New observational window on the high-scale origin of matter-antimatter asymmetry

The Poltergeist mechanism

(A mechanism to enhance the induced gravitational wave)

Poltergeist

mechanism for gravitational wave production

Logo by ©Takahiro Terada

Inomata, Kohri, Nakama, Terada, arXiv:1904.12879

Inomata, Kawasaki, Mukaida, Terada, Yanagida, arXiv:2003.10455

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Abstract

We can confirm the existence of the early matter-dominated epoch by precisely-observing

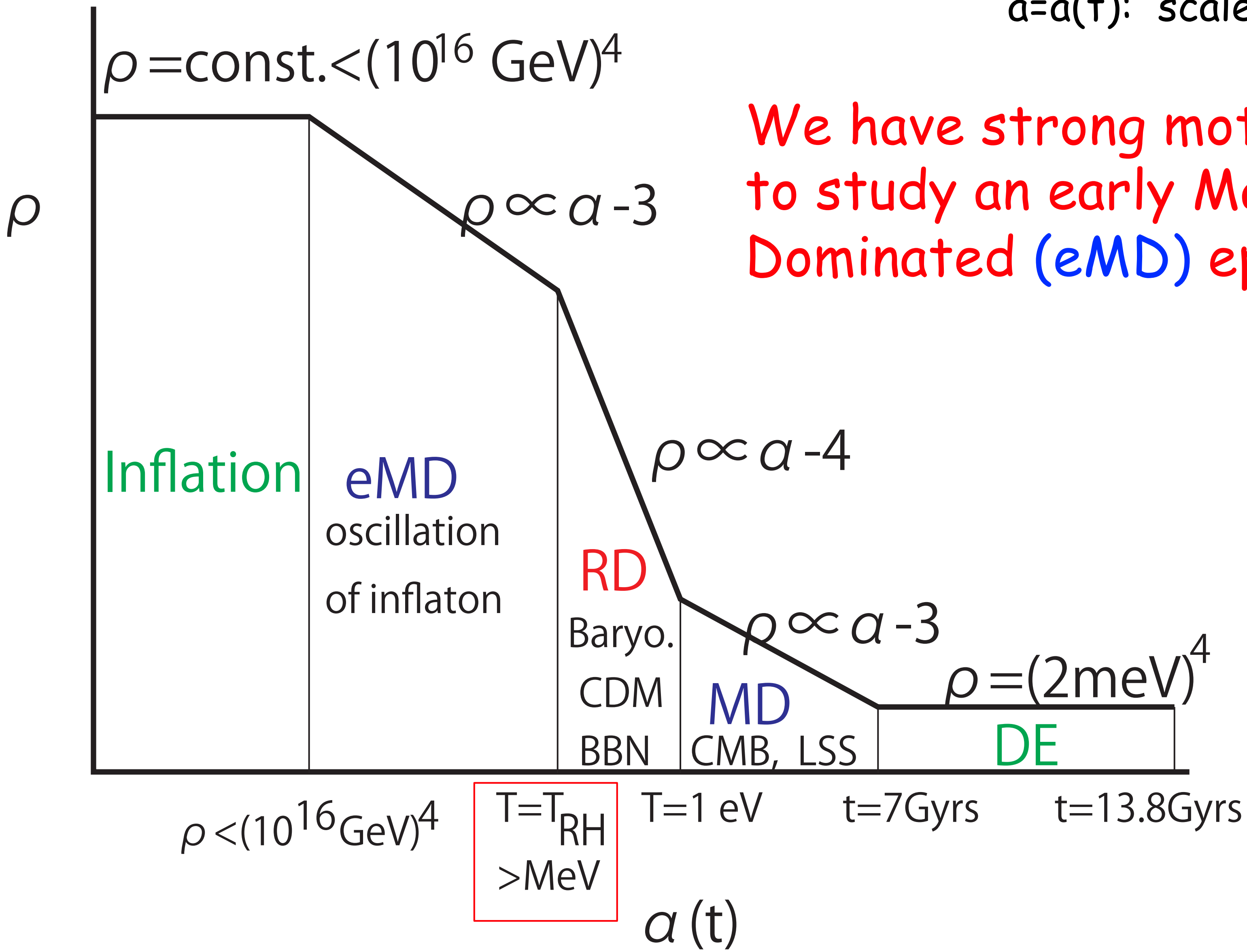
1. Spectrum of the (induced) gravitational wave background

2. Effective number of neutrino species ($N_\nu < 3$)

3. Spin of the primordial black holes ($a_* \sim 1$)

Cosmic history of energy density

a=a(t): scale factor



We have strong motivation to study an early Matter Dominated (eMD) epoch

1) Stochastic gravitational wave produced in the early Matter Dominated epoch

The induced gravitational wave

Ananda, Clarkson, and Wands (2006)
Baumann, Steinhardt, Takahashi, and Ichiki (2007)
Assadullahi and Wands (2009)
Saito and Yokoyama (2009)
Bugaev and Klimai (2009)
Alabidi, Kohri, Sendouda, Sasaki(2012, 2013)
Espinosa, Rocco and Riotto (2018)
Kohri and Terada (2018)
Inomata, Kohri, Nakama,Terada (2019a)(2019b)
Cai, Pi, Sasaki (2019)
Inomata, Kawasaki, Mukaida, Terada, Yanagida (2020)
Papanikolaou, Vennin, Langlois, (2020)
Domenech, Lin, Sasaki(2021)

The poltergeist mechanism was proposed by those authors (in the red-colored publications)

The induced 2nd order GWs

- It is not the primordial 1st order GW produced from inflation

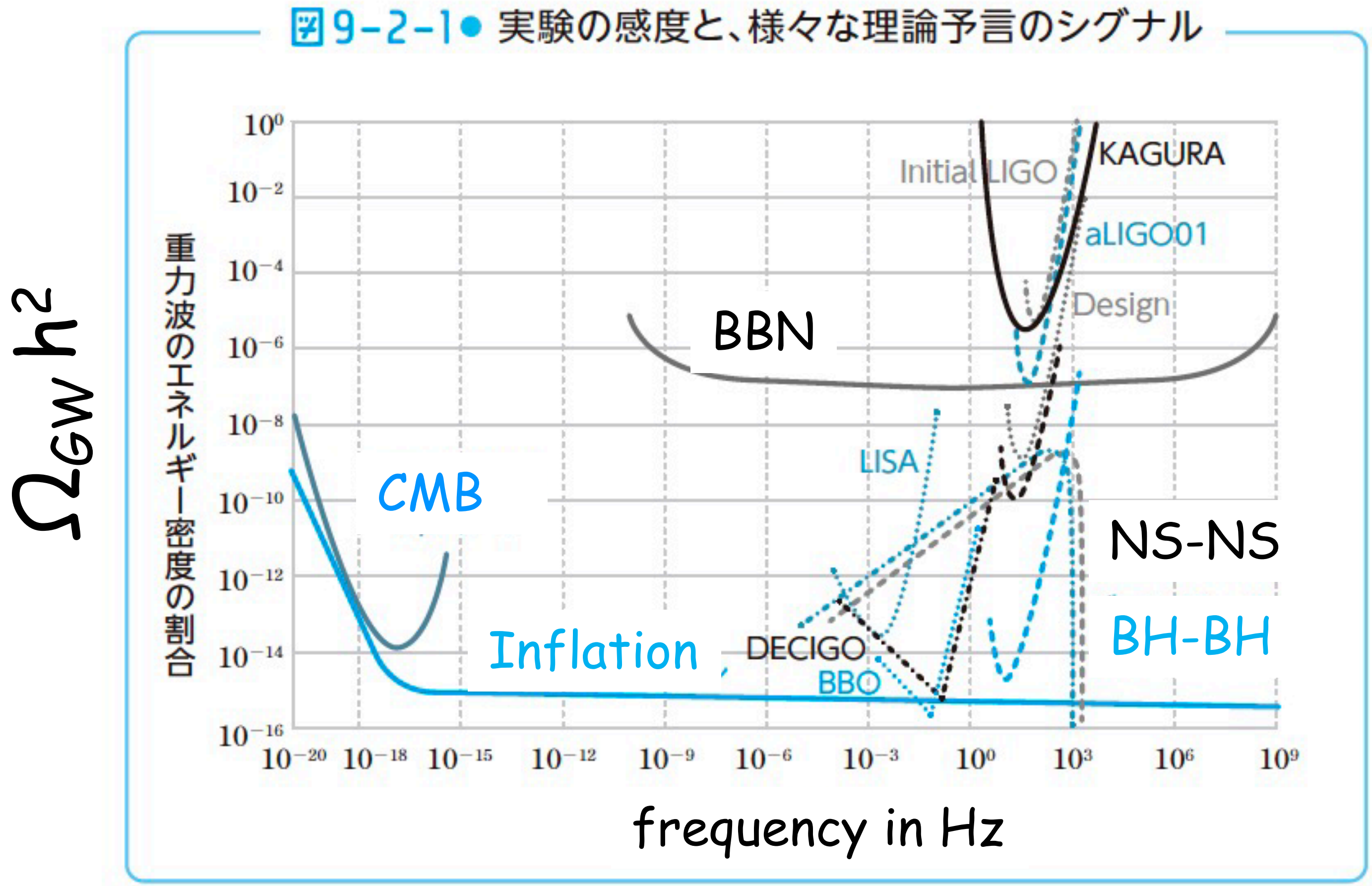
$$\Omega_{\mathrm{GW}}^{(1)}(k_{\mathrm{R}} > k > k_{\mathrm{EQ}}) \sim 10^{-5} r \mathcal{P}_\zeta(k)$$

$$r = \left.\frac{\mathcal{P}_h}{\mathcal{P}_\zeta}\right|_{k_0}$$

- It is nonlinearly-induced by 2nd order effects of large scalar (curvature) perturbation at small scales

$$\Omega_{\mathrm{GW}}^{(2)}(k) \gtrsim \mathcal{P}_\zeta^2(k)$$

Future sensitivities on the primordial GW produced by inflation

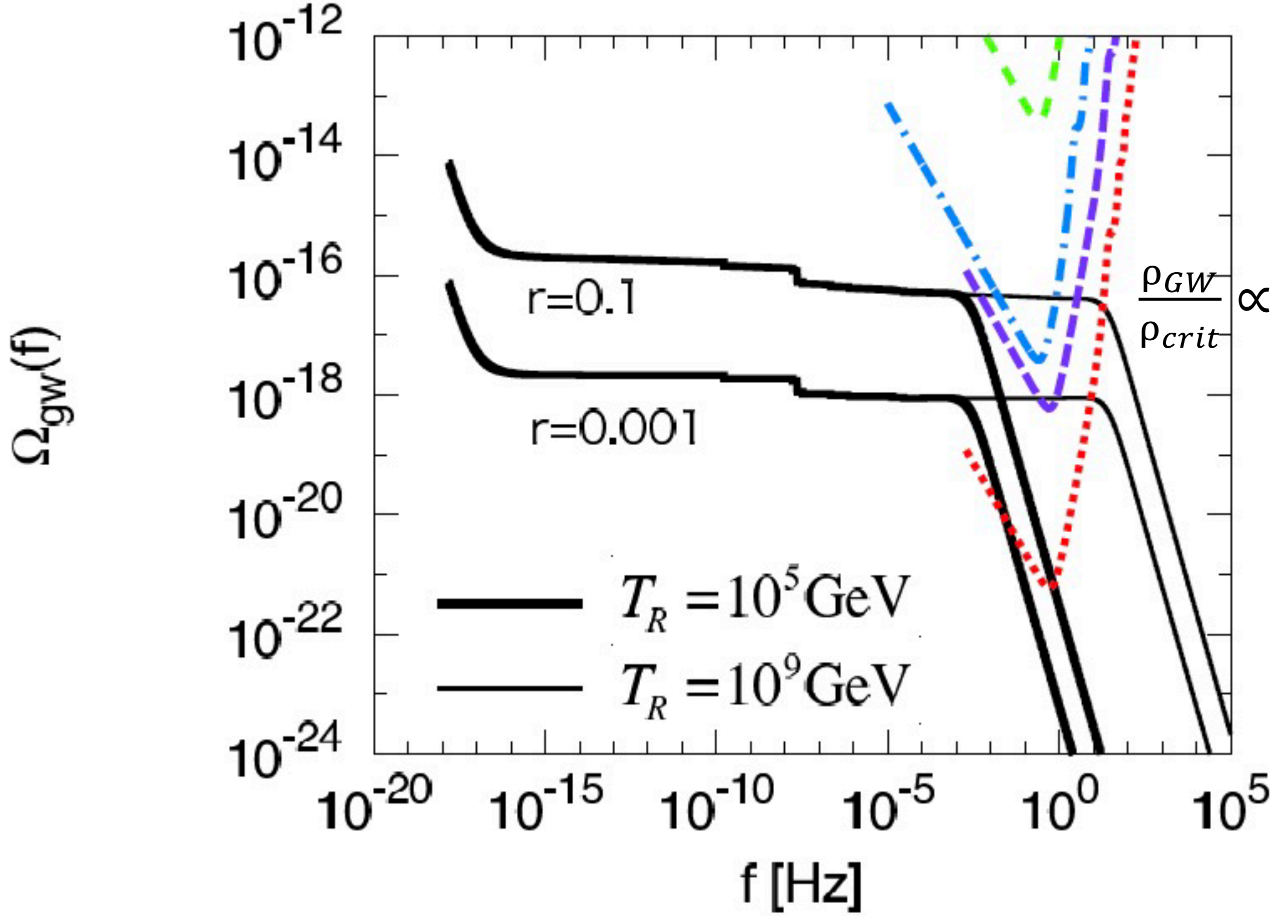


下に平行に伸びているのがインフレーション起源の重力波のシグナルの可能性。将来のCMB実験、DECIGO実験、BBO実験に感度がある。

Kazunori Kohri, "Neutrino and Gravitational wave", Beret Press (2021)

The break point of $\Omega^{(1)}_{GW}$ marks the reheating temperature after inflation

Naoki Seto, Jun'ichi Yokoyamam, arXiv:gr-qc/0305096
Kazunori Nakayama, Shun Saito, Yudai Suwa, Jun'ichi Yokoyama, arXiv:0804.1827



$$\frac{\rho_{GW}}{\rho_{crit}} \propto \frac{a^{-4}}{a^{-3}} \propto a^{-1} \propto f^{-2}$$

Tensor perturbation

- Metric

$$ds^2 = a^2\left(-(1+2\Phi)d\eta^2 + \left((1-2\Psi)\delta_{ij} + \frac{1}{2}h_{ij}\right)dx^i dx^j\right)$$

- Gravitational potential

$$\Phi$$

- Spatial curvature in Newton gauge

$$\Psi = \Phi \sim O(\zeta)$$

- Tensor perturbation (gravitational wave)

$$h_{ij}$$

Energy density of induced tensor modes

- Tensor perturbation (gravitational wave)

$$h_{ij}(\boldsymbol{x},\eta) = \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \left[h^{+}_{\boldsymbol{k}}(\eta)\mathrm{e}^{+}_{ij}(\boldsymbol{k}) + h^{\times}_{\boldsymbol{k}}(\eta)\mathrm{e}^{\times}_{ij}(\boldsymbol{k})\right]$$

- Power spectrum of the tensor perturbation

$$\langle h^{r}_{\boldsymbol{k}}(\eta)h^{s}_{\boldsymbol{k}'}(\eta)\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_h(k,\eta)\delta(\boldsymbol{k}+\boldsymbol{k}')\delta^{rs},$$

- Omega parameter well inside the horizon

$$\Omega_{\mathrm{GW}}(k,\eta) = \frac{1}{3}\left(\frac{k}{\mathcal{H}}\right)^2 \mathcal{P}_h(k,\eta).$$

- Substituting the solution into this

$$\Omega_{\mathrm{GW}}(k,\eta) = \frac{2}{3}\left(\frac{k}{a\mathcal{H}}\right)^2 \iint_{|\mu_{12}|<1} \mathrm{d}k_1\mathrm{d}k_2\, \mathcal{P}_\Psi(k_1)\mathcal{P}_\Psi(k_2)(1-\mu_{12}^2)\left(\frac{k_1k_2}{k^2}\right)^2 I(k,k_1,k_2,\eta)^2, \tag{25}$$

here

$$I(k,k_1,k_2,\eta) \equiv k\int^{\eta} \mathrm{d}\tilde{\eta}\; a(\tilde{\eta})g_k(\eta;\tilde{\eta})f(k_1,k_2,\tilde{\eta}), \tag{26}$$

Relevant when $P_\psi \gg r$

An example for Inflation models to produce large curvature perturbation at small scales

PBH formation and Inflation models

- Multi-field Inflaiton

  Kawasaki,Sugiyama,Yanagida (1998)

- At the end of inflation

  Lyth, Malik, Sasaki, Zabarra (2006)

- Preheating

  Green and Malik (1999)
  Taruya (1998)

- Blue-tilted spectral

  Kohri, Lyth and Melchiorri (2007)

- Curvaton

  Kawasaki, Kitajima, Yanagida (2012)
  Kohri, Lin, Matsuda (2012)

- …

Simple parameterization of running of spectral indexes of curvature perturbation

- Curvature perturbation

$$P_\zeta(k) = A_\mathrm{s}\left(\frac{k}{k_*}\right)^{n_\mathrm{s}-1+\frac{\alpha_\mathrm{s}}{2}\ln\left(\frac{k}{k_*}\right)+\frac{\beta_\mathrm{s}}{6}\left(\ln\left(\frac{k}{k_*}\right)\right)^2}$$

$$A_s \equiv P_\zeta|_* \sim \left.\frac{V}{m_\mathrm{pl}^4\varepsilon}\right|_* \sim (\delta T/T)^2$$

$$\varepsilon \equiv \frac{1}{2}\left(m_\mathrm{pl}\frac{V'}{V}\right)^2$$

- spectral index

$$n_s - 1 = dP_\zeta/d\ln k = 2\eta - 6\varepsilon$$

$$\eta \equiv m_\mathrm{pl}^2\frac{V''}{V}$$

- running of n_s

$$\alpha_s = dn_s/d\ln k = -24\varepsilon^2 + 16\varepsilon\eta - \xi^{(2)}$$

$$\xi^{(2)} \equiv m_\mathrm{pl}^4\frac{V'V'''}{V^2}$$

- running of running of n_s

$$\sigma^{(3)} \equiv m_\mathrm{pl}^6\frac{(V')^2V''''}{V^3}$$

$$\beta_s = d\alpha_s/d\ln k = 192\varepsilon^3 + 192\varepsilon^2\eta - 32\varepsilon\eta^2 + (-24\varepsilon + 2\eta)\xi^{(2)} + 2\sigma^{(2)}$$

Type-III Hilltop inflation models

German, Ross, Sarkar (01)
KK, Lin and Lyth (07)

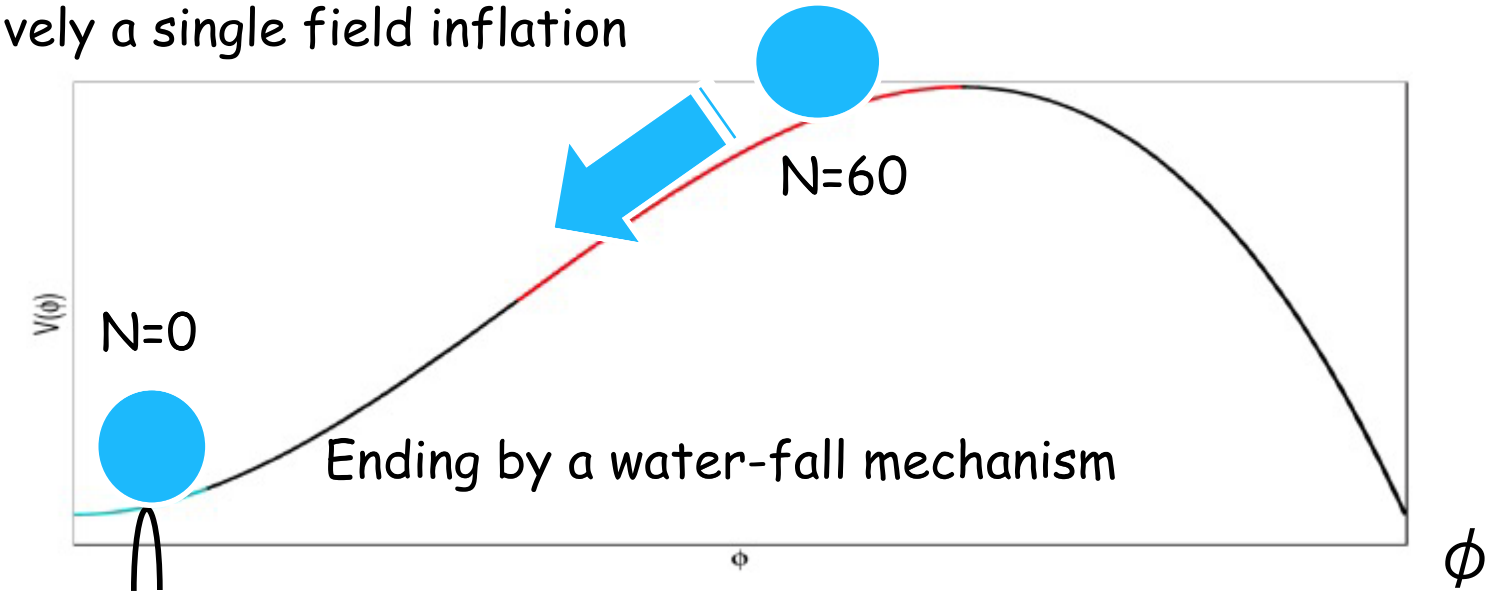
- Potential in supergravity, e.g.,

$$V(\phi) \;=\; V_0 + \frac{1}{2}m^2\phi^2 - \lambda\frac{\phi^p}{M_{\rm P}^{p-4}} + \cdots$$

$$W = C\frac{\phi^\rho}{M_{\rm pl}^{p-3}}, \quad \lambda \sim C m_{3/2}/M_{\rm pl}$$ in SUGRA

Allahverdi, Kusenko, Mazumdar (06)

Effectively a single field inflation



Large running spectral index

KK, Lin and Lyth (07)

- Spectrum

$$P_\zeta \sim \frac{V}{m_{\rm pl}^4 \varepsilon}$$

- Enhanced curvature perturbation at small scales due to a large running of running

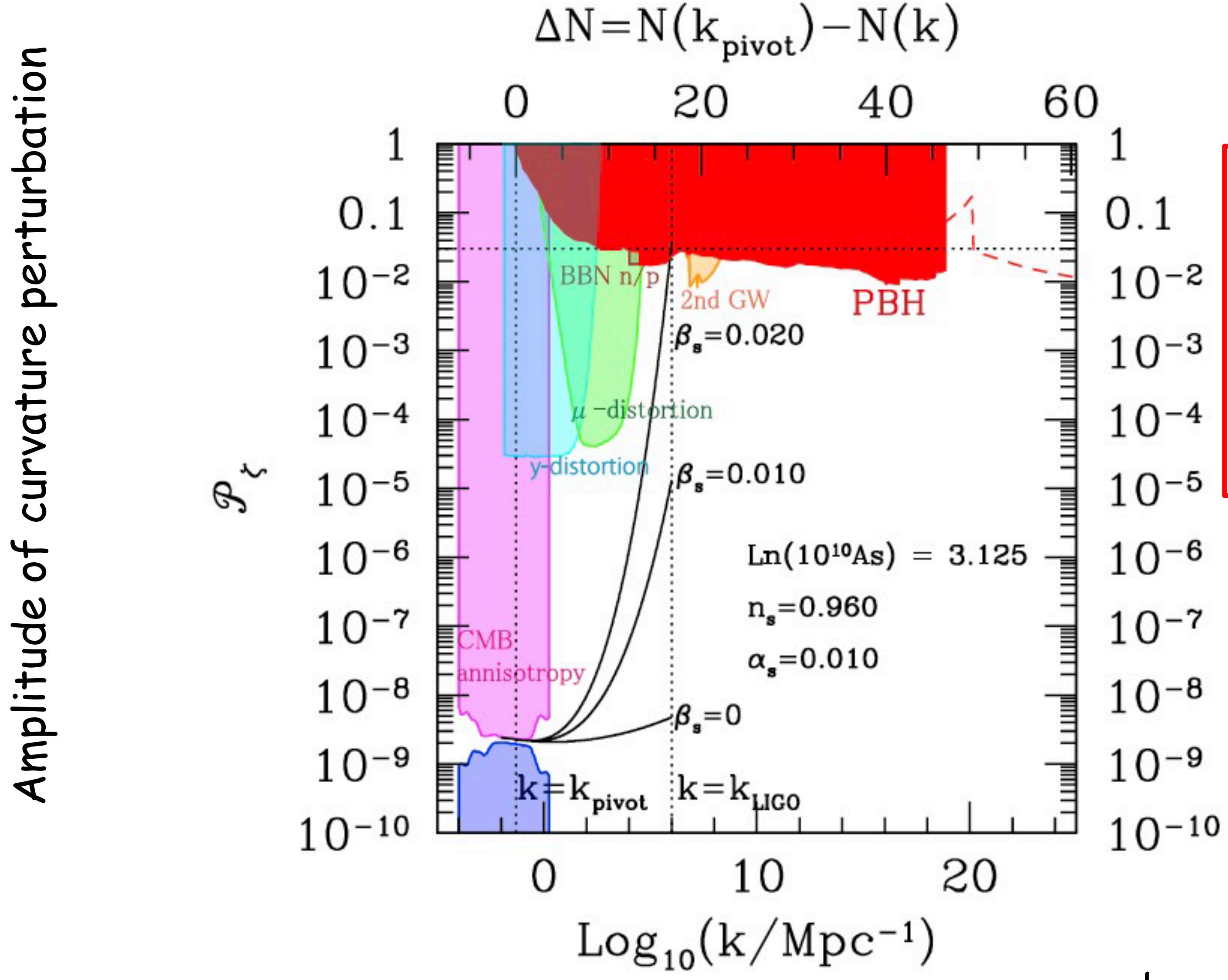
$$\varepsilon \equiv \frac{1}{2}\left(m_{\rm pl}\frac{V'}{V}\right)^2 \to 0 \text{ for } \phi \downarrow$$

$$\beta_s = \frac{d^3 P_\zeta}{d(\ln k)^3} = 192\epsilon^3 + 192\epsilon^2\eta - 32\epsilon\eta^2 + (-24\epsilon + 2\eta)\boxed{\xi^{(2)}} + 2\boxed{\sigma^{(3)}}$$

Could be large!

Curvature perturbation $P_\zeta(k)$

Kohri and Terada, 2018
Alabidi, Kohri, Sendouda, Sasaki, 2013



Planck (2015)

$n_s = 0.9586 \pm 0.0056,$

$\alpha_s = 0.009 \pm 0.010,$

$\beta_s = 0.025 \pm 0.013.$

at 68% C.L.

