Tidal Deformation and Dissipation of Rotating Black Holes

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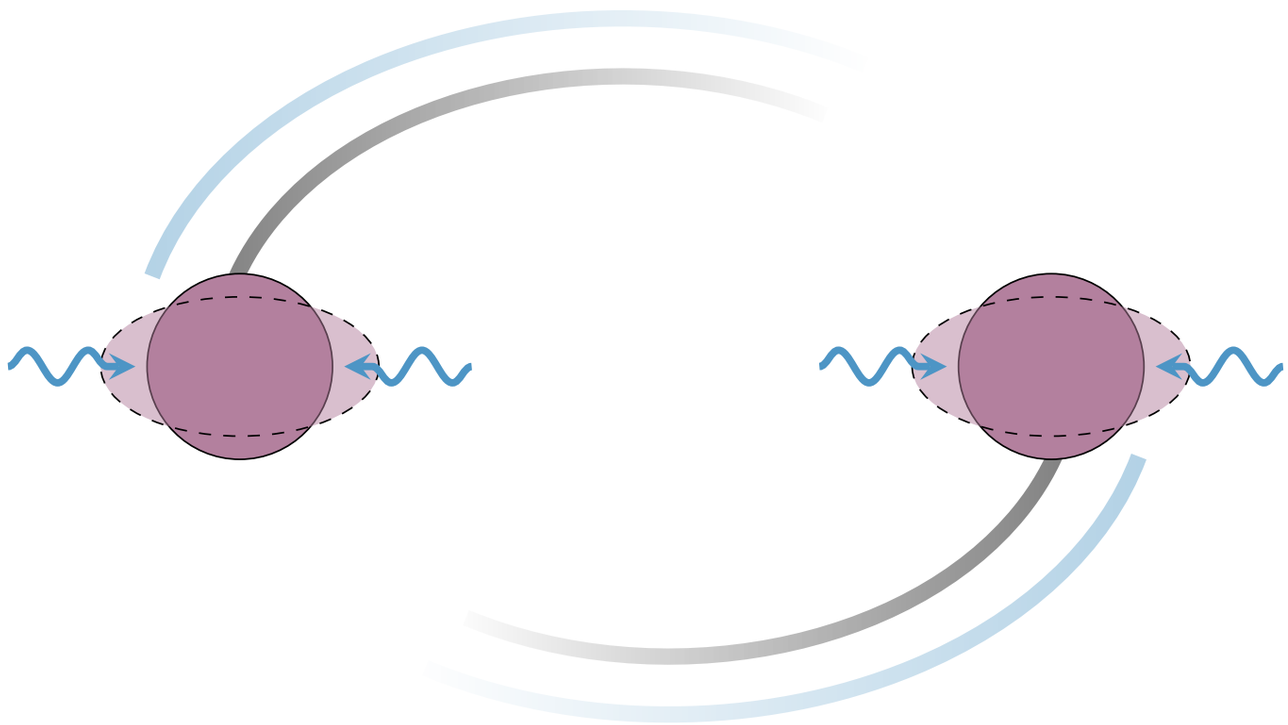
Tidal Effects in Binary Systems

