The evolutions of the innermost stable circular orbits in dynamical spacetimes

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Introduction

In 2019, the Event Horizon Telescope Published the first image of a black hole at the center of the M87 galaxy [1]. In the image, one can see a shadow region which is called the black hole shadow, and the black hole lies in the shadow. One can also see a ring-like structure that corresponds to the accretion disk, and the study of the innermost stable circular orbit (ISCO) plays a vital role in analyzing this image.
Up to date, there are many studies based on the effective potential to study ISCO in spacetime. On the one hand, ISCO has many important properties. For example, it is the inner edge of an accretion disk; it is also the boundary between the stable orbits and the unstable orbits, and the accretion flow changes dramatically across the ISCO in a thin disk. On the other hand, ISCO has many applications. Such as, for a rotating black hole, the radius of ISCO is a key fit parameter to measure the spin of the black hole, and there are many other studies about the ISCOs in Kerr-like spacetimes. In the modified gravitational theories, ISCOs may also exist. Also, ISCO may have some applications in AdS/CFT. In recent years, some studies suggest that ISCO should describe field theory long-lived excitations that do not thermalize like typical excitations.
Through the effective potential method, one can efficiently study the ISCOs in static and stationary spacetimes. But, this method is not suitable for dynamical spacetimes because the effective potential cannot be defined in dynamical spacetimes.
In this paper, we get a method to study the evolutions of the ISCOs in dynamical spacetimes. As examples, we studied the ISCOs in Vaidya spacetime, Vaidya-AdS spacetime and the slow rotation limit of Kerr-Vaidya spacetime. The results given by these examples are reasonable and have similar evolution curves to photon spheres in dynamical spacetimes [3].

Conclusion

In this paper, we reviewed the two methods to get the ISCO in Schwarzschild spacetime. We domenstrated the extremum method is equivalent to the effective potential method in static and stationary spacetimes. We verify this equivalence in general spherically symmetric spacetimes and Kerr spacetime. We then generalized the extremum method into dynamical spacetimes. From this generalization, we studied the evolutions of the ISCOs in Vaidya spacetime, Vaidya-AdS spacetime, and Kerr-Vaidya spacetime under the limit of slow rotation. These examples are all giving reasonable resluts.

Reference

[1] K. Akiyama *et al.* [Event Horizon Telescope Collaboration], Astrophys. J. **875**, no. 1, L1 (2019) [arXiv:1906.11238 [astro-ph.GA]].
[2] T. Berry, A. Simpson and M. Visser, Universe **7**, no.1, 2 (2020) doi:10.3390/universe7010002 [arXiv:2008.13308 [gr-qc]].
[3] A. K. Mishra, S. Chakraborty and S. Sarkar, Phys. Rev. D **99**, no.10, 104080 (2019) doi:10.1103/PhysRevD.99.104080 [arXiv:1903.06376 [gr-qc]].

lessons from Schwarzschild spacetime

we get two critical properties of the ISCO in Schwarzschild spacetime:

(1). For a general circular orbit, it does not evolve in time, i.e.,

$$dr_o/d\tau = d^2 r_o/d\tau^2 = 0\ . \tag{1}$$

(2). For a family of circular orbits, ISCO has a minimal orbital angular momentum, i.e.,

$$\delta l_o/\delta r_o = \delta l_o^2/\delta r_o = 0\ , \tag{2}$$

where l_o should be regarded as a function of r_o

If the solution of eq.(2) is single-valued, it is an ISCO. If the solution of eq.(2) is double-valued, such as Schwarzschild-dS spacetime, Kerr-dS spacetime and so on, the one with $\delta^2 l_o/\delta r_o^2 > 0$ is ISCO, and the one with $\delta^2 l_o/\delta r_o^2 < 0$ is OSCO (outermost stable circular orbit). Below, we only treat single-valued cases.