For inflation models with a big running, see Kohri, Lin, Lyth (2008)

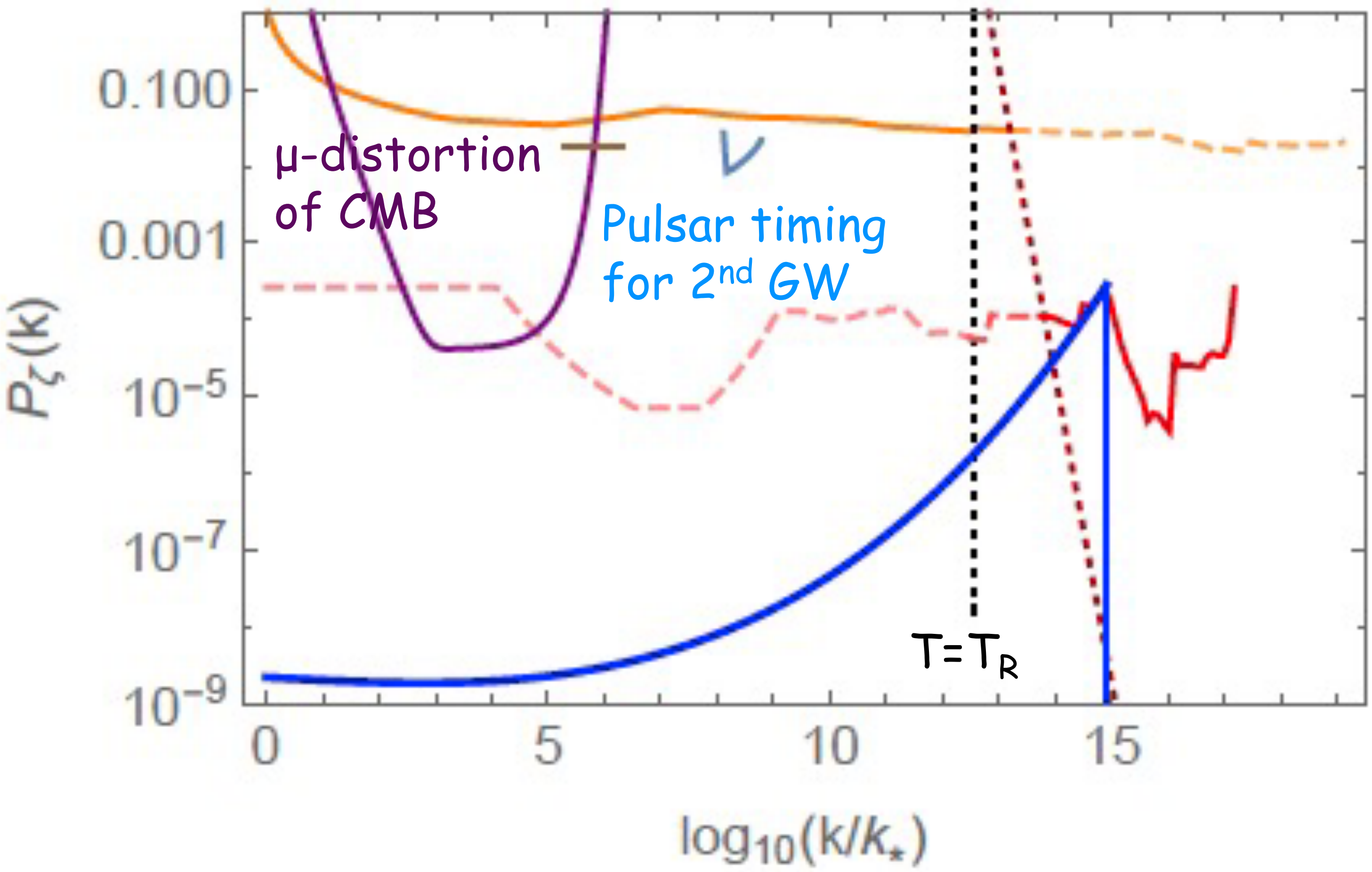
wave number k = p x a

100 % Dark Matter by PBHs with 10^{17} g masses

KK and T.Terada, 2018

$$T_{\rm R} = 10^4 {\rm GeV},$$

$$n_{\rm s} = 0.96, \alpha_{\rm s} = 0, \beta_{\rm s} = 0.0019485.$$

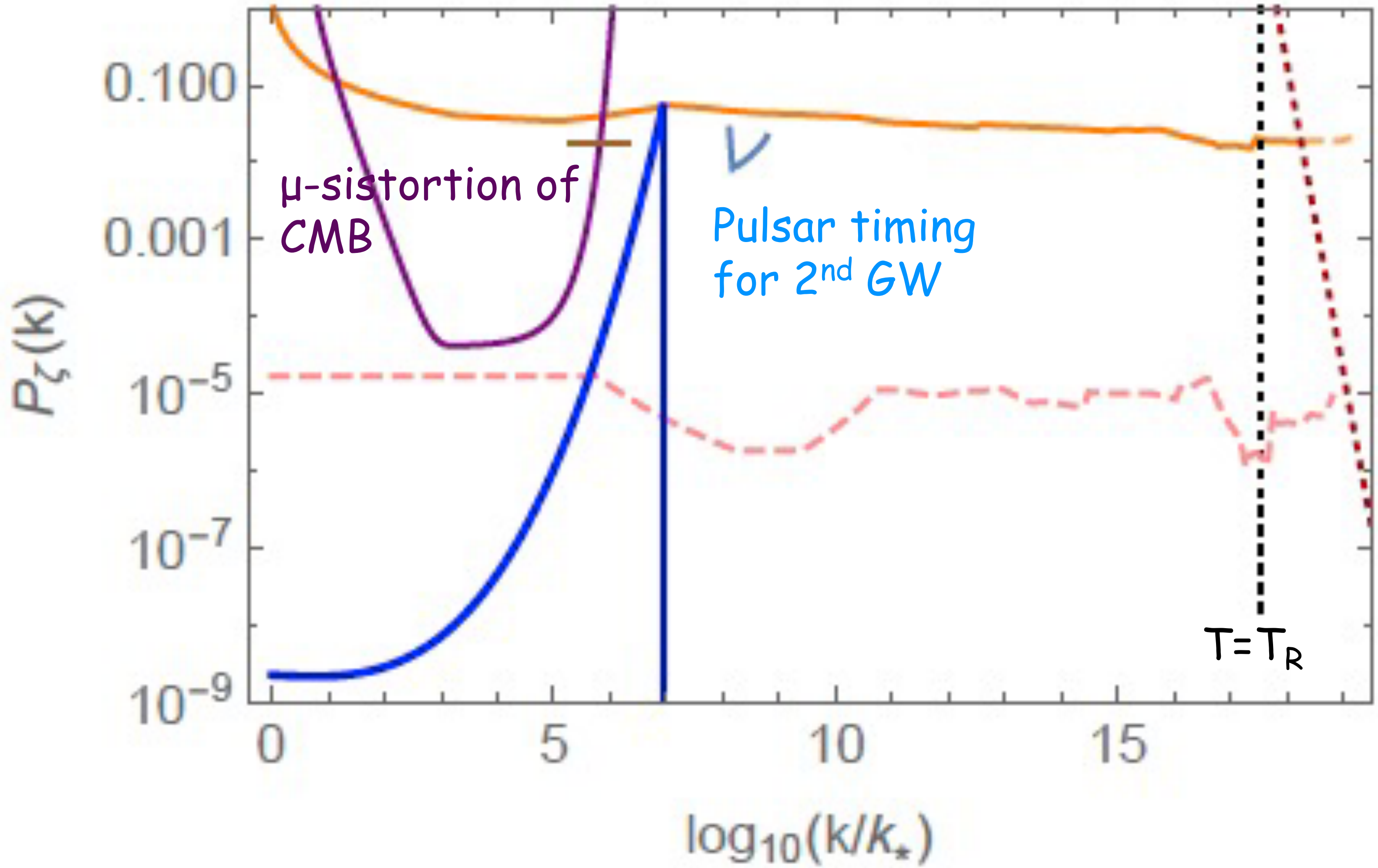


LIGO/VIRGO event with 30 Msolar

KK and T.Terada, 2018

$T_R = 10^9\mathrm{GeV}$

$n_s = 0.96, \alpha_s = 0, \beta_s = 0.026.$



The induced gravitational wave

Time evolution of gravitational potential Φ

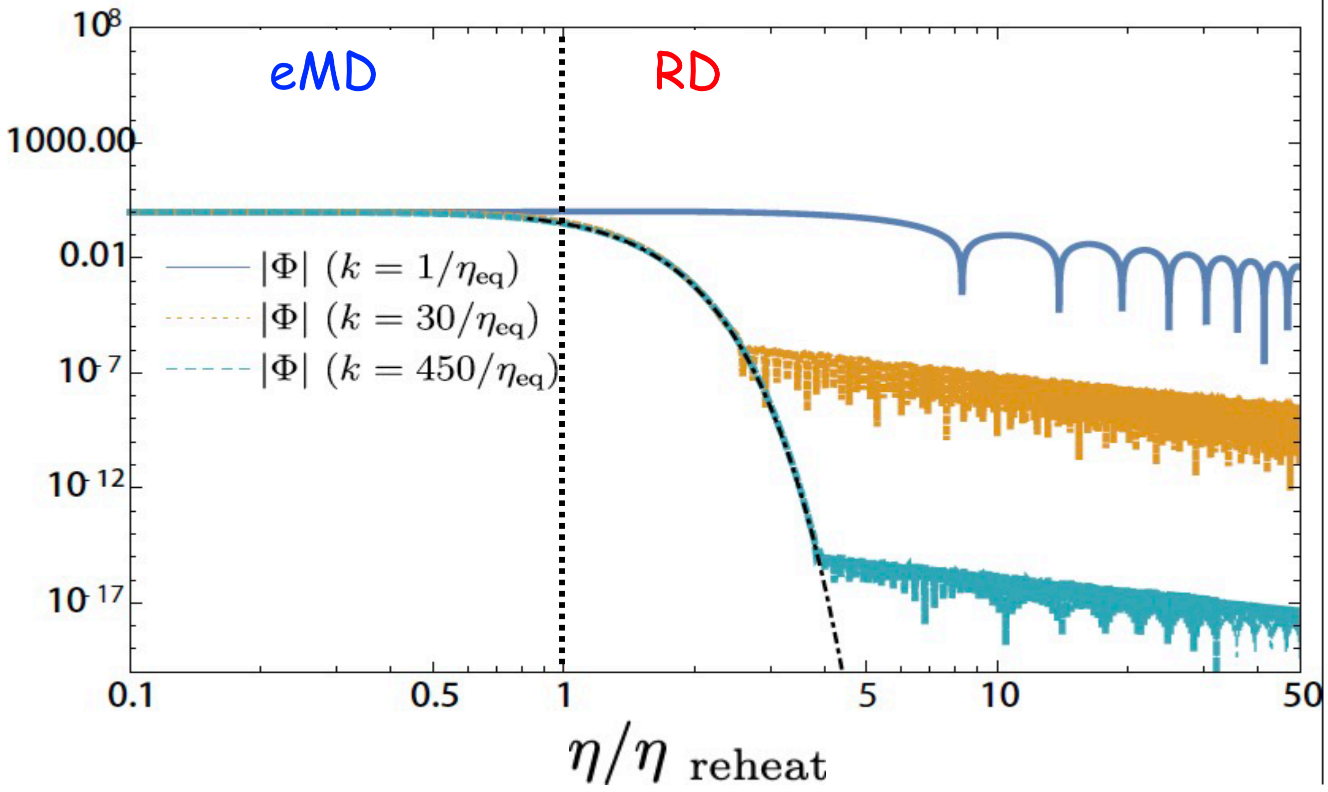
Inomata, Kohri, Nakama, Terada, arXiv:1904.12878
Inomata, Kohri, Nakama, Terada, arXiv:1904.12879

$$\Phi'' + 3(1+w)\mathcal{H}\Phi' + wk^2\Phi = 0 \qquad \omega = \rho/p$$

$$\Phi(x, x_{\mathrm{R}}) = \begin{cases} 1 & (\eta < \eta_{\text{ reheat}}) \\ \text{dumped oscillation} & (\eta > \eta_{\text{ reheat}}) \end{cases}$$

early Matter Domination (eMD)
Radiation Domination (RD)

Φ



$$\eta \sim \frac{1}{a(t)H(t)}$$

2^{nd} order GWs with gradual transition (normal exponential decay) from MD to RD

Inomata, Kohri, Nakama, Terada, JCAP10(2019)071, arXiv:1904.12878

$$\Omega_{\rm GW}(\eta,k) = \frac{\rho_{\rm GW}(\eta,k)}{\rho_{\rm tot}(\eta)} = \frac{1}{24}\left(\frac{k}{\mathcal{H}(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta,k)}$$

$\overline{\mathcal{P}_h(\eta,k)}$ ~ $\int\int f^2(u,v,x,x_R)$

$$f(u,v,\bar{x},x_{\rm R}) = \frac{3\left(2(5+3w)\Phi(u\bar{x})\Phi(v\bar{x}) + 4\mathcal{H}^{-1}(\Phi'(u\bar{x})\Phi(v\bar{x}) + \Phi(u\bar{x})\Phi'(v\bar{x})) + 4\mathcal{H}^{-2}\Phi'(u\bar{x})\Phi'(v\bar{x})\right)}{25(1+w)}$$

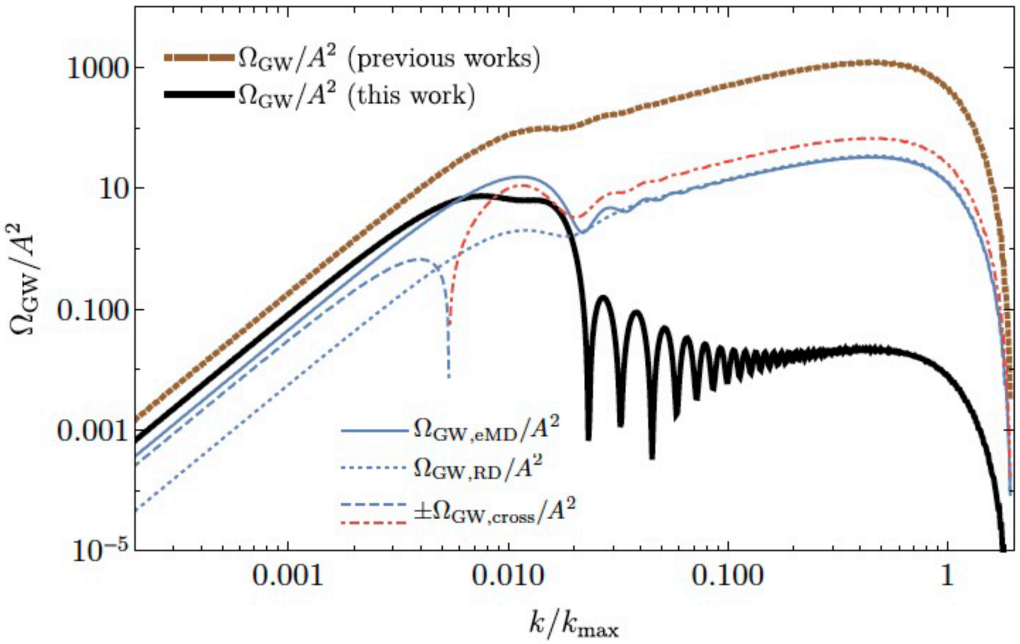
k-mode of gravitational potential $\Phi_k(t)$

$$\Phi_{\mathbf{k}} = \Phi(k\eta)\,\phi_{\mathbf{k}}$$

transfer function

primordial value $\phi_k = \Phi_k(t = t_{initial})$

$$\langle \phi_{\mathbf{k}}\phi_{\mathbf{k}'}\rangle = \delta(\mathbf{k}+\mathbf{k}')\frac{2\pi^2}{k^3}\left(\frac{3+3w}{5+3w}\right)^2 \mathcal{P}_\zeta(k)$$



The poltergeist mechanism

Inomata, Kohri, Nakama, Terada, arXiv:1904.12879
Inomata, Kawasaki, Mukaida, Terada, Yanagida, arXiv:2003.10455

Poltergeist
mechanism for gravitational wave production

Logo by ©Takahiro Terada

Sudden transition from eMD to RD by $\Gamma|_{t=\tau} \gg H$ (faster than exponential decay)

Inomata, Kohri, Nakama, Terada, Phys. Rev. D 100, 043532 (2019), arXiv:1904.12879

- Horizon-in during eMD, i.e., $k \gg (aH)_{\text{reheat}}$
- Induced GWs were produced during the RD
- Φ is dumping with rapid oscillation $\omega \sim k$ in
- Then, sudden decay ($\Gamma|_{t=\tau} \gg H$) gives a huge enhancement

$$\mathcal{H}^{-2}\Phi'\Phi' \gg \Phi^2$$

2^{nd} order GWs enhanced at a sudden transition from MD to RD

Inomata, Kohri, Nakama, Terada, Phys. Rev. D 100, 043532 (2019), arXiv:1904.12879

$\overline{\mathcal{P}_h(\eta,k)}$ ~ $\int\int f^2(u,v,x,x_R)$

$$f(u,v,\bar{x},x_{\rm R}) = \frac{3\left(2(5+3w)\Phi(u\bar{x})\Phi(v\bar{x}) + 4\mathcal{H}^{-1}(\Phi'(u\bar{x})\Phi(v\bar{x}) + \Phi(u\bar{x})\Phi'(v\bar{x})) + \boxed{4\mathcal{H}^{-2}\Phi'(u\bar{x})\Phi'(v\bar{x})}\right.}{25(1+w)}$$

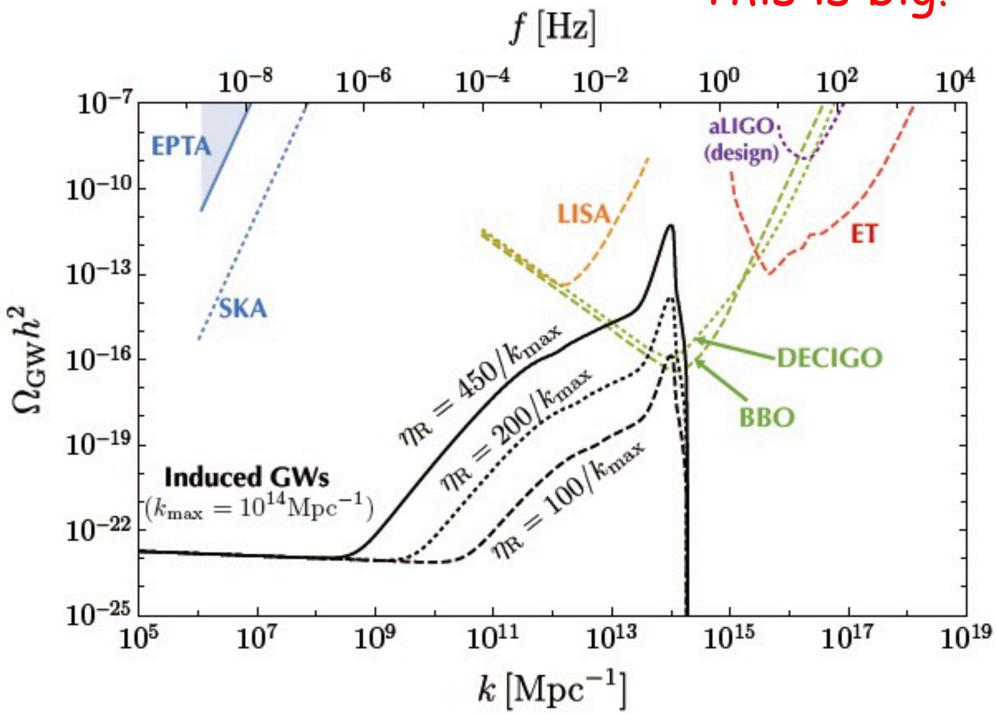
This is big!

- Enhancement at T_R

$$\mathcal{H}^{-2}\Phi'\Phi' \sim (k\eta_{\rm R})^2\Phi^2 \gg \Phi^2$$

Remarks:

- Amplitude should be less than unity
- The transition occurs in a finite time



Sudden decay from ϕ → 2χ only when M > 2 m_χ~√λ/2 τ

Inomata, Kohri, Nakama, Terada, Phys. Rev. D 100, 043532 (2019), arXiv:1904.12879

- Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}\partial^\mu\chi\partial_\mu\chi - \frac{1}{2}\partial^\mu\tau\partial_\mu\tau - V,$$
$$V = \frac{1}{2}M^2\phi^2 + \frac{1}{2}m^2\tau^2 + \frac{\lambda}{4}\tau^2\chi^2 + \frac{c}{2}M\phi\chi^2,$$

τ field ↘

- Effective mass of χ

$$m_{\chi,\mathrm{eff}}^2 = \langle\lambda\tau^2/2\rangle$$ ↘

- Decay rate

$$\Gamma = \frac{c^2 M}{32\pi}\sqrt{1 - \frac{m_{\chi,\mathrm{eff}}^2}{(M/2)^2}}\,\Theta\left(M^2 - 4m_{\chi,\mathrm{eff}}^2\right)$$

Γ ↗

Applications of this mechanism (The poltergeist mechanism)

Poltergeist

mechanism for gravitational wave production

Logo by ©Takahiro Terada

- Evaporating PBHs with their domination

Keisuke Inomata, Masahiro Kawasaki, Kyohei Mukaida, Takahiro Terada, Tsutomu T. Yanagida, arXiv:2003.10455 [astro-ph.CO]

- Poisson fluctuation of evaporating PBHs themselves with their domination

Guillem Domènech, Chunshan Lin, Misao Sasaki, arXiv:2012.08151 [gr-qc]

Guillem Domènech, Volodymyr Takhistov, Misao Sasaki, arXiv:2105.06816 [astro-ph.CO]

Extension from Theodoros Papanikolaou, Vincent Vennin, David Langlois, arXiv:2010.11573 [astro-ph.CO]

- Sudden decays of Q-balls

Graham White, Lauren Pearce, Daniel Vagie, Alex Kusenko, arXiv:2105.11655 [hep-ph]

- …

GWs produced in the early matter-dominated (MD) epoch by formation of halos, evaporation of halos, and turbulence after reheating

Tomohiro Nakama, Phys. Rev. D 101, 063519 (2020)
https://journals.aps.org/prd/abstract/10.1103/PhysRevD.101.063519

- Halo formation

$$\Omega_{\mathrm{GW,c}} = \frac{G^4 M^4}{c^7 R^5} T = \left(\frac{r_g}{2R}\right)^4 \frac{cT}{R}$$

See also Karsten Jedamzik (LPTA), Martin Lemoine (IAP), Jerome Martin (IAP), arXiv:1002.3278 [astro-ph.CO]

- Halo evaporation

$$\Omega_{\mathrm{GW,c}} \simeq \frac{9 \cdot 3^{5/6} \pi^{23/6} (\delta_c^{\mathrm{lin}})^{3/2} \sigma^{7/2}}{16 \cdot 2^{1/6} (\delta_m^{\mathrm{lin}})^5 (\delta_{\mathrm{vir}}^{\mathrm{nonlin}})^{1/2}} \simeq 12 \sigma^{7/2}.$$

- Turbulence

$$\Omega_{\mathrm{GW}} \sim 2 \times 10^{-7} \left(\frac{R}{c H_c^{-1}}\right)^3 \simeq 5 \times 10^{-10} \sigma^{3/2},$$

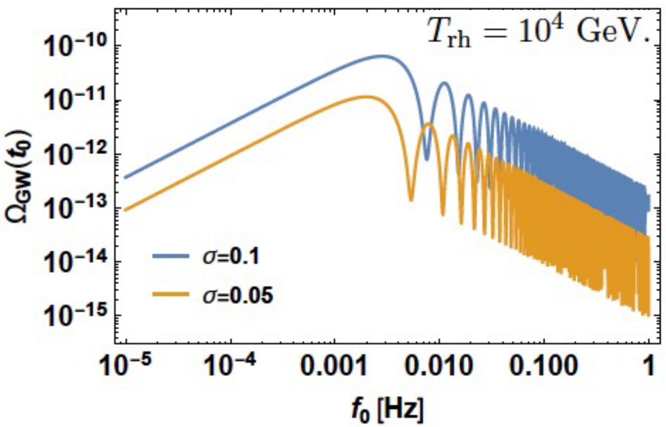
Gravitational Waves from evolving density Perturbations in an Early Matter Domination Era

Ioannis Dalianis, Chris Kouvaris, arXiv:2012.09255 [astro-ph.CO]

$$
\begin{aligned}
\Omega_{\rm GW}(t_0, f_0) = & \frac{1}{\rho_{\rm crit}} \int_0^\infty \int_{-\infty}^{\alpha} \int_{-\infty}^{\beta} d\alpha d\beta d\gamma \, \frac{4\pi G}{5c^5} \frac{2\pi f_0}{54\pi^2} M^2 \left(\frac{2\alpha\delta_{\rm L}}{\alpha+\beta+\gamma}\right)^2 \\
& \times \left[1 + \left(\frac{\beta}{\alpha}\right)^4 + \left(\frac{\gamma}{\alpha}\right)^4 - \left(\frac{\gamma}{\alpha}\right)^2 - \left(\frac{\beta}{\alpha}\right)^2 \left(1 + \left(\frac{\gamma}{\alpha}\right)^2\right)\right] \\
& \times \left| {\rm Ei}\left[\frac{1}{3}, i t_{\rm max} 2\pi f_0 (1+z_{\rm col})\right] - 2{\rm Ei}\left[\frac{1}{3}, i t_{\rm col} 2\pi f_0 (1+z_{\rm col})\right] \right|^2 \\
& \times \Theta\left(t_{\rm rh} - t_{\rm col}(\alpha,\beta,\gamma)\right) \left(\frac{4\pi}{3} q^3\right)^{-1} \mathcal{F}_{\rm D}(\alpha,\beta,\gamma).
\end{aligned}
$$

$$
I_{ij} = \frac{1}{5} M \begin{pmatrix} r_2^2 + r_3^2 & 0 & 0 \\ 0 & r_1^2 + r_3^2 & 0 \\ 0 & 0 & r_1^2 + r_2^2 \end{pmatrix}
$$

$$
Q_{ij} = -I_{ij}(t) + \frac{1}{3}\delta_{ij} {\rm Tr} I(t)
$$



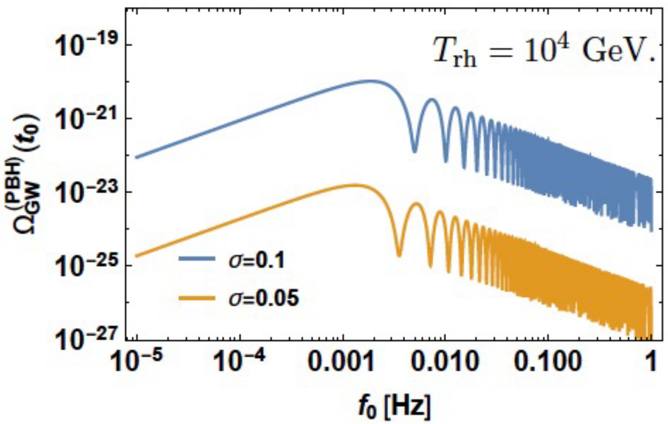
$$
\frac{dE_e}{dt} = \frac{G}{5c^5} \sum_{ij} \dddot{Q}_{ij}(t) \dddot{Q}_{ji}(t)
$$

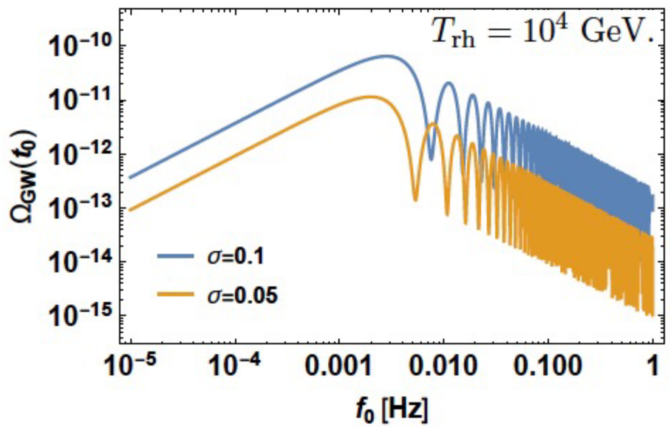
$$
\begin{aligned}
\Omega_{\rm GW}(t_0, f_0) = & \frac{1}{\rho_{\rm crit}(t_0)} \int_0^\infty \int_{-\infty}^{\alpha} \int_{-\infty}^{\beta} d\alpha d\beta d\gamma \sum_N \frac{1}{1+z_N} \frac{4\pi G}{5c^5} \sum_{ij} |\tilde{Q}_{ij}^N (2\pi f_0 (1+z_N))|^2 \\
& \times (2\pi f_0 (1+z_N))^7 \Theta\left(t_{\rm rh} - t_{\rm col}(\alpha,\beta,\gamma)\right) \left(\frac{4\pi}{3} q^3\right)^{-1} \mathcal{F}_{\rm D}(\alpha,\beta,\gamma).
\end{aligned}
$$

Gravitational Waves from Density Perturbations in an Early Matter Domination Era

Ioannis Dalianis, Chris Kouvaris, arXiv:2012.09255 [astro-ph.CO]

$$\Omega_{\rm GW}(t_0,f_0) = \frac{1}{\rho_{\rm crit}}\int_0^\infty\int_{-\infty}^{\alpha}\int_{-\infty}^{\beta} d\alpha d\beta d\gamma\,\frac{4\pi G}{5c^5}\frac{2\pi f_0}{54\pi^2}M^2\left(\frac{2\alpha\delta_{\rm L}}{\alpha+\beta+\gamma}\right)^2$$
$$\times\left[1+\left(\frac{\beta}{\alpha}\right)^4+\left(\frac{\gamma}{\alpha}\right)^4-\left(\frac{\gamma}{\alpha}\right)^2-\left(\frac{\beta}{\alpha}\right)^2\left(1+\left(\frac{\gamma}{\alpha}\right)^2\right)\right]$$
$$\times\left|{\rm Ei}\left[\frac{1}{3}, it_{\rm max}2\pi f_0(1+z_{\rm col})\right]-2{\rm Ei}\left[\frac{1}{3}, it_{\rm col}2\pi f_0(1+z_{\rm col})\right]\right|^2$$
$$\times\,\Theta\left(t_{\rm rh}-t_{\rm col}(\alpha,\beta,\gamma)\right)\left(\frac{4\pi}{3}q^3\right)^{-1}\mathcal{F}_{\rm D}(\alpha,\beta,\gamma).$$





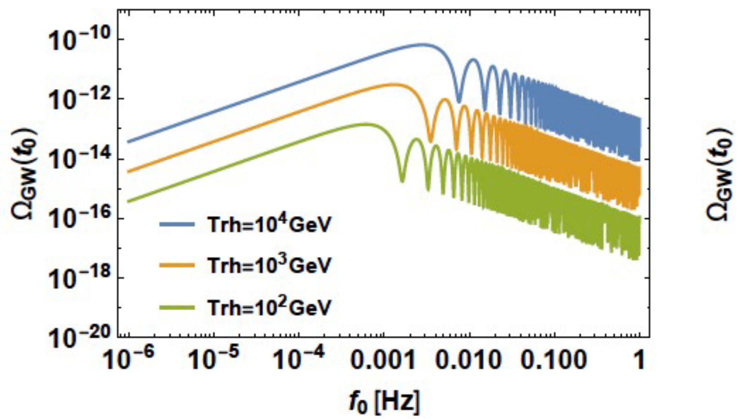


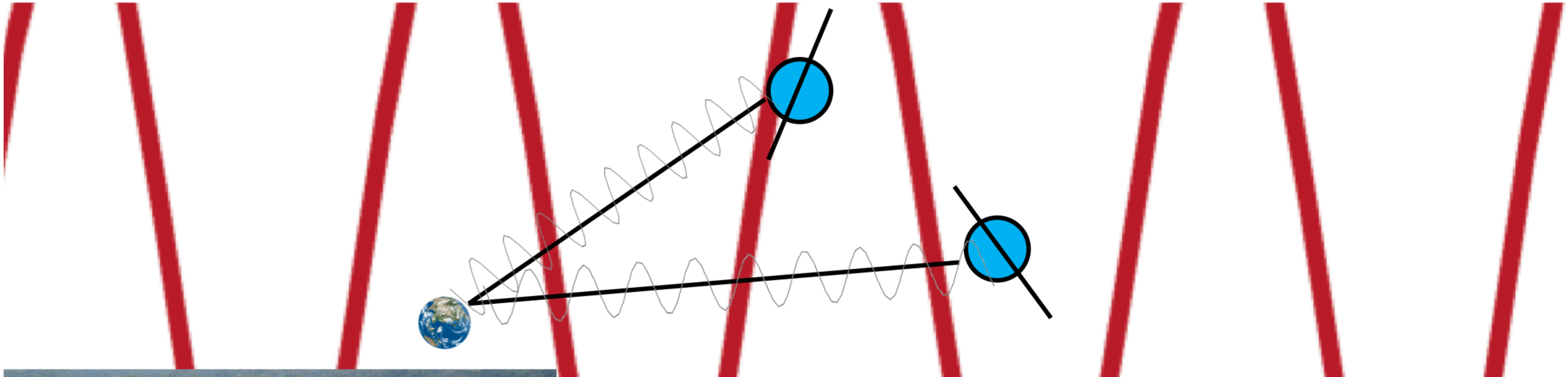
Figure 3: *Left panel.* The GW signal for $\sigma = 0.1$

Zaven Arzoumanian, et al, The NANOGrav Collaboration, arXiv:2009.04496

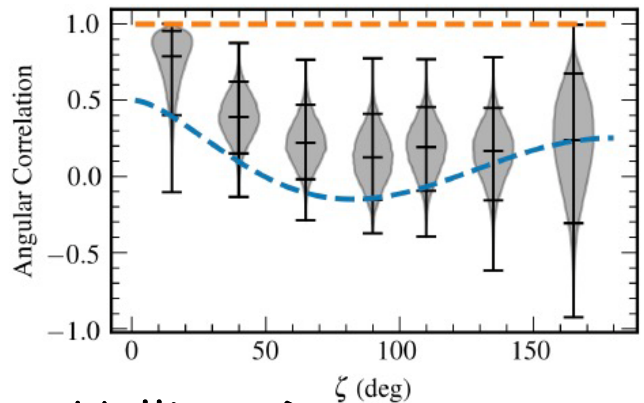
NANOGrav 12.5 yr

(North American Nanohertz Observatory for Gravitational Waves)

found stochastic GWs through pulsar timing ?



