**AstroDark-2021**: December 7- 10 (Japan)

# Non-linear electrodynamics: a model of Inflationary Universe

Payel Sarkar, **Prasanta Kumar Das** and Gauranga Charan Samanta

BITS Pilani, K K Birla Goa Campus, NH-17B, Zuarinagar, Goa-403726

**Journal Reference**: Physica Scripta. 96, 065305 (2021)

**Department of Physics**
**BITS Pilani**
K K Birla Goa Campus

## Abstract

We study the inflationary expansion in the early Universe within the framework of non linear electrodynamics(NLED). We propose a NLED lagrangian which is characterised by two parameters $\alpha$(dimensionless) and $\beta$(dimensionful). The classical stability and causality aspects of the model requires the speed of the sound wave $c_s^2 > 0$ and $0 \le c_s \le 1$ provided $1.828 \le \beta B^2 \le 1.469$ with $\alpha = 1$. The magnetic field necessary to trigger the inflation is found to be $B \simeq \sqrt{\frac{0.4\rho_B^{(max)}}{0.65}} = 4\times 10^{51}$ Gauss, where $\rho_B^{(max)}(\sim \rho_{inf}) = 10^{64}$ GeV$^4$. The model also predicts the desired e-fold number $N = 71$ in the parameter space of $\alpha$ and $\beta$. With $\alpha = 0.3$ and $\beta B^2 = 0.3974$, we find the scalar spectral index $n_s = 0.9649$, which is consistent with the PLANCK 2018 data and also predict the tensor-to-scalar ratio $r = 0.1417$ and the tensor spectral index $n_T = -0.0177$ corresponding to $n_s = 0.9649$.

## Introduction

The cosmic evolution of the early Universe was largely controlled by the non-linear electromagnetic(NLED) field. This NLED field, which acts as a source of gravity, can lead to negative pressure and thus drive the inflationary expansion of the Universe [1].

## A model of non-linear electrodynamics

The NLED Lagrangian, we propose, is given by

$$\mathcal{L} = -\frac{\mathcal{F}e^{-\beta\mathcal{F}}}{(\mathcal{F}\beta+\alpha)^2}$$

where $\mathcal{F} = \frac14 F_{\mu\nu}F^{\mu\nu} = \frac12(B^2-E^2)$. One finds the energy-momentum tensor of the NLED field,

$$T_{\mu\nu} = \frac{\partial\mathcal{L}}{\partial\mathcal{F}}F_{\mu\alpha}F_\nu^\alpha - g_{\mu\nu}\mathcal{L}$$

The early Universe being magnetic(where we set $E = 0$ as the electric field is screened off due to the charged plasma), we find the energy density($\rho(=\rho_B)$) and pressure($P(=P_B)$)

$$\rho_B = \frac{2B^2e^{-\beta B^2/2}}{(\beta B^2+2\alpha)^2},\quad P_B = -\frac{4B^2e^{-\beta B^2/2}}{3(\beta B^2+2\alpha)^3}\left[\beta^2B^4+\beta B^2\left(\frac72+2\alpha\right)-\alpha\right]$$

### Inflation in non-linear electrodynamics

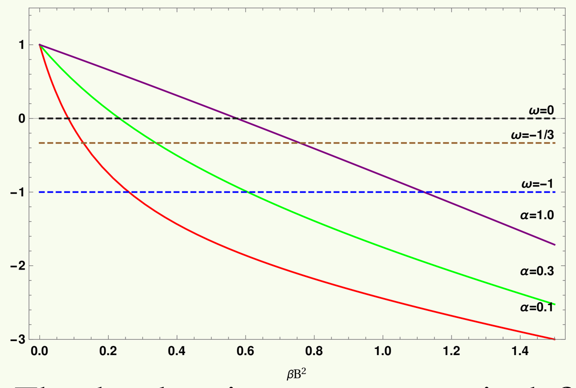
The Friedmann and the Roychaudhury equation are given by,

$$\left(\frac{\dot a}{a}\right)^2 = \frac13\rho_B,\quad \frac{\ddot a}{a} = -\frac16(\rho+3P_B)$$

where we set $8\pi G = 1$.

- The EoS parameter $\omega$ of the NLED field with $B = B_0/a^2$, is given by

$$\omega = \frac{P_B}{\rho_B} = -\frac23\left(\frac{\beta B_0^2}{a^4}+2\alpha\right)^{-1}\left[\frac{\beta^2B_0^4}{a^8}+\frac{\beta B_0^2}{a^4}\left(\frac72+2\alpha\right)-\alpha\right]$$




- $\omega$ is plotted against $\beta B^2$ for different $\alpha$.
- $\omega$ crosses $-1$(negative pressure $\to$ acceleration) at $\beta B^2 = 0.25(1.112)$ corresponding to $\alpha = 0.1(1.0)$.

