a vs c in the landscape of 4d SCFTs

HirosiFest @Kavli IPMU

Based on:

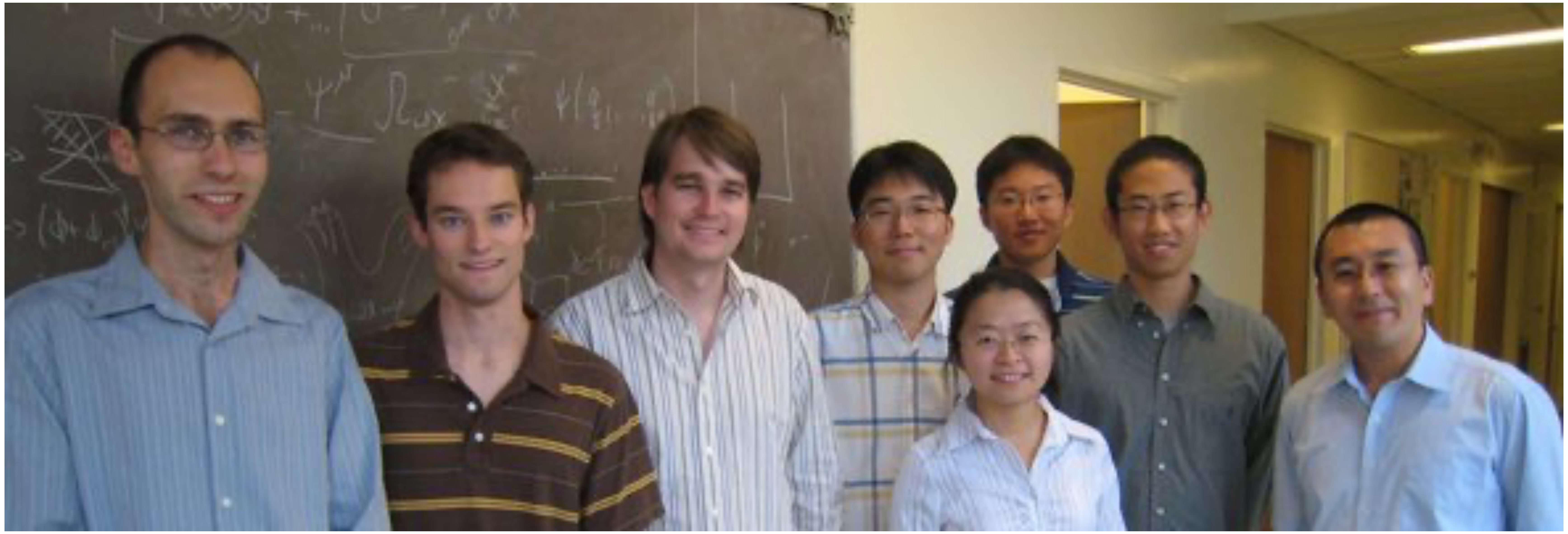
Work with Prarit Agarwal, Monica Kang, Craig Lawrie, Ki-Hong Lee, Matteo Sacchi
[1912.12881][2007.16165][2106.12579]
[2111.12092][2207.05764][2210.06497] + in progress

Korea Advanced Institute of Science and Technology

Jaewon Song

Oct. 20th, 2022

About 15 years ago…



(From Hirosi’s webpage)

IPMU in 2008:

(From the blog “CONFESSIONS OF A CLASSLESS AMERICAN GOON”)

It was a great place for research!

There was a regular tea time with delicious cookies.

It had awesome, high-tech electronic whiteboards!

arXiv:0806.4733
CALT-68-2692
IPMU-08-0038

Current Correlators for General Gauge Mediation

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My very first research paper (and the only paper co-authored with Hirosi so far) was written at the IPMU container box. (With the help of e-whiteboard!)

Just like many other graduate students, the first paper had a deep impact on my research direction in physics. But his influence on me goes far beyond this single paper.

About 10 years ago…

(From Hirosi's and Caltech Theory group's webpage)

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Conformal anomalies (central charges) a & c.

Central charges of 4d CFT

- **Conformal anomalies** of a 4d CFT are parametrized by two parameters (central charges) a & c:

$$\langle T^\mu_\mu \rangle = \frac{c}{16\pi^2} W^2 - \frac{a}{16\pi^2} E$$

- It is now well-established that a-function is a monotonically decreasing function along the RG flow (a-theorem): [Komargodski, Schwimmer]

$$a_{IR} < a_{UV}$$

- One can think of the a-function as a quantity that measures degrees of freedom.

- The c-function, on the other-hand, does ***not*** always decrease along the RG flow.

Hofman-Maldacena bound on central charges

- The ratio a/c of central charges is bounded by **unitarity**: [Hofman, Maldacena]

$$\frac{1}{2} \leq \frac{a}{c} \leq \frac{31}{18}$$ (lower/upper bound saturated by free scalar/free vector)

- For superconformal theory:
  - $\mathcal{N}$=1 SCFT: $\frac{1}{2} \leq \frac{a}{c} \leq \frac{3}{2}$ (lower/upper bound saturated by free chiral/free vector)
  - $\mathcal{N}$=2 SCFT: $\frac{1}{2} \leq \frac{a}{c} \leq \frac{5}{4}$ (lower/upper bound saturated by free hyper/free vector)
  - $\mathcal{N}$=3 or $\mathcal{N}$=4 SCFT: $a = c$ [Aharony, Evtikhiev]

The role of *a* & *c*

- Any **holographic** theories have $a = c$ (for large *N*). [Henningson, Skenderis]
- When $a \neq c$, there is a correction to the celebrated **entropy-viscosity ratio** bound of [Kovtun, Son, Starinet] to [Katz, Petrov][Buchel, Myers, Sinha]

$$\frac{\eta}{s} \geq \frac{1}{4\pi}\left(1 - \frac{c-a}{c} + \cdots\right)$$

- Appears in the Cardy-like (**high-temperature**) limit of superconformal index:

$$I(p = q = e^{-\beta}) \to \exp\left(\# \frac{3c - 2a}{\beta^2}\right)$$

[J. Kim, S. Kim, JS]
[Cabo-Bizet, Cassani, Martelli, Murthy]
[Cassani, Komargodski]

This formula accounts for the **entropy of supersymmetric black holes** in AdS_5. [Choi, Kim, Kim, Nahmgoong] [Benini, Milan] [Cabo-Bizet, Cassani, Martelli, Murthy]

- $c - a$ appears in the universal part of **entanglement entropy**. [Perlmutter, Rangamani, Rota]

Large N scaling behavior of a and c

Large *N* scaling behavior of *a* and *c*

- Typical 4d gauge theories (of rank *N*) have

$$a \sim c \sim \mathcal{O}(N^2), \quad \text{and} \quad c - a \sim \mathcal{O}(N)$$

so that $a = c$ in the **large *N*** limit, but **not for a finite *N***. (satisfying the necessary condition for it to be holographic)

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- Is this true in general?
  - Is the above **scaling behavior** for *a* and *c* true in general?
  - Any **universality** for the sign of $c - a$?

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- Is this true in general?
  - Is the above **scaling behavior** for *a* and *c* true in general?
  - Any **universality** for the sign of $c - a$?
  - Is it possible to have $a = c$ for **finite *N***? (for $\mathcal{N}$=0, 1, 2 SUSY)

Non-universal of scaling behavior of central charges a & c

(From classification of Large N superconformal gauge theories)

Classifying SUSY large N theories

[Agarwal, Lee, JS]

- Let us classify all possible supersymmetric **large *N* *gauge theories*** in 4d with the following conditions:
  - The gauge group is **simple**: G=SU(N), SO(N), Sp(N)
  - The **flavor symmetry is fixed** as we take large N limit.
  - No superpotential (except for the flipper of the decoupled operators).
- In the context of AdS/CFT:
  flavor symmetry of the boundary CFT = **gauge symmetry in the bulk**.

See [Bhardwaj, Tachikawa] for the full classification of $\mathcal{N}$=2 gauge theories.

Constraints on matter multiplets

- Gauge anomaly should be absent:

$$\sum_i \mathcal{A}(\mathbf{R}_i) = 0$$

- Asymptotic freedom (negative beta function)

$$b_0 = \left(3h^\vee - \sum_i T(\mathbf{R}_i)\right) \geq 0$$

- Above condition (and also large *N*) restrict the matter representations to fundamental, adjoint, (trace-less) symmetric, anti-symmetric.
- Let us restrict ourselves to the gauge theories flow to superconformal theories.
- cf) Conformal gauge theories ($b_0 = 0$, finite *N*) classified by [Razamat, Sabag, Zafrir]

Superconformal fixed point

- Necessary condition: Non-anomalous **U(1) R-symmetry**

$$\mathrm{Tr}RT^aT^b = 0 \leftrightarrow T(\text{adj}) + \sum_i T(R_i)(r_i - 1) = 0$$

- Due to the superconformal symmetry, the conformal anomalies are fixed by the trace anomalies of R-symmetry.

