

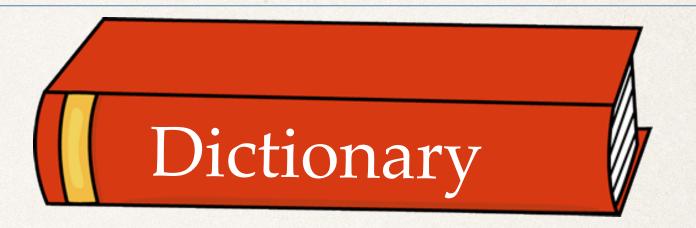
Gravitational Wave Gastronomy

New Observational Windows on High-Scale Physics Workshop, IPMU

David Dunsky, Anish Ghoshal, Hitoshi Murayama, Yuki Sakakihara, and Graham White

Gravitational Wave Gastronomy

Definition:



- A savory variation of Gravitational Wave Astronomy.
- The search for gravitational wave signatures from the "eating" of one topological defect by another. Observed to occur in hybrid topological defects (i.e. domain walls bounded by strings, and strings bounded by monopoles.)
- Seasoned with equal parts of gravitational wave physics, early universe cosmology, and grand unification.

Outline

- *Appetizers*: Gravitational Wave (GWs) basics, as probes of the early universe, and relationship to Grand Unified Theories (GUTs)
- Entree: Dynamics and the GW signatures from hybrid topological defects
- Dessert: Outlook for distinguishing GUT symmetry breaking chains by such GW signatures

Gravitational Wave Basics: Emission

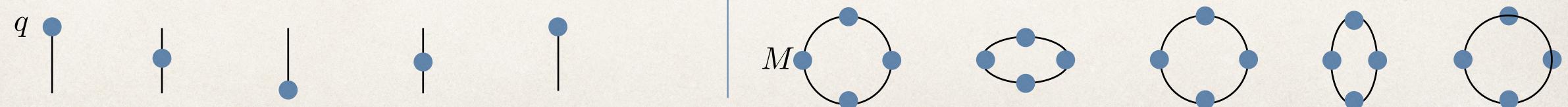
Electromagnetism

Accelerated electric charges — Electromagnetic radiation

increases

If $q \& \gamma$ increase Emitted power

Electromagnetic waves oscillate charges up and down



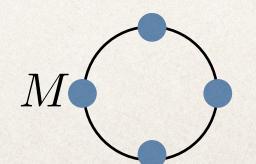
Gravity

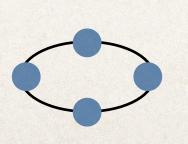
Accelerated masses Gravitational radiation

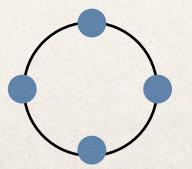


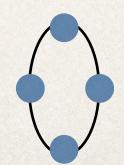
Emitted power If $M & \gamma$ increase \longrightarrow increases

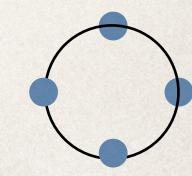
Gravitational waves stretch and squeeze space





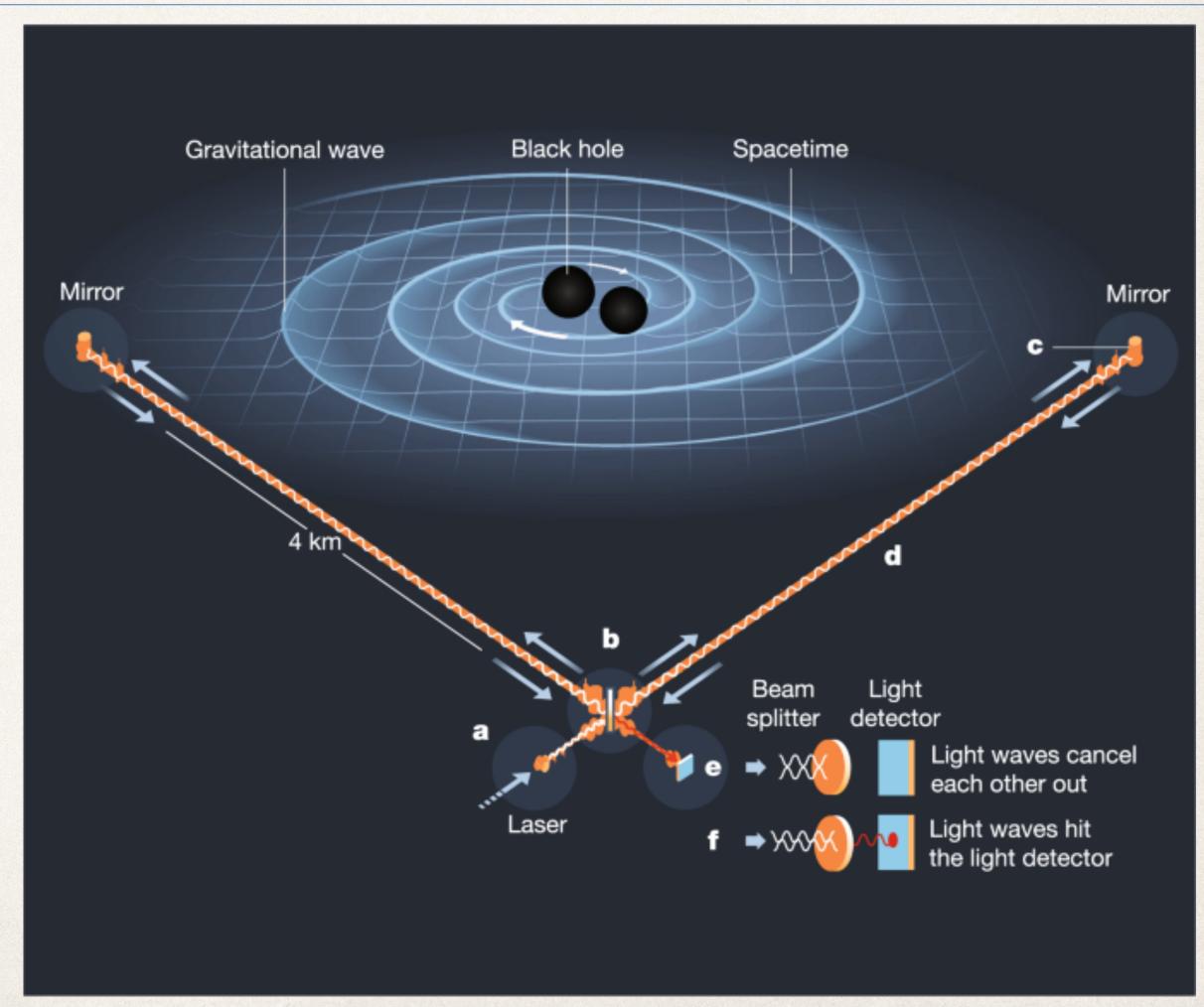






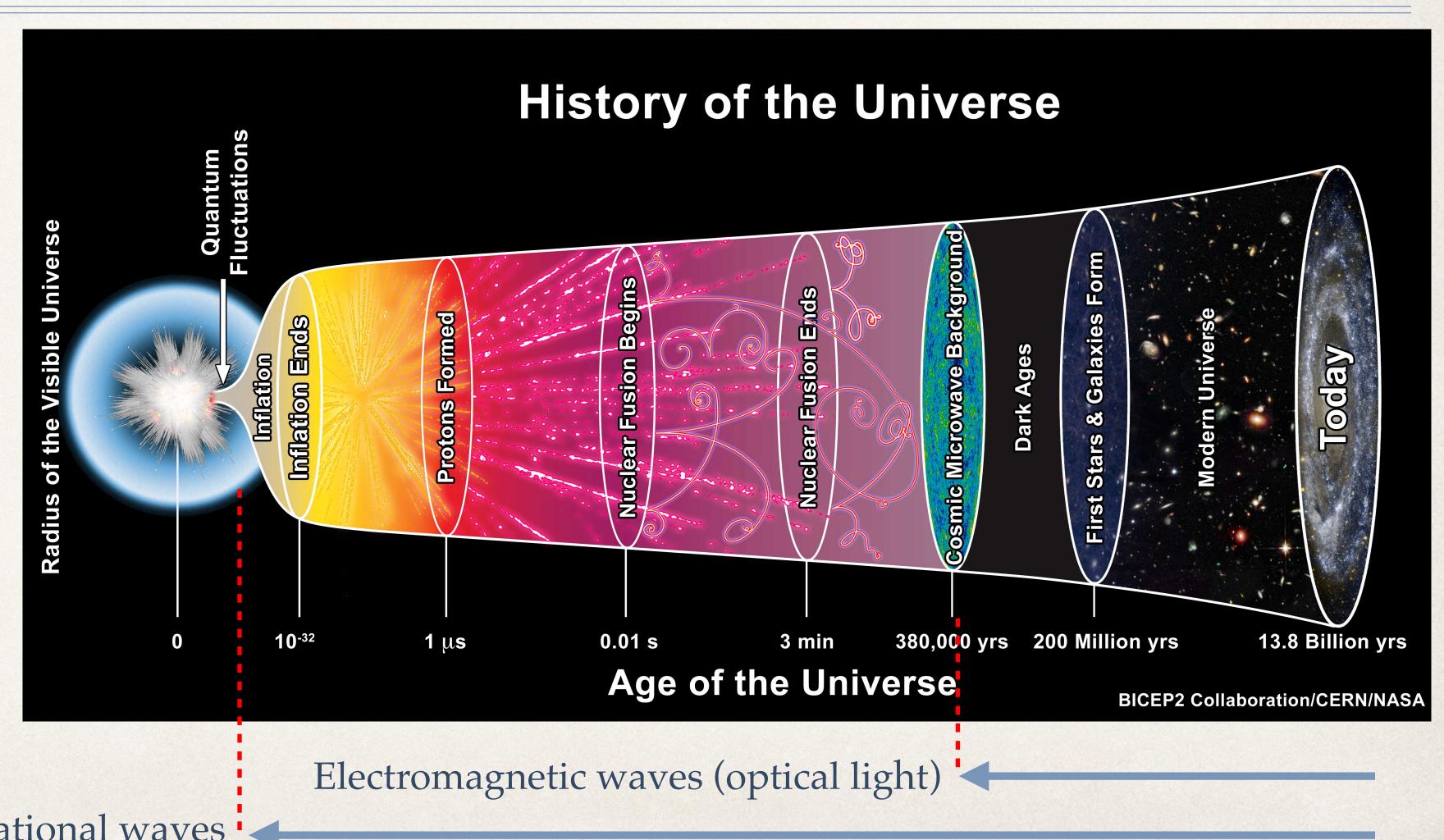
Gravitational Wave Basics: Detection

- Most GW detectors take advantage of this incredibly small stretch and squeeze of space via laser interferometry
 - For ex, Laser Interferometer
 Gravitational-Wave
 Observatory (LIGO)