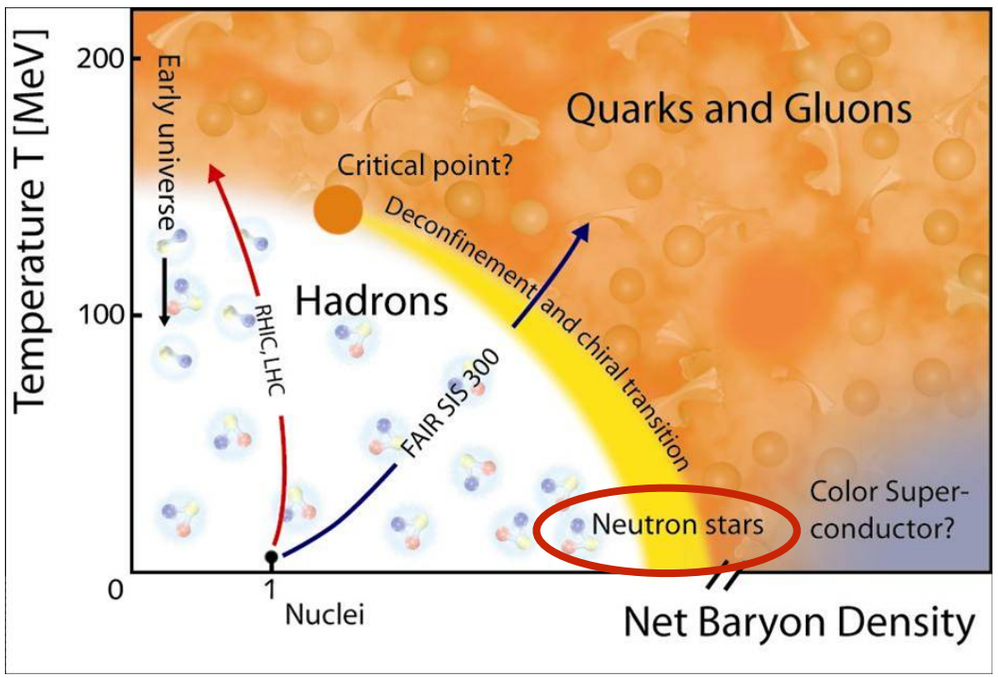
At leading order, the dynamics of binary systems are modeled as two orbiting point particles endowed with spin vectors.

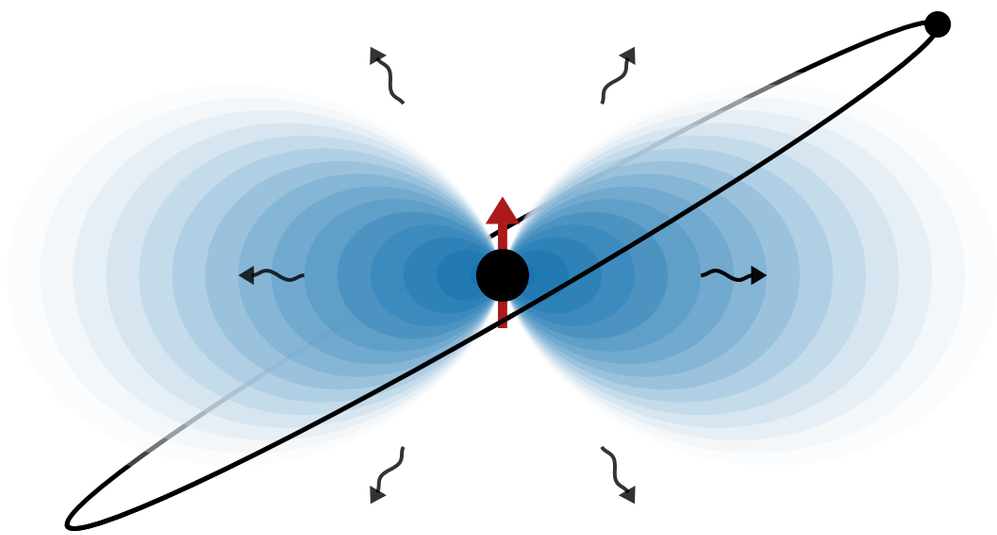


Beyond the point particle approximation, conservative and dissipative “finite-size” effects of the binary components also impact the binary dynamics.

Probing Binary Constituents with Tidal Effects

For binary neutron stars, these tidal effects probe the high-density and low-temperature regime of the QCD phase diagram.





Measurements of these tidal effects could also provide hints for the existence of new types of compact objects, e.g. superradiant clouds, boson stars, etc.