[Anselmi, Freedman, Grisaru, Johansen]

$$a = \frac{3}{32}\left(3\mathrm{Tr}R^3 - \mathrm{Tr}R\right)\ , \quad c = \frac{1}{32}\left(9\mathrm{Tr}R^3 - 5\mathrm{Tr}R\right)$$

- The R-symmetry is not always determined via anomaly constraint. There can be a family of candidate R-symmetries.

- The superconformal R-symmetry is fixed by '**a-maximization**':

$$\frac{\partial a_{\text{trial}}}{\partial R} = 0\ , \quad \frac{\partial^2 a_{\text{trial}}}{\partial R^2} < 0$$

[Intriligator, Wecht]

Decoupling of operators along the RG flow

- **Important caveat** in a-maximization: accidental symmetry
- Some of the gauge invariant operators may seem to violate the **unitarity bound**: $\Delta \geq 1$.
- Plausible scenario: such an operator gets **decoupled along the RG flow** and becomes free with $\Delta_{\mathcal{O}} = 1$. [Kutasov, Parnavhev, Sahakyan]
- One can remove the decoupled free field by introducing a '**flip field**' X and a superpotential coupling $W = X\mathcal{O}$. [Barnes, Intriligator, Wecht, Wright] [Benvenuti, Giacomelli][Maruyoshi, Nardoni, JS]
- Redo the *a*-maximization until no operator gets decoupled.

The full list of SU(*N*) theories with large *N* limit. (4+16 classes of theories)

Theory	β_{matter}	chiral	dense	conformal window
$1\ \mathbf{Adj} + N_f\ (\square + \overline{\square})$	$\sim N$	N	Y	$1 \le N_f < 2N$
$1\ \square\!\square + 1\ \overline{\square\!\square} + N_f\ (\square + \overline{\square})$	$\sim N$	N	Y	$0 \le N_f < 2N-2$
$1\ \begin{smallmatrix}\square\\\square\end{smallmatrix} + 1\ \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + N_f\ (\square + \overline{\square})$	$\sim N$	N	Y	$4 \le N_f < 2N+2$
$1\ \square\!\square + 1\ \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + 8\ \overline{\square} + N_f\ (\square + \overline{\square})$	$\sim N$	Y	Y	$0 \le N_f \le 2N-4^*$
$2\ \square\!\square + 2\ \overline{\square\!\square} + N_f\ (\square + \overline{\square})$	$\sim 2N$	N	N	$0 \le N_f < N-4$
$1\ \square\!\square + 2\ \overline{\square\!\square} + 1\ \begin{smallmatrix}\square\\\square\end{smallmatrix} + 8\ \square + N_f\ (\square + \overline{\square})$	$\sim 2N$	Y	N	$0 \le N_f < N-6$
$1\ \square\!\square + 1\ \overline{\square\!\square} + 1\ \begin{smallmatrix}\square\\\square\end{smallmatrix} + 1\ \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + N_f\ (\square + \overline{\square})$	$\sim 2N$	N	N	$0 \le N_f \le N^*$
$1\ \square\!\square + 1\ \begin{smallmatrix}\square\\\square\end{smallmatrix} + 2\ \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + 8\ \overline{\square} + N_f\ (\square + \overline{\square})$	$\sim 2N$	Y	N	$0 \le N_f < N-2$
$2\ \square\!\square + 2\ \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + 16\ \overline{\square} + N_f\ (\square + \overline{\square})$	$\sim 2N$	Y	N	$0 \le N_f < N-8$
$1\ \mathbf{Adj} + 1\ \square\!\square + 1\ \overline{\square\!\square} + N_f\ (\square + \overline{\square})$	$\sim 2N$	N	N	$0 \le N_f < N-2$
$2\ \begin{smallmatrix}\square\\\square\end{smallmatrix} + 2\ \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + N_f\ (\square + \overline{\square})$	$\sim 2N$	N	N	$0 \le N_f < N+4$
$1\ \mathbf{Adj} + 1\ \square\!\square + 1\ \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + 8\ \overline{\square} + N_f\ (\square + \overline{\square})$	$\sim 2N$	Y	N	$0 \le N_f \le N-4^*$
$1\ \mathbf{Adj} + 1\ \begin{smallmatrix}\square\\\square\end{smallmatrix} + 1\ \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + N_f\ (\square + \overline{\square})$	$\sim 2N$	N	N	$0 < N_f < N+2$
$2\ \mathbf{Adj} + N_f\ (\square + \overline{\square})$	$\sim 2N$	N	N	$0 \le N_f \le N^*$
$1\ (\square\!\square + \overline{\square\!\square}) + 2\ (\begin{smallmatrix}\square\\\square\end{smallmatrix} + \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}) + N_f\ (\square + \overline{\square})$	$\sim 3N$	N	N	$0 \le N_f < 2$
$3\ \begin{smallmatrix}\square\\\square\end{smallmatrix} + 3\ \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + N_f\ (\square + \overline{\square})$	$\sim 3N$	N	N	$0 \le N_f < 6$
$1\ \mathbf{Adj} + 2\ \begin{smallmatrix}\square\\\square\end{smallmatrix} + 2\ \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + N_f\ (\square + \overline{\square})$	$\sim 3N$	N	N	$0 \le N_f < 4$
$1\ \mathbf{Adj} + (\begin{smallmatrix}\square\\\square\end{smallmatrix} + \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}) + (\square\!\square + \overline{\square\!\square})$	$\sim 3N$	N	N	$*$
$2\ \mathbf{Adj} + 1\ \begin{smallmatrix}\square\\\square\end{smallmatrix} + 1\ \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + N_f\ (\square + \overline{\square})$	$\sim 3N$	N	N	$0 \le N_f \le 2^*$
$3\ \mathbf{Adj}$	$\sim 3N$	N	N	$*$

Theory	β_{matter}	dense spectrum	conformal window
$1\,\square\!\square + N_f\,\square$	$\sim N$	Y	$0 \leq N_f \leq 2N - 8^*$
$1\,\begin{smallmatrix}\square\\ \square\end{smallmatrix} + N_f\,\square$	$\sim N$	Y	$1 \leq N_f \leq 2N - 4^*$
$2\,\square\!\square + N_f\,\square$	$\sim 2\,N$	N	$0 \leq N_f \leq N - 10^*$
$1\,\square\!\square + 1\,\begin{smallmatrix}\square\\ \square\end{smallmatrix} + N_f\,\square$	$\sim 2\,N$	N	$0 \leq N_f \leq N - 6^*$
$2\,\begin{smallmatrix}\square\\ \square\end{smallmatrix} + N_f\,\square$	$\sim 2\,N$	N	$0 \leq N_f < N - 2$
$3\,\begin{smallmatrix}\square\\ \square\end{smallmatrix}$	$\sim 3\,N$	N	$*$

SO(N) theories

Theory	β_{matter}	dense spectrum	conformal window
$1\,\square\!\square + 2N_f\,\square$	$\sim N$	Y	$1 \leq N_f \leq 2N + 2^*$
$1\,\begin{smallmatrix}\square\\ \square\end{smallmatrix} + 2N_f\,\square$	$\sim N$	Y	$4 \leq N_f < 2N + 4$
$2\,\square\!\square + 2N_f\,\square$	$\sim 2N$	N	$0 \leq N_f < N + 1$
$1\,\square\!\square + 1\,\begin{smallmatrix}\square\\ \square\end{smallmatrix} + 2N_f\,\square$	$\sim 2N$	N	$0 \leq N_f \leq N + 3^*$
$2\,\begin{smallmatrix}\square\\ \square\end{smallmatrix} + 2\,N_f\,\square$	$\sim 2N$	N	$0 \leq N_f \leq N + 5^*$
$2\,\square\!\square + 1\,\begin{smallmatrix}\square\\ \square\end{smallmatrix} + 2N_f\,\square$	$\sim 2N$	N	$0 \leq N_f < 2$
$1\,\square\!\square + 2\,\begin{smallmatrix}\square\\ \square\end{smallmatrix} + 2N_f\,\square$	$\sim 2N$	N	$0 \leq N_f \leq 4^*$
$3\,\begin{smallmatrix}\square\\ \square\end{smallmatrix} + 2N_f\,\square$	$\sim 3N$	N	$0 \leq N_f \leq 6^*$
$3\,\square\!\square$	$\sim 3N$	N	$*$

Sp(N) theories

Example: ‘Simplest’ Large N SCFT

[Agarwal, JS 1912]

Matter contents:

	$SU(N)$	$U(1)_B$	$U(1)_A$	R
Q	N	1	N	$1-NR_\Phi$
$\tilde{Q}$	$\bar{N}$	-1	N	$1-NR_\Phi$
Φ	adj	0	-1	R_Φ

Gauge invariant operators:

- Coulomb branch operators: $\Phi^n, \ 2 \leq n \leq N$
- dressed mesons: $Q\Phi^n\widetilde{Q}, \ 0 \leq n \leq N-1$
- ‘baryon’: $Q(\Phi Q)(\Phi^2 Q)\dots(\Phi^{N-1}Q)$
- ‘anti-baryon’: $\widetilde{Q}(\Phi\widetilde{Q})(\Phi^2\widetilde{Q})\dots(\Phi^{N-1}\widetilde{Q})$

It looks like any other gauge theories with a sparse low-lying spectrum.

