AstroDark-2021

Cosmological Tests of the Early Matter-dominated Epoch by Observing Cosmic Neutrino Background, Gravitational Wave Background and Primordial Black Holes

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Abstract

We can confirm the existence of the early matter-dominated epoch by observing

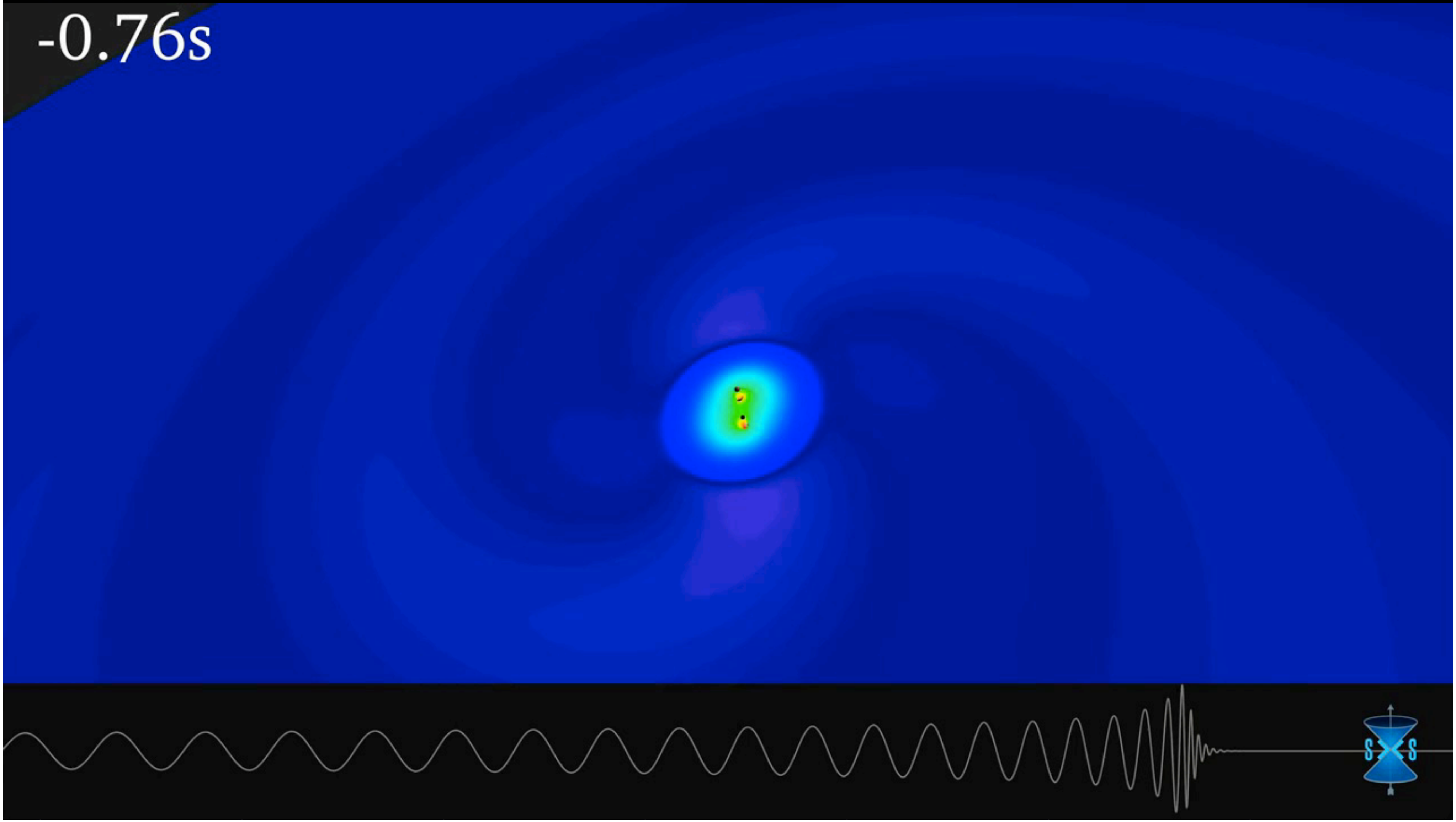
- Spin of the primordial black holes ($a_* \sim 1$)
- Characteristic signals of stochastic gravitational wave (the Poltergeist mechanism)
- Effective number of neutrino species ($N_\nu < 3$)

Why PBHs?

- We can probe high-energy physics, the early Universe, and gravity with PBHs through recent and future gravitational wave observations

LIGO and Virgo have detected gravitational wave signals from Binary Black Holes

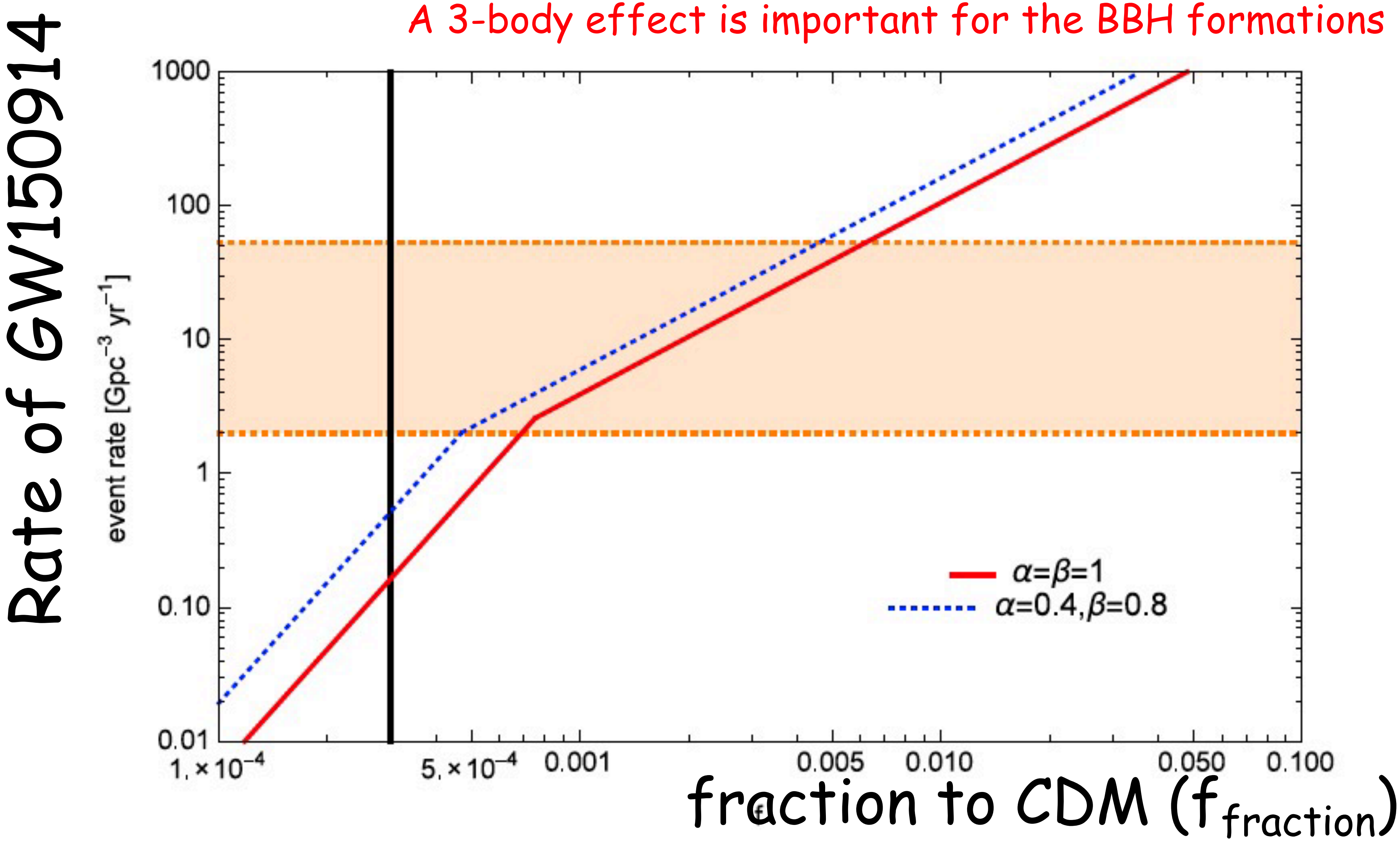
https://www.youtube.com/watch?v=1agm33iEAuo



GW150914 and its merger rates for 30 M_{solar} masses BBH

M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama (2016).

A 3-body effect is important for the BBH formations



How to produce the binary black holes (BBH) with masses of O(10) $M_{\odot}$?

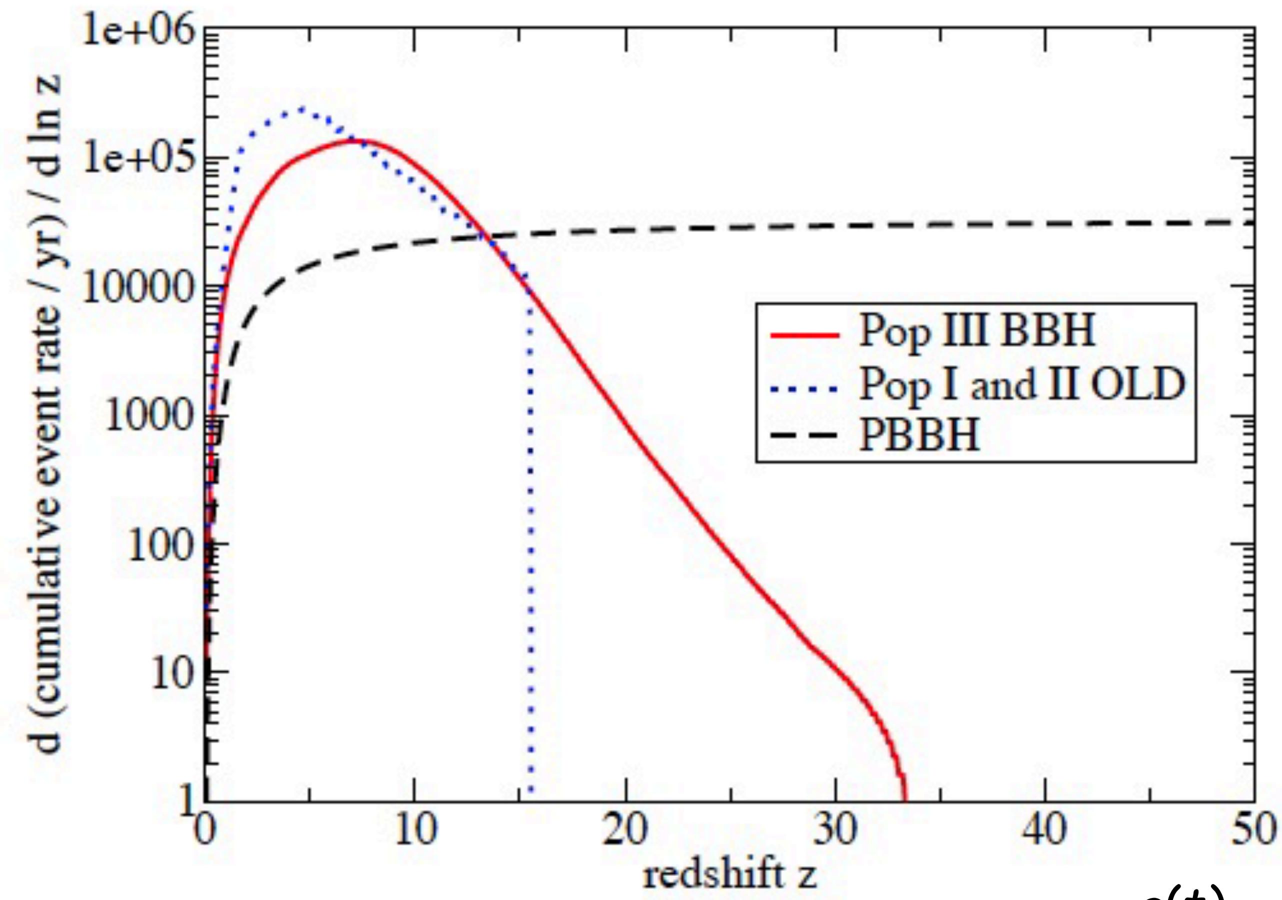
- Astrophysics: Large uncertainties on gravitational frictions through common envelope phases, mechanisms of supernovae (SNe) and appropriate kick velocities after SNe for Pop III/Pop II stars [astrophysically-model dependent]

- Cosmology: large uncertainties on numbers of PBHs, which depend on inflation models

  [cosmologically-model dependent]

DECIGO discriminates BPBHs from the normal BBHs

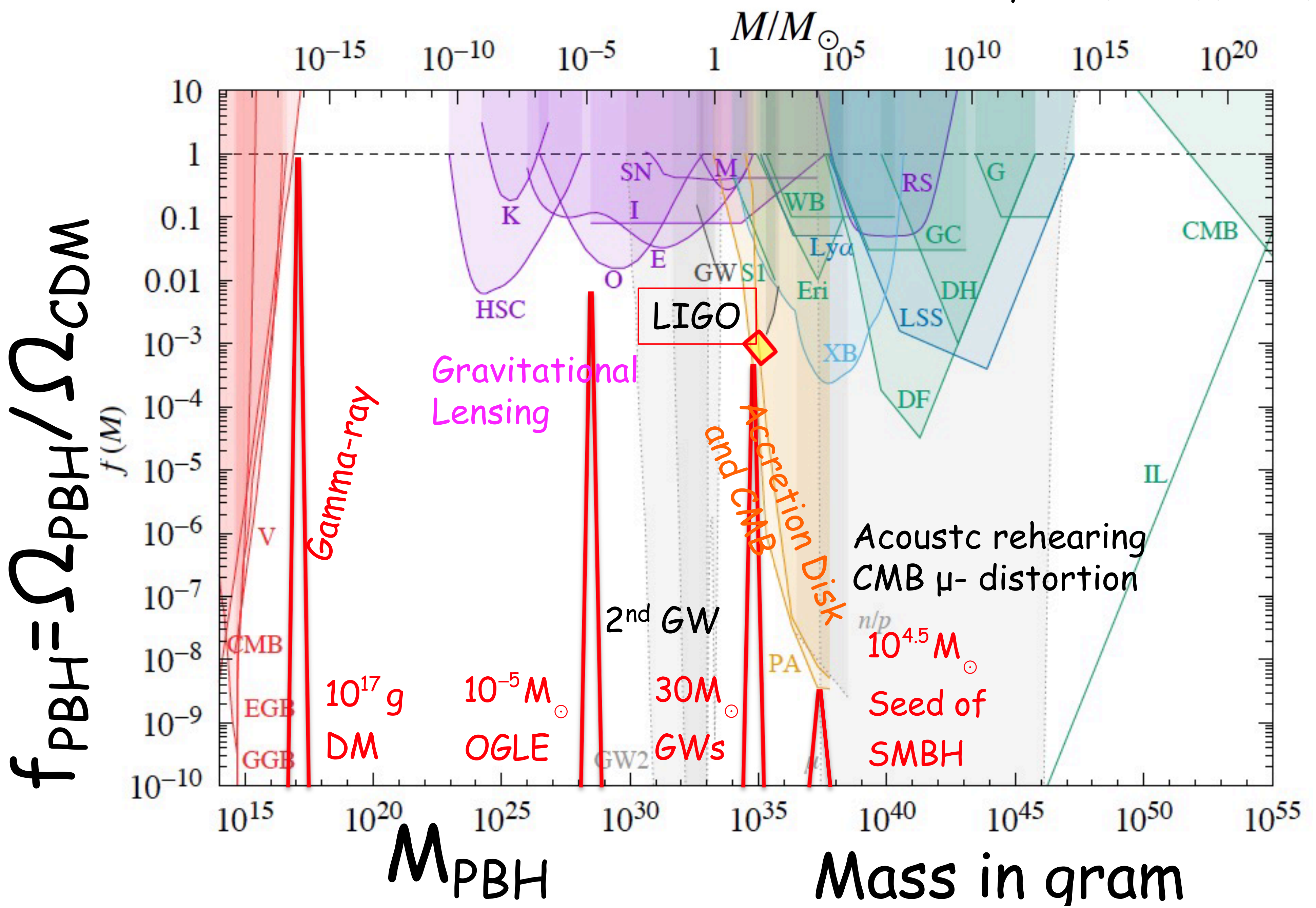
Takashi Nakamura et al, arXiv:1607.00897 [astro-ph.HE]



$$1/z \sim \frac{a(t)}{a(t_0)} \sim \left(t/10\text{Gyr}\right)^{2/3}$$

Upper bounds on the fraction to CDM

Carr, Kohri, Sendouda, J.Yokoyama (2009)(2020)



