non-perturbative Quantum field theories

Hitoshi Murayama (Berkeley, Kavli IPMU)

HirosiFest @ KavliIPMU, Oct 20, 2022


60
祝還暦!!
BCCスイーツ

Colleagues 3 times

- UTokyo 1996-1998
- Berkeley 1994-2000
- Kavli IPMU 2007-

Oct 15, 2018

NEW DIRECTOR

Slide on the day of transition

- Fantastic scientist
  - renowned string theorist
  - Eisenbud Prize AMS
  - Hamburg Prize
  - Nishina Prize
- Management experience
  - President, Aspen CfP
  - Director, WB Institute
- Active in popularization
- 50:50 Caltech vs Kavli IPMU

Hirosi Ooguri (Caltech)

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quant-ph | astro-ph | hep-ph | hep-th

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2008-present
SENIOR, Tokyo U., IPMU

1995-present
SENIOR, UC, Berkeley

1995-present
SENIOR, LBL, Berkeley

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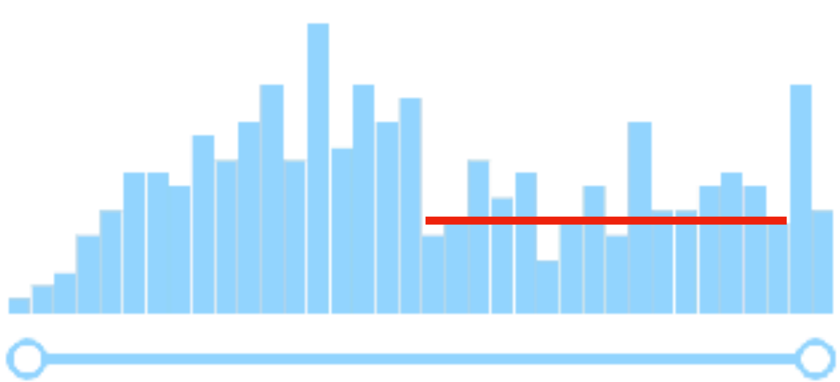

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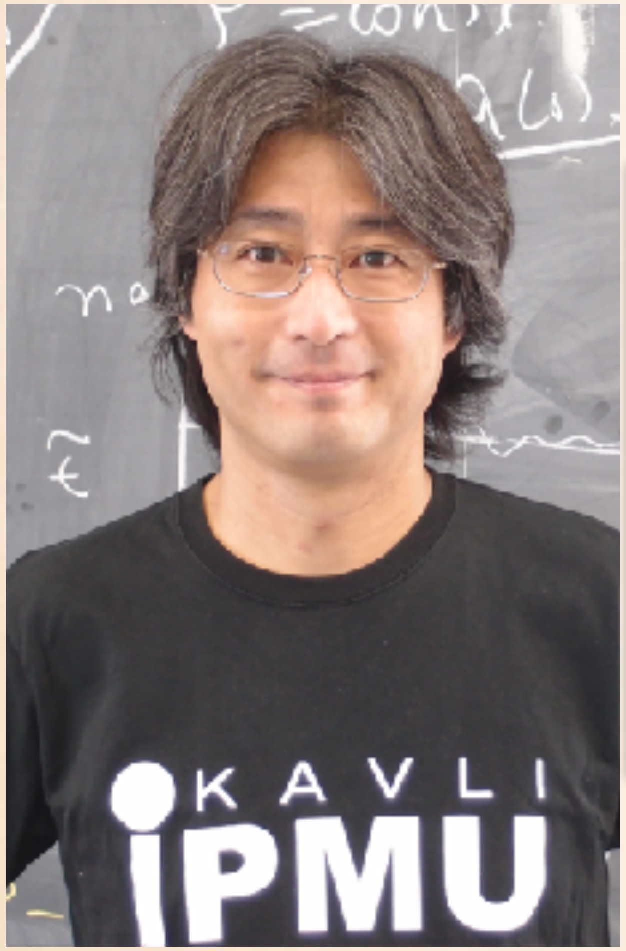

Oct 15, 2018


2018 2022
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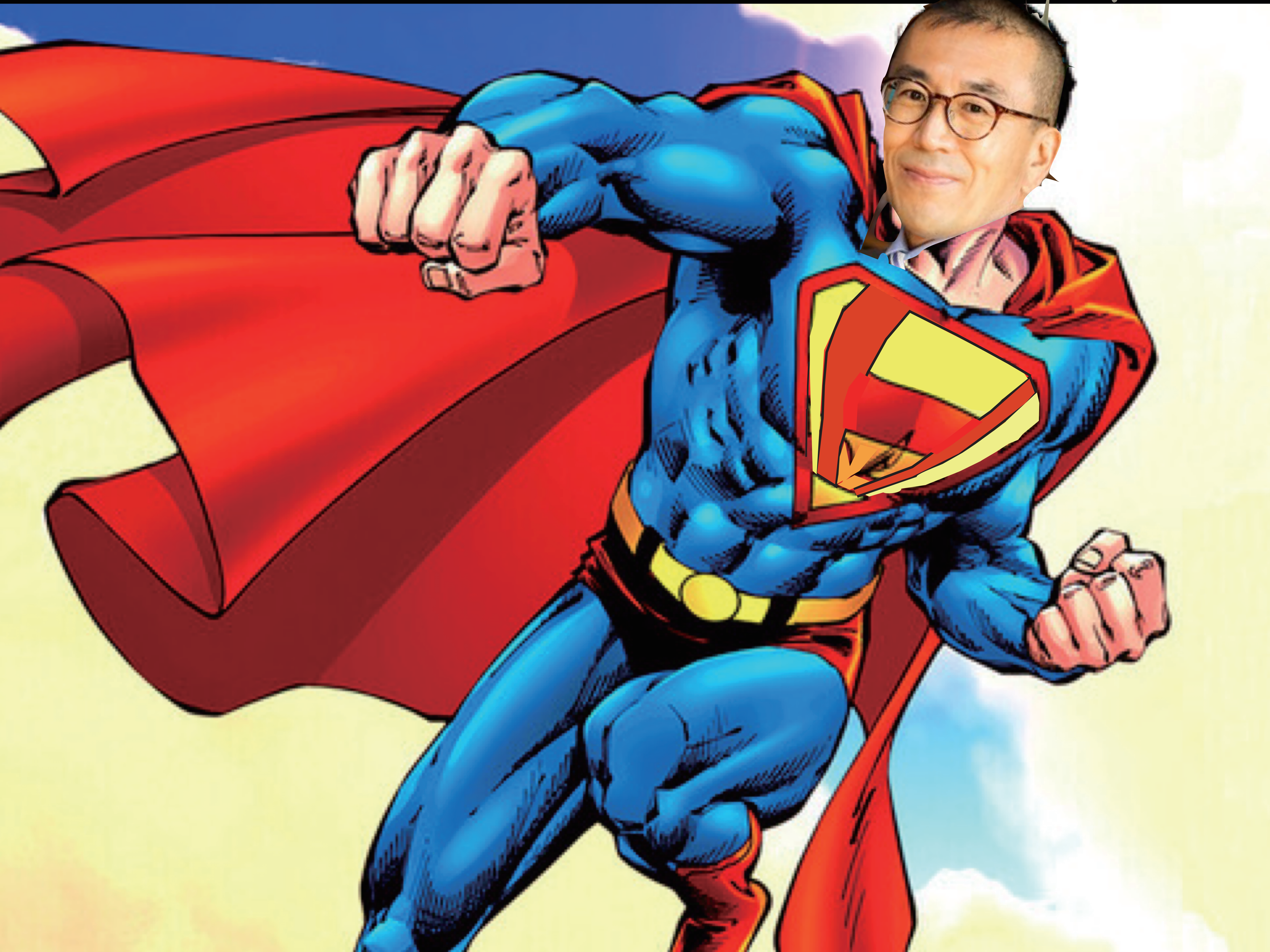


non-perturbative Quantum field theories

Hitoshi Murayama (Berkeley, Kavli IPMU)

HirosiFest @ KavliIPMU, Oct 20, 2022





references

vector-like

chiral

condensed matter

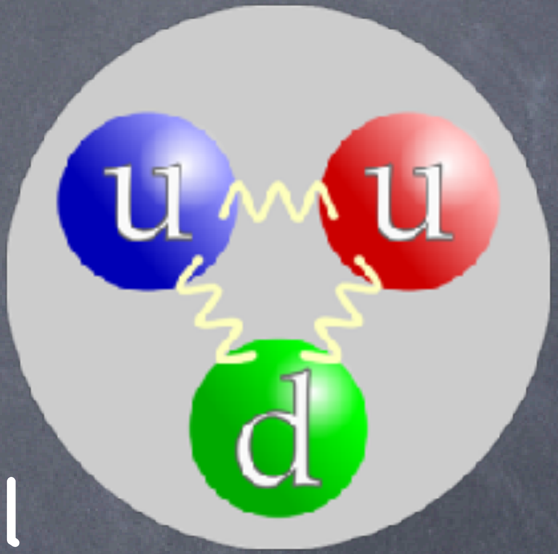
- HM, 2104.01179, PRL
- Csáki, HM, Telem, 2104.10171, PRD
- Csáki, HM, Telem, 2105.03444, PRD
- Csáki, Gomes, HM, Telem, 2106.10288, PRL
- Csáki, Gomes, HM, Telem, 2107.02813, PRD
- HM, Noether, Varier, 2111.09690
- Leedom, HM, Suter, in preparation
- Csáki, Gomes, HM, Noether, Telem, Varier, in preparation
- Kondo, HM, Sylber, 2209.09287
- HM and Wang, in preparation





QCD

I felt cheated by QCD



- When we first learn about quarks, we get told
  - Here, look at this proton.
  - There are actually colorful, beautiful quarks inside.
  - But you can never see them, you can never take them out.
  - Nonetheless, believe, they are in there!
- Who would buy that? Internet Scam?

Dear friend,
I am Andre Ouedraogo, a banker by profession from Burkina Faso in West Africa and currently holding the post of Director Auditing and Accounting unit of the bank. It's my urgent need for a foreign partner that made me to contact you for this business. I have the opportunity of transferring the left over funds ($11.5 million) of one of my bank clients who died along with his entire family on 31 July 2000 in a plane crash. You can confirm the genuineness of the deceased death by clicking on this website.
http://news.bbc.co.uk/1/hi/world/europe/859479.stm
I need a foreign partner who will support me because i can not claim this money alone without a foreign partner since the deceased client (the owner of the fund) was a foreigner.

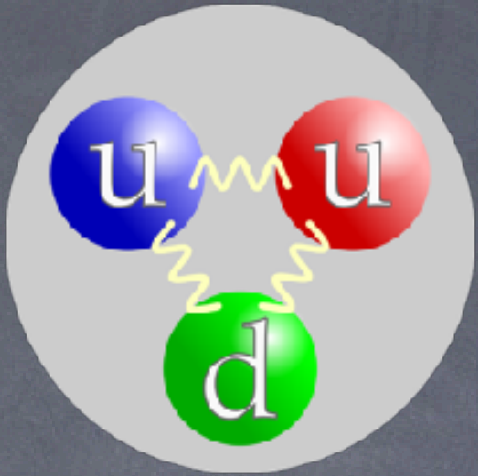
This fund ($11.5 million) will be shared between us in the ratio of 60/40. I agreed that 40% of this money will be for you as a respect to the provision of a foreign account while 60% will be for me and I want to assure you that this transaction is absolutely legal and risk free since i work in this bank and i have all the necessary information that might be needed. Before we proceed, i would like to know your ability to handle this over there in your country.
Please tell me more about the political/economic stability/monetary policy of your country. I need to know all these because i don't want to have problem with the Government of your country.

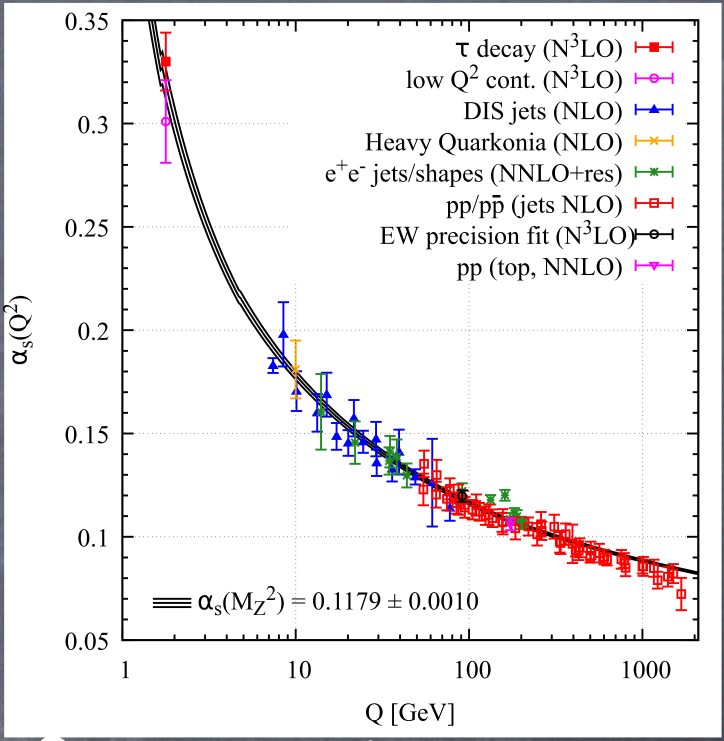
Kindly update me with the
following information because i want to know you more before we proceed on this transaction. Hope you will understand the importance of this request.

1. Your full name........................
2. Your age/sex ....
3. your occupation ....
4. Your residential address ....
5. Your nationality ....
6. Your private phone number ....
7. Your fax number ....

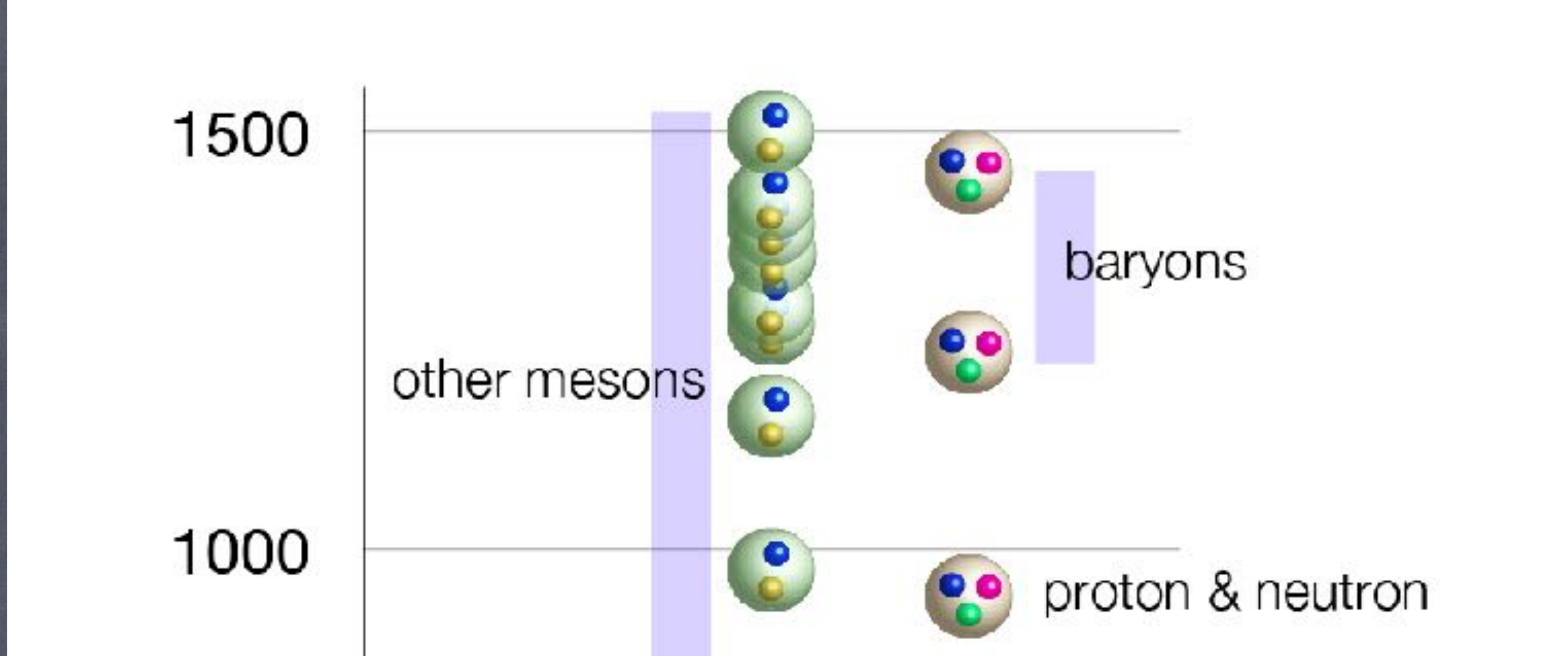
I will be waiting for your response.
Thanks for your understanding.
Have a great day.
Yours.
Andre Ouedraogo

Can we solve QCD?