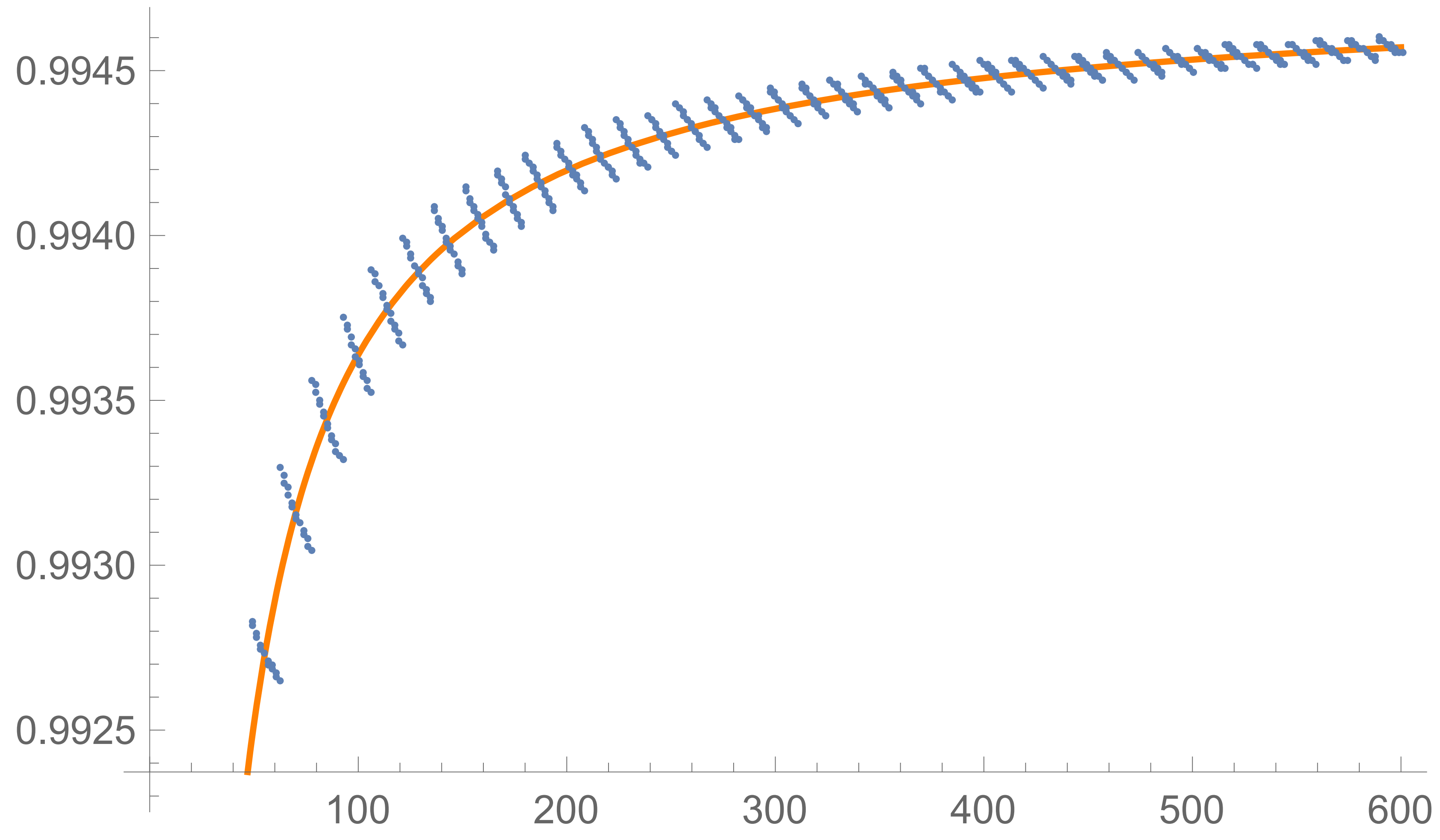
This theory flows to a superconformal fixed point in the IR.

	$SU(N)$	$U(1)_B$	$U(1)_A$	R
Q	N	1	N	$1-NR_\Phi$
$\tilde{Q}$	$\bar{N}$	-1	N	$1-NR_\Phi$
Φ	adj	0	-1	R_Φ

- Coulomb branch operators: $\Phi^n,\ 2 \le n \le N$
- dressed mesons: $Q\Phi^n\widetilde{Q},\ 0 \le n \le N-1$
- 'baryon': $Q(\Phi Q)(\Phi^2 Q)\ldots(\Phi^{N-1}Q)$
- 'anti-baryon': $\widetilde{Q}(\Phi\widetilde{Q})(\Phi^2\widetilde{Q})\ldots(\Phi^{N-1}\widetilde{Q})$

- This simple theory flows to a superconformal fixed point with a number of decoupled free fields.
- Some of the Coulomb branch operators $\mathrm{Tr}\Phi^i$ and the dressed mesons $\tilde{Q}\Phi^i Q$ decouple along the RG flow.
- None of the 'baryons' decouple. $\Delta_B \sim O(N)$
- The decoupled field can be removed by introducing flip field (X) and the superpotential coupling $W = X\mathcal{O}$. "$\mathcal{O} \leftrightarrow X$"