Formation

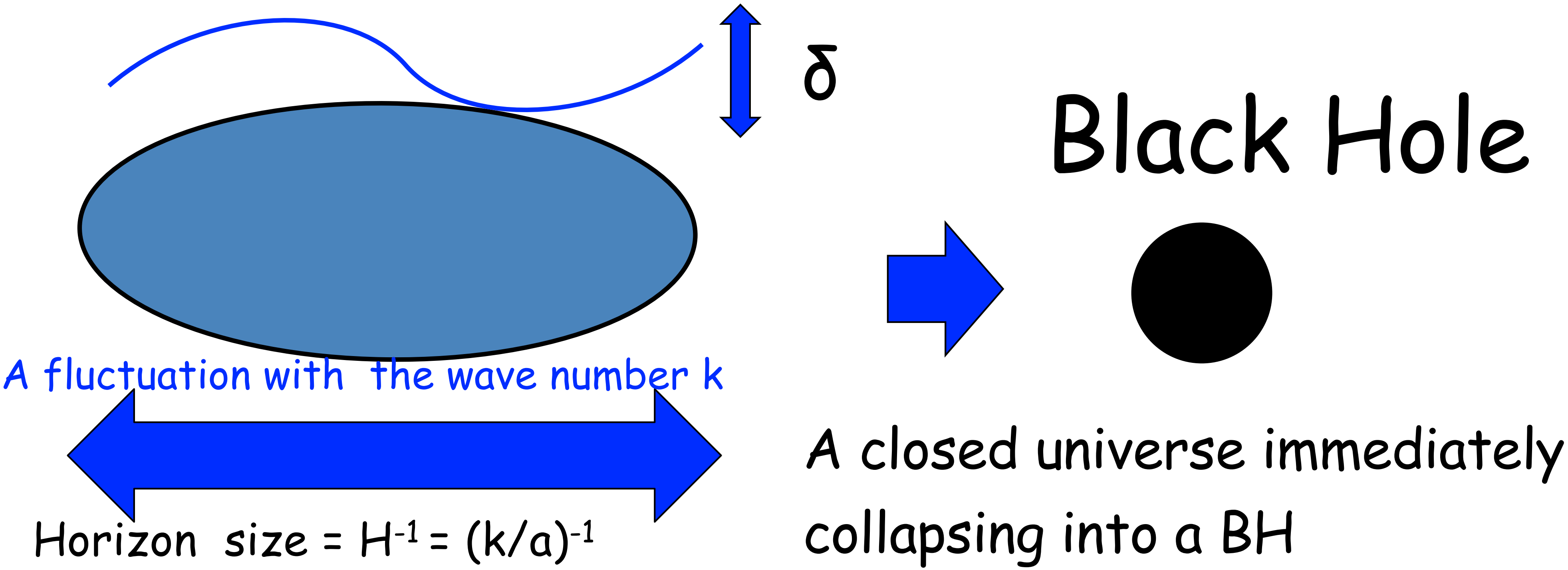
Conditions for a PBH formation in Radiation dominated (RD) Universe

Zel'dovich and Novikov (1967), Hawking (1971), Carr (1975)

Harada,Yoo and KK (2013)

- Gravity could be stronger than pressure

$$\delta > \delta_c \sim p / \rho \sim c_s^2 = w = 1/3$$



P_ζ (k) and PBH abundance β(M)

- Fraction of PBH to the total with Press Schechter formalism

For Peak Statistics,
e.g., see Yoo, Harada, KK et al (2018)(2020)

$$\beta(M) \equiv \frac{\rho_{\rm PBH}(M)}{\rho_{\rm tot}} = \int_{\delta_{\rm th}}^{\infty} d\delta \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) = {\rm erfc}\left(\frac{\delta_{\rm th}}{\sqrt{2}\sigma}\right)$$

$\delta_{\rm th}$: $\sim 1/3 - 0.5$

For analytical derivations, see Harada, Yoo, KK (2013)

$\sigma \sim \overline{\delta\rho/\rho}$

- Relation between β and fluctuation σ (or β and Ω)

$$\beta(M) \sim {\rm erfc}\left(\frac{\delta_{\rm th}}{\sqrt{2}\sigma}\right) \simeq \sqrt{\frac{2}{\pi}}\frac{\sigma}{\delta_{\rm th}} \exp\left(-\frac{\delta_{\rm th}^2}{2\sigma^2}\right)$$

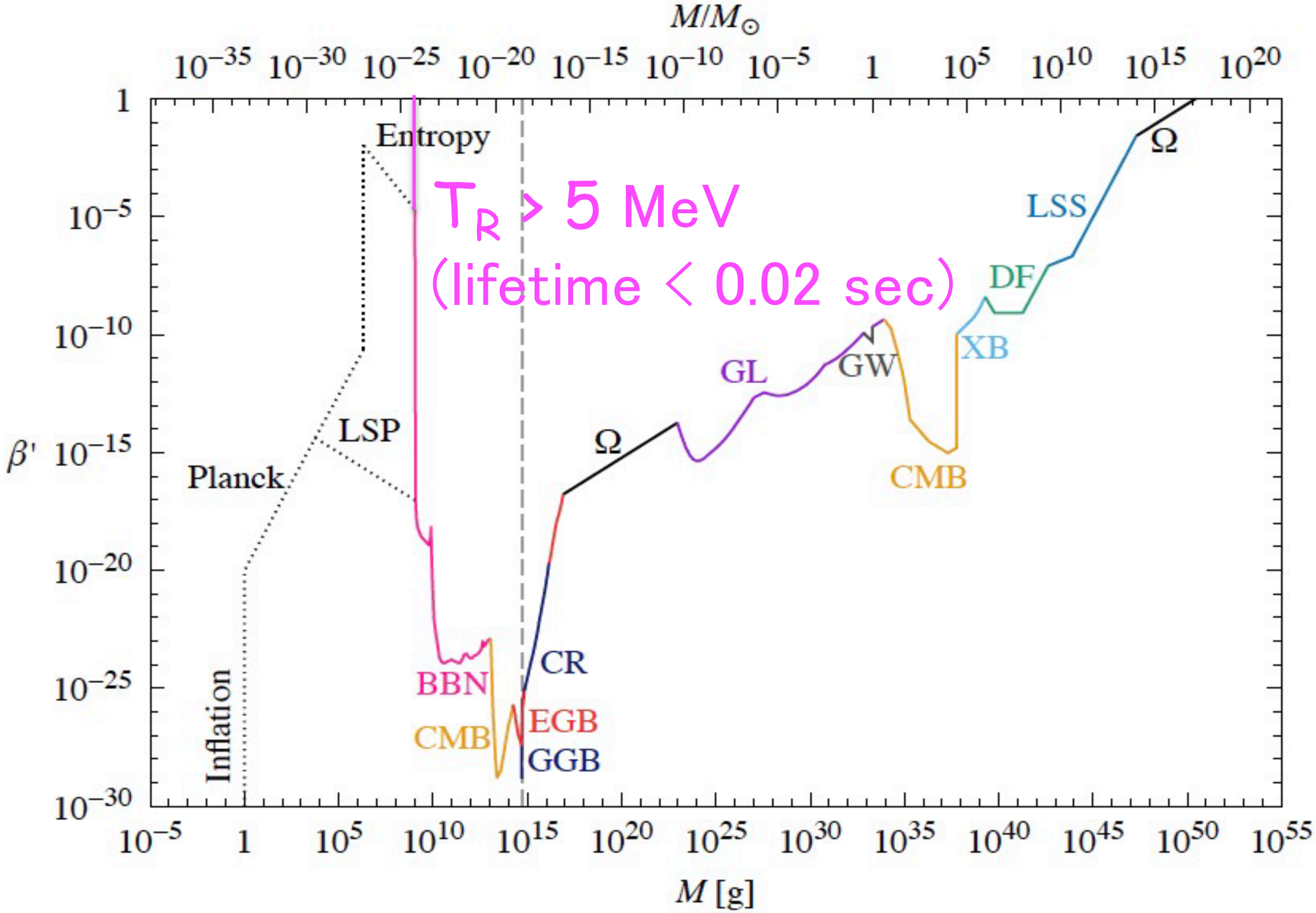
σ^2: $\sim P_\zeta$

$$= 1.5 \times 10^{-18} \left(\frac{m_{\rm PBH}}{10^{15} g}\right)^{1/2} \left(\frac{\Omega_{\rm PBH} h^2}{0.1}\right)$$

$\beta=\rho_{PBH}/\rho_{tot}$ vs M_{PBH}

Carr, Kohri, Sendouda, J.Yokoyama (2009)(2020)

$\beta=\rho_{PBH}/\rho_{tot}$



M_{PBH} (g)

Typical quantities of PBHs in RD

- Mass (horizon mass $=\rho(t_{form})\, H(t_{form})^{-3}$)

$$M_{\rm PBH} \sim \rho\left(H_{\rm form}^{-1}\right)^3 \sim M_{pl}^2 t_{\rm from} \sim \frac{M_{pl}^3}{T_{\rm form}^2} \sim 10^{15} g \left(\frac{T_{\rm form}}{3\times 10^8 {\rm GeV}}\right)^{-2} \sim 5{\times}10^4 M_\odot \left(\frac{T_{\rm form}}{\rm MeV}\right)^{-2}$$

- Lifetime

$$\tau_{\rm PBH} \sim \frac{M_{\rm PBH}^3}{M_{pl}^4} \sim 4\times 10^{17}\,{\rm sec}\left(\frac{M_{\rm PBH}}{10^{15}\ {\rm g}}\right)^3 \sim 1\,{\rm sec}\left(\frac{M_{\rm PBH}}{10^9\ {\rm g}}\right)^3$$

- Hawking Temperature

$$T_{\rm PBH} \sim \frac{M_{\rm pl}^2}{M_{\rm PBH}} \sim 10\,{\rm MeV}\left(\frac{M_{\rm PBH}}{10^{15}{\rm g}}\right)^{-1} \sim 2\times 10^{-9}{\rm K}\left(\frac{M_{\rm PBH}}{30 M_\odot}\right)^{-1}$$

- Wave number of horizon length

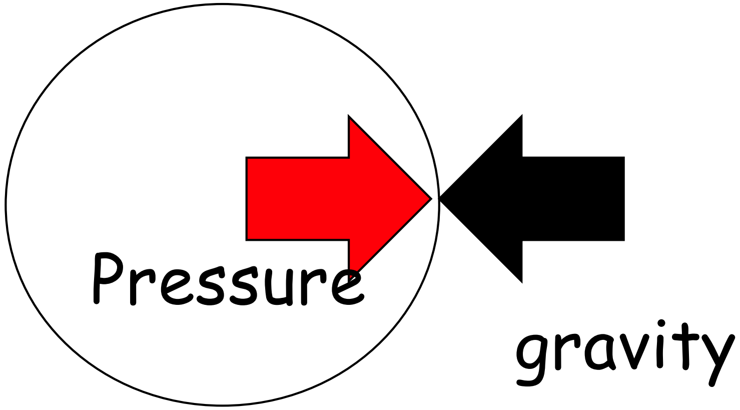
$$k = aH \sim 10^5{\rm Mpc}^{-1}\left(\frac{M_{\rm PBH}}{5\times 10^4 M_\odot}\right)^{-1/2} \sim 10^5{\rm Mpc}^{-1}\left(\frac{T_{\rm form}}{\rm MeV}\right)^{+1}$$

- Fraction to CDM

$$f_{\rm fraction} \equiv \frac{\Omega_{\rm PBH}}{\Omega_{\rm CDM}} \sim \left(\frac{\beta}{10^{-18}}\right)\left(\frac{M_{\rm PBH}}{10^{15} g}\right)^{-1/2} \sim \left(\frac{\beta}{10^{-8}}\right)\left(\frac{M_{\rm PBH}}{30 M_\odot}\right)^{-1/2} \sim 10^8 \left(\frac{M_{\rm PBH}}{30 M_\odot}\right)^{-1/2} \sqrt{P_\delta}\, \exp\left[-\frac{1}{18 P_\delta}\right]$$

Features of PBH formations in RD

- Spherical due to radiation pressure



$(w \equiv p/\rho \sim 1/3)$

- Negligible evolutions of density perturbations
- Quite a small angular momentum