Gravity Waves as Probes of the Early Universe

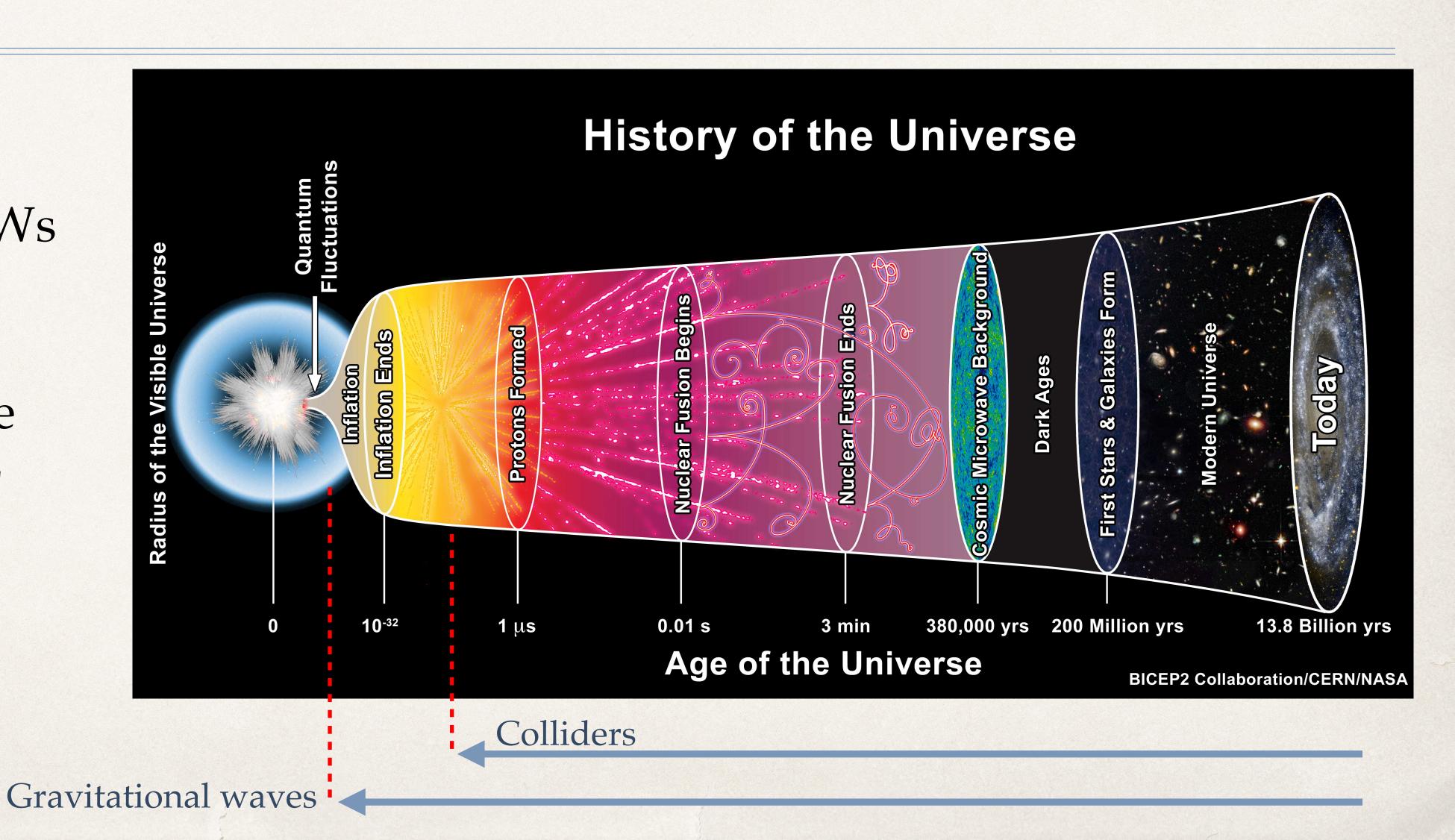
- Early universe transparent to GWs
- Potentially remarkable probe of moments after Big Bang



Gravitational waves •

Gravity Waves as Probes of the Early Universe

- Early universe transparent to GWs
- Potentially
 remarkable probe
 of moments after
 Big Bang



Gravitational Waves and GUTs

- Grand Unified Theories (GUTs) intimately tied to gravitational waves:
 - True symmetries of nature are restored at high temperatures in early universe
 - As the universe expands and cools, gauge symmetries describing nature spontaneously break

$$G o G_i o G_{i+1} o \cdots o G_{\mathrm{SM}}$$

$$\bullet \qquad \qquad \text{Colliders}$$
Universe

Gravitational Waves and GUTs

- Grand Unified Theories (GUTs) intimately tied to gravitational waves:
 - When breaking, many gauge groups leave behind topological relics if nontrivial vacuum $\pi_{2-D}(G_i/G_{i+1}) \neq 0$

$$D = 0 \rightarrow \text{Monopoles form}$$

$$D=1 \to \text{Cosmic Strings form}$$

$$D=2 \rightarrow \text{Domain Walls form}$$









$$G \xrightarrow{\text{monopoles}} H \times U(1)$$

$$G \xrightarrow{\text{monopoles}} H \times U(1) \xrightarrow{\text{strings}} H$$



$$G \xrightarrow{\text{monopoles}} H \times U(1) \xrightarrow{\text{strings}} H$$



$$G \xrightarrow{\text{strings}} H \times Z_2$$



$$G \xrightarrow{\text{monopoles}} H \times U(1) \xrightarrow{\text{strings}} H$$



$$G \xrightarrow{\text{strings}} H \times Z_2 \xrightarrow{\text{walls}} H$$



• For some chains, strings bounded by monopoles, or walls bounded by strings

$$G \xrightarrow{\text{monopoles}} H \times U(1) \xrightarrow{\text{strings}} H$$

$$G \xrightarrow{\text{strings}} H \times Z_2 \xrightarrow{\text{walls}} H$$



• Tension force of string or wall causes (relativistic) oscillations

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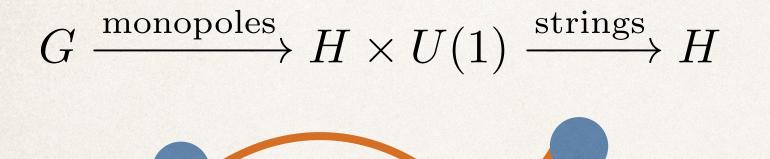
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Gravitational radiation (Unique fingerprints of chains)

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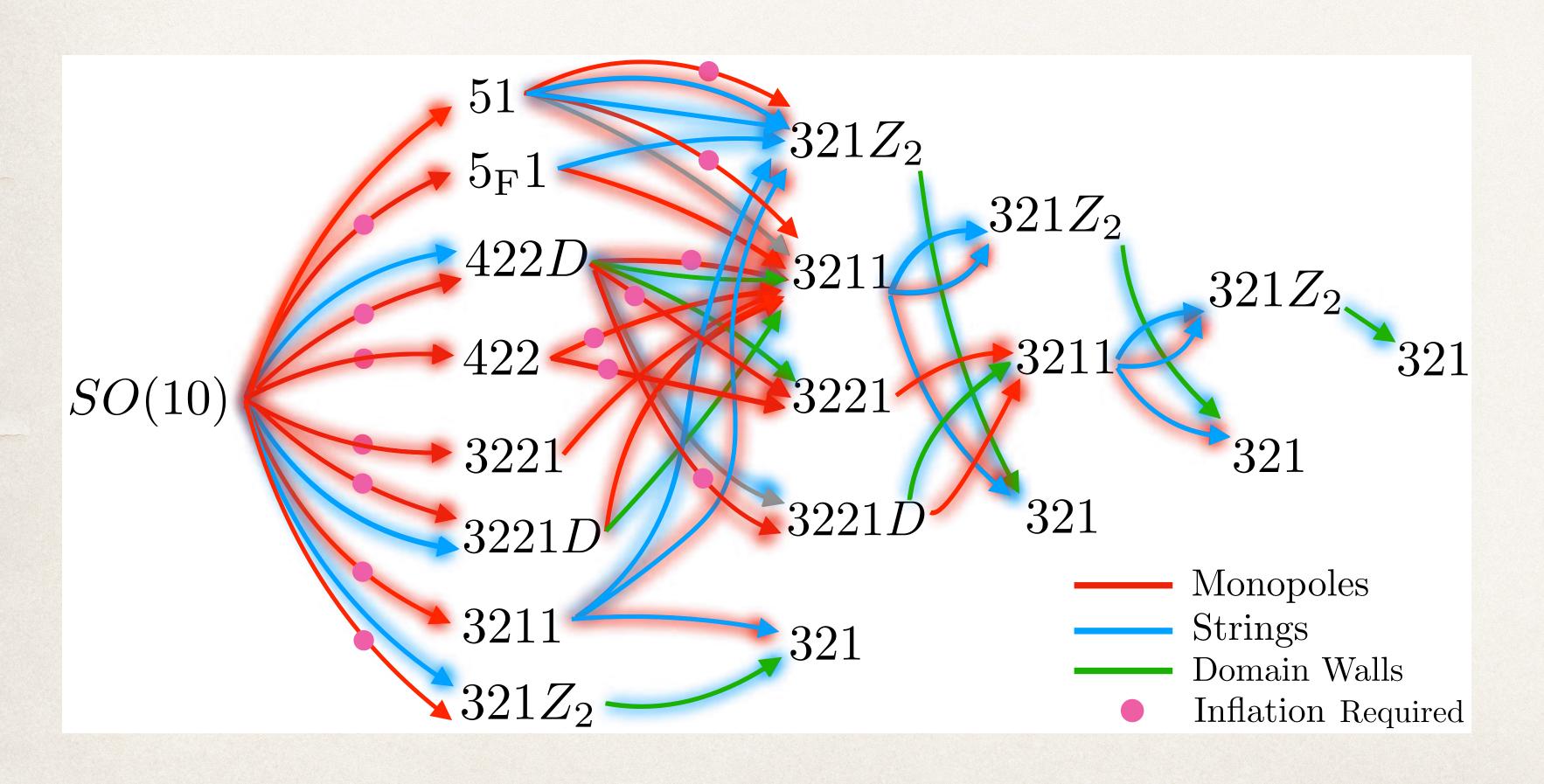
Gravitational radiation (Unique fingerprints of chains)

Decay

Four Course Menu

- Monopoles eating strings (nucleation)
 See previous talk by Valerie Domcke!
- Strings eating monopoles (collapse)
- Strings eating walls (nucleation)
- Walls eating strings (collapse)