The 305-meter dish of the William E. Gordon Telescope, The Arecibo Obs.

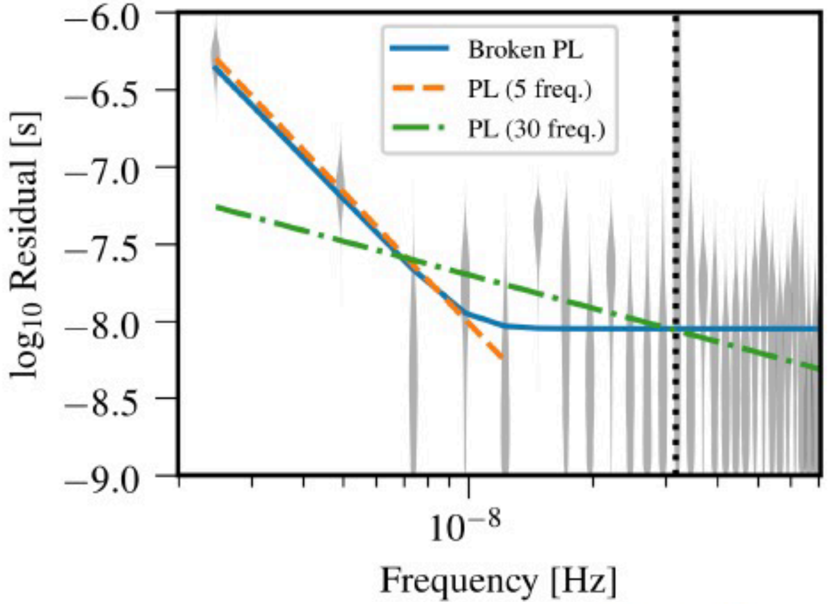


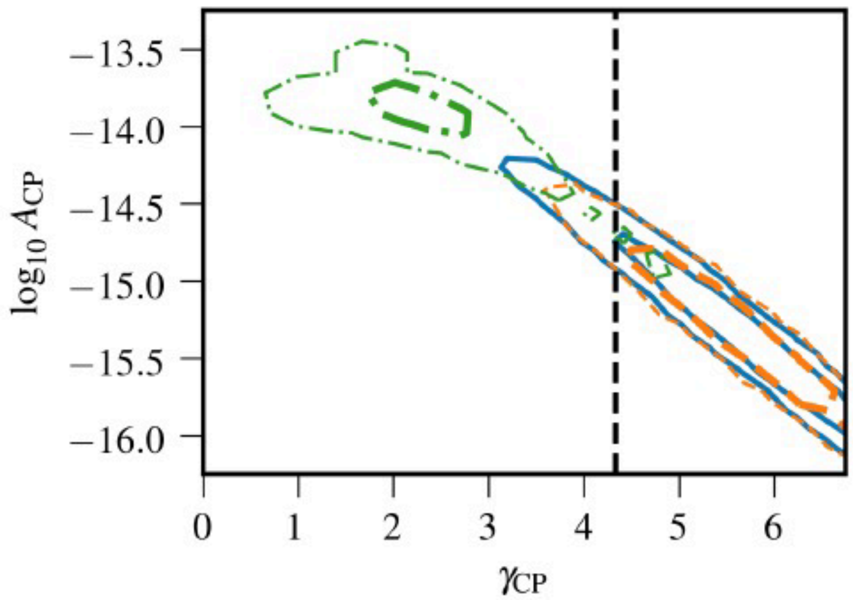
Hellings-Downs curve

The 100-meter Green Bank Telescope

The NANOGrav 12.5-year Pulsar-timing Data for An Isotropic Stochastic Gravitational-Wave Background

Zaven Arzoumanian, et al, The NANOGrav Collaboration, arXiv:2009.04496





$$h_c(f) = A_{\rm GWB}\left(\frac{f}{f_{\rm yr}}\right)^{\alpha}$$

$$\Omega(f) = \frac{2\pi}{3H_0^2} f^2 h_c(f)^2 = \Omega_{yr}\left(\frac{f}{f_{\rm yr}}\right)^{\beta}$$

$$\gamma = 3 - 2\alpha = 5 - \beta$$

$$\beta = 5 - \gamma$$

Models to fit NANOGrav 12.5 yr

1. Astrophysics

- Binary SMBHs

2. Early Universe

- 2ndary GWs from large ζ at small scales
- Cosmic strings produced by GUT phase transition
- 1st order phase transition induced by a new scalar
- …

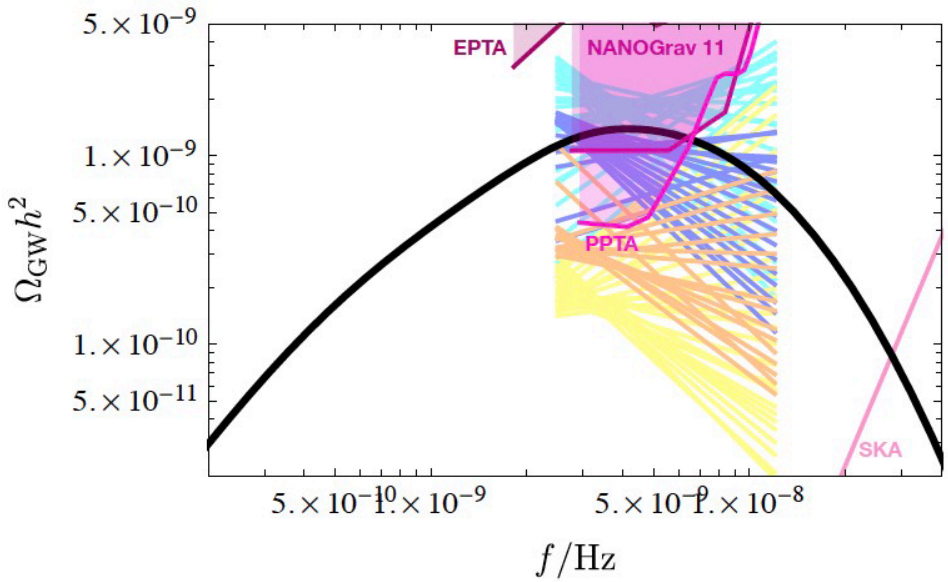
Secondary GWs from large curvature perturbation ζ at small scales

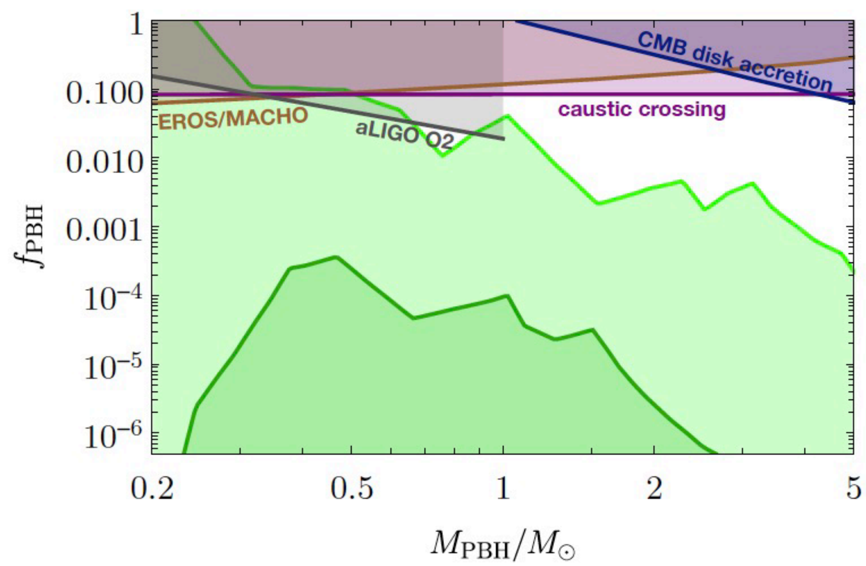
K. Kohri and T. Terada, arXiv:arXiv:2009.11853

Large P_ζ at $k \sim 10^7 \mathrm{Mpc}^{-1}$ produces both GWs at nHz and O(1) $M_\odot$ PBHs

R.Saito and J.Yokoyama, Phys. Rev. Lett. 102 (2009) 161101
K. Kohri and T. Terada, arXiv:arXiv:2009.11853

$$M_{\mathrm{PBH}} \sim O(1) M_\odot \left(\frac{k}{10^7 \mathrm{Mpc}^{-1}} \right)^{-2} \sim O(1) M_\odot \left(\frac{5 \times 10^{-9} \mathrm{Hz}}{f} \right)^2$$



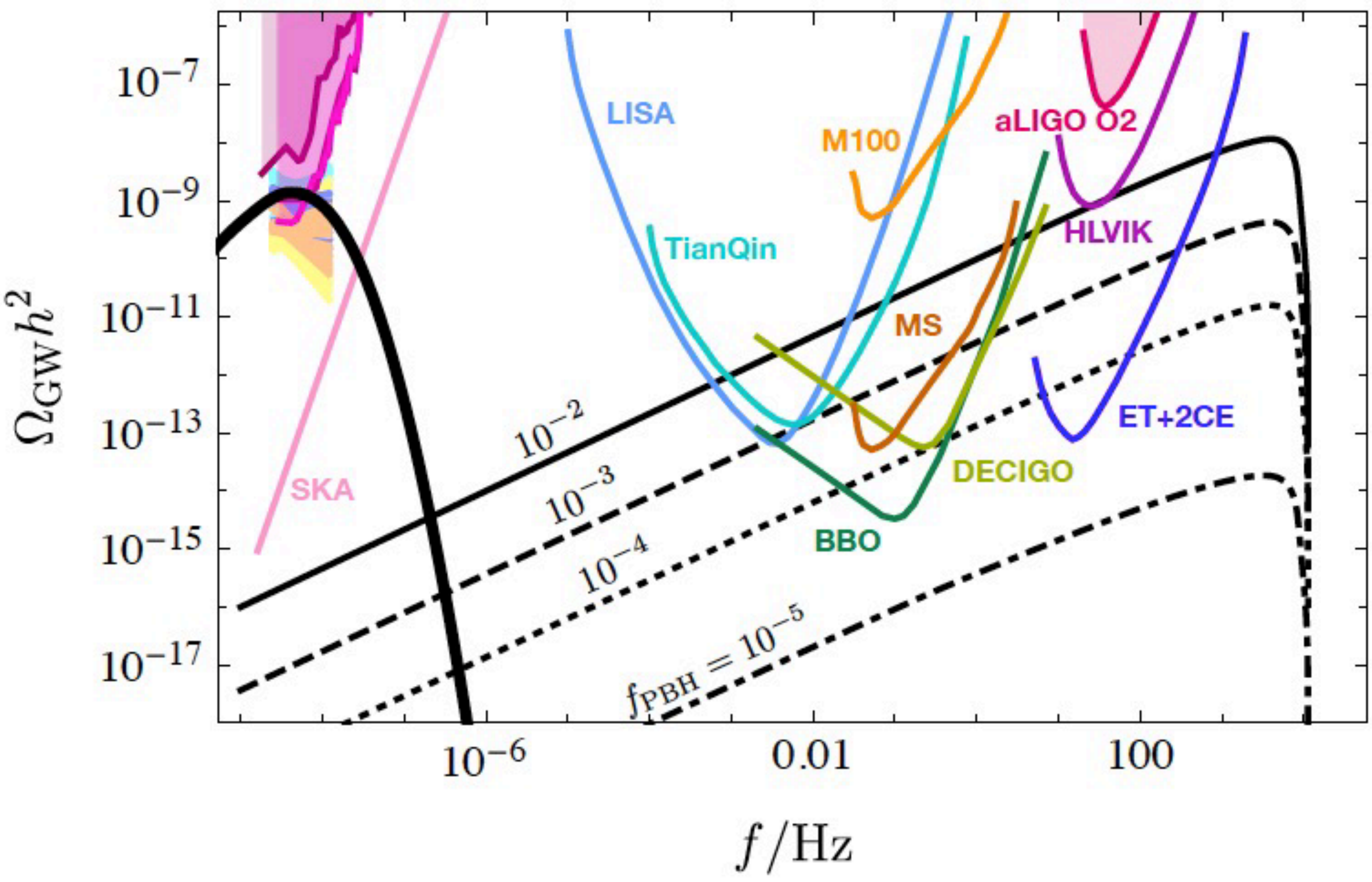


GWs at nHz

PBHs of O(1) $M_\odot$

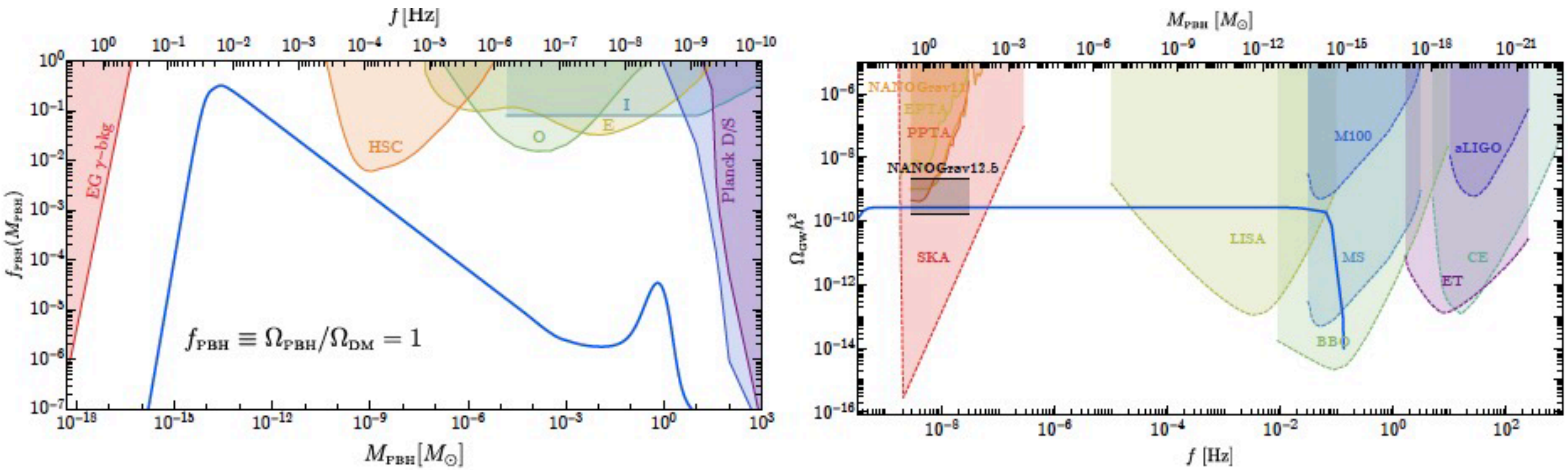
NANOGrav12.5yr and solar mass PBHs

K. Kohri and T. Terada, arXiv:arXiv:2009.11853



NANOGrav Hints to Primordial Black Holes as Dark Matter

V. De Luca, G. Franciolini, A. Riotto, arXiv:2009.08268 [astro-ph.CO]



$$\mathcal{P}_\zeta(k) \approx A_\zeta\,\Theta(k_s - k)\Theta(k - k_l), \qquad k_s \gg k_l \qquad (12)$$

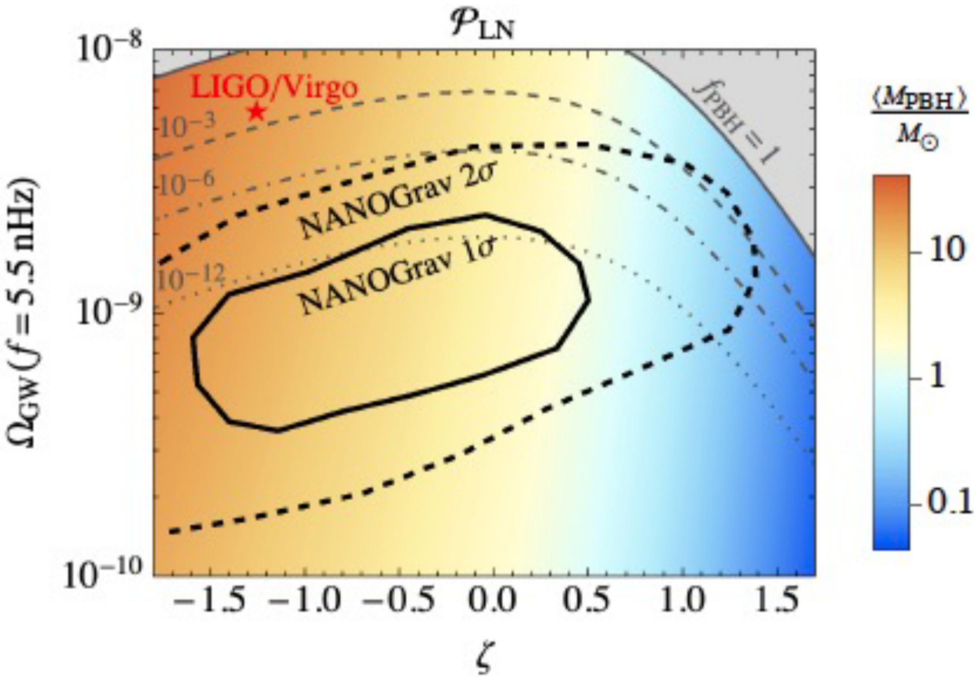
$$M_H \simeq 33\left(\frac{10^{-9}\,\mathrm{Hz}}{f}\right)^2 M_\odot.$$

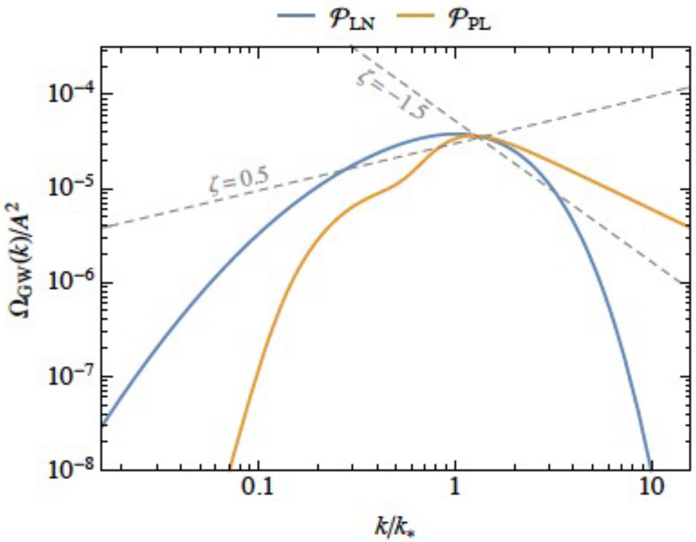
$$M_{\rm PBH} = \kappa M_H(\delta - \delta_c)^{\gamma_c}$$

PBHs seed for SMBHs and NANOGrav12.5yr

Ville Vaskonen and Hardi Veerae, arXiv:2009.07832v2

The potential GW signal can be fitted by a power-law $\Omega_{\rm GW} \propto f^{\zeta}$ with an amplitude $\Omega_{\rm GW}(f = 5.5\,{\rm nHz}) \in (3\times10^{-10}, 2\times10^{-9})$ and exponent $\zeta \in (-1.5, 0.5)$ at 1σ confidence level and with a small positive correlation between the amplitude and the exponent.

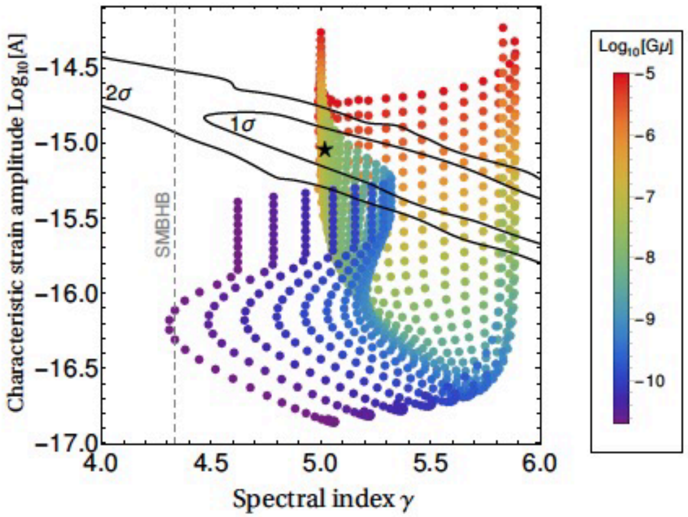


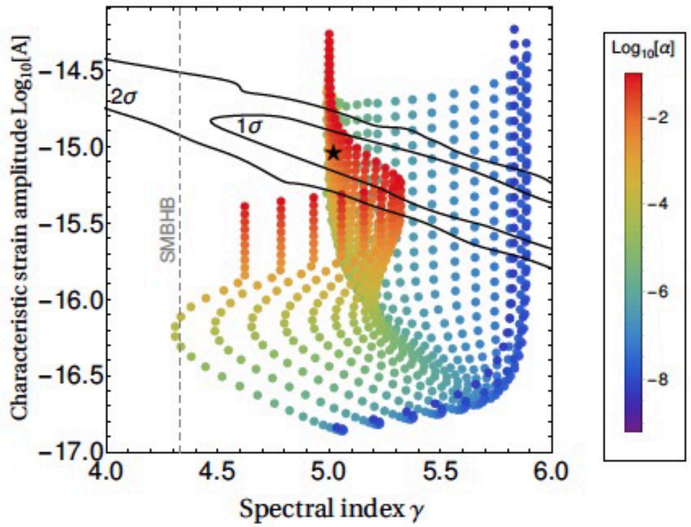


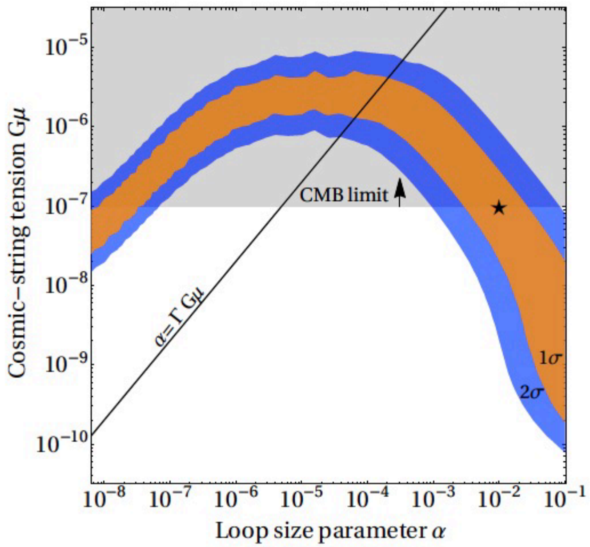
Note added: – Soon after the first version of this work, Ref. [74] claimed that the NANOGrav signal may be consistent with the LIGO/Virgo PBH scenario attributing the discrepancy between our conclusions to the choice of window function used in Eq. (9). We improved our PBH abundance estimate following [54] and found that our conclusions remain in tact. The differences between our conclusions and Ref. [74] can be resolved when we omit the non-linear relation between the density contrast and curvature perturbations. Additionally, Ref. [75] appeared pointing out a potential scenario for light PBH DM consistent with NANOGrav, which was not considered here.

