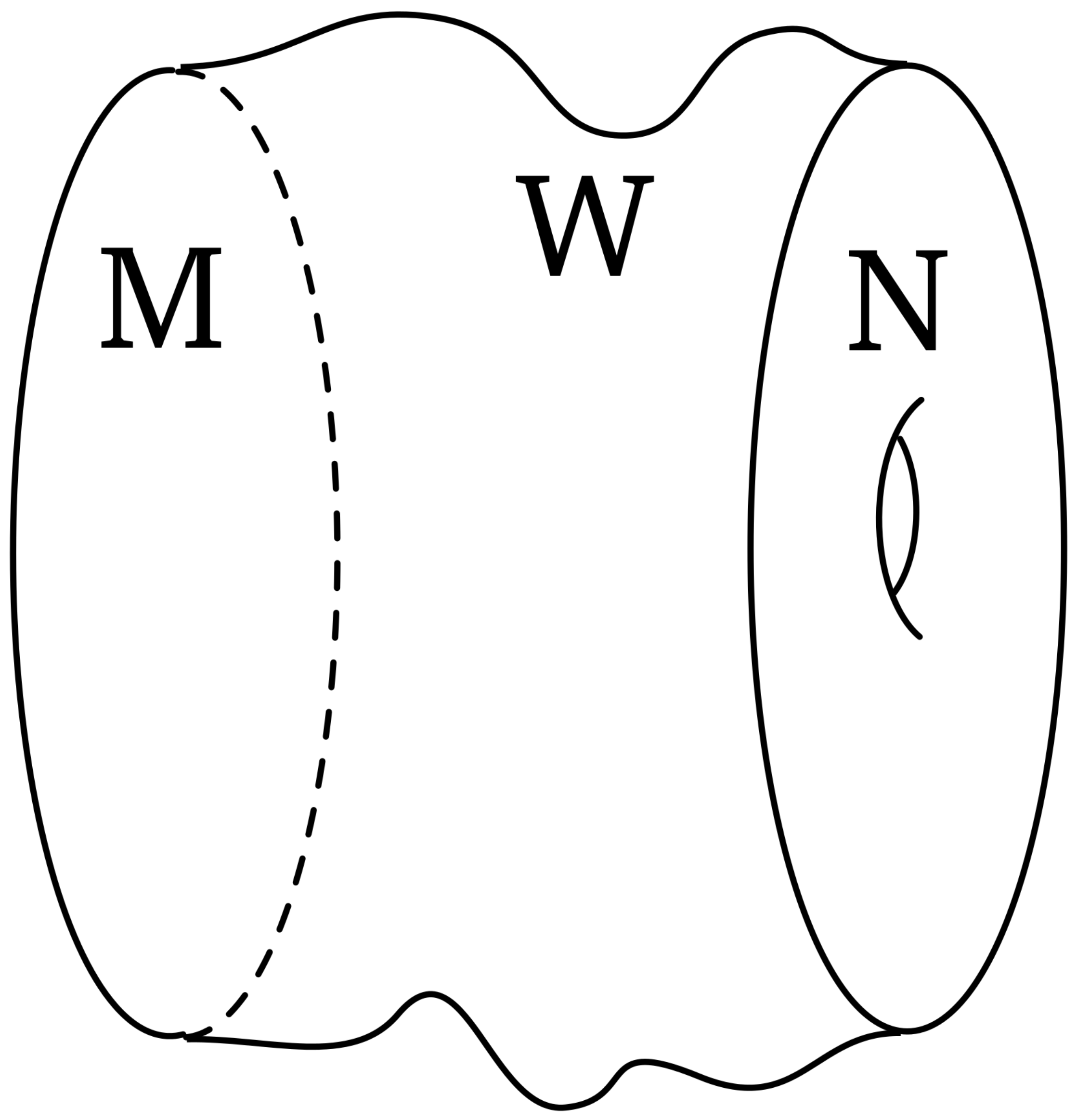
$(M_{\rm Pl} = \sqrt{\frac{1}{8\pi G_N}})$

No Labels in QG

Charge as a **label** we can assign to a *state,* which cannot be altered by continuous variations of the state.

Extend to labels on regions of different dimension, even all spacetime.
In quantum gravity, everything deformable to everything else.

"Cobordism Hypothesis"



Example: **instanton number** is a label on gauge field configurations in spacetime.

Should be forbidden!

arXiv:1909.10355, Jake McNamara, Cumrun Vafa

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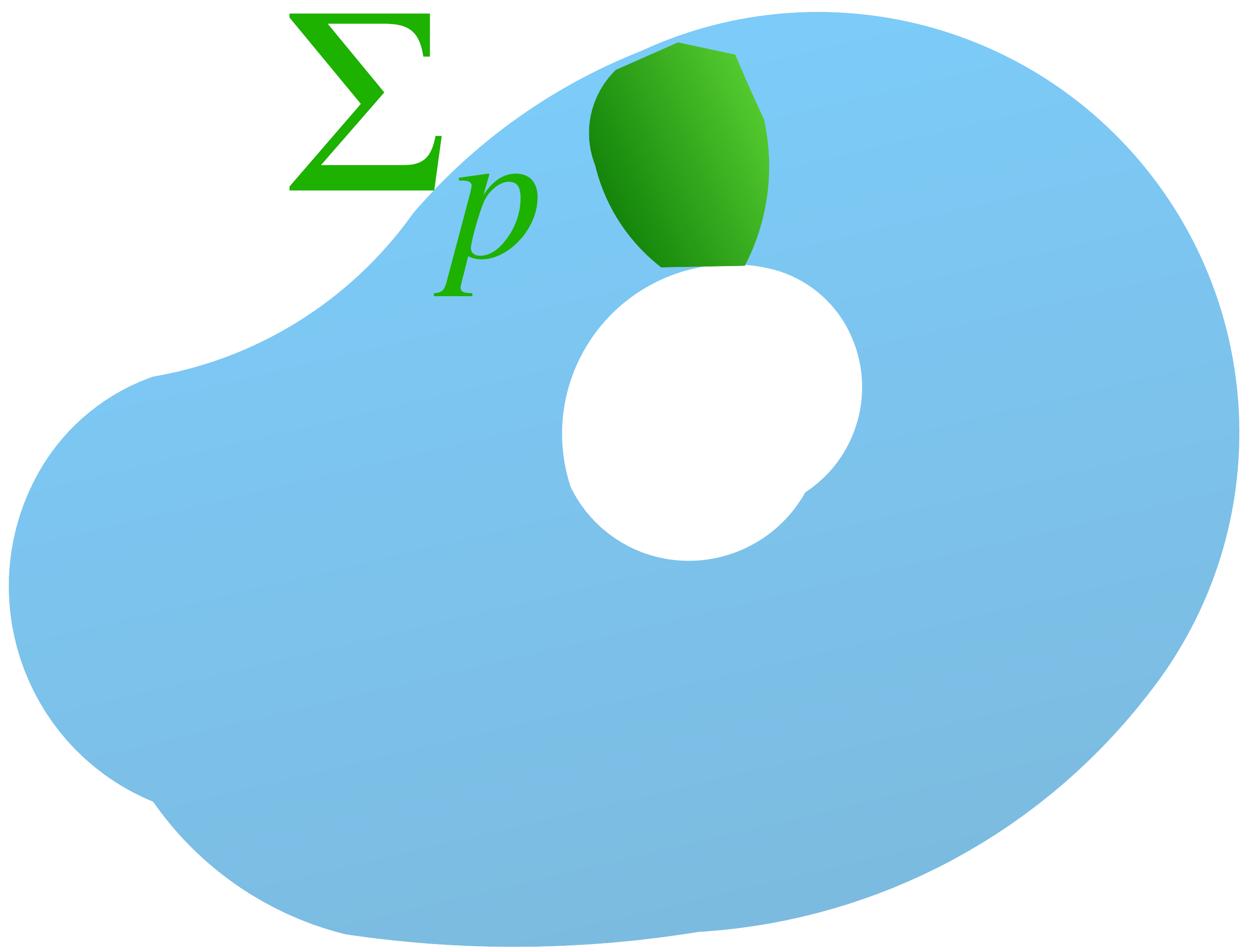
The Ubiquitous Axion: Lamppost or Principle?