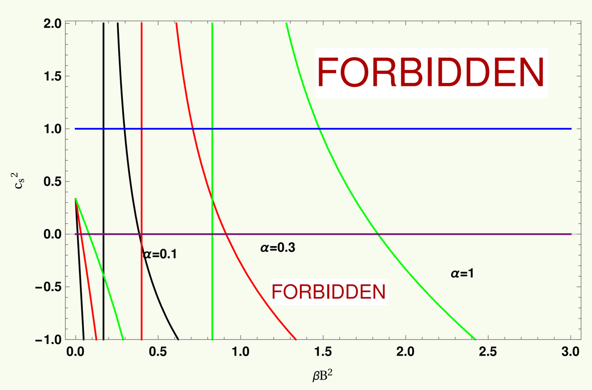
- The deceleration parameter q is defined as

$$q = -\frac{a\ddot a}{\dot a^2} = \frac12(1+3\omega) = \frac{2\alpha-3\beta B^2-2\alpha\beta B^2-\beta^2B^4}{2\alpha+\beta B^2} \to 1 \text{ as B} \to 0$$

So, $q \to 1 \to \ddot a < 0 \Longrightarrow$ *No acceleration without the magnetic field*!!

- Causality and Classical stability of $c_s$: The speed of the sound wave $c_s$ is given by

$$c_s^2 = \frac{dP_B}{d\rho_B} = -1-\frac23\beta B^2+\frac{8\alpha}{2\alpha+\beta B^2}+\frac{16\alpha-2\beta B^2(2\alpha+2)}{6\alpha\left(\frac{\beta B^2}{2}-1\right)+3\beta B^2\left(\frac{\beta B^2}{2}+1\right)}$$




- $c_s^2$ is plotted against $\beta B^2$ for different $\alpha$.
- For $\alpha = 0.1(1.0)$, we see that the classically stability condition $0 \le c_s^2 \le 1$ requires $0.382(1.828) \le \beta B^2 \le 0.288(1.469)$.
- The regions corresponding to $c_s^2 < 0$ and $c_s^1 > 1$ are forbidden.

### Effective potential $V_{eff}$, the inflation energy density $\rho_B$ and the e-fold number($N$)

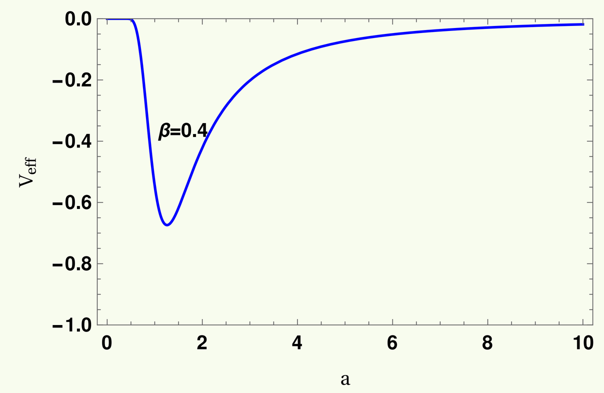
- The Friedmann equation reads

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{\rho_B}{3} \to \dot a^2 + V_{eff}(a) = 0$$

where the effective potential $V_{eff}(a) = -\frac{B_0^2}{6a^2}e^{-\beta B_0^2/2a^4}\left(\frac{\beta B_0^2}{2a^4}+\alpha\right)^{-2}$.

- Taking a derivative of the Friedman eqn, we find

$$2\dot a\ddot a+\dot a\frac{dV_{eff}}{da} = 0 \to \ddot a+\frac12\frac{dV_{eff}}{da} = 0$$




- $V_{eff}$ is plotted against $a$ for $\alpha = 0.3, \beta = 0.4$ and $B_0 = 1$ and it has a minima at $a_c = 1.69$ (the point where the inflation stops).
- On the left of $a_c$, we see $\frac{dV_{eff}}{da} < 0$ i.e. $\ddot a > 0 \to$ *acceleration*, whereas, on the right of $a_c$, $\frac{dV_{eff}}{da} > 0$ i.e. $\ddot a < 0 \to$: *deceleration*.

- Inflation requires $\dot\rho_B = 0 \to \rho_B + P_B = 0$ and one finds

$$\beta^2B^4+\beta B^2(2+2\alpha)-4\alpha = 0$$

This gives $\beta B^2 = 0.4$ for $\alpha = 0.3$.

- The energy density $\rho_B(=\rho_B^{(max)})$ during inflation is calculated as

$$\rho_B = \frac{2\beta B^2e^{-\beta B^2/2}}{\beta(2\alpha+\beta B^2)^2} = 0.65/\beta = \rho_B^{max} \to \beta\rho_B^{max} = 0.65$$

- Now $\beta B^2 = 0.4 \to B \simeq \sqrt{\frac{0.4}{\beta}} = \sqrt{\frac{0.4\rho_B^{max}}{0.65}} = 4\times10^{51}$ G with $\rho_B^{max}(\rho_{inf}) = 10^{64}$ GeV$^4$.
- The e-fold number($N$) is defined as

$$N = \int_t^{t_{end}} H dt = \ln\frac{a_{end}}{a} = ln\sqrt{\frac{B_{start}}{B_{end}}}$$

With $B_{end} = 10^{-10}$ G and $B_{start} = 4\times10^{51}$ G, we find

$$N = \frac12 ln(4\times10^{61}) = 71$$

.