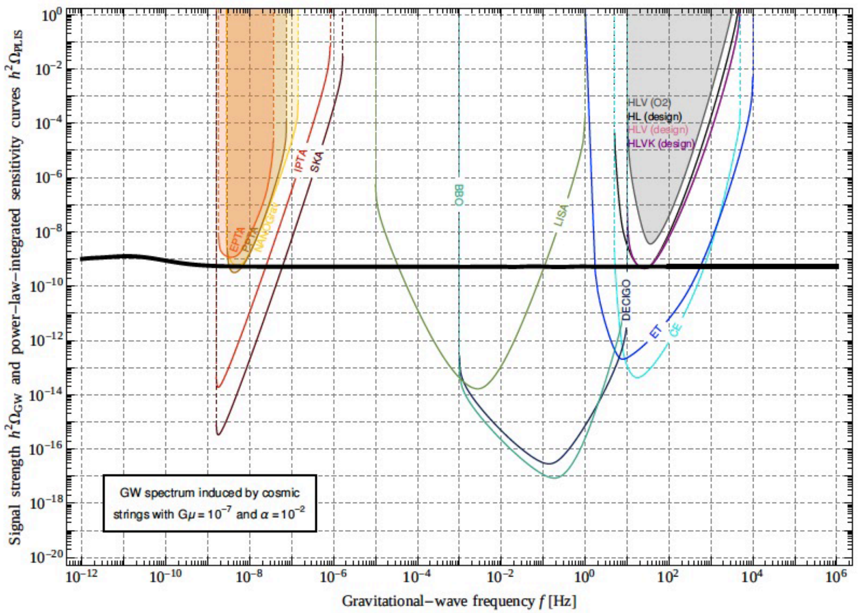
NanoGRAV found GWs from cosmic string

Simone Blasi, Vedran Brdar, Kai Schmitz, arXiv:2009.06607 [astro-ph.CO]



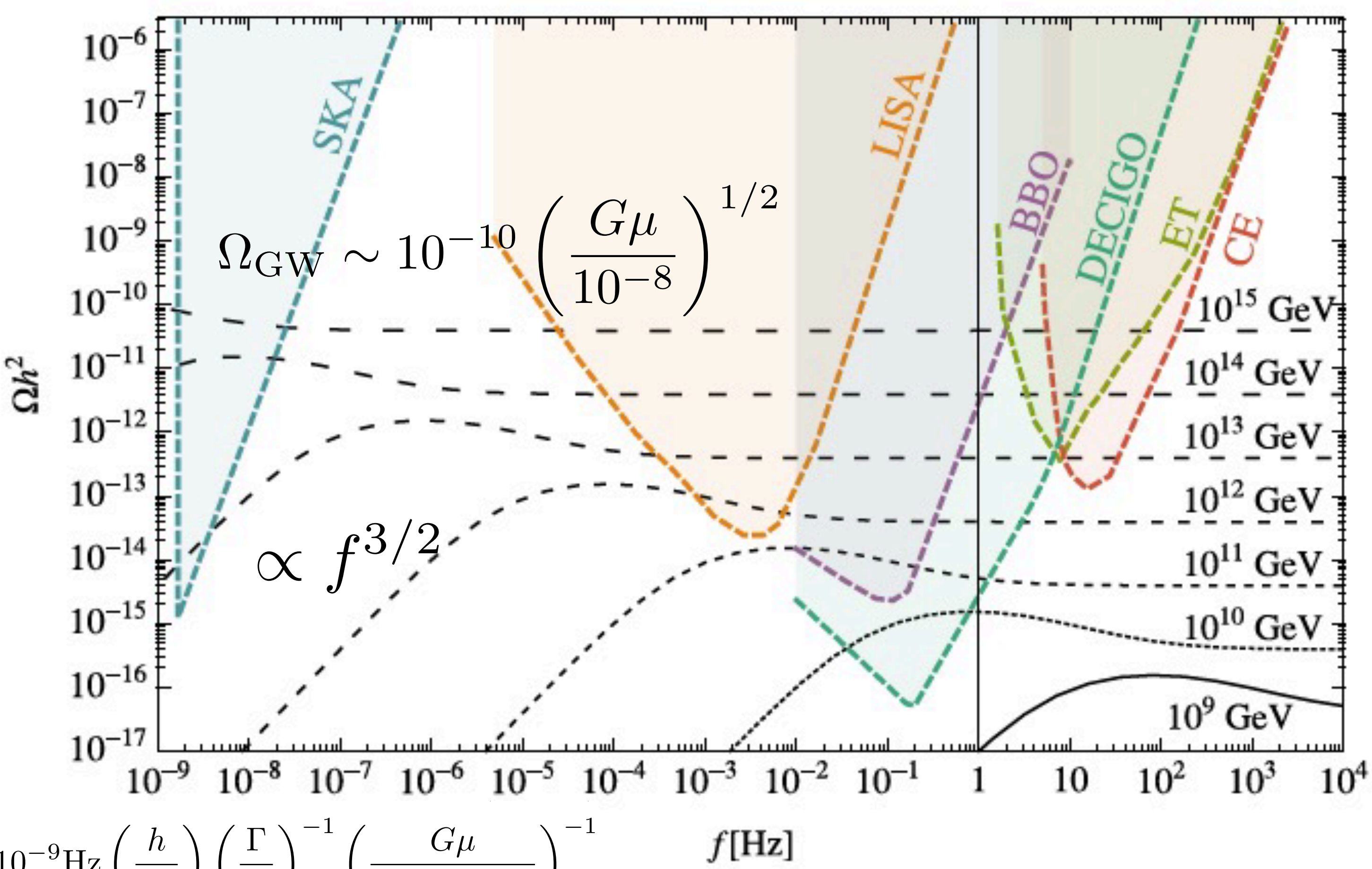






GW emitted from Cosmic string which was produced after GUT phase transition

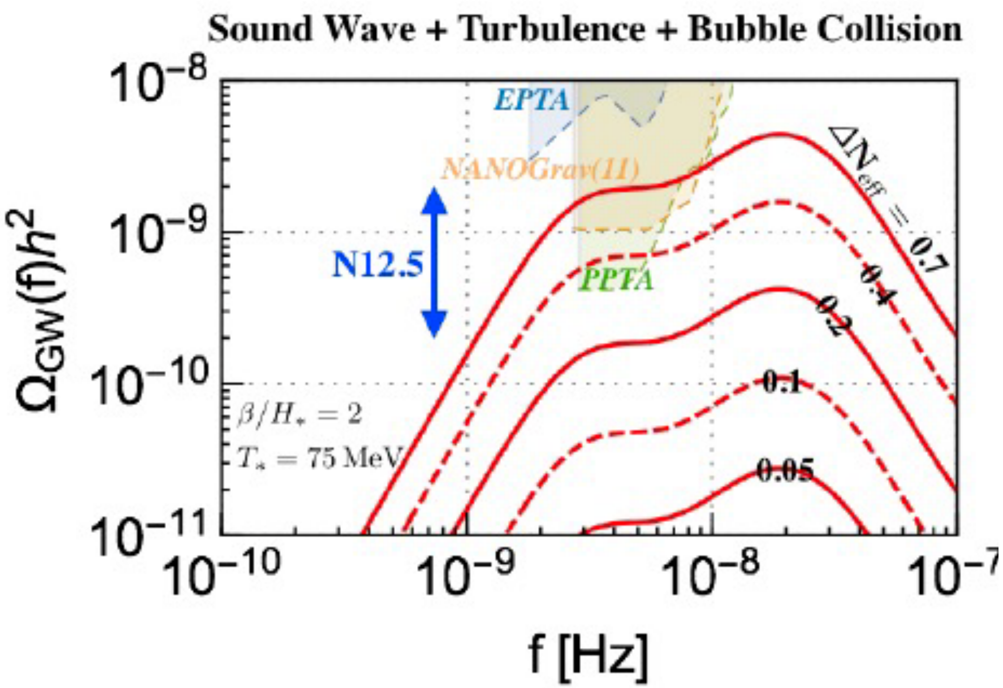
Jeff A. Dror, Takashi Hiramatsu, Kazunori Kohri, Hitoshi Murayama, Graham White, arXiv:1908.03227 [hep-ph]

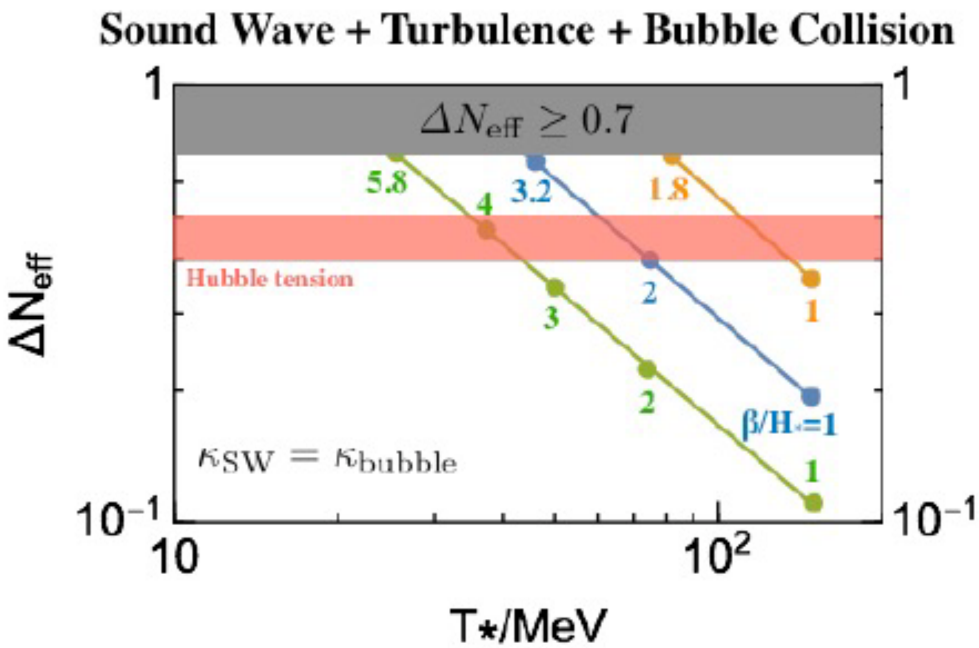


$$f_b \sim 0.7\times 10^{-9}\mathrm{Hz}\left(\frac{h}{0.7}\right)\left(\frac{\Gamma}{50}\right)^{-1}\left(\frac{G\mu}{0.7\times 10^{-10}}\right)^{-1}$$

Gravitational Waves and Dark Radiation from Dark Phase Transition: Connecting NANOGrav Pulsar Timing Data and Hubble Tension

Yuichiro Nakai, Motoo Suzuki, Fuminobu Takahashi, Masaki Yamada,
arXiv:2009.09754 [astro-ph.CO]





2) $N_\nu < 3 \rightarrow T_R \sim O(1)$ MeV

Kawasaki, Kohri, and Sugiyama, 1998; 2000
Ichikawa, Kawasaki, F. Takahashi, 2002
T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012

3) a_*~1→ PBHs produced with σ<<1 in the eMD

Harada, Yoo, KK, Nakao (2017)

Summary

We can confirm the existence of the early matter-dominated epoch by precisely-observing

1. Stochastic GW produced from inflation at the level of $\Omega_{GW} \sim 10^{-16}$ or secondary GW nonlinearly-induced through the poltergeist mechanism by large curvature perturbation at small scales

2. Effective number of neutrino species ($N_\nu < 3$)

3. Spin of the primordial black holes ($a_* \sim 1$)

2) MeV-scale reheating temperature

Kawasaki, Kohri, and Sugiyama, 1998; 2000
Ichikawa, Kawasaki, F. Takahashi, 2002
T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012

Freeze out of weak interaction between n and p at T ~ O(1) MeV

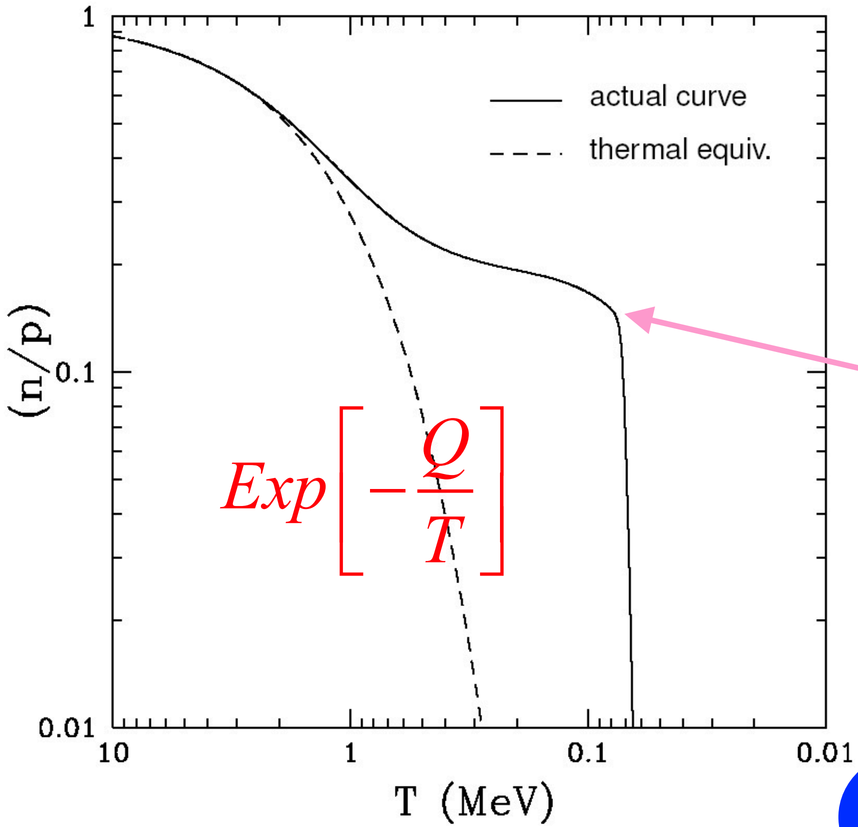
- Weak interaction between p and n

$$n \leftrightarrow p + e^- + \bar{\nu}_e \,,$$

$$e^+ + n \leftrightarrow p + \bar{\nu}_e \,,$$

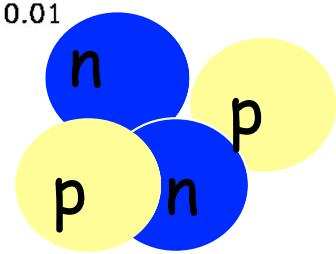
$$\nu_e + n \leftrightarrow p + e^- \,,$$

He4 mass fraction Y



$$\left(\frac{n_n}{n_p}\right)_{\text{freezeout}} \approx \frac{1}{7}$$

$^4\mathrm{He}$



$$n_{^4\mathrm{He}} = n_n / 2$$

$$Y_p \equiv \frac{\rho_{^4\mathrm{He}}}{\rho_B} \approx \frac{4 \times \cancel{m_N} \times n_{^4\mathrm{He}}}{\cancel{m_N} \times (n_n + n_p)} \approx \frac{2(n_n / n_p)_{\text{freezeout}}}{(n_n / n_p)_{\text{freezeout}} + 1} \approx 0.25$$

Helium 4 mass fraction Y

- Primordial value of mass fraction Y

$$Y_p \sim \frac{1}{4} + 0.01\Delta N_\nu + 0.01\ln(\eta_{10}/6) + 0.1\left(\frac{\tau_n - 880\,sec}{880\,sec}\right)$$

小玉、井岡、郡著 「宇宙物理学」(2014)

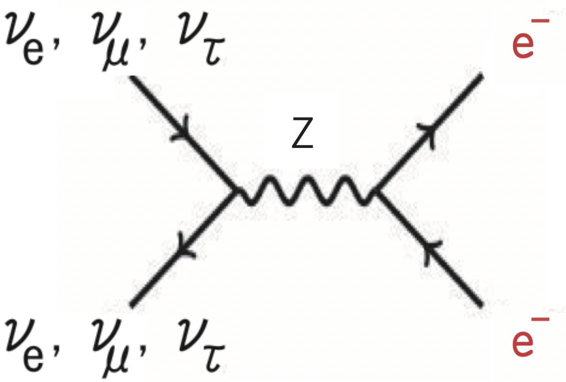
$$\Delta N_\nu = N_\nu - 3$$

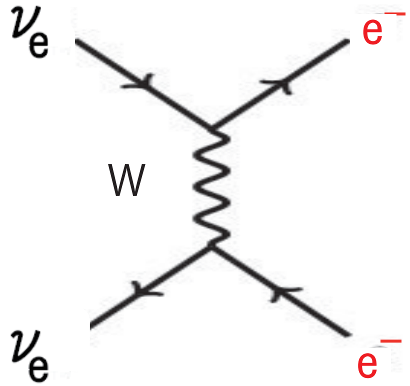
$$\eta = \frac{n_B}{n_\gamma} = \eta_{10} \times 10^{-10}$$: baryon to photon ratio

τ_n : neutron life time

Neutrino decoupling

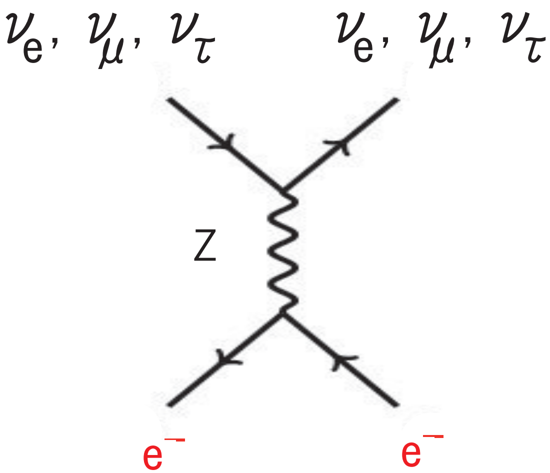
- Production and Annihilation ($\nu + \nu \leftrightarrow e^+ + e^-$)

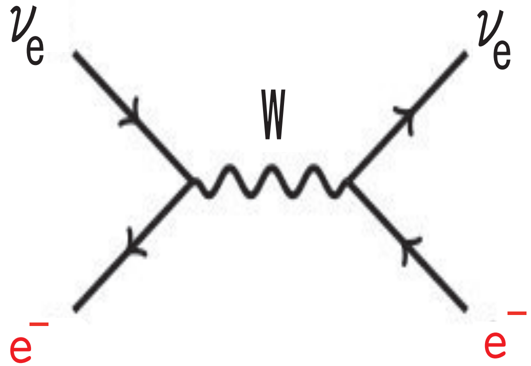




ν_e has stronger interactions with plasma through $W^\pm$

- Scattering ($\nu + e \leftrightarrow \nu + e$)



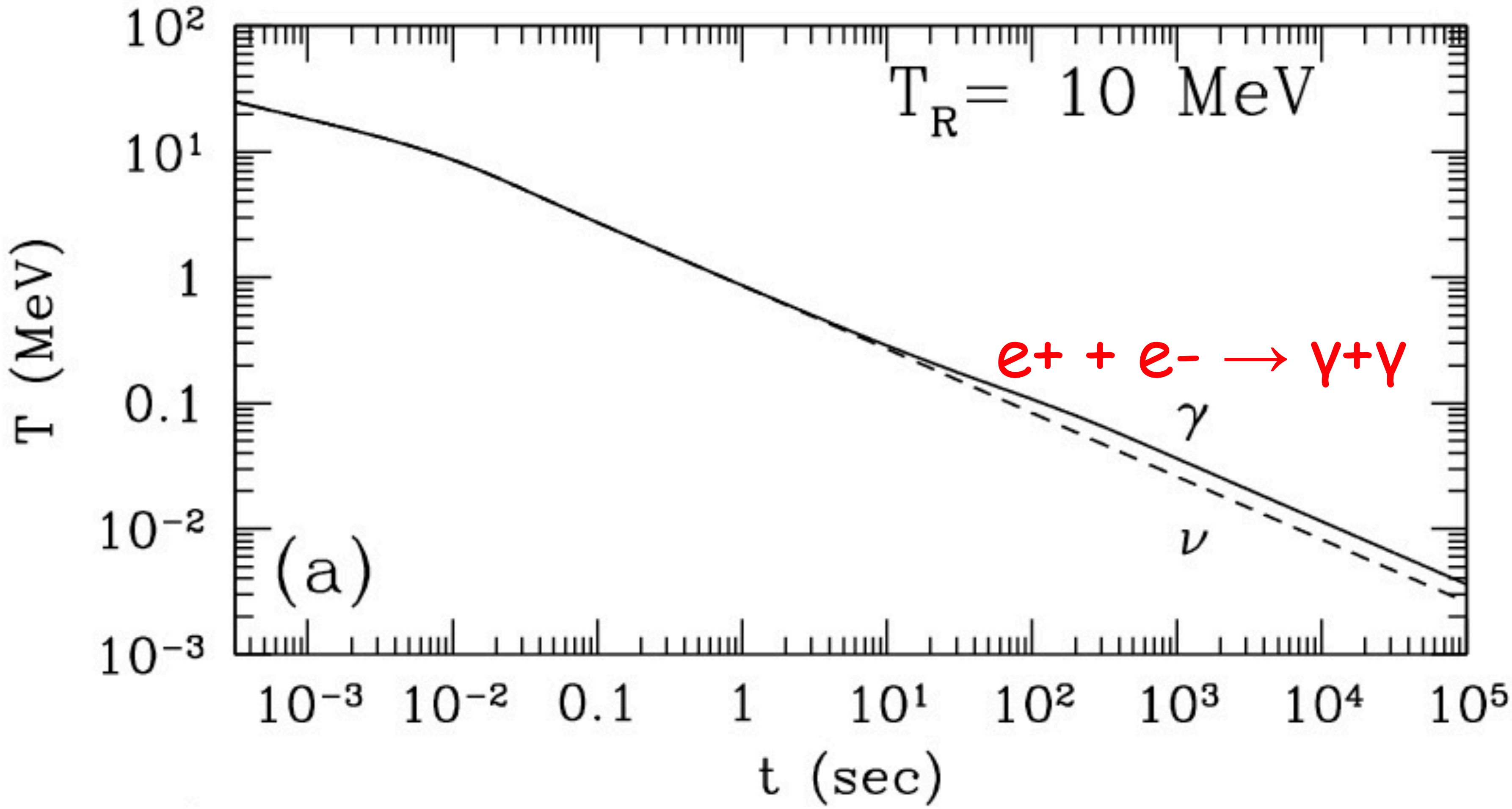


DECOUPLING:
3 MeV for ν_μ and ν_τ
2 MeV for ν_e

Time evolution of neutrinos

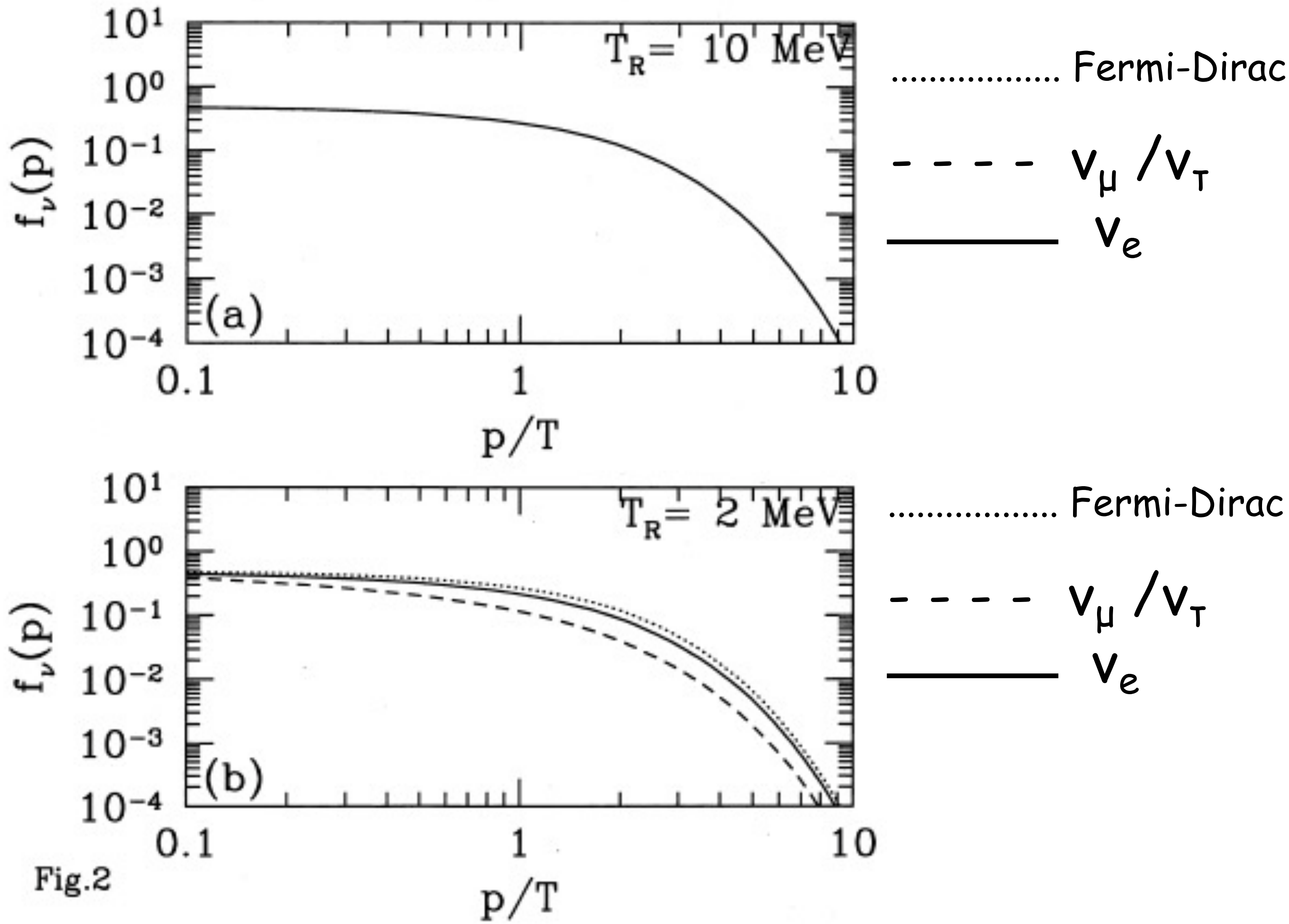
Kawasaki, Kohri, and Sugiyama, 1998; 2000

Only photons can be heated by e+e- annihilation at T = 0.511 MeV



Imperfect thermalization of neutrinos by MeV-scale reheating

Kawasaki, Kohri, and Sugiyama, 1998; 2000



Neutrino IMPERFECT thermalisation and Big Bang Nucleosynthesis

- Modifications on interaction rates due to MeV reheating or oscillations among $\nu \leftrightarrow \nu$ and/or ν_τ

$$\Gamma_{n\nu_e \to pe^-} = K \int_0^\infty dp_{\nu_e} \left[\sqrt{(p_{\nu_e}+Q)^2 - m_e^2}\,(p_{\nu_e}+Q) \frac{p_{\nu_e}^2}{1+e^{-(p_{\nu_e}+Q)/T_\gamma}} f_{\nu_e}(p_{\nu_e}) \right]$$

$$\Delta\Gamma_{n\leftrightarrow p} < 0$$

$$\boxed{\Delta Y \simeq +0.19(-\Delta\Gamma_{n\leftrightarrow p}/\Gamma_{n\leftrightarrow p})}$$

- Modifications on energy density

$$N_\nu^{\rm eff} \equiv \frac{\rho_{\nu_e} + \rho_{\nu_\mu} + \rho_{\nu_\tau}}{\rho_{\rm STD}} < 3 \qquad \to \Delta\rho_{\rm tot} < 0$$

$$\boxed{\Delta Y \simeq -0.10\,(-\Delta\rho_{\rm tot}\,/\rho_{\rm tot}\,)}$$

Neutrino oscillations

- Vacuum oscillation

$$P(\nu_i \to \nu_j) = \sin^2 2\theta_{ij} \sin^2\left[\frac{L\delta m_{ij}^2}{4E}\right]$$

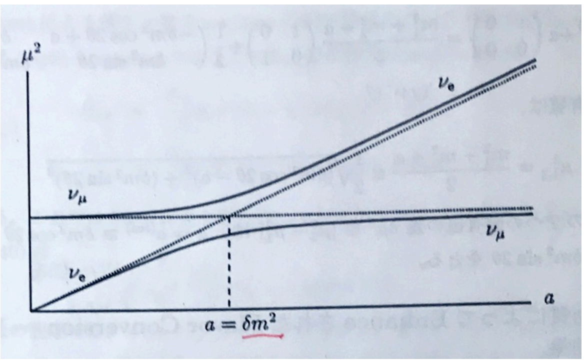
$\delta m_{ij}^2 = m_j^2 - m_i^2$

L : distance

E: energy

θ_{ij} : mixing angle

- MSW (matter effect)



$$P(\nu_i \to \nu_j) = 1 - \exp\left[-\pi \frac{\sin^2 2\theta_{ij}}{\cos 2\theta_{ij}} \frac{\delta m_{ij}^2}{4E} \frac{dt}{d\log n_e}\right]$$

$a = 2\sqrt{2}G_F n_e E$

n_e : electron #density

dt : time derivative

Neutrino oscillation in the early Universe

Quantum Kinetic Equation

$$\frac{d\varrho_p}{dt} = \frac{\partial\varrho_p}{\partial t} - Hp\frac{\partial\varrho_p}{\partial p} = \boxed{-i\left[\mathcal{H}_p, \varrho_p\right]} \boxed{+ C\left(\varrho_p\right)}$$

ν oscillation $\quad$ ν production/collision

density matrix for ν (2 flavor) ν_e - ν_x $\quad$ $e^- + e^+ \rightarrow \nu_\alpha + \nu_\alpha$

$$\varrho_p = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{ex}^* & \rho_{xx} \end{pmatrix}$$

diagonal: ν distribution

off-diagonal: flavor coherence

$$\mathcal{H}_p = \boxed{\frac{\mathrm{M}^2}{2p}} - \frac{8\sqrt{2}G_{\mathrm{F}}p}{3}\left[\frac{\boldsymbol{E}_l}{m_W^2} + \frac{\boldsymbol{E}_\nu}{m_Z^2}\right]$$

matter effect

$$\mathrm{M}^2 = U\mathcal{M}^2U^\dagger \quad \mathcal{M}^2 = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \quad U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\boldsymbol{E}_l \sim \mathrm{diag}(\rho_e, 0) \quad \boldsymbol{E}_\nu \sim \mathrm{diag}(\rho_{\nu_e}, \rho_{\nu_x})$$

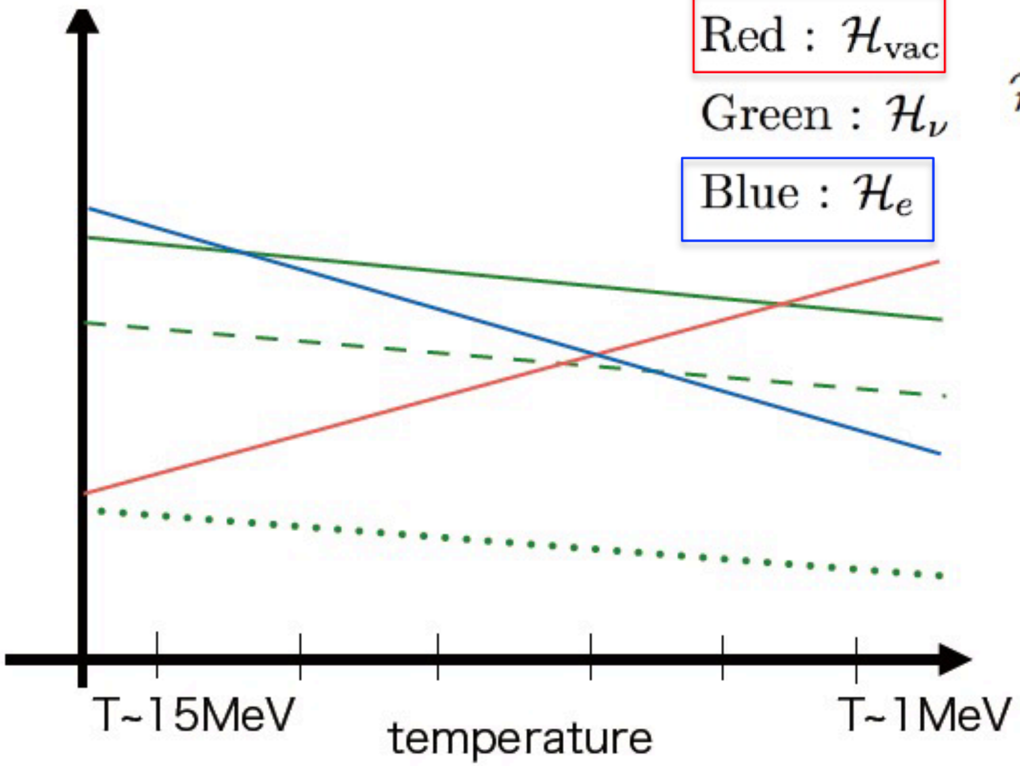
MSW-like effective mass difference in the early Universe

$$\frac{\delta m_M^2}{2p} = \sqrt{\left(\frac{\delta m^2}{2p}\right)^2 \sin^2 2\theta + (\mathcal{H}_{\rm vac} + \mathcal{H}_{\rm mat})^2}$$

$$\sin^2 2\theta_M = \frac{\left(\frac{\delta m^2}{2p}\right)^2 \sin^2 2\theta}{\left(\frac{\delta m^2}{2p}\right)^2 \sin^2 2\theta + (\mathcal{H}_{\rm vac} + \mathcal{H}_{\rm mat})^2}$$