Raithel, Özel, Psaltis [1803.07687], Chatziioannou [2006.03168]

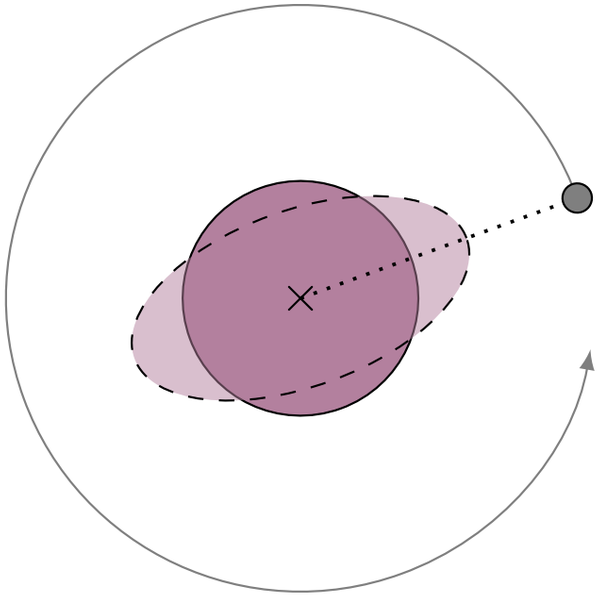
Figures adapted from Aarts [1512.05145] & Baumann, HSC, Porto, Stout [1912.04932]

Static Tidal Response

In Newtonian gravity, a body would respond to a **static** external tidal field by acquiring the induced mass moments:

$$\delta Q_{\ell m} \propto 2k_{\ell m}\mathcal{E}_{\ell m}$$

where the proportionality constants $k_{\ell m}$ are called the Love numbers.



Love (1912)

Poisson, Will (Gravity textbook)

Time-Dependent Tidal Response

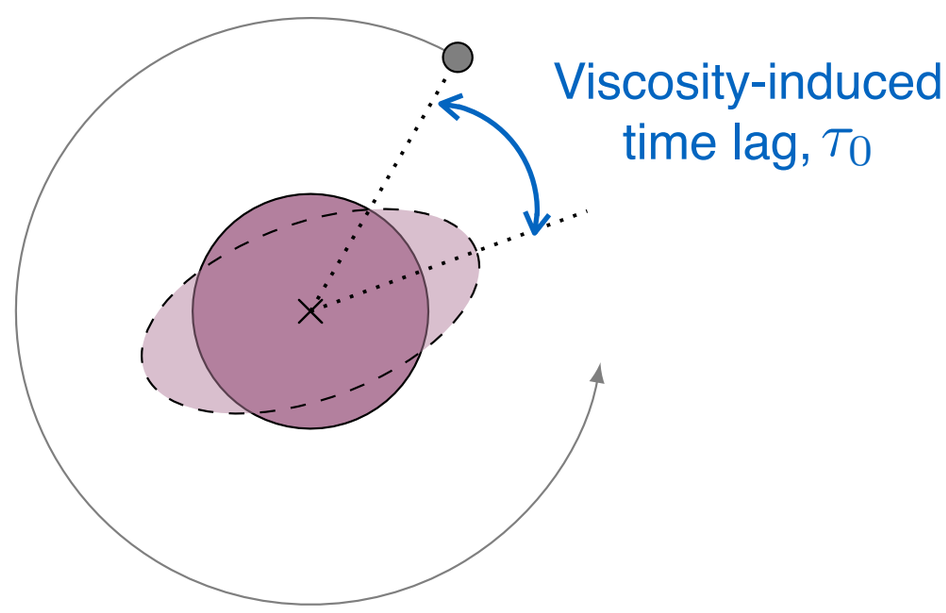
For a **slowly-varying** external tidal field, the induced response is

$$\delta Q_{\ell m}(t) = \int_0^\infty \mathrm{d}\tau\, F_{\ell m}(\tau)\mathcal{E}_{\ell m}(t-\tau)\,,$$

$$\propto 2k_{\ell m}\,\mathcal{E}_{\ell m}(t) - \tau_0\nu_{\ell m}\,\dot{\mathcal{E}}_{\ell m}(t) + \cdots$$

static tides tidal dissipation* dynamical tides

where $\nu_{\ell m}$ are the dissipation numbers associated to the object's viscosity.



