123. Dan Kondo
SO(N) model as Pion like SIDM from effective range theory

Background (Dark Matter)

Various problems for CCDM

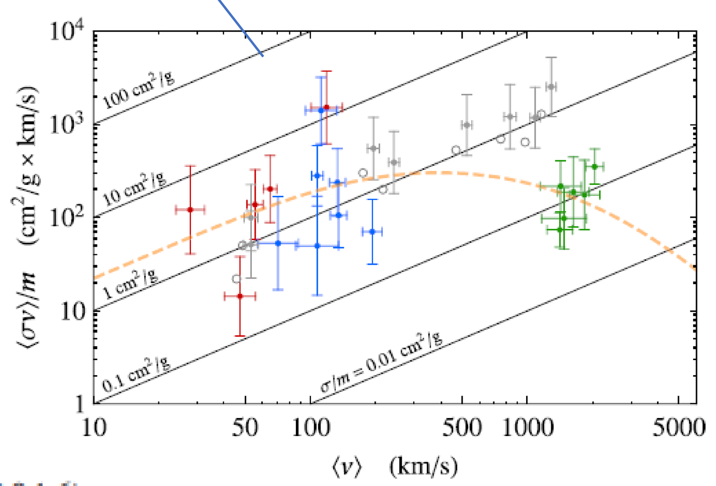
Cusp-Core problem
Diversity problem
Too big to fail problem

Self-interacting dark matter

How should they have self-interaction?
→Simulation

Velocity independent line (const)

$\frac{\langle \sigma v\rangle}{m_{DM}}$ [cm²·km/s]

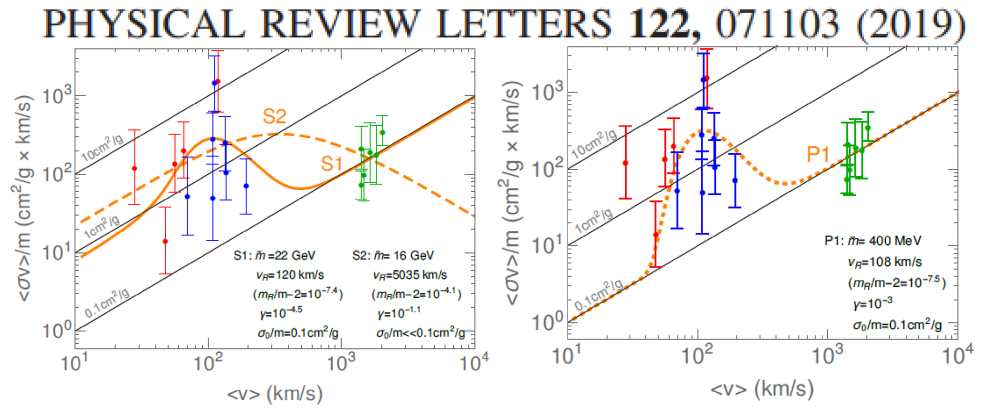


PRL **116**, 041302 (2016)

Average velocity $\langle v\rangle$ [km/s]

RSIDM(Resonant Self Interacting Dark Matter)

PHYSICAL REVIEW LETTERS **122**, 071103 (2019)

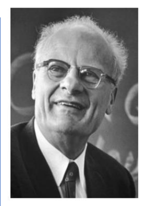


By introducing velocity resonance, cross section varies by scale (dwarf, cluster, etc)
→may explain DM distribution?

Abstract

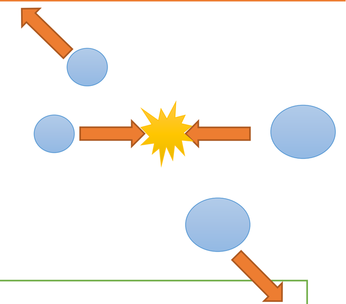
* Using effective range theory, we studied what parameters fit well to SIDM simulation result.
* We studied what parameters imply using ϕ^4-theory. It seems that bound state with mass degeneracy $m_\sigma \simeq 2m_\pi$ needed. It is possible!
* We propose SO(N) models and calculate parameters to produce relic abundances. They are viable.