ISCOs in static and stationary spacetimes

In the general static and stationary spacetimes, eq.(1) is obviously valid. Below we will demonstrate that eq.(2) is also valid in some conditions.
In the general static and stationary spacetimes, suppose one can define the effective potential as $V_l(r)$, where l is the conserved orbital angular momentum. Consider a free point particle, and for a given circular orbit, one always has the following relation

$$V'_{l_o}(r_o) = 0\ , \tag{3}$$

and for this circular orbit, l_o is a constant. Considering a family of circular orbits and varying eq.(3), one can get the following equation,

$$0 = \frac{\delta V'_{l_o}(r_o)}{\delta r_o} = V''_{l_o}(r_o) + \frac{\partial V'_{l_o}(r_o)}{\partial l_o}\frac{\delta l_o}{\delta r_o}\ . \tag{4}$$

Here, l_o should regard as a function of r_o. Then, one have the following relation

$$V''_{l_o}(r_o) = -\frac{\partial V'_{l_o}(r_o)}{\partial l_o}\frac{\delta l_o}{\delta r_o}\ , \tag{5}$$

where we have assumed that $\partial V'_{l_o}(r_o)/\partial l_o\big|_{r_{isco},l_{isco}} \neq 0$. In general, this assumption can be satisfied. So, for an ISCO, the condition $V''_{l_{isco}}(r_{isco}) = 0$ is equivalent to $\delta l_o/\delta r_o = 0$ [2], and the stable circular orbits should satisfy the condition that $\delta l_o/\delta r_o \geq 0$.

The evolutions of the ISCOs in dynamical spacetimes

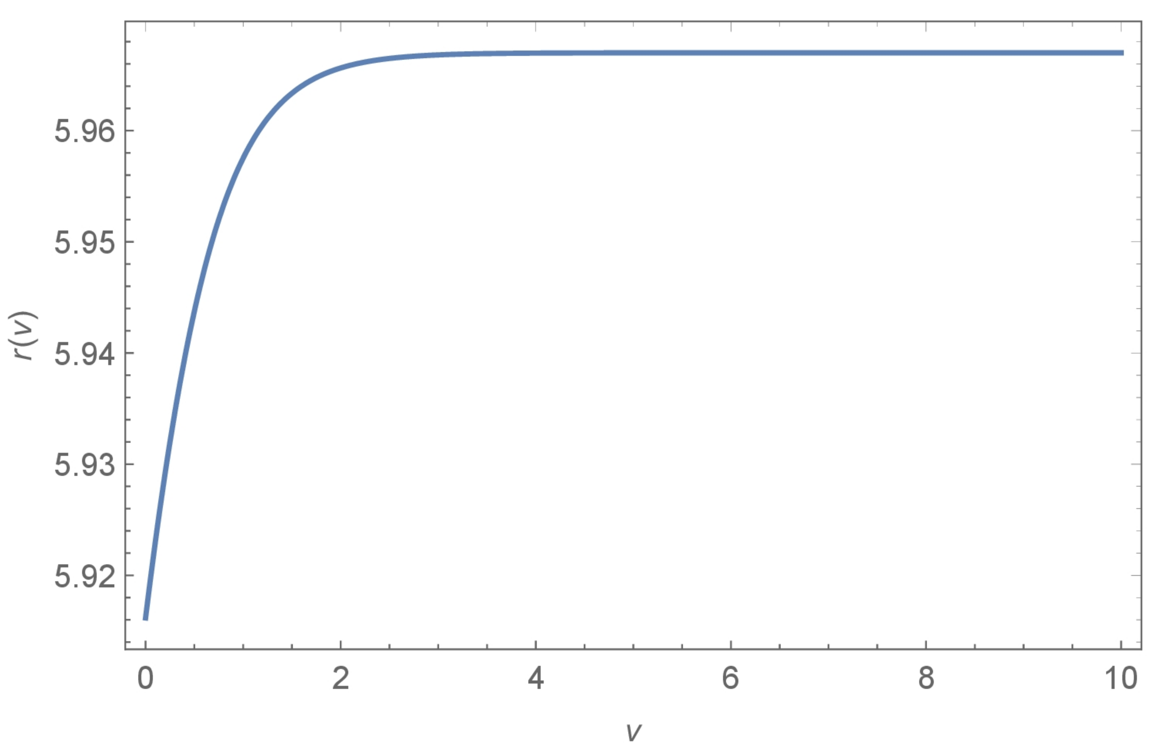
In general dynamical spacetimes, condition (1) does not hold anymore. Enlightened by [3], we assume