Symmetry Breaking Chains



Group abbreviations

$$51 = SU(5) \times U(1)_{X}$$

$$5_{\rm F}1 = SU(5)_{\rm flipped} \times U(1)_{\rm flipped}$$

$$422 = SU(4)_{c} \times SU(2)_{L} \times SU(2)_{R}$$

$$3221 = SU(3)_{c} \times SU(2)_{L} \times SU(2)_{R} \times U(1)_{B-L}$$

$$3211 = SU(3)_{c} \times SU(2)_{L} \times U(1)_{Y} \times U(1)_{X}$$

$$321 = SU(3)_{c} \times SU(2)_{L} \times U(1)_{Y}$$

$$D = \text{D-Parity}$$

Gravitational Wave Basics: Computing Signal

- Measure strain through $\Omega_{\rm GW} \equiv \frac{d\rho_{\rm GW}}{d\ln f} \frac{1}{\rho_{\rm crit}} \propto {\rm Strain}^2$
- Recipe:

$$P_{\rm GW} \sim G\langle \ddot{Q}^2 \rangle \longrightarrow$$

$$P_{\rm GW} \sim G\langle \ddot{Q}^2 \rangle \longrightarrow \rho_{\rm GW} \sim P_{\rm GW} \times \Delta t \times n_{\rm sources} \longrightarrow$$

$$\Omega_{\mathrm{GW}} \sim \left. \frac{\rho_{\mathrm{GW}}}{\rho_{\mathrm{BG}}} \right|_{\mathrm{Emission}} \times \Omega_{\mathrm{rad}}$$



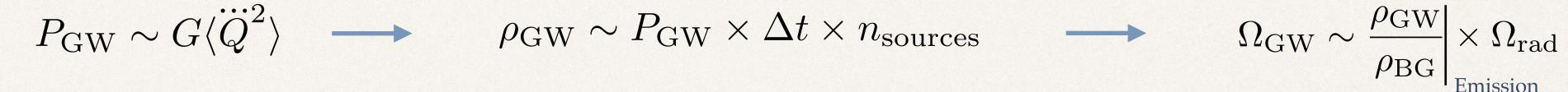
 $f \sim \text{scale size of system}^{-1} \times \text{Redshift}$ Emission

Gravitational Wave Basics: Computing Signal

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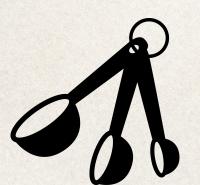
$$P_{\rm GW} \sim G\langle \ddot{Q}^2 \rangle \longrightarrow$$

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$$f \sim \text{scale size of system}_{-1}^{-1} \times \text{Redshift}$$



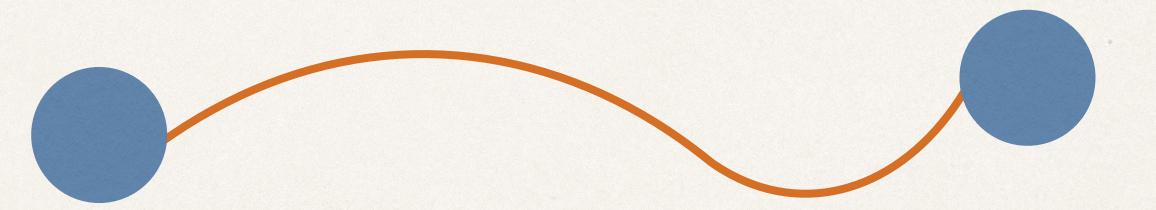


Exact:
$$P_{\rm GW}(f) \sim G |T_{\mu\nu}^T(f)|^2$$
 Weinberg '72

$$\frac{d\rho_{\mathrm{GW}}(t)}{df} = \int dt' \frac{a(t')^4}{a(t)^4} \int dl \frac{dn(l,t')}{dl} \frac{dP(l,t')}{df'} \frac{df'}{df}$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$
Expansion Source Density Power Redshift

Strings Bounded by Monopoles



See previous talk by Valerie Domcke!

Occurs in following scenario

Monopole formation

Occurs in following scenario

Monopole formation — Inflation

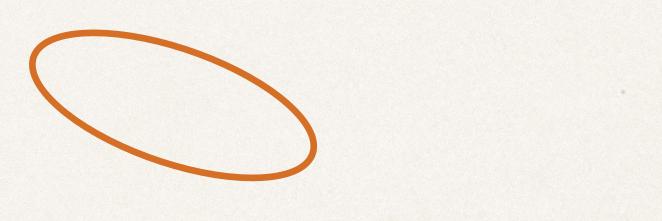
Occurs in following scenario

Monopole formation — Inflation — String formation

Occurs in following scenario

Monopole formation → Inflation → String formation → Wait...





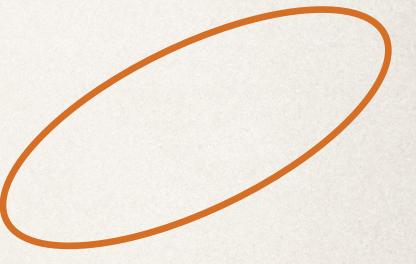


Occurs in following scenario

Monopole formation --> Inflation --> String formation --> Nucleation

 String rest mass converted to monopole kinetic energy





Decay via gravitational radiation

· Before nucleation, strings evolve as pure string network

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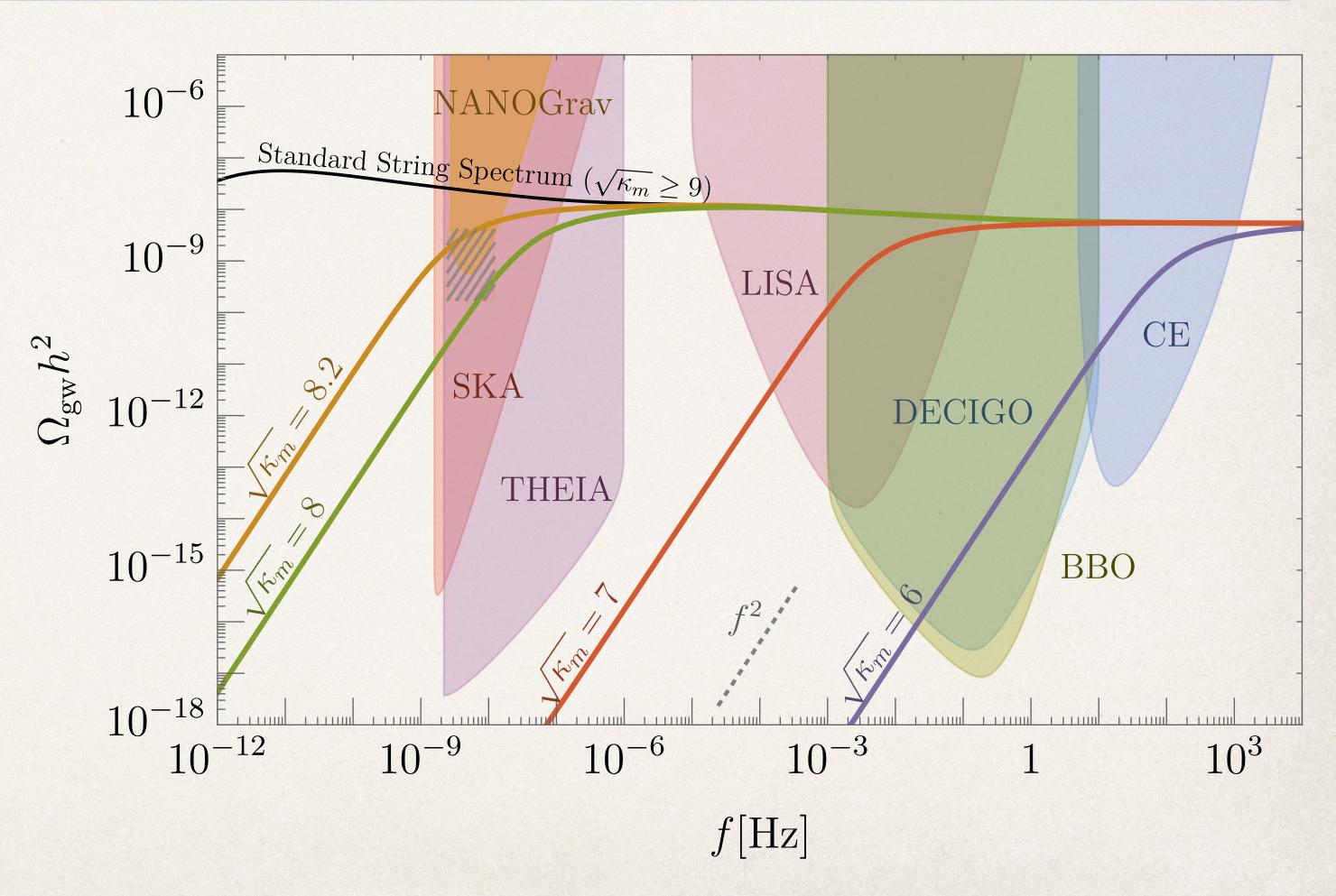
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Decay via gravitational radiation

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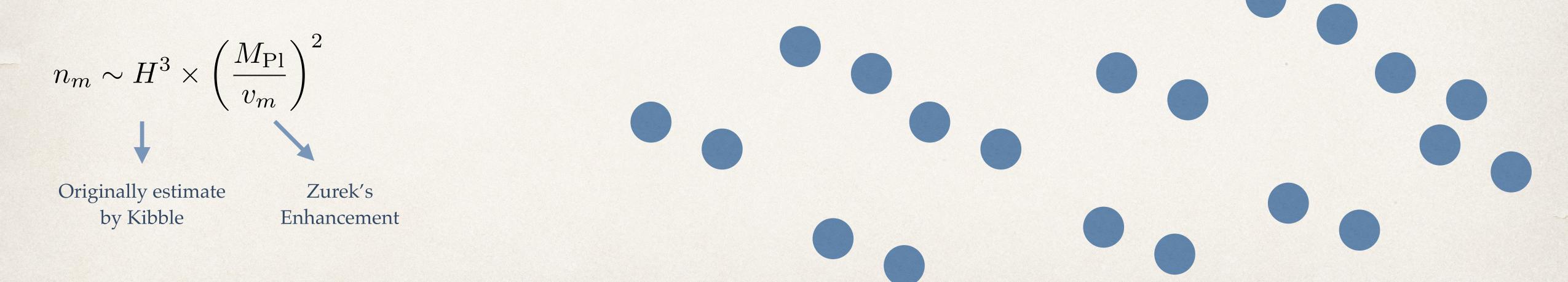
Monopole Nucleation Spectrum

• Standard, flat, string spectrum at high frequencies, f^2 decaying spectrum at lower frequencies