Back ground (Effective Range Theory)



Phys.Rev.76.38

H.A.Bethe
Hans Bethe - Biographical (nobelprize.org)
Hans Bethe - Work on low energy systems (115/158) - YouTube



Nonrelativistic nuclei 2 body resonant scatterings can be characterized by 2 parameters =scattering length a+effective range r

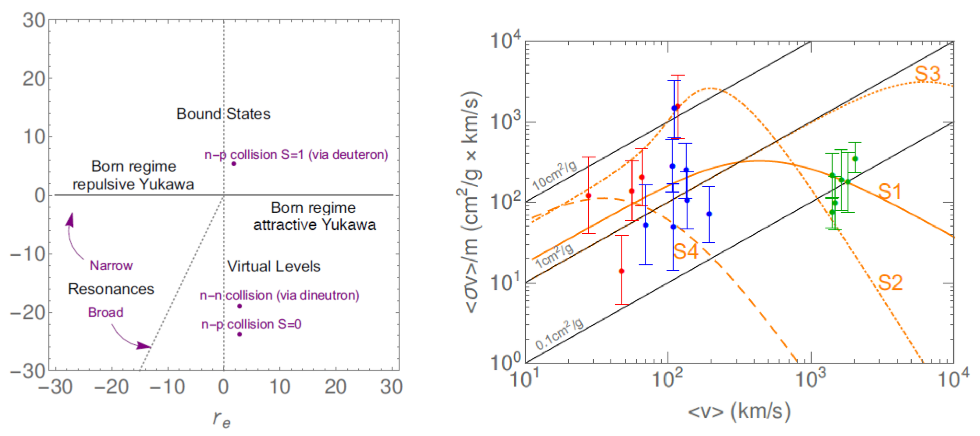
$$f(k,\theta)=\sum_{\ell=0}^{\infty}(2l+1)f_\ell(k)P_\ell(\cos\theta),$$

$$\text{with}\quad f_\ell(k)\equiv\frac{e^{2i\delta_\ell(k)}-1}{2ik}=\frac{1}{k\left(\cot\delta_\ell(k)-i\right)}$$

$$k^{2\ell+1}\cot\delta_\ell(k)\simeq-\frac{1}{a_\ell^{2\ell+1}}+\frac{1}{2r_{e,\ell}^{2\ell-1}}k^2$$

Nuclei-nuclei scattering states are characterized by a and r.

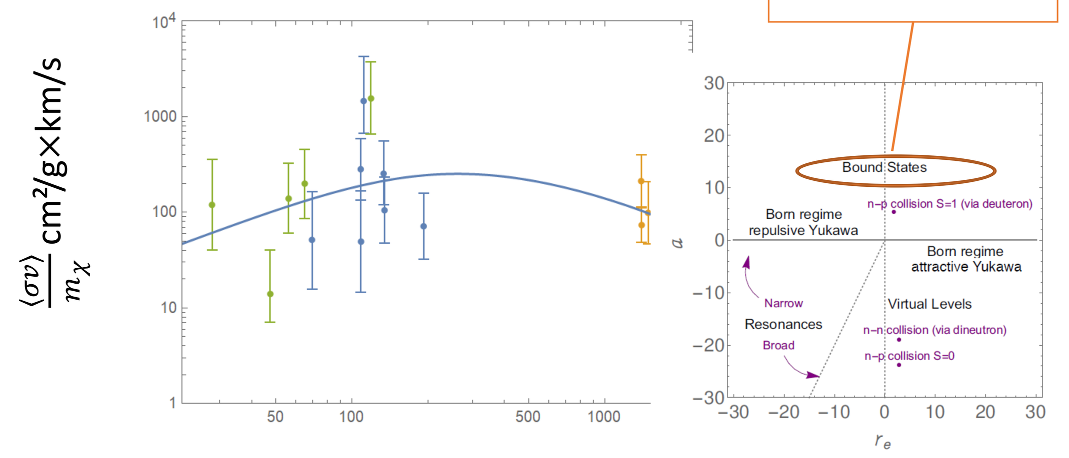
JCAP 06 (2020) 043



fitting

S-wave ($m_\chi \simeq 16$ GeV)