MSW-type resonance (oscillation) in the early Universe

$\log_{10}\mathcal{H}_\alpha \;\; (\alpha = \mathrm{vac}, e, \nu)$



$$\mathcal{H} = \boxed{\mathcal{H}_{\rm vac}} + \boxed{\mathcal{H}_e} + \mathcal{H}_\nu$$
$$= \boxed{\frac{\delta m^2}{2E}\mathrm{B}} - \boxed{\frac{8\sqrt{2}G_F E \varrho_{e^\pm}}{3m_W^2}\mathrm{L}}$$
$$+ \frac{\sqrt{2}G_F}{2\pi^2}\int dE' E'^2 \left[\rho(E') - \bar{\rho}^*(E')\right]$$

$$\mathrm{B} = \mathrm{U}\left(\mathrm{diag}\left[-1/2, 1/2\right]\right)\mathrm{U}^\dagger, \mathrm{L} = \mathrm{diag}\left[1, 0\right]$$

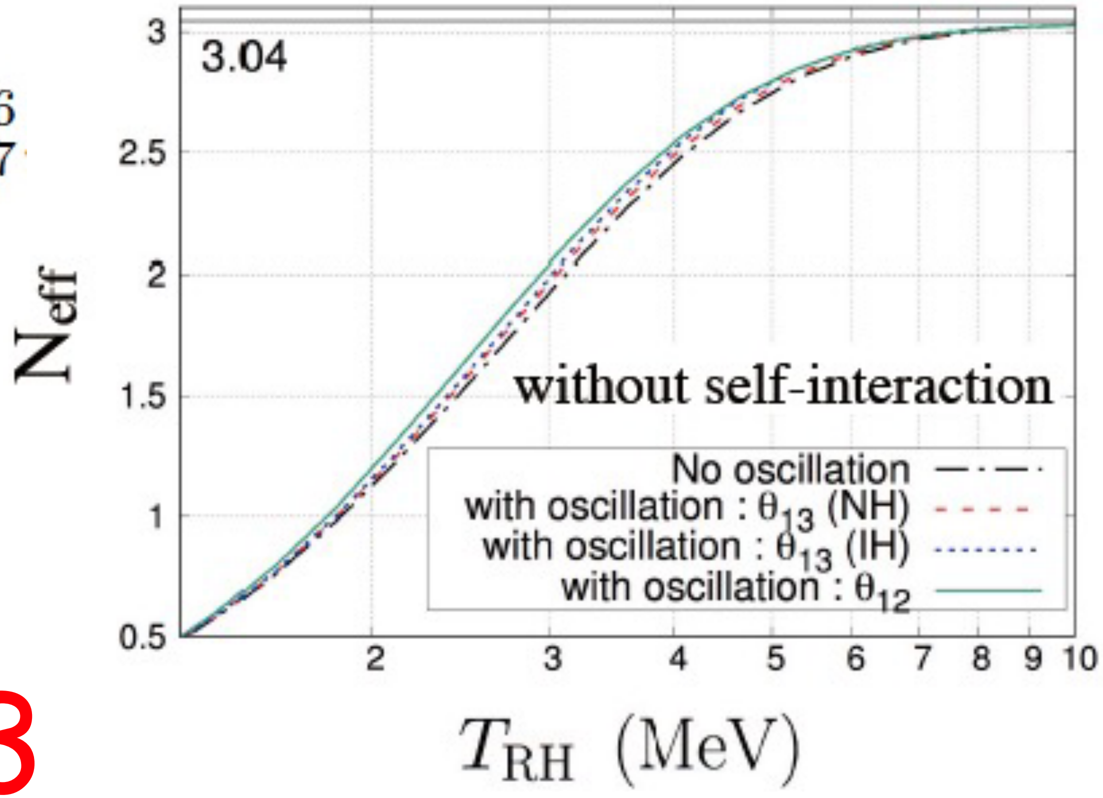
$$|\mathcal{H}_{\rm mat}| \approx |\mathcal{H}_{\rm vac}| \;\Rightarrow$$
$$T_c \approx G_F^{-1/3}(\delta m^2 \cos 2\theta)^{1/6} \approx \begin{cases} 3\,\mathrm{MeV}\left(\dfrac{\delta m_{12}^2}{2.5\times10^{-3}\,\mathrm{eV}^2}\right)^{1/6} \\ 5\,\mathrm{MeV}\left(\dfrac{\delta m_{13}^2}{7.5\times10^{-5}\,\mathrm{eV}^2}\right)^{1/6} \end{cases}$$

Thermalization of three active neutrinos

T. Hasegawa, Hiroshima, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012

See Planck 2018

$$N_\nu^{\rm eff} = 2.92^{+0.36}_{-0.37}$$



Nν < 3

$T_{\rm RH}$ ↘ $N_{\rm eff}$ ↘

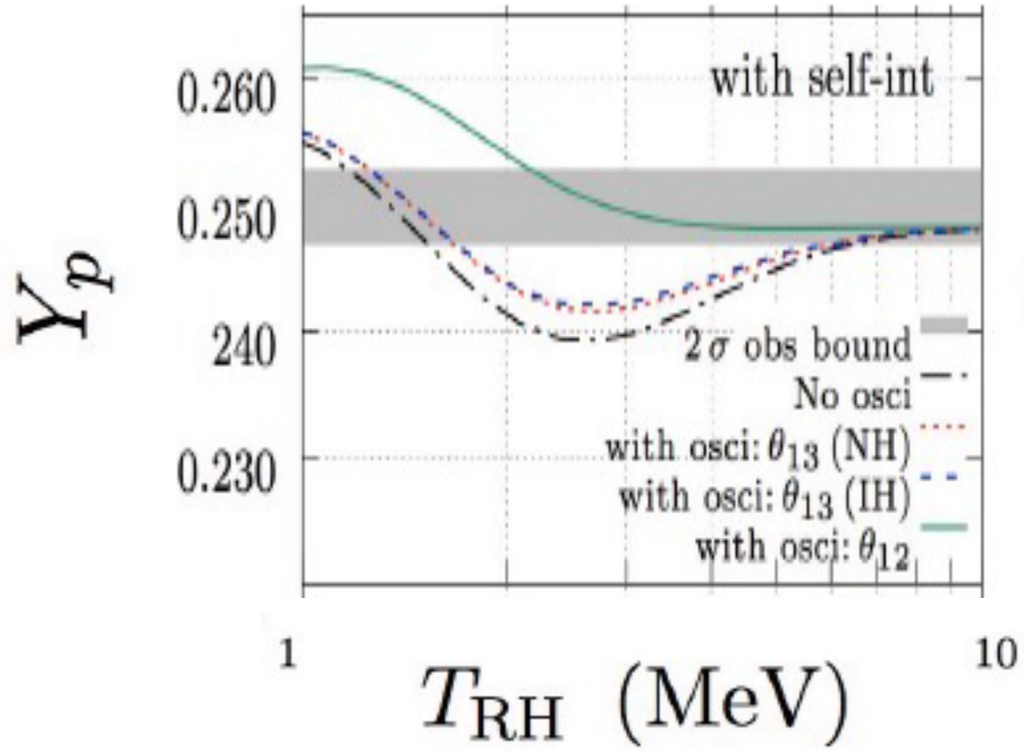
with ν oscillation or self-int. ➡ $N_{\rm eff}$ ↗

Observational Helium 4 abundance Yp

$$Y_p = 0.2449 \pm 0.0040 \quad (68\% \text{ C.L.})$$

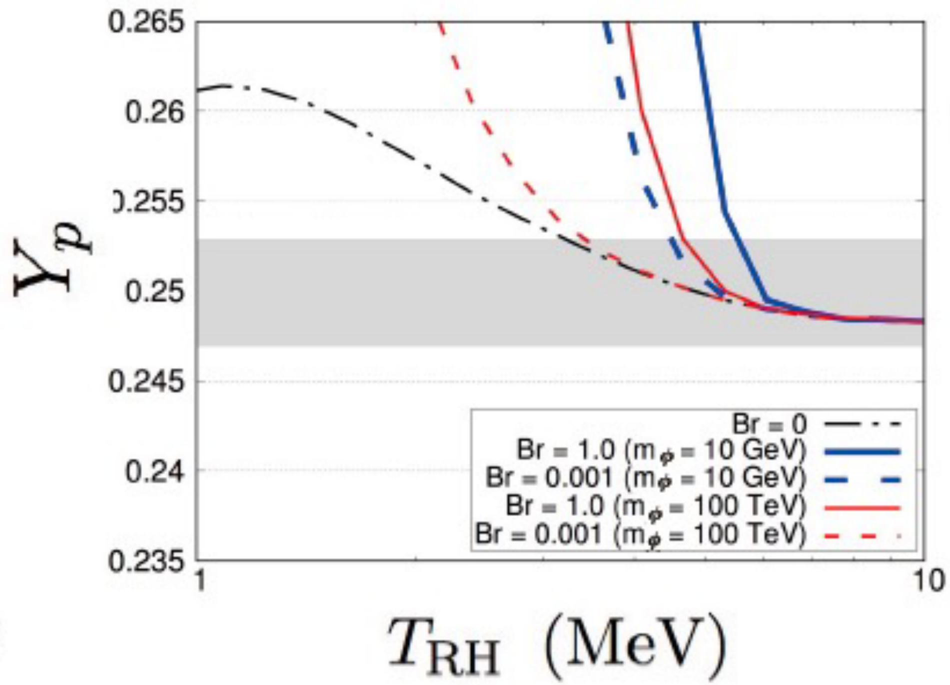
T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012

Radiative decay



$\rightarrow \gamma, e^{\pm} \rightarrow \nu$'s

Hadronic decay



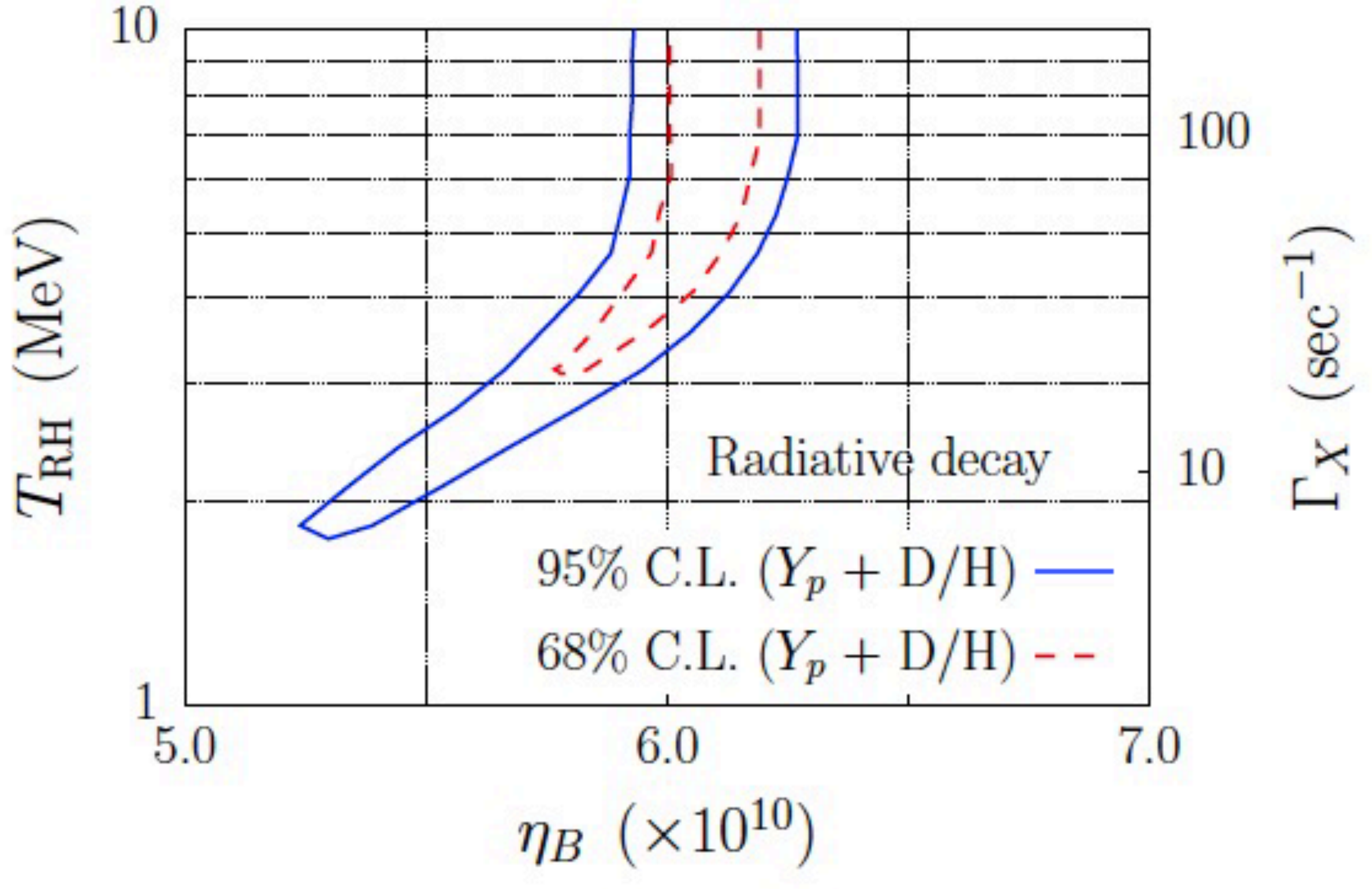
$\rightarrow$qq, gluon$\rightarrow \pi$'s

$$n + \pi^+ \rightarrow p + \pi^0$$
$$p + \pi^- \rightarrow n + \pi^0$$

n/p ↗

Lower bound on T_{RH} for radiative decay

T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012

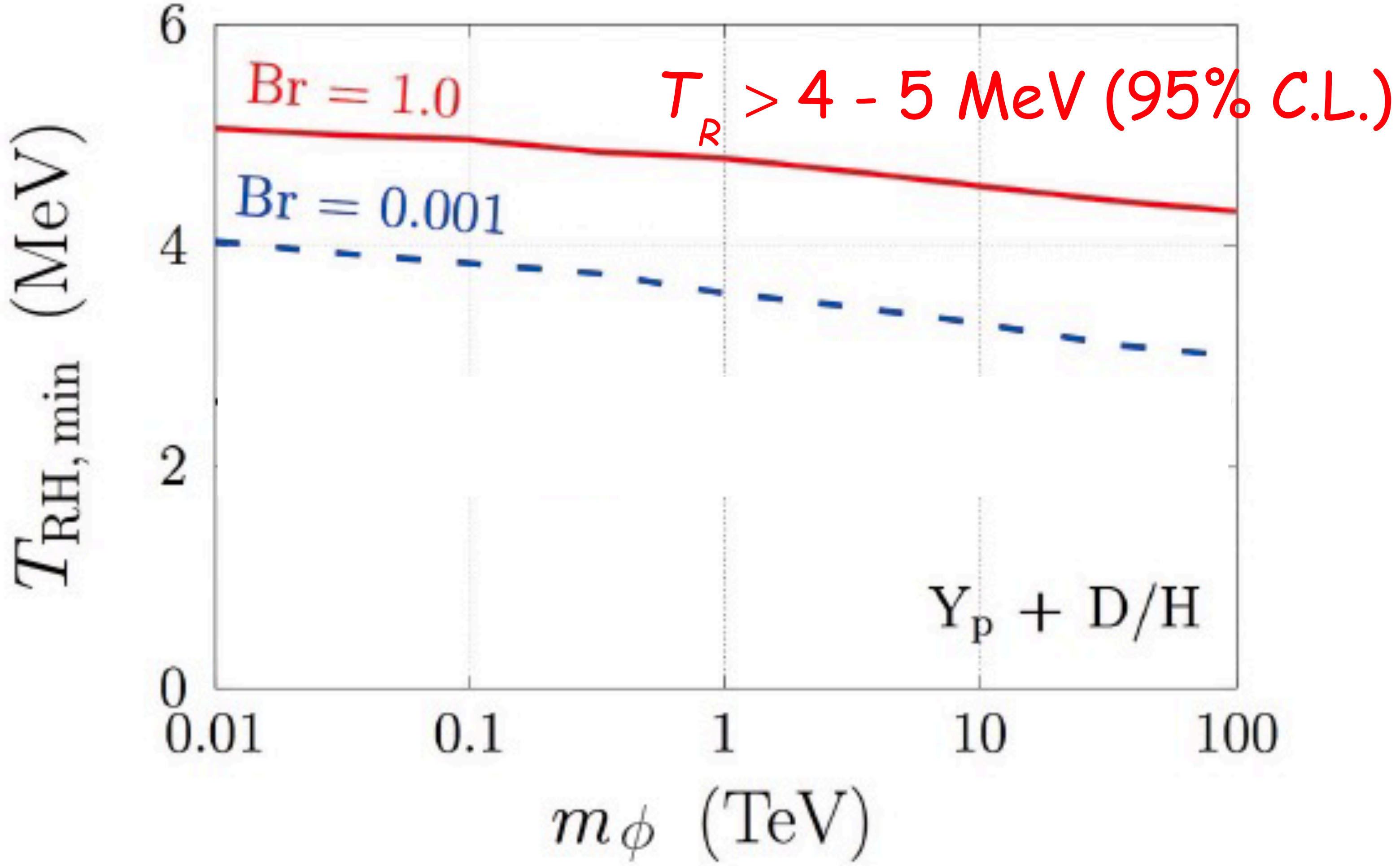


$$T_R > 1.8 \text{ MeV (95\% C.L.)}$$

$\eta_B = \frac{n_B}{n_\gamma} = \eta_{10} \times 10^{-10}$: baryon to photon ratio

Lower bounds on T_{RH} for hadronic decay

T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012



Lower bounds on Reheating temperature

T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012

- Radiative decay

$$T_R > 1.8 \text{ MeV } (95\% \text{ C.L.})$$

ΔNnu <~ -2

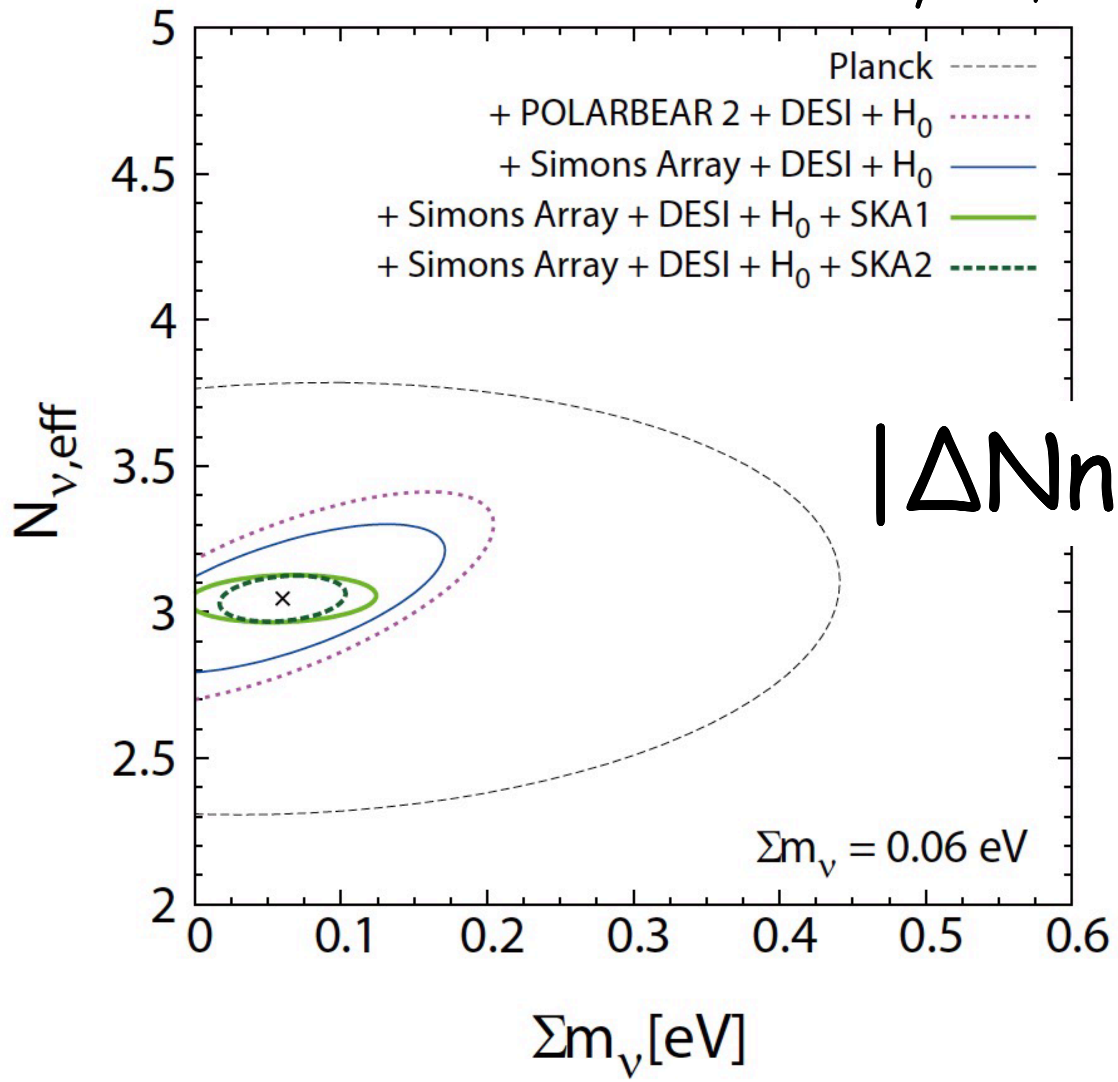
- Hadronic decay ($B_H = 1$)

$$T_R > 4 - 5 \text{ MeV } (95\% \text{ C.L.})$$

ΔNnu <~ -0.3

Future constraints on neutrino species and mass by 21cm, CMB, and BAO

Oyama, Kohri, Hazumi (2015)



$|\Delta N\mathrm{nu}| < 0.05$

From CMB with He4 only for radiative decay

P.F. de Salas, M. Lattanzi, G. Mangano, G. Miele, S. Pastor, O. Pisanti, arXiv:1511.00672

$$T_R > 4.7\mathrm{MeV}(95\%\mathrm{C.L.})$$

2) $N_\nu < 3 \rightarrow T_R \sim O(1)$ MeV

Kawasaki, Kohri, and Sugiyama, 1998; 2000
Ichikawa, Kawasaki, F. Takahashi, 2002
T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012

3) Formation of PBHs in the early Matter Dominated epoch

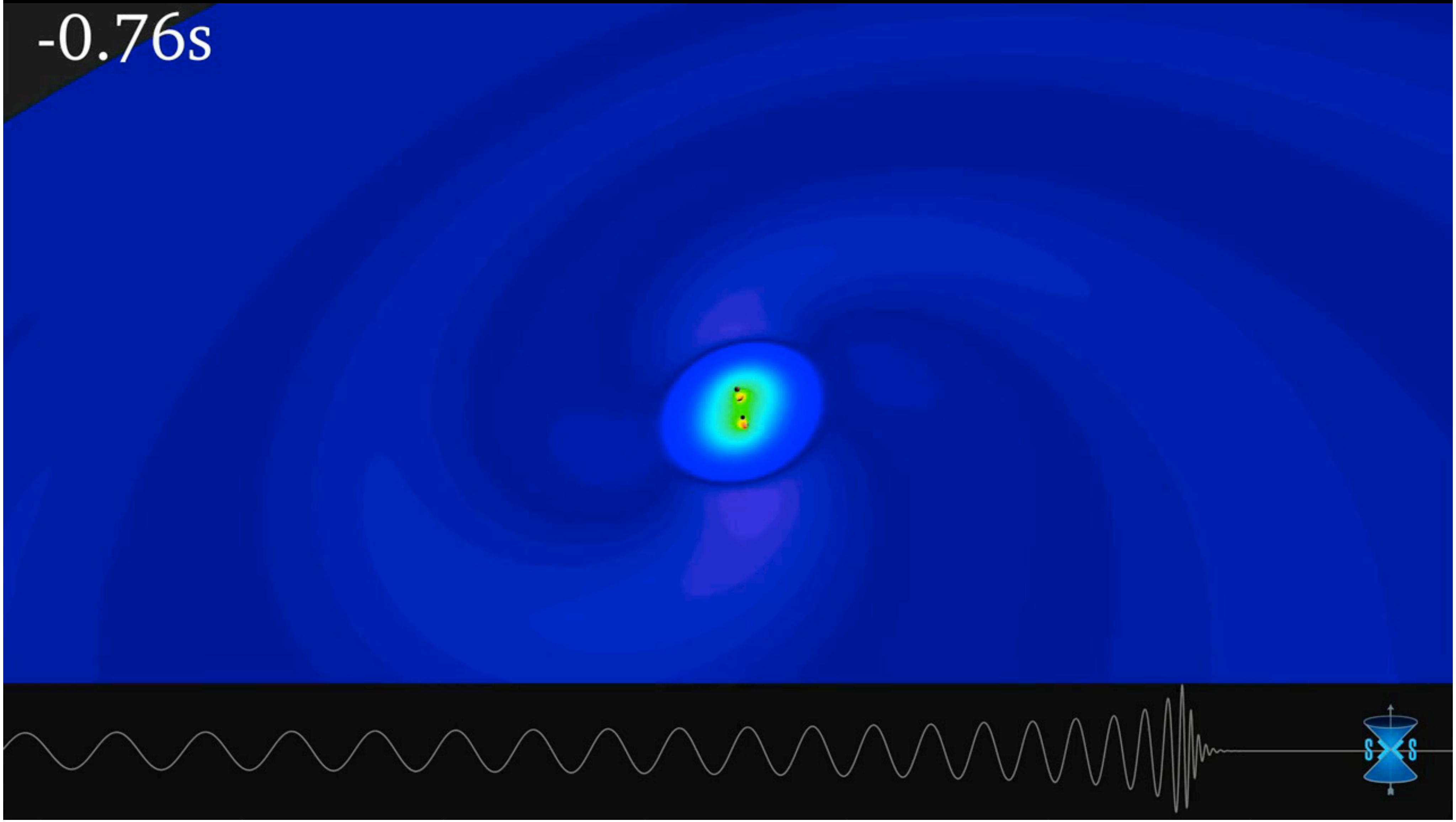
Harada, Yoo, KK, Nakao (2017)

Why PBHs?