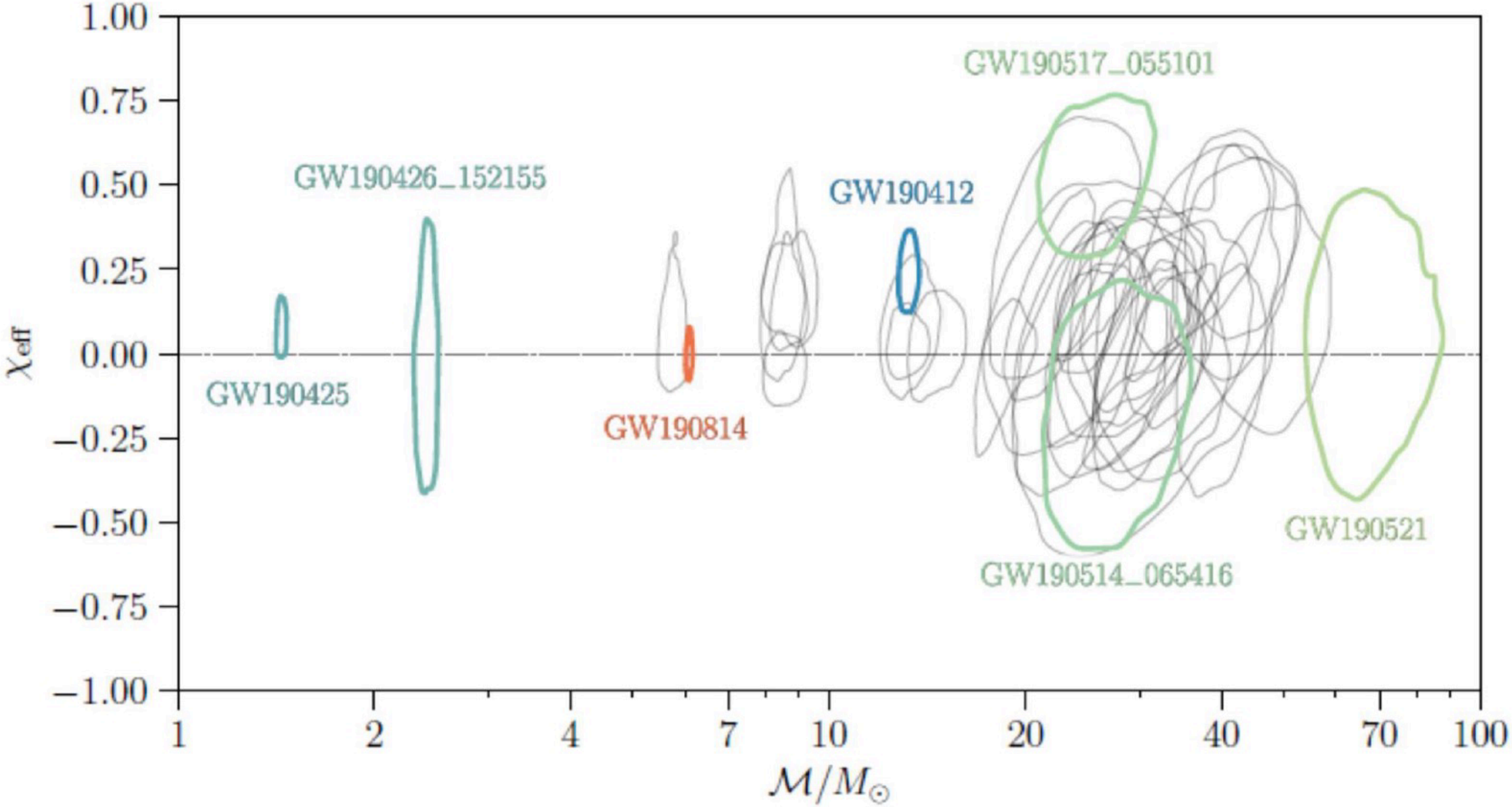
See, T.Chiba and S.Yokoyama, 2017
De Luca et al, 2019
Minxi He and Suyama, 2019
Harada, Yoo, Kohri, Koga and Monobe, 2020

(dimensionless Kerr parameter)

$$\sqrt{\langle a_*^2 \rangle} \simeq 6.5 \times 10^{-4} \left(\frac{M}{M_H} \right)^{-1/3}$$

Effective inspiral spin parameter of the observed BHs

$$\chi_{\rm eff} = \frac{m_1\chi_1\cos\theta_1 + m_2\chi_2\cos\theta_2}{m_1+m_2}$$

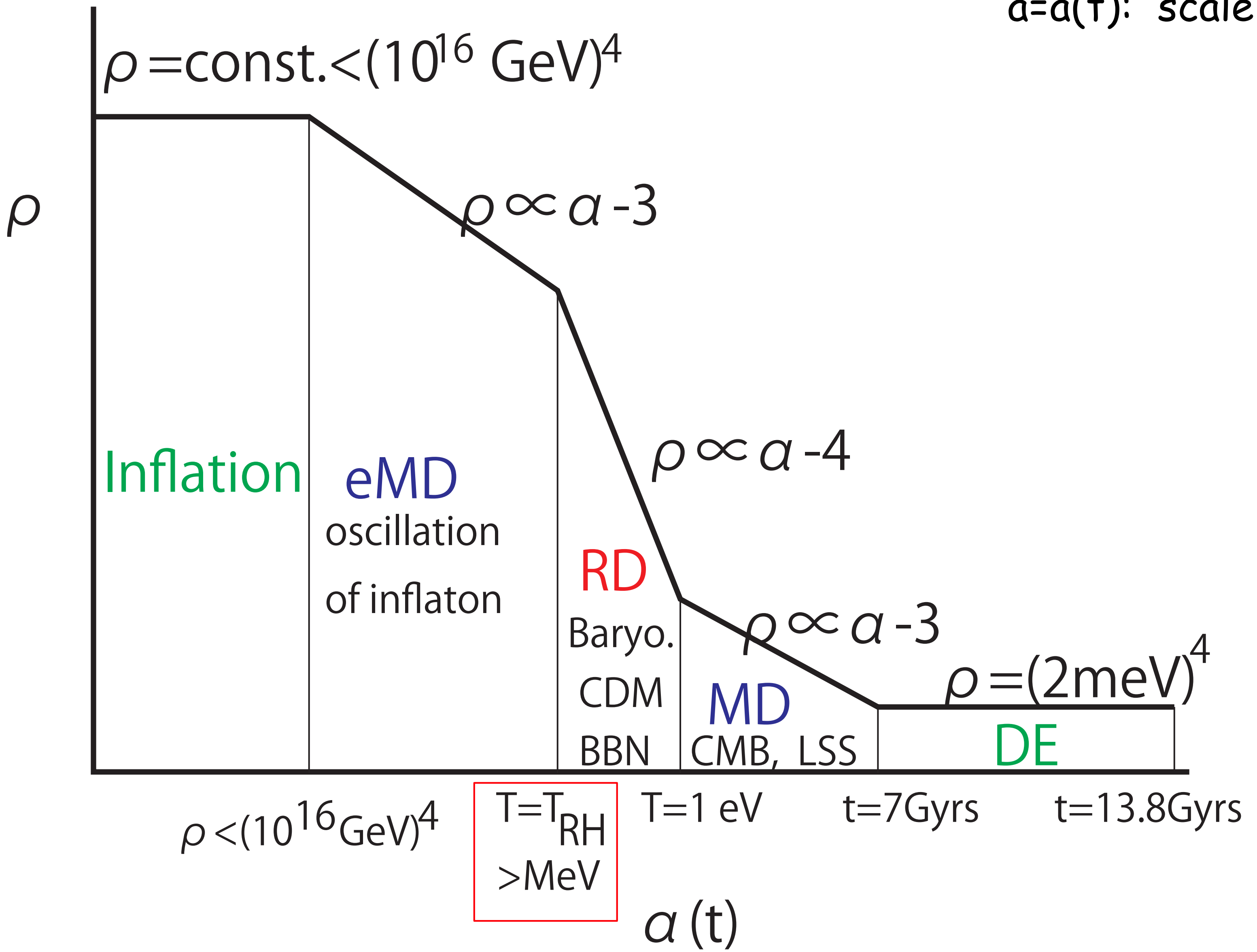


Credible region contours for all candidate events in the plane of chirp mass $\mathcal{M}$ and effective inspiral spin \chieff. Each contour represents the 90\% credible region for a different event. We highlighted the previously published candidate events (cf.\ Fig.~\ref{fig:mtotqpost}), as well as \protect\NAME{GW190517A}\ and \protect\NAME{GW190514A}, which have the highest probabilities of having the largest and smallest \chieff respectively.

R. Abbott, et al, LSC P&P Committee, arXiv:2010.14527 [gr-qc]

Cosmic history of energy density

a=a(t): scale factor



1) MeV-scale reheating temperature

Freeze out of weak interaction between n and p at T ~ O(1) MeV

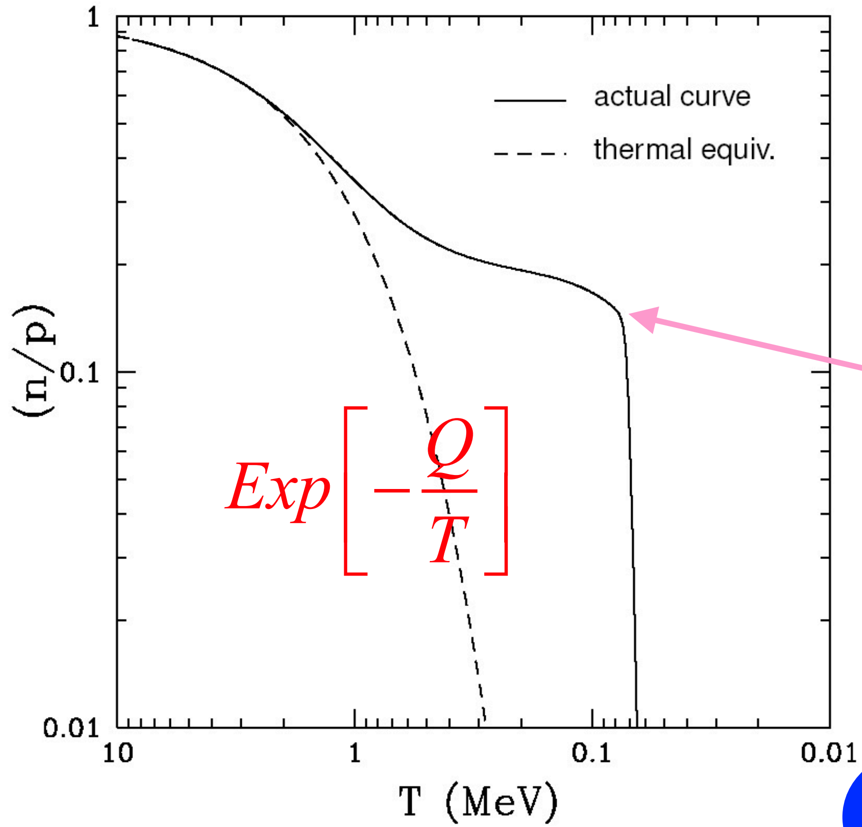
- Weak interaction between p and n

$$n \leftrightarrow p + e^- + \bar{\nu}_e \, ,$$

$$e^+ + n \leftrightarrow p + \bar{\nu}_e \, ,$$

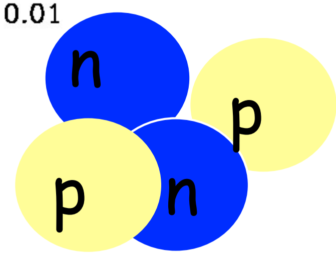
$$\nu_e + n \leftrightarrow p + e^- \, ,$$

He4 mass fraction Y



$$\left(\frac{n_n}{n_p}\right)_{\text{freezeout}} \approx \frac{1}{7}$$

$^4\mathrm{He}$



$$n_{^4\mathrm{He}} = n_n / 2$$

$$Y_p \equiv \frac{\rho_{^4\mathrm{He}}}{\rho_B} \approx \frac{4\times m_N \times n_{^4\mathrm{He}}}{m_N \times (n_n + n_p)} \approx \frac{2(n_n / n_p)_{\text{freezeout}}}{(n_n / n_p)_{\text{freezeout}} + 1} \approx 0.25$$

Helium 4 mass fraction Y

- Primordial value of mass fraction Y

$$Y_p \sim \frac{1}{4} + 0.01\Delta N_\nu + 0.01\ln(\eta_{10}/6) + 0.1\left(\frac{\tau_n - 880sec}{880sec}\right)$$

小玉、井岡、郡著 「宇宙物理学」 (2014)

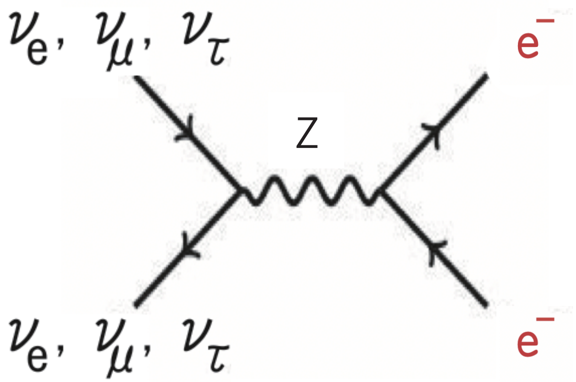
$$\Delta N_\nu = N_\nu - 3$$

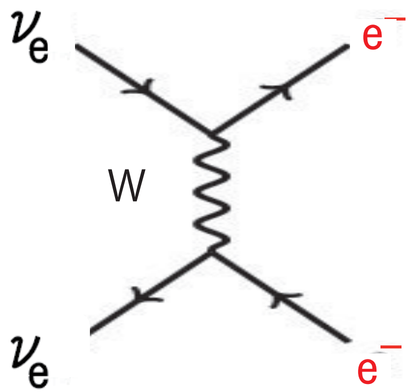
$$\eta = \frac{n_B}{n_\gamma} = \eta_{10} \times 10^{-10}$$: baryon to photon ratio

τ_n : neutron life time

Neutrino decoupling

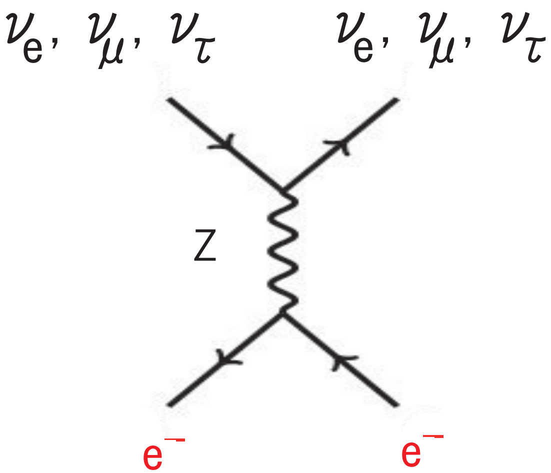
- Production and Annihilation (ν + ν ⟷ e+ + e-)

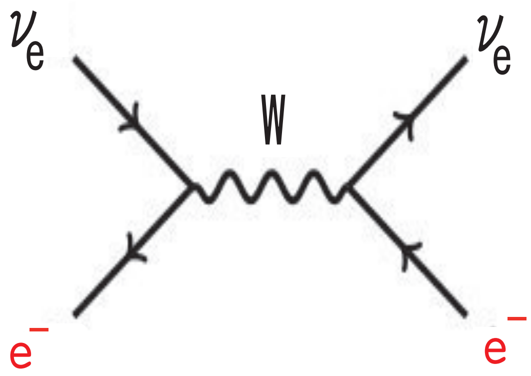




ν_e has stronger interactions with plasma through $W^{\pm}$

- Scattering (ν + e ⟷ ν + e)