Bound state



$a\simeq 20$ fm, $x=\frac{r}{a}=-\frac{1}{10}$ $\frac{\chi^2_{all}}{d.o.f}=2.17$ $\frac{\chi^2_{w.o.clu}}{d.o.f}=1.47$

Naive estimate…

$\frac{\hbar c}{m_\chi}\simeq\frac{0.197}{15}$[GeV·fm/GeV]=0.013[fm]≪20[fm]

Q: Why the scattering length is so long?
(Personally, I did not know how to calculate scattering length a)

Study by Linear Sigma model

$$H=\int d^3x\left(\frac{1}{2}\dot{\Phi}^{i2}+\frac{1}{2}\nabla\Phi^{i2}+V(\Phi)\right)$$

$$V(\Phi)=\frac{1}{2}m^2\Phi^2+\frac{\lambda}{4}(\Phi^2)^2$$

Add $\Delta V(\Phi)=-\alpha\Phi^N$

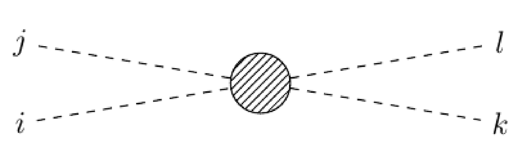
When $m^2=-\mu^2<0$, Φ obtain VEV (say $\langle\Phi^N\rangle$)→S.S.B.

$$V=\left(\frac{1}{2}\lambda v^2-\frac{1}{2}\mu^2\right)\pi^{i2}+\left(\frac{3}{2}\lambda v^2-\frac{1}{2}\mu^2\right)\sigma^2+\lambda v\pi^{i2}\sigma+\frac{\lambda}{4}\left(\pi^{i2}\right)^2+\frac{\lambda}{2}\pi^{i2}\sigma^2+\lambda v\sigma^3+\frac{\lambda}{4}\sigma^4$$

→ π obtained mass $m_\pi^2=\lambda v^2-\mu^2$, and
σ obtained mass $m_\sigma^2=3\lambda v^2-\mu^2$
↔ $\lambda v^2=\frac{m_\sigma^2-m_\pi^2}{2}$, $\mu^2=\frac{m_\sigma^2-3m_\pi^2}{2}$

Massive $\pi+\sigma$

Consider amplitude (tree +running width)



$$M=-4\lambda^2v^2\left(\frac{\delta_{ij}\delta_{kl}}{s-m_{\sigma^2}+im_\sigma\Gamma}+\frac{\delta_{ik}\delta_{jl}}{t-m_\sigma^2}+\frac{\delta_{il}\delta_{jk}}{u-m_\sigma^2}\right)-2\lambda(\delta_{ij}\delta_{kl}+\delta_{il}\delta_{jk}+\delta_{ik}\delta_{jl})$$

Γ is running width $m_\sigma\Gamma=\frac{(N-1)\lambda^2v^2}{8\pi}\sqrt{1-\frac{4m_\pi^2}{p^2}}=\frac{(N-1)\lambda^2v^2}{8\pi}\beta$

At threshold $s\simeq 4m_\pi^2$, $t\simeq 0$, $u\simeq 0$, $\Gamma\simeq 0$

$$M\simeq-2\lambda\delta_{ij}\delta_{kl}\left(\frac{s+im_\sigma\Gamma-m_\pi^2}{s-m_\sigma^2+im_\sigma\Gamma}\right)-2\lambda\left(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}\right)\left(\frac{m_\pi^2}{m_{\sigma^2}}\right)$$
$$=-2\lambda\delta_{ij}\delta_{kl}\left(\frac{3m_\pi^2+im_\sigma\Gamma}{4m_\pi^2-m_\sigma^2+im_\sigma\Gamma}\right)-2\lambda\left(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}\right)\left(\frac{m_\pi^2}{m_{\sigma^2}}\right)$$

If there is a relation $m_\sigma\simeq 2m_\pi$, scattering length can be very large give reason for large scattering length $a\simeq 20$[fm]≫ 0.013[fm]

I assume that in such condition, 1st term is more dominant than 2nd term
→Compare with ERT (plot result)