Feature 1: The O(N) degrees of freedom



$$a \simeq 0.500819\, N - 0.692539$$

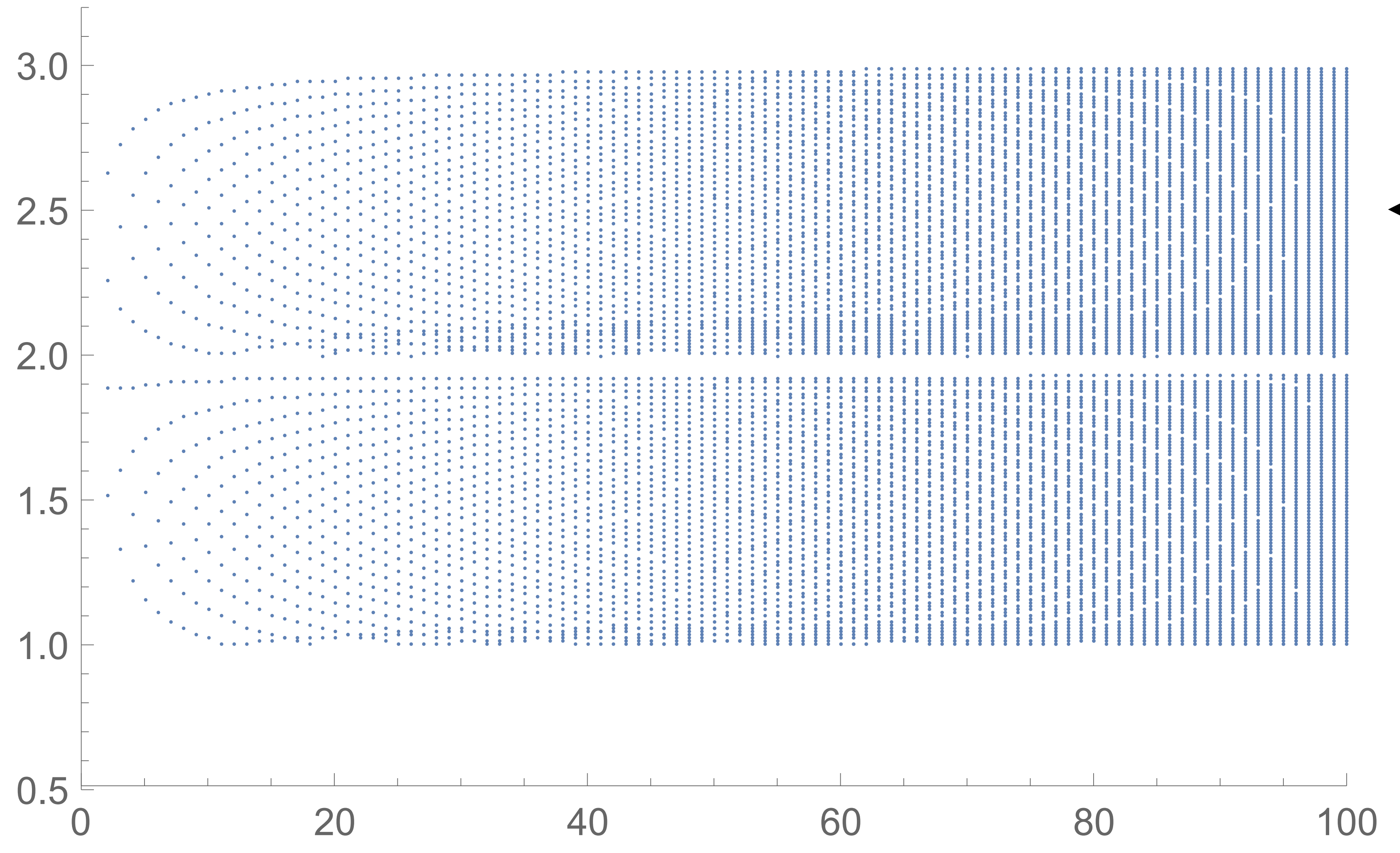
$$c \simeq 0.503462\, N - 0.640935$$

$$a/c \sim 0.994757 \; - 0.111888/N$$

The degrees of freedom grows as $O(N^1)$ instead of
the natural matrix degrees of freedom $O(N^2)$!

The ratio *a/c* asymptotes to a value close to 1, but *not exactly*.

Feature 2: Dense spectrum



Flip fields for $\mathrm{Tr}\Phi^i$ small *i*

$\mathrm{Tr}\Phi^i,\ Q\Phi^i\widetilde{Q}$

$R_\Phi \simeq 0.712086/N$

$R_Q \simeq 0.284372 + 0.609971/N$

The spectrum of chiral operators form a **dense band**, instead of being sparse!
(analog of the Liouville theory? Decompactification?)

It does not seem to exhibit confinement/deconfinement transition.

Deformation of SU(N) + 1 adj + N_f=1 by flipping

Matter contents:

	$SU(N)$	$U(1)_B$	$U(1)_A$	R
Q	N	1	N	$1-NR_\Phi$
$\tilde{Q}$	$\bar{N}$	-1	N	$1-NR_\Phi$
Φ	adj	0	-1	R_Φ

Superpotential:

$$W=\sum_{i=1}^{N-1}\left(X_i \mathrm{Tr}\Phi^{i+1}+M_i\tilde{Q}\Phi^{i-1}Q\right)$$

Chiral operators:

$M_i, X_i \ (i=1,\ldots,N-1)$ “Coulomb branch op”

$X\equiv Q^N\Phi^{N(N-1)/2}, Y\equiv \tilde{Q}^N\Phi^{N(N-1)/2}, Z\equiv \tilde{Q}\Phi^{N-1}Q$ “Higgs branch op”

$XY=Z^N$ $\mathscr{M}_H=\mathbb{C}^2/\mathbb{Z}_N$

This theory flows to the (A_1, A_{2n-1}) Argyres-Douglas theory, which is a ‘non-Lagrangian’ $\mathcal{N}$=2 SCFT.

[Maruyoshi, JS 1606]
[Maruyoshi, JS 1607]

$$a = \frac{12N^2 - 5N - 5}{24(N+1)}\ , \ c = \frac{3N^2 - N - 1}{6(N+1)}$$

Central charges grow linearly in N.

$$a/c \to 1$$

$$\Delta_{M_i} = \frac{2N - i + 1}{N+1}, \quad \Delta_{X_i} = \frac{3N - i + 2}{N+1}$$

(M_i, X_i) form an $\mathcal{N}$=2 chiral multiplet ($\mathscr{E}$).

The spectrum becomes dense at large N.
Fill the band between 1 and 3.

$$\Delta_X = \Delta_Y = N\ , \ \Delta_Z = 2$$

$$\frac{\Delta_X^2}{B^2} = 1 < \frac{9C_T}{40C_{V,B}} = \frac{3N^2 - N - 1}{2N^2}$$

(The Weak Gravity Conjecture holds.)

[Nakayama, Nomura]

Dense/O(N) theories behave similar to the Argyres-Douglas theories!
("N=1 AD theories")

Feature 3: Multiple bands

eg) SU(N) + 1 adj + N_f=2

[Agarwal, Lee, JS]

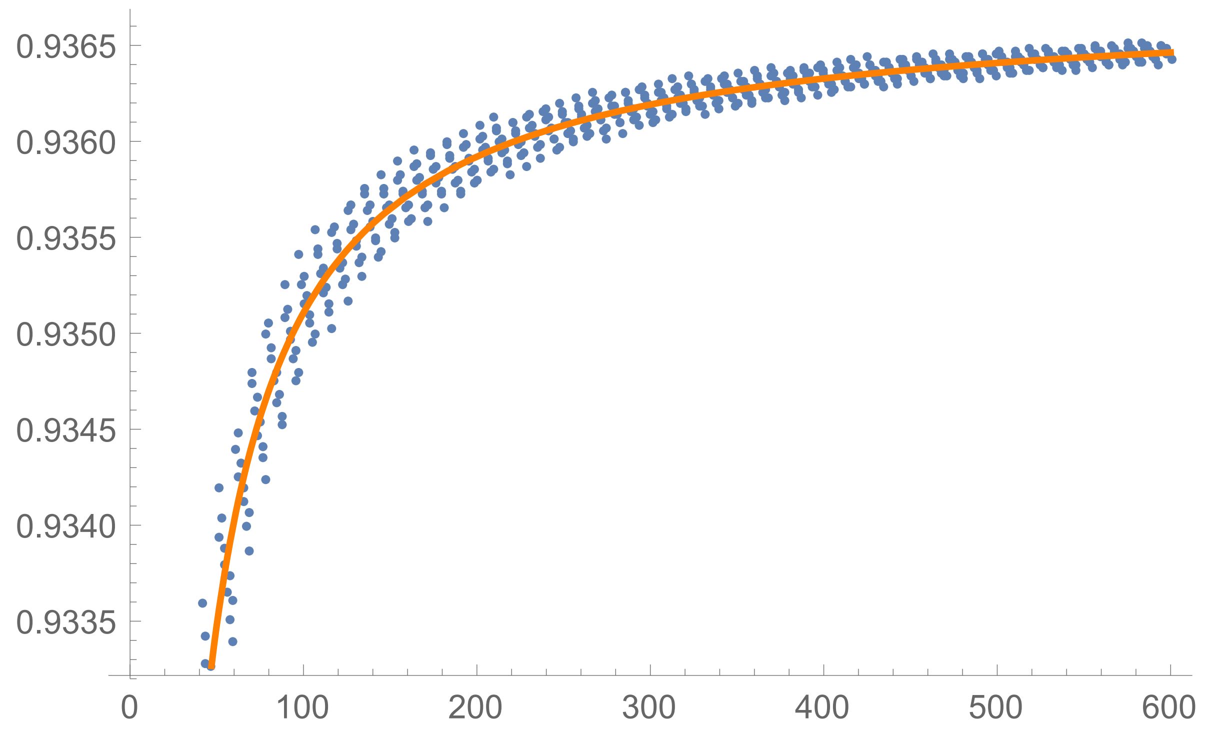


Figure 6: Plot of a/c vs N for the $SU(N)$ theory with 1 adjoint and $N_f = 2$. The orange curve fits the plot with $a/c \sim 0.936734 - 0.162684/N$.