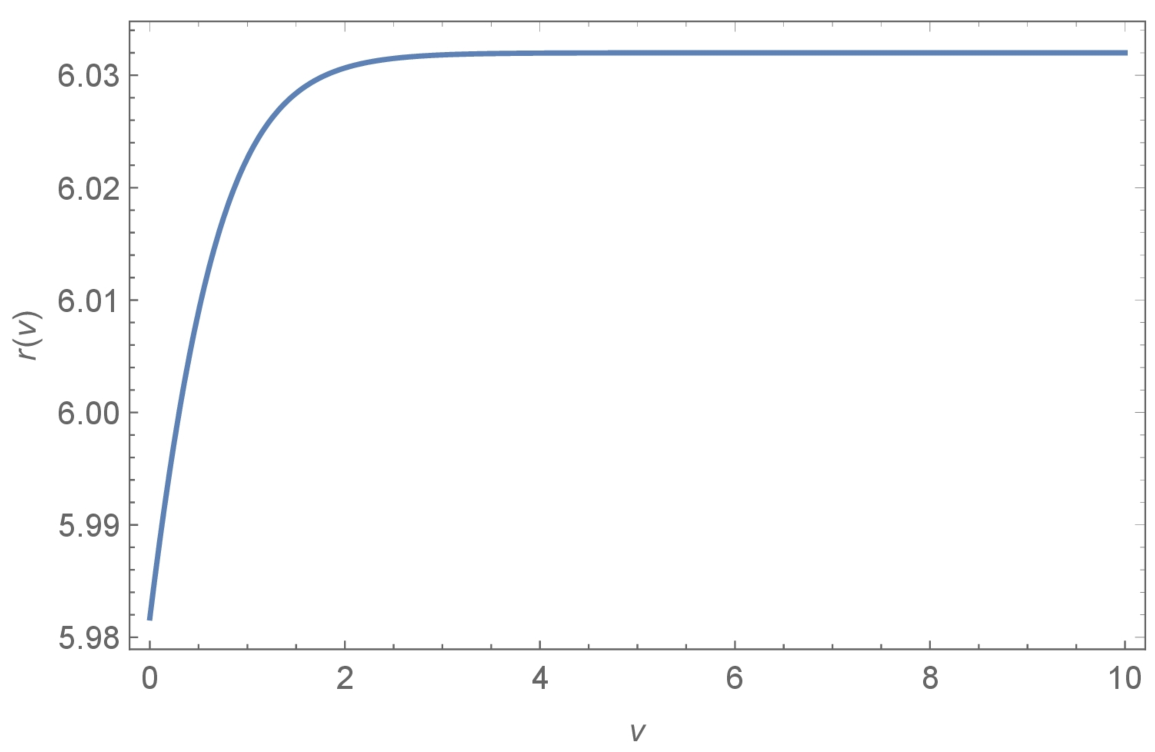
$$dr_o(t) = \frac{\partial r_o(t)}{\partial t}dt = \dot{r}_o(t)dt\ , \tag{6}$$

where t is the coordinate time and a dot stands for the derivative with respect to this coordinate time. As for condition (2), we generalize it to the following equation

$$\frac{\delta l_o}{\delta r_o(t)} = \frac{\delta l_o^2}{\delta r_o(t)} = 0\ . \tag{7}$$

Here, l_o should regard as a function of $r_o(t)$. The evolution of the ISCO of Kerr-Vaidya spacetime in the slow rotation limit is shown below.





The left figure is correspond to the “direct” rotation and The right figure is correspond to the “retrograde” rotation. Thses results are similar to the evolutions of the photon sphere in Kerr-Vaidya spacetime under the slow rotation limit [3].