Poisson, Will (Gravity textbook)

*also called tidal heating, tidal torquing, tidal acceleration etc.

Tidal Response of a Newtonian Body

In Fourier space, the tidal response of a Newtonian body is

$$\delta Q_{\ell m}(\omega) = F_{\ell m}(\omega)\mathcal{E}_{\ell m}(\omega)\,,$$

where ω is the frequency of the external tidal field, and

$$\boxed{F_{\ell m}(\omega) = 2k_{\ell m} + i\omega\tau_0\nu_{\ell m} + \cdots}$$

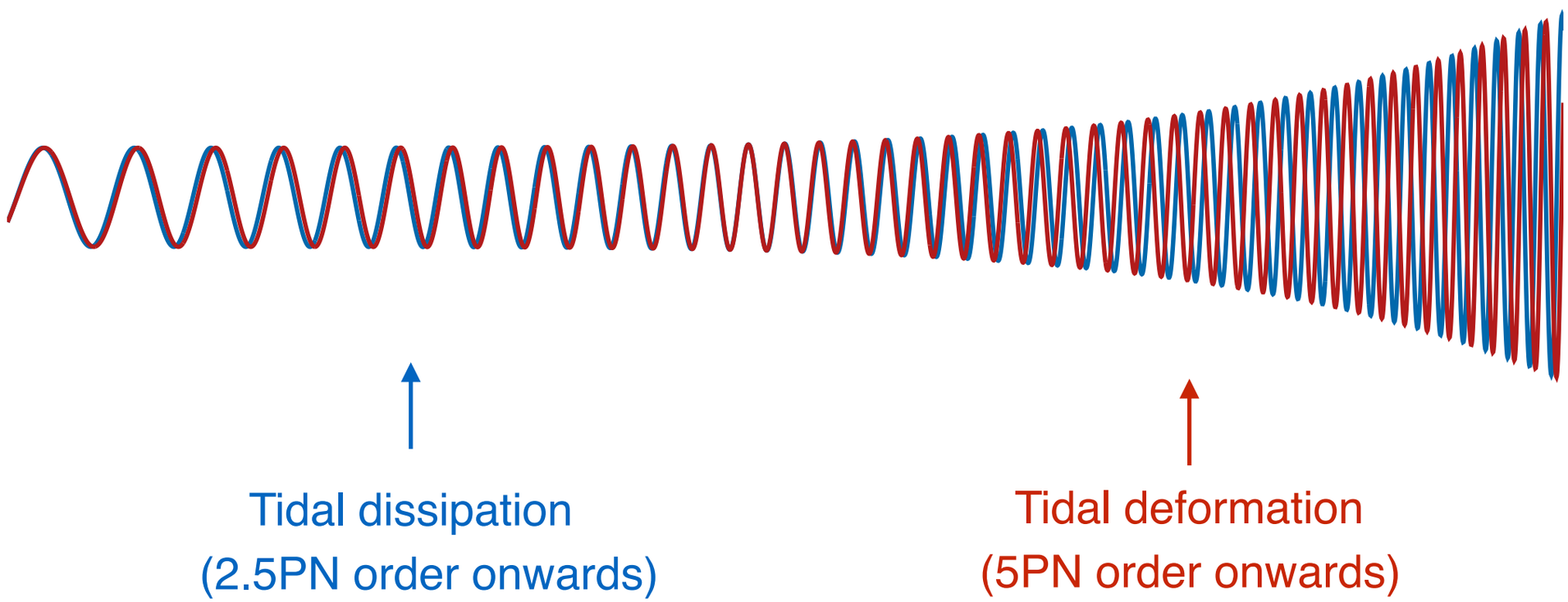
real part =
conservative effect

imaginary part =
dissipative effect

Goldberger, Rothstein [0409156, 0511133]
Chakrabarti, Delsate, Steinhoff [1306.5820]

Phase Imprints of Tidal Effects

Tidal deformation and dissipation are **different physical phenomena**.



They affect the phases of gravitational waveforms at **different PN orders**, and can be measured precisely in current and future GW detectors.