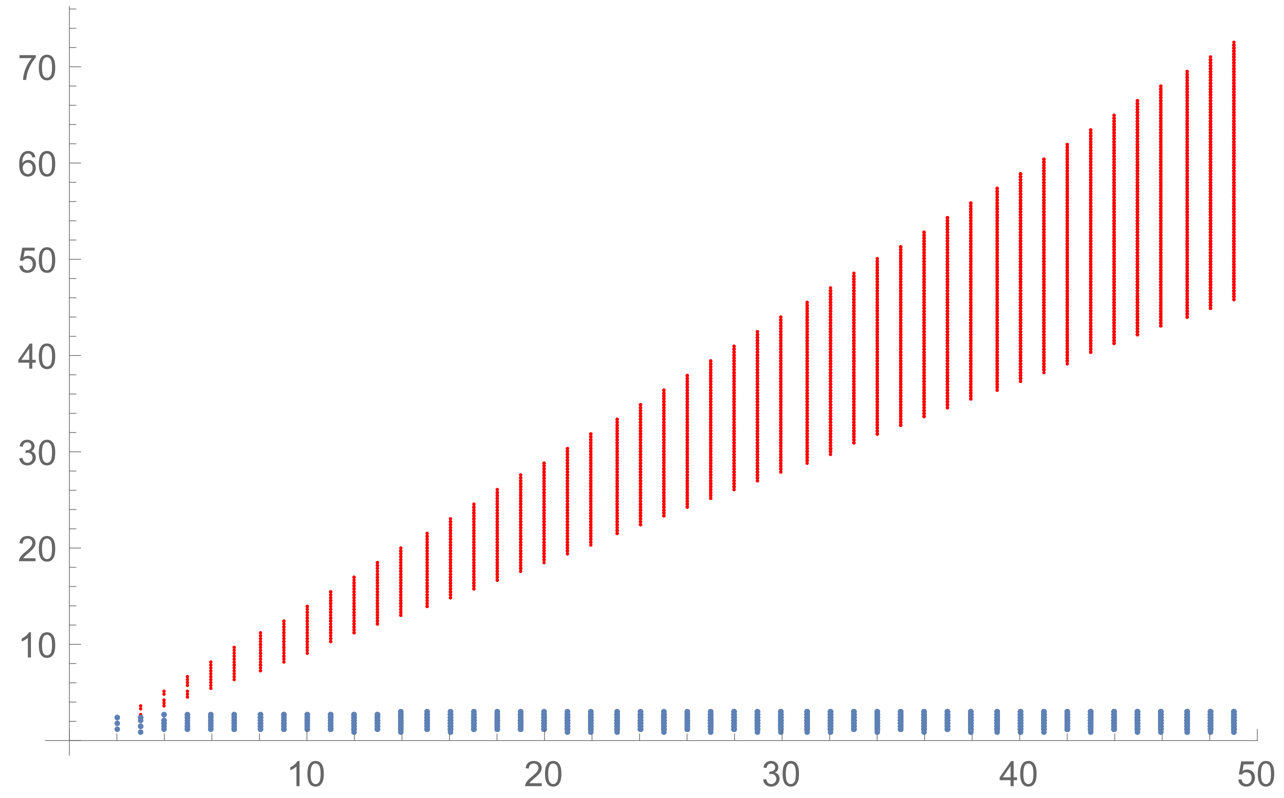


Figure 8: Dimensions of single-trace gauge-invariant operators including baryons in $SU(N)$ + 1 **Adj** + 2 ($\square + \overline{\square}$) theory. The baryons(red) form another band above the band of Coulomb branch operators and mesons.

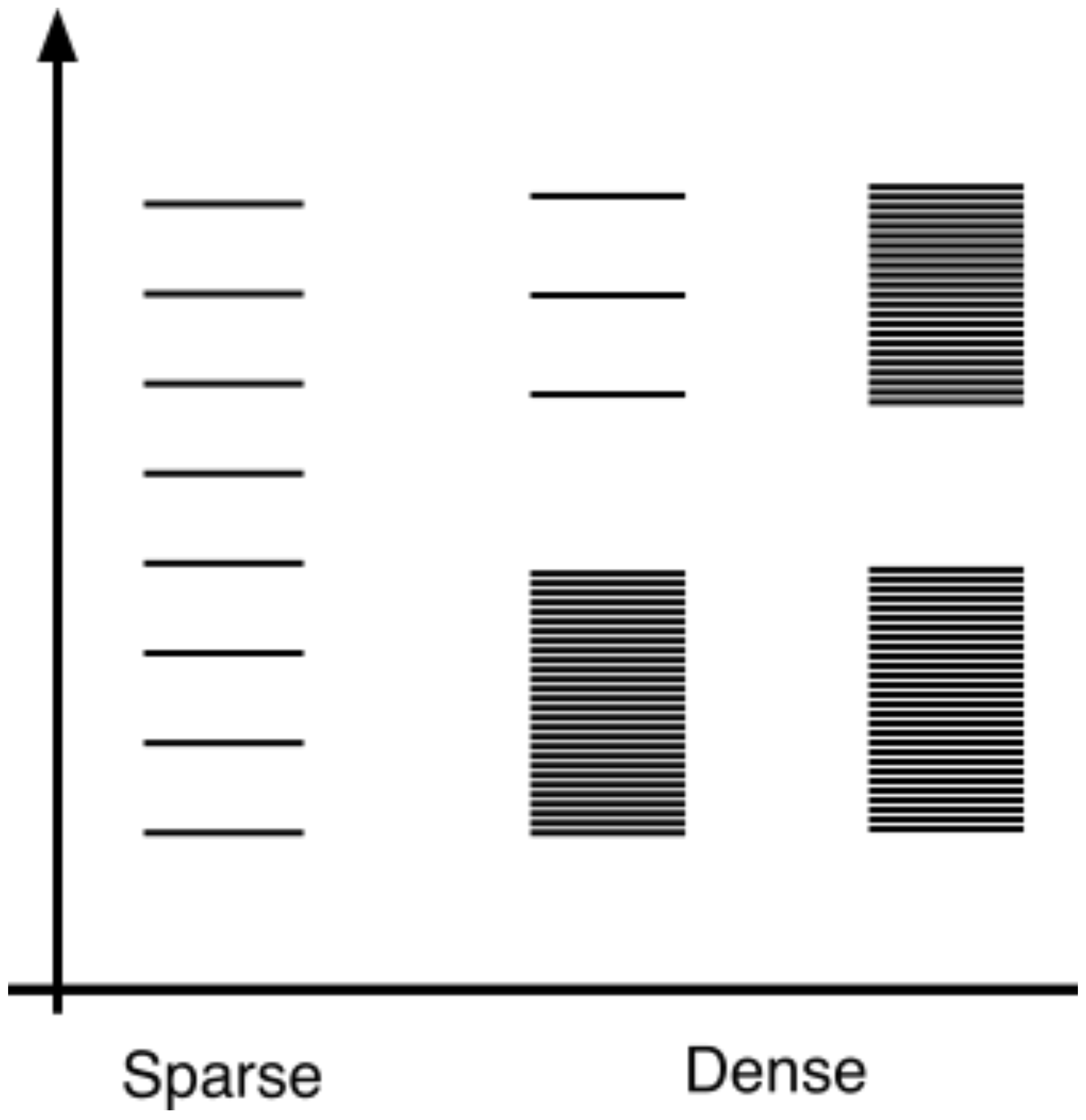
The ratio of central charges *a/c* does not go to 1 in large N.

We see the secondary band of size O(*N*). They are formed by 'baryons'.

- 'baryon': $Q(\Phi Q)(\Phi^2 Q)\ldots(\Phi^{N-1}Q)$
- 'anti-baryon': $\widetilde{Q}(\Phi\widetilde{Q})(\Phi^2\widetilde{Q})\ldots(\Phi^{N-1}\widetilde{Q})$

Supersymmetric analog of 'band' theory?

Sparse vs Dense spectrum



Out of 35 classes of all possible large *N* gauge theories, 8 of them have **dense spectrum** and the rest have sparse spectrum.

Sparse: The gap is O(1). $a = c$ at large N.

$$a \sim c = O(N^2)$$

Dense: The gap is O(1/*N*). $a \neq c$ at large N.

$$a \sim c = O(N^1)$$

$c - a$ can have either sign.

No universality!

(We also checked that the AdS version of the Weak Gravity Conjecture holds for all* the cases)

[Agarwal, Lee, JS][Cho, Choi, Lee, JS in progress]

Can we have 4d CFTs with $a = c$ even at finite N? (with $\mathcal{N}$=0, 1, 2 SUSY)

*$\mathcal{N}$=3, 4 SCFTs ***must*** have a=c.