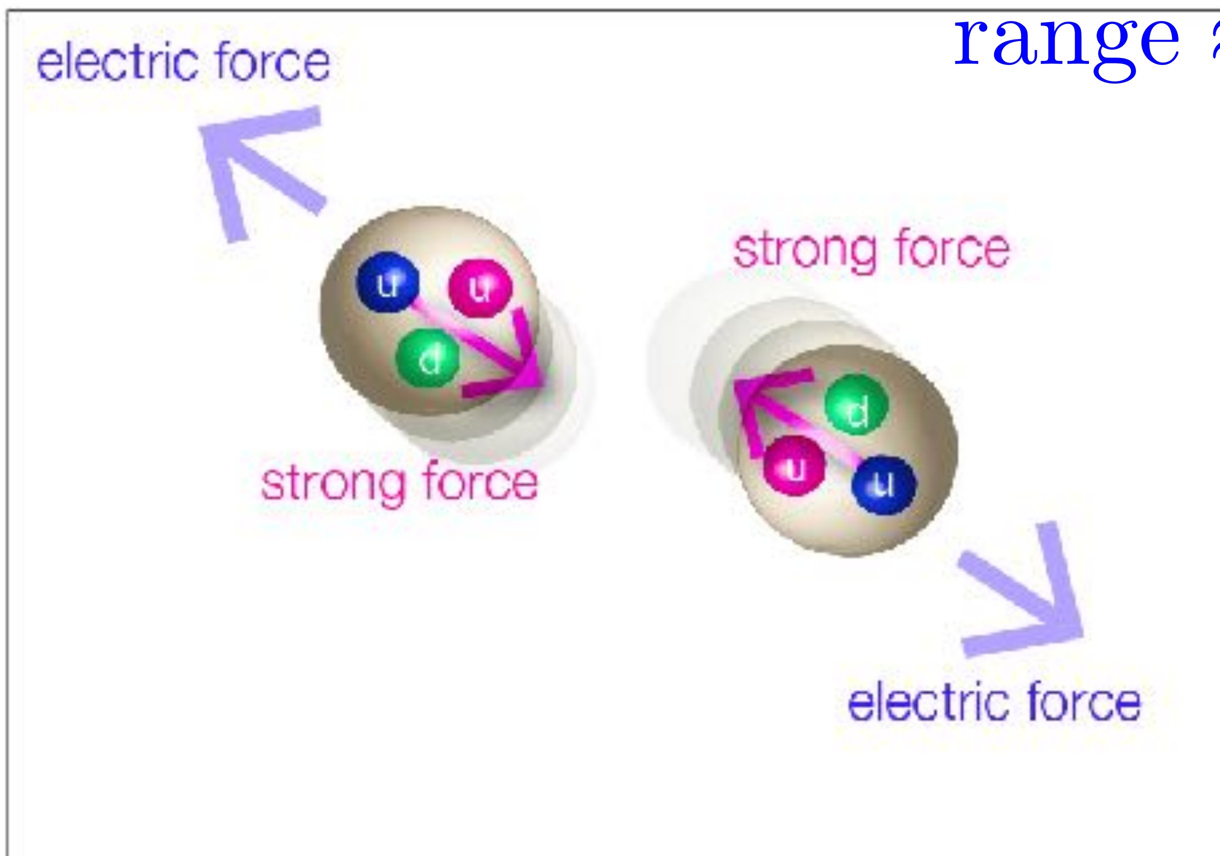




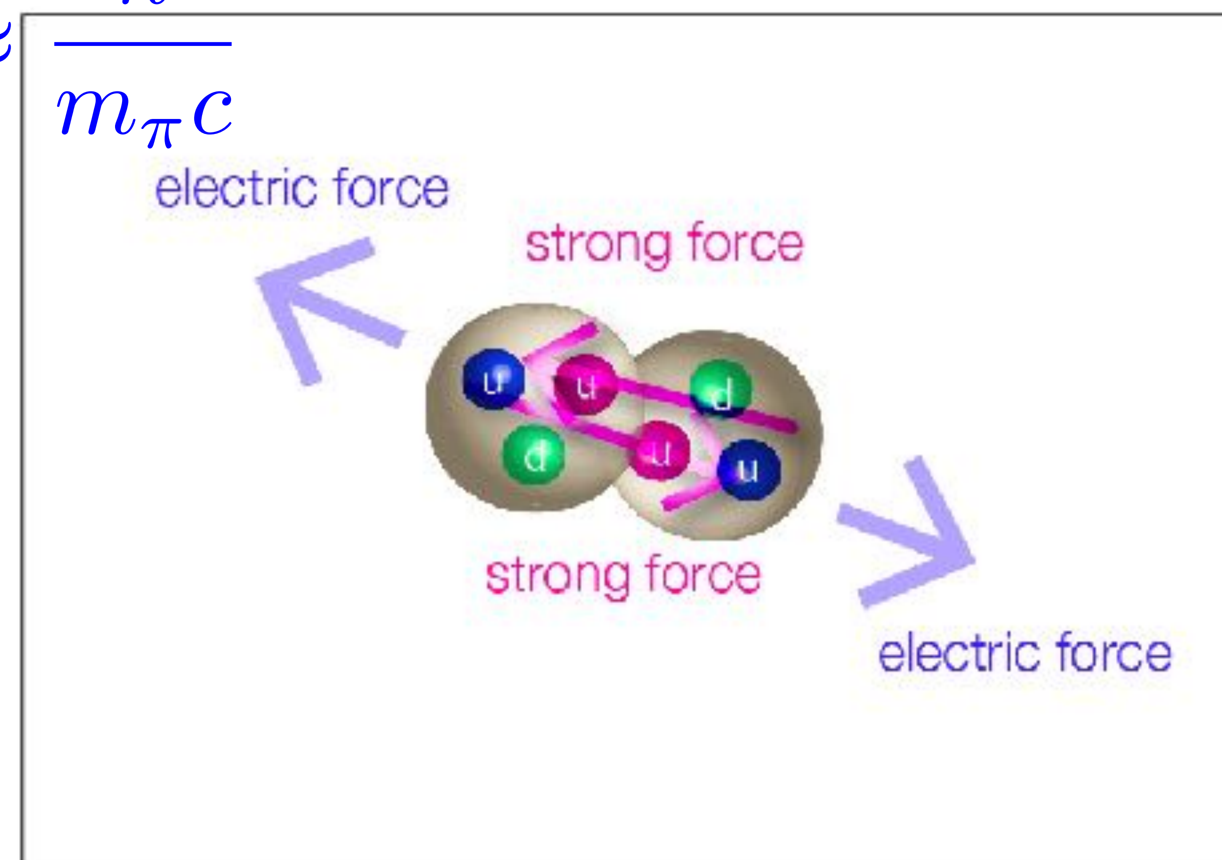
- When we first learn about quarks, we get told we can never see them
  - Internet Scam?
  - Confinement!
  - $\beta(g^2)<0$ and asymptotic freedom
  - only suggestive, doesn't prove confinement
- Another puzzle: proton and pion are made of the same quarks
  - why pion ≈ massless ≪ proton?
- very mysterious!
- Very little known about chiral gauge theories



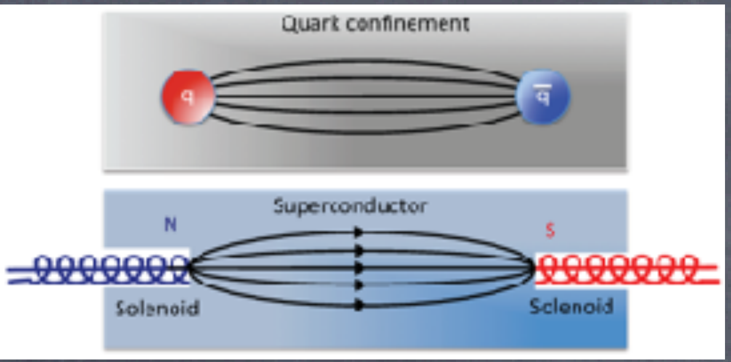
$$\text{range} \approx \frac{\hbar}{m_\pi c}$$



If pions are heavy



With real light-weight pions

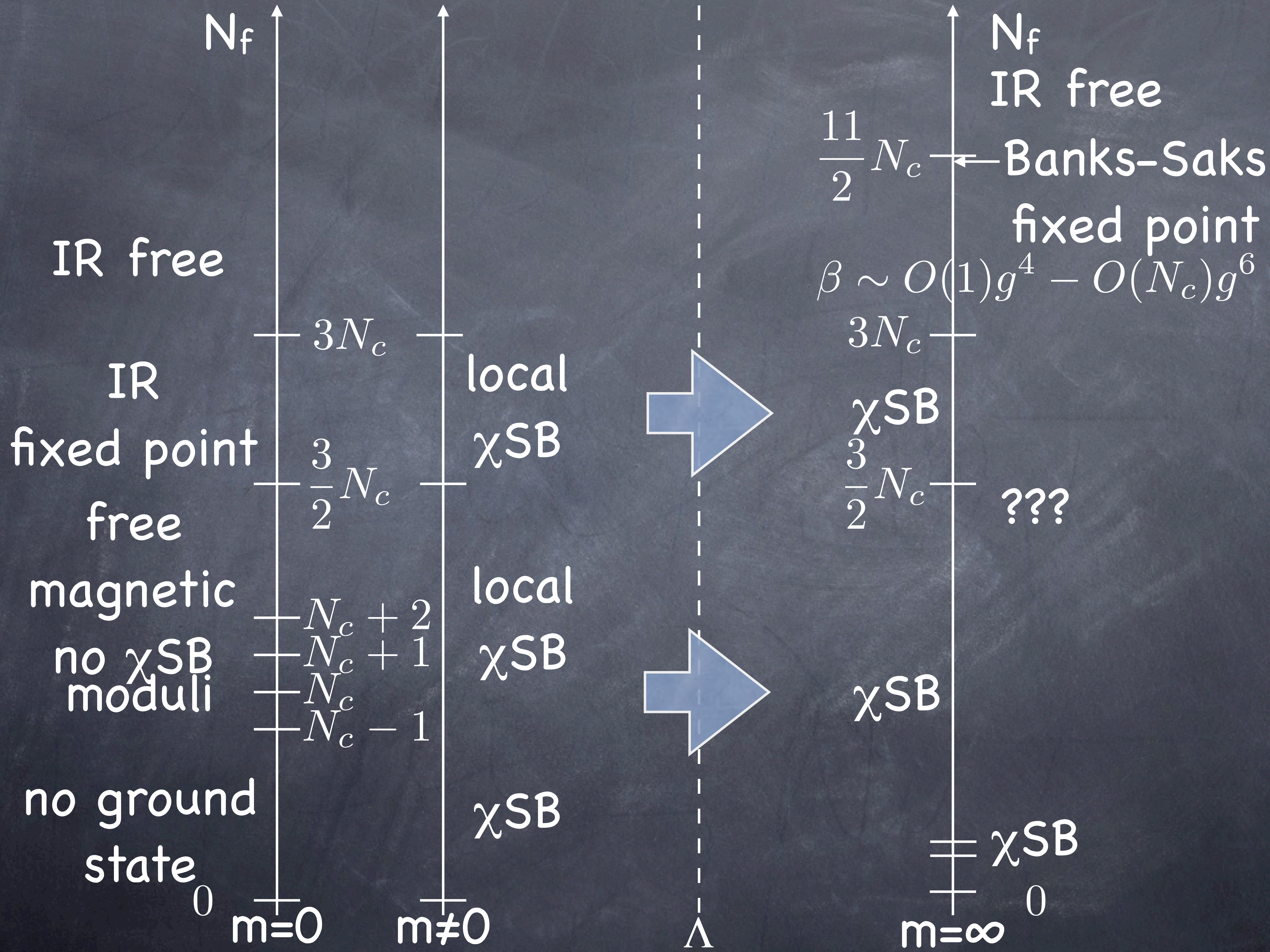


Feeling better

- Qualitative picture makes us feel better
- Confinement
  - dual Meißner effect (Mandelstam)
  - assume monopole condensation
  - quarks confined by electric flux tube
- Chiral symmetry breaking (Nambu)
  - massless QCD invariant under $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$
  - assume broken to $SU(N_f)_V \times U(1)_B$
  - pion = Nambu-Goldstone boson = massless
- Are there really monopoles?
- How do we know $\langle \bar{q} q \rangle \neq 0$?