- We can probe high-energy physics, the early Universe, and gravity with PBHs through recent and future gravitational wave observations

LIGO and Virgo have detected gravitational wave signals from Binary Black Holes

https://www.youtube.com/watch?v=1agm33iEAuo

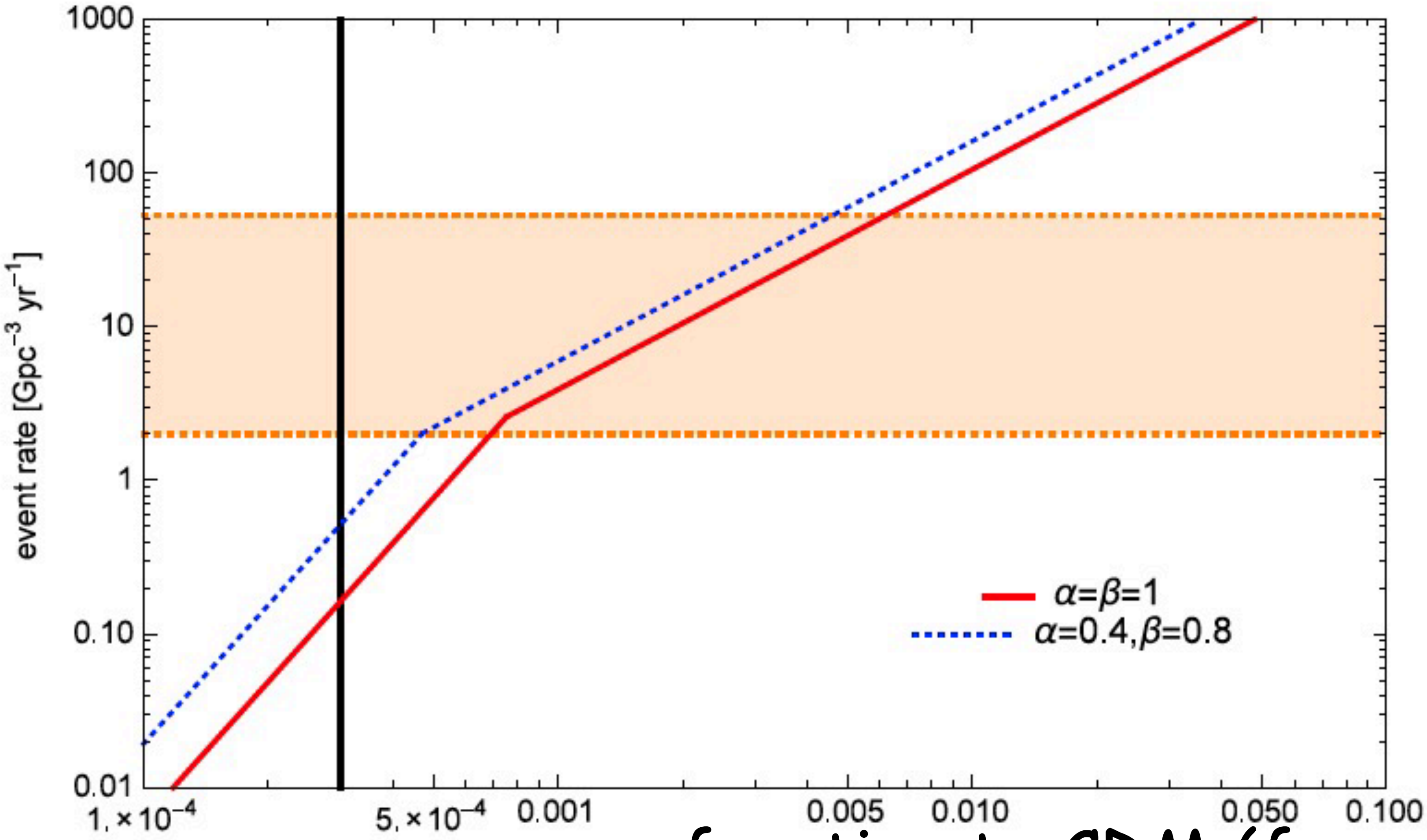


GW150914 and its merger rates for 30 M_{solar} masses BBH

M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama (2016).

A 3-body effect is important for the BBH formations

Rate of GW150914



fraction to CDM ($f_{fraction}$)

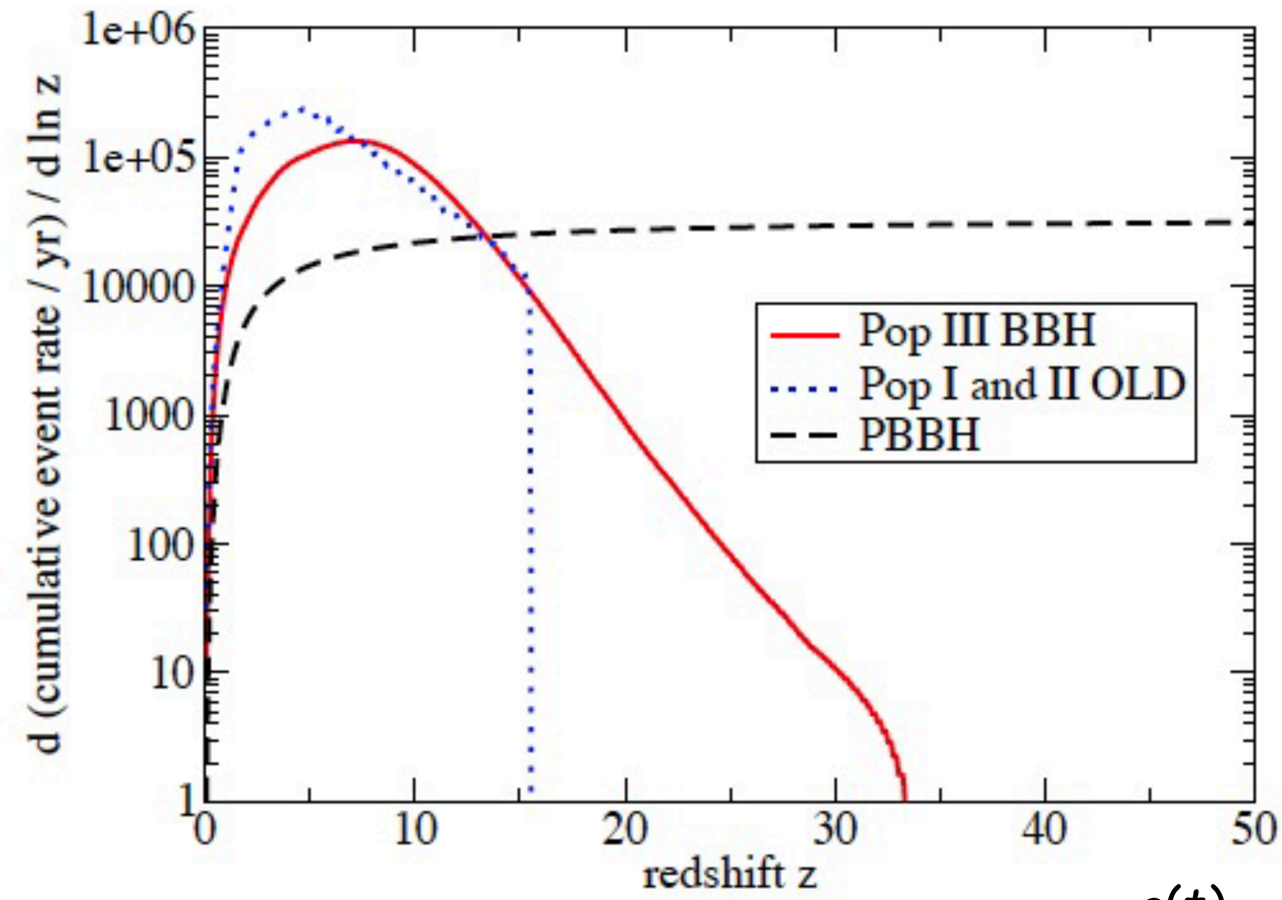
How to produce the binary black holes (BBH) with masses of O(10) $M_{\odot}$?

- Astrophysics: Large uncertainties on gravitational frictions through common envelope phases, mechanisms of supernovae (SNe) and appropriate kick velocities after SNe for Pop III/Pop II stars [astrophysically-model dependent]

- Cosmology: large uncertainties on numbers of PBHs, which depend on inflation models [cosmologically-model dependent]

DECIGO discriminates BPBHs from the normal BBHs

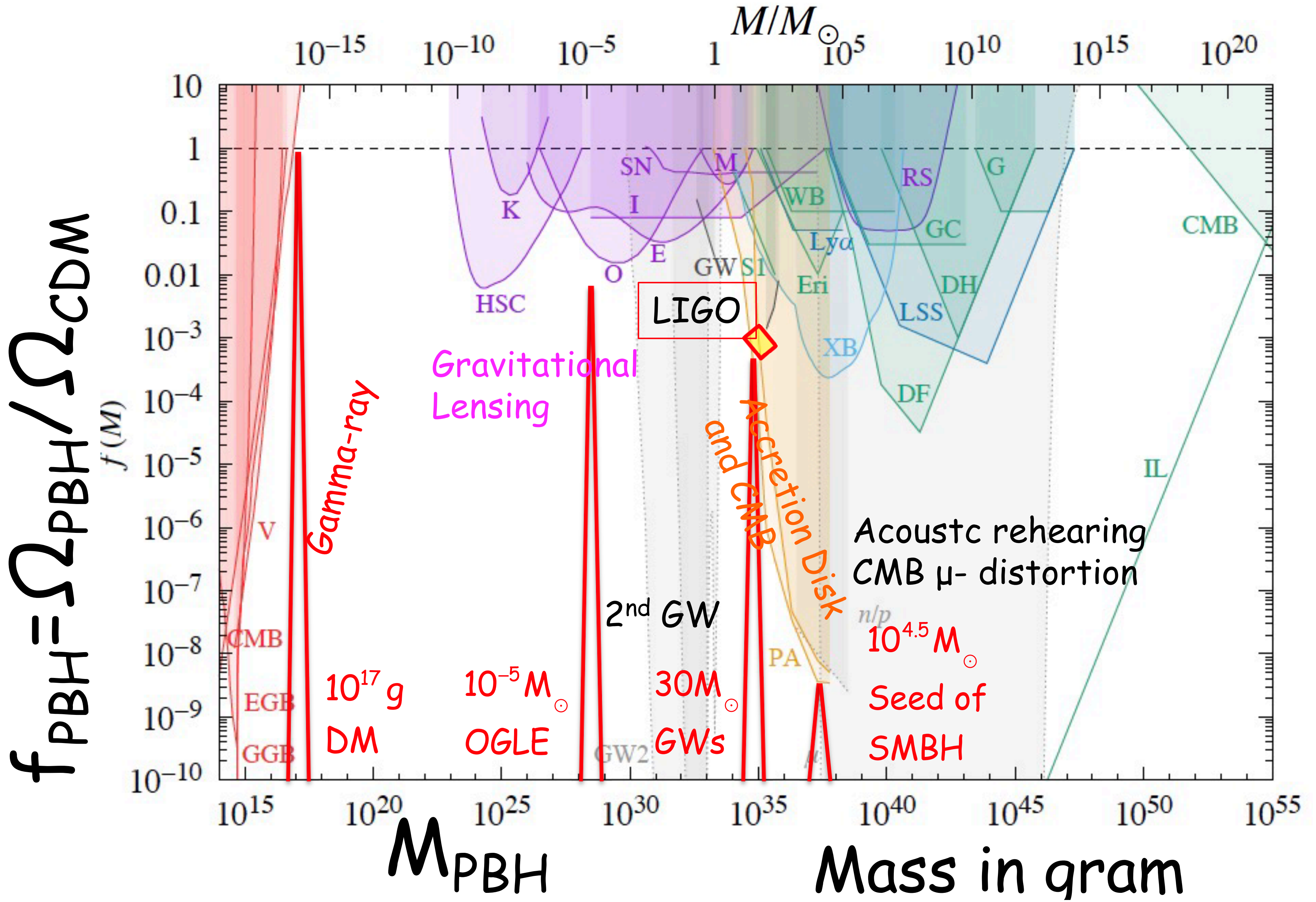
Takashi Nakamura et al, arXiv:1607.00897 [astro-ph.HE]



$$1/z \sim \frac{a(t)}{a(t_0)} \sim \left(t/10\mathrm{Gyr}\right)^{2/3}$$

Upper bounds on the fraction to CDM

Carr, Kohri, Sendouda, J.Yokoyama (2009)(2020)



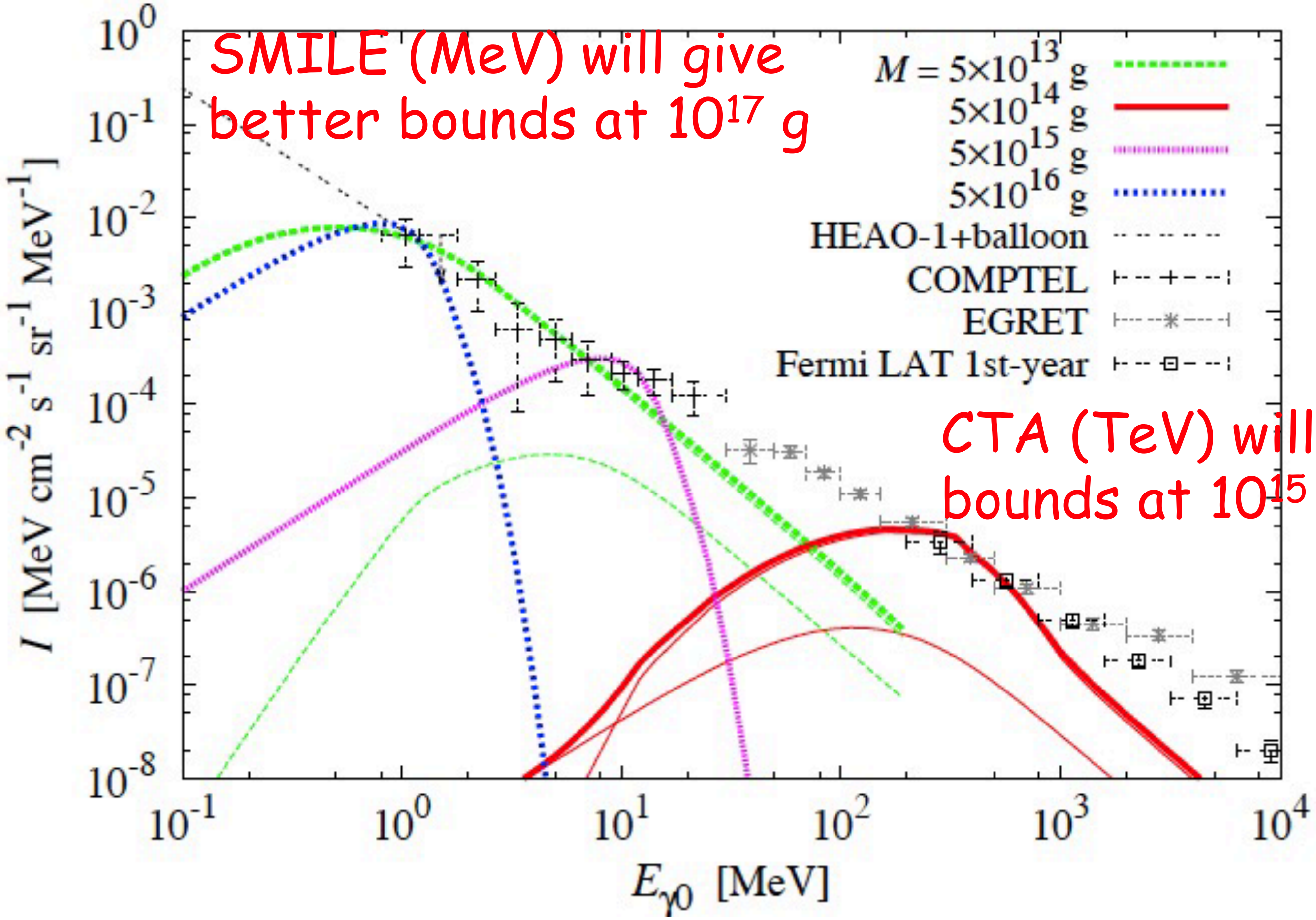
Observational constraints on PBHs

High-energy particles from evaporating PBHs :	$M < 10^{-16}\ M_\odot$
To be dark matter :	$10^{-16} < M < 10^{-10}\ M_\odot$
Gravitational lensing :	$10^{-10} < M < 10^{0}\ M_\odot$
GWs from merging PBHs :	$10^{-4} < M < 10^{1}\ M_\odot$
Emission from accretion disk around a PBH :	$10^{0} < M < 10^{4}\ M_\odot$
CMB distortions by dissipating density perturbation :	$10^{4} < M < 10^{11}\ M_\odot$
Secondary GWs produced from large perturbation :	$10^{-8} < M < 10^{0}\ M_\odot$
…	

Evaporating PBHs through Hawking Process

Carr, Kohri, Sendouda and Yokoyama (2010)

$$d\dot{N}_s = \frac{dE}{2\pi}\frac{\Gamma_s}{e^{E/T_{\rm BH}} - (-1)^{2s}}$$



M31 lensing on PBHs modified by size-distribution and finite-size effects on bright star sources

Nolan Smyth, Stefano Profumo, Samuel English, Tesla Jeltema, Kevin McKinnon, Puragra Guhathakurta, arXiv:1910.01285 [astro-ph.CO]

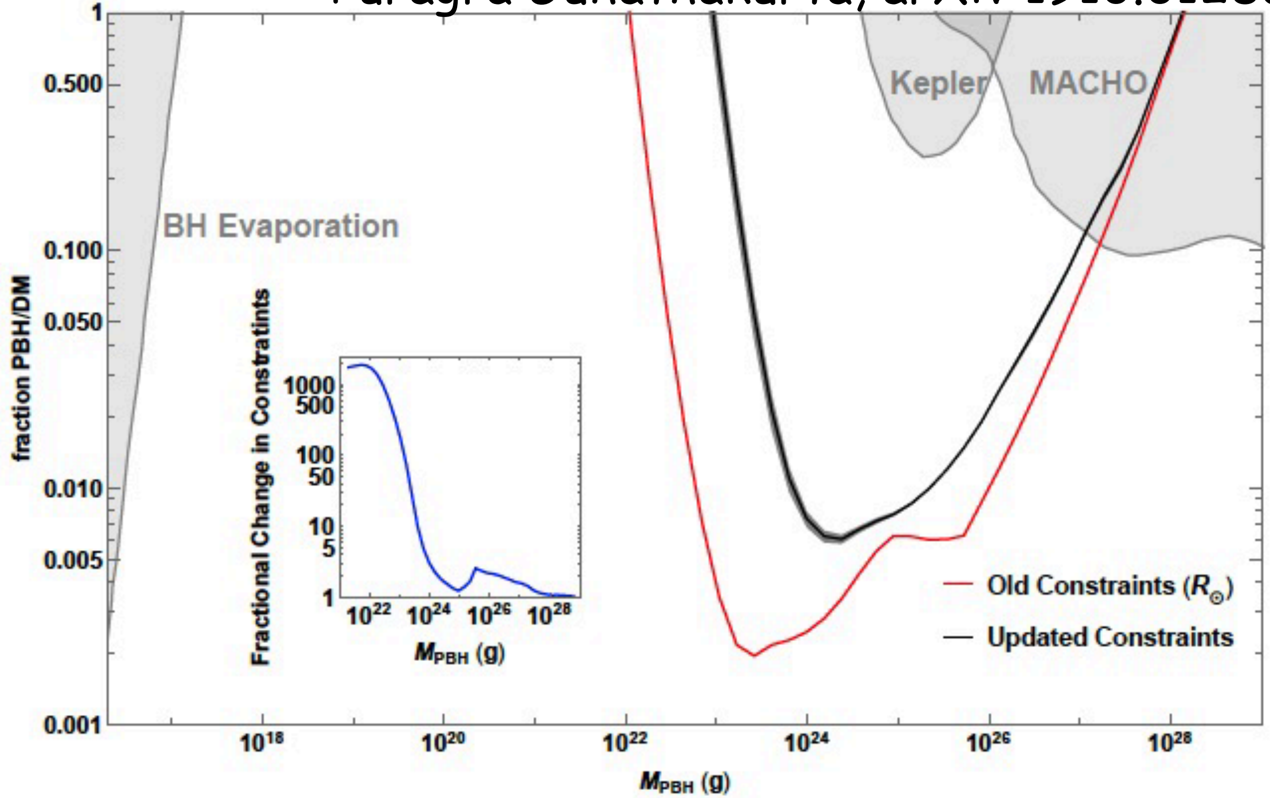
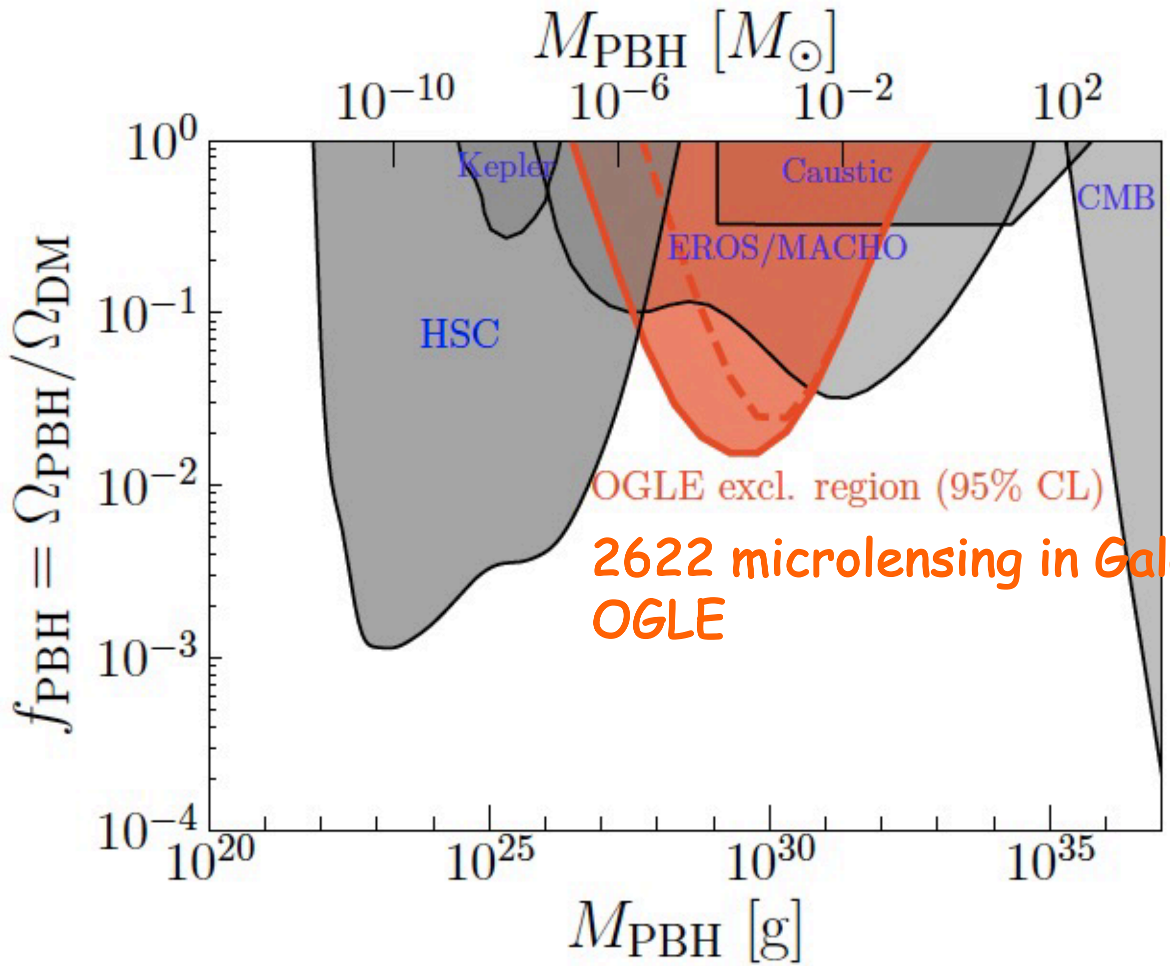


Figure 2. The constraints on primordial black holes as dark matter. The black line is the benchmark constraint and the primary result of this paper. The gray shading comes from the uncertainty in determining the stellar size distribution. The red line is

Gravitational lensing constrains on PBHs

Hiroko Niikura, Masahiro Takada, Shuichiro Yokoyama, Takahiro Sumi, Shogo Masaki,
arXiv:1901.07120 [astro-ph.CO]

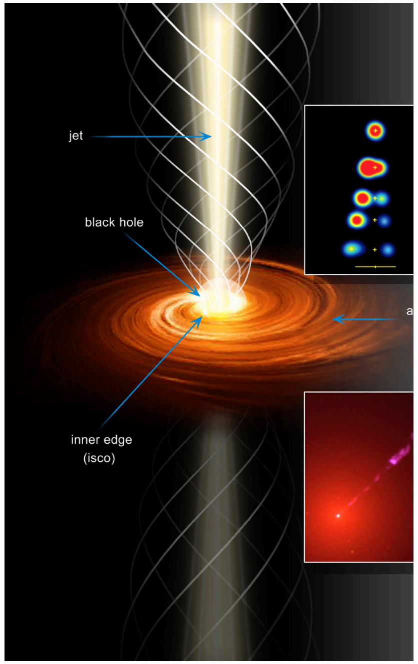


2622 microlensing in Galactic bulge by OGLE

CMB bound on PBHs by disk-accretion in the late MD epoch

Poulin, Serpico, Calore, Clesse, KK (2017)

- A non-spherical accretion disk (ADAF(slim) + Standard disk) around a PBH caused by an angular momentum emits radiation



$$\dot{M}_{\rm HB} \equiv 4\pi\lambda\,\rho_\infty v_{\rm eff} r_{\rm HB}^2 \equiv 4\pi\lambda\,\rho_\infty \frac{(GM)^2}{v_{\rm eff}^3}$$

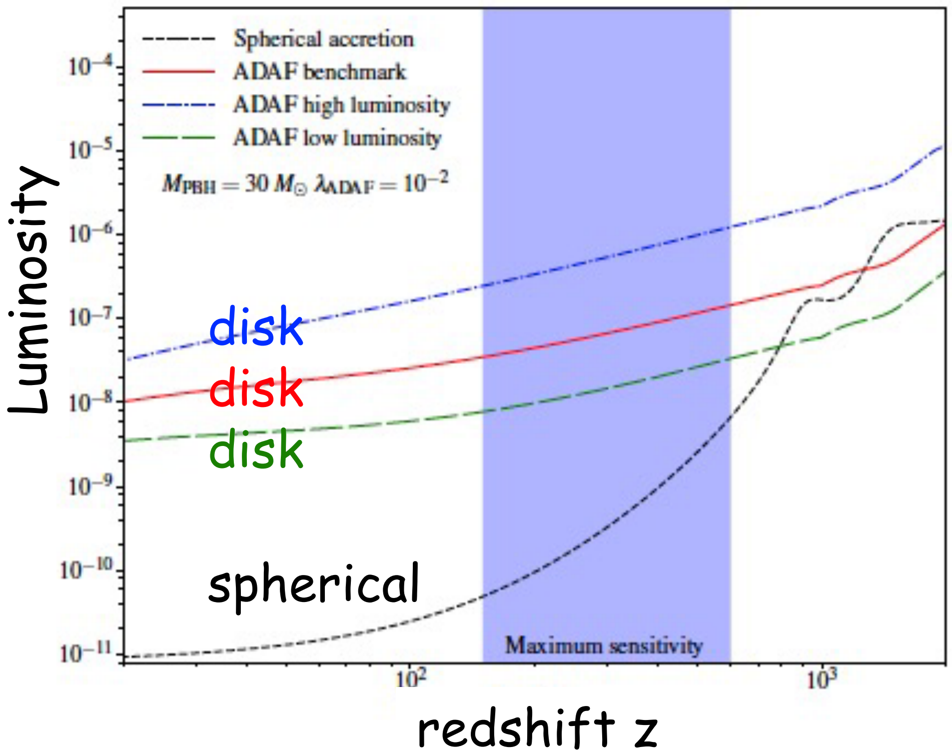
$$l \simeq \omega\, r_{\rm HB}^2 \simeq \left(\frac{\delta\rho}{\rho} + \frac{\delta v}{v_{\rm eff}}\right) v_{\rm eff} r_{\rm HB}$$

- CMB anisotropies are affected

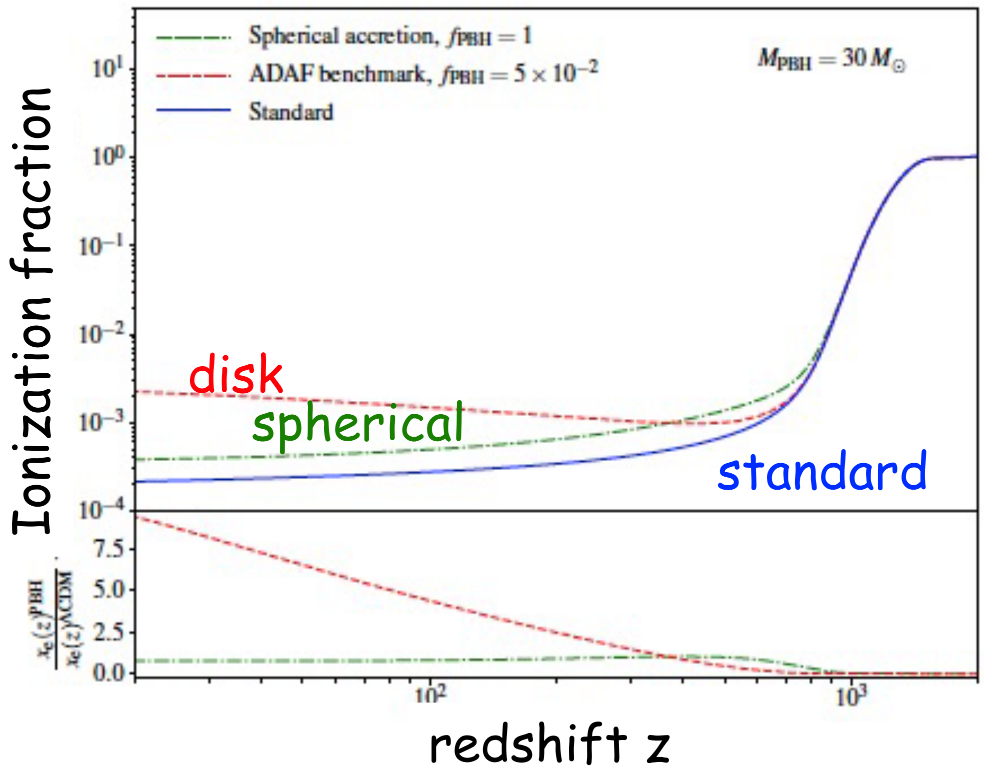
- From observations, we can constrain the number density of PBHs

Modified CMB anisotropy

Poulin, Serpico, Calore, Clesse, Kohri (2017)