There is a large ***Landscape*** of known, consistent quantum gravity theories containing gauge fields. (String compactifications.)

Almost always couple to axions via $\theta\, \mathrm{tr}(F \wedge F)$ interactions!



Often higher-dimensional gauge fields C_p with Chern-Simons couplings $C_p \wedge \mathrm{tr}(F \wedge F)$, and $\theta = \int_{\Sigma_p} C_p$.

Is it a generic prediction, or an accident of our current abilities?

Moduli and Axions for Gauge Couplings

In string theory, the gauge kinetic function is often a *dynamical field*:

$$\frac{1}{16\pi \mathrm{i}}\int d^2\theta\, \tau(x)\, \mathscr{W}^\alpha(x)\mathscr{W}_\alpha(x)$$

$$\tau(x) = \frac{1}{2\pi}\theta(x) + 4\pi \mathrm{i} S(x), \quad \langle S \rangle = \frac{1}{g^2}$$

axion

"saxion" or scalar modulus

Note: I am *not* assuming TeV-scale SUSY! Just compactification-scale SUSY.

Aspects of Moduli Fields

The limit where $g \to 0$, i.e., $S \to \infty$, lies at infinite distance.
No global symmetries: cannot send gauge couplings to zero.

(cf. Ooguri/Vafa “Swampland Distance Conjecture”; Arkani-Hamed/Motl/Nicolis/Vafa “Weak Gravity Conjecture”)

Have in mind Lagrangians like:

$$\mathscr{L} \supset M_*^2 \partial_\mu (\log S) \partial^\mu (\log S) + \frac{M_*^2}{S^2} \partial_\mu \theta \partial^\mu \theta$$

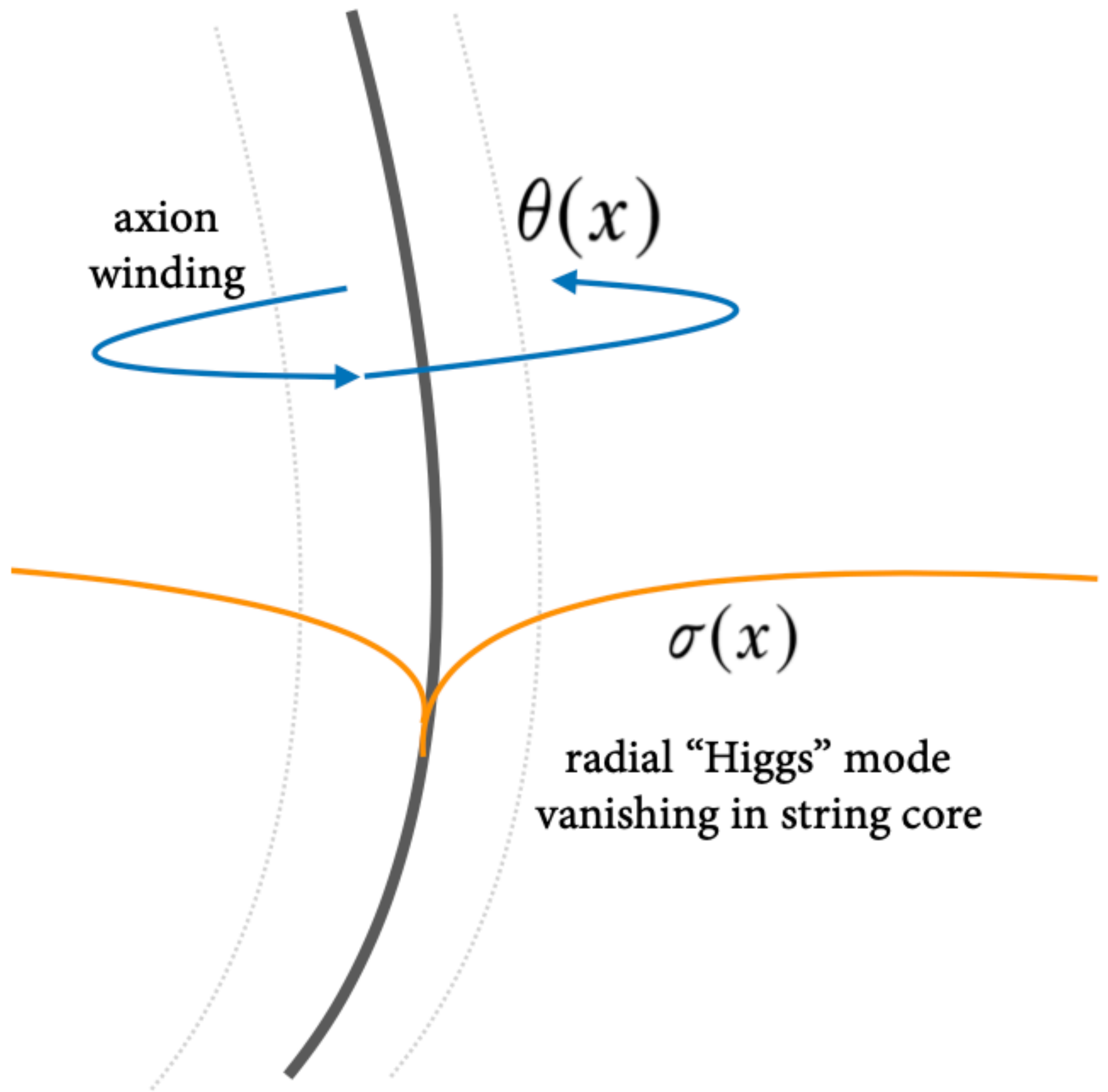
(can be more complicated in multi-field cases).