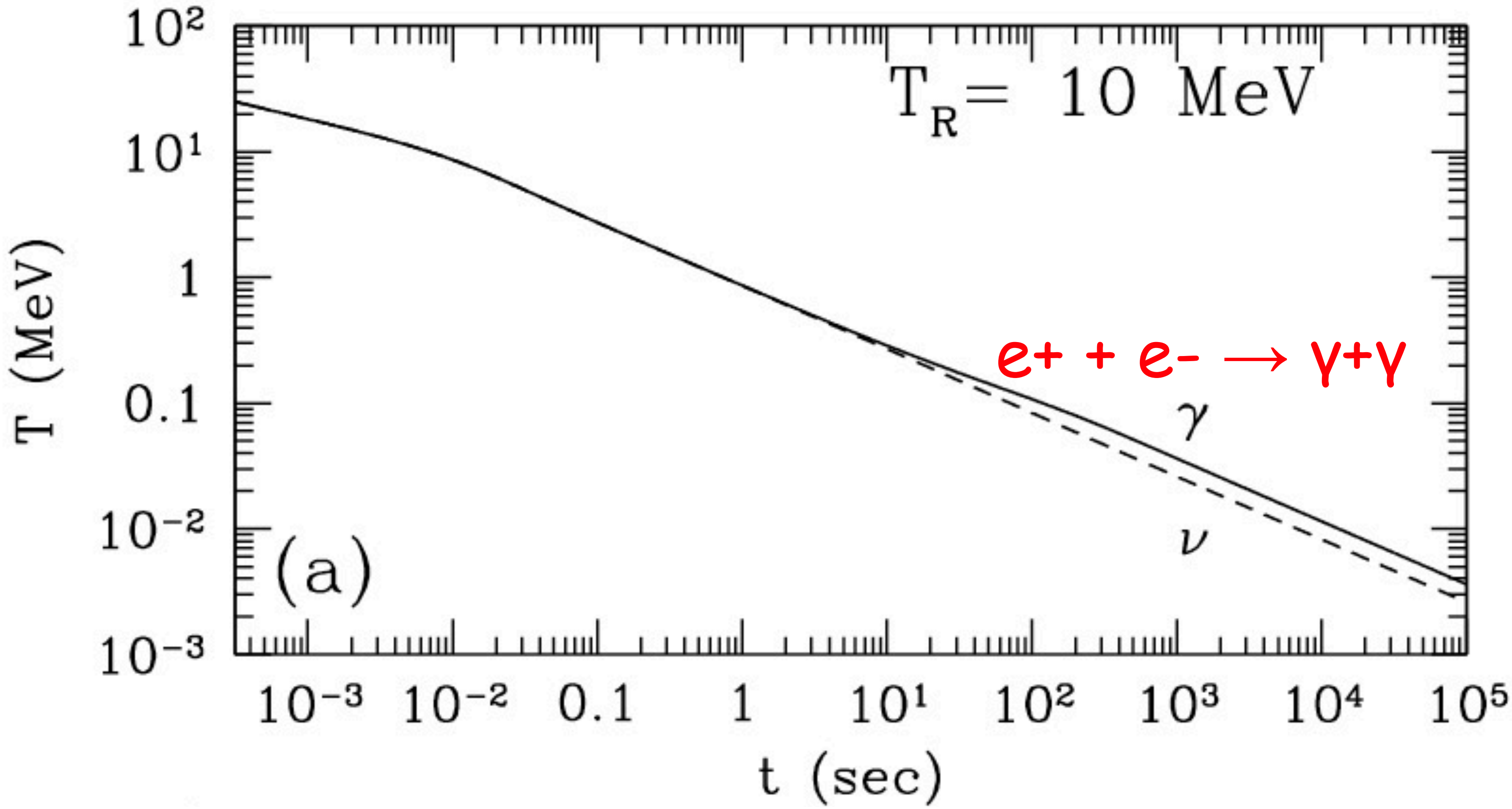


DECOUPLING:
3 MeV for ν_μ and ν_τ
2 MeV for ν_e

Time evolution of neutrinos

Kawasaki, Kohri, and Sugiyama, 1998; 2000

Only photons can be heated by e+e- annihilation at T = 0.511 MeV



Imperfect thermalization of neutrinos by MeV-scale reheating

Kawasaki, Kohri, and Sugiyama, 1998; 2000

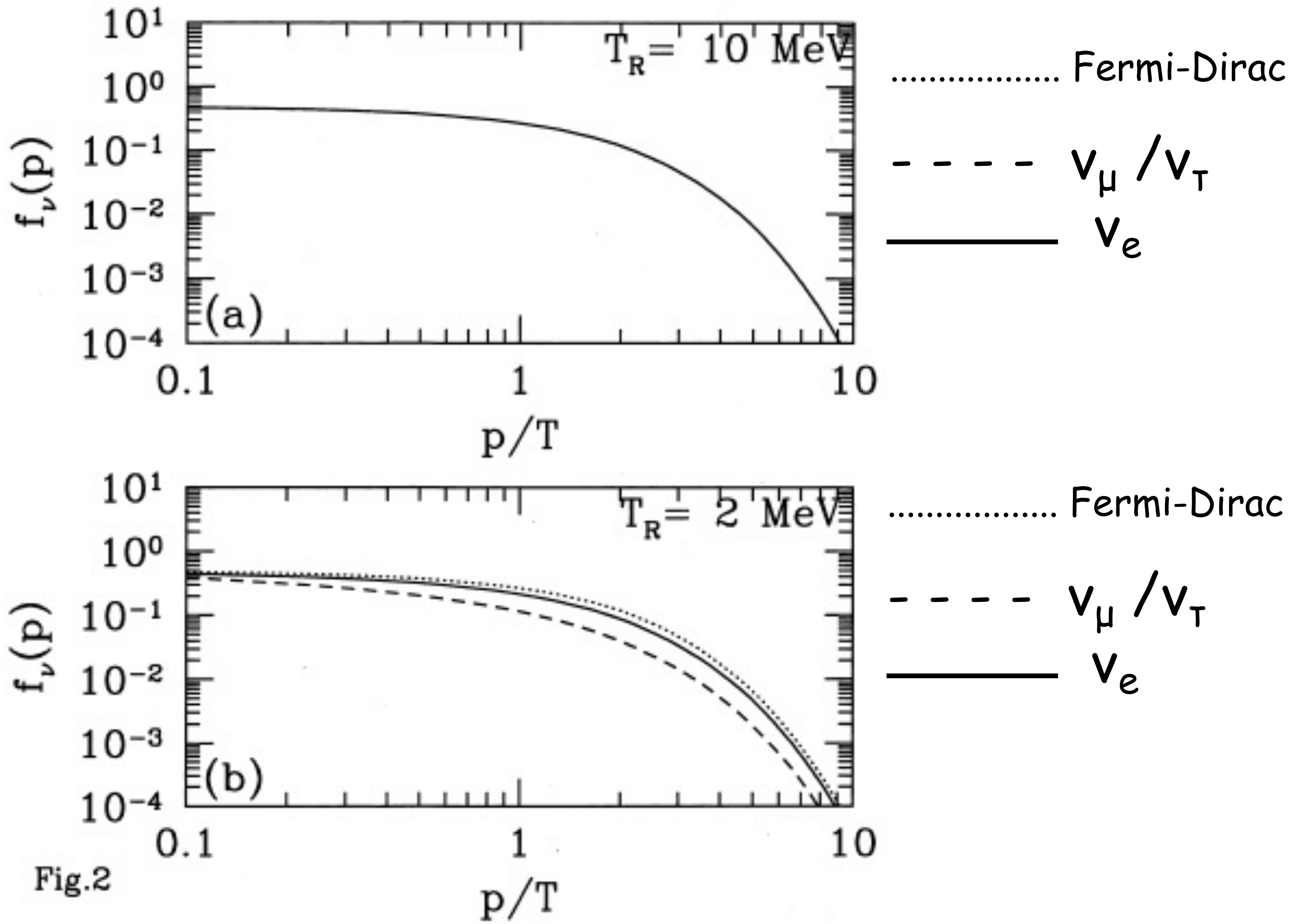


Fig.2

Neutrino IMPERFECT thermalisation and Big Bang Nucleosynthesis

- Modifications on interaction rates due to MeV reheating or oscillations among $\nu \leftrightarrow \nu$ and/or ν_τ

$$\Gamma_{n\nu_e \to pe^-} = K \int_0^\infty dp_{\nu_e} \left[\sqrt{(p_{\nu_e}+Q)^2 - m_e^2}\,(p_{\nu_e}+Q) \frac{p_{\nu_e}^2}{1+e^{-(p_{\nu_e}+Q)/T_\gamma}} f_{\nu_e}(p_{\nu_e}) \right]$$

$$\Delta\Gamma_{n\leftrightarrow p} < 0$$

$$\Delta Y \simeq +0.19(-\Delta\Gamma_{n\leftrightarrow p}/\Gamma_{n\leftrightarrow p})$$

- Modifications on energy density

$$N_\nu^{\rm eff} \equiv \frac{\rho_{\nu_e}+\rho_{\nu_\mu}+\rho_{\nu_\tau}}{\rho_{\rm STD}} < 3 \quad \rightarrow \Delta\rho_{tot} < 0$$

$$\Delta Y \simeq -0.10\,(-\Delta\rho_{\rm tot}\,/\rho_{\rm tot}\,)$$

Neutrino oscillations

- Vacuum oscillation

$$P(\nu_i \to \nu_j) = \sin^2 2\theta_{ij} \sin^2\left[\frac{L\delta m_{ij}^2}{4E}\right]$$

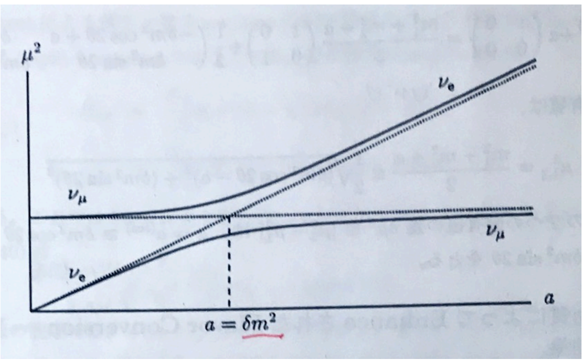
$\delta m_{ij}^2 = m_j^2 - m_i^2$

L : distance

E: energy

θ_{ij} : mixing angle

- MSW (matter effect)



$$P(\nu_i \to \nu_j) = 1 - \exp\left[-\pi \frac{\sin^2 2\theta_{ij}}{\cos 2\theta_{ij}} \frac{\delta m_{ij}^2}{4E} \frac{dt}{d\log n_e}\right]$$

$a = 2\sqrt{2} G_F n_e E$

n_e : electron #density

dt : time derivative

Neutrino oscillation in the early Universe

Quantum Kinetic Equation

$$\frac{d\varrho_p}{dt} = \frac{\partial \varrho_p}{\partial t} - Hp\frac{\partial \varrho_p}{\partial p} = -i\left[\mathcal{H}_p, \varrho_p\right] + C(\varrho_p)$$

ν oscillation $\quad$ ν production/collision

density matrix for ν (2 flavor) ν_e - ν_x $\qquad$ $e^- + e^+ \to \nu_\alpha + \nu_\alpha$

$$\varrho_p = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{ex}^* & \rho_{xx} \end{pmatrix}$$

diagonal: ν distribution

off-diagonal: flavor coherence

$$\mathcal{H}_p = \frac{\mathrm{M}^2}{2p} - \frac{8\sqrt{2}G_{\mathrm{F}}p}{3}\left[\frac{\boldsymbol{E}_l}{m_W^2} + \frac{\boldsymbol{E}_\nu}{m_Z^2}\right]$$

matter effect

$$\mathrm{M}^2 = U\mathcal{M}^2U^\dagger \quad \mathcal{M}^2 = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \quad U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

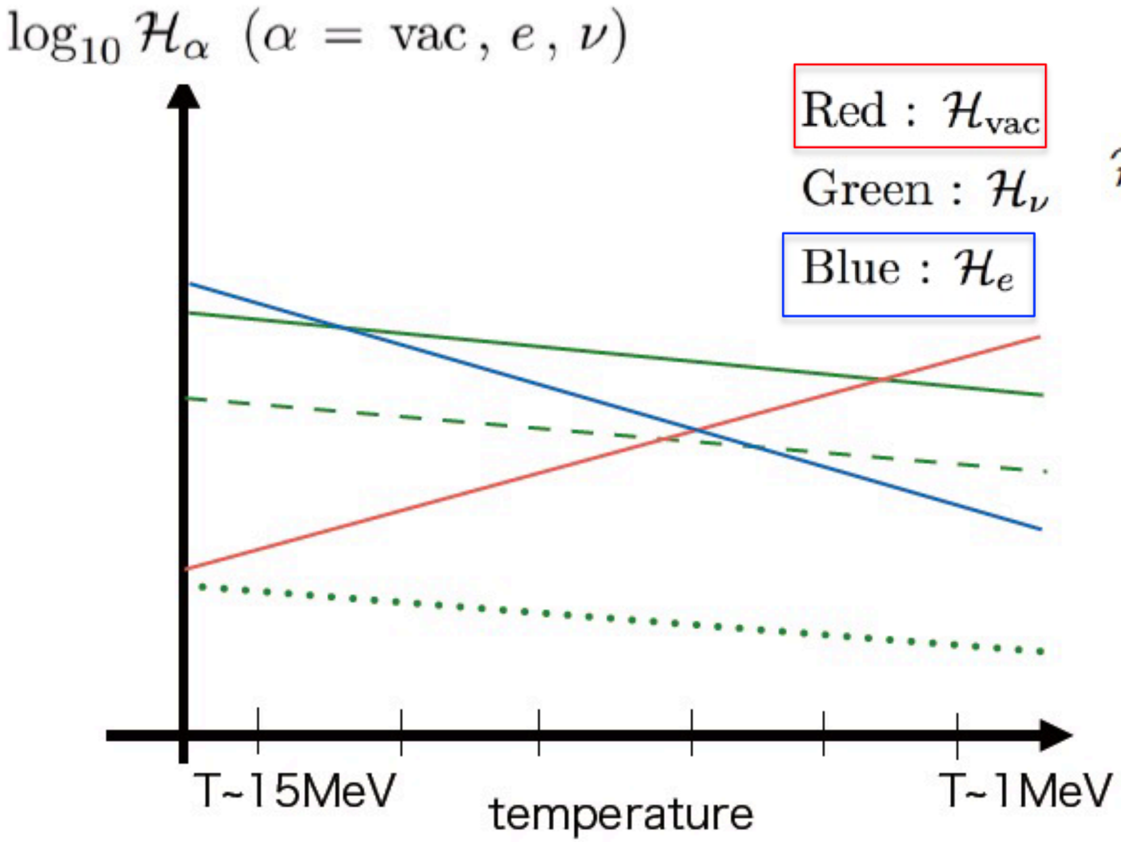
$$\boldsymbol{E}_l \sim \mathrm{diag}(\rho_e, 0) \quad \boldsymbol{E}_\nu \sim \mathrm{diag}(\rho_{\nu_e}, \rho_{\nu_x})$$

