Testing Extended Gravity with Neutrino Oscillations [1]

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Abstract

Neutrino oscillations are studied in **extended theories of gravity.** The oscillation probability formula is derived for propagation in static spacetimes described by actions quadratic in the curvature invariants. Calculations are carried for both vacuum and matter oscillations, showing that the neutrino phase is sensitive to the *strong equivalence principle* violation. The possibility to constrain extended theories is discussed.

Introduction

Neutrinos are the most enigmatic elementary particles in the Standard Model. Since their prediction to be existent, they fascinate physicists, attracting even more attention after the discovery of **flavor oscillations**.

While being largely investigated in quantum theory on flat spacetime, neutrino oscillations in Einstein's gravity have only recently been analyzed by Stodolsky, highlighting non-trivial peculiar effects [2]. However, it is commonly thought that Einstein's theory might not be the ultimate description of gravity, because of its shortcomings at both small and large scales. This has paved the way for a strenuous search of new models [3].

Among all the Extended Theories (ET's) formulated over the years, the most straightforward and predictive approaches are the so-called **quadratic theories**, which consist in generalizing the Einstein-Hilbert action by including contributions quadratic in the curvature invariants. Understanding which of these formalisms provides the best candidate for GR extension is a challenging task.

Here, we consider neutrino oscillations as a stress-test for quadratic theories. Using the covariant approach of [2], we compute corrections to the oscillation probability formula induced by the extra terms in the gravitational action. Calculations are developed for both vacuum and matter oscillations. We show that the modified neutrino phase is sensitive to the violation of the *strong equivalence principle* (SEP), offering the possibility of constraining the proposed models based on the consistency with experimental data. Throughout the work, we consider a simplified model involving two neutrino flavors. Extension to three generations is straightforward.

1 Neutrino oscillations in curved spacetime

Since Pontecorvo's pioneering idea of flavor oscillation [4], it is known that a neutrino of given flavor (say electron) at the production point A can transmute into a different flavor (say muon) after traveling a distance $L_p = |x_B - x_A|$ in vacuum with probability

$$\mathcal{P}_{e\to\mu}(x_B) = \sin^2(2\theta)\,\sin^2\left(\frac{\varphi_{12}}{2}\right), \tag{1}$$

where θ is the mixing angle of corresponding mass states. For relativistic neutrinos of common energy E, the phase shift reads $\varphi_{12} \equiv |\varphi_1 - \varphi_2| \simeq \frac{\Delta m^2}{2E} L_p$,where $\Delta m^2 \equiv |m_2^2 - m_1^2|$ is the neutrino mass-difference.

Equation (1) holds in flat space. For a generic static curved spacetime $g_{\mu\nu}$, it remains formally unchanged, provided that the neutrino phase is replaced by the covariant definition [2]

$$\varphi_i \to \Phi_i = \int_{\lambda_A}^{\lambda_B} P_{\mu,i}\,\frac{dx^\mu_{null}}{d\lambda}d\lambda\,, \quad i = 1,2\,, \tag{2}$$

where $dx^\mu_{null}/d\lambda$ is the tangent vector to the neutrino worldline parameterized by λ and $P_{\mu,i}$ is the generator of spacetime translations for the i^{th}-neutrino mass eigenstate. Henceforth, we shall assume that both mass neutrinos approximately propagate along a null trajectory. We then have

$$P_\mu \frac{dx^\mu_{null}}{d\lambda} = \left(\frac{M^2}{2} - \frac{dx^\mu_{null}}{d\lambda} A_\mu P_L\right), \tag{3}$$

where M is the diagonal mass matrix and $P_L \equiv (1-\gamma_5)/2$. The vector potential is defined by $A^\mu = \frac{1}{4} g^{1/2} e^\mu_{\hat{a}} \epsilon^{\hat{a}\hat{b}\hat{c}\hat{d}} \left(e_{\hat{b}\nu,\sigma} - e_{\hat{b}\sigma,\nu}\right) e^\nu_{\hat{c}} e^\sigma_{\hat{d}}$ for vacuum propagation and $A^\mu \equiv \begin{pmatrix} -\sqrt{2}\,G_F\,N_e^\mu & 0 \\ 0 & 0 \end{pmatrix}$ for propagation in a background fluid, where N_e^μ is the number current of the fluid.