- New menu items: 1) Enhanced Kibble-Zurek mechanism, 2) Monopole-antimonopole annihilation, and 3) GW spectrum

 Langacker-Pi '80, Holman '92, Vilken '96 and '97



Occurs in following scenario

Inflation — Monopole formation

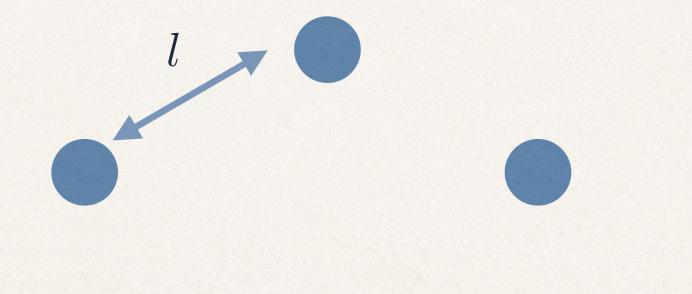
$$n_m(T) \approx 10^{-5} \frac{v_m}{M_{\rm Pl}} T^3$$
 (Freeze-Out Abundance)

Occurs in following scenario

Inflation — Monopole formation — String formation — Annihilation

$$l \approx \frac{1}{n_m (T = v_\mu)^{1/3}}$$

 String rest mass converted to monopole kinetic energy





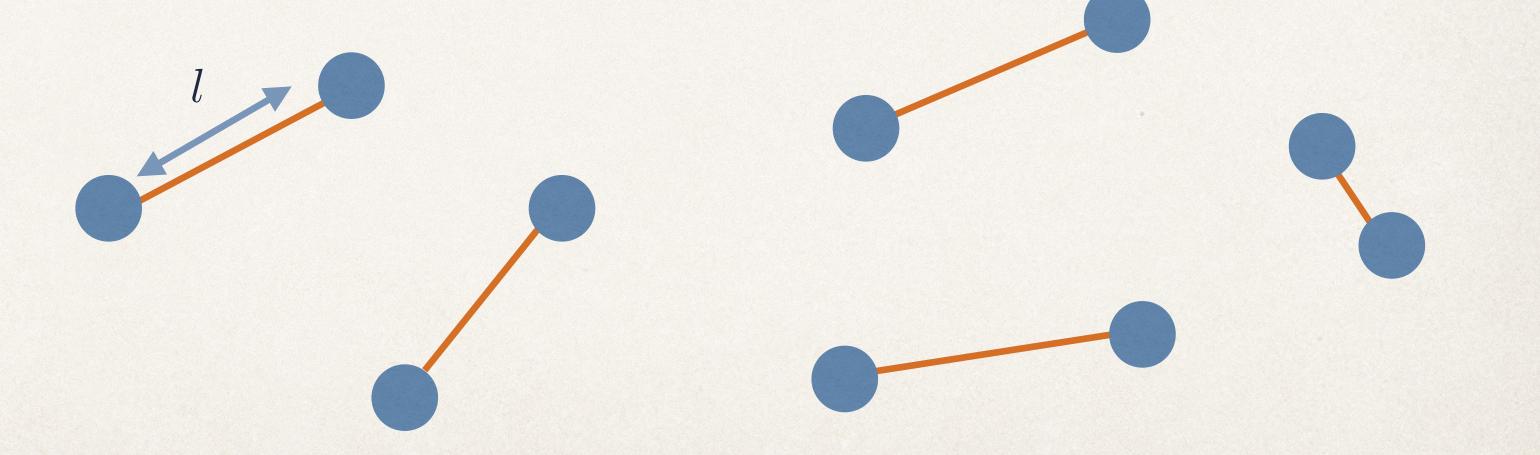


Occurs in following scenario

Inflation — Monopole formation — String formation — Annihilation

$$l \approx \frac{1}{n_m (T = v_\mu)^{1/3}}$$

 String rest mass converted to monopole kinetic energy



Monopole Network Evolution

String rest mass converted to monopole kinetic energy

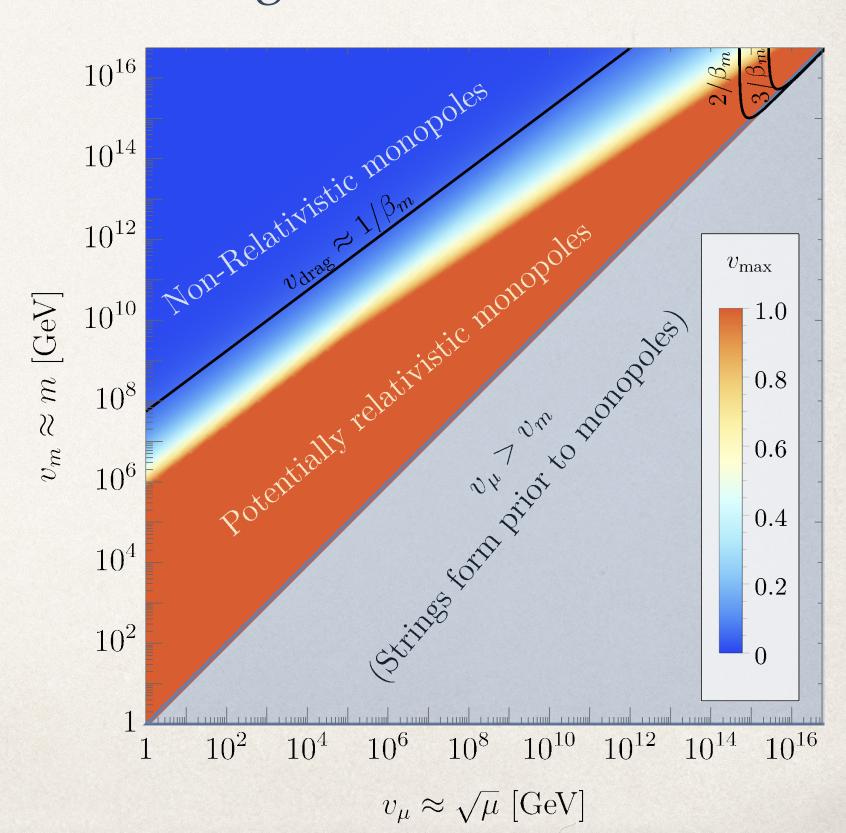
$$\mu l \approx 2 \times \frac{1}{2} m v_{\text{max}}^2$$

- Potential gravitational wave signal if $v_{\rm max} \sim 1$
- String induced monopole motion generates friction with plasma

$$F_{
m string} \sim \mu$$
 balance $F_{
m friction} \sim T^2 v$ $V_{
m drag} \sim \frac{\mu}{T^2}$ Vilenkin '82

$$\tau \approx \frac{E}{|P_f|} \approx \frac{\mu l}{\beta T^2 v^2}$$
 (Monopole-bounded string lifetime)