## Spectral index analysis

The scalar spectral index($n_s$), tensor-to-scalar ratio ($r$) and the tensor spectral index ($n_T$) in NLED are defined as

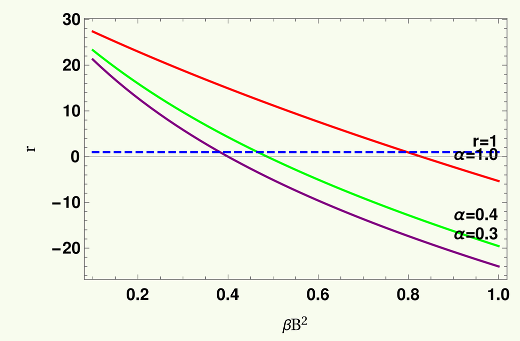
$$n_s = 1-\frac{4(4\alpha-2\beta B^2-\beta B^2(\beta B^2+2\alpha))}{\beta B^2+2\alpha},$$

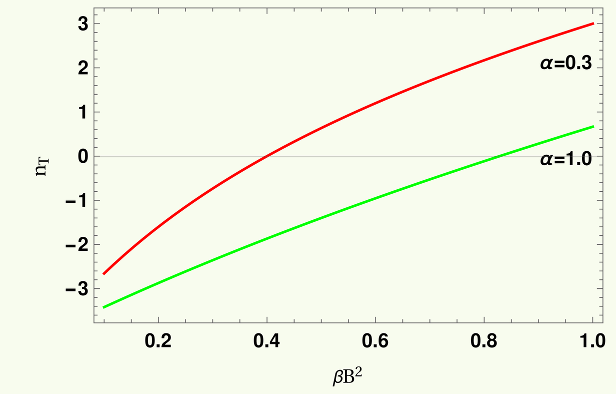
$$r = \frac{16(4\alpha-2\beta B^2-\beta B^2(\beta B^2+2\alpha))}{(\beta B^2+2\alpha)}$$

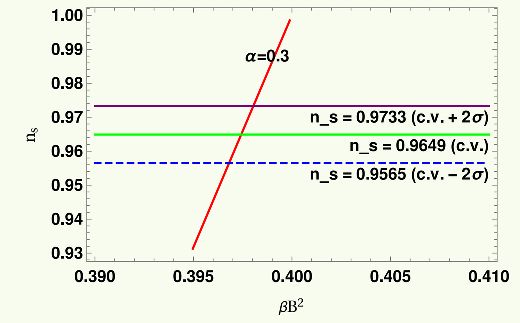
and

$$n_T = -\frac{r}{8} = -2\left[\frac{4\alpha-2\beta B^2-\beta B^2(\beta B^2+2\alpha)}{(\beta B^2+2\alpha)}\right]$$

Results:






- $n_s$, $r$ and $n_T$ are plotted as a function of $\beta B^2$ for different $\alpha$ values.
- Also shown are the scalar spectral index $n_s = 0.9649 \pm 0.0042$ (Planck 2018 CMB data)[2].

- We find that $\alpha = 0.3$ and $\beta B^2 = 0.3974$ are consistent with the scalar spectral index data $n_s = 0.9649 \pm 0.0042$(Planck 2018).
- With these $\alpha, \beta$, we predict the tensor-to-scalar ratio $r = 0.1417, 0.1089, 0.1744$ and the tensor spectral index $n_T = -0.0177, -0.0136, -0.0218$ corresponding to c.v. and c.v.$\pm 2\sigma$ values of $n_s$.

## Results & Conclusions

- We find that a highly non-linear magnetic field $B = 4\times10^{51}$ G can trigger the inflationary expansion of the early Universe.
- Our NLED model predicts the e-fold number $N = 71$ in the space of nonlinear parameters $\alpha$ and $\beta$.
- We find that the nonlinear parameters $\alpha = 0.3$ and $\beta B^2 = 0.3974$ are consistent with the scalar spectral index $n_s = 0.9649 \pm 0.0042$(Planck 2018) and we also predict the tensor-to-scalar ratio $r = 0.1417$ and tensor spectral index $n_T = -0.0177$.

## Acknowledgements

PS thanks to Department of Science and Technology(DST), Government of India for the Inspire fellowship (No. DST/INSPIRE Fellowship/2017/IF170807). The work of PKD and GCS is supported by CSIR Grant No.25(0260)/17/EMR-II .

## References

1 S. I. Kruglov, Int. J. Mod. Phys. A 32 No.13 (2017) 1750071.

2 Y. Akrami (2018), arXiV:1807.06211