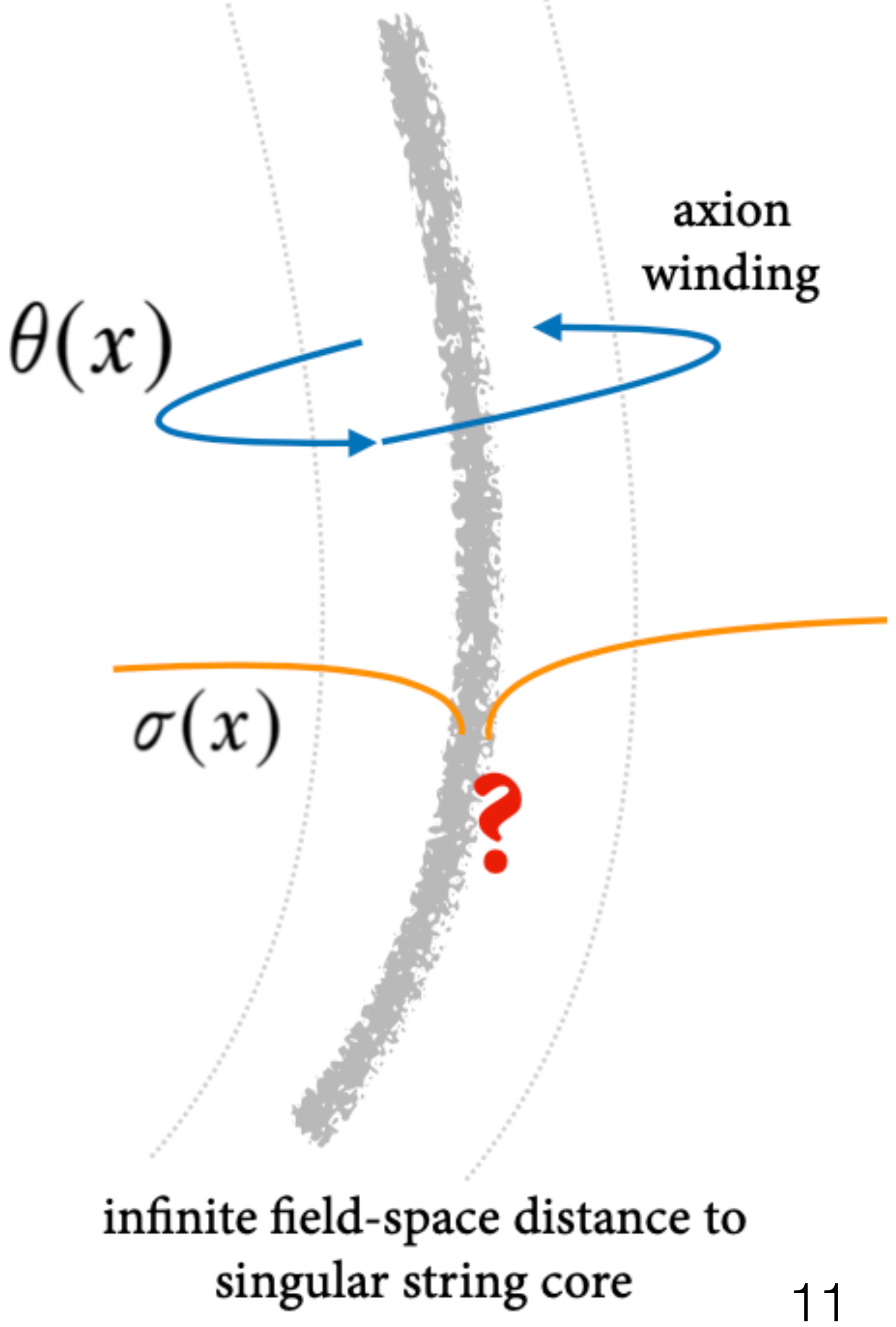
Aspects of Axion Fields

decay constant f^2

$$\mathscr{L} \supset M_*^2 \partial_\mu (\log S) \partial^\mu (\log S) + \frac{M_*^2}{S^2} \partial_\mu \theta \partial^\mu \theta$$

Axion decay constant is S-dependent, and never zero at finite distance.
"Fundamental axion": PQ symmetry is never restored.





Axion strings are fundamental objects (e.g., F-string, wrapped D-brane).

Conventional Axions vs Fundamental Axions

Conventional axion:

- Pseudo-Goldstone boson for $U(1)_{PQ}$
- PQ phase transition forms EFT strings
- f_a is an ordinary 4d scale, string tension typically $\sim f_a^2$

Fundamental axion:

- Pseudo-Goldstone boson only for $\partial_\mu \theta$
- No phase transition, axion strings fundamental
- f_a a UV scale (~KK scale), string tension potentially as large as $f_a M_{\mathrm{Pl}}$

Axions Remove Instanton Number Label

The axion has a job to do in QG:

instanton number density

$$\frac{1}{2} f_a^2 (\partial\theta)^2 + \frac{\theta}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \quad \Rightarrow \quad \partial^\mu (f_a^2 \partial_\mu \theta) = \frac{1}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{a\mu\nu}$$

Gauss law constraint! Axion causes would-be invariant in spacetime (instanton number) to vanish: integral of total derivative.

The axion serves to **gauge** a would-be (-1)-form global $U(1)$ symmetry. But this is qualitative! Can we guide experiments more?

arXiv:2012.00009, Ben Heidenreich, Jake McNamara, Miguel Montero, MR, Tom Rudelius, Irene Valenzuela

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Weak Gravity Conjecture (WGC)

Exists electrically charged object with:

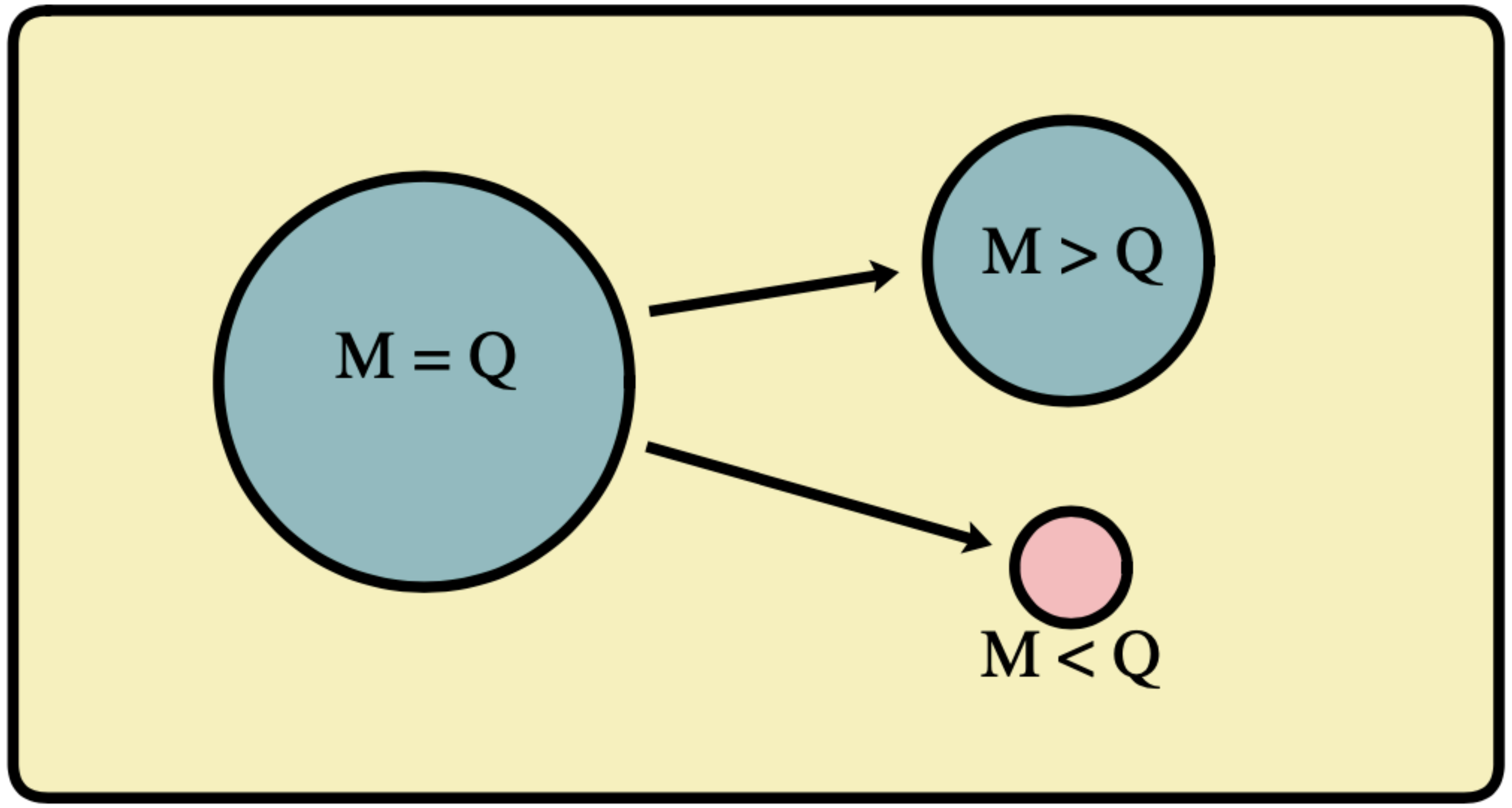
$$m < \sqrt{2} e q M_{\rm Pl}$$

Electric/Magnetic duality
⇒ exists magnetically charged object with:

$$m_{\rm mag} < \sqrt{2} \frac{2\pi}{e} q_{\rm mag} M_{\rm Pl}$$

(⇒ UV cutoff $\lesssim eM_{\rm Pl}$)

hep-th/0601001, Arkani-Hamed, Motl, Nicolis, Vafa



Necessary condition for discharge of extremal black holes.