Feeling even better

- Progress in understanding QCD
- Confinement (Seiberg-Witten)
  - N=2 SYM has Coulomb branch $u=\mathrm{Tr}\Phi^2$
  - singularities = massless monopole/dyon
  - N=1 perturbation $W = \mu u - (u-\Lambda^2)M^+M^-$
  - $M^+=M^-=\sqrt{\mu}\neq 0$: monopole condensation!
  - can further perturb to N=0 with $m_\lambda \neq 0$
- Chiral symmetry breaking
  - N=2 doesn't have χS $W = \sqrt{2}\tilde{Q}_i \Phi Q^i$
  - N=1 (Seiberg) has too unusual phases
- Chiral gauge theories
  - Standard Model is one of them
  - But even definition is not clear
  - Very little known about dynamics



Nf
IR free
IR
fixed point
free
magnetic
no χSB
moduli
no ground
state
0
m=0
$3N_c$
$\frac{3}{2}N_c$
N_c+2
N_c+1
N_c
N_c-1
local
χSB
local
χSB
χSB
m≠0
Λ
Nf
IR free
$\frac{11}{2}N_c$
Banks-Saks
fixed point
$\beta \sim O(1)g^4 - O(N_c)g^6$
$3N_c$
χSB
$\frac{3}{2}N_c$
???
χSB
χSB
0
m=∞

Main message

- Supersymmetric QCD is "solved" exactly by Seiberg in the 90s
  - but far removed from the real world
- Adding small SUSY breaking via anomaly mediation still allows for exact solution*
- derive non-perturbative behavior analytically
  - chiral symmetry breaking
  - monopole condensation and confinement
- can solve chiral gauge theories exactly, too!
- Sometimes phase transitions to non-SUSY limit, but local minima are still useful

*Not always

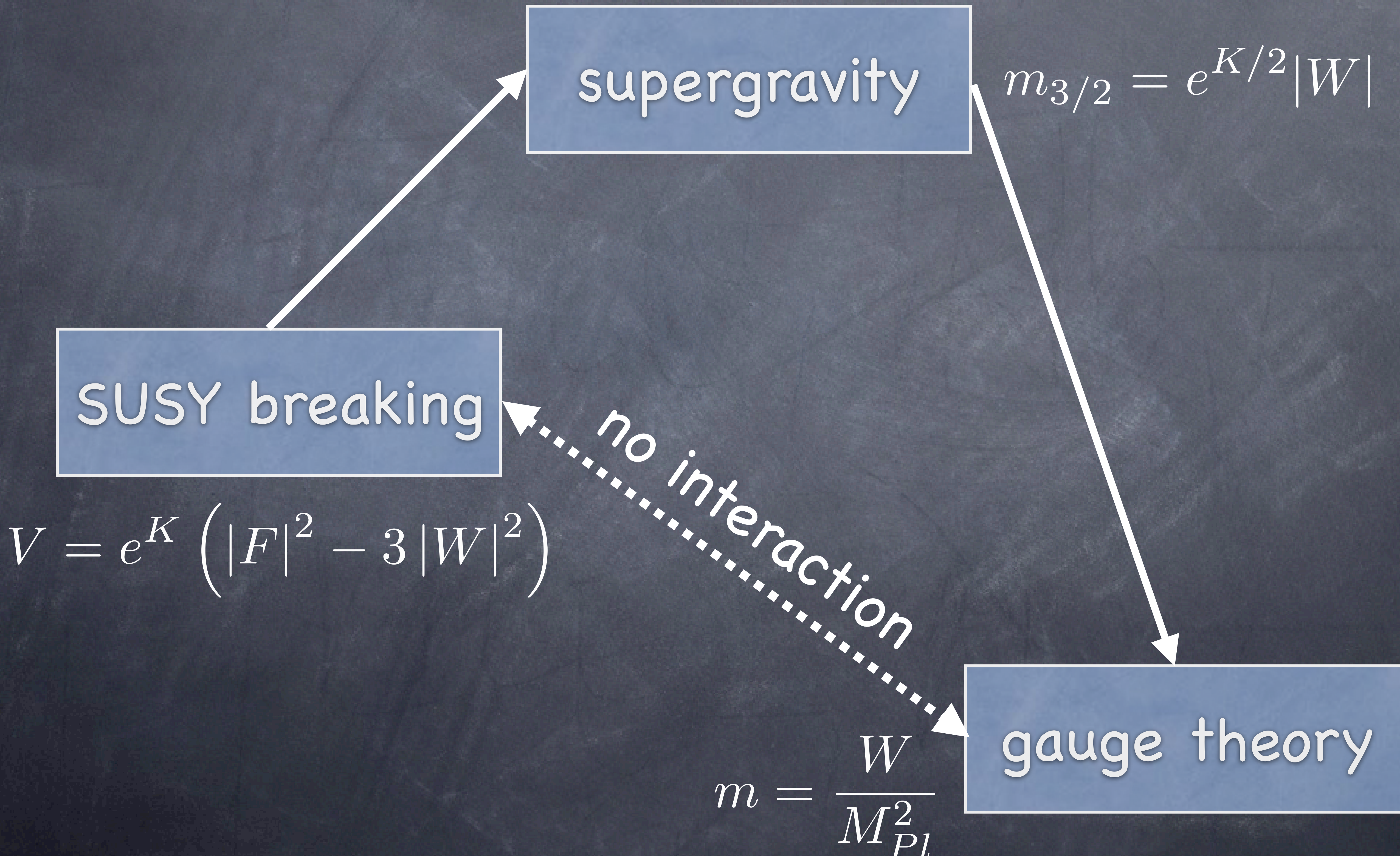
Anomaly-Mediated Supersymmetry Breaking

Randall, Sundrum (1998)
Giudice, Luty, HM, Rattazzi (1998)

Our Needs

- We'd like to connect N=1 SUSY results by Seiberg to non-SUSY gauge theories
  - decouple gauginos and squarks!
  - SUSY breaking m_λ and $m_{\tilde{Q}}$
- But we need to deal with composites such as mesons and baryons
- SUSY breaking effects on composites may be non-trivial
- Anomaly mediation!

Sequestering



Weyl compensator

- "superspacetime background"

$$\int d^4\theta \mathcal{E}^* \mathcal{E} K - \int d^2\theta \mathcal{E}^3 W$$

$$\mathcal{E} = 1 + \theta^2 m$$

- all effects of SUSY breaking encoded in $\mathcal{E}$
- it can be removed by conformal transformation $\phi \to \phi/\mathcal{E}$ if no mass params

$$\mathcal{L} = \int d^4\theta \mathcal{E}^* \mathcal{E} \phi^* \phi - \int d^2\theta \mathcal{E}^3 \lambda \phi^3$$

$$\to \mathcal{L} = \int d^4\theta \phi^* \phi - \int d^2\theta \lambda \phi^3$$

Weyl compensator

- "superspacetime background"

$$\int d^4\theta \mathcal{E}^* \mathcal{E} K - \int d^2\theta \mathcal{E}^3 W$$

$$\mathcal{E} = 1 + \theta^2 m \qquad \phi \to \phi/\mathcal{E}$$

- dimensionful parameters receive SUSY breaking

$$\mathcal{L} = \int d^4\theta \mathcal{E}^* \mathcal{E} \phi^* \phi - \int d^2\theta \mathcal{E}^3 \left(\frac{1}{2} M \phi^2 + \lambda \phi^3 \right)$$

$$\to \mathcal{L} = \int d^4\theta \phi^* \phi - \int d^2\theta \left(\frac{1}{2} M \mathcal{E} \phi^2 + \lambda \phi^3 \right)$$

$$V_{\rm AMSB} = -m \left(\phi \frac{\partial W}{\partial \phi} - 3W \right)$$

Superconformal Anomaly

- Cutoff scale also acquires SUSY breaking

$$\frac{1}{g^2}\left(\frac{\mu}{M}\right) \to \frac{1}{g^2}\left(\frac{\mu}{M\mathcal{E}}\right) = \frac{1}{g^2} - \theta^2 \frac{\beta(g^2)}{g^4} m$$

$$\int d^2\theta \frac{1}{g^2}\left(\frac{\mu}{M\mathcal{E}}\right) W_\alpha W^\alpha \supset -\frac{\beta(g^2)}{4g^4} m\lambda\lambda$$

$$m_\lambda = -\frac{\beta(g^2)}{2g^2} m$$

- determined only by physics at the energy scale of interest
- UV insensitivity!

Superconformal Anomaly

- Cutoff scale also acquires SUSY breaking

$$Z\left(\frac{\mu}{M}\right) \to \mathcal{Z}\left(\frac{\mu}{M\mathcal{E}}\right) = Z\left(1 + \gamma\frac{1}{2}\ln\frac{\mu^2}{M^*\mathcal{E}^*M\mathcal{E}} + \frac{1}{2}\dot{\gamma}\frac{1}{4}\ln^2\frac{\mu^2}{M^*\mathcal{E}^*M\mathcal{E}} + \cdots\right)$$

$$\int d^4\theta \mathcal{Z}\phi^*\phi = Z\left(F^*F + \gamma\frac{1}{2}(m^*\phi^*F + m\phi F^*) + \frac{1}{2}\dot{\gamma}m^*m\phi^*\phi\right)$$

$$V = -\frac{1}{4}\dot{\gamma}_i m^* m\phi_i^*\phi_i - \frac{1}{2}(\gamma_i + \gamma_j + \gamma_k)m\lambda_{ijk}\phi_i\phi_j\phi_k + c.c.$$

- determined only by physics at the energy scale of interest
- UV insensitivity!

AMSB Summary

- Tree-level piece on dimensionful parameters

$$V_{\text{AMSB}} = -m\left(\phi\frac{\partial W}{\partial \phi} - 3W\right)$$

- loop-level piece from running

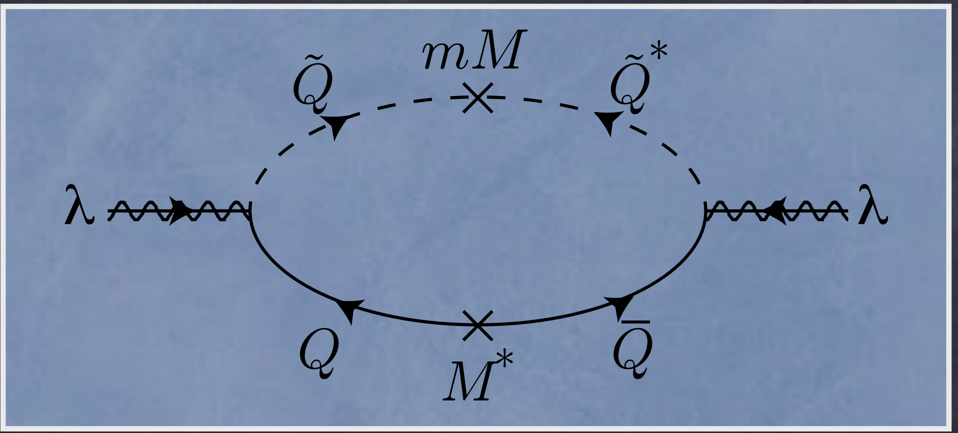
$$M_i = -\frac{\beta_i(g^2)}{2g_i^2}m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4}m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k)m_{3/2}$$

- determined only by physics at the energy scale of interest
- UV insensitivity!

UV insensitivity

$$M_i = -\frac{\beta_i(g^2)}{2g_i^2} m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4} m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

- Surprising result: AMSB depends only on physics at the energy scale of interest
- No matter how complicated the UV physics is, they all disappear from low-energy soft SUSY breaking
- e.g., decouple a massive matter field:
  - Changes the beta function
  - one-loop threshold correction precisely account for the change in gaugino mass



UV insensitivity cont.

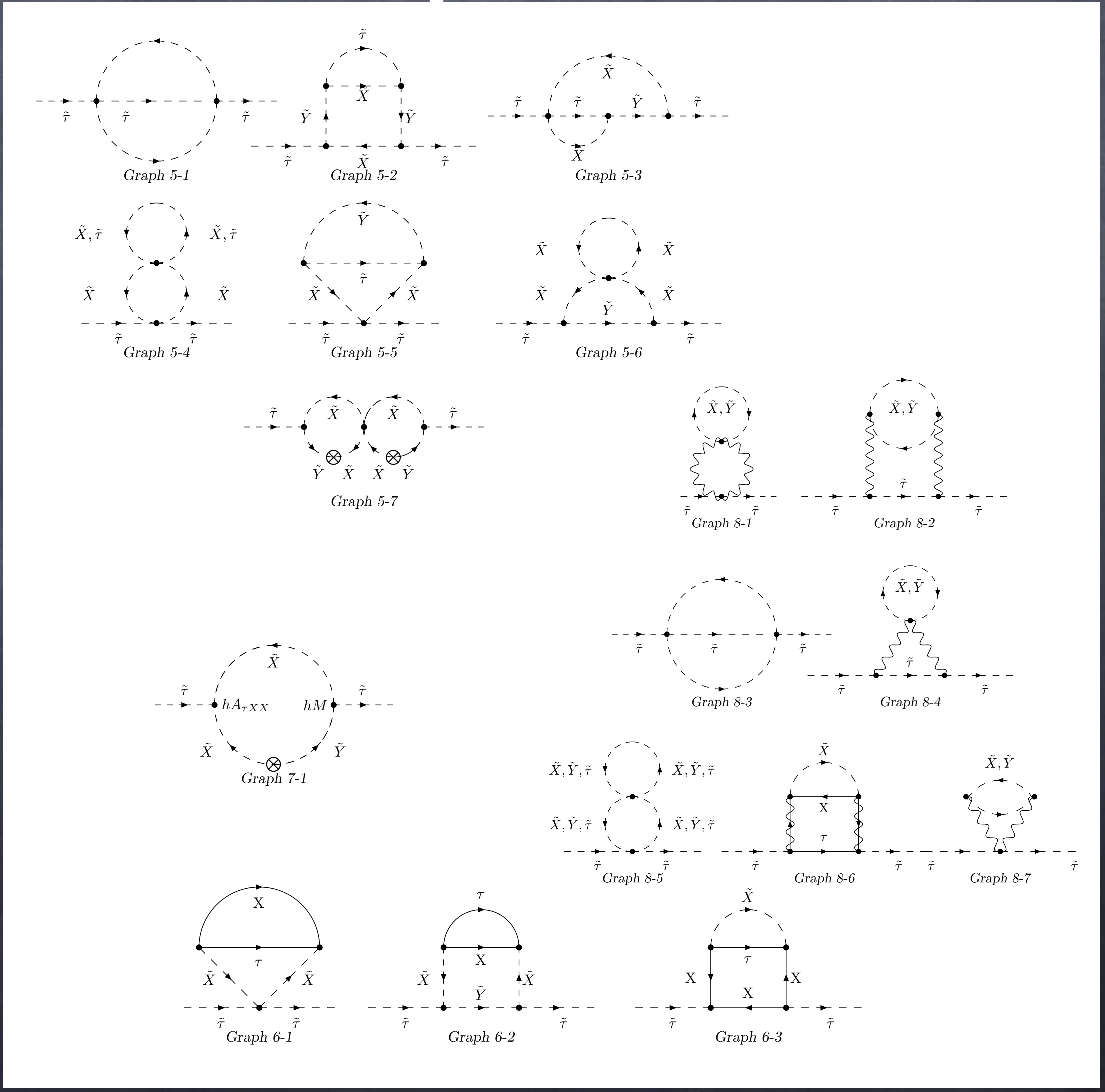
- decouple a massive matter field
- two-loop threshold correction precisely account for the change in the anomalous dimension and hence the scalar mass