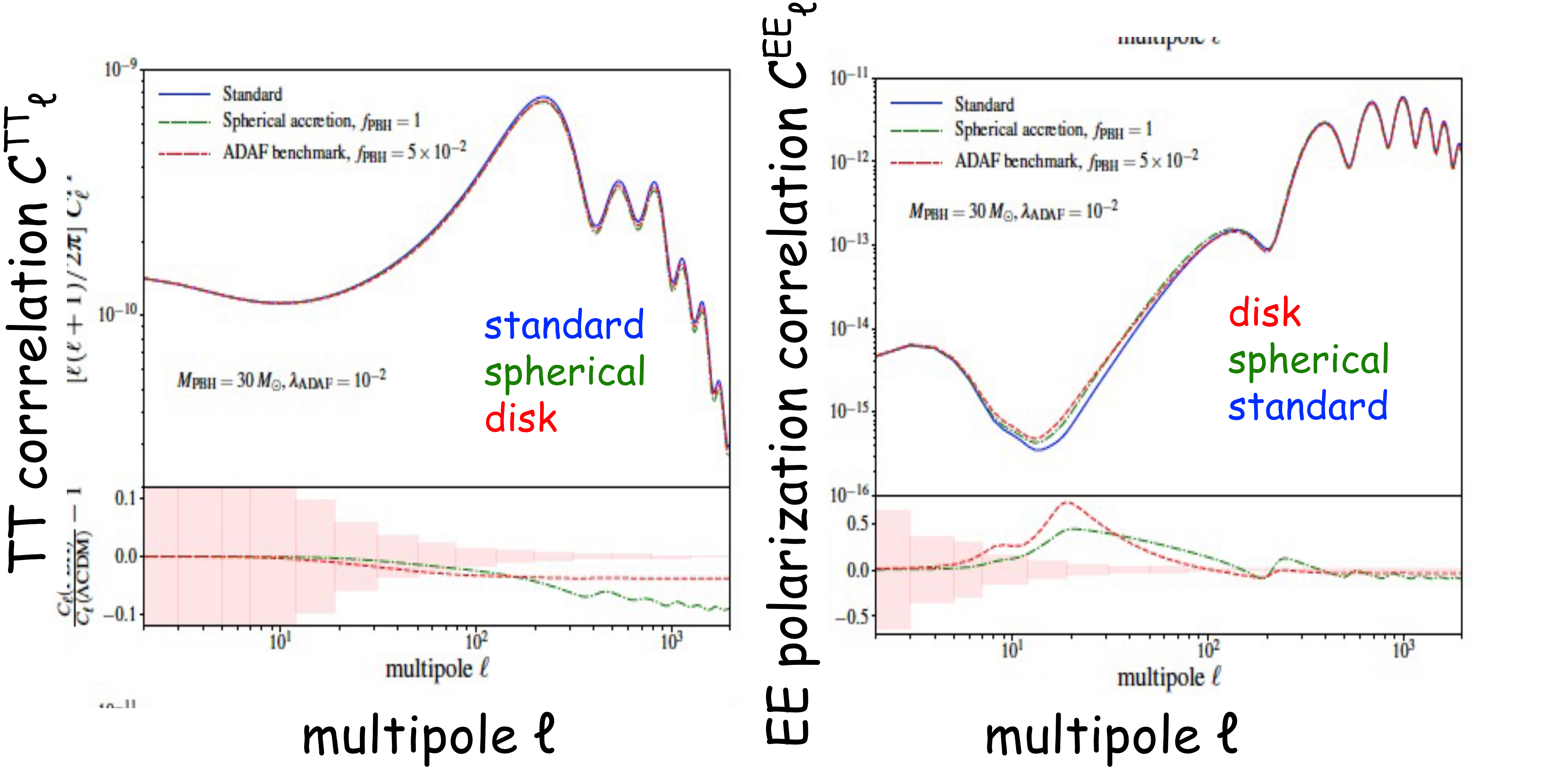
Luminosity



Ionization fraction

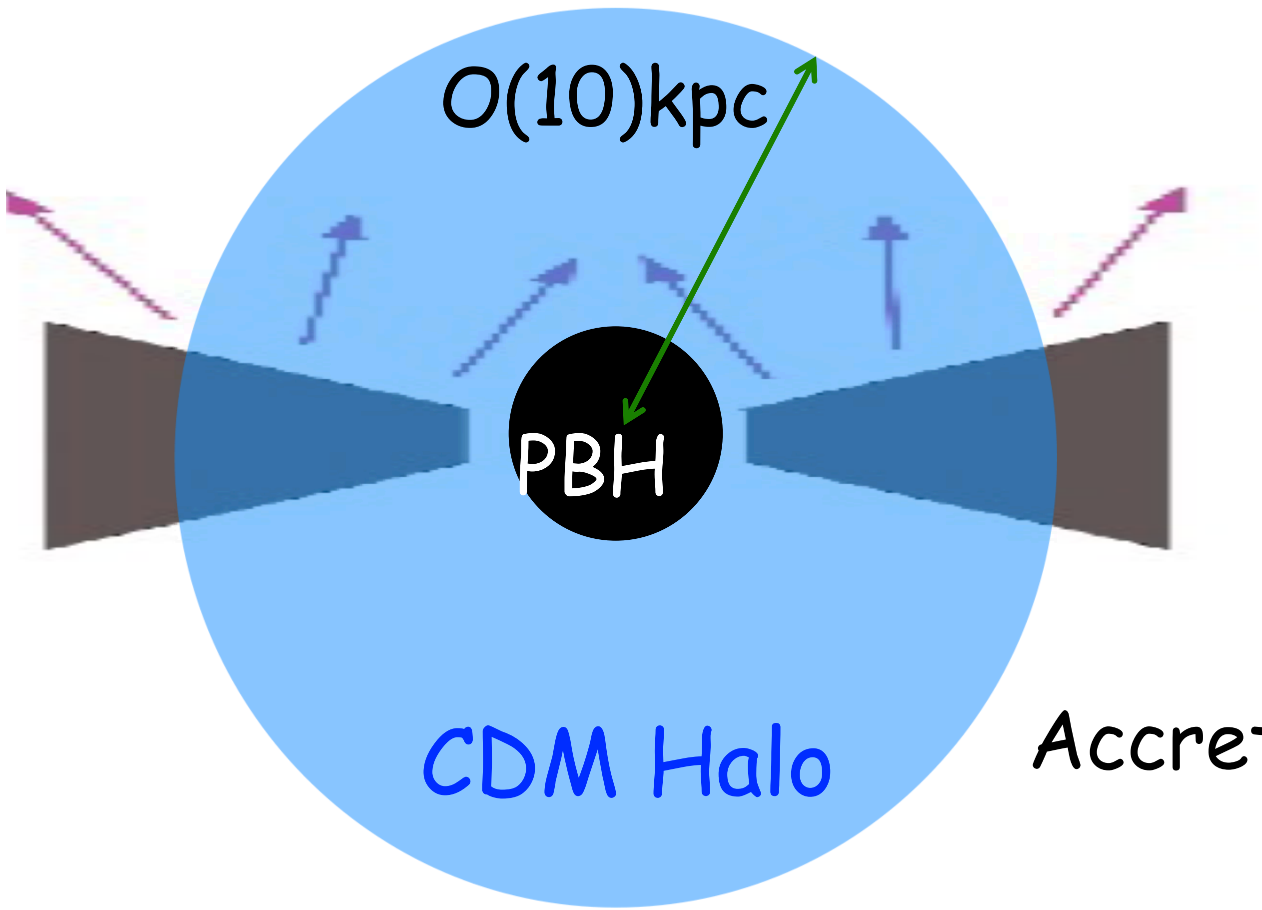
Modified CMB anisotropy

Poulin, Serpico, Calore, Clesse, Kohri (2017)



Cosmological baryon accretion onto the PBH + CDM halo system

Poulin, Serpico, Inman, Kohri (2020)



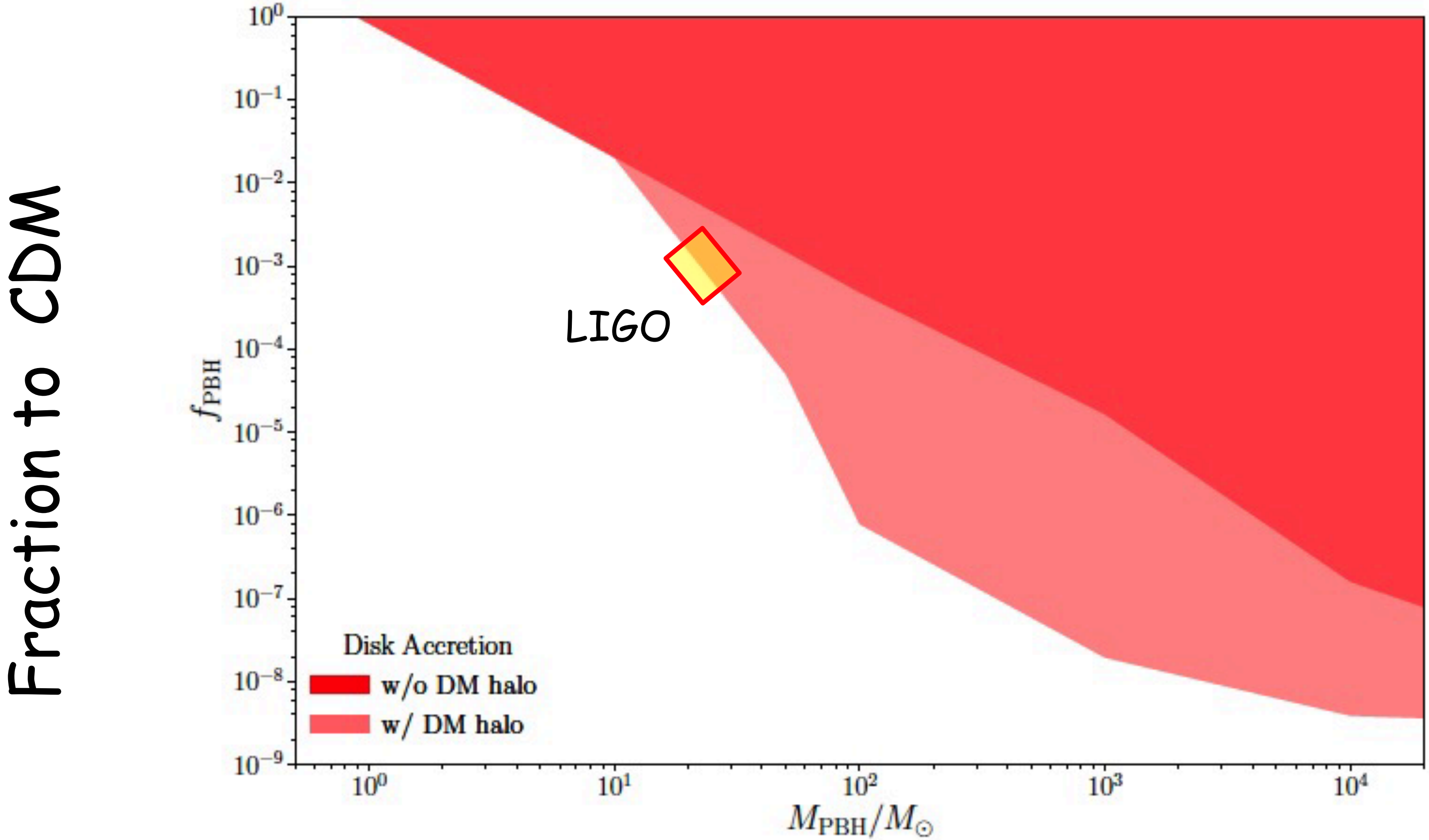
$\rho \propto r^{-2.25}$

Accretion onto CDM + PBH

z ~30 -- 100

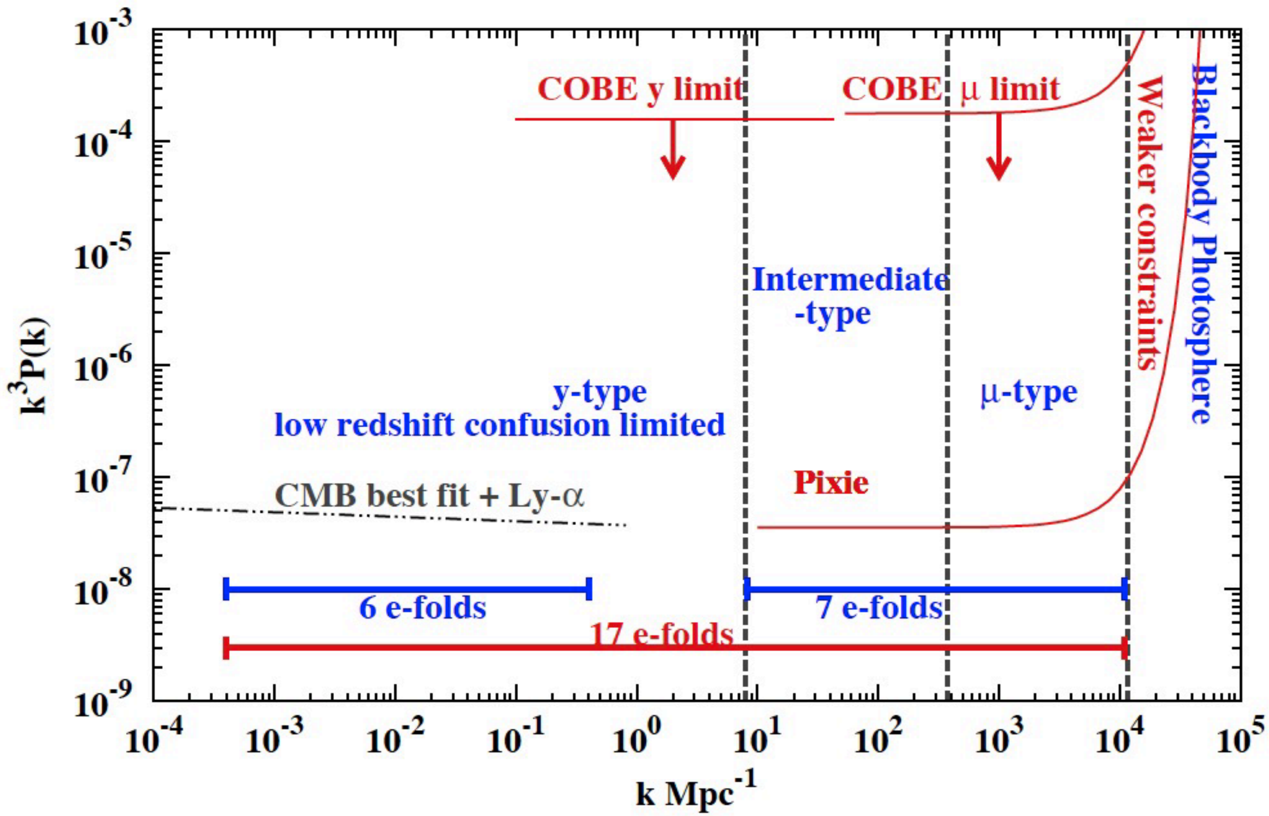
CMB bound by disk-accretion in the latest MD epoch

Poulin, Serpico, Inman, Kohri (2020)



CMB μ- and y- distortion by Large P_ζ

- Large amplitude of fluctuation will be dissipated and produce μ- and y- distortion of CMB after decoupling of double-Compton scatterings



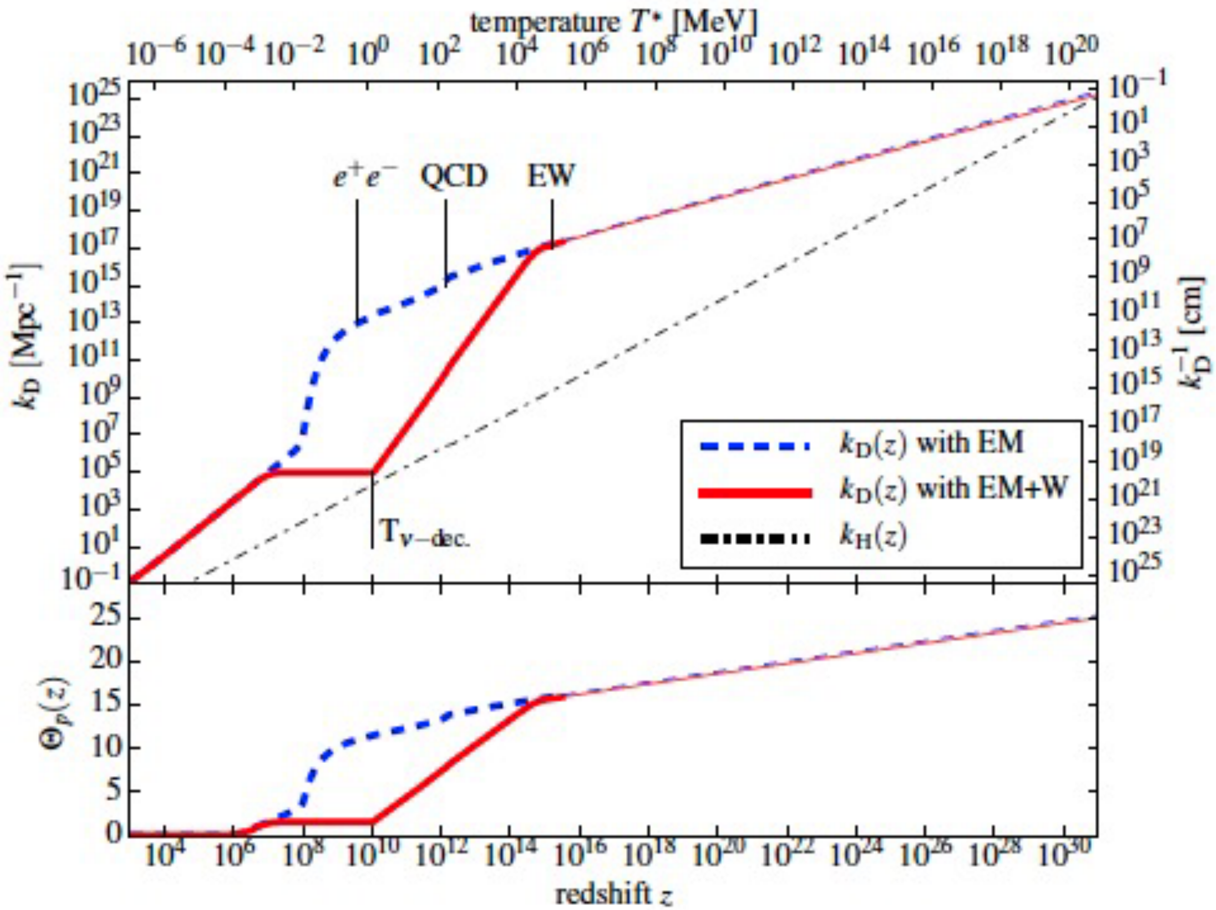
Khatri (2013)

Acoustic reheating

- Nonthermal heating by Silk dumping which can be constrained by BBN and CMB

Jeong,Kamionkowksi, Chluba and Pradler (2014)
Nakama, Suyama, Yokoyama (2014)
Inomata, Tada, Kawasaki (2016)

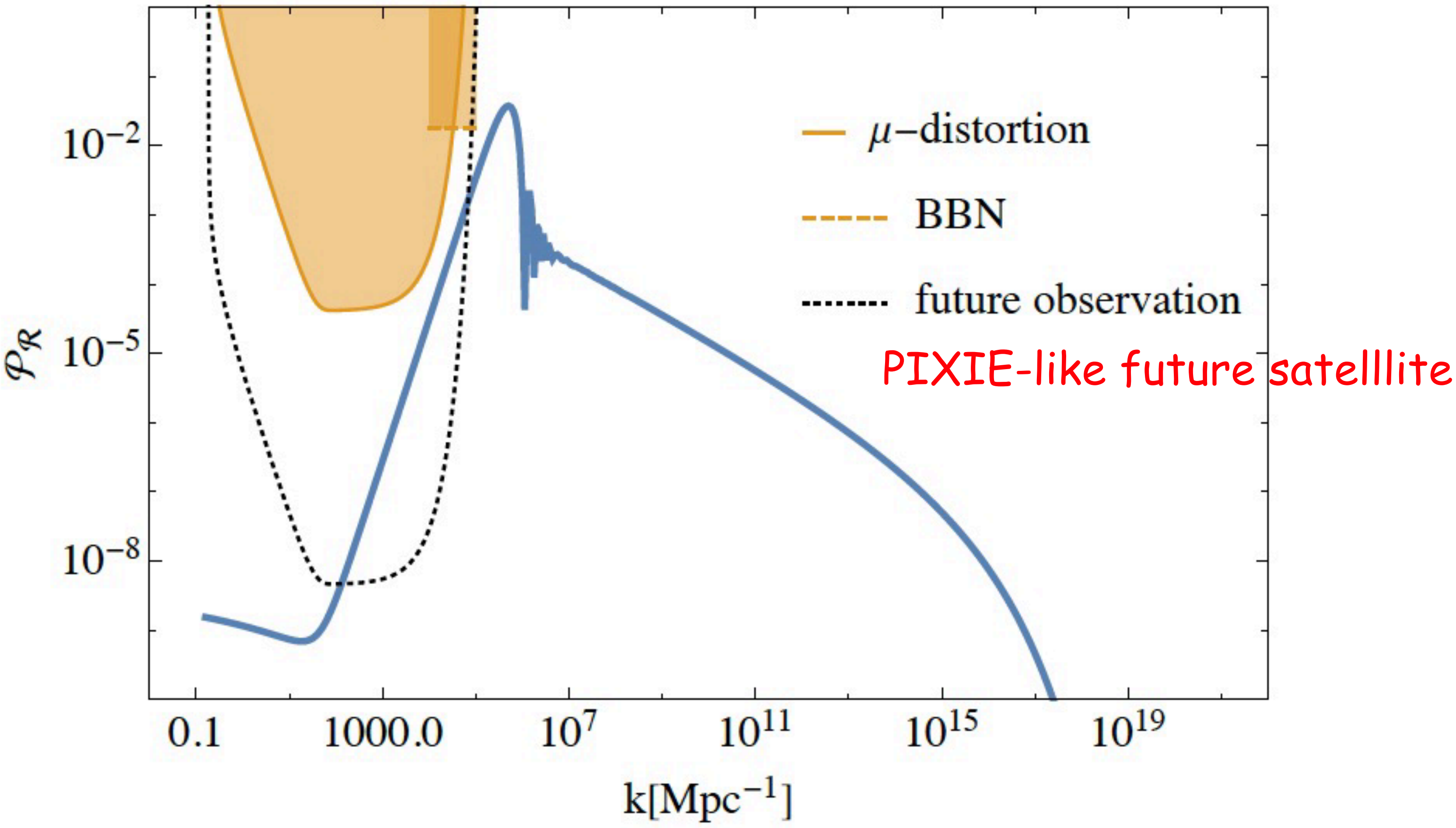
Diffusion scale



μ-distortion and acoustic reheating

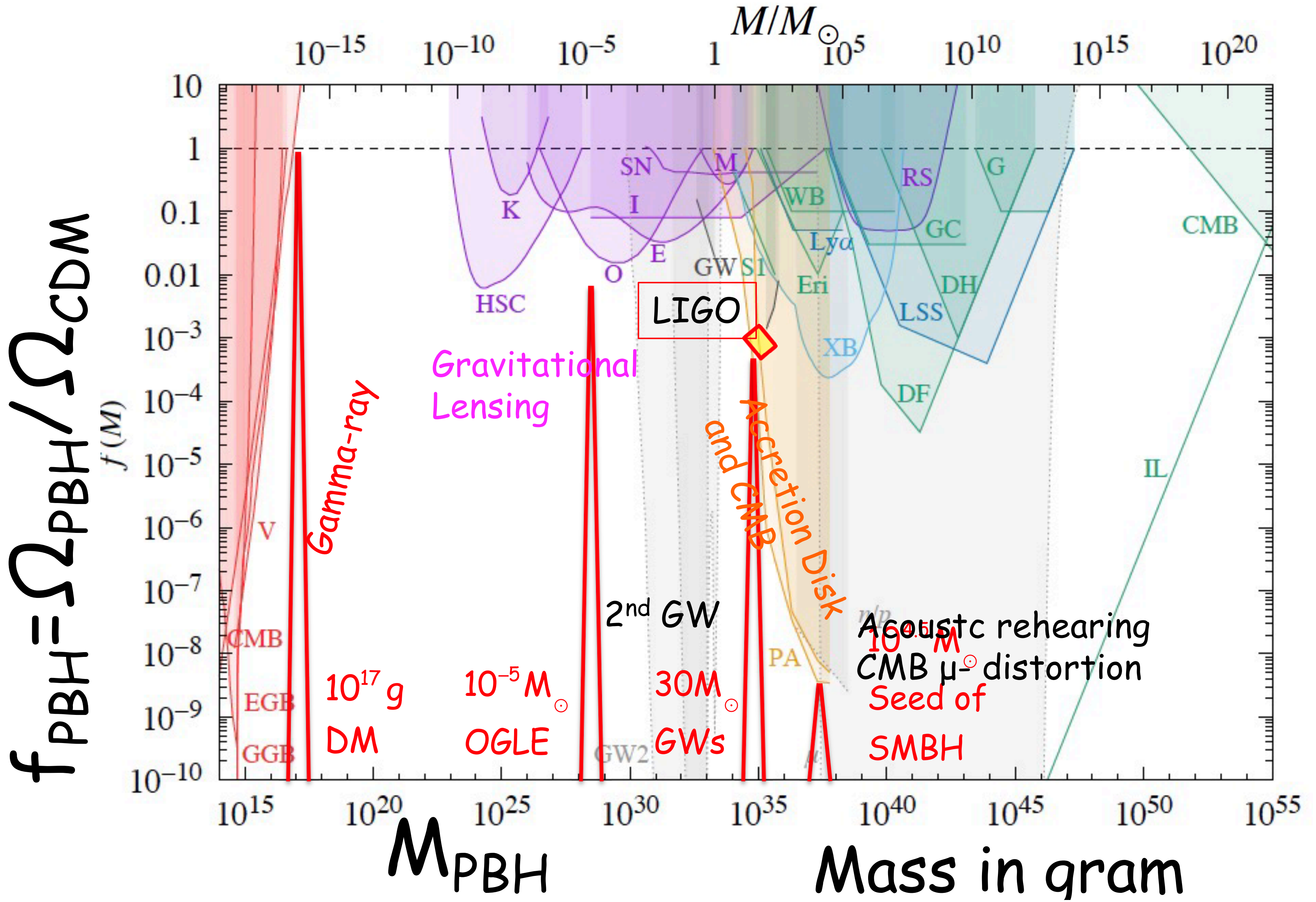
Kohri, Nakama, Suyama (2014)

Inomata, Kawasaki, Mukaida, Tada, Yanagida (2017)



Upper bounds on the fraction to CDM

Carr, Kohri, Sendouda, J.Yokoyama (2009)(2020)



Formation

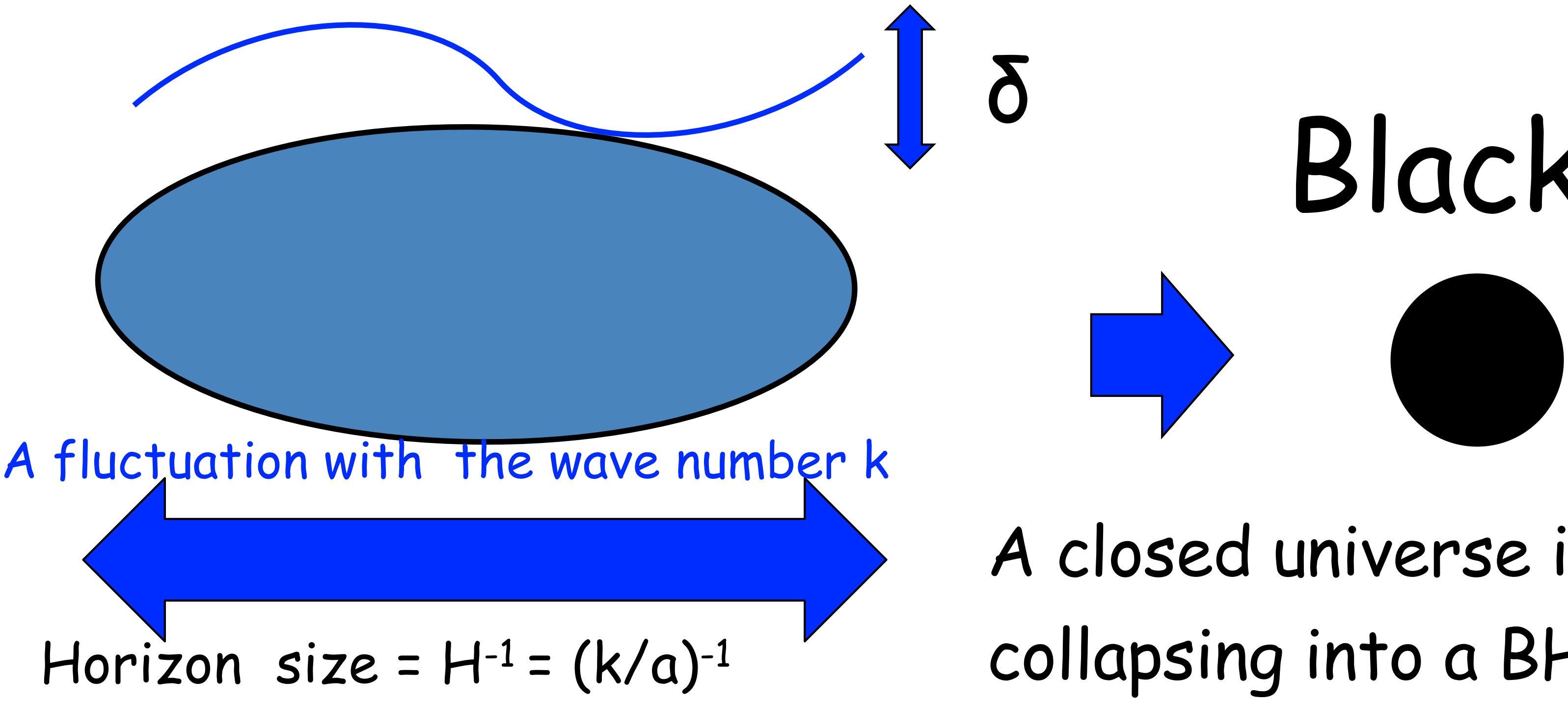
Conditions for a PBH formation in Radiation dominated (RD) Universe

