Long Baseline Oscillation Probability Approximation in a Model for Light Sterile Neutrinos

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A Model for Light Sterile Neutrinos

The Lagrangian

$$-\mathcal{L} = Y_{\alpha i}\bar{N}_i H L_\alpha + \frac{1}{2}\bar{N}_i M^N_{ij} N^C_j + \lambda_{ai}\bar{N}_i \phi \nu_a + h.c.$$

ϕ Scalar field, charge 2 under Z_3

$H = i\sigma_2\phi^*_H$ the Higgs field

N_i Heavy Majorana Neutrinos

α a Lepton flavor ; light sterile indices

The Mass Matrices

Denoting the light sterile neutrinos by ν_s and working in the basis of (ν_a, ν_s, N^C_i), the neutrino mass matrix is written as follows (left) upon SSB, where $M_D = Y v_H$

$$\begin{pmatrix} 0 & 0 & M_D \\ 0 & 0 & \lambda v_\phi \\ M_D^T & \lambda^T v_\phi & M^N \end{pmatrix} \quad \mathfrak{M} = -\sum_j \begin{pmatrix} \frac{Y_{\alpha j}Y_{\beta j}v_H^2}{M_j} & \frac{Y_{\alpha j}\lambda_{bj}v_H v_\phi}{M_j} \\ \frac{\lambda_{aj}Y_{\beta j}v_H v_\phi}{M_j} & \frac{\lambda_{aj}\lambda_{bj}v_\phi^2}{M_j} \end{pmatrix}$$

Upon Seesaw mechanism, the mass matrix of the active and light sterile neutrino in the flavor space is also given above (right)

The Mixings

The effective "PMNS" matrix U under this model is given in a relation with the mass matrices as

$$\mathfrak{M} = UPMPU^T$$

Where we can also write U in separate rotations.

Cosmology and More…

This model allows for interactions in the forms of $\frac{A\phi^2}{2}\nu_s\nu_s + h.c.$ or sterile-active/sterile interactions of strength $G_s = \frac{g^2_{\nu_s\nu_s\phi}}{m^2_\phi}$ or $G_s\sin^2(\theta_{4\alpha})$, which can be used to reconcile with cosmological bounds. Stay tuned for details in our paper!

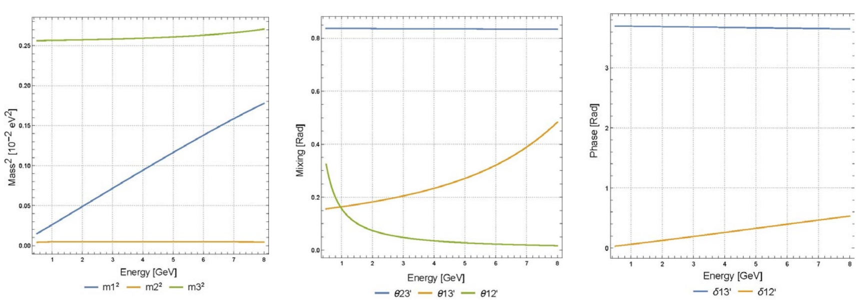
We provide a model for light sterile neutrinos, explained only briefly here.

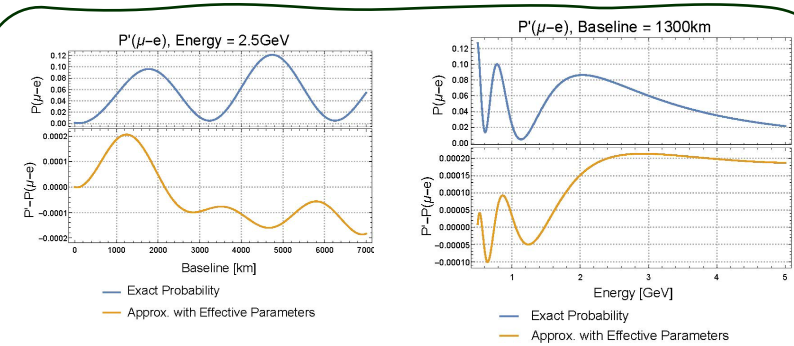
Summary

We introduce an oscillation probability approximation by sequentially approximating the Hamiltonian matrix elements, effective mixing angles and masses.

We test the validity and accuracy of this new method

Effective mass differences, mixings, and phases as functions of energy





In general, matter effect is being incorporate by diagonalizing the effective Hamiltonian:

$$H_{\text{eff}} \propto U\text{diag}(m_1^2, m_2^2, m_3^2)U^\dagger + \text{diag}(A, 0, 0)$$

Our procedure starts with the idea of obtaining the effective PMNS matrix: $U_{\text{eff}} = R'_4 R'_{123}$

Where the original mixing is denoted by rotations: $R_{123} = R_{23}R_{13}R_{12}, \ R_4 = R_{34}R_{24}R_{14}$

After making appropriate approximations, we have effective mass as $\tilde{M}^2 \approx \ R'^\dagger_{123}(R_{123}M^2R^\dagger_{123} + R^\dagger_4 \mathcal{A} R_4)R'_{123}$

Starting from here, we can start sequentially obtaining the effective mixing angles by simply reading off from this effective square mass difference matrix:

The same can be performed to obtain the effective masses from the effective Hamiltonian.

$$H^{(1)} = H^{(1)}_R + H^{(1)}_A, \quad H^{(1)}_R = R_{123}M^2R^\dagger_{123}, \quad H^{(1)}_A = R^\dagger_4 \mathcal{A} R_4$$

Read off → $\theta'_{23} = \frac{1}{2}\arctan\frac{2|H^{(1)}[2,3]|}{H^{(1)}[3,3] - H^{(1)}[2,2]}$

Plug in →

$$H^{(2)}_A[1,1] = Ac^2_{14} - A'c^2_{24}s^2_{14}, \quad H^{(2)}_A[1,3] = -A'e^{-i\delta_{14}}c_{24}s_{14}s_{23}s_{24}, \quad H^{(2)}_A[3,3] = -A's^2_{23}s^2_{24}$$

Read off → $\theta'_{13} = \frac{1}{2}\arctan\frac{2|H^{(2)}[1,3]|}{H^{(2)}[3,3] - H^{(2)}[1,1]}$, $\delta'_{13} = -\arg\left((m_3^2 - m_2^2 s_{12}^2)c_{13}s_{13}e^{-i\delta_{13}} - A'c_{24}s_{14}s_{23}s_{24}e^{-i\delta_{14}}\right)$

Plug in →

$$H^{(3)}_A[1,1] = \ldots, \quad H^{(3)}_A[1,2] = \ldots, \quad H^{(3)}_A[2,2] = -A'c^2_{23}s^2_{24}$$

Read off → $\theta'_{12} = \frac{1}{2}\arctan\frac{2|H^{(3)}[1,2]|}{H^{(3)}[2,2] - H^{(3)}[1,1]}$, $\delta'_{12} = \ldots$

Plugging in the effective parameters into the oscillation probability formula in vacuum, we test the difference between the exact calculation and our method: the approximation reaches an accuracy of ≤ 0.0002 while for lower energies it reduces to ≤ 0.0001.