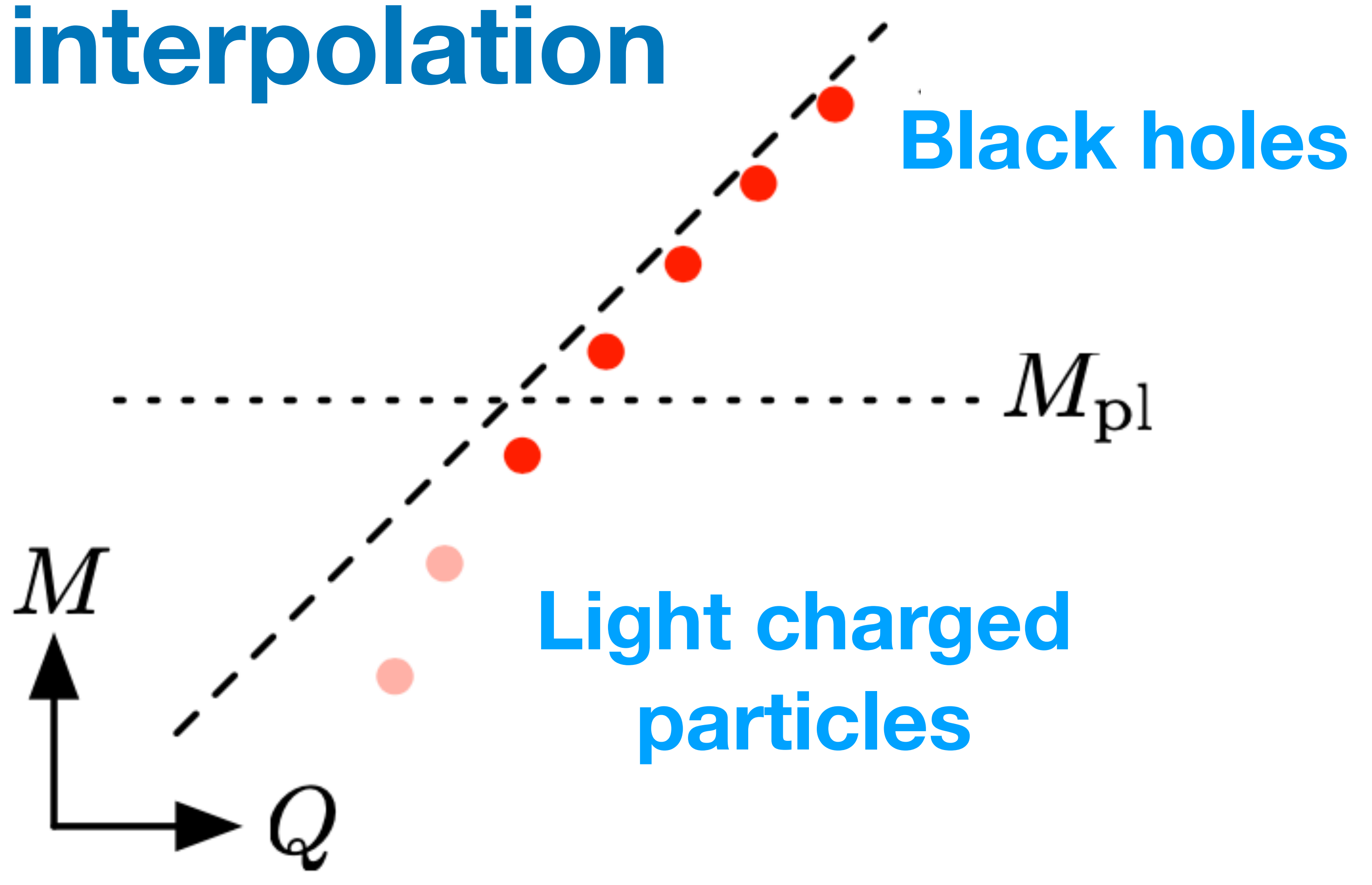
Tower Weak Gravity Conjecture

$\Lambda \lesssim eM_{\rm Pl}$ is our cutoff energy. But what happens there?

Internal consistency under dimensional reduction / examples:

There is always an infinite *tower* of charged particles of different charge q, *each of which* obeys the bound $m < \sqrt{2}eqM_{\rm Pl}$.

Smooth interpolation



2015-2017: Ben Heidenreich, MR, Tom Rudelius

(related: Montero, Shiu, Soler ’16; Andriolo, Junghans, Noumi, Shiu ’18)

p-Form Weak Gravity Conjecture

General (p-form) case: $-\frac{1}{4e_p^2}F^2_{\mu_1\cdots\mu_{p+1}}$, exists a charged $(p-1)$-brane with tension

$$T_p \lesssim e_p q M_{\rm Pl}$$

by analogy (or dimensional reduction),

Axion (0-form) case: $\frac{1}{2}f_a^2(\partial_\mu\theta)^2$, exists a charged **instanton** with action

$$S \lesssim \frac{q}{f_a}M_{\rm Pl}$$

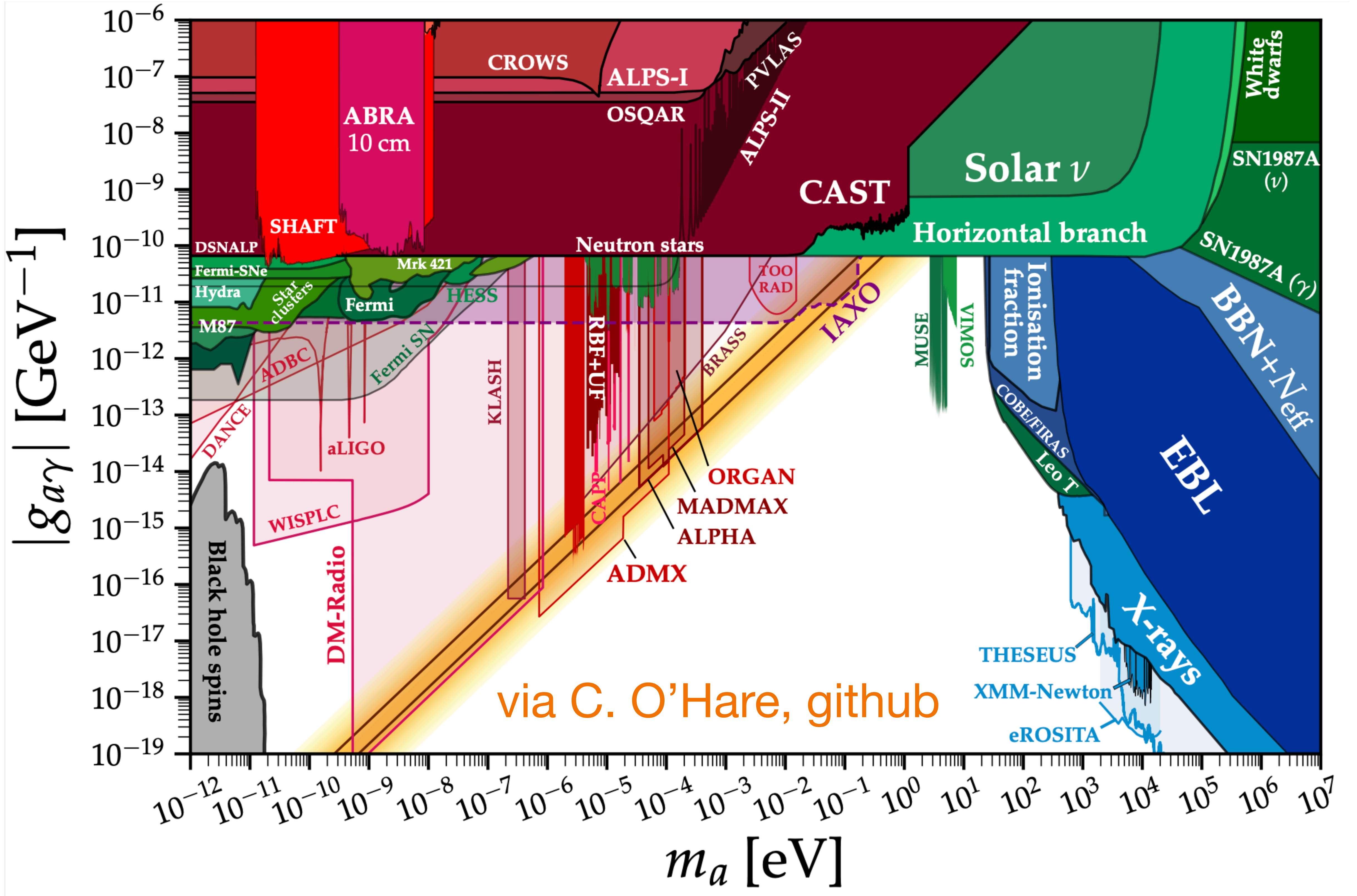
Axions and the WGC

Axion as “0-form gauge field”: $S_{\text{inst}} \lesssim \frac{1}{f_a} M_{\text{Pl}}$.