2 Quadratic theories of gravity

The most general parity-invariant and torsion-free action around maximally symmetric backgrounds reads [5]

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{\mathcal{R} + \frac{1}{2}\Big[\mathcal{R}\,\mathcal{F}_1(\Box)\mathcal{R} + \mathcal{R}_{\mu\nu}\mathcal{F}_2(\Box)\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{F}_3(\Box)\mathcal{R}^{\mu\nu\rho\sigma}\Big]\right\}, \tag{4}$$

where $\kappa = 1/M_p$ is the inverse of the reduced Planck mass and $\mathcal{F}_j(\Box) = \sum_{n=0}^{N} f_{j,n}\Box^n$, $j = 1,2,3$.

In our perturbative approach we study neutrino oscillations in the presence of a weak gravitational field described by the linearized metric

$$ds^2 = (1+2\phi)dt^2 - (1-2\psi)(dr^2 + r^2 d\Omega^2), \tag{5}$$

where m_s denotes the source mass and the two metric potentials in Fourier-transform are given by [1]

$$\phi(r) = -\frac{4Gm_s}{\pi r}\int_0^\infty dk\left[\frac{f-2g}{f(f-3g)}\frac{\sin(kr)}{k}\right], \qquad \psi(r) = \frac{4Gm_s}{\pi r}\int_0^\infty dk\left[\frac{g}{f(f-3g)}\frac{\sin(kr)}{k}\right], \tag{6}$$

for given functions $f \equiv f(k^2)$ and $g \equiv g(k^2)$.

3 Neutrino oscillations in quadratic gravity

Let us investigate how the oscillation phase (2) entering the probability (1) appears for neutrino propagation between the production (r_A) and detection (r_B) points in the spacetime (5).

3.1 Oscillations in vacuum

For neutrino vacuum oscillations, the phase-difference can be split as [1]

$$\Phi_{12} \equiv |\Phi_1 - \Phi_2| = \Phi_0 + \Phi_{GR} + \Phi_Q\,, \tag{7}$$

where Φ_0 is formally the same as the flat phase φ_{12} for $m_s = 0$, $\Phi_{GR} = \frac{\Delta m^2 L_p}{2E_\ell}\left[\frac{G\,m_s}{r_B} - \frac{G\,m_s}{L_p}\ln\left(\frac{r_B}{r_A}\right)\right]$ is the contribution pertaining to GR gravity and

$$\Phi_Q = \frac{\Delta m^2 L_p}{2E_\ell}\left[\frac{1}{L_p}\int_{r_A}^{r_B} dr\,\phi_Q(r) - \phi_Q(r_B)\right] \tag{8}$$

is the correction we are interested in, as it includes quadratic gravity terms. Here, we have recast the metric potentials as $\phi(r) = \phi_{GR}(r) + \phi_Q(r)$ and $\psi(r) = \psi_{GR}(r) + \psi_Q(r)$.

Consistently with our assumptions, we always have either $|\Phi_0| > |\Phi_{GR}| \gtrsim |\Phi_Q|$ or $|\Phi_0| > |\Phi_Q| \gtrsim |\Phi_{GR}|$. By requiring consistency with current experimental bounds on neutrino oscillations, such inequalities could allow to constrain the free parameters featuring quadratic theories of gravity [6].