Decay via friction & gravitational radiation

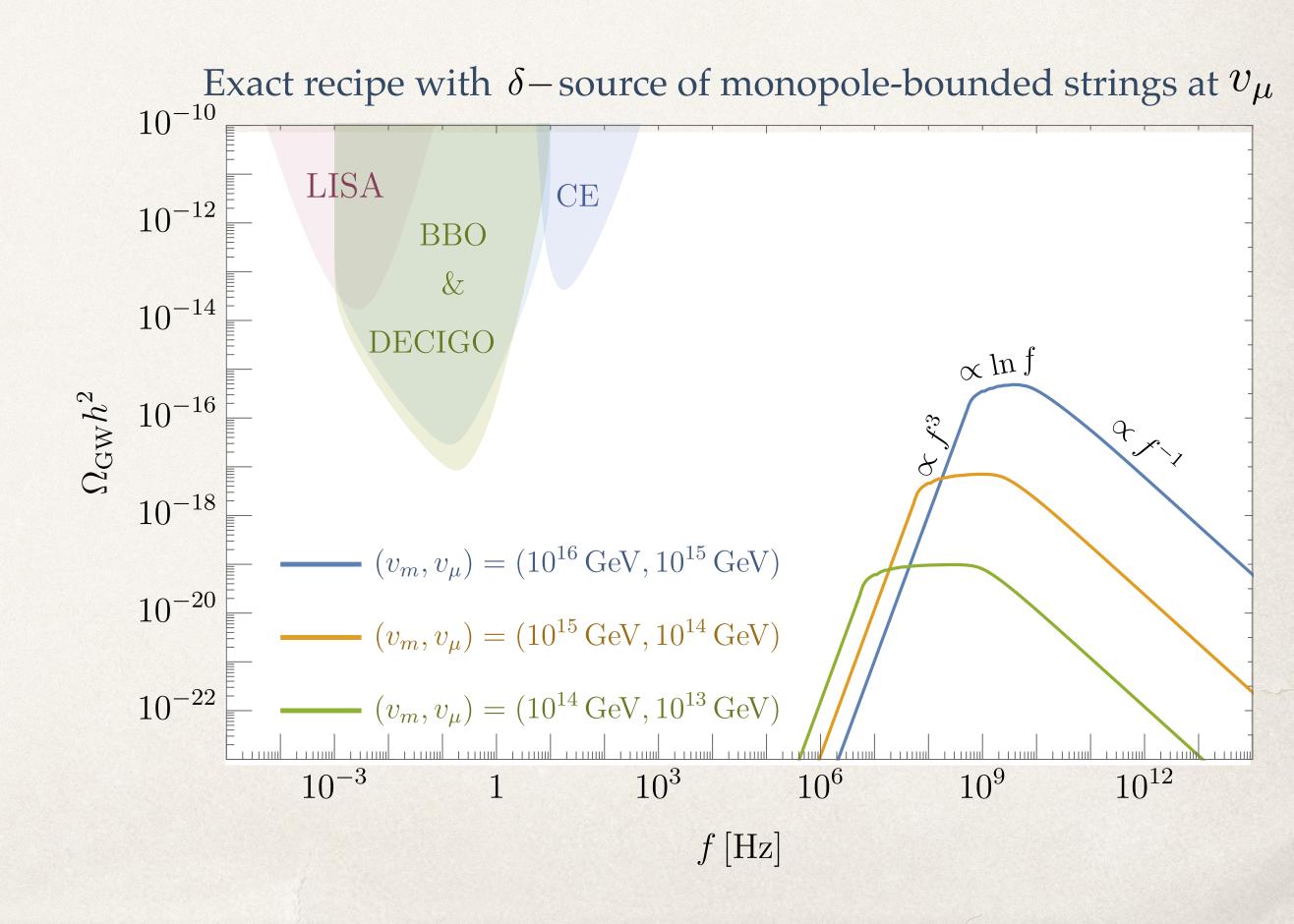


Monopole Burst Spectrum

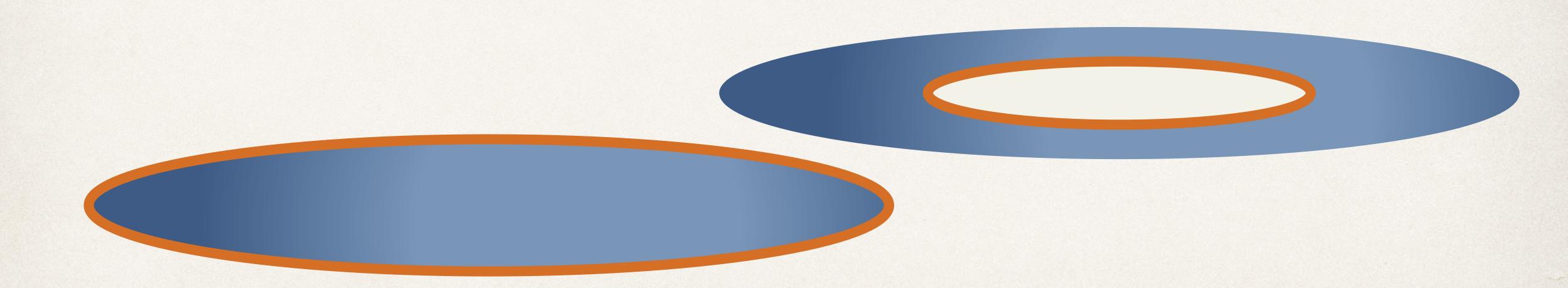
- $ho_{
 m GW,burst} pprox n_m(v_\mu) imes P_{
 m GW} imes au$ $\sim G \mu^2 \; ext{(Roughly same as pure strings)} \; ext{Leblond '09}$
- $\Omega_{\rm GW,burst} \approx \frac{\rho_{\rm GW,burst}}{\rho_{\rm BG}(v_{\mu})} \times \Omega_{\rm rad} \sim 10^{-4} (G\mu)^{4/3}$

$$f_{\text{burst}} \sim \frac{1}{l} \frac{a(v_{\mu})}{a(t_0)} \sim 10^8 \text{Hz} \left(\frac{v_m}{10^{15} \text{GeV}}\right)^{1/3}$$

• $\Omega_{\rm GW} \propto f^{-1}$ and $\ln f$ at high frequencies, f^3 at lower frequencies



Domain Walls Bounded by Strings



Occurs in following scenario

String formation

Occurs in following scenario

String formation — Inflation

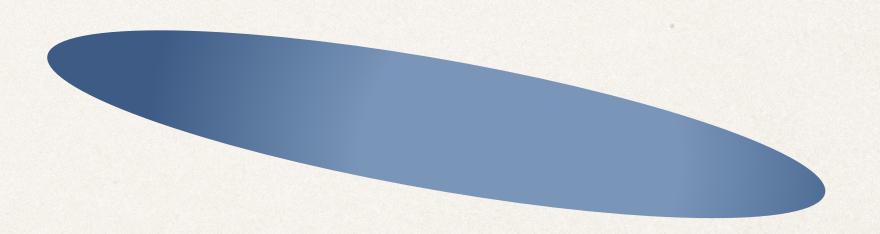
Occurs in following scenario

String formation — Inflation — Wall formation

Occurs in following scenario

String formation — Inflation — Wall formation — Wait...

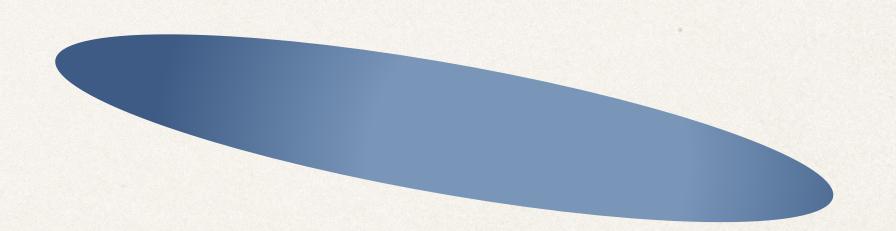




Occurs in following scenario

String formation — Inflation — Wall formation — Nucleation

 Wall rest mass converted to string kinetic energy (string expands and eats wall)

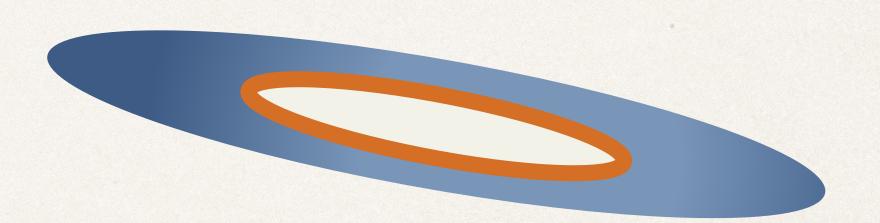


• Before nucleation, walls evolve as pure wall network

Occurs in following scenario

String formation — Inflation — Wall formation — Nucleation

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Pure Wall Network Evolution

• One-scale model and "infinite" (superhorizon) wall network

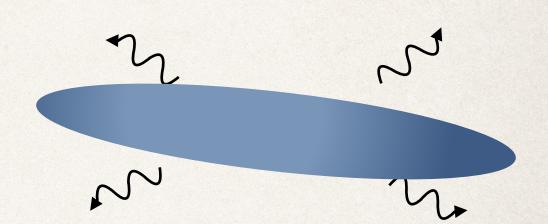
Typical separation and wall curvature same scale $R \longrightarrow \rho_{\infty} \sim \frac{\sigma R^2}{R^3} = \frac{\sigma}{R}$ $\sigma \approx v_w^3 \quad \text{(Wall mass per unit area)}$

• Network reaches scaling regime, similar to infinite string network R/t = O(1) constant

• Domain wall energy density grows with respect to background energy density! (Leads to "domain wall problem")

Domain Wall Gravitational Wave Amplitude

• Gravitational wave power sets amplitude of spectrum



$$P_{\text{GW}} \approx G \sum_{i,j} \langle \ddot{Q}_{ij} \ddot{Q}_{ij} \rangle \sim G(M_w R^2 \omega^3)^2 = \mathcal{B} G \sigma M_w.$$

 $\mathcal{B} \sim 1$ for walls in scaling regime Hiramatsu 2014

$$\frac{d\rho_{\rm GW}}{dt} + 4H\rho_{\rm GW} = -n_{\rm DW}P_{\rm GW} \simeq \frac{G\sigma^2}{t}\theta(t_{\Gamma} - t) \longrightarrow \rho_{\rm GW}(t) \approx \begin{cases} G\sigma^2 & t \leq t_{\Gamma} \\ G\sigma^2 \left(\frac{a(t_{\Gamma})}{a(t)}\right)^4 & t > t_{\Gamma} \end{cases}$$

$$\Omega_{\rm GW,max} \approx \frac{\rho_{\rm GW}(t_{\Gamma})}{\rho_c(t_{\Gamma})} \Omega_{\rm rad} \sim (G\sigma t_{\Gamma})^2 \Omega_{\rm rad}$$
 $f_{\rm peak} \approx \frac{1}{t_{\Gamma}} \frac{a(t_{\Gamma})}{a(t_0)}$ (Observed peak frequency)

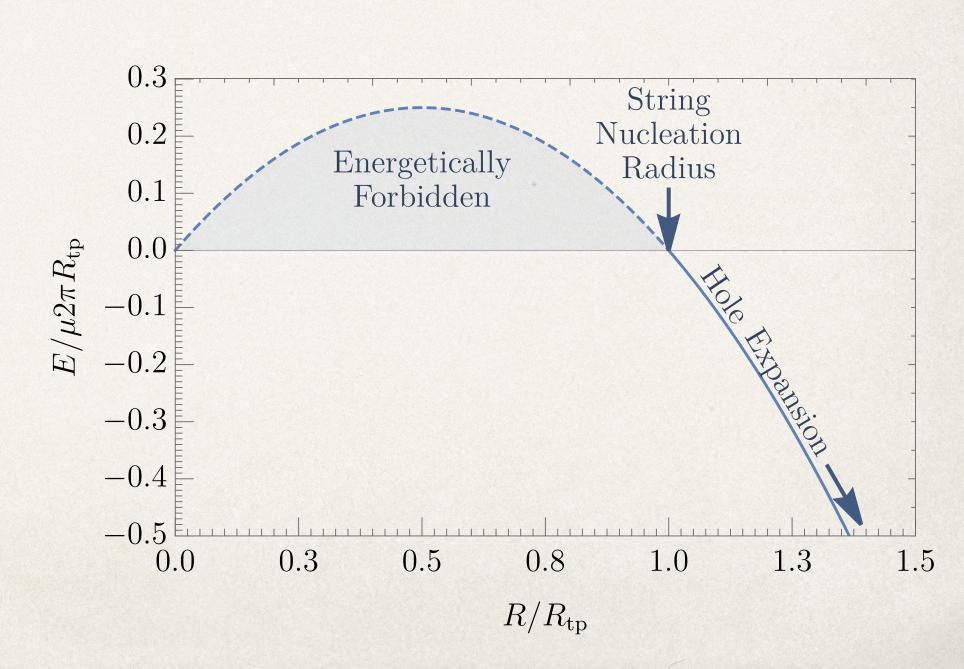
String Nucleation Rate

- We saw domain wall size grows with time $R^2 \sim t^2$
- Eventually, energetically favorable to lose wall mass and nucleate strings

$$E = \mu 2\pi R - \sigma \pi R^2$$

• WKB tunneling rate per unit area

$$\Gamma_d \propto \sigma e^{-S_E} \;, \qquad S_{\rm E} = \int_0^{R_{\rm tp}} dr \sqrt{\mu r E} \propto \frac{\mu^3}{\sigma^2}$$