Zel'dovich and Novikov (1967), Hawking (1971), Carr (1975)

Harada, Yoo and KK (2013)

- Gravity could be stronger than pressure

$$\boxed{\delta > \delta_c \sim p/\rho \sim c_s^2 = w = 1/3}$$



A closed universe immediately collapsing into a BH

P_ζ (k) and PBH abundance β(M)

- Fraction of PBH to the total with Press Schechter formalism

For Peak Statistics,
e.g., see Yoo, Harada, KK et al (2018)(2020)

$$\beta(M) \equiv \frac{\rho_{\mathrm{PBH}}(M)}{\rho_{\mathrm{tot}}} = \int_{\delta_{\mathrm{th}}}^{\infty} d\delta \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) = \mathrm{erfc}\left(\frac{\delta_{\mathrm{th}}}{\sqrt{2}\sigma}\right)$$

$\boxed{\sim 1/3 - 0.5}$

For analytical derivations, see Harada, Yoo, KK (2013) $\sigma \sim \overline{\delta\rho/\rho}$

- Relation between β and fluctuation σ (or β and Ω)

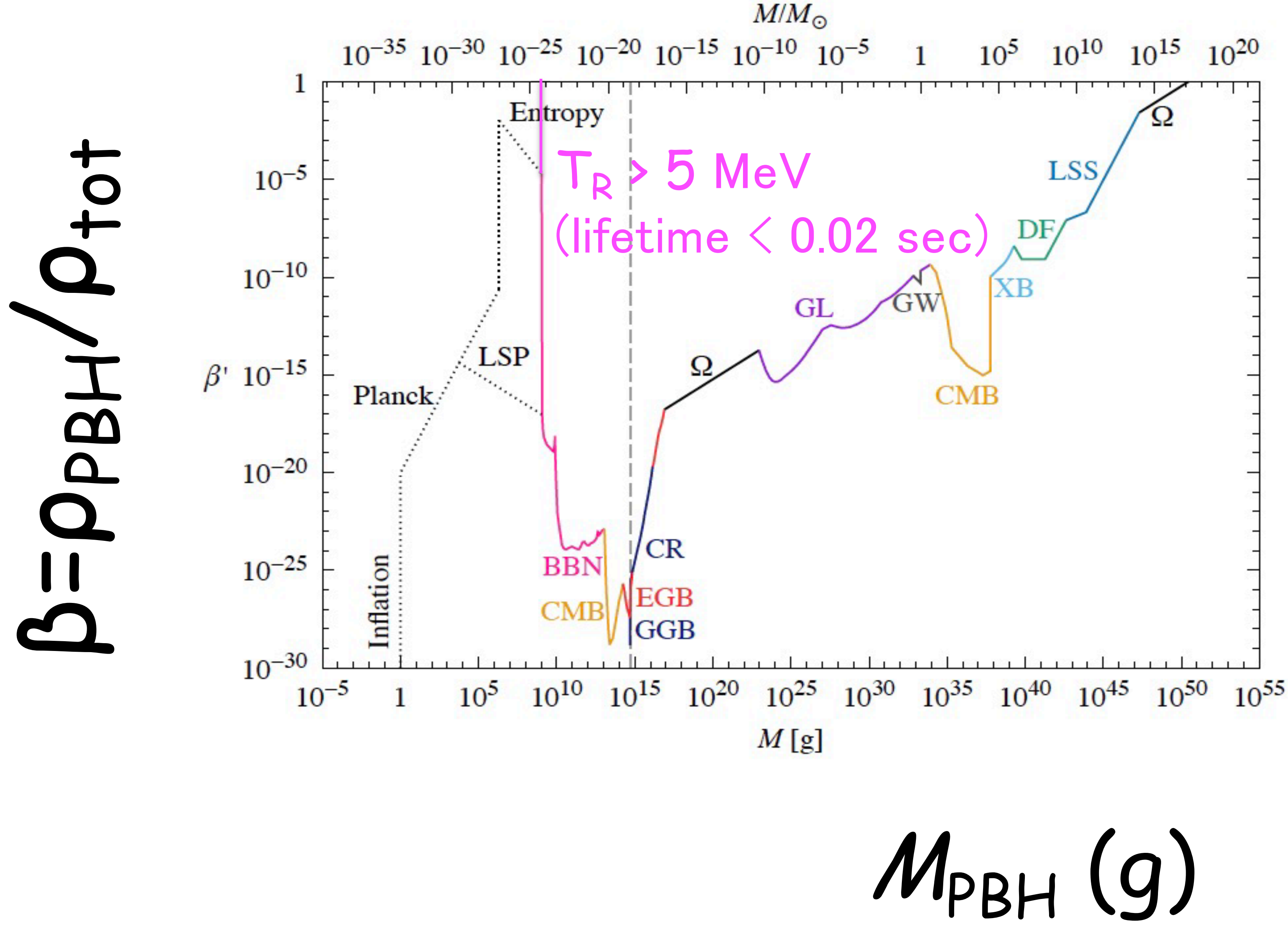
$$\beta(M) \sim \mathrm{erfc}\left(\frac{\delta_{\mathrm{th}}}{\sqrt{2}\sigma}\right) \simeq \sqrt{\frac{2}{\pi}}\frac{\sigma}{\delta_{\mathrm{th}}} \exp\left(-\frac{\delta_{\mathrm{th}}^2}{2\sigma^2}\right)$$

$$= 1.5 \times 10^{-18} \left(\frac{m_{\mathrm{PBH}}}{10^{15} g}\right)^{1/2} \left(\frac{\Omega_{\mathrm{PBH}} h^2}{0.1}\right)$$

$\boxed{\sim P_\zeta}$

$\beta=\rho_{PBH}/\rho_{tot}$ vs M_{PBH}

Carr, Kohri, Sendouda, J.Yokoyama (2009)(2020)



Typical quantities of PBHs in RD

- Mass (horizon mass $=\rho(t_{form})\, H(t_{form})^{-3}$)

$$M_{\rm PBH} \sim \rho \left(H_{\rm form}^{-1}\right)^3 \sim M_{pl}^2 t_{\rm from} \sim \frac{M_{pl}^3}{T_{\rm form}^2} \sim 10^{15} g \left(\frac{T_{\rm form}}{3\times 10^8 {\rm GeV}}\right)^{-2} \sim 5{\times}10^4 M_\odot \left(\frac{T_{\rm form}}{\rm MeV}\right)^{-2}$$

- Lifetime

$$\tau_{\rm PBH} \sim \frac{M_{\rm PBH}^3}{M_{pl}^4} \sim 4\times 10^{17}\,{\rm sec}\left(\frac{M_{\rm PBH}}{10^{15}\ {\rm g}}\right)^3 \sim 1\,{\rm sec}\left(\frac{M_{\rm PBH}}{10^9\ {\rm g}}\right)^3$$

- Hawking Temperature

$$T_{\rm PBH} \sim \frac{M_{\rm pl}^2}{M_{\rm PBH}} \sim 10\,{\rm MeV}\left(\frac{M_{\rm PBH}}{10^{15}{\rm g}}\right)^{-1} \sim 2\times 10^{-9}{\rm K}\left(\frac{M_{\rm PBH}}{30 M_\odot}\right)^{-1}$$

- Wave number of horizon length

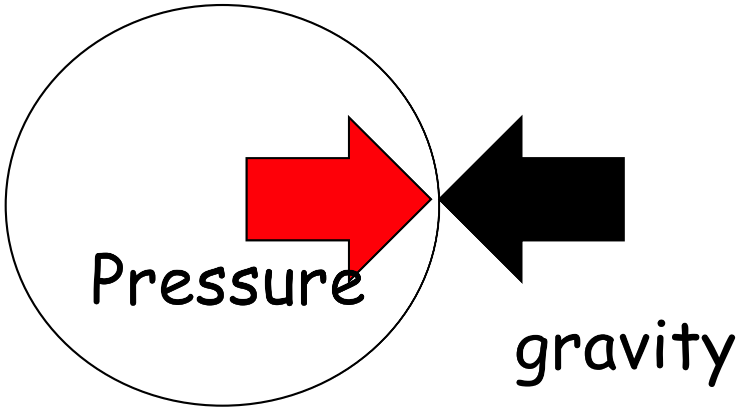
$$k = aH \sim 10^5 {\rm Mpc}^{-1}\left(\frac{M_{\rm PBH}}{5\times 10^4 M_\odot}\right)^{-1/2} \sim 10^5 {\rm Mpc}^{-1}\left(\frac{T_{\rm form}}{\rm MeV}\right)^{+1}$$

- Fraction to CDM

$$f_{\rm fraction} \equiv \frac{\Omega_{\rm PBH}}{\Omega_{\rm CDM}} \sim \left(\frac{\beta}{10^{-18}}\right)\left(\frac{M_{\rm PBH}}{10^{15} g}\right)^{-1/2} \sim \left(\frac{\beta}{10^{-8}}\right)\left(\frac{M_{\rm PBH}}{30 M_\odot}\right)^{-1/2} \sim 10^8 \left(\frac{M_{\rm PBH}}{30 M_\odot}\right)^{-1/2} \sqrt{P_\delta}\, \exp\left[-\frac{1}{18 P_\delta}\right]$$

Features of PBH formations in RD

- Spherical due to radiation pressure



$(w \equiv p/\rho \sim 1/3)$

- Negligible evolutions of density perturbations
- Quite a small angular momentum

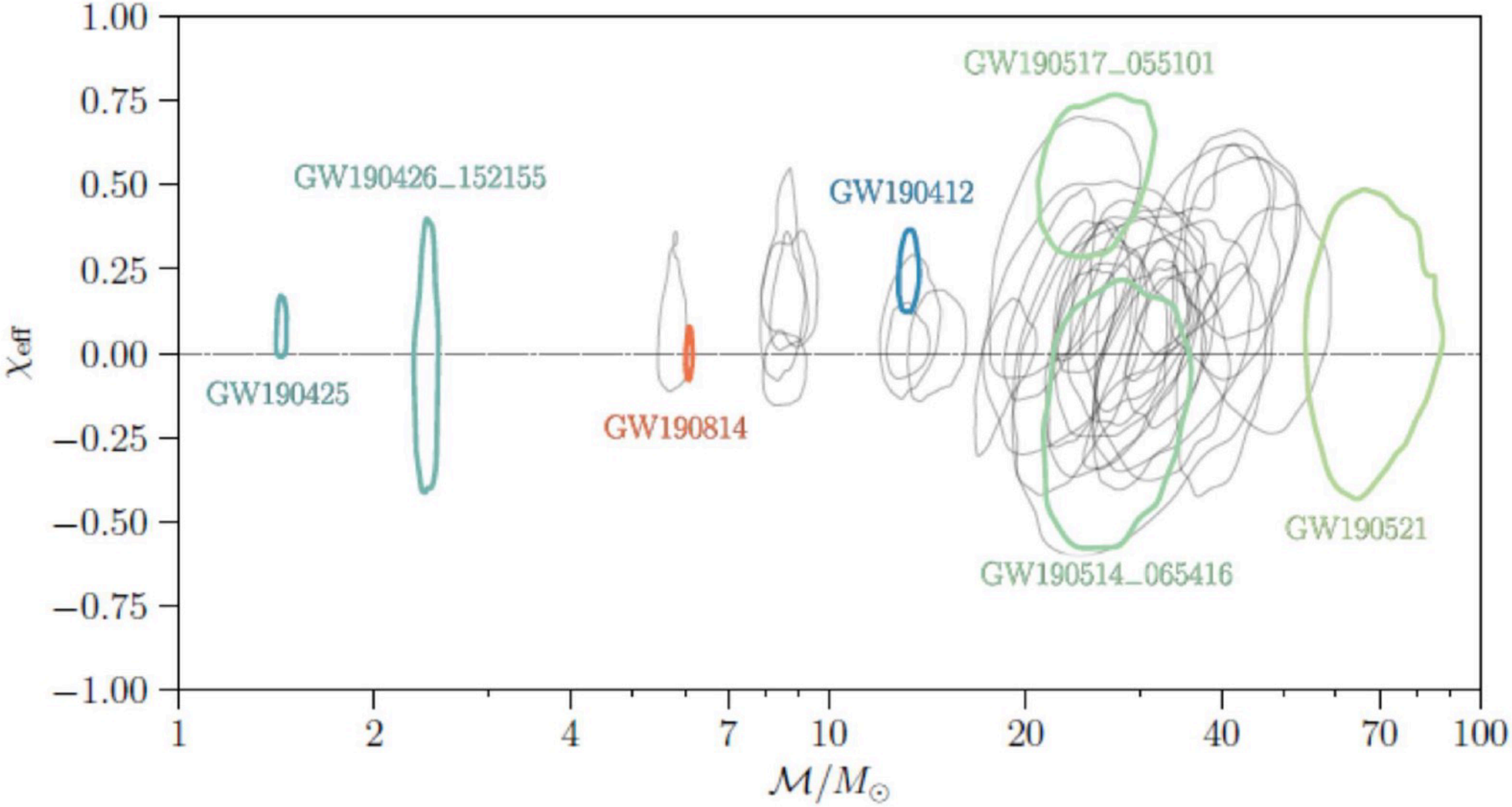
See, T.Chiba and S.Yokoyama, 2017
De Luca et al, 2019
Minxi He and Suyama, 2019
Harada, Yoo, Kohri, Koga and Monobe, 2020

(dimensionless Kerr parameter)

$$\sqrt{\langle a_*^2 \rangle} \simeq 6.5 \times 10^{-4} \left(\frac{M}{M_H} \right)^{-1/3}$$

Effective inspiral spin parameter of the observed BHs

$$\chi_{\rm eff} = \frac{m_1\chi_1\cos\theta_1 + m_2\chi_2\cos\theta_2}{m_1+m_2}$$



Credible region contours for all candidate events in the plane of chirp mass $\mathcal{M}$ and effective inspiral spin \chieff. Each contour represents the 90\% credible region for a different event. We highlighted the previously published candidate events (cf.\ Fig.~\ref{fig:mtotqpost}), as well as \protect\NAME{GW190517A}\ and \protect\NAME{GW190514A}, which have the highest probabilities of having the largest and smallest \chieff respectively.

R. Abbott, et al, LSC P&P Committee, arXiv:2010.14527 [gr-qc]

PBH formation at the (early) matter dominated (MD) Universe

Polnarev and Khlopov (1982)

Harada, Yoo, KK, Nakao, Jhingan (2016)

1. Pressure is negligible, which could induce an immediate collapse and producing more PBHs?

2. Density perturbations can evolve, which produces non-spherical objects and cannot be enclosed by the Horizon. That means less PBHs can be produced?

Matter Domination

- Three radius in Lagrangian coordinate q_i

Zel'dovich Approximation

$$r_1 = (a - \alpha b)q_1$$
$$r_2 = (a - \beta b)q_2$$
$$r_3 = (a - \gamma b)q_3$$

- Eccentricity $$e^2 = 1 - \left(\frac{r_2(t_c)}{r_3(t_c)}\right)^2 = 1 - \left(\frac{\alpha-\beta}{\alpha-\gamma}\right)^2$$
- Hoop with 2nd Elliptic funciton E(x)

$$\mathcal{C} = 16\left(1 - \frac{\gamma}{\alpha}\right) E\left(\sqrt{1 - \left(\frac{\alpha-\beta}{\alpha-\gamma}\right)^2}\right) r_f$$

- Hoop conjecture for PBH production

$$\mathcal{C} \lesssim 2\pi r_g.$$

Abundance of PBHs formed in MD

- Probability distribution by peak statistics (BBKS)

Doroshkevich (1970)

$$
\begin{aligned}
&w(\alpha,\beta,\gamma)d\alpha d\beta d\gamma \\
&= -\frac{27}{8\sqrt{5}\pi\sigma_3^6}\exp\left[-\frac{1}{10\sigma_3^2}(\alpha+\beta+\gamma)^2-\frac{1}{4\sigma_3^2}\{(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2\}\right] \\
&\cdot(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)d\alpha d\beta d\gamma.
\end{aligned}
$$

$$\sigma_H = \sqrt{5}\sigma_3$$

- Probability

$$\beta_0 = \int_0^\infty d\alpha \int_{-\infty}^{\alpha} d\beta \int_{-\infty}^{\beta} d\gamma\ \theta(1-h(\alpha,\beta,\gamma))w(\alpha,\beta,\gamma)$$

$$h(\alpha,\beta,\gamma) = \frac{2}{\pi}\frac{\alpha-\gamma}{\alpha^2}E\left(\sqrt{1-\left(\frac{\alpha-\beta}{\alpha-\gamma}\right)^2}\right)$$

$$h(\alpha,\beta,\gamma) := \mathcal{C}/(2\pi r_g)$$

Angular momentum produced by perturbations

Harada, Yoo, KK, and Nakao (2017)

- Angular momentum

1st order effects for nonspherical V

2nd order effects

$$\mathbf{L}_c = \int_{a^3V} \rho \mathbf{r} \times \mathbf{v} d^3\mathbf{r} = \rho_0 a^4 \left(\int_V \mathbf{x} \times \mathbf{u} d^3\mathbf{x} + \int_V \mathbf{x}\delta \times \mathbf{u} d^3\mathbf{x} \right)$$

- Density perturbation δ
- (Peculiar) Velocity perturbation

$$\mathbf{u} := aD\mathbf{x}/Dt$$

$$\mathbf{u}_1 = -\frac{t}{a}\nabla\psi_1$$

- Potential perturbation

$$\psi := \Psi - \Psi_0$$

Effects by finite angular momentum

Harada, Yoo, KK, Nakao (2017)

- Probability distribution

$$a_* := L/(GM^2/c)$$

$$f_{\mathrm{BH}(2)}(a_*)da_* \propto \frac{1}{a_*^{5/3}} \exp\left(-\frac{1}{2\sigma_H^{2/3}}\left(\frac{2}{5}\mathcal{I}\right)^{4/3}\frac{1}{a_*^{4/3}}\right) da_*$$

- Probability

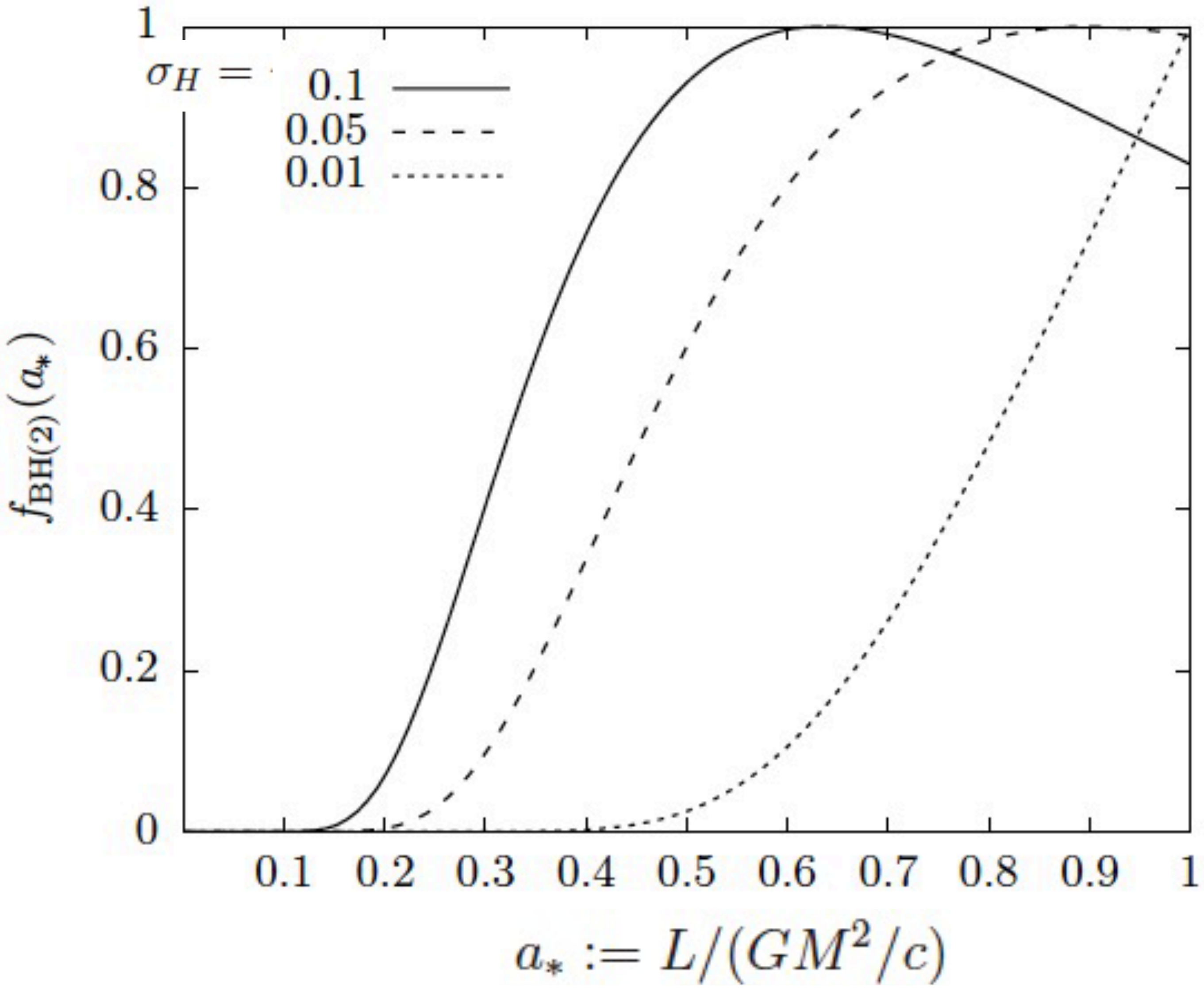
$$\beta_0 \simeq \int_0^\infty d\alpha \int_{-\infty}^{\alpha} d\beta \int_{-\infty}^{\beta} d\gamma \theta[\delta_H(\alpha,\beta,\gamma)-\delta_{\mathrm{th}}]\theta[1-h(\alpha,\beta,\gamma)]w(\alpha,\beta,\gamma)$$

$$\delta_H(\alpha,\beta,\gamma) = \alpha+\beta+\gamma \qquad \delta_{\mathrm{th}} := \left(\frac{2}{5}\mathcal{I}\sigma_H\right)^{2/3}$$

Spin distribution

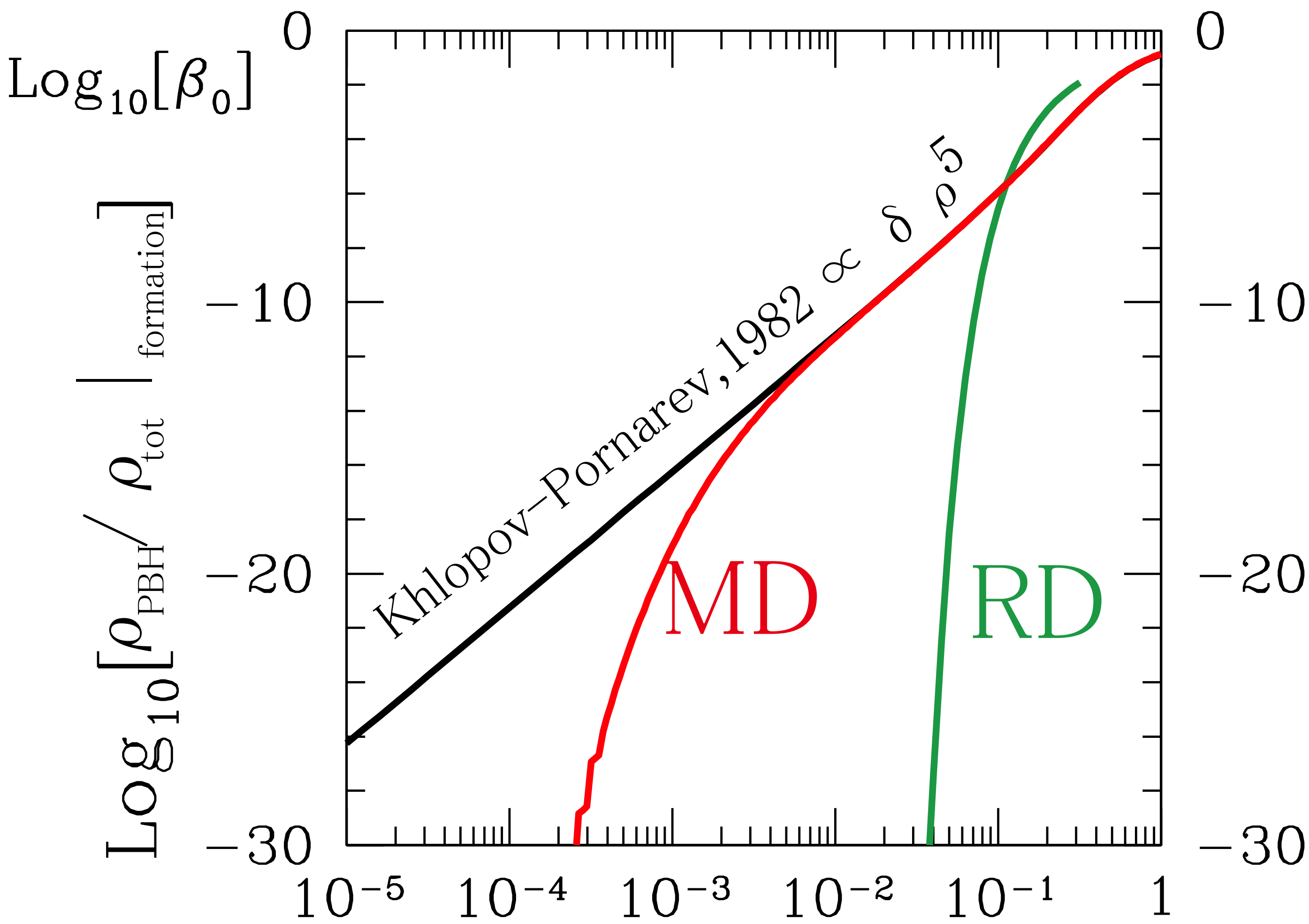
More highly-spinning halos cannot collapse into PBHs, which means that the PBHs produced tend to have high spins in MD

Harada, Yoo, KK, Nakao (2017)



Beta in matter-domination

Harada, Yoo, KK, Nakao (2017)



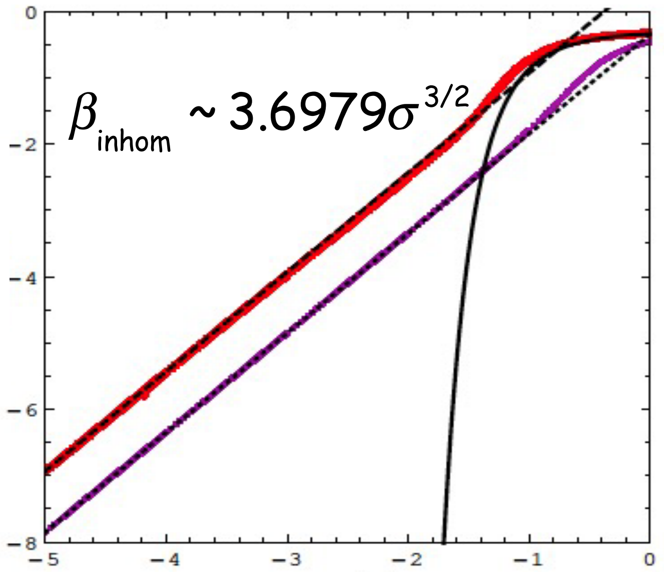
$\sigma_H = \sqrt{5}\sigma_3 = \delta\rho/\rho$

Effects of Inhomogeneity on PBH formations in Matter Domination

T.Kokubu, K.Kyutoku, K.Kohri, T.Harada, arXiv:1810.03490

Singularity should be enclosed by (apparent) horizon

$$\beta \equiv \frac{\rho_{PBH}}{\rho_{tot}}$$



$\mathrm{Log}_{10}[\sigma]$

$$\beta_{aniso} \simeq 0.05556\sigma^5$$

$\sigma \sim \overline{\delta\rho/\rho}$

$$\beta_{inhom+aniso} \simeq \beta_{inhom} \times \beta_{aniso} = 0.2055\sigma^{13/2}$$

3) $a_* \sim 1 \rightarrow$ PBHs produced with $\sigma \ll 1$ in the eMD

Harada, Yoo, KK, Nakao (2017)