3.2 SEP violation

With the aid of Eqs. (7) and (8), the oscillation probability formula (1) can be elegantly written as [1]

$$\mathcal{P}_{e\to\mu} = \sin^2(2\theta)\sin^2\left[\frac{\Delta m^2}{4E}(r_B - r_A) + \frac{\Phi_{SEP}}{2}\right], \tag{9}$$

where we have introduced the shorthand notation

$$\Phi_{SEP} = \frac{\Delta m^2}{2E}\int_{r_A}^{r_B} dr\left[\phi_Q(r) - \psi_Q(r)\right]. \tag{10}$$

It is easy to show that the difference $\phi_Q - \psi_Q$ is directly related to the Nordtvedt parameter η through the relation $\eta = 1 - \gamma$, where $\gamma = 1 - \frac{\phi_Q - \psi_Q}{\phi}$ [1]. Since departure of η from zero quantifies SEP violation, a non-vanishing value of Φ_{SEP} recognizable by a suitable modification of the measured oscillation probability could be a signal of SEP violation induced by a quadratic gravity model. As expected, in the case of GR we simply have $\Phi_{SEP} = 0$, since $\phi_Q = \psi_Q = 0$.

3.3 Oscillations in matter

For the case of neutrino oscillations in matter background, the oscillation formula (1) gets modified as

$$\mathcal{P}_{e\to\mu} = \sin^2\left(\frac{\tilde{\Phi}_{12}}{2}\right), \tag{11}$$

where we have assumed the resonance condition $\tilde{\theta} = \pi/4$. Here we have defined

$$\tilde{\Phi}_{12} = \frac{\Delta\mu^2}{2E}\int_{r_A}^{r_B} dr\left[1 + \phi(r) - \psi(r) + \alpha\,\phi(r)\right] \tag{12}$$

$$\alpha \equiv \frac{\Delta m^2\left(\Delta m^2 + v\cos 2\theta\right)}{\left(\Delta\mu^2\right)^2} - 1\,, \tag{13}$$

$$\Delta\mu^2 = \sqrt{\left(\Delta m^2\right)^2 + v^2 - 2\,\Delta m^2\,v\,\cos 2\theta}\,. \tag{14}$$

where $v = \Delta m^2 \cos 2\theta$ under our assumptions [1].

As before, it is convenient to rewrite $\tilde{\Phi}_{12}$ in such a way as to separate out the SEP violation term. This yields

$$\tilde{\Phi}_{12} = \frac{\Delta\mu^2}{2E}(r_B - r_A) + \tilde{\Phi}_{SEP} + \frac{\alpha\,\Delta\mu^2}{2E}\int_{r_A}^{r_B} dr\,\phi(r)\,, \quad \tilde{\Phi}_{SEP} \equiv \frac{\Delta\mu^2}{2E}\int_{r_A}^{r_B} dr\left[\phi_Q(r) - \psi_Q(r)\right]. \tag{15}$$

The result (9) is recovered by removing the matter background ($v \to 0$).

4 Results

We now determine Φ_Q and Φ_{SEP} for vacuum oscillations in several quadratic theories (corresponding quantities for propagation in matter can be derived by using the recipe (12)-(15)). Results are summarized below (for the definition of the free parameters, refer to [1]):