$\mathscr{D}_p[G]$ theory

[Cecotti, Del Zotto]
[Cecotti, Del Zotto, Giacomelli]
[Xie][Wang, Xie]

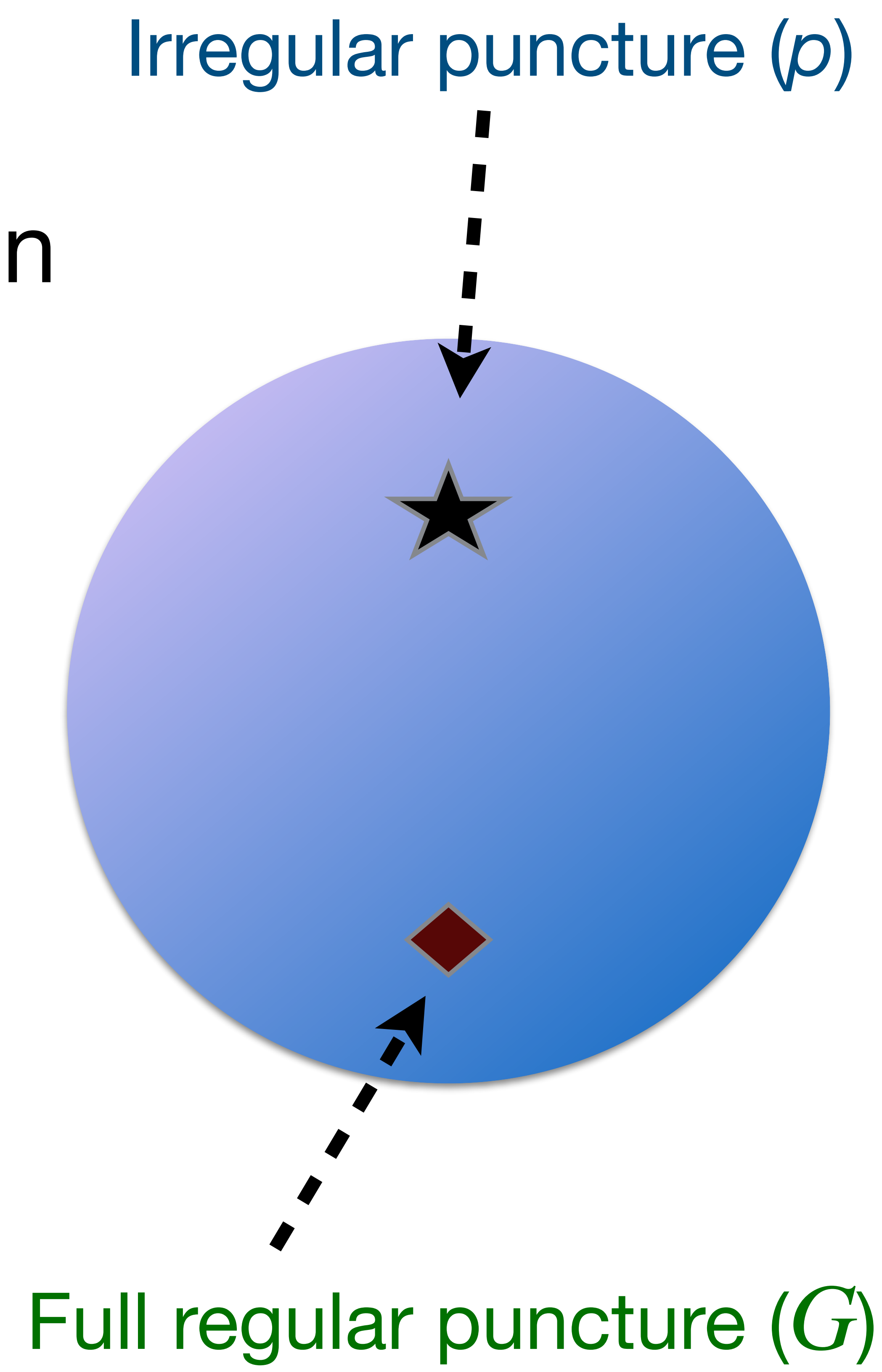
- It is a 4d $\mathcal{N}$=2 SCFT (Argyres-Douglas type) with **flavor symmetry G** (or larger).

- It can be realized as the 6d $\mathcal{N}$=(2, 0) theory of type G compactified on a **sphere** with one **irregular puncture** (p) and one **full regular puncture** (flavor G).

- The flavor symmetry is **exactly** G for *some choice* of p, when the irregular puncture does not possess extra flavor symmetry.

- The flavor central charge for G: $$k_G = \frac{2(p-1)}{p} h_G^\vee$$

G	$SU(N)$	$SO(2N)$	E_6	E_7	E_8
No additional symmetry	$(p, N) = 1$	$p \notin 2\mathbb{Z}_{>0}$	$p \notin 3\mathbb{Z}_{>0}$	$p \notin 2\mathbb{Z}_{>0}$	$p \notin 30\mathbb{Z}_{>0}$



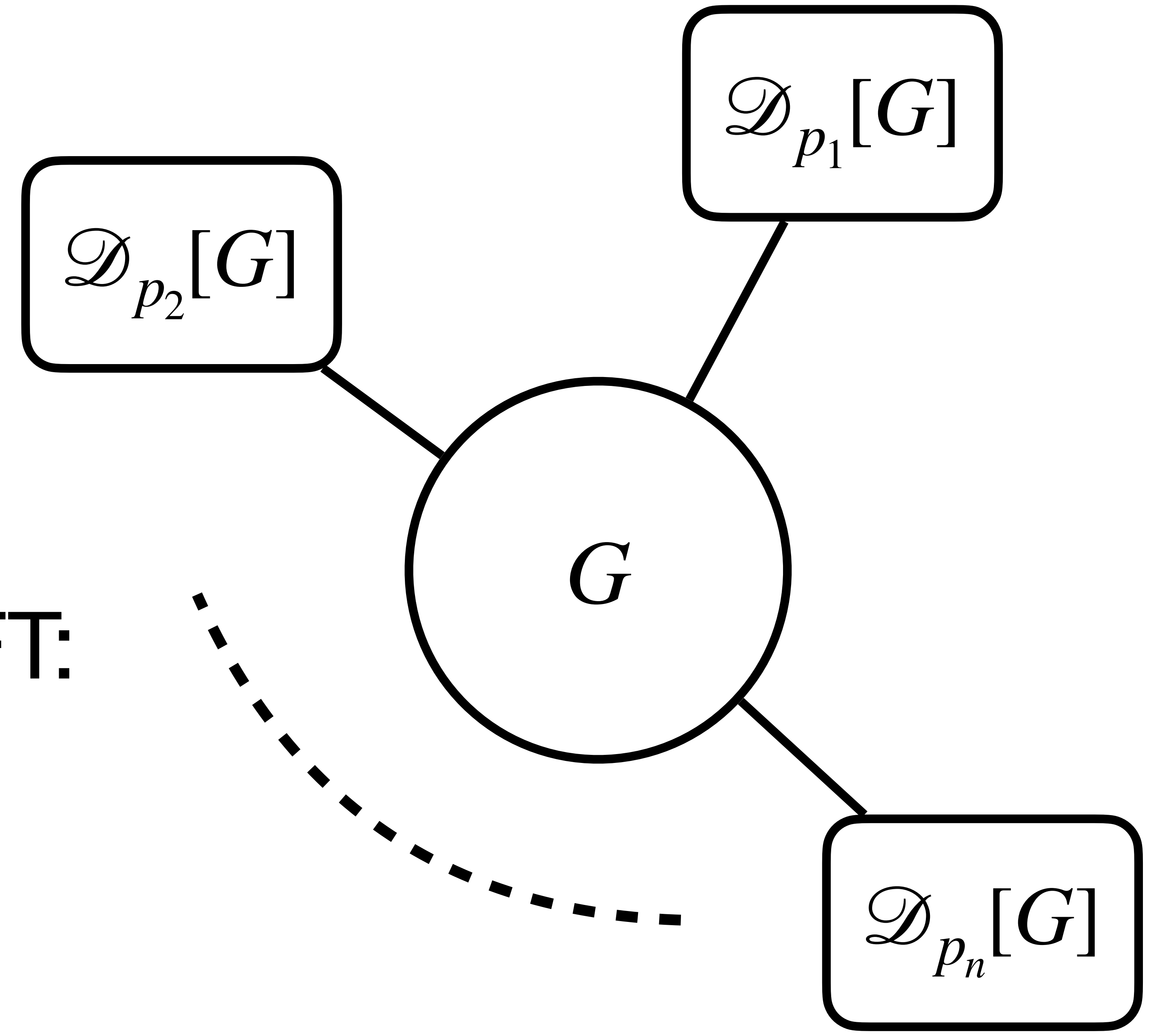
Gauging $\mathscr{D}_p[G]$ theories

[Cecotti, Del Zotto, Giacomelli]
[Closset, Giacomelli, Schafer-Nameki, Wang]
[Kang, Lawrie, JS]

- In order to gauge the flavor and obtain SCFT, the 1-loop beta function for the gauge group should vanish:

$$\beta_G = 0 \quad \leftrightarrow \quad \sum_i k_i = 4h_G^\vee$$

flavor central charges k_i : “matter” contribution to the beta function.