String Nucleation Rate

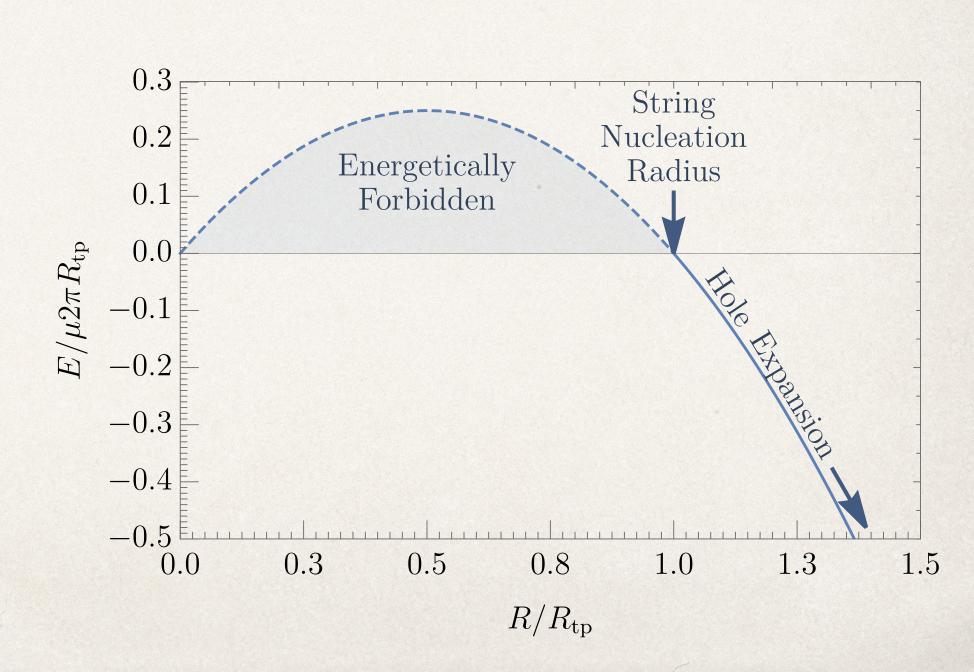
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$$E = \mu 2\pi R - \sigma \pi R^2$$

Bounce action formalism

$$\Gamma_d \sim \sigma \exp(-\frac{16\pi}{3}\kappa_s)$$
 $\kappa_s = \frac{\mu^3}{\sigma^2}$

 Exponentially sensitive! Large separation of string and domain wall scales → long-lived wall network



String Nucleation on Walls Spectrum

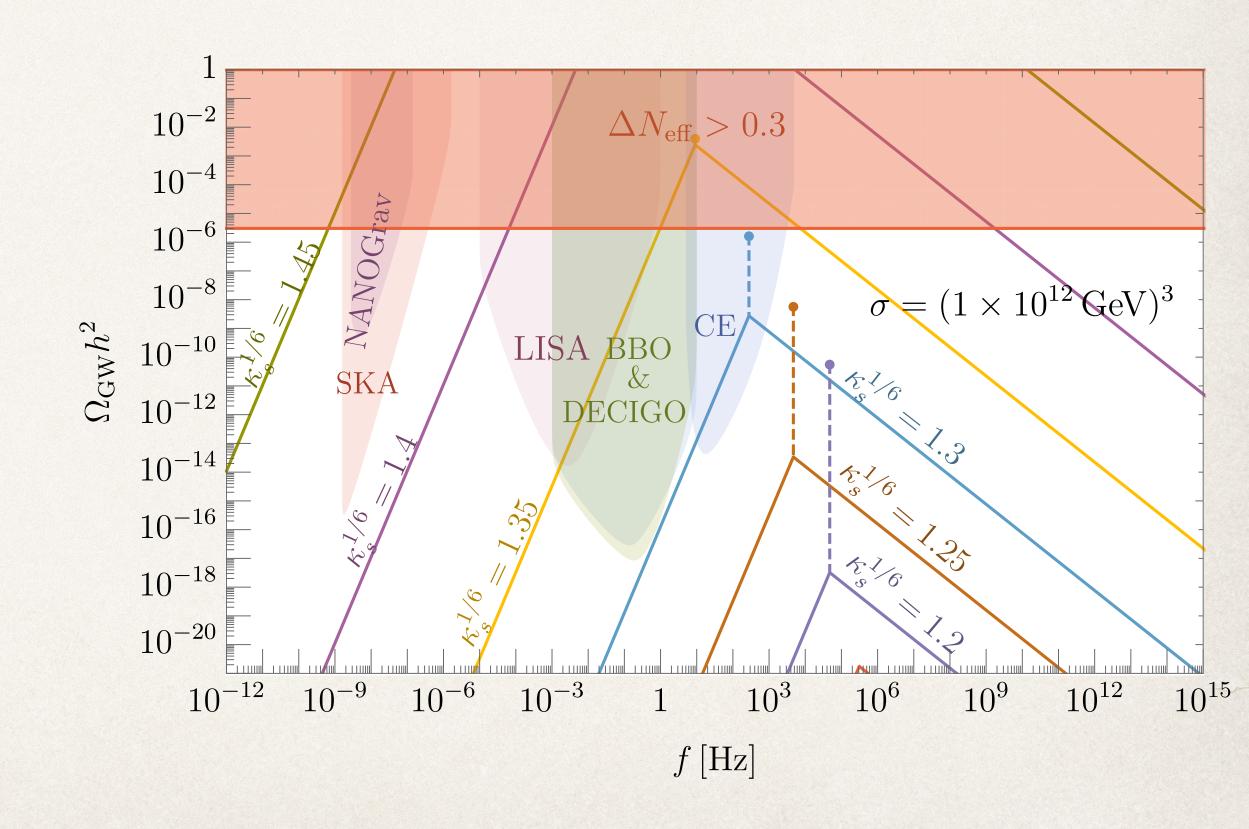
• Walls eaten when $A > A_{\rm max} \sim \Gamma_d t^2$

$$t_{\Gamma} \sim \frac{1}{A \times \Gamma_d} \sim \frac{1}{\sigma^{1/3}} \exp \frac{16\pi \kappa_s}{9}$$

$$\Omega_{\text{GW}}(f) = \frac{f}{\rho_c} \frac{d\rho_{\text{GW}}(t_0, f)}{df}$$

$$\approx \Omega_{\text{GW,max}} \begin{cases} \left(\frac{f}{f_{\text{peak}}}\right)^{-1} & f > f_{\text{peak}} \\ \left(\frac{f}{f_{\text{peak}}}\right)^3 & f \leq f_{\text{peak}} \end{cases}$$

• f^{-1} spectrum at high frequencies, f^3 decaying spectrum at lower frequencies *Hiramatsu* 2014



• Occurs in following scenario

Inflation --> String formation

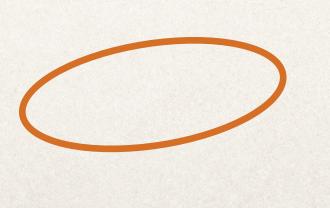
Occurs in following scenario

Inflation --> String formation -->

Wall formation — Eventual collapse

Wall rest mass converted to string kinetic energy

$$f_{
m string} pprox \mu \longrightarrow R_c \equiv \frac{\mu}{\sigma}$$
 $f_{
m DW} pprox \sigma R$



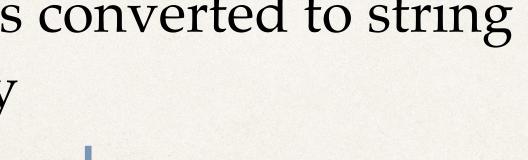


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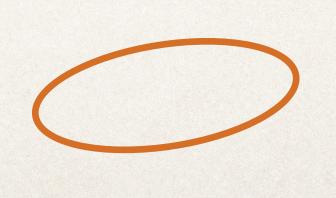
Inflation --> String formation -->

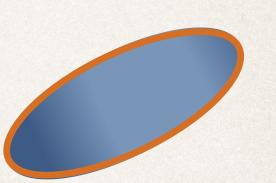
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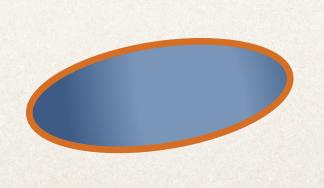
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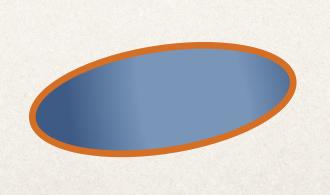
Inflation --> String formation -->

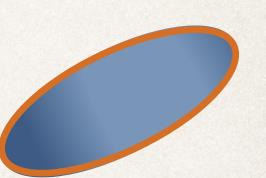
Wall formation — Eventual collapse

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Dynamics of Domain Wall Bounded Strings

• Combine domain wall and string Nambu-Goto actions

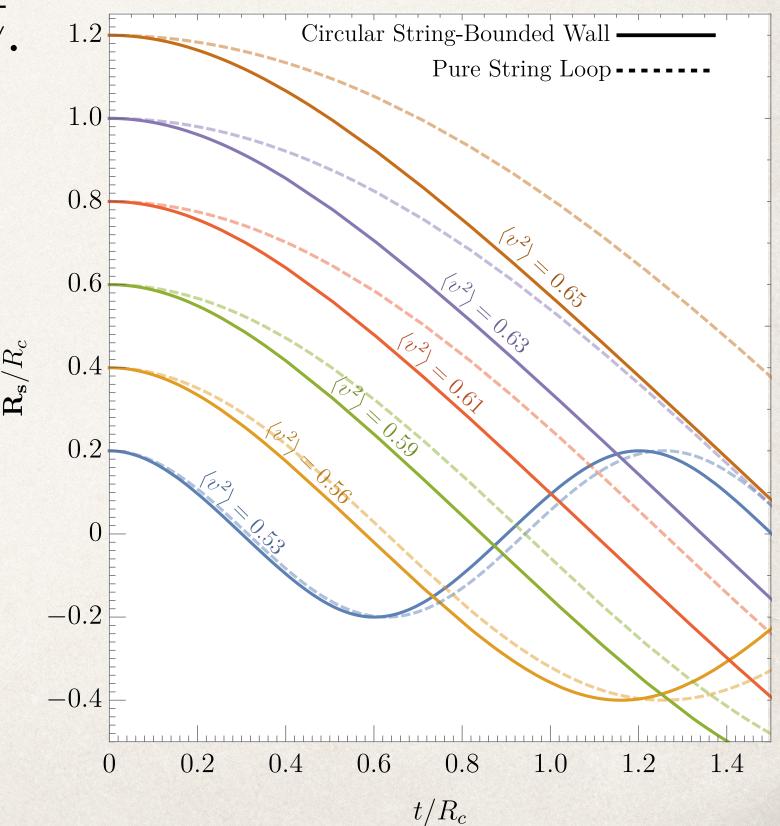
$$S = -\mu \int d^2\xi \sqrt{-\gamma} - \sigma \int d^3\xi \sqrt{-\gamma}.$$
 1.2