Given $\theta\, \text{tr}(F \wedge F)$,
S_{inst} from usual QCD instantons:

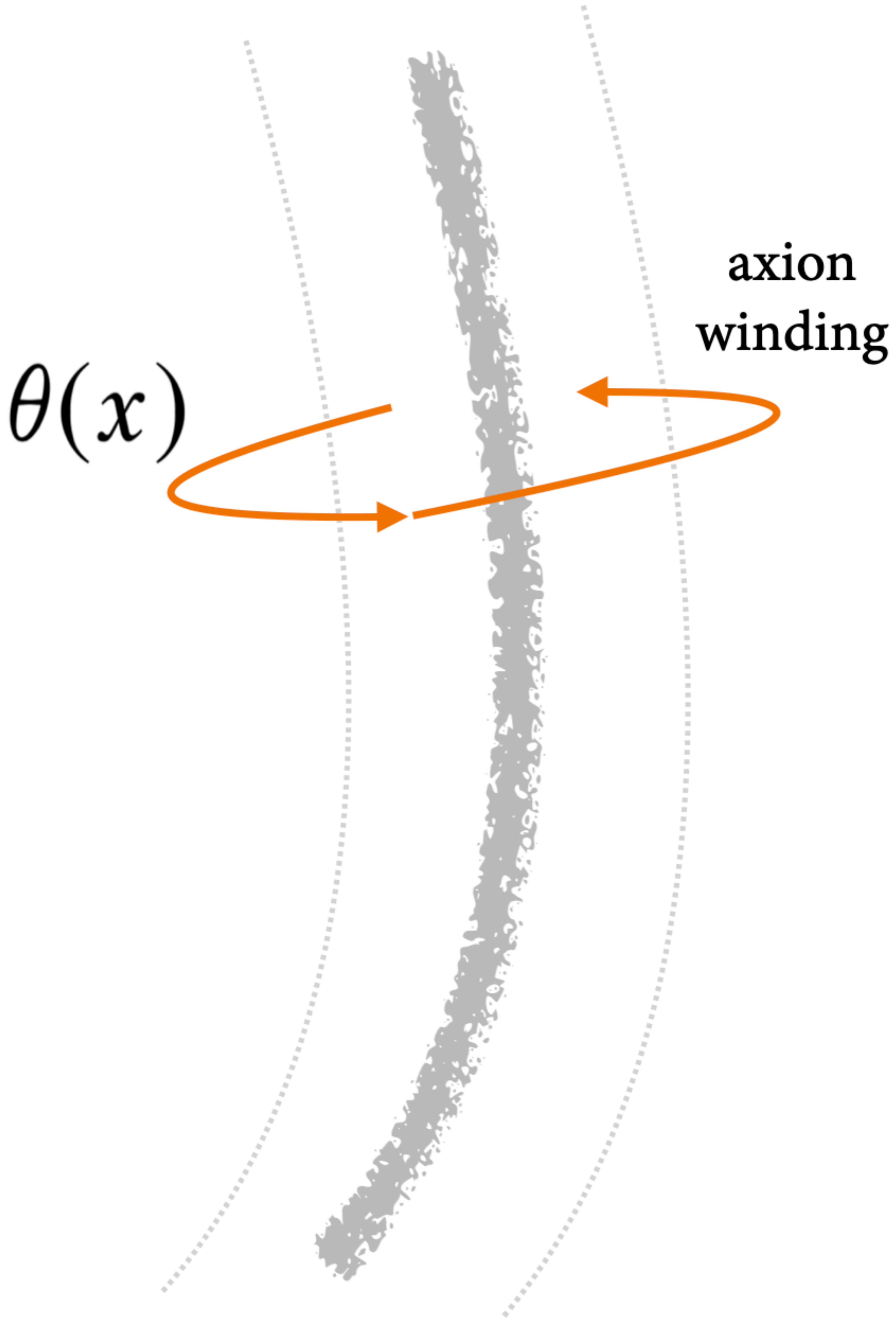
$$f_a \lesssim \frac{g^2}{8\pi^2} M_{\text{Pl}}$$

Nontrivial phenomenological prediction!
QCD axion with $f_a \lesssim 1.5 \times 10^{16}\,\text{GeV}$.



Axion Strings

arXiv:2108.11383 Ben Heidenreich, MR, Tom Rudelius



Assume $\theta F \wedge F$ coupling.

4d axion has a “magnetic dual” 2-form B-field: $\partial^\mu \theta \sim \epsilon^{\mu\nu\rho\sigma}\partial_{[\nu}B_{\rho\sigma]}$

Magnetic axion WGC: string tension

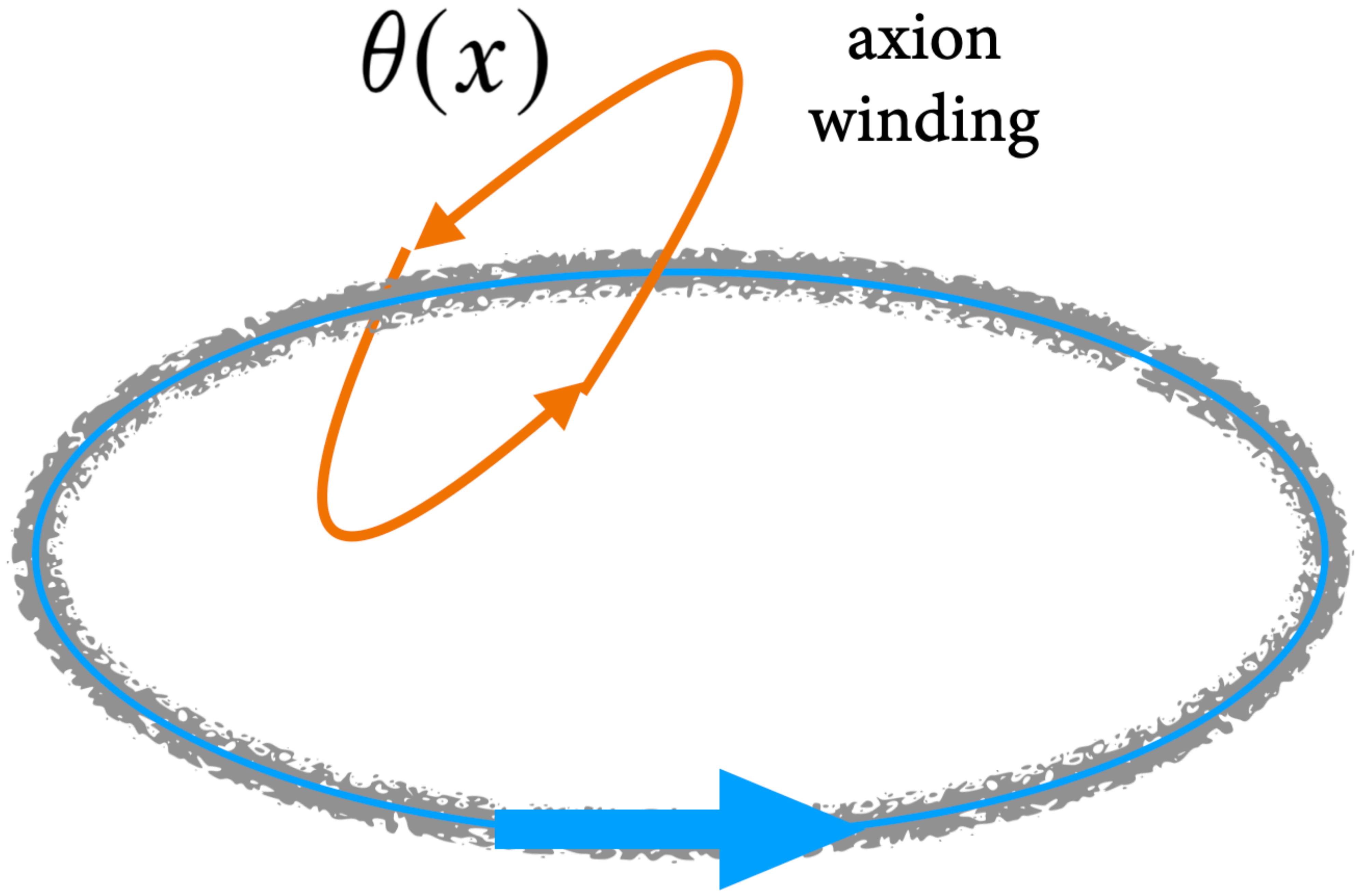
$$T \lesssim 2\pi f_a M_{\rm Pl} \lesssim \frac{g^2}{4\pi} M_{\rm Pl}^2$$