Model	Φ_Q	Φ_{SEP}
$f(\mathcal{R})$	$\frac{\Delta m^2 L_p}{2E_\ell}\left\{\frac{Gm_s\,e^{-m_0 r_B}}{3r_B} - \frac{Gm_s}{3L_p}\Big[\mathrm{Ei}\,(-m_0 r)\Big]_{r_A}^{r_B}\right\}$	$\frac{\Delta m^2\,Gm_s}{3E_\ell}\Big[\mathrm{Ei}\,(-m_0 r)\Big]_{r_A}^{r_B}$
Stelle	$\frac{\Delta m^2 L_p}{2E_\ell}\left\{\frac{Gm_s\,e^{-m_0 r_B}}{3r_B} - \frac{4Gm_s\,e^{-m_2 r_B}}{3r_B} - \frac{Gm_s}{3L_p}\Big[\mathrm{Ei}\,(-m_0 r)\Big]_{r_A}^{r_B} + \frac{4Gm_s}{3L_p}\Big[\mathrm{Ei}\,(-m_2 r)\Big]_{r_A}^{r_B}\right\}$	$\frac{\Delta m^2\,Gm_s}{3E_\ell}\Big[\mathrm{Ei}\,(-m_0 r) - \mathrm{Ei}\,(-m_2 r)\Big]_{r_A}^{r_B}$
Sixth-ord.	$\frac{\Delta m^2 L_p}{2E_\ell}\left\{\frac{Gm_s\,e^{-m_0 r_A}}{3r_B}\cos\left[m_0 r_B\right] - \frac{4Gm_s\,e^{-m_2 r_B}}{3r_B}\cos\left[m_2 r_B\right]\right.$ $\left. -\frac{Gm_s}{6L_p}\Big[\mathrm{Ei}\,(k_1 m_0 r) + \mathrm{Ei}\,(k_2 m_0 r)\Big]_{r_A}^{r_B} + \frac{2Gm_s}{3L_p}\Big[\mathrm{Ei}\,(k_1 m_2 r) + \mathrm{Ei}\,(k_2 m_2 r)\Big]_{r_A}^{r_B}\right\}$	$\frac{\Delta m^2\,Gm_s}{3E_\ell}\left\{\Big[\mathrm{Ei}\,(k_1 m_2 r) + \mathrm{Ei}\,(k_2 m_2 r)\Big]_{r_A}^{r_B}\right.$ $\left.-\Big[\mathrm{Ei}\,(k_1 m_0 r) + \mathrm{Ei}\,(k_2 m_0 r)\Big]_{r_A}^{r_B}\right\}$
IDG	$\frac{\Delta m^2 L_p}{2E_\ell}\left\{-\frac{Gm_s}{r_B}\mathrm{Erfc}\left[\frac{M_s r_B}{2}\right] + \frac{Gm_s}{L_p}\ln\left(\frac{r_B}{r_A}\right) - \frac{Gm_s}{L_p}\left[\frac{M_s r}{\sqrt{\pi}}{}_2F_2\left(\frac{1}{2},\frac{1}{2};\frac{3}{2},\frac{3}{2};-\frac{M_s^2 r^2}{4}\right)\right]_{r_A}^{r_B}\right\}$	0
Non-local (1)	$\frac{\alpha}{3\alpha-1}\,\Phi_{GR}$	$-\frac{\Delta m^2 Gm_s}{E_\ell}\,\frac{\alpha}{3\alpha-1}\ln\left(\frac{r_B}{r_A}\right)$
Non-local (2)	$\frac{\Delta m^2 L_p}{2E_\ell}\left\{\frac{Gm_s}{3r_B}\left(1 - e^{-\sqrt{3\beta}r_B}\right) - \frac{Gm_s}{3L_p}\ln\left(\frac{r_B}{r_A}\right) + \frac{Gm_s}{3L_p}\left[\mathrm{Ei}\left(-\sqrt{3\beta}r\right)\right]_{r_A}^{r_B}\right\}$	$\frac{\Delta m^2 Gm_s}{3E_\ell}\left\{\left[\mathrm{Ei}\left(-\sqrt{3\beta}r\right)\right]_{r_A}^{r_B} - \ln\left(\frac{r_B}{r_A}\right)\right\}$

5 Conclusions

Gravity effects on neutrino oscillation probability have been studied in quadratic theories. Corrections induced by extra terms in the Einstein-Hilbert action have been calculated explicitly for several models, showing that the ensuing neutrino phase is sensitive to the SEP violation. Besides its intrinsic theoretical interest, we stress that the present study opens up the possibility of constraining quadratic theories with neutrino oscillations measurements. Indeed, recent studies have proved that the oscillation phase of neutrinos is peculiarly affected in strong-gravity regime. Thus, consistency between experimental data and our predictions may allow to shed some light on the current zoo of quadratic theories, either validating or ruling out them at a fundamental level. Work in this direction is under active investigation [6].

References

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