Flanagan, Hinderer [0709.1915], Binnington, Poisson [0906.1366], Damour, Nagar [0906.0096]
Hartle (1973), Poisson, Sasaki [9412027], Tagoshi, Mano, Takasugi [9711072]

Tidal Response of Schwarzschild Black Holes

To obtain the tidal response of black holes, we solve the **Teukolsky equation**. For the Schwarzschild black hole,

$$\psi_4^{\rm Schw} \propto r^{\ell-2}\left[(1+\cdots) + F_{\ell m}^{I,\rm Schw}\left(\frac{2M}{r}\right)^{2\ell+1}(1+\cdots)\right], r \gg 2M$$

$$F_{\ell m}^{I,\rm Schw}(\omega) = 0 + i\omega(2M)\nu_{\ell m}^{\rm Schw} + \cdots$$

Well-known result in the literature

We derive the **general expression** for the dissipation numbers $\nu_{\ell m}^{\rm Schw}$, which recovers known results for the first few orders of ℓ in the literature.

HSC [2010.07300], Poisson [2012.10184]

Binnington, Poisson [0906.1366], Damour, Nagar [0906.0096], Kol, Smolkin [1110.3764]

Goldberger, Rothstein [0511133]

Newtonian vs Schwarzschild Responses

Newtonian object:

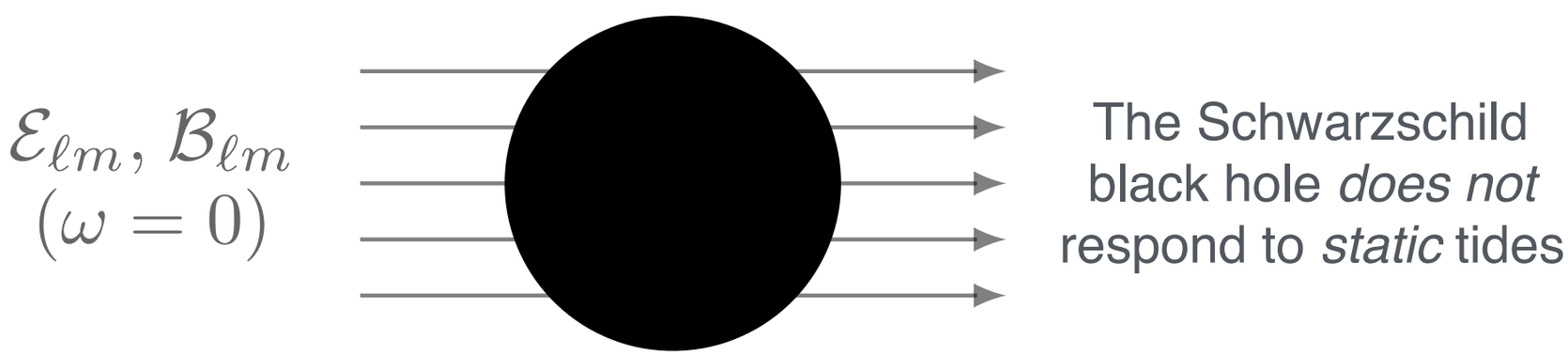
$$F_{\ell m}(\omega) = 2k_{\ell m} + i\omega\tau_0\nu_{\ell m} + \cdots$$

Schwarzschild black hole:

$$F^{I,\mathrm{Schw}}_{\ell m}(\omega) = 0 + i\omega(2M)\nu^{\mathrm{Schw}}_{\ell m} + \cdots$$

where $\tau_0 = 2M$ is the black hole light crossing time.

At $\omega = 0$, tidal dissipation of the Schwarzschild black hole vanishes as well (not true for the Kerr black hole!)