$$M_i = -\frac{\beta_i(g^2)}{2g_i^2} m_{3/2}$$

$$m_i^2 = -\frac{\dot{\gamma}_i}{4} m_{3/2}^2,$$

$$A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

Boyda, HM, Pierce 2001



It is sometimes convenient to introduce a complex gravitino mass, defined as

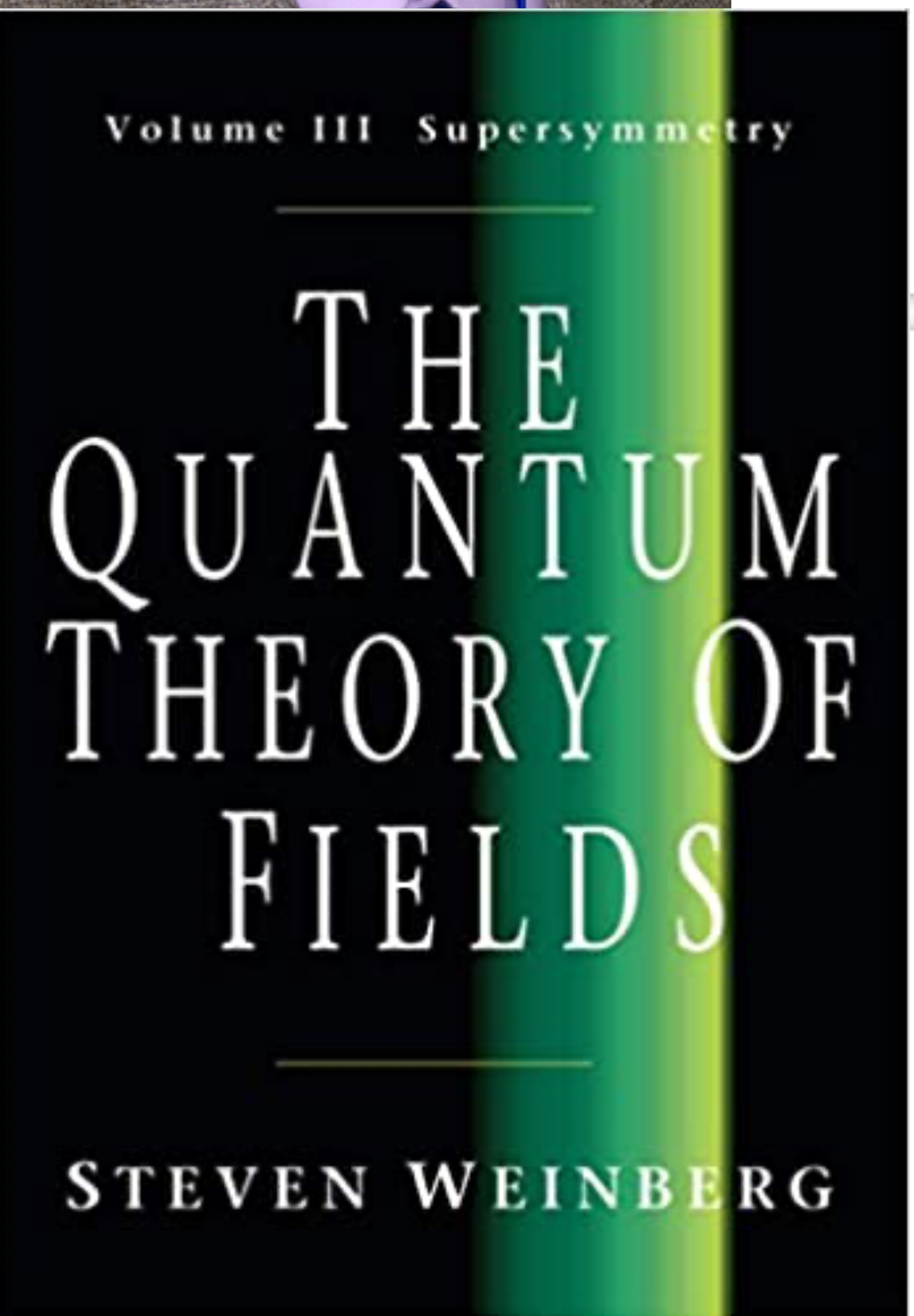
$$\tilde{m}_g \equiv \frac{2\kappa}{3}\Big(\langle s\rangle + i\langle p\rangle\Big)\,, \tag{31.3.20}$$

whose absolute magnitude is the physical gravitino mass (31.3.19).

31.4 Anomaly-Mediated Supersymmetry Breaking

In Section 28.3 the possibility was raised that supersymmetry may be broken in some sort of hidden sector of superfields that do not carry the $SU(3) \times SU(2) \times U(1)$ quantum numbers of the standard model, and communicated to observable particles gravitationally. In this section we will deal with one class of supersymmetry-breaking effects in the minimum supersymmetric standard model, those of first order in $\kappa \equiv \sqrt{8\pi G}$. This includes the gaugino masses and the parameters A_{ij} and B in the Lagrangian density (28.4.1). Other supersymmetry-breaking effects such as squark and slepton squared masses are of second order in κ, and will be taken up in Section 31.7, when we consider gravity-mediated supersymmetry breaking using the general supergravity formalism described in Section 31.6.

We can find the effects of gravity-mediated supersymmetry breaking to first order in κ by simply replacing the component fields of the gravitational supermultiplet in the interaction (31.1.34) with their expectation values. The only ones of these component fields that can acquire non-vanishing vacuum expectation values from the spontaneous breakdown



I must acknowledge my special intellectual debt to colleagues at the University of Texas, notably Luis Boya, Phil Candelas, Bryce and Cecile De Witt, Willy Fischler, Daniel Freed, Joaquim Gomis, Vadim Kaplunovsky, and especially Jacques Distler. Also, Sally Dawson, Michael Dine, Michael Duff, Lawrence Hall, Hitoshi Murayama, Joe Polchinski, Edward Witten, and Bruno Zumino gave valuable help with special topics. Jonathan Evans read through the manuscript of this volume, and made many valuable suggestions. For pointing out various errors in the first printing of this book, I am greatly indebted to Stephen Adler, Jose Espinora, Tony Gherghetta, and San Fu Tuan. For corrections to the first printing of this volume I am indebted to several colleagues, especially Stephen Adler. Thanks are due to Alyce Wilson, who prepared the illustrations, to Terry

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STEVEN WEINBERG

Austin, Texas
May, 1999

QCD-like theories $SU(N_c)$, $SO(N_c)$, $Sp(2N_c)$

HM, 2104.01179, PRL 126 (2021) 25, 251601

Csáki, Gomes, HM, Telem, 2106.10288, PRL

Csáki, Gomes, HM, Telem, 2107.02813, PRD

HM, Noether, Varier, 2111.09690

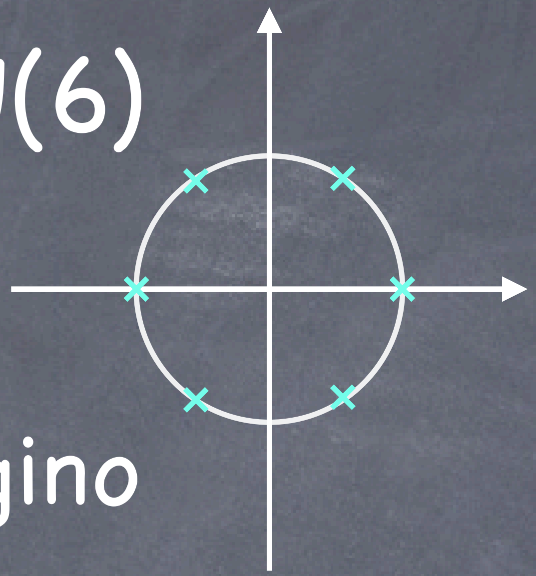
Csáki, Gomes, HM, Noether, Telem, Varier

in preparation

QCD-like theories

- SU(N_c) theory with N_f $Q(N_c)$ and $\bar{Q}(N_c^*)$
  - Expect $SU(N_f)_Q \times SU(N_f)_{\tilde{Q}} \to SU(N_f)_V$
- SO(N_c) theory with N_f $Q(N_c)$
  - Expect $SU(N_f)_Q \to SO(N_f)_Q$
- Sp($2N_c$) theory with $2N_f$ $Q(2N_c)$
  - Expect $SU(2N_f)_Q \to Sp(2N_f)_Q$

SU(6)



N_f=0

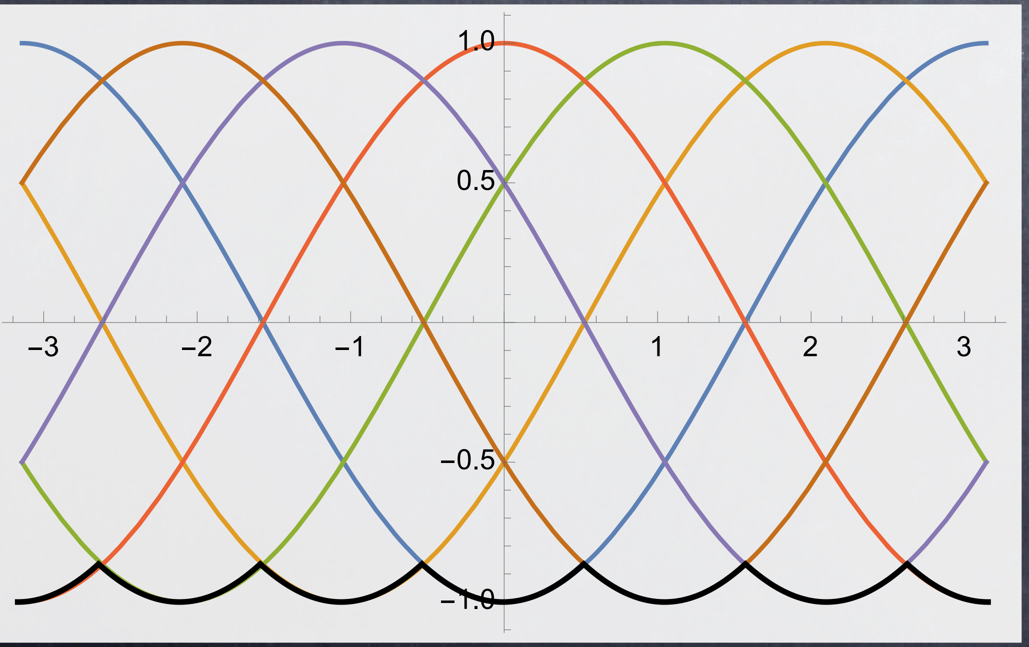
- Pure SU(N_c) YM: gauge boson and gaugino

$$\Lambda^{3N_c} = \exp\left(-\frac{8\pi^2}{g^2} - i\theta\right)$$

- Only possible non-perturbative superpotential

$$W = (\Lambda^{3N_c})^{1/N_c} = e^{2\pi i/N_c} \exp\frac{1}{N_c}\left(-\frac{8\pi^2}{g^2} - i\theta\right)$$

- N_c possible values of gaugino condensate



$$\langle\lambda\lambda\rangle = e^{2\pi i/N_c}\Lambda^{N_c}$$

$$V = -3mW = -6m\left|\Lambda\right|^3 \cos\left(\theta + k\frac{2\pi}{N_c}\right)$$

θ

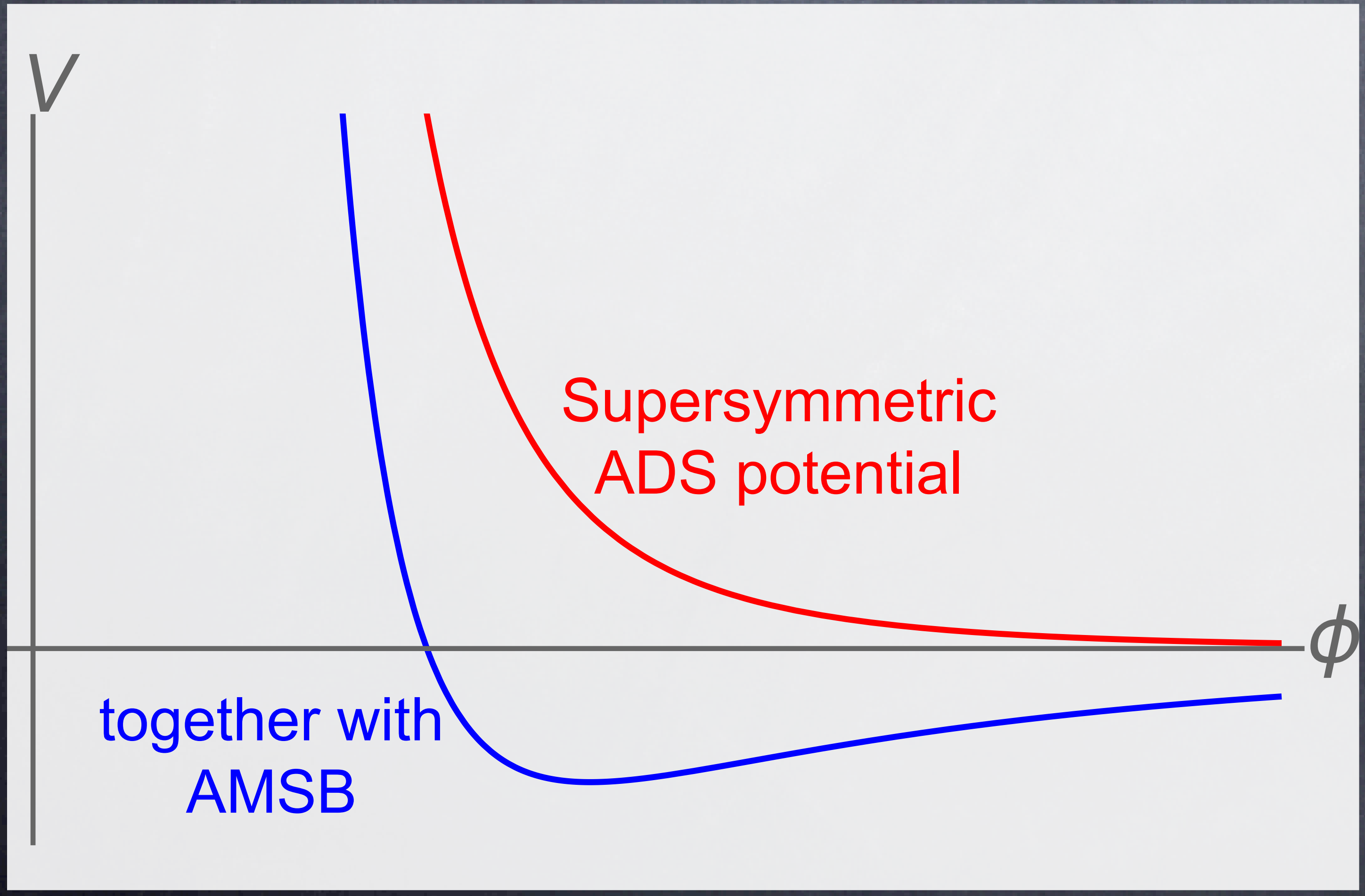
$$k = 0, 1, \cdots, N_c - 1$$

$N_f < N_c$

- run-away superpotential for $M^{ij} = \tilde{Q}^i Q^j$

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c-N_f}}{\det M}\right)^{1/(N_c-N_f)} \qquad M^{ij} = \delta^{ij}\phi^2$$

$$V = \left|2N_f \frac{1}{\phi}\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)}\right|^2 - (3N_c - N_f)m\left(\frac{\Lambda^{3N_c-N_f}}{\phi^{2N_f}}\right)^{1/(N_c-N_f)} + c.c.$$



$$M_{ij} = \Lambda^2\left(\frac{4N_f(N_c+N_f)}{3N_c-N_f}\frac{\Lambda}{m}\right)^{(N_c-N_f)/N_c}\delta_{ij}$$