LSM vs ERT

LSM

$$M\simeq-2\lambda\delta_{ij}\delta_{kl}\left(\frac{3m_\pi^2+im_\sigma\Gamma}{4m_\pi^2-m_\sigma^2+im_\sigma\Gamma}\right)$$

vs

ERT

$$\frac{1}{-\frac{1}{a}+\frac{r_ep^2}{2}-ip}$$

$$-2A\lambda\delta_{ij}\delta_{kl}\left(\frac{3m_\pi^2+im_\sigma\Gamma}{4m_\pi^2-m_\sigma^2+im_\sigma\Gamma}\right)=\frac{1}{-\frac{1}{a}+\frac{r_ep^2}{2}-ip}$$

A is proportional constant
A= $\frac{1}{32\pi m_\pi}$ (Weinberg : Phys Rev Lett.17.616)

$im_\sigma\Gamma$ in numerator can be neglected because $\Gamma\simeq 0$ at threshold

Compare real part and imaginary part in denominator, respectively
and term proportional to p^2 or constant term

$$-\frac{1}{a}=-\frac{16\pi}{3m_\pi\lambda}(4m_\pi^2-m_\sigma^2)\qquad a=\frac{3m_\pi\lambda}{16\pi(4m_\pi^2-m_\sigma^2)}$$

$$\frac{r}{2}=-\frac{64\pi}{3m_\pi\lambda}$$

$$p=-\frac{16\pi}{3m_\pi\lambda}m_\sigma\Gamma$$

I set $m_\sigma=(2-\epsilon)m_\pi$
→ $m_\sigma^2\simeq 4(1-\epsilon)m_\pi$
$a\simeq\frac{3\lambda}{64\pi m_\pi\epsilon}$ can be very large as expected

$\epsilon\simeq 7.7\times10^{-6}$…small enough
$\lambda\simeq 0.83$
Assumption was not bad.

$$\lambda=-\frac{128\pi}{3m_\pi r}=-\frac{128\pi}{3m_\pi ax}$$
$$\epsilon=\frac{3\lambda(\hbar c)^2}{64\pi m_\pi a}=-\frac{2(\hbar c)^2}{m_\pi^2a^2x}$$

In the real world

Our very fine-tuning ($m_\sigma\simeq 2m_\pi$) can be realized in real world (!?)
→It is possible as a dark matter candidate.

Fact (by PDG) $m_{a_0}\simeq m_{f_0}\simeq 2m_{K^\pm}$

$K^\pm$ $I(J^P)=1/2(0^-)$	$K^\pm$ MASS	493.677 ± 0.016 MeV (S = 2.8)
$a_0(980)$ $I^G(J^{PC})=1^-(0^{++})$	$f_0(980)$ $I^G(J^{PC})=0^+(0^{++})$	

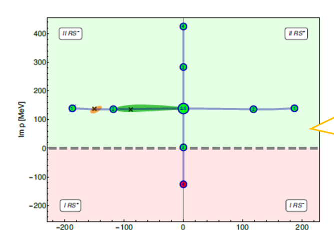
arxiv:1510.00653

From controversy to precision on the sigma meson:
a review on the status of the non-ordinary $f_0(500)$ resonance
José R. Peláez
Departamento de Física Teórica II. Universidad Complutense. 28040 Madrid. SPAIN

① not ordinary quark-antiquark meson (!?)
Tetra-quark or molecule or something
→ σ is $\pi-\pi$ bound state possible

② Lattice QCD simulation of $N_f=2$ σ-resonance

arxiv:1804.10225



σ for $m_\pi\geq 2m_\pi$ can be on the imaginary axis, bound state.