MSW-like effective mass difference in the early Universe

$$\frac{\delta m_M^2}{2p} = \sqrt{\left(\frac{\delta m^2}{2p}\right)^2 \sin^2 2\theta + (\mathcal{H}_{\rm vac} + \mathcal{H}_{\rm mat})^2}$$

$$\sin^2 2\theta_M = \frac{\left(\frac{\delta m^2}{2p}\right)^2 \sin^2 2\theta}{\left(\frac{\delta m^2}{2p}\right)^2 \sin^2 2\theta + (\mathcal{H}_{\rm vac} + \mathcal{H}_{\rm mat})^2}$$

MSW-type resonance (oscillation) in the early Universe



$$\mathcal{H} = \mathcal{H}_{\mathrm{vac}} + \mathcal{H}_e + \mathcal{H}_\nu$$
$$= \frac{\delta m^2}{2E}\mathrm{B} - \frac{8\sqrt{2}G_F E \varrho_{e^\pm}}{3m_W^2}\mathrm{L}$$
$$+ \frac{\sqrt{2}G_F}{2\pi^2}\int dE' E'^2 \left[\rho(E') - \bar{\rho}^*(E')\right]$$

$$\mathrm{B} = \mathrm{U}\left(\mathrm{diag}\left[-1/2, 1/2\right]\right)\mathrm{U}^\dagger, \mathrm{L} = \mathrm{diag}\left[1, 0\right]$$

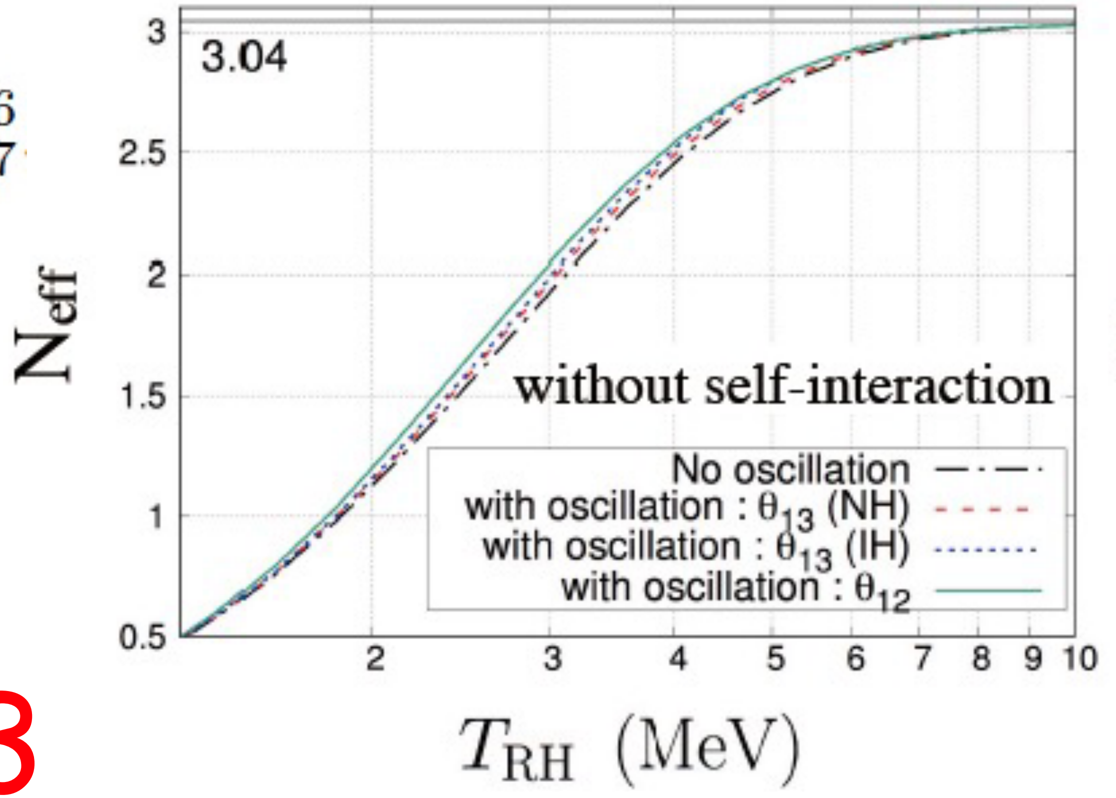
$$|\mathcal{H}_{\mathrm{mat}}| \approx |\mathcal{H}_{\mathrm{vac}}| \ \Rightarrow$$
$$T_c \approx G_F^{-1/3}(\delta m^2 \cos 2\theta)^{1/6} \approx \begin{cases} 3\,\mathrm{MeV}\left(\frac{\delta m_{12}^2}{2.5\times 10^{-3}\,\mathrm{eV}^2}\right)^{1/6} \\ 5\,\mathrm{MeV}\left(\frac{\delta m_{13}^2}{7.5\times 10^{-5}\,\mathrm{eV}^2}\right)^{1/6} \end{cases}$$

Thermalization of three active neutrinos

T. Hasegawa, Hiroshima, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012

See Planck 2018

$$N_\nu^{\rm eff} = 2.92^{+0.36}_{-0.37}$$



Nν < 3

$T_{\rm RH}$ ↘ $N_{\rm eff}$ ↘

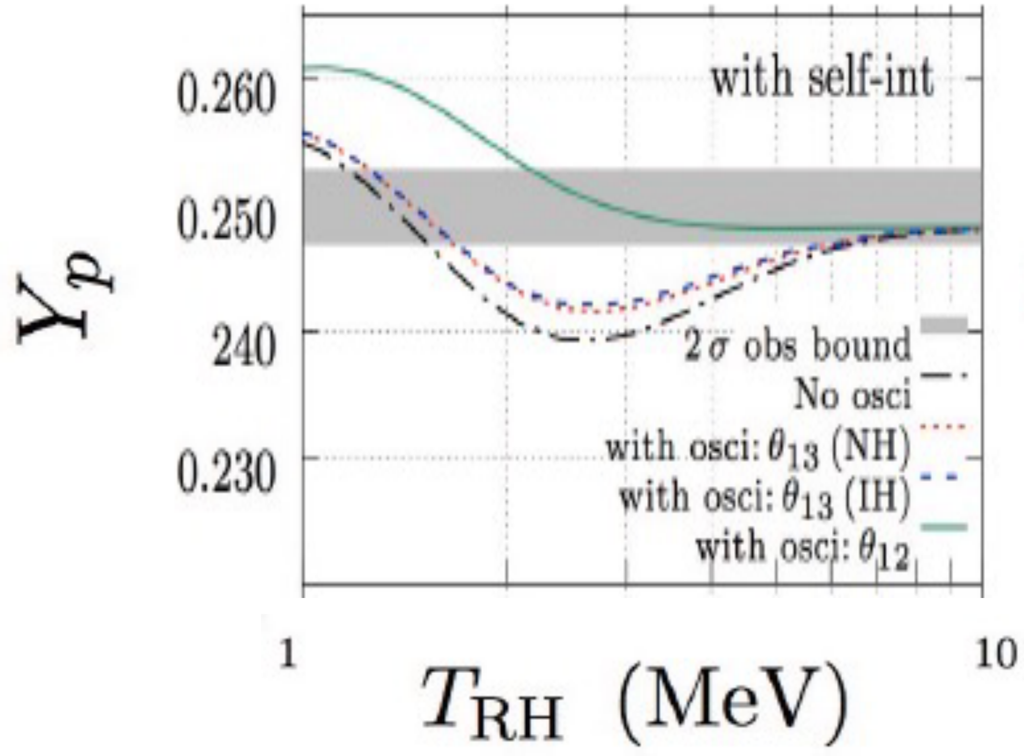
with ν oscillation or self-int. → $N_{\rm eff}$ ↗

Observational Helium 4 abundance Yp

$$Y_p = 0.2449 \pm 0.0040 \quad (68\%\ \mathrm{C.L.})$$

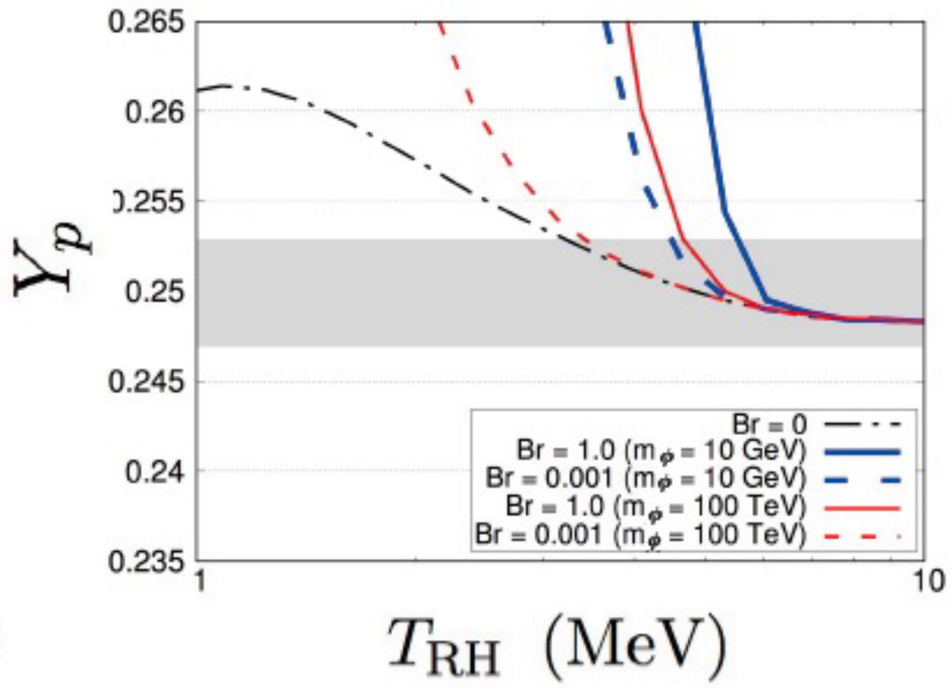
T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012

Radiative decay



$\rightarrow \gamma, e^{\pm} \rightarrow \nu$'s

Hadronic decay



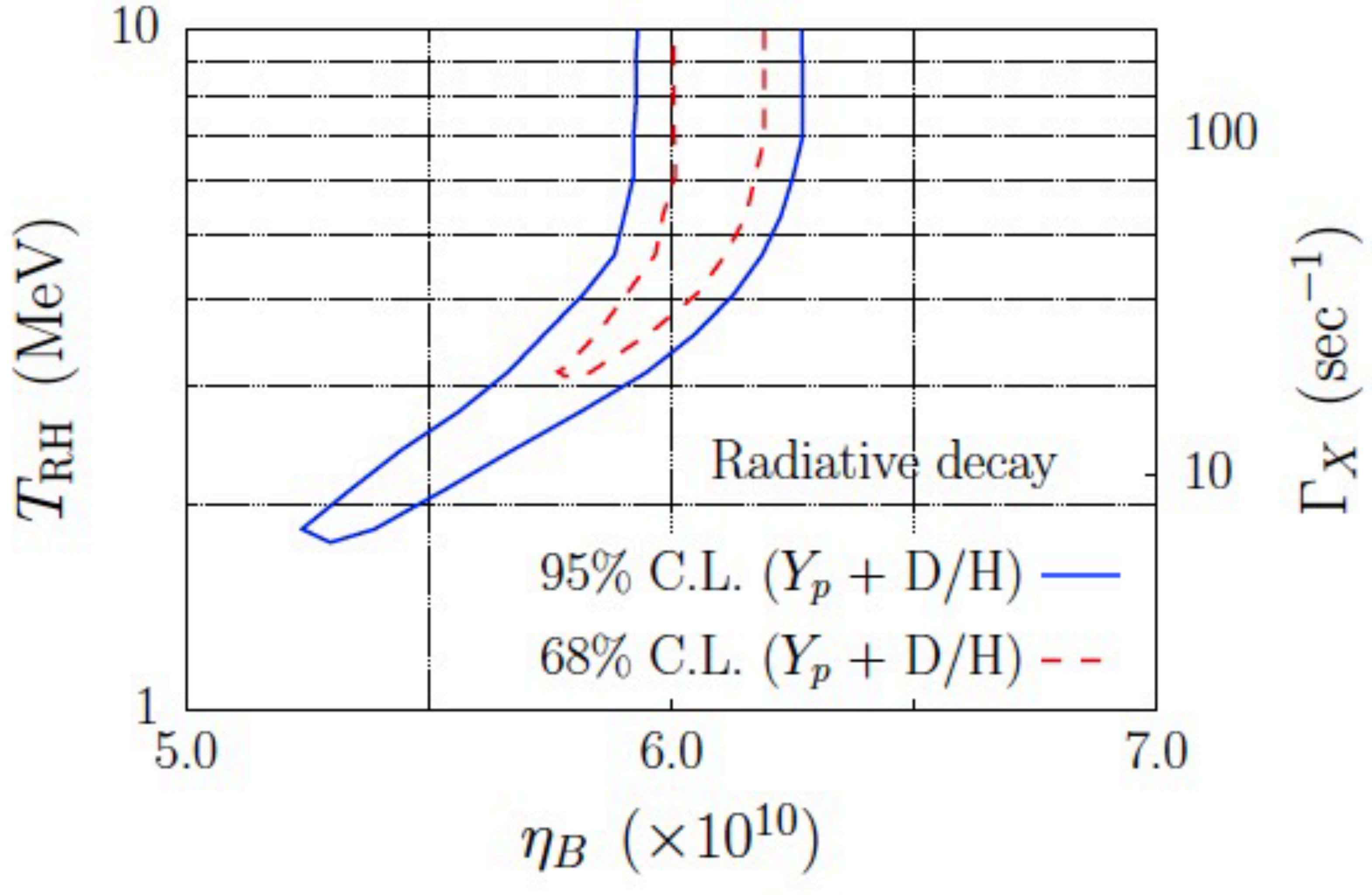
$\rightarrow$qq, gluon$\rightarrow \pi$'s