- Consider gluing a number of $\mathscr{D}_p[G]$ theories to form $\mathcal{N}$=2 SCFT:

$$\sum_{i=1}^{n} \frac{2(p_i-1)}{p_i} h_G^\vee = 4h_G^\vee \quad \to \quad \sum_{i=1}^{n} \frac{1}{p_i} = n-2$$

- **Only 4 non-trivial solutions**: (2, 2, 2, 2), (3, 3, 3), (2, 4, 4), (2, 3, 6)

$\hat{\Gamma}(G)$ theory with $\Gamma = D_4, E_6, E_7, E_8$

[Kang, Lawrie, JS]

(p_1, p_2, p_3, p_4)	$\widehat{\Gamma}(G)$	Quivers via gauging $\mathcal{D}_p(G)$s	$a = c$
$(2,2,2,2)$	$\widehat{D}_4(G)$	$\begin{tikzcd} & \mathcal{D}_2(G) \arrow[d, dash] & \\ \mathcal{D}_2(G) \arrow[r, dash] & G \arrow[d, dash] & \mathcal{D}_2(G) \arrow[l, dash] \\ & \mathcal{D}_2(G) & \end{tikzcd}$	$\frac{1}{2}\dim(G)$
$(1,3,3,3)$	$\widehat{E}_6(G)$	$\begin{tikzcd} & \mathcal{D}_3(G) \arrow[d, dash] & \\ \mathcal{D}_3(G) \arrow[r, dash] & G & \mathcal{D}_3(G) \arrow[l, dash] \end{tikzcd}$	$\frac{2}{3}\dim(G)$
$(1,2,4,4)$	$\widehat{E}_7(G)$	$\begin{tikzcd} & \mathcal{D}_2(G) \arrow[d, dash] & \\ \mathcal{D}_4(G) \arrow[r, dash] & G & \mathcal{D}_4(G) \arrow[l, dash] \end{tikzcd}$	$\frac{3}{4}\dim(G)$
$(1,2,3,6)$	$\widehat{E}_8(G)$	$\begin{tikzcd} & \mathcal{D}_2(G) \arrow[d, dash] & \\ \mathcal{D}_3(G) \arrow[r, dash] & G & \mathcal{D}_6(G) \arrow[l, dash] \end{tikzcd}$	$\frac{5}{6}\dim(G)$

We get $a = c$ is when the largest comark α_Γ of Γ satisfies

$$\gcd(h_G^\vee, \alpha_\Gamma) = 1 \quad \Longrightarrow \quad a = c.$$

$$\alpha_{D_4} = 2, \ \alpha_{E_6} = 3, \ \alpha_{E_7} = 4, \ \alpha_{E_8} = 6.$$

$a = c$ without any symmetry constraints! Genuinely $\mathcal{N}$=2.

In holography, it prevents $R^2_{\mu\nu\rho\sigma}$ correction in the effective supergravity action.

Genuinely $\mathcal{N}$=2 SCFTs with $a = c$ in $\hat{\Gamma}(G)$

- $a = c$ without any symmetry constraints.
- Some of these theories have class-$\mathcal{S}$ realization, but most of $\hat{\Gamma}(G)$ theories are not found in class-$\mathcal{S}$.
- The $\hat{\Gamma}(G)$ theory with $a = c$ has **no flavor symmetry**.
- They all have 1 exactly marginal coupling.
- They all have center 1-form symmetry $Z(G)$.

$\widehat{\Gamma}(G)$	$a = c$
$\widehat{D}_4(SU(2\ell+1))$	$2\ell(\ell+1)$
$\widehat{E}_6(SU(3\ell\pm1))$	$2\ell(3\ell\pm2)$
$\widehat{E}_6(SO(6\ell))$	$2\ell(6\ell+1)$
$\widehat{E}_6(SO(6\ell+4))$	$2(2\ell+1)(3\ell+2)$
$\widehat{E}_7(SU(4\ell\pm1))$	$6\ell(2\ell\pm1)$
$\widehat{E}_8(SU(6\ell\pm1))$	$10\ell(3\ell\pm1)$

The full list of $a = c$ theories in $\hat{\Gamma}(G)$

$\mathcal{N}$=4 SYM and $\hat{\Gamma}(G)$ theory

- The Schur index of $\hat{\Gamma}(G)$ theory is **identical to that of the $\mathcal{N}$=4 SYM** upon rescaling!

$$I_{\hat{\Gamma}(G)}(q) = I_G^{\mathcal{N}=4}(q^{\alpha_\Gamma}; q^{\alpha_\Gamma/2-1})$$

- For the $\hat{D}_4(SU(2\ell+1))$ theory, we find the index can be written in terms of MacMahon's generalized 'sum-of-divisor' function which is **quasi-modular**:

$$I_{\widehat{D}_4(SU(2k+1))}(q) = q^{-k(k+1)} A_k(q^2)$$

$$I^{\mathcal{N}=4}_{SU(2k+1)}(q) = q^{-\frac{k(k+1)}{2}} A_k(q)$$

$$A_k(q) = \sum_{0<m_1<m_2\cdots<m_k} \frac{q^{m_1+\cdots m_k}}{(1-q^{m_1})^2\cdots(1-q^{m_k})^2}$$

- There is an isomorphism between **associated VOA**s as a graded vector space. [Buican, Nishinaka]

- More connections to $\mathcal{N}$=4 SYM: 1 exactly marginal coupling, S-duality, 1-form center symmetry *Z(G)*.

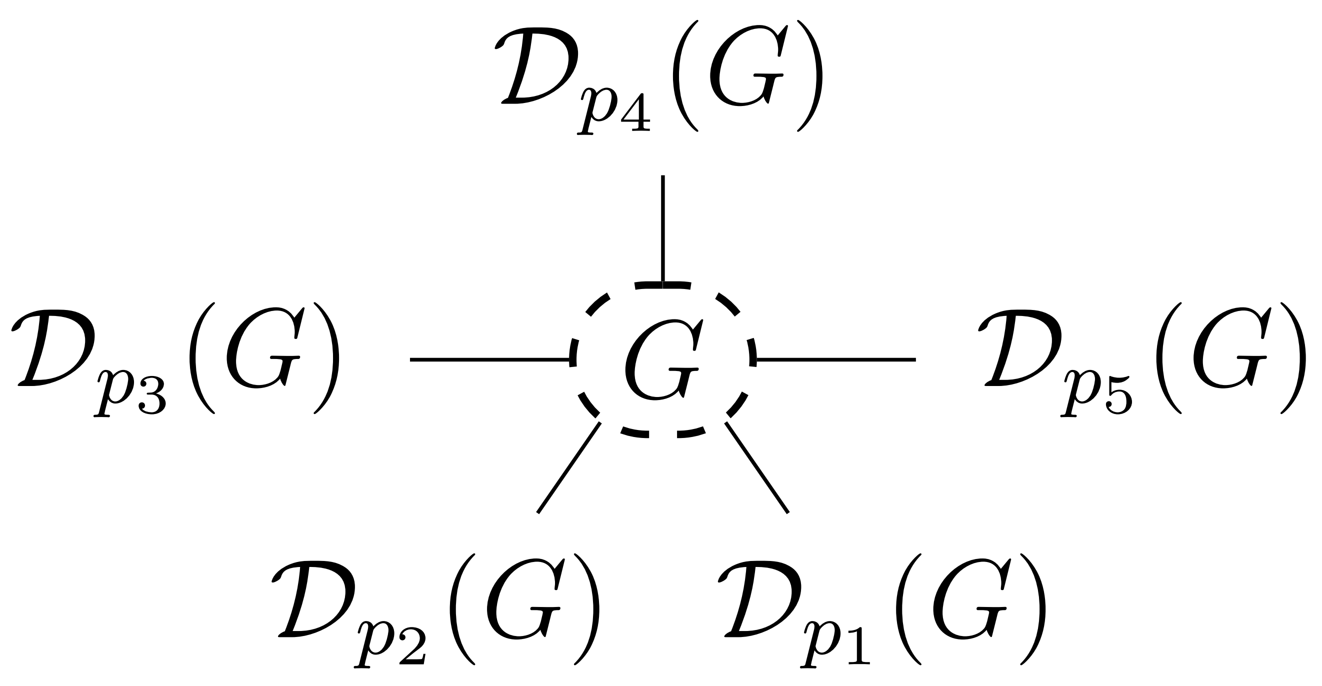
$\mathcal{N}$=1 SCFTs with $a = c$

[Kang, Lawrie, Lee, JS]

- One can construct even larger set of $a = c$ theories with **minimal supersymmetry**.
- Consider a number of $\mathscr{D}_p[G]$ theories gauged via $\mathcal{N}$**=1 vector** multiplet.
- It modifies the condition to be a CFT in the IR, since the theory now **RG flows**. From **asymptotic freedom** bound:

$$\sum_{i=1}^{N} \frac{2(p_i - 1)}{p_i} h_G^\vee < 6 h_G^\vee \qquad \sum_{i=1}^{N} \frac{1}{p_i} > N - 3$$

- The IR SCFT has a number of U(1) **flavor symmetry** originates from broken R-symmetry of each block.