Identify boundary of domain wall with string radius

E.O.M. of circular string boundary

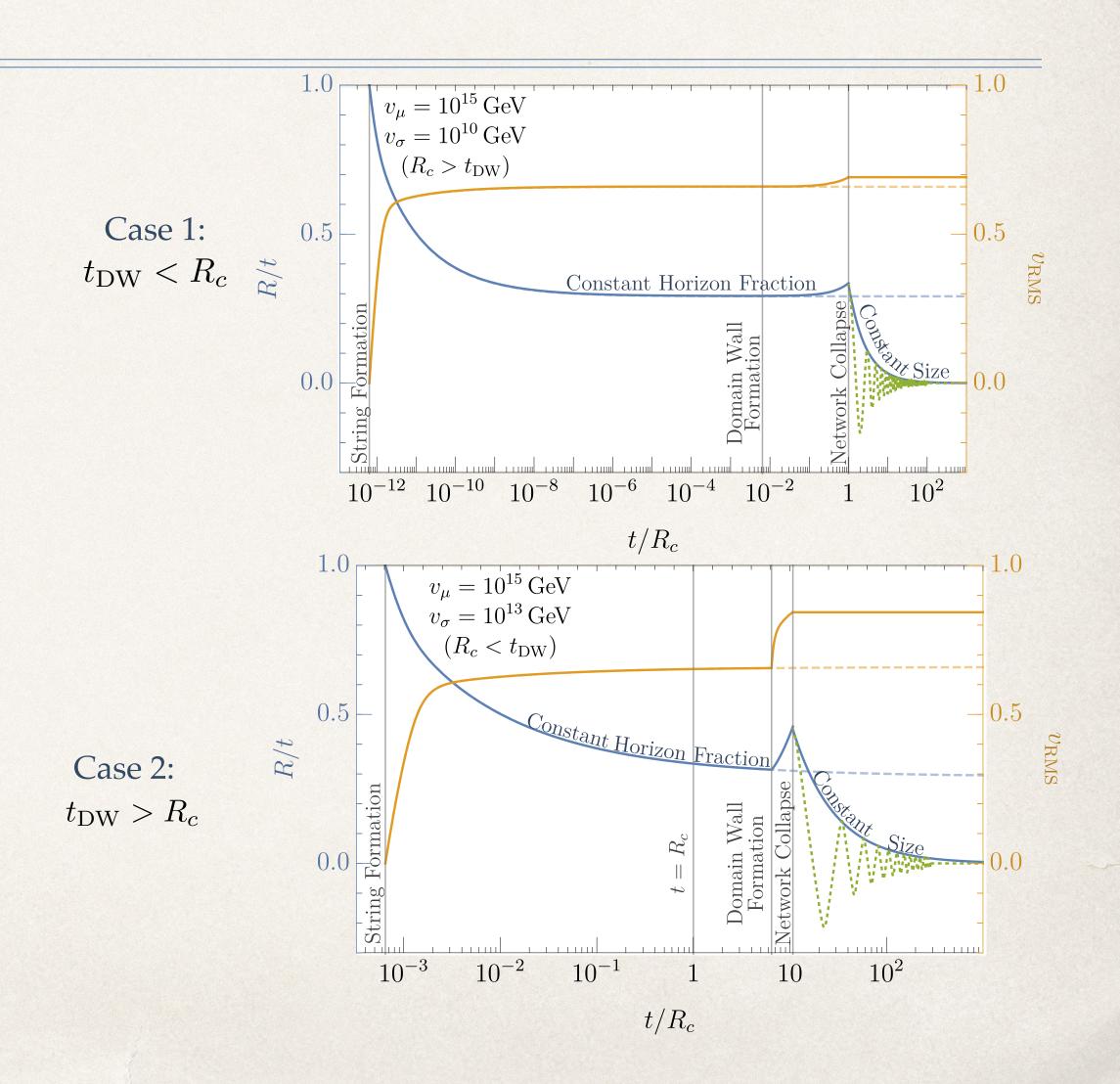
String dominates dynamics for $|\mathbf{r}_s| < R_c$

Domain wall dominates dynamics for $|\mathbf{r}_s| > R_c$



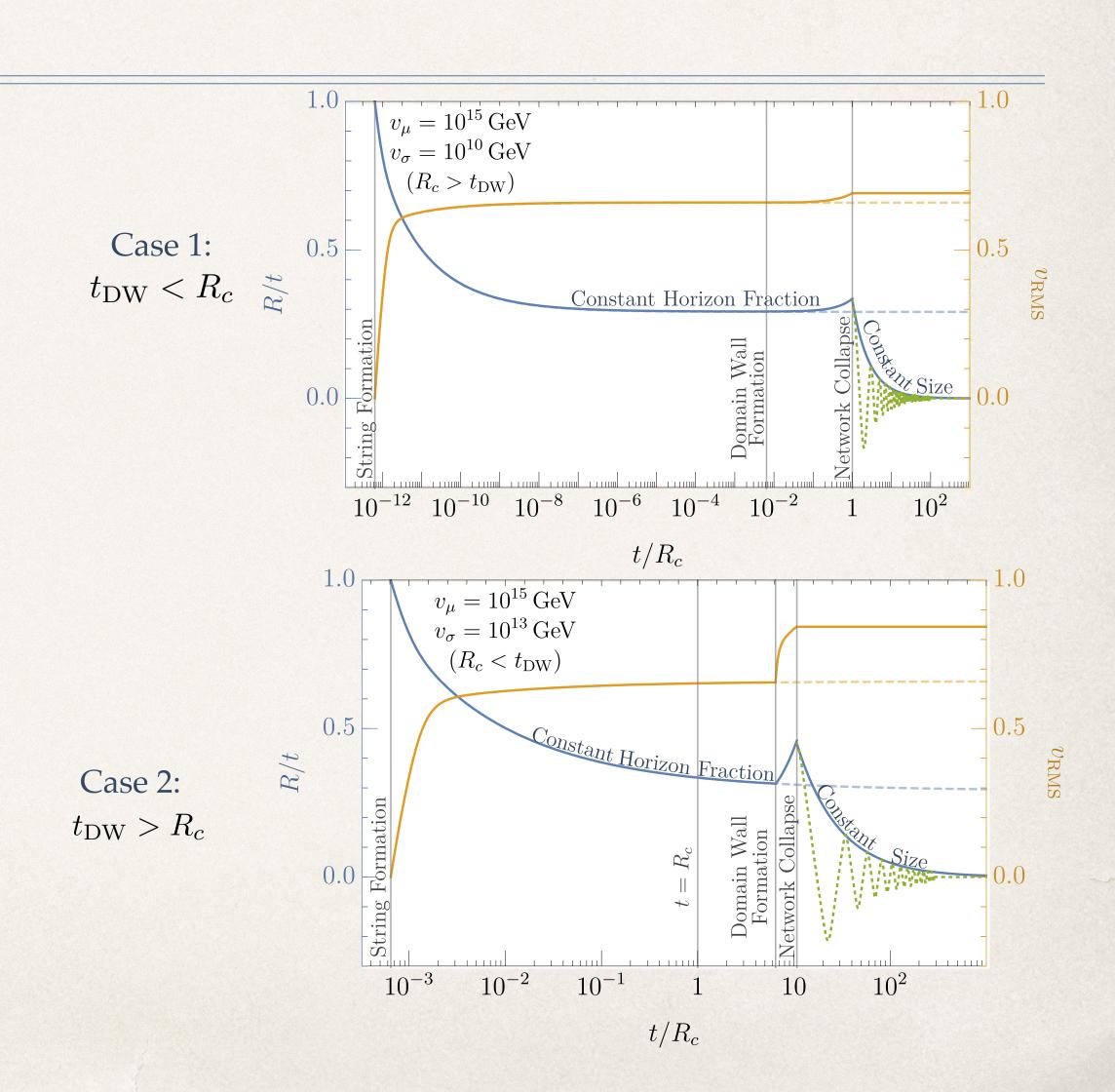
Evolution of String-Wall Network

- Effect of wall tension on the infinite string network?
- Recipe:
 - 1a) Solve velocity averaged Nambu-Goto E.O.M one-scale model ($\rho_{\infty} = \frac{\mu R}{R^3} = \frac{\mu}{R^2}$) Martins, Shellard '96
 - 1b) Once wall tension begins affecting evolution, infinite strings of curvature radius ${\cal R}$ behave like wall bounded strings of radius ${\cal R}$
 - 2) Piecewise connect with solution of circular string-bounded wall



Evolution of String-Wall Network

- After attaching to strings, network evolves like usual string network if $R < R_c$
- After $R > R_c$, domain wall tension dominates, collapses (eating begins)
- Network collapse at $t_* \approx \text{Max}(R_c, t_{\text{DW}})$, as first proposed by Everett, Vilenkin '82 and Martin, Vilenkin '96
- Domain wall bounded strings oscillate with constant mass before decaying via GW



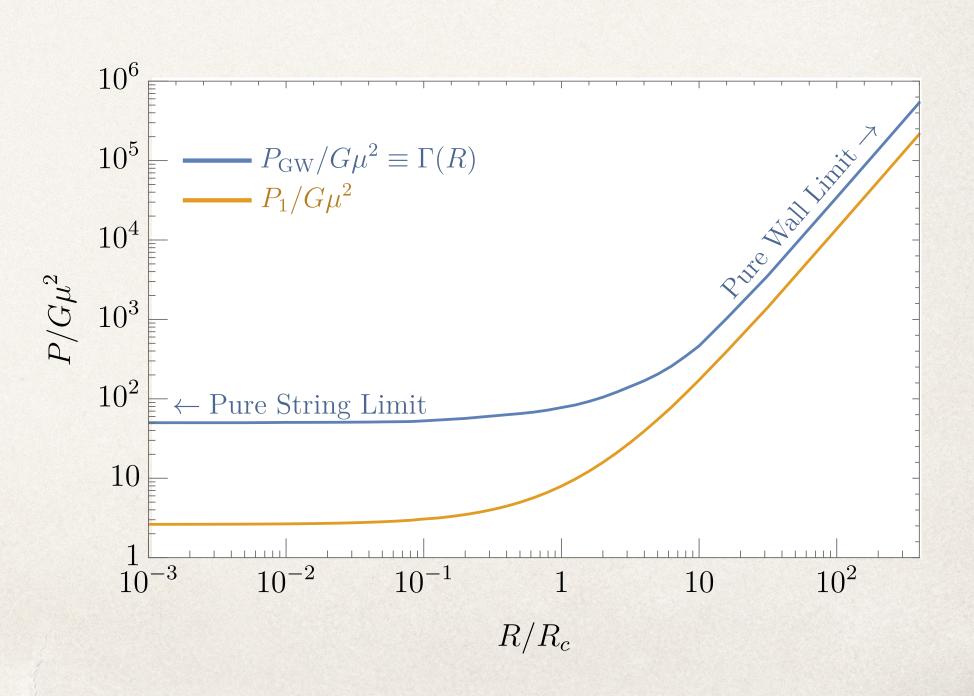
Gravitational Power of String-Bounded Wall