$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$

χSB! Proving Nambu

Fermion loop→WZW term

$N_f=1$ special

no NGB, gapped

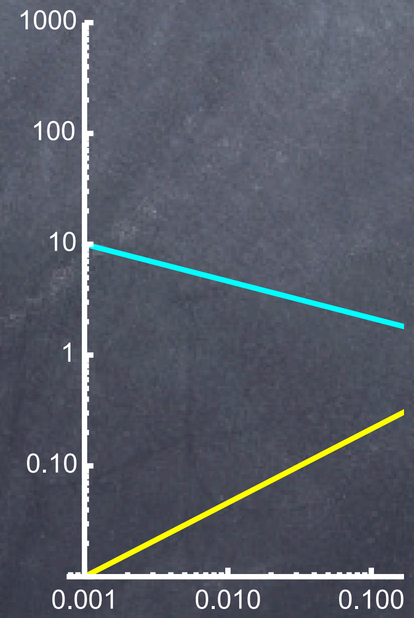
fermion bilinear

$$M^{ij} = \tilde{q}_L^{i*}\tilde{q}_R^j + \theta^2 \bar{q}_L^i q_R^j$$

$$\tilde{q}_R^{i*}\tilde{q}_L^j \sim \left(m^{N_f - N_c}\Lambda^{3N_c - N_f}\right)^{1/N_c}$$

$$\bar{q}_R^i q_L^j \sim m\left(m^{N_f - N_c}\Lambda^{3N_c - N_f}\right)^{1/N_c}$$

$$N_c = 3, N_f = 2$$

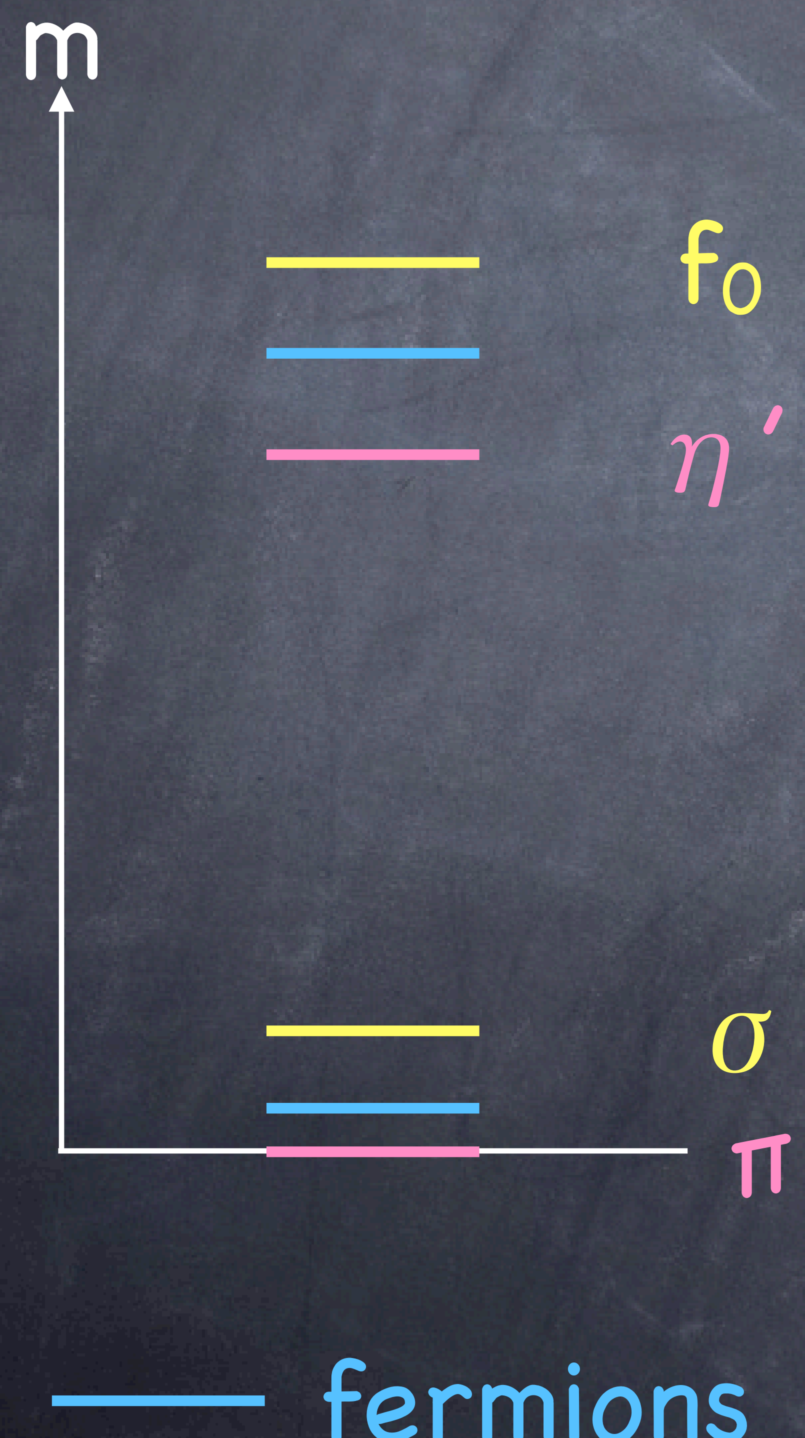


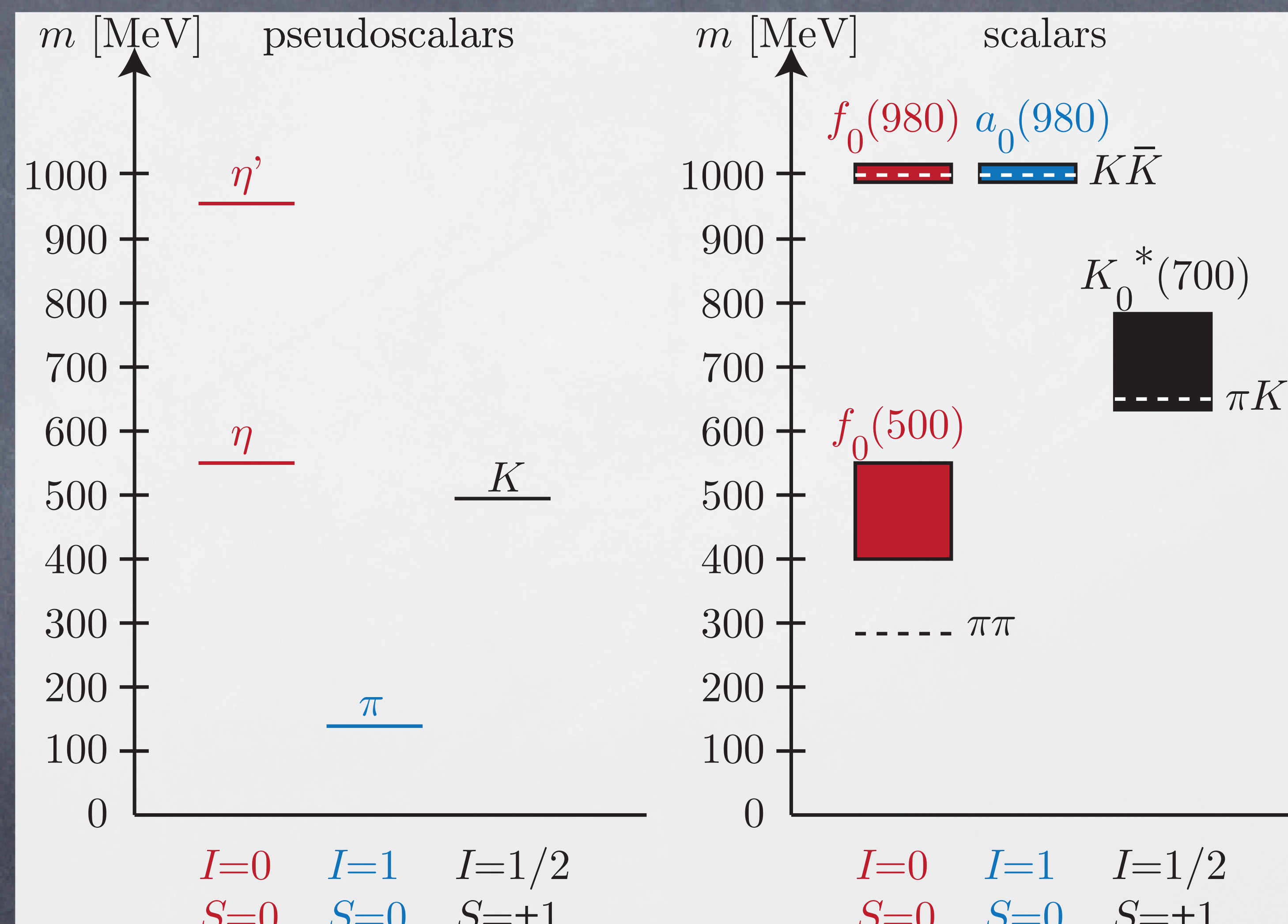
Λ

$$\Lambda^3_{\mathrm{non-SUSY}} = \left(m^{3N_c+N_f}\Lambda_{\mathrm{SUSY}}^{9N_c-3N_f}\right)^{3/(11N_c-2N_f)}$$

m

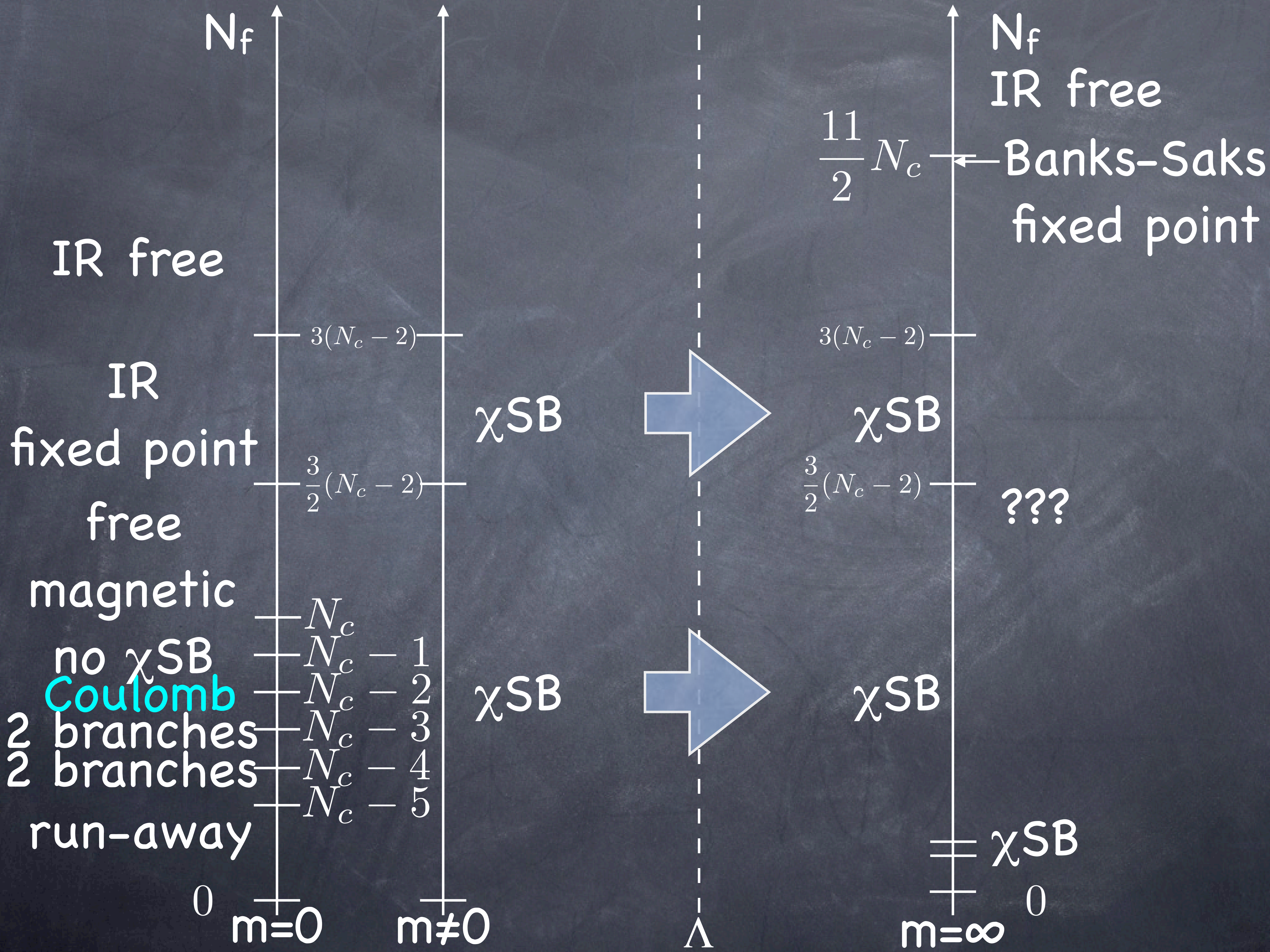
Light spectrum





confinement vs screening

- We've "derived" χSB in SU(N_c) QCD
  - it has no confinement
  - massless quarks in the fundamental rep can screen any color charges
  - Wilson loop is perimeter law
- SO(N_c) QCD with quarks in vector rep
  - cannot screen Z_2 center (e.g. spinor rep)
  - rigorous definition of confinement
  - can we see an interplay with χSB?



N_f
IR free
IR fixed point
free magnetic
no χSB
Coulomb
2 branches
2 branches
run-away
$3(N_c-2)$
$\frac{3}{2}(N_c-2)$
N_c
N_c-1
N_c-2
N_c-3
N_c-4
N_c-5
0
m=0
m≠0
χSB
χSB
Λ
N_f
IR free
$\frac{11}{2}N_c$
Banks-Saks fixed point
$3(N_c-2)$
χSB
$\frac{3}{2}(N_c-2)$
???
χSB
χSB
0
m=∞

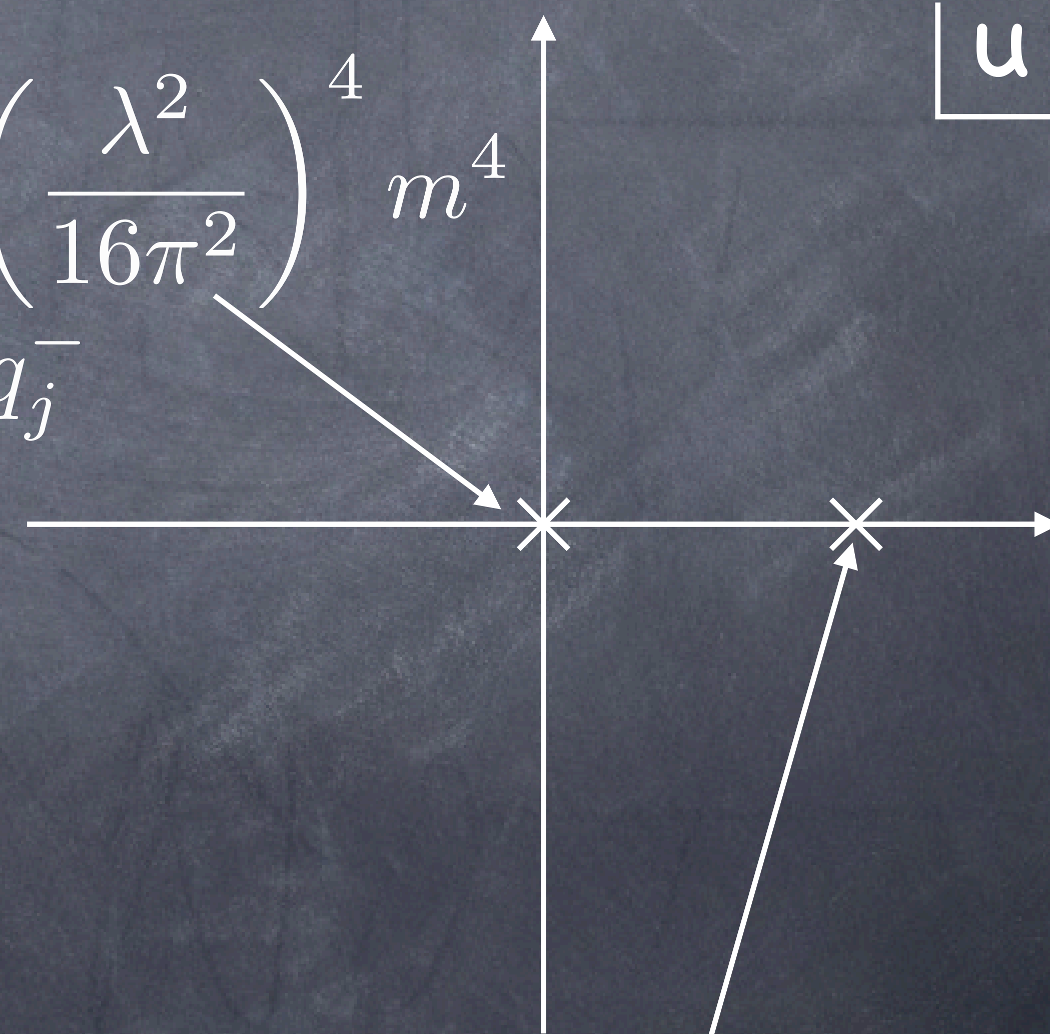
$N_f = N_c - 2$

$$Q = \begin{pmatrix} v & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & v \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{pmatrix}$$

- for $M^{ij}=Q^iQ^j \neq 0$ with rank $M=N_f$, $SO(N_c)$ is broken to $SO(2)=U(1)$
- 't Hooft Polyakov monopoles!
- Coulomb branch $u = \det M$
- two singularities
- $u = \det M = 0$
  - dyons: $q_i^{\pm}$ $\quad W = \frac{1}{\mu} M^{ij} q_i^+ q_j^-$
- $u = \det M = \Lambda^{2N_f}$
  - monopoles: $W = (u - \Lambda^{2N_f}) E^+ E^-$

    $$|E^{\pm}| = (m\Lambda)^{1/2}$$
  - both monopoles and meson condense!