HSC [2010.07300]
Poisson [2012.10184]

Tidal Response of Kerr Black Holes

We also solve the response function of the Kerr black hole:

$$F^{I,\mathrm{Kerr}}_{\ell m}(\omega, \Omega_H) = 0 - i(m\Omega_H - \omega)\left[2Mr_+/(r_+ - r_-)\right]\nu^{\mathrm{Kerr}}_{\ell m} + \cdots$$

- **Love numbers of rotating black holes for static tides are zero**
  - true for all spins, all $\{\ell, m\}$, and both electric-type and magnetic-type tides
  - generalizes partial results known in the literature

- Tidal dissipation is proportional to the **superradiance** factor

$$\propto m\Omega_H - \omega$$

which can either be negative (energy loss) or positive (energy extraction)

HSC [2010.07300],
Poisson [1411.4711], Pani, Gualtieri, Maselli, Ferrari [1503.07365], Le Tiec, Casals [2007.00214]
Goldberger, Li, Rothstein [2012.14869], Charalambous, Dubovsky, Ivanov [2102.08917]

Why Does Love Only Vanish at D=4?

Remarkably, the black hole Love numbers are only zero in 4 space-time dimensions, but not at higher dimensions. The logarithmic running of the Love numbers also only vanish at D=4.

$$S^{\text{eff}} = S_{\text{pp}} + \int \mathrm{d}\tau \mathrm{d}x^4\, \delta^{(4)}(x - x(\tau))\, \left(2k\, \mathcal{E}_L \mathcal{E}^L + \cdots\right)$$

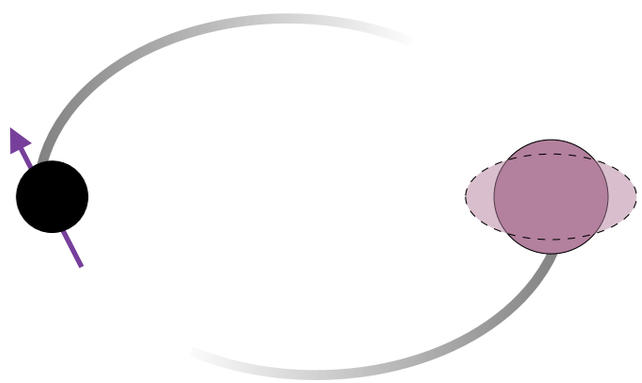
Love numbers = Wilsonian coefficients of worldline EFT

From a worldine EFT perspective, the vanishing Love numbers (Wilsonian coefficients) and their vanishing runnings present a **fine-tuning puzzle**.

Kol, Smolkin [1110.3764], Porto [1606.08895]
Charalambous, Dubovsky, Ivanov [2102.08917, 2103.01234]
Hui, Joyce, Penco, Santoni, Solomon [2010.00593]

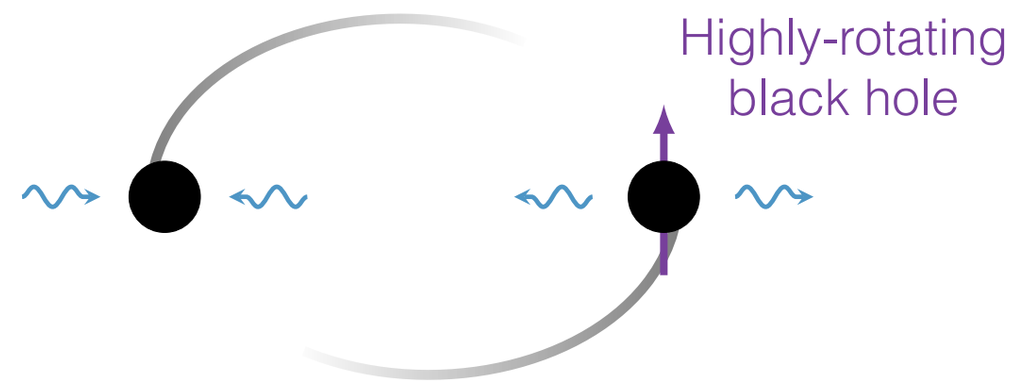
Thank you for your attention!

Tidal deformation



Rotating black holes
have **zero Love numbers**

Tidal dissipation



Black hole dissipation can induce
mode absorption and **amplification**