String excitations $M_{\rm string} \lesssim g M_{\rm Pl}$

— at the ordinary *gauge field's* WGC scale!

Tower WGC Modes from Axion Strings

arXiv:2108.11383 Ben Heidenreich, MR, Tom Rudelius



String excitations $M_{\text{string}} \lesssim g M_{\text{Pl}}$.

In fact, these can can carry A gauge charge!

"**Anomaly inflow**" (Callan, Harvey 1985)

$\theta F \wedge F$ interaction $\Rightarrow$ nontrivial gauge invariance, $A \mapsto A + \mathrm{d}\lambda, B \mapsto B + \frac{1}{4\pi}\lambda F$.

Charged modes on string cancel the λF.

Tower WGC automatic, via axion physics!

What about abelian case? What instantons?

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New Origin of Axion Potential

It is well known that for axion coupling to non-Abelian gauge group, instantons generate a potential for the axion.

Yet for axion coupling to ***abelian*** gauge fields, the axion could still acquire a potential through ***loops of magnetic monopoles***.
(Fan, Fraser, MR, Stout 2021, just published in Phys.Rev.Lett.)

Existence of magnetic monopoles: "***completeness hypothesis***"
Polchinski 2003

The Witten Effect

Add the time-reversal odd term in the action: $\frac{\theta}{8\pi^2}\int F \wedge F$

Then, derive the modified Maxwell equations.

Electric Gauss's law: $\nabla \cdot \mathbf{E} + \frac{e^2}{4\pi^2}\theta(\nabla \cdot \mathbf{B}) = 0$

Consider a magnetic monopole, which sources **B** $\Rightarrow$ $\frac{Q_E}{e} = -\frac{\theta}{2\pi}$

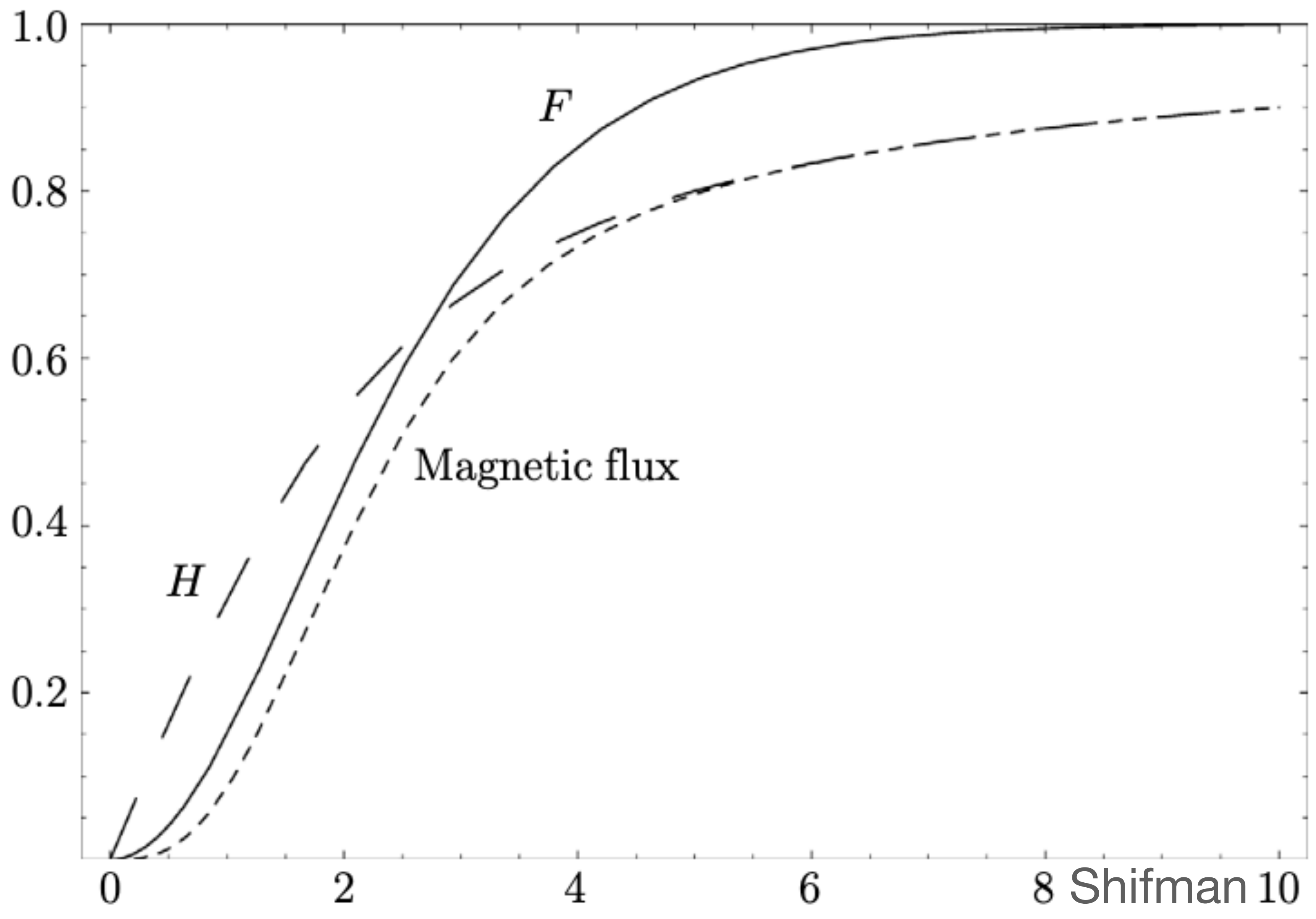
Magnetic monopole acquires an electric charge!

Edward Witten, 1979

Magnetic monopole provides boundary condition allowing effect. We haven't seen one (yet), so no experimental probe of this T-violating effect.