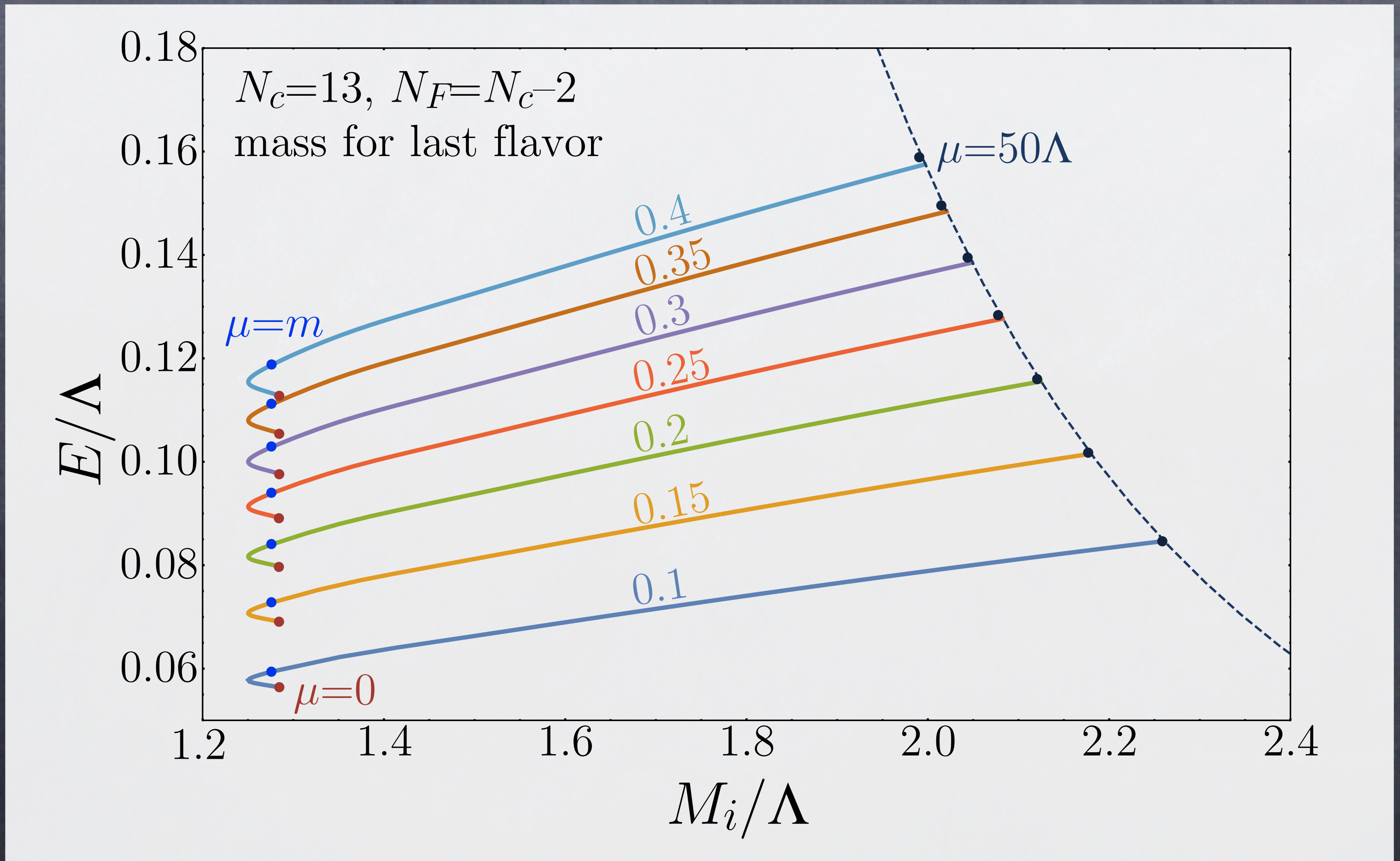
$$V \approx -\left(\frac{\lambda^2}{16\pi^2}\right)^4 m^4$$



$$V = -N_f m^2 \Lambda^2$$

$N_f < N_c - 2$

- add mass m_q to some of the quarks
- can show monopole VEVs persist $m_q \to \infty$
- demonstration of confinement and chiral symmetry breaking for all $N_f \leq N_c - 2$



$N_f = N_c$

- "Quantum Modified Moduli Space"

$$\det M - \tilde{B}B = \Lambda^{2N_c} \qquad B = \det Q, \tilde{B} = \det\tilde{Q}$$

- non-trivial 't Hooft anomaly matching implies Kähler potential is regular for M, B
- go to canonical normalization for M, B

$$W = X\left(\lambda\frac{\det M}{\Lambda^{N_c-2}} - \kappa\tilde{B}B - \Lambda^2\right)$$

$$M^{ij} = \lambda^{-1/N_c}\Lambda\delta^{ij}, \qquad B = \tilde{B} = 0,$$

$$X = \lambda^{-2/N_c}m, \qquad V = -N_c\lambda^{-2/N_c}m^2\Lambda^2$$

- Naive dimensional analysis: $\lambda \approx (4\pi)^{N_c/2}, \quad \kappa \approx 4\pi$
- $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$
- χSB! Proving Nambu for SU(3) with u, d, s
- massless pions, massive baryons!

  a loose end: need to assume $\kappa > \lambda^{2/N_c}$

$N_f = N_c+1$

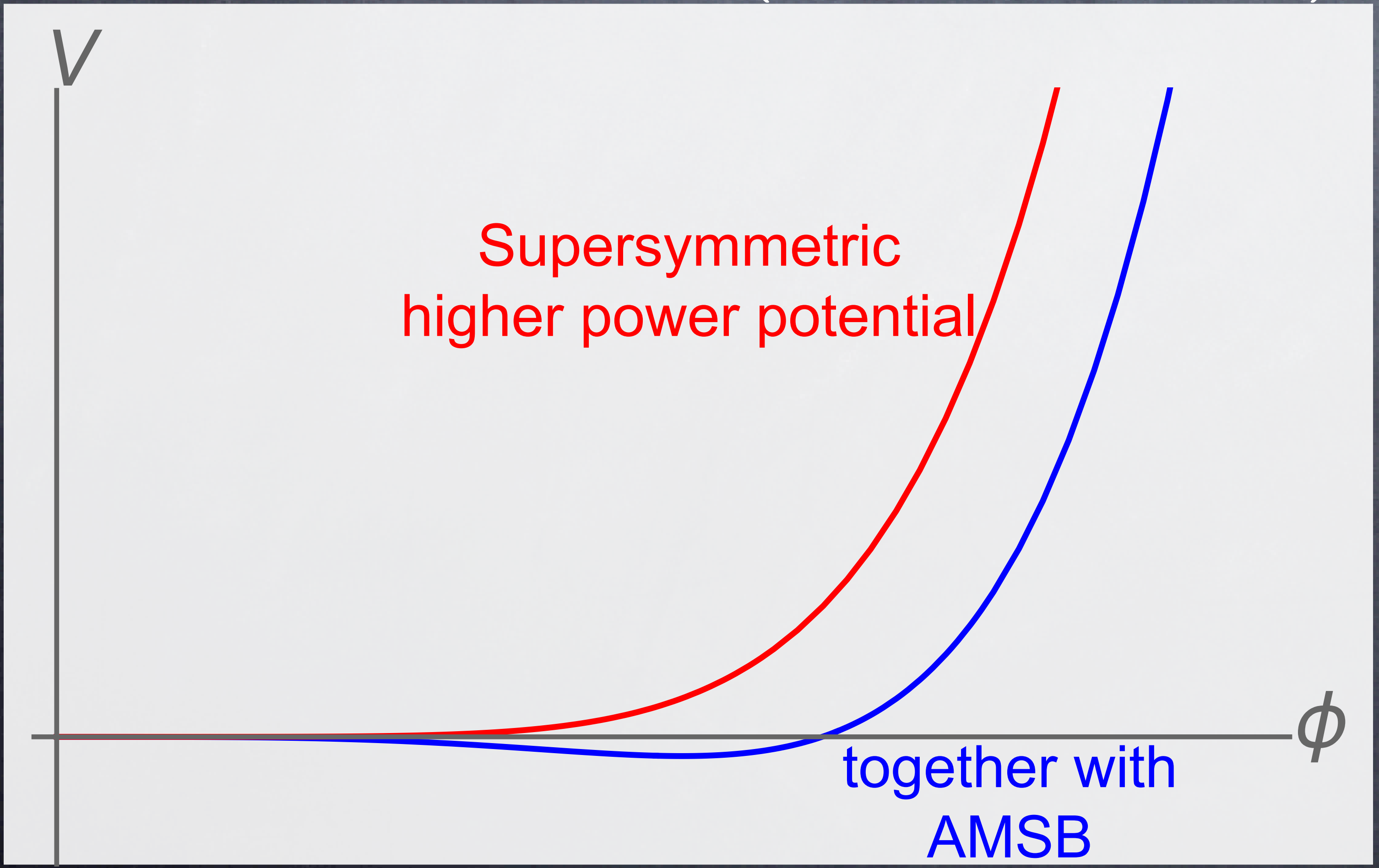
- "Confinement without χSB"

$$W = \frac{\det M - \tilde{B} M B}{\Lambda^{2N_c-1}}$$

$$W = \lambda \frac{\det M}{\Lambda^{N_f-3}} - \kappa \tilde{B} M B \qquad B_i = \epsilon_{i j_1 \cdots j_{N_c}} Q^{j_1} \cdots Q^{j_{N_c}}$$

$$V = N_f \lambda^2 \frac{|\phi|^{2N_f-2}}{\Lambda^{2N_f-6}} - \lambda (N_f - 3) m \phi^{N_f} + c.c.$$

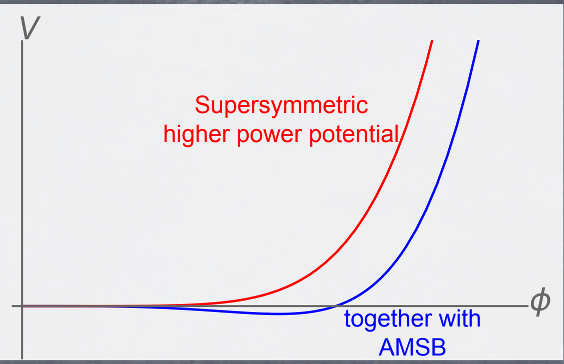
$$\phi = \kappa^{-1} \Lambda \left(\frac{2N_f - 3N_c}{N_c} \frac{m}{\Lambda} \right)^{(N_f - N_c)/(2N_c - N_f)} \ll \Lambda$$



$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$

- χSB!
- massless pions

$N_c+2 \leq N_f < 3N_c/2$



- "magnetic" IR-free $SU(N_f-N_c)$ theory

$$W = \frac{1}{\mu} M^{ij} q_i \tilde{q}_j \to \lambda \tilde{M}^{ij} q_i \tilde{q}_j$$

- go along the meson direction with rank $M=N_f$
- integrate out dual quarks with $M^{ij} = \phi \delta^{ij}$
- pure $SU(N_f-N_c)$ YM forms gaugino condensate

$$W = (N_f - N_c) \left(\frac{\kappa^{N_f} \det M}{\Lambda^{3N_c - 2N_f}} \right)^{1/(N_f - N_c)}$$

$$V = N_f \Lambda^4 \left| \frac{\kappa\phi}{\Lambda} \right|^{2N_c/(N_f-N_c)} - (2N_f - 3N_c) m \Lambda^3 \left(\frac{\kappa\phi}{\Lambda} \right)^{N_f/(N_f-N_c)} + c.c.$$

$$\phi = \kappa^{-1} \Lambda \left(\frac{2N_f - 3N_c}{N_c} \frac{m}{\Lambda} \right)^{(N_f-N_c)/(2N_c-N_f)} \ll \Lambda$$

- $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$ $\quad V \approx -\Lambda^4 \left(\frac{m}{\Lambda} \right)^{2N_c/(2N_c-N_f)}$
- deeper minimum with $U(1)_B$ breaking

Andrea Luzio, Ling-Xiao Xu2202.01239 Csaki, Gomes, HM, Noether, Telem, Varier, in prep