$$n + \pi^{+} \rightarrow p + \pi^{0}$$
$$p + \pi^{-} \rightarrow n + \pi^{0}$$

n/p ↗

Lower bound on T_{RH} for radiative decay

T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012

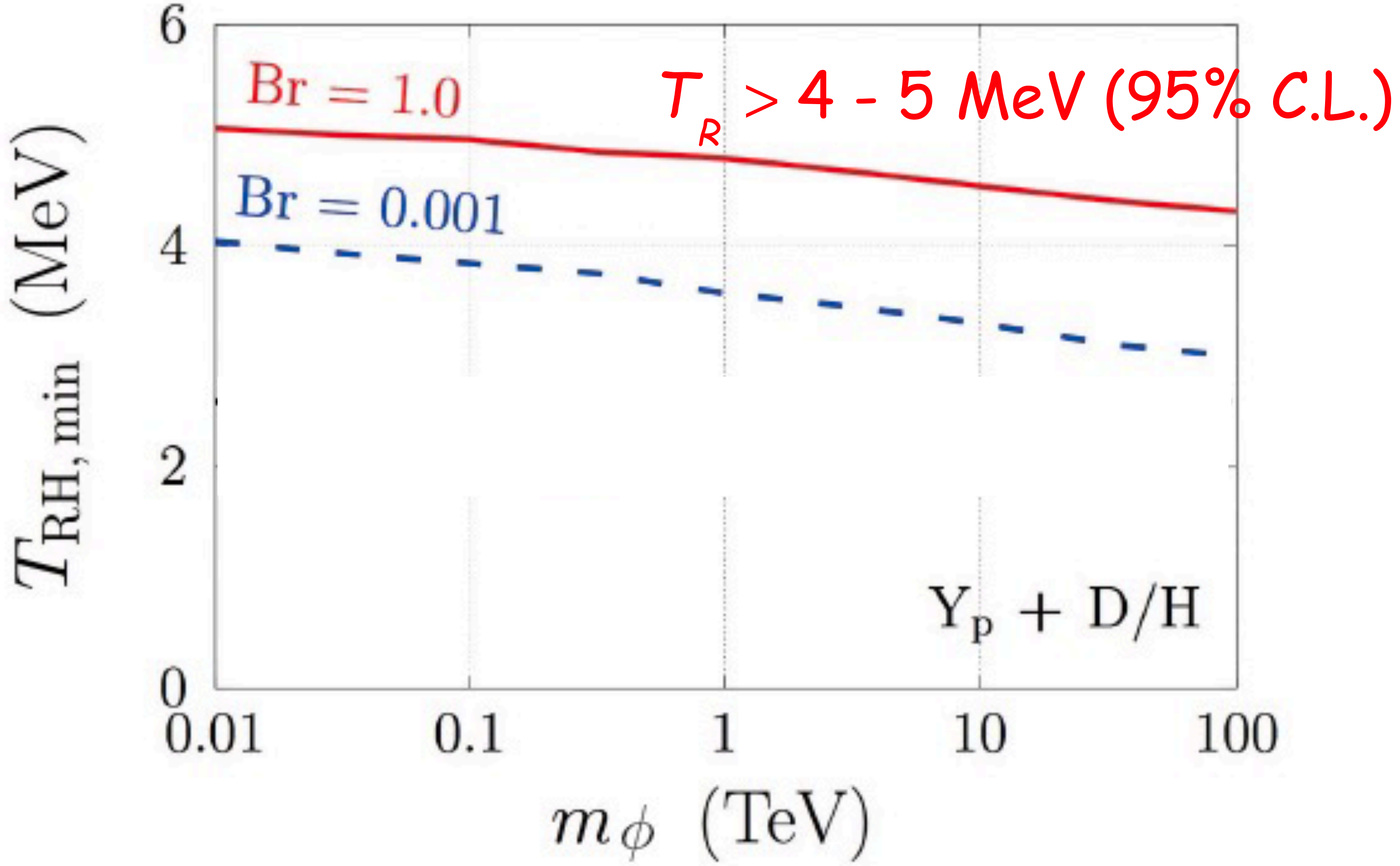


$T_R > 1.8$ MeV (95% C.L.)

$\eta_B = \frac{n_B}{n_\gamma} = \eta_{10} \times 10^{-10}$: baryon to photon ratio

Lower bounds on T_{RH} for hadronic decay

T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012



Lower bounds on Reheating temperature

T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012

- Radiative decay

$$T_R > 1.8 \text{ MeV (95\% C.L.)}$$

ΔNnu <~ -2

- Hadronic decay ($B_H = 1$)

$$T_R > 4 - 5 \text{ MeV (95\% C.L.)}$$

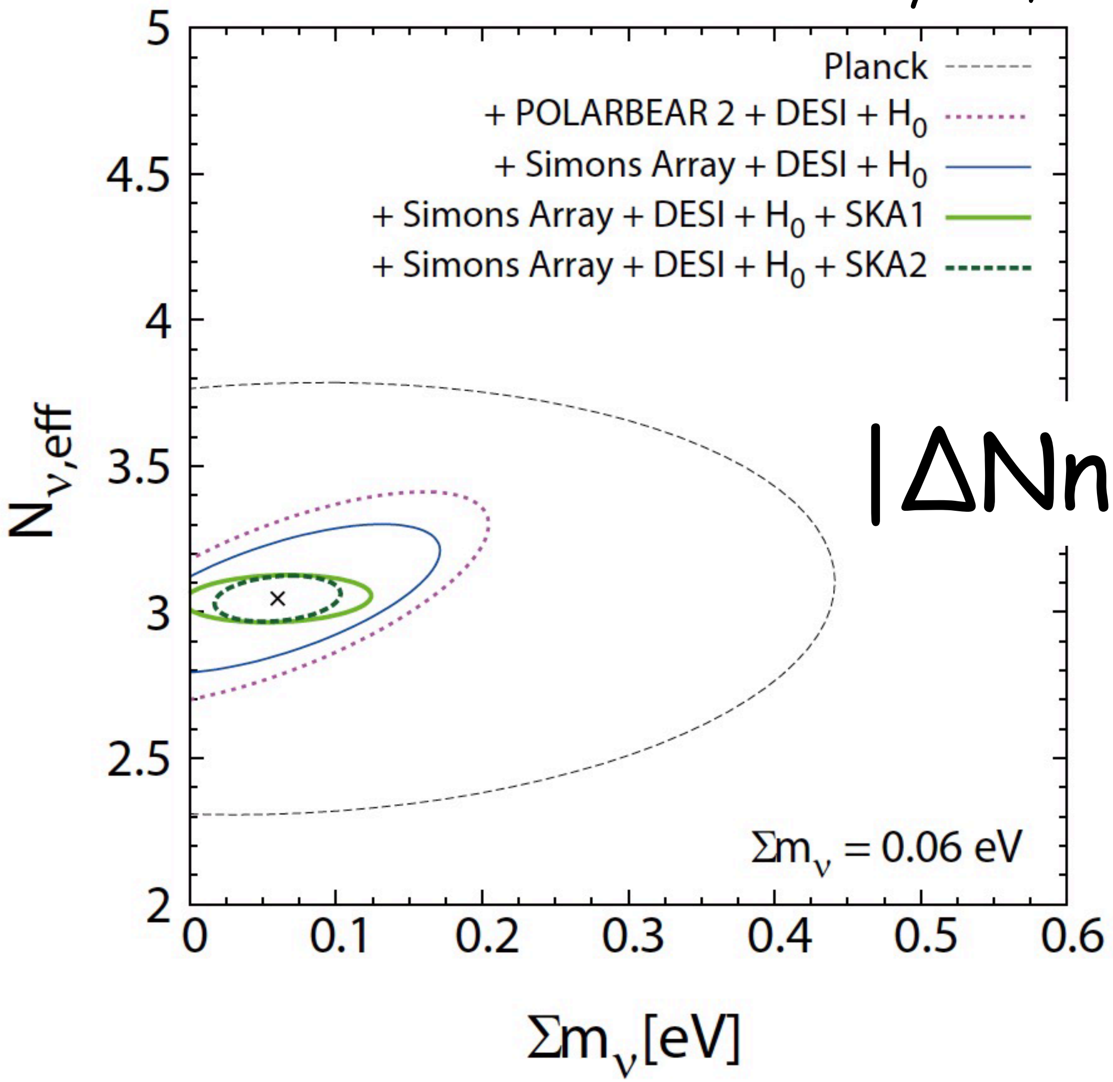
ΔNnu <~ -0.3

See also the talk by Gordan Krnjaic for a positive $\Delta N\nu$

See also the CMB bound, T_R > 4.7 MeV, P.F. de Salas, M. Lattanzi, G. Mangano, G. Miele, S. Pastor, O. Pisanti, arXiv:1511.00672

Future constraints on neutrino species and mass by 21cm, CMB, and BAO

Oyama, Kohri, Hazumi (2015)



$|\Delta N_{nu}| < 0.05$

1) $N\nu < 3 \rightarrow T_R \sim O(1)$ MeV

2) Formation of PBHs in the early Matter Dominated epoch

PBH formation at the (early) matter dominated (MD) Universe

Polnarev and Khlopov (1982)

Harada, Yoo, KK, Nakao, Jhingan (2016)

1. Pressure is negligible, which could induce an immediate collapse and producing more PBHs?

2. Density perturbations can evolve, which produces non-spherical objects and cannot be enclosed by the Horizon. That means less PBHs can be produced?

Matter Domination

- Three radius in Lagrangian coordinate q_i

Zel'dovich Approximation

$$r_1 = (a - \alpha b) q_1$$
$$r_2 = (a - \beta b) q_2$$
$$r_3 = (a - \gamma b) q_3$$

- Eccentricity $$e^2 = 1 - \left(\frac{r_2(t_c)}{r_3(t_c)}\right)^2 = 1 - \left(\frac{\alpha - \beta}{\alpha - \gamma}\right)^2$$
- Hoop with 2nd Elliptic funciton E(x)

$$\mathcal{C} = 16\left(1 - \frac{\gamma}{\alpha}\right) E\left(\sqrt{1 - \left(\frac{\alpha - \beta}{\alpha - \gamma}\right)^2}\right) r_f$$

- Hoop conjecture for PBH production

$$\mathcal{C} \lesssim 2\pi r_g.$$

Abundance of PBHs formed in MD

- Probability distribution by peak statistics (BBKS)

Doroshkevich (1970)

$$w(\alpha,\beta,\gamma)d\alpha d\beta d\gamma$$
$$= -\frac{27}{8\sqrt{5}\pi\sigma_3^6}\exp\left[-\frac{1}{10\sigma_3^2}(\alpha+\beta+\gamma)^2-\frac{1}{4\sigma_3^2}\{(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2\}\right]$$
$$\cdot(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)d\alpha d\beta d\gamma.$$

$$\sigma_H = \sqrt{5}\sigma_3$$

- Probability

$$\beta_0 = \int_0^\infty d\alpha \int_{-\infty}^{\alpha} d\beta \int_{-\infty}^{\beta} d\gamma\ \theta(1-h(\alpha,\beta,\gamma))w(\alpha,\beta,\gamma)$$

$$h(\alpha,\beta,\gamma) = \frac{2}{\pi}\frac{\alpha-\gamma}{\alpha^2}E\left(\sqrt{1-\left(\frac{\alpha-\beta}{\alpha-\gamma}\right)^2}\right)$$

$$h(\alpha,\beta,\gamma) := \mathcal{C}/(2\pi r_g)$$

Angular momentum produced by perturbations

Harada, Yoo, KK, nad Nakao (2017)

- Angular momentum

1st order effects for nonspherical V

2nd order effects

$$\mathbf{L}_c = \int_{a^3V} \rho \mathbf{r} \times \mathbf{v} d^3\mathbf{r} = \rho_0 a^4 \left(\int_V \mathbf{x} \times \mathbf{u} d^3\mathbf{x} + \int_V \mathbf{x}\delta \times \mathbf{u} d^3\mathbf{x} \right)$$

- Density perturbation δ
- (Peculiar) Velocity perturbation

$$\mathbf{u} := aD\mathbf{x}/Dt$$

$$\mathbf{u}_1 = -\frac{t}{a}\nabla\psi_1$$

- Potential perturbation

$$\psi := \Psi - \Psi_0$$

Effects by finite angular momentum

Harada, Yoo, KK, Nakao (2017)

- Probability distribution

$$a_* := L/(GM^2/c)$$

$$f_{\mathrm{BH}(2)}(a_*)da_* \propto \frac{1}{a_*^{5/3}} \exp\left(-\frac{1}{2\sigma_H^{2/3}}\left(\frac{2}{5}\mathcal{I}\right)^{4/3}\frac{1}{a_*^{4/3}}\right) da_*$$

- Probability

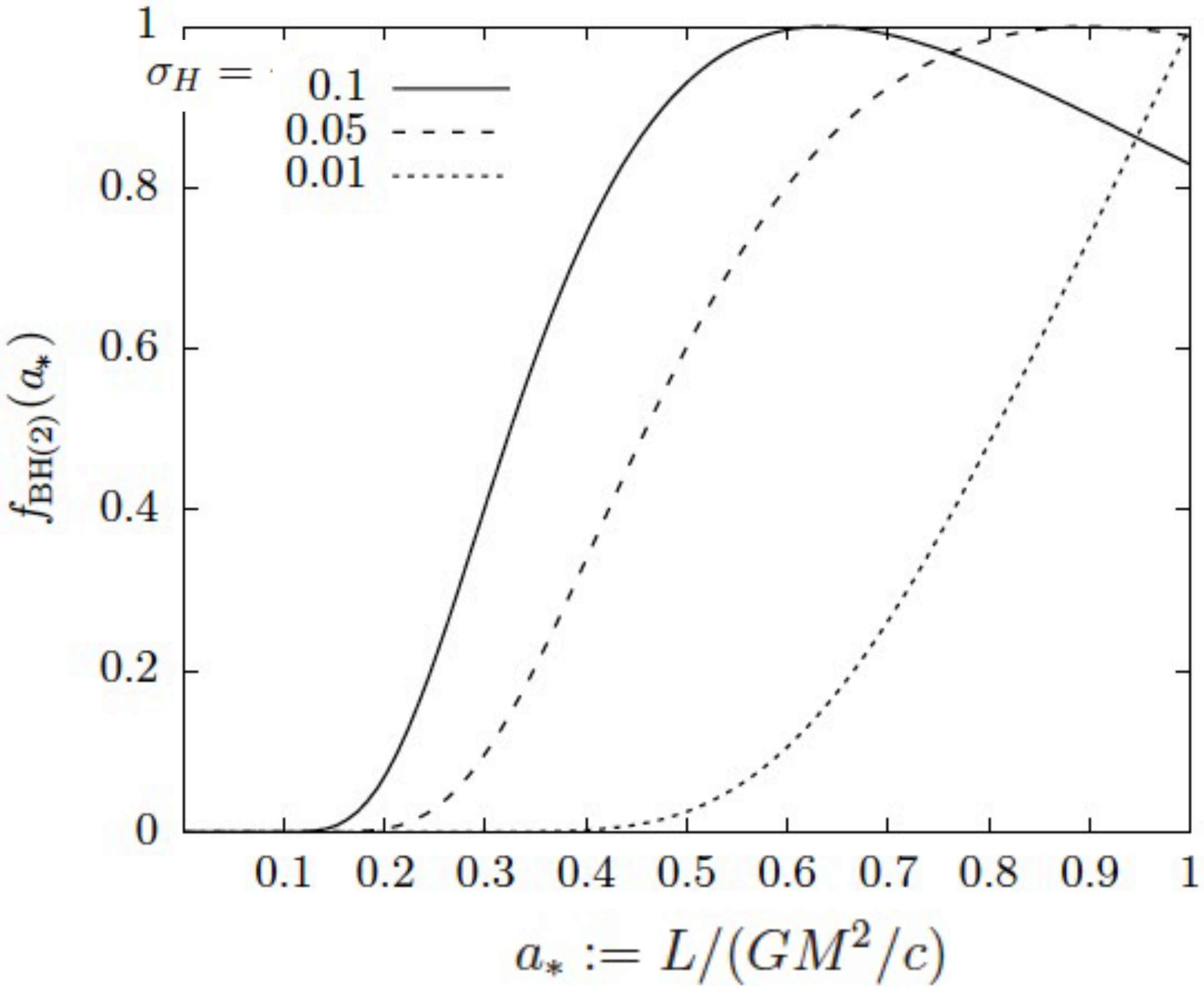
$$\beta_0 \simeq \int_0^\infty d\alpha \int_{-\infty}^{\alpha} d\beta \int_{-\infty}^{\beta} d\gamma \theta[\delta_H(\alpha,\beta,\gamma)-\delta_{\mathrm{th}}]\theta[1-h(\alpha,\beta,\gamma)]w(\alpha,\beta,\gamma)$$

$$\delta_H(\alpha,\beta,\gamma) = \alpha+\beta+\gamma \qquad \delta_{\mathrm{th}} := \left(\frac{2}{5}\mathcal{I}\sigma_H\right)^{2/3}$$