Monopole Refresher: ’t Hooft-Polyakov

$SU(2) \to U(1)$ symmetry broken by an adjoint vev: classical solution of ’t Hooft-Polyakov (’t H-P) monopole.



$$\phi^a = v\hat{r}^a H(r), \quad A_i^a = \epsilon^{aij}\frac{1}{r}\hat{r}^j F(r)$$

$$r \to \infty : \quad H(r) \to 1,\ F(r) \to 1$$

$$r \to 0 : \quad H(r) \to 0,\ F(r) \to 0$$

The solution has **4 zero modes** (collective coordinates): 3 translations, 1 U(1) (large gauge transformation, not vanishing at infinity).

review: Shifman, *Advanced Topics in Quantum Field Theory,* Chapter 4

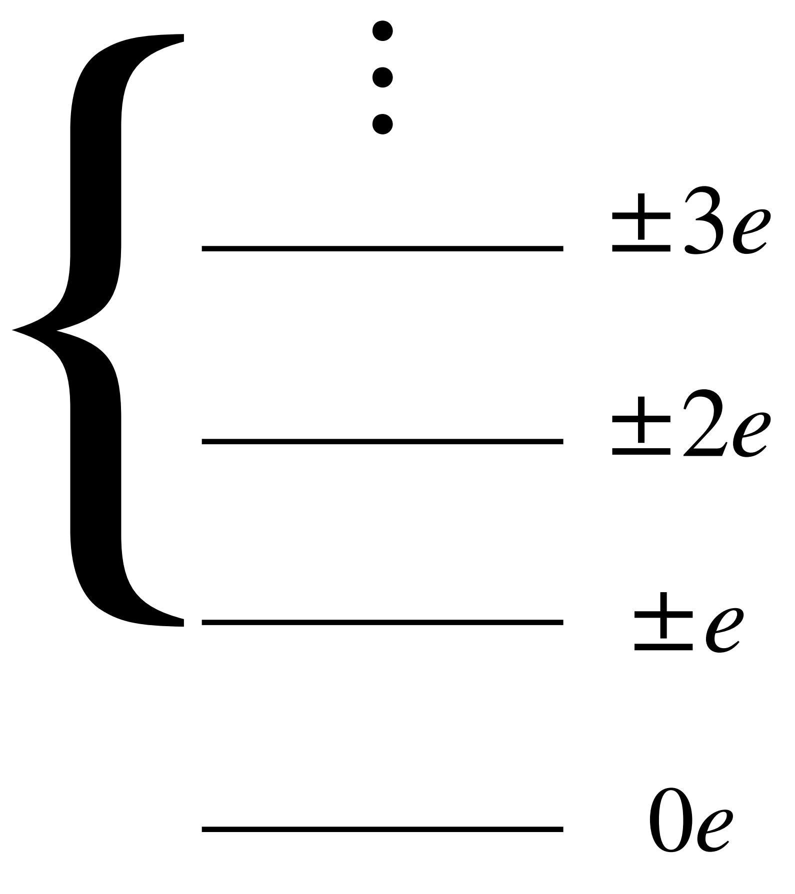
The Dyon Tower

Possible charged states: not only magnetic monopoles, but also ***dyons*** (particles with both magnetic and electric charges).

Monopole worldline EFT: compact scalar $\sigma \cong \sigma + 2\pi$ (dyonic collective coordinate). **Generic consequence** of $\theta F \wedge F$ (anomaly inflow).

Quantum particle on a circle: spectrum labeled by integers (charges!)

Dyon tower



excited states $m_n^2 = m_M^2 + m_\Delta^2 n^2$

ground state $m_0^2 = m_M^2$

The Witten Effect for Dyons

Modified charges: $$\frac{Q_E}{e} = n - \frac{\theta}{2\pi}, \quad n = 0, \pm 1, \pm 2, \cdots$$

σ: dyonic collective coordinate $$L = \frac{1}{2}\dot{\sigma}^2 + \frac{\theta}{2\pi}\dot{\sigma}$$

Conjugate momentum: $$\Pi_\sigma = \dot{\sigma} + \frac{\theta}{2\pi}$$

Hamiltonian:

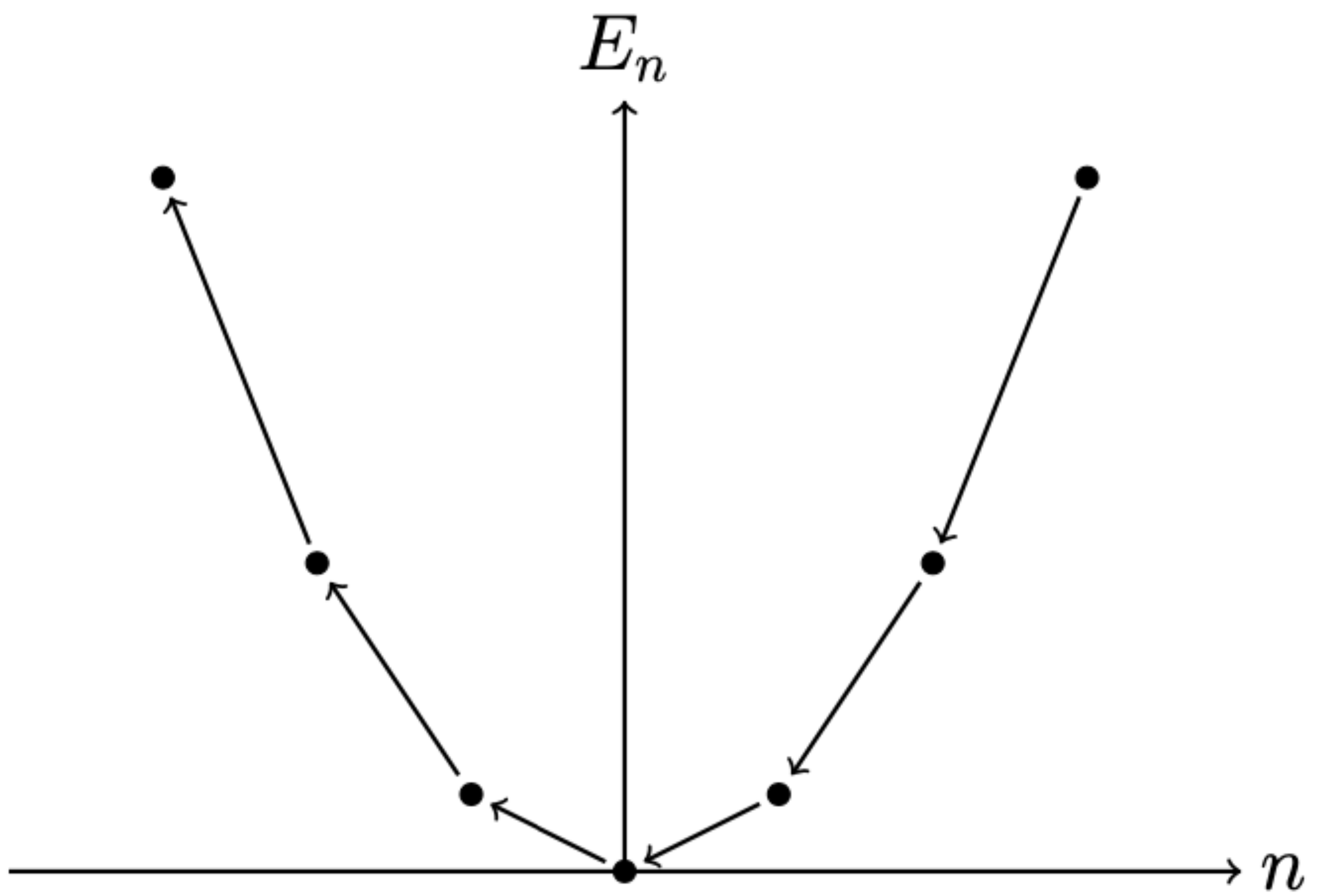
$$H = \frac{1}{2}\left(\Pi_\sigma - \frac{\theta}{2\pi}\right)^2 \quad \Rightarrow \quad E_n = \frac{1}{2}\left(n - \frac{\theta}{2\pi}\right)^2$$

$$\frac{1}{2}\left(-i\partial_\sigma - \frac{\theta}{2\pi}\right)^2 \psi_n = E_n \psi_n$$

The corresponding energy spectrum

$$m_n^2 - m_M^2 = m_\Delta^2 \left(n - \frac{\theta}{2\pi} \right)^2$$

ground state monopole mass at $\theta = 0$



periodicity through **"monodromy"** or rearrangement of the eigenstates:

$$n \to n+1, \quad \theta \to \theta + 2\pi$$

Integrating out these states $\Rightarrow$ vacuum potential for the axion θ!