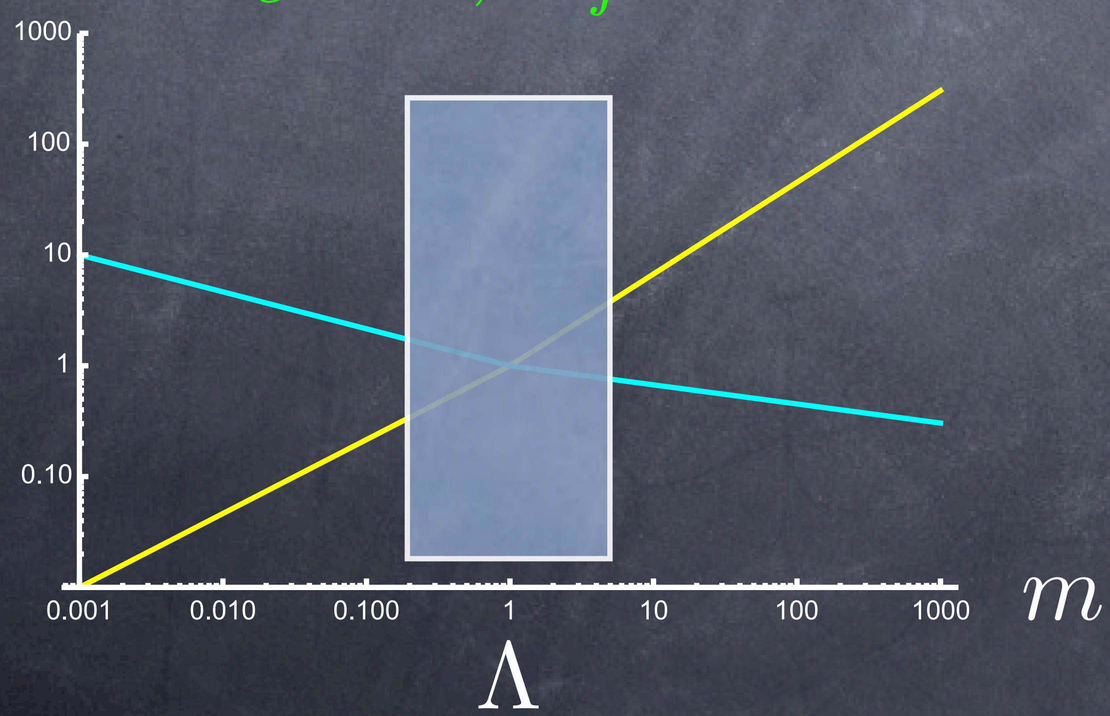
fermion bilinear

$$M^{ij} = \tilde{q}_L^{i*}\tilde{q}_R^j + \theta^2 \bar{q}_L^i q_R^j$$

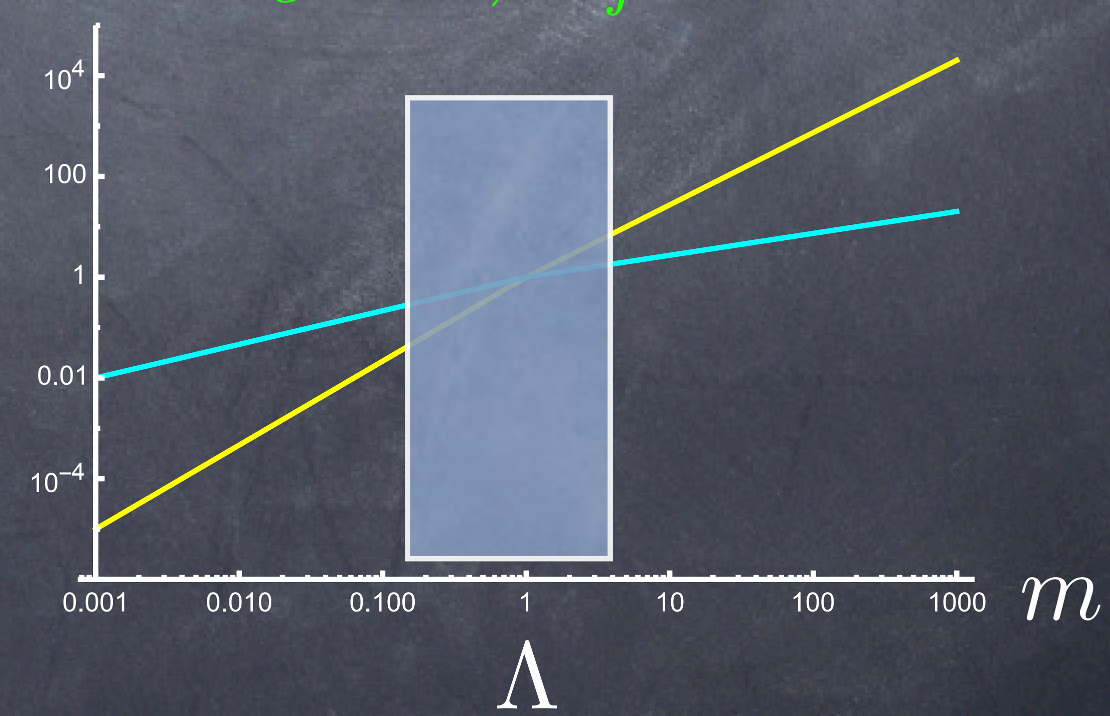
$$\tilde{q}_R^{i*}\tilde{q}_L^j \sim \left(m^{N_f - N_c}\Lambda^{3N_c - N_f}\right)^{1/N_c}$$

$$\bar{q}_R^i q_L^j \sim m\left(m^{N_f - N_c}\Lambda^{3N_c - N_f}\right)^{1/N_c}$$

$N_c = 3, N_f = 2$



$N_c = 3, N_f = 5$



$3N_c/2 < N_f < 3N_c$

- integrate q out like in free magnetic phase

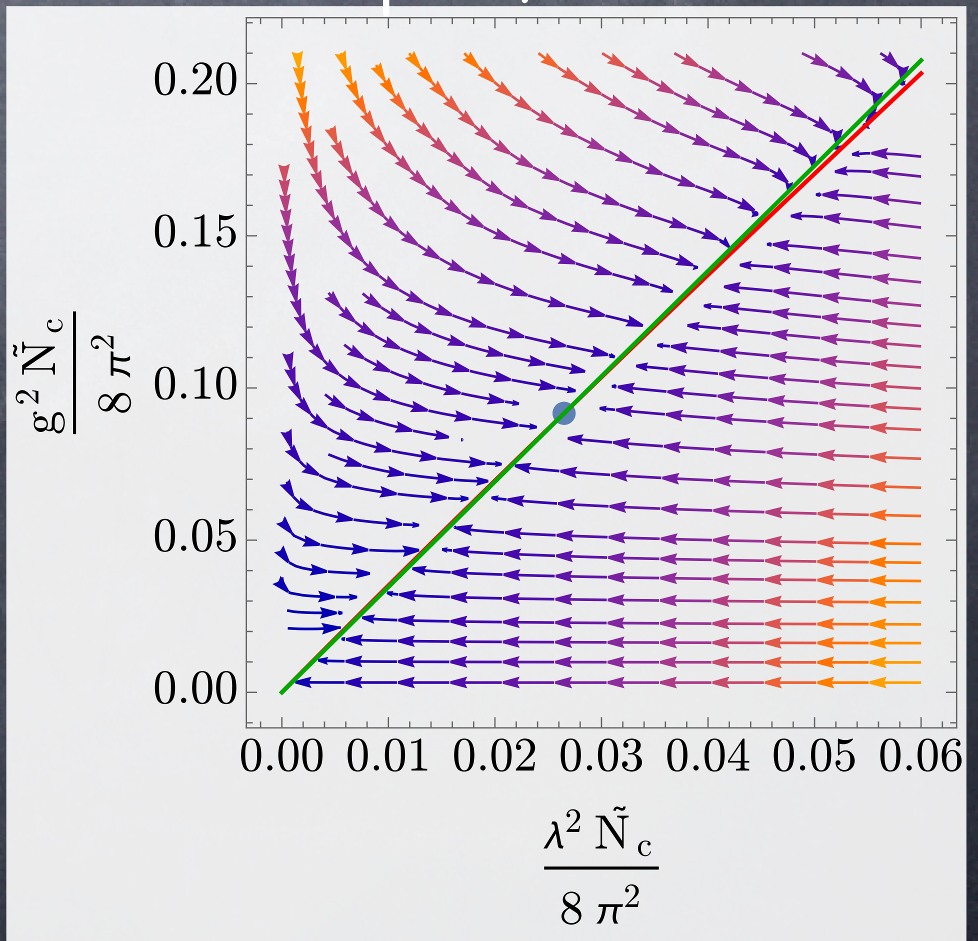
$$W = (N_f - N_c)\left(\frac{\kappa^{N_f}\det M}{\Lambda^{3N_c-2N_f}}\right)^{1/(N_f-N_c)}$$

- No stable minimum for $N_f > 2N_c$?
- Actually remember $Z_M(\mu) = \left(\frac{\mu}{\Lambda}\right)^{(6N_c-4N_f)/N_f}$
- Effective potential

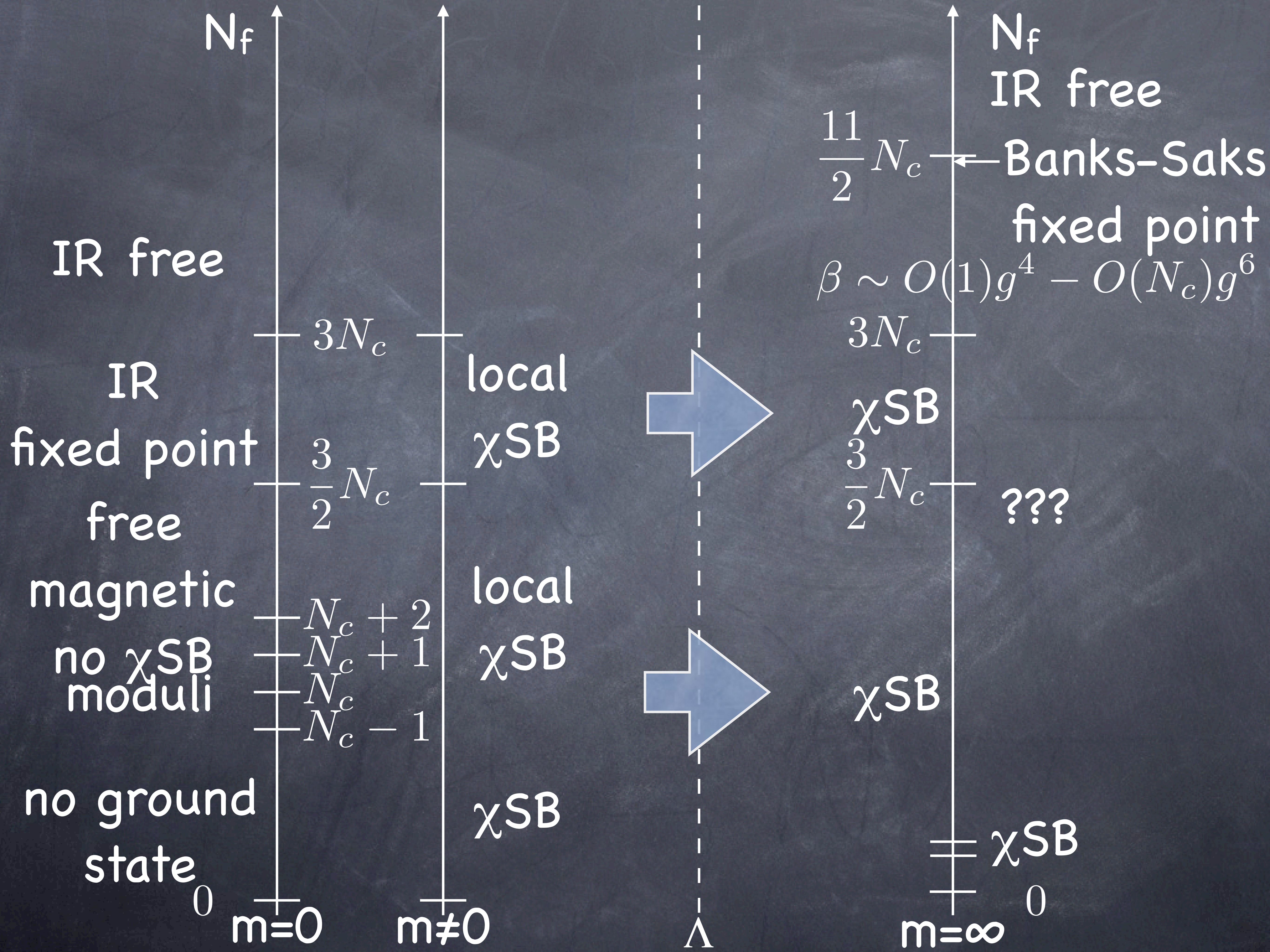
$$V = Z_M(\phi)\left|\frac{\partial W}{\partial M}\right|^2 + (N_f-3)mW$$

- Well-defined local minimum
- χSB up to $N_f < 3N_c$
- Loose end: we need the initial. condition below the fixed line

Friedland, de Gouvêa, HM
hep-th/9810020



HM, Bea Noether, Digvijay Roy Varier
arxiv:2111.09690



N_f
IR free
IR
fixed point
free
magnetic
no χSB
moduli
no ground
state
$3N_c$
$\frac{3}{2}N_c$
N_c+2
N_c+1
N_c
N_c-1
0
m=0
local
χSB
local
χSB
χSB
m≠0
Λ
N_f
IR free
$\frac{11}{2}N_c$
Banks-Saks
fixed point
$\beta \sim O(1)g^4 - O(N_c)g^6$
$3N_c$
χSB
$\frac{3}{2}N_c$
???
χSB
χSB
0
m=∞

SO(10) with 16's

Dan Kondo, HM, Cameron Sylber, 2209.09287

Chiral gauge theories

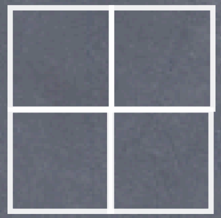
- It has been very difficult to formulate them on lattice because of fermion doubling
- Recent progress: domain wall fermions and "overlap lattice Dirac operator"
- Still challenging numerical problems
- Never simulated on lattice
- Only hand-waving ideas called "tumbling"
  - Theory breaks itself due to fermion bilinear condensates in the MAC

SO(10) with N_f 16's

- Yoshio Kikukawa: most likely chiral gauge theory to be simulated on lattice
- SO(10): smallest anomaly-free group with complex representations
- 16: smallest complex representation
  - N_f=1: dynamical SUSY breaking, gapped
  - N_f=2: gapped, SU(2) unbroken
  - N_f=3: SU(3) broken dynamically to SO(3)
  - N_f=4: SU(4)=SO(6) broken dynamically to SO(3)×SO(3) or SO(5)
- No massless composite fermions

Superpotentials

$$S_{\alpha\beta;\gamma\delta} = \frac{1}{3!} A^{IJK}_{\alpha\beta} A^{IJK}_{\gamma\delta} = S_{\gamma\delta;\alpha\beta}$$

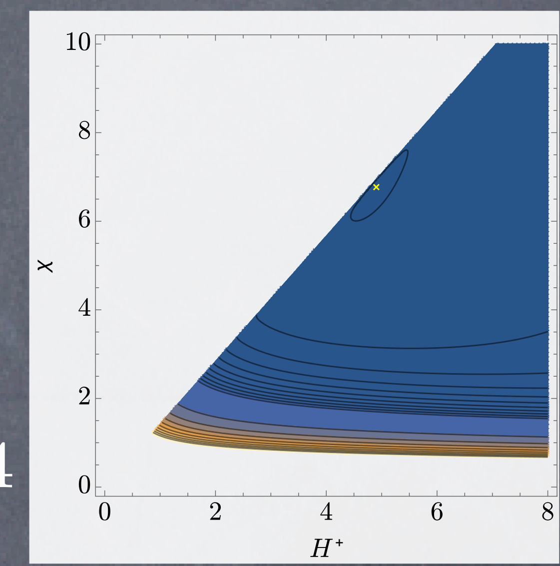


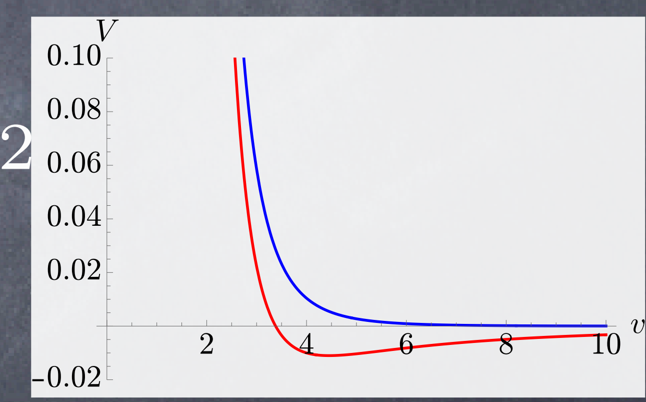
$$A^{IJK}_{\alpha\beta} = \psi^T_\alpha \mathcal{C}_{10} \Gamma^{IJK} \psi_\beta = -A^{IJK}_{\beta\alpha}$$