SO(N) Model Proposal and Relic abundances

Model1: SIMPlest model (heavier than original)

arxiv:1411.3727

SIMPlest model
SU(4) ≃SO(6) →Sp(4)≃ SO(5)
6 flavors→1 obtain VEV (become σ)
5 pions…2 of them are charged under U(1)'

4 fundamental (dark) quarks charged under U(1)'

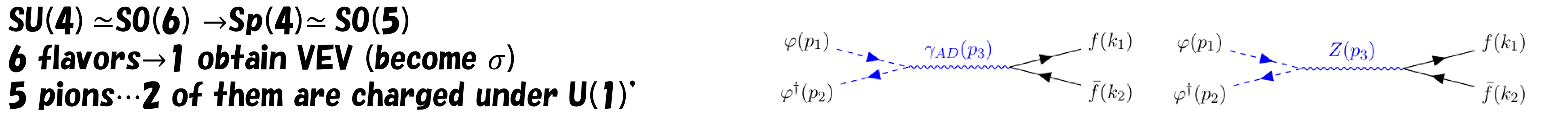
+	+	-	-

Out of $_4C_2=6$, 3 are neutral. (1 is σ)
(++,--,0,0,0)

While freeze out (decrease the amount),
(0,0,0) compensate for (++,--)
because of thermal equilibrium

After freezeout, (0,0,0) are relics
(no direct detection bound)

Abundance is determined by freeze out
$m_{AD}>m_{DM}\simeq 16$[GeV]



I neglected the fermion mass m_f

Among 5 PiDM, only 2 contributes
→Combinatoric factor $_5C_2=10$

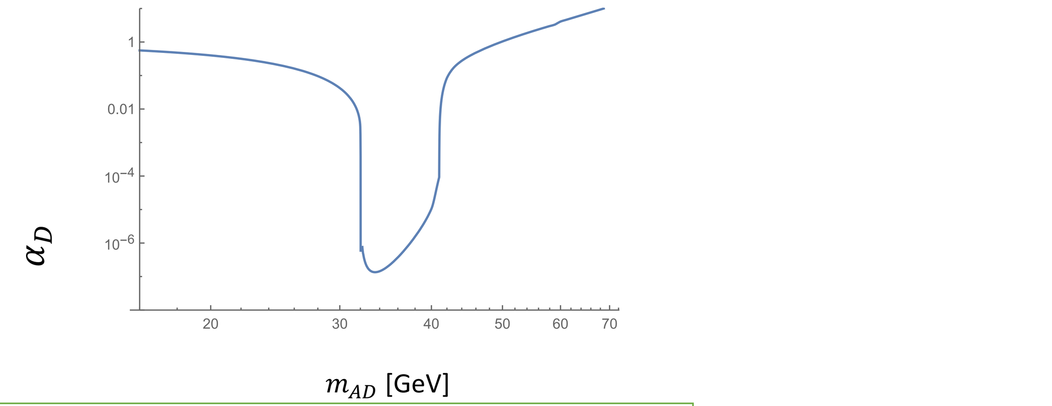
$\langle\sigma v\rangle$ is

$$\frac{1}{3}\sqrt{\frac{\pi}{2}}Q^2\epsilon_f^2\alpha\alpha_D\frac{1}{m_{DM}^{\frac{1}{2}}T^{\frac{3}{2}}}\int_0^\infty dy\frac{(y+2)^{\frac{5}{2}}\left((y+2)^2-4\right)^{\frac{3}{2}}}{\left((y+2)^2-\frac{m_{AD}^2}{m_{DM}^2}\right)^2+\frac{1}{9}[Q^2\epsilon_f^2\alpha(y+2)^2]^2}e^{-\frac{m_{DM}}{T}y}\quad\text{for } m_{DM}<m_{AD}<2m_{DM}$$

$$\frac{1}{3}\sqrt{\frac{\pi}{2}}Q^2\epsilon_f^2\alpha\alpha_D\frac{1}{m_{DM}^{\frac{1}{2}}T^{\frac{3}{2}}}\int_0^\infty dy\frac{(y+2)^{\frac{5}{2}}\left((y+2)^2-4\right)^{\frac{3}{2}}}{\left((y+2)^2-\frac{m_{AD}^2}{m_{DM}^2}\right)^2+\frac{1}{9}[Q^2\epsilon_f^2\alpha(y+2)^2+\alpha_D((y+2)^2-4)]^2}e^{-\frac{m_{DM}}{T}y}\quad\text{for } 2m_{DM}<m_{AD}$$

Possible ϵ_f and α_D ?