Note: ***different*** from the axion potential generated by ***monopole and anti-monopole plasma***! Fischler, Preskill 1983; Kawasaki, Takahashi, Yamada 2015; Nomura, Rajendran, Sanches 2015; …

A plasma of monopoles and anti-monopoles could be generated through the Kibble-Zurek mechanism in the early Universe.

Here we talk about the axion potential from the ***virtual*** effects of monopole (dyon) loops.

Two Viewpoints

1. Integrate out the dyons to get a Coleman-Weinberg potential for axion.

2. Do the path integral over all monopole loops.

Related by Poisson resummation

invariant length

transition amplitude $\langle x | x \rangle_\tau$

$$V_{\text{eff}} = -\int_0^\infty \frac{\mathrm{d}\tau}{2\tau} \frac{1}{2(2\pi\tau)^2} \exp\left(-\frac{m^2\tau}{2}\right)$$

$$m_n^2 = m_{\text{M}}^2 + m_\Delta^2 \left(n - \frac{\theta}{2\pi}\right)^2$$

$$-\sum_{n\in\mathbb{Z}} \int_0^\infty \frac{\mathrm{d}\tau}{4\tau\,(2\pi\tau)^2} \exp\left(-\frac{m_{\text{M}}^2\tau}{2} - \frac{m_\Delta^2\tau}{2}\left(n - \frac{\theta}{2\pi}\right)^2\right).$$

Poisson resum

$$\sum_{n\in\mathbb{Z}} \mathrm{e}^{-\frac{1}{2}m_\Delta^2\tau\left(n-\frac{\theta}{2\pi}\right)^2} = \sum_{\ell\in\mathbb{Z}} \sqrt{\frac{2\pi}{m_\Delta^2\tau}} \exp\left(-\frac{2\pi^2\ell^2}{m_\Delta^2\tau} + i\ell\theta\right).$$

$$V_{\text{eff}}(\theta) = -\sum_{\ell=1}^{\infty} \frac{m_\Delta^2 m_{\text{M}}^2}{32\pi^4 \ell^3} \mathrm{e}^{-2\pi\ell m_{\text{M}}/m_\Delta} \cos(\ell\theta) \times \left(1 + \frac{3m_\Delta}{2\pi\ell m_{\text{M}}} + \frac{3m_\Delta^2}{(2\pi\ell m_{\text{M}})^2}\right),$$

winding number in the σ direction

Fan, Fraser, MR, Stout, 2021

$$V_{\text{eff}}(\theta) = -\sum_{\ell=1}^{\infty} \frac{m_\Delta^2 m_{\text{M}}^2}{32\pi^4 \ell^3} e^{-2\pi\ell m_{\text{M}}/m_\Delta} \cos(\ell\theta) \times \left(1 + \frac{3m_\Delta}{2\pi\ell m_{\text{M}}} + \frac{3m_\Delta^2}{(2\pi\ell m_{\text{M}})^2}\right),$$

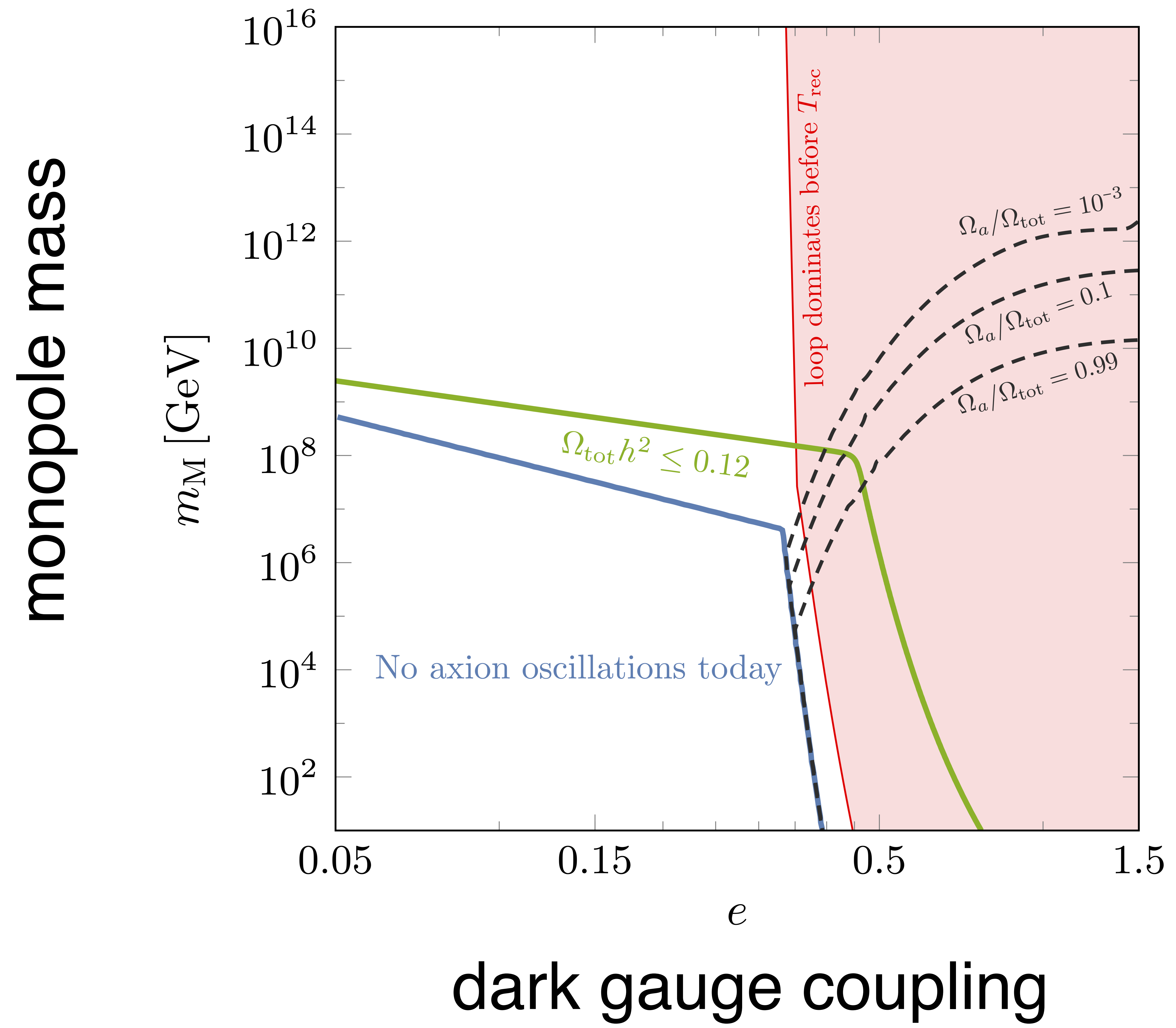
winding number in the σ direction

$e^{-S_{\text{inst}}} \sim e^{-8\pi^2/g^2}$ in 't H-P model: same instanton action as in YM theory!

Fan, Fraser, MR, Stout, 2021

In a **hidden** gauged $U(1)$ sector with an axion and monopoles: both axion and monopole contribute to DM

$$m_a(T) = m_a^{\text{loop}} + m_a^{\text{plasma}}(T)$$



Caveat:

Assumes no light charged fermions!

Work in progress for SM case (w/ Fan, Fraser, Stout, Telem)

Summary

Axions have a job to do in quantum gravity: eliminating a global (*instanton number*) symmetry by gauging it.

Fundamental axions need not be ordinary pseudo-Nambu-Goldstone bosons: no point in field space where Peccei-Quinn is restored.

Charged modes on fundamental axion strings (Callan-Harvey) are the predicted Weak Gravity Conjecture towers, when $\theta F \wedge F$ couplings are present.

The localized worldline fields on **virtual magnetic monopoles** lead to axion potentials.

Minimum mass for axion coupled to photons? Depends on subtleties about fermion mass dependence. Work in progress (w/ Fan, Fraser, Stout, Telem)

Thank You!