• Quadrupole formula, $P_{\rm GW} \approx \frac{G}{45} \langle \ddot{Q}_{ij} \ddot{Q}_{ij} \rangle$, for string loop $P_{\rm GW}^{(\rm string)} \sim G \mu^2$ and wall $P_{\rm GW}^{(\rm wall)} \sim G \sigma M_{\rm DW}$

$$P_{
m GW}^{
m (wall)} \gtrsim P_{
m GW}^{
m (string)}$$
 when $R > R_c$

• In reality, expect highly relativistic string to dominate GW emission

• Numerically compute $P_n = \frac{G\omega_n^2}{\pi} \int d\Omega |T_{\mu\nu}^T(\mathbf{k},\omega_n)|^2$ for circular string-bounded wall solution *Weinberg '72*



Gravitational Wave Amplitude

- Can estimate $\Omega_{\rm GW}$ and frequency of GW spectrum similar to strings
- Energy density of largest wall bounded strings after collapse evolves as

$$\rho_{\rm DW}(t) = \rho(t_*)_{\rm DW} \left(\frac{a(t*)}{a(t)}\right)^3 \longrightarrow \Omega_{\rm DW} \approx \frac{\rho_{\rm DW}(t_{\rm decay})}{\rho_{\rm BG}(t_{\rm decay})} \Omega_{\rm rad} \qquad t_{\rm decay} \approx \frac{M_{\rm DW}}{\Gamma(R)G\mu^2} \stackrel{R \gtrsim R_c}{\longrightarrow} \sim \frac{1}{G\sigma}$$

$$\frac{\Omega_{
m DW}}{\Omega_{
m strings}} \sim \sqrt{\frac{t_*}{R_c}}$$
 (Growth over string spectrum requires $t_{
m DW} \gg R_c$)

$$f_{\rm decay} \sim \frac{1}{R(t_*)} \frac{a(t_\Gamma)}{a(t_0)} \sim 10^3 \, \rm Hz \left(\frac{t_*}{R_c}\right)^{-1} \left(\frac{\sigma}{(10^7 {\rm GeV})^3}\right)^{1/2} \left(\frac{\mu}{(10^{14} {\rm GeV})^2}\right)^{-1} \qquad \text{(Relatively high frequencies)}$$

Gravitational Wave Amplitude

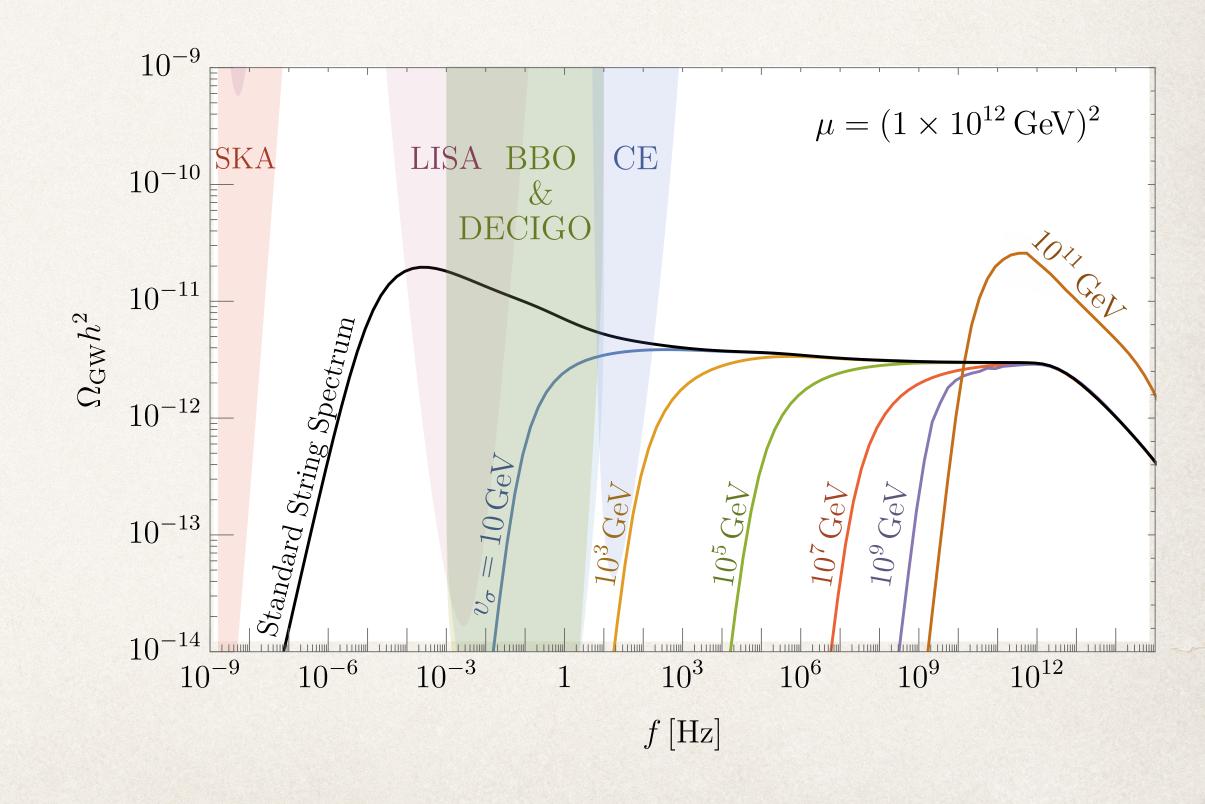
Can confirm simple estimates more precisely

$$\Omega_{\rm GW}h^2 = f\frac{d\rho_{\rm GW}}{df}\frac{h^2}{\rho_c}$$

$$\frac{d\rho_{\rm GW}(t)}{df} = \int_{t_{\rm sc}}^t dt' \frac{a(t')^4}{a(t)^4} \int dl \frac{dn(l,t')}{dl} \frac{dP_l(l,t')}{df'} \frac{df'}{df}$$

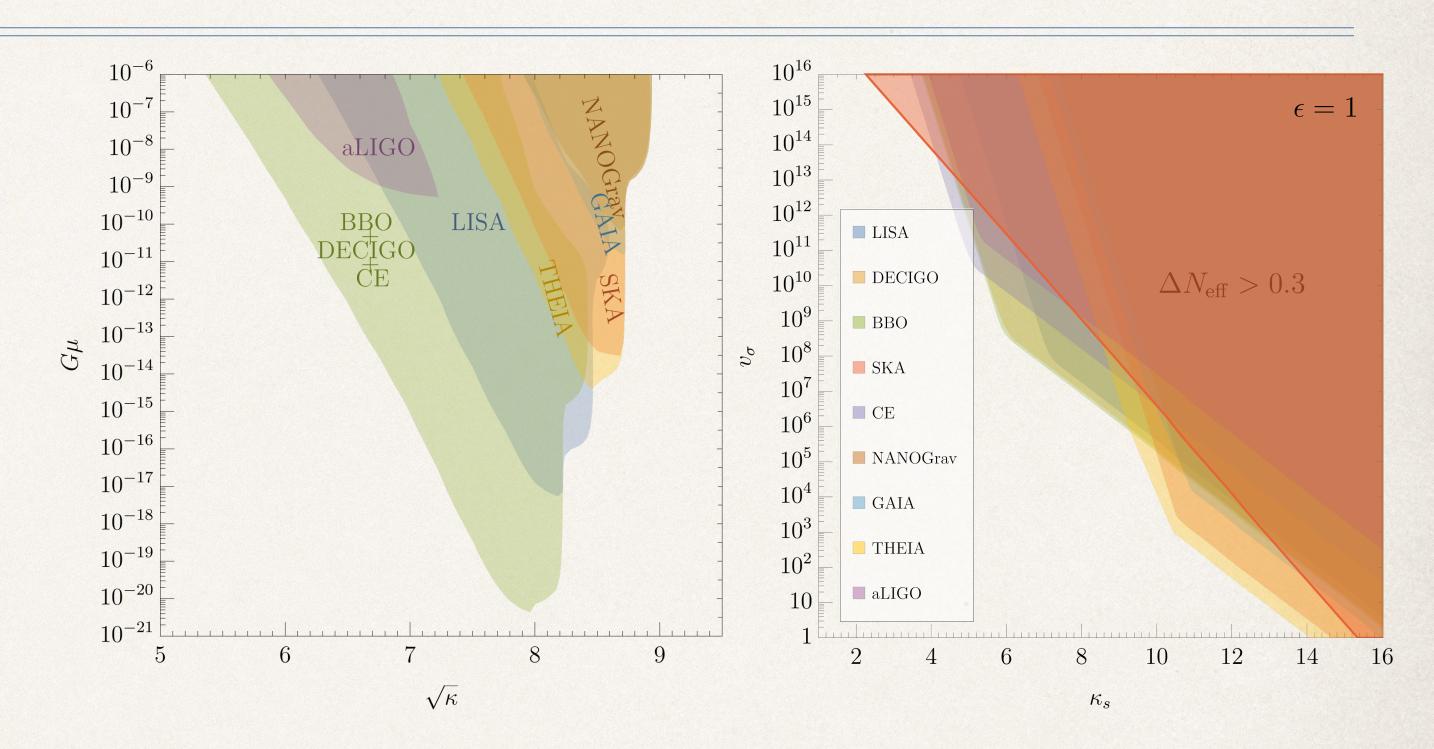
$$\uparrow \qquad \uparrow \qquad \uparrow$$
 Expansion One-scale model Power Redshift

- Standard, flat, string spectrum at high frequencies, f^3 decaying spectrum at lower frequencies
- Domain wall "bump" before decay if $t_{\rm DW} \gg R_c$



Overview of Spectra

	IR	UV	Generally high frequency signal?
Monopoles eating strings (Nucleation)	f^2	f^0	
Strings eating monopoles (Collapse)	f^3	$\ln f \to f^{-1}$	
Strings eating domain walls (Nucleation)	f^3	f^{-1}	
Domain walls eating strings (Collapse)	f^3	f^0	

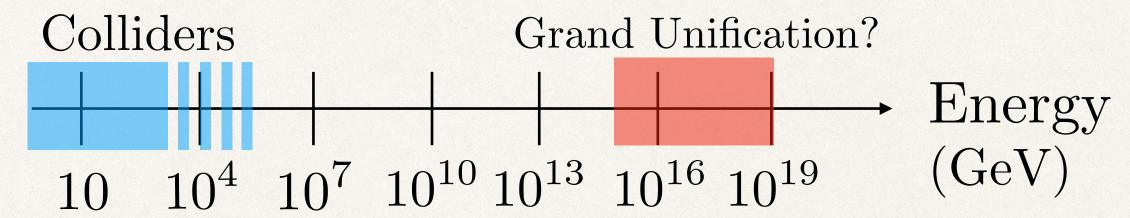


Monopole nucleation on strings

String nucleation on monopoles

Conclusions

 Gravitational waves provide amazing view into early universe and high energy physics



- GUT symmetry chains with hybrid topological defects produce unique gravitational wave fingerprints upon being "eaten"
- These signatures may allow us to understand our universe at a fundamental level

Thank you!