It is difficult for me to determine α_D and ϵ_f once.
→Fix $\epsilon_f=10^{-1.5}$ for optimistic case and study what values α_D can take.

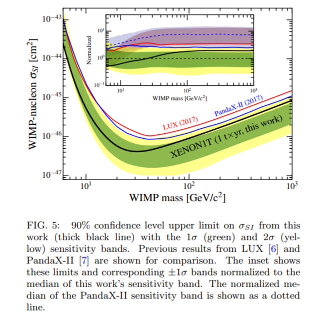


Model2: SO(N) model

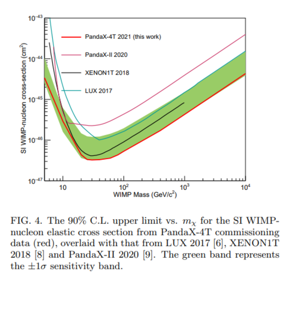
2 Weyl fermions in vector representation
N_f fermions: Coset space is SU(N_f)/SO(N_f)
Now $N_f=2$ →SU(2)/U(1)≃SO(3)/SO(2)
$\psi_1\left(\frac{1}{2}\right)$ and $\psi_2\left(-\frac{1}{2}\right)$ under SO(2)≃U(1) dark photon
Condensate $\langle\psi_1\psi_2\rangle\neq 0$
NGBs: $\psi_1\psi_1(1)$, $\psi_2\psi_2(-1)$

Abundance

Direct detection bound is very strong restriction.



XENON1T
arxiv:1805.12562



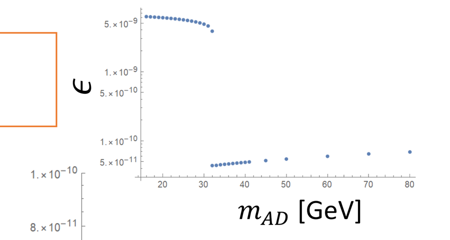
PandaX4T
arxiv:2107.13438

Estimate of cross section though dark photon and nuclear recoil

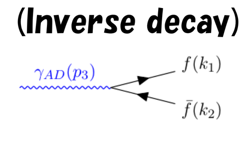
$$\sigma\sim\frac{\alpha\alpha_D\epsilon_f^2}{(m_{AD}^2)^2}\left(\frac{m_pm_{DM}}{m_p+m_{DM}}\right)^2\sim\alpha\alpha_D\epsilon_f^2\frac{m_p^2}{m_{AD}^4}$$

Abundance…freeze-in

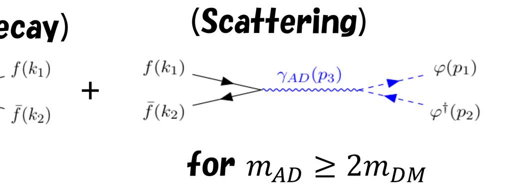
Total look (→)
There is a gap because…



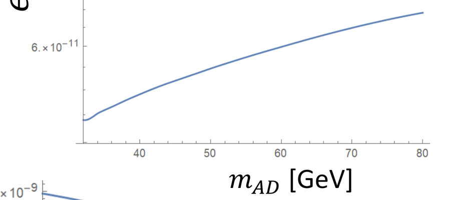
(Inverse decay)



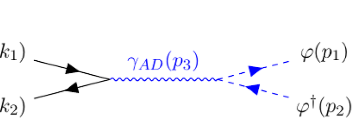
(Scattering)



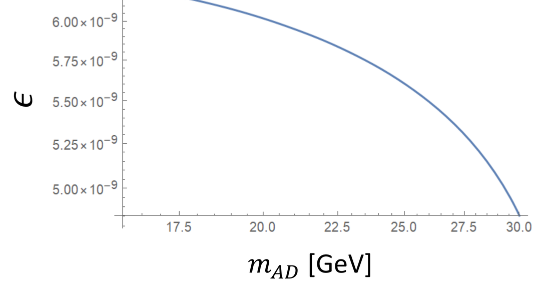
for $m_{AD}\geq 2m_{DM}$



(Only Scattering)



for $m_{AD}<2m_{DM}$



It may be possible for very light dark photon to be detected directly even in freeze-in.