Spin distribution

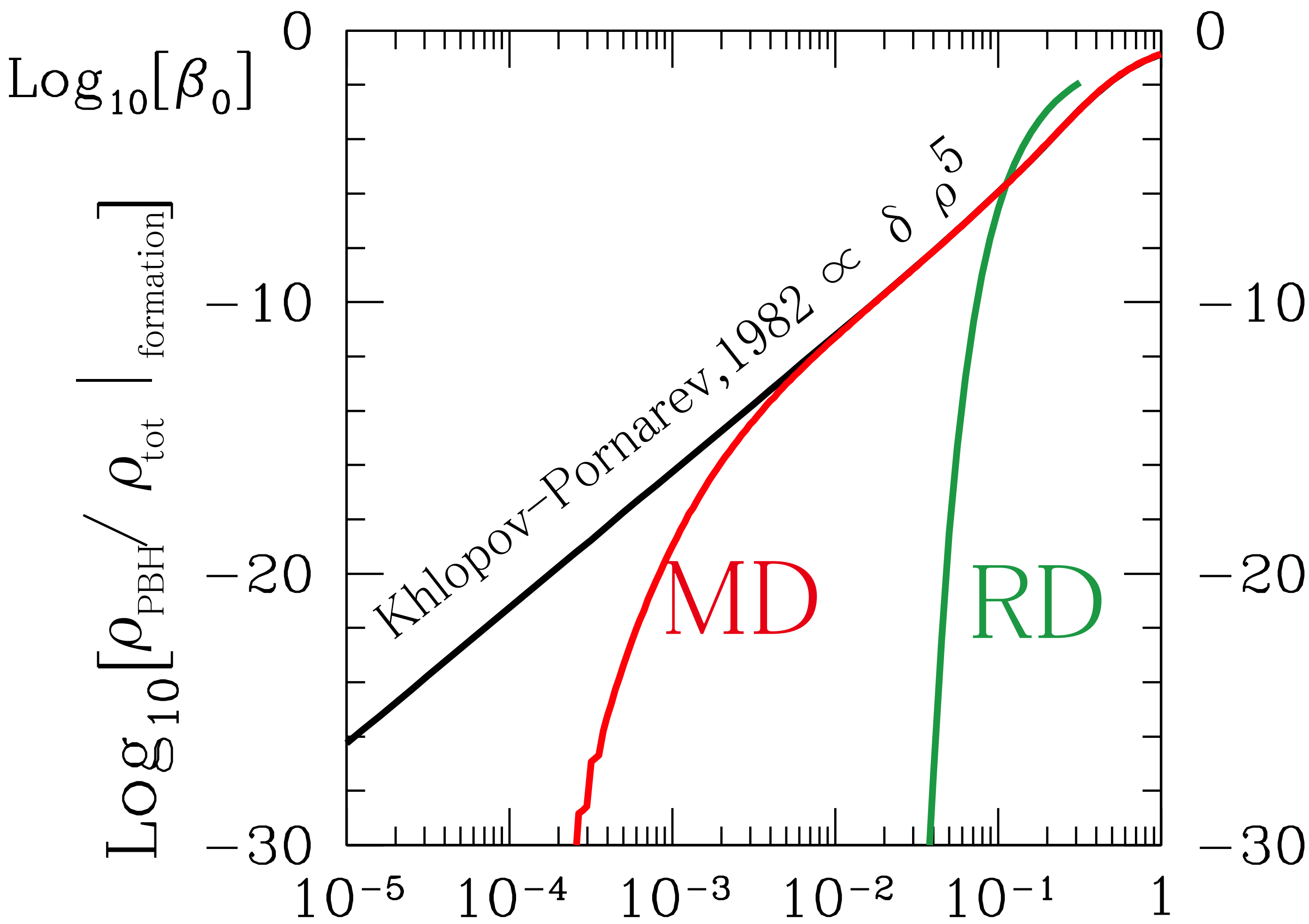
More highly-spinning halos cannot collapse into PBHs, which means that the PBHs produced tend to have high spins in MD

Harada, Yoo, KK, Nakao (2017)



Beta in matter-domination

Harada, Yoo, KK, Nakao (2017)



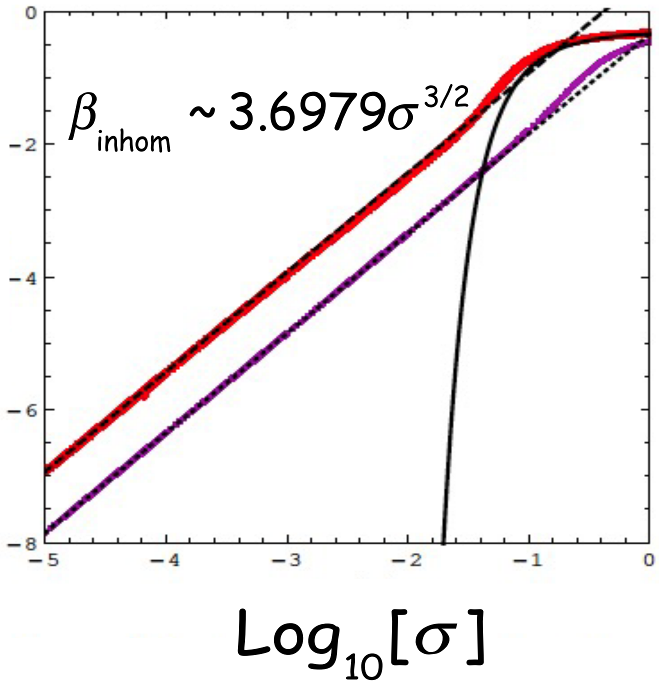
$\sigma_H = \sqrt{5}\sigma_3 = \delta\rho/\rho$

Effects of Inhomogeneity on PBH formations in Matter Domination

T.Kokubu, K.Kyutoku, K.Kohri, T.Harada, arXiv:1810.03490

Singularity should be enclosed by (apparent) horizon

$$\beta \equiv \frac{\rho_{PBH}}{\rho_{tot}}$$



$$\beta_{aniso} \simeq 0.05556\sigma^5$$

$$\sigma \sim \overline{\delta\rho/\rho}$$

$$\beta_{inhom+aniso} \simeq \beta_{inhom} \times \beta_{aniso} = 0.2055\sigma^{13/2}$$

2) $a_* \sim 1 \rightarrow$

PBHs produced with $\sigma \ll 1$

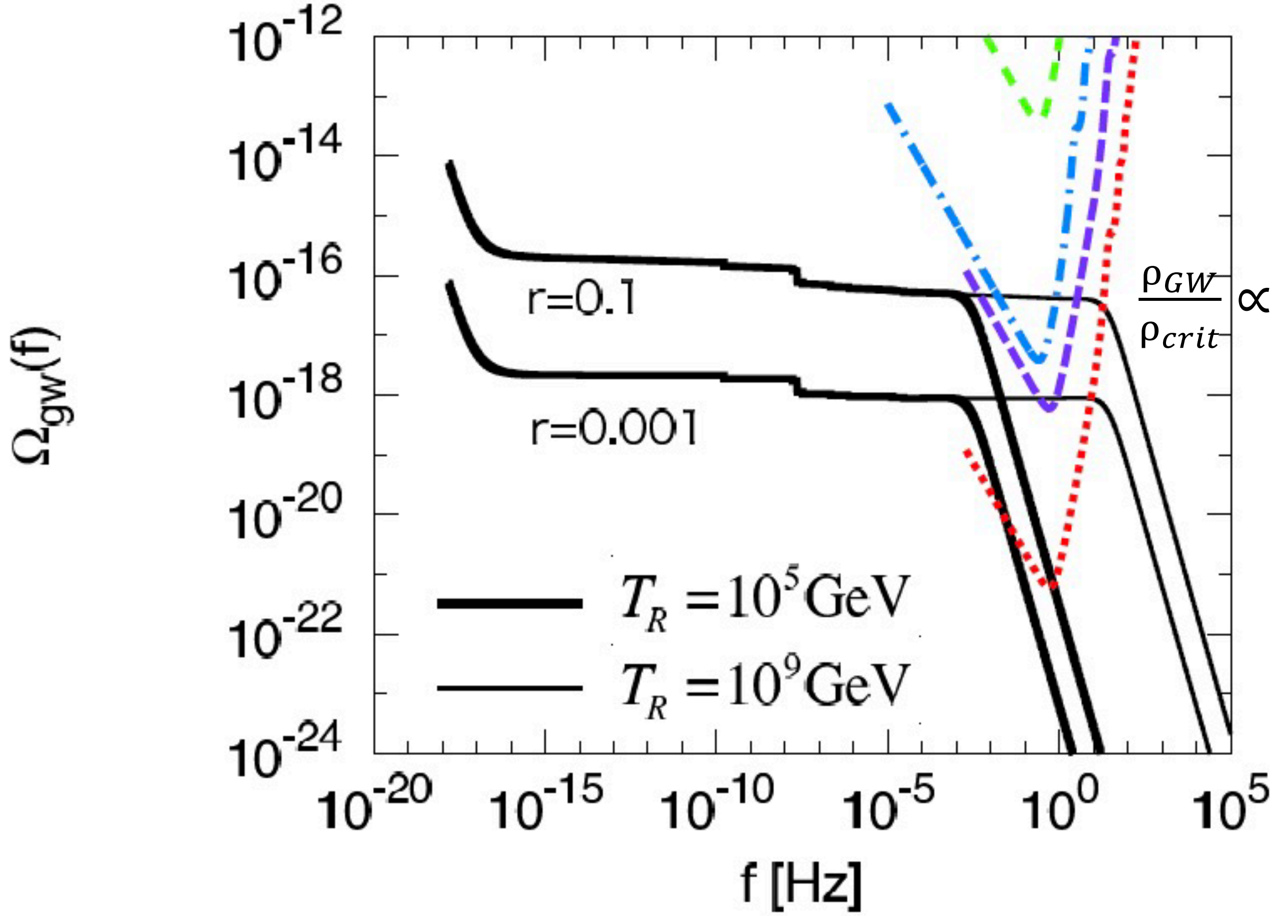
in the eMD

3) Stochastic gravitational wave produced in the early Matter Dominated epoch

The break point of Ω_{GW} marks the reheating temperature after inflation

Naoki Seto, Jun'ichi Yokoyamam, arXiv:gr-qc/0305096

Kazunori Nakayama, Shun Saito, Yudai Suwa, Jun'ichi Yokoyama, arXiv:0804.1827 [astro-ph]



$$\frac{\rho_{GW}}{\rho_{crit}} \propto \frac{a^{-4}}{a^{-3}} \propto a^{-1} \propto f^{-2}$$

The 2nd order GWs with gradual transition from MD to RD

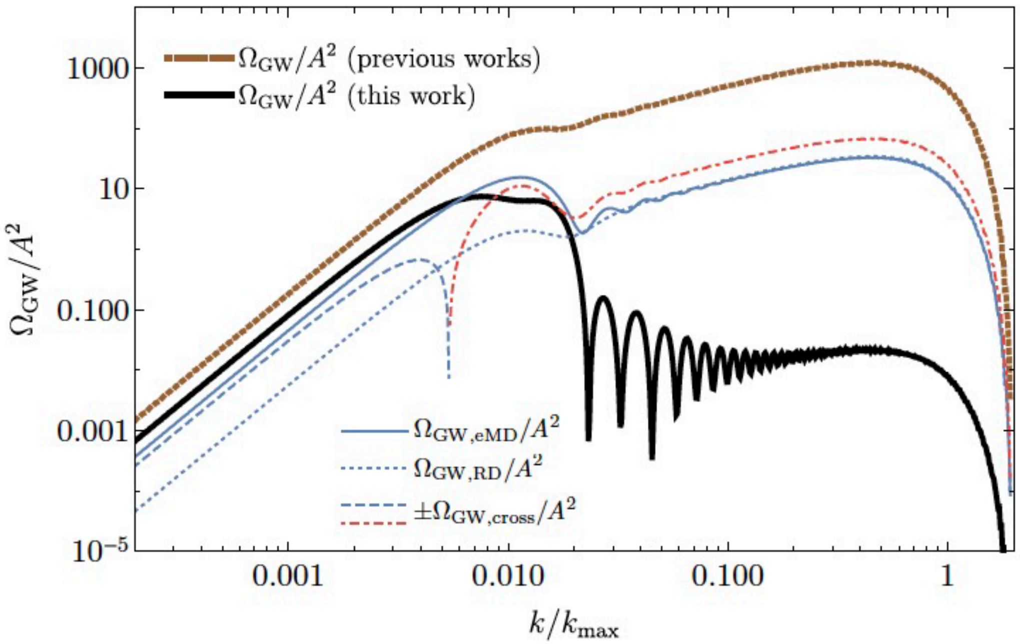
Inomata, Kohri, Nakama, Terada, JCAP10(2019)071, arXiv:1904.12878 [astro-ph.CO]

$$\Omega_{\rm GW}(\eta,k) = \frac{\rho_{\rm GW}(\eta,k)}{\rho_{\rm tot}(\eta)} = \frac{1}{24}\left(\frac{k}{\mathcal{H}(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta,k)}$$

$$\overline{\mathcal{P}_h(\eta,k)} = 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1+v^2-u^2)^2}{4vu}\right)^2 \overline{I^2(u,v,k,\eta,\eta_{\rm R})}\mathcal{P}_\zeta(uk)\mathcal{P}_\zeta(vk).$$

$\overline{\mathcal{P}_h(\eta,k)}$ ~ $\int\int f^2(u,v,x,x_R)$

$$f(u,v,\bar{x},x_{\rm R}) = \frac{3\left(2(5+3w)\Phi(u\bar{x})\Phi(v\bar{x}) + 4\mathcal{H}^{-1}(\Phi'(u\bar{x})\Phi(v\bar{x}) + \Phi(u\bar{x})\Phi'(v\bar{x})) + 4\mathcal{H}^{-2}\Phi'(u\bar{x})\Phi'(v\bar{x})\right)}{25(1+w)}$$



The 2nd order GWs enhanced at a sudden transition from MD to RD

Inomata, Kohri, Nakama, Terada, Phys. Rev. D 100, 043532 (2019), arXiv:1904.12879

$\overline{\mathcal{P}_h(\eta,k)} \sim \int\int f^2(u,v,x,x_R)$

$$f(u,v,\bar{x},x_R) = \frac{3\left(2(5+3w)\Phi(u\bar{x})\Phi(v\bar{x}) + 4\mathcal{H}^{-1}(\Phi'(u\bar{x})\Phi(v\bar{x}) + \Phi(u\bar{x})\Phi'(v\bar{x})) + 4\mathcal{H}^{-2}\Phi'(u\bar{x})\Phi'(v\bar{x})\right)}{25(1+w)}$$

This is big!