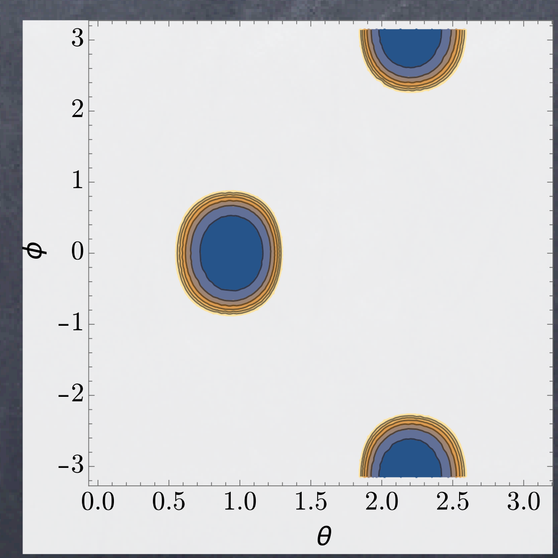
- N_f=1: Dynamical SUSY breaking (HM, hep-th/9505082), both goldstino and $U(1)_R$ NGB acquire mass by AMSB
- N_f=2: S is SU(2) singlet $W = 4\left(\frac{\Lambda^{20}}{S^2}\right)^{1/4}$
- N_f=3: S is SU(3) $\overline{\square\square}$ $W = 2\left(\frac{\Lambda^{18}}{\det \bar{S}}\right)^{1/2}$
- N_f=4: S is SO(6) $S_{ij}=S_{ji}$

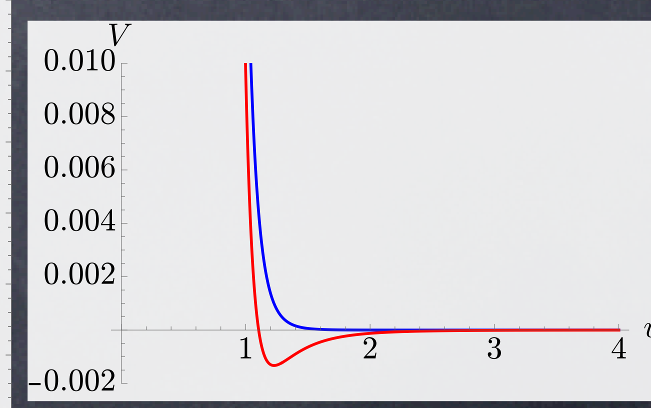
  $4\mathrm{Tr}\, S^4 - (\mathrm{Tr}\, S^2)^2 - \Lambda^{16}$

  SO(3)xSO(3) or SO(5)

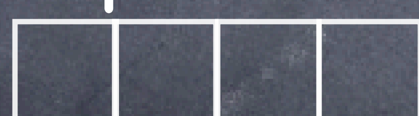
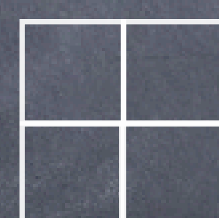








Reasonable

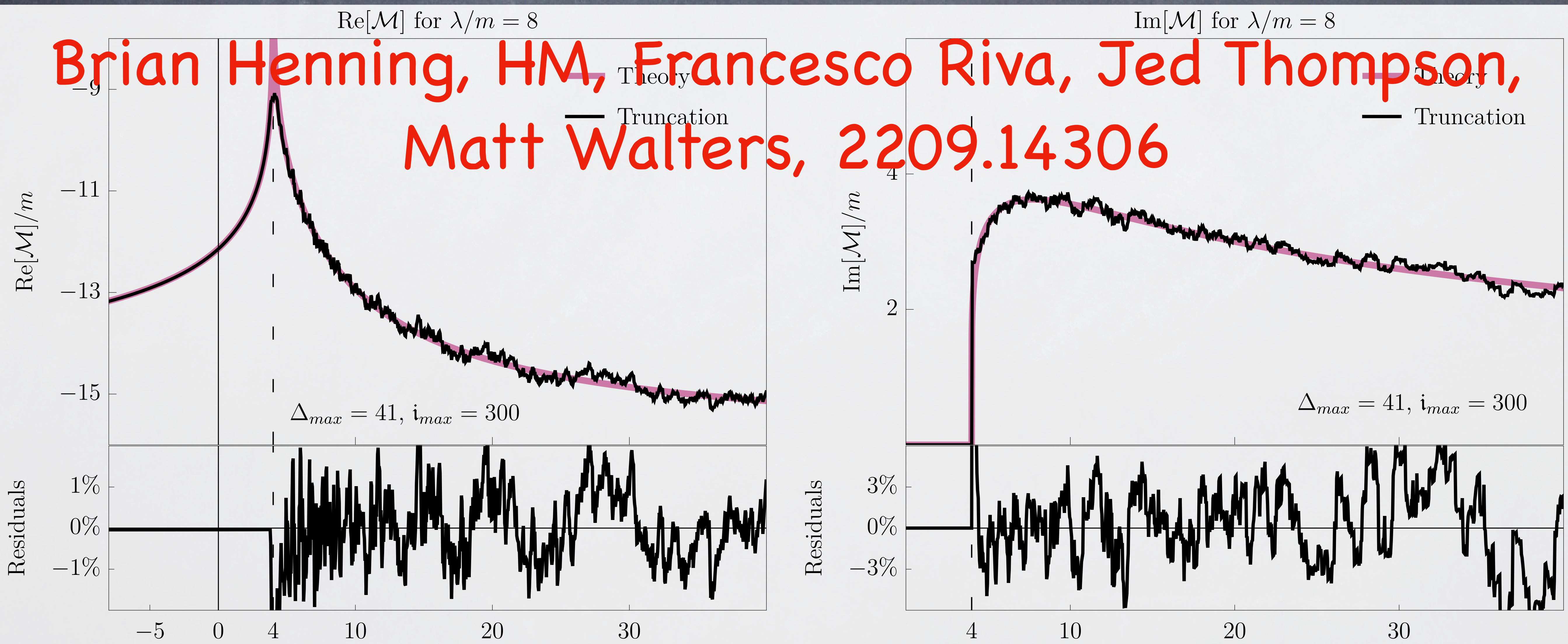
- In the non-SUSY limit, it is clear there is no massless fermion
- 16 is odd under Z_2 center
- Gauge-invariant operators are even in 16
- No way to match $SU(N_f)^3$ anomaly
- It has to be broken to an anomaly-free subgroup
- 16^4 is  or 
- MAC is $S_{\alpha\beta} = \psi_\alpha^T \mathcal{C}\Gamma_i \psi_\beta = S_{\beta\alpha}$ $SU(N_f)/SO(N_f)$
- But $Sp(N_f)$ is also conceivable for even N_f
- no useful description of SUSY limit N_f>4

Satisfying!

- N=1 SUSY + AMSB: a great tool to study non-SUSY gauge theories
- vector-like theories
  - Can show $\langle \bar{q} q \rangle \neq 0$
  - Monopole condensation for $SO(N_c)$
  - For $N_f > N_c$, local minima seem useful
- Chiral gauge theories can also be solved
  - Concrete predictions for SO(10) with 16's
  - Also for $SU(N_c)$ with A+Fbar, S+Fbar
  - Need to understand general symmetry breaking patterns with more examples

Hamiltonian truncation

- One can approximate QFT with a finite-dimensional Hilbert space w/o Wick rotation
- Can study scattering problems directly
- O(N) sigma model in 2+1 dim with non-perturbative Wilson-Fischer fixed point

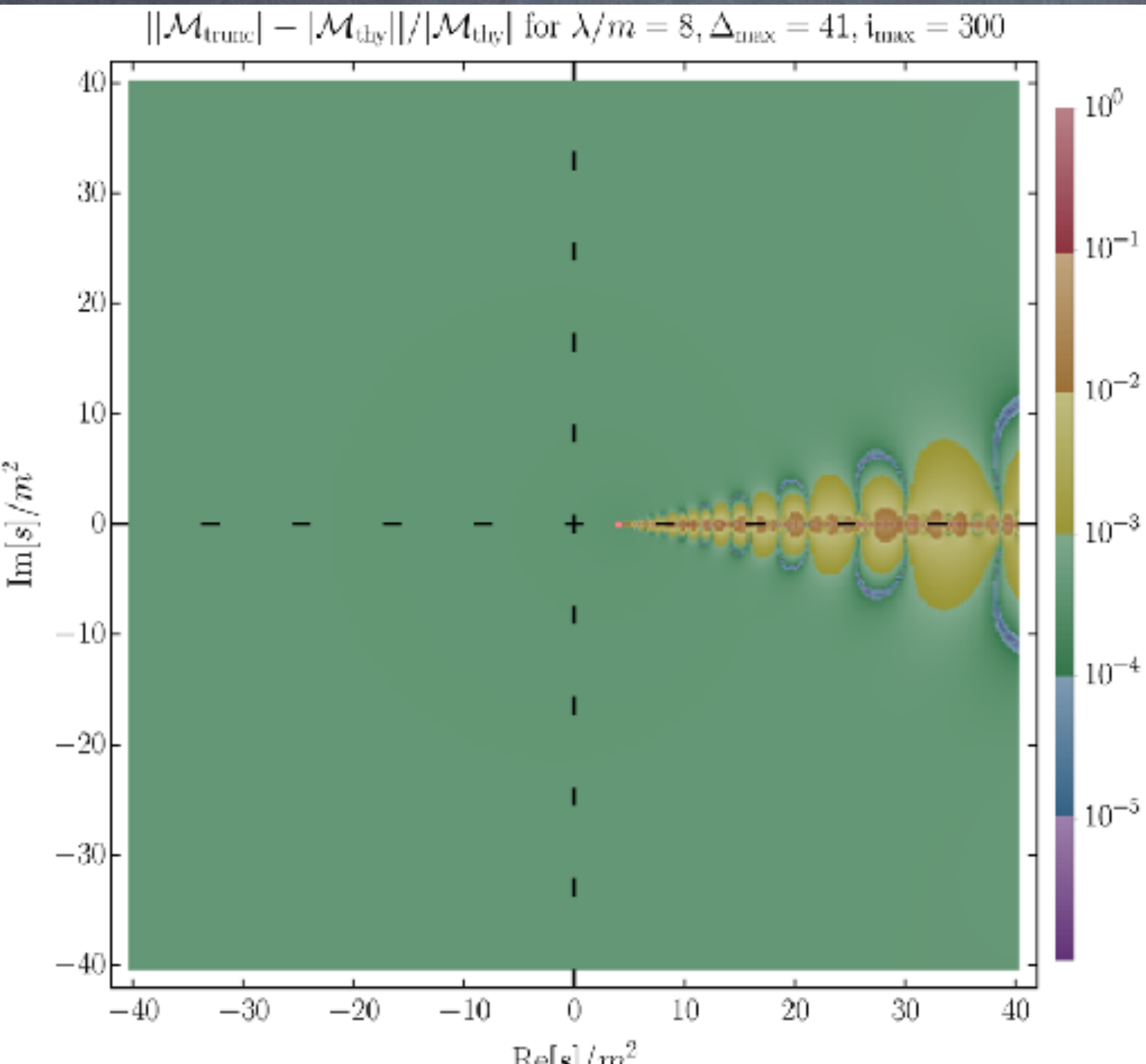


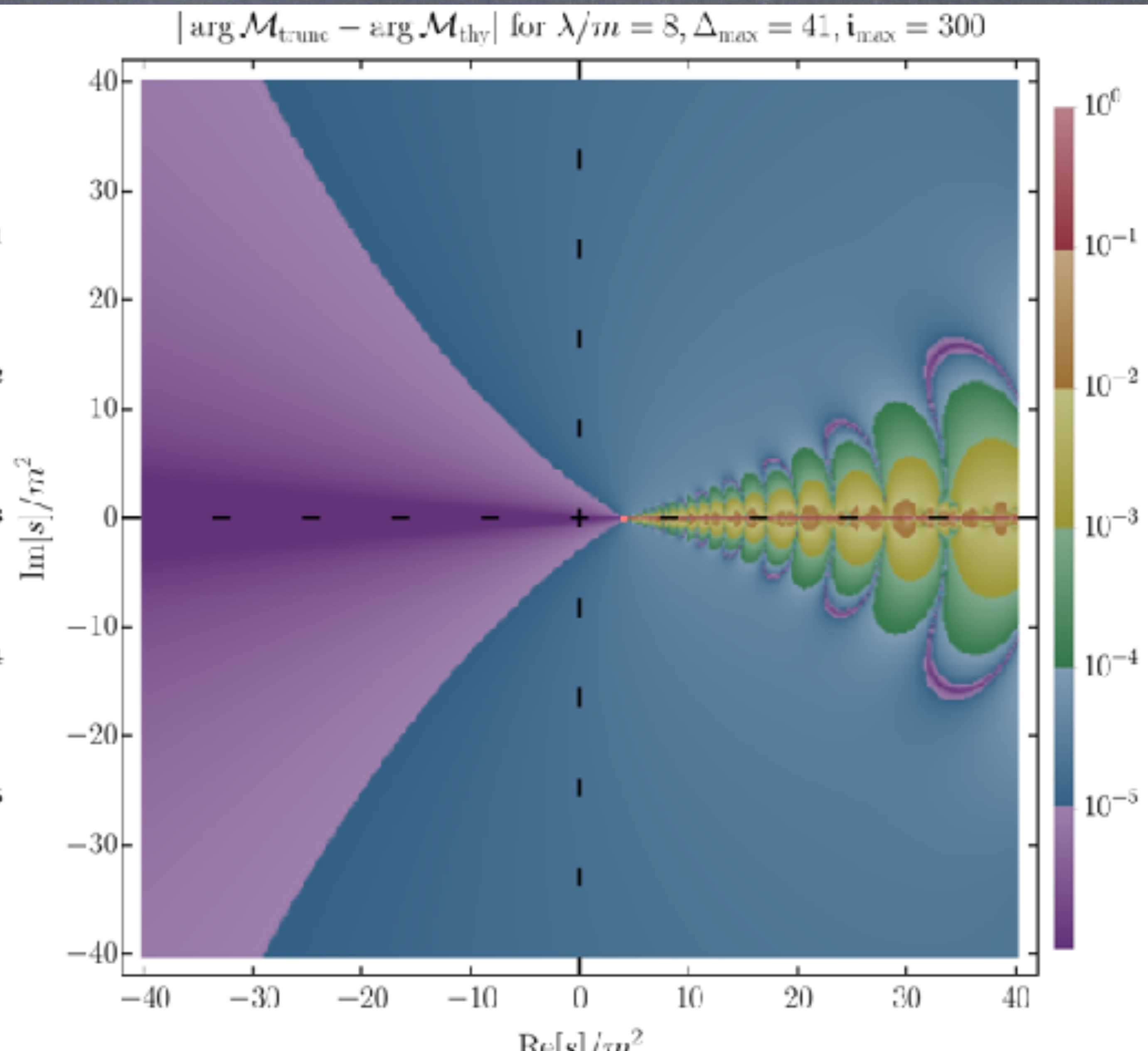
Brian Henning, HM, Francesco Riva, Jed Thompson, Matt Walters, 2209.14306

Residuals

$$\mathcal{M}^{ijk\ell}(s,t) = \frac{1}{N}\Big(\mathcal{M}(s)\delta^{ij}\delta^{k\ell} + \mathcal{M}(t)\delta^{ik}\delta^{j\ell} + \mathcal{M}(u)\delta^{i\ell}\delta^{jk}\Big) + O\Big(\frac{1}{N^2}\Big),$$

$$\mathcal{M}(s) = -\frac{2\lambda}{1+\frac{\lambda}{8\pi\sqrt{s}}\left[\log\left(\frac{\sqrt{s}+2m}{\sqrt{s}-2m}\right)+i\pi\right]}$$



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祝還暦!!
BCCスイーツ