- Gravitational potential

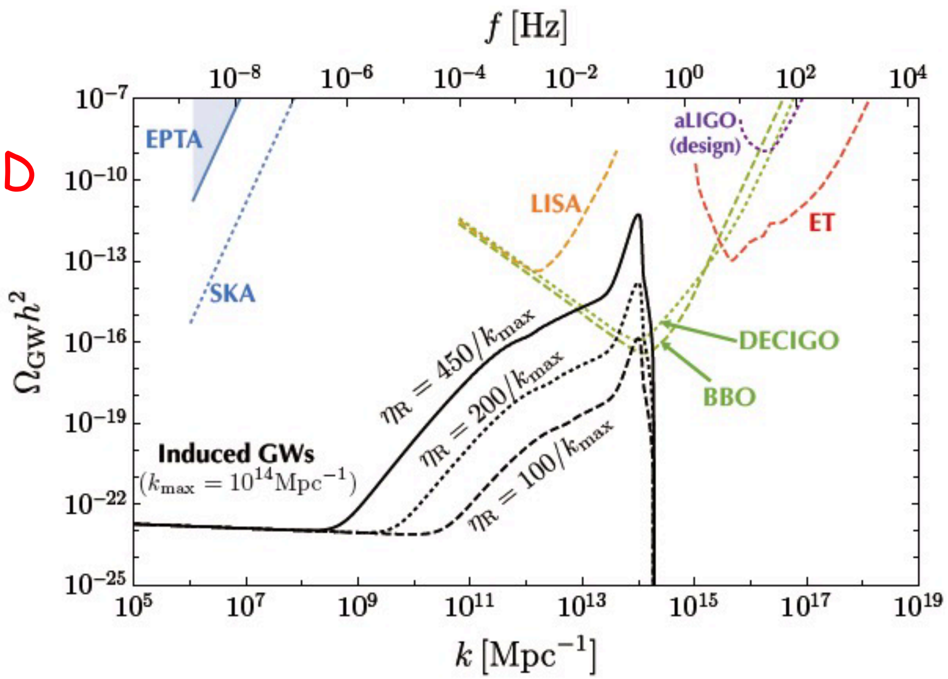
$$\Phi(x,x_R) = \begin{cases} 1 & (\text{for } x \le x_R), \\ A(x_R)\mathcal{J}(x) + B(x_R)\mathcal{Y}(x) & (\text{for } x \ge x_R), \end{cases}$$

eMD
RD

- Enhancement at T_R

$$\mathcal{H}^{-2}\Phi'\Phi' \sim (k\eta_R)^2\Phi^2 \gg \Phi^2$$

Amplitude should be less than unity
The transition occurs in a finite time



Sudden decay from ϕ → 2χ only when M > 2 m_χ~√λ/2 τ

- Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - \frac{1}{2}\partial^\mu\tau\partial_\mu\tau - V,$$
$$V = \frac{1}{2}M^2\phi^2 + \frac{1}{2}m^2\tau^2 + \frac{\lambda}{4}\tau^2\chi^2 + \frac{c}{2}M\phi\chi^2,$$

- Decay rate

$$\Gamma = \frac{c^2 M}{32\pi}\sqrt{1 - \frac{m_{\chi,\mathrm{eff}}^2}{(M/2)^2}}\,\Theta\left(M^2 - 4m_{\chi,\mathrm{eff}}^2\right)$$

- Effective mass of χ

$$m_{\chi,\mathrm{eff}}^2 = \langle \lambda\tau^2/2 \rangle$$

Applications of this mechanism (The poltergeist mechanism)

Poltergeist
mechanism for gravitational wave production

Logo by ©Takahiro Terada

- Evaporating PBHs with their domination

Keisuke Inomata, Masahiro Kawasaki, Kyohei Mukaida, Takahiro Terada, Tsutomu T. Yanagida, arXiv:2003.10455 [astro-ph.CO]

- Poisson fluctuation of evaporating PBHs themselves with their domination

Guillem Domènech, Chunshan Lin, Misao Sasaki, arXiv:2012.08151 [gr-qc]
Guillem Domènech, Volodymyr Takhistov, Misao Sasaki, arXiv:2105.06816 [astro-ph.CO]
Extension from Theodoros Papanikolaou, Vincent Vennin, David Langlois, arXiv:2010.11573 [astro-ph.CO]

- Sudden decays of Q-balls

Graham White, Lauren Pearce, Daniel Vagie, Alex Kusenko, arXiv:2105.11655 [hep-ph]

- …

GWs produced in the early matter-dominated (MD) epoch by formation of halos, evaporation of halos, and turbulence after reheating

Tomohiro Nakama, Phys. Rev. D 101, 063519 (2020)
https://journals.aps.org/prd/abstract/10.1103/PhysRevD.101.063519

- Halo formation

$$\Omega_{\rm GW,c} = \frac{G^4 M^4}{c^7 R^5} T = \left(\frac{r_g}{2R}\right)^4 \frac{cT}{R}$$

See also Karsten Jedamzik (LPTA), Martin Lemoine (IAP), Jerome Martin (IAP), arXiv:1002.3278 [astro-ph.CO]

- Halo evaporation

$$\Omega_{\rm GW,c} \simeq \frac{9 \cdot 3^{5/6} \pi^{23/6} (\delta_c^{\rm lin})^{3/2} \sigma^{7/2}}{16 \cdot 2^{1/6} (\delta_m^{\rm lin})^5 (\delta_{\rm vir}^{\rm nonlin})^{1/2}} \simeq 12\sigma^{7/2}.$$

- Turbulence

$$\Omega_{\rm GW} \sim 2 \times 10^{-7} \left(\frac{R}{cH_c^{-1}}\right)^3 \simeq 5 \times 10^{-10} \sigma^{3/2},$$

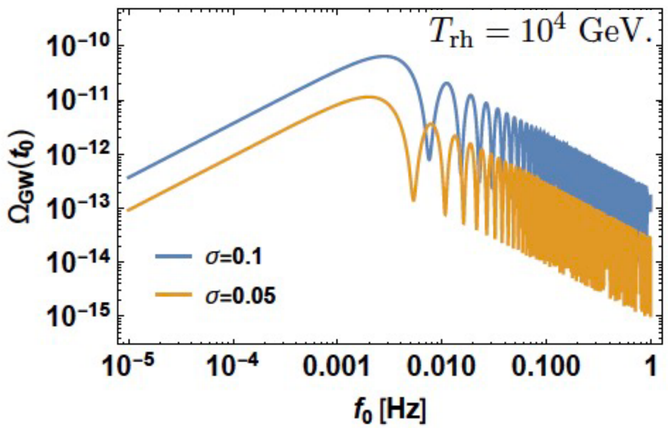
Gravitational Waves from evolving density Perturbations in an Early Matter Domination Era

Ioannis Dalianis, Chris Kouvaris, arXiv:2012.09255 [astro-ph.CO]

$$\begin{aligned}\Omega_{\rm GW}(t_0, f_0) =& \frac{1}{\rho_{\rm crit}} \int_0^\infty \int_{-\infty}^{\alpha} \int_{-\infty}^{\beta} d\alpha d\beta d\gamma \, \frac{4\pi G}{5c^5} \frac{2\pi f_0}{54\pi^2} M^2 \left(\frac{2\alpha \delta_{\rm L}}{\alpha+\beta+\gamma} \right)^2 \\ & \times \left[1 + \left(\frac{\beta}{\alpha}\right)^4 + \left(\frac{\gamma}{\alpha}\right)^4 - \left(\frac{\gamma}{\alpha}\right)^2 - \left(\frac{\beta}{\alpha}\right)^2 \left(1 + \left(\frac{\gamma}{\alpha}\right)^2\right) \right] \\ & \times \left| {\rm Ei}\left[\frac{1}{3}, i t_{\rm max} 2\pi f_0 (1+z_{\rm col})\right] - 2{\rm Ei}\left[\frac{1}{3}, i t_{\rm col} 2\pi f_0 (1+z_{\rm col})\right] \right|^2 \\ & \times \Theta\left(t_{\rm rh} - t_{\rm col}(\alpha,\beta,\gamma)\right) \left(\frac{4\pi}{3} q^3\right)^{-1} \mathcal{F}_{\rm D}(\alpha,\beta,\gamma).\end{aligned}$$

$$I_{ij} = \frac{1}{5} M \begin{pmatrix} r_2^2 + r_3^2 & 0 & 0 \\ 0 & r_1^2 + r_3^2 & 0 \\ 0 & 0 & r_1^2 + r_2^2 \end{pmatrix}$$

$$Q_{ij} = -I_{ij}(t) + \frac{1}{3}\delta_{ij} {\rm Tr} I(t)$$

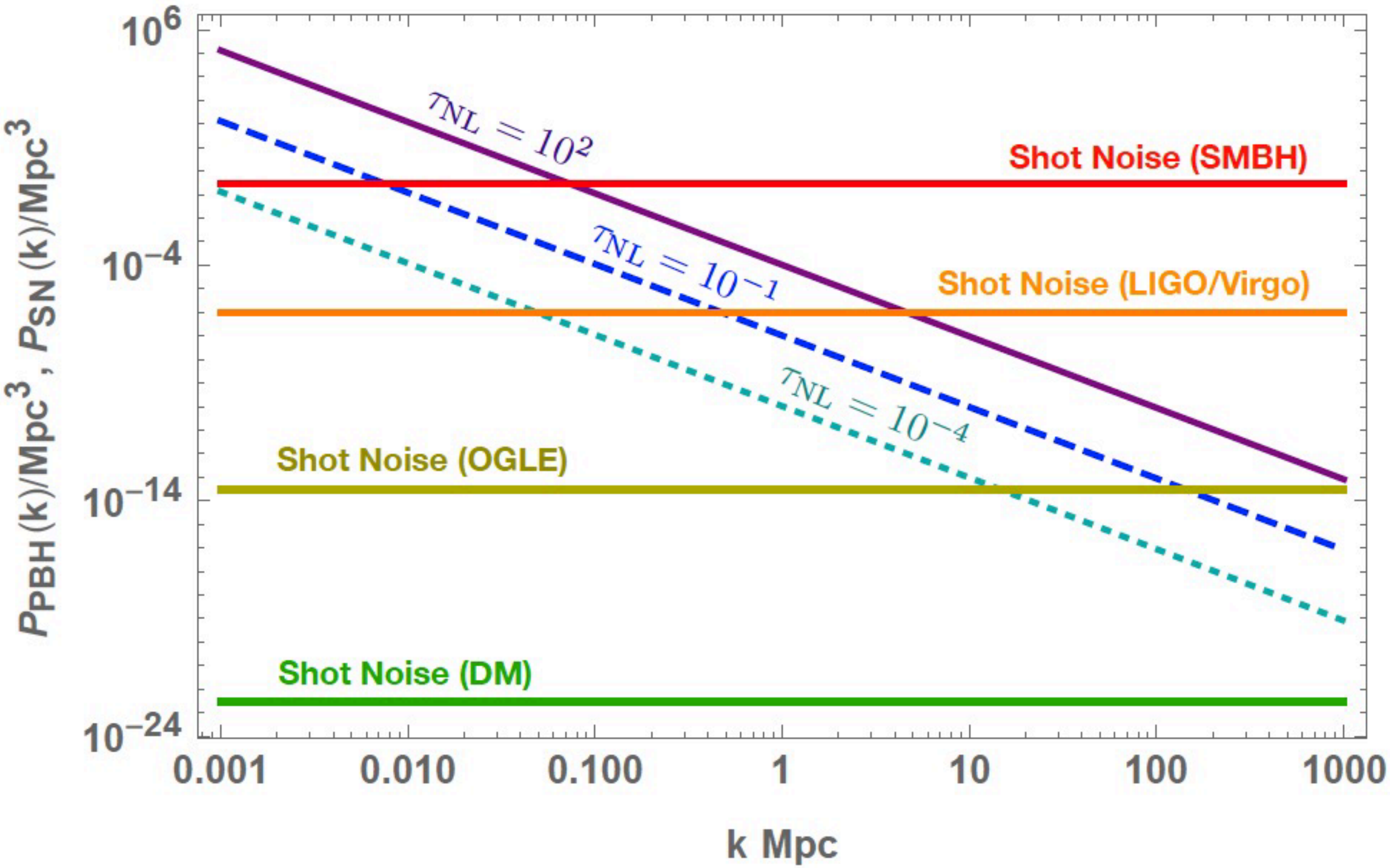


$$\frac{dE_e}{dt} = \frac{G}{5c^5} \sum_{ij} \dddot{Q}_{ij}(t) \dddot{Q}_{ji}(t)$$

$$\begin{aligned}\Omega_{\rm GW}(t_0, f_0) =& \frac{1}{\rho_{\rm crit}(t_0)} \int_0^\infty \int_{-\infty}^{\alpha} \int_{-\infty}^{\beta} d\alpha d\beta d\gamma \sum_N \frac{1}{1+z_N} \frac{4\pi G}{5c^5} \sum_{ij} |\tilde{Q}_{ij}^N (2\pi f_0 (1+z_N))|^2 \\ & \times (2\pi f_0 (1+z_N))^7 \Theta\left(t_{\rm rh} - t_{\rm col}(\alpha,\beta,\gamma)\right) \left(\frac{4\pi}{3} q^3\right)^{-1} \mathcal{F}_{\rm D}(\alpha,\beta,\gamma). \qquad (\end{aligned}$$

PBHs produced in RD or MD are clustering?

Matsubara, Terada, Kohri, S. Yokoyama, 1909.06048
See also, Suyama and S. Yokoyama (2019)
Tada and S.Yokoyama (2015)



Summary

We can confirm the existence of the early matter-dominated epoch by observing

- Effective number of neutrino species ($N_\nu < 3$)
- Spin of the primordial black holes ($a_* \sim 1$)
- Characteristic signals of stochastic GW produced from inflation at the level of $\Omega_{GW} \sim 10^{-16}$ or secondary GW nonlienary-produced by large curvature perturbation at small scales (The Poltergeist mechanism)