p_1	p_2	p_3	p_4	p_5
1	1	1	1	p_5
1	1	1	p_4	p_5
1	1	p_3	p_4	p_5
1	2	2	p_4	p_5
1	2	3	≤ 6	p_5
1	2	3	7	≤ 41
1	2	3	8	≤ 23
1	2	3	9	≤ 17

p_1	p_2	p_3	p_4	p_5
1	2	3	10	≤ 14
1	2	3	11	≤ 13
1	2	4	4	p_5
1	2	4	5	≤ 19
1	2	4	6	≤ 11
1	2	4	7	≤ 9
1	2	5	5	≤ 9
1	2	5	6	≤ 7

p_1	p_2	p_3	p_4	p_5
1	3	3	3	p_4
1	3	3	4	≤ 11
1	3	3	5	≤ 7
1	3	4	4	≤ 5
2	2	2	2	p_5
2	2	2	3	3
2	2	2	3	4
2	2	2	3	5

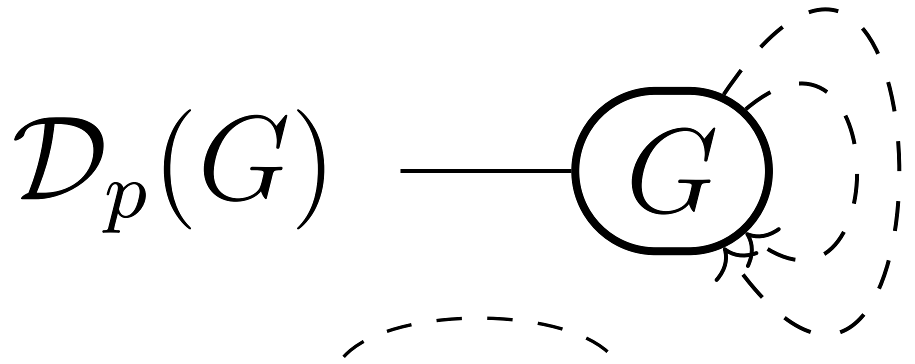
Tuples of (p_i)'s satisfying the asymptotic freedom bound.

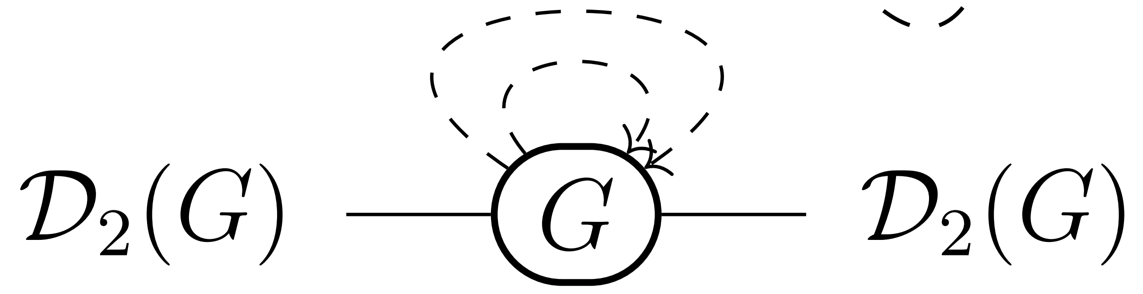
Landscape of $\mathcal{N}$=1 SCFTs with $a = c$

[Kang, Lawrie, Lee, JS, in progress]

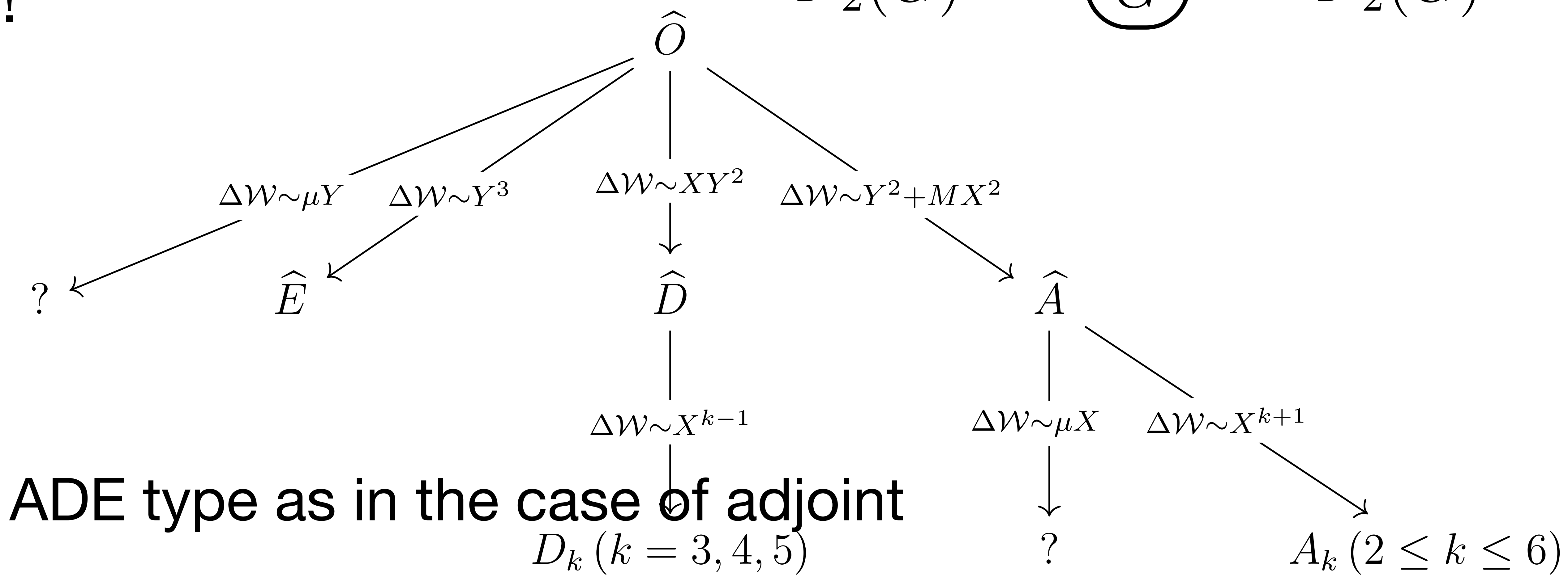
- $a = c$ theories with less SUSY not only exists, but rather common!
- One can add 1 or 2 **adjoint chiral** multiplets on top of the previous setup.
- 1 adjoint: can attach up to 4 $\mathscr{D}_p[G]$ theories.

$$p_i = (p_1, p_2),\ (2,2,p_3),\ (2,3,\leq 6),\ (2,4,4),\ (3,3,3),\ (2,2,2,2)$$





- 2 adjoints: One can even have zero $\mathscr{D}_p[G]$ theories!
    - **The simplest *Lagrangian* model** with $a = c$: $\mathcal{N}$=1 gauge theory with 2 adjoints.
    - Can attach up to 2 $\mathscr{D}_p[G]$'s



- One can consider **superpotential deformations** of ADE type as in the case of adjoint SQCD. [Intriligator, Wecht]
- How common are the $a = c$ theories in the landscape of 4d CFTs?

Holographic dual of a=c theories?

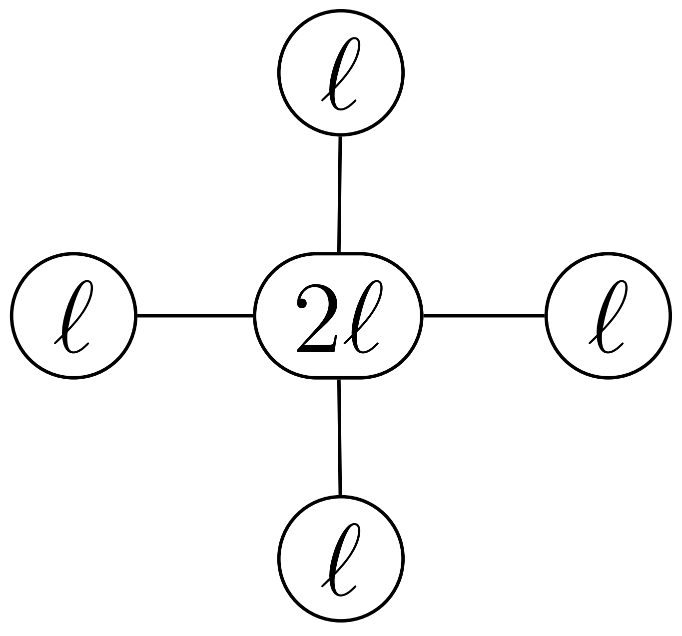
Lagrangian $\hat{\Gamma}(G)$ theory with $\Gamma = D_4, E_6, E_7, E_8$

- What is the **holographic dual** of such $a = c$ theories? It should forbid particular type of corrections in SUGRA action without any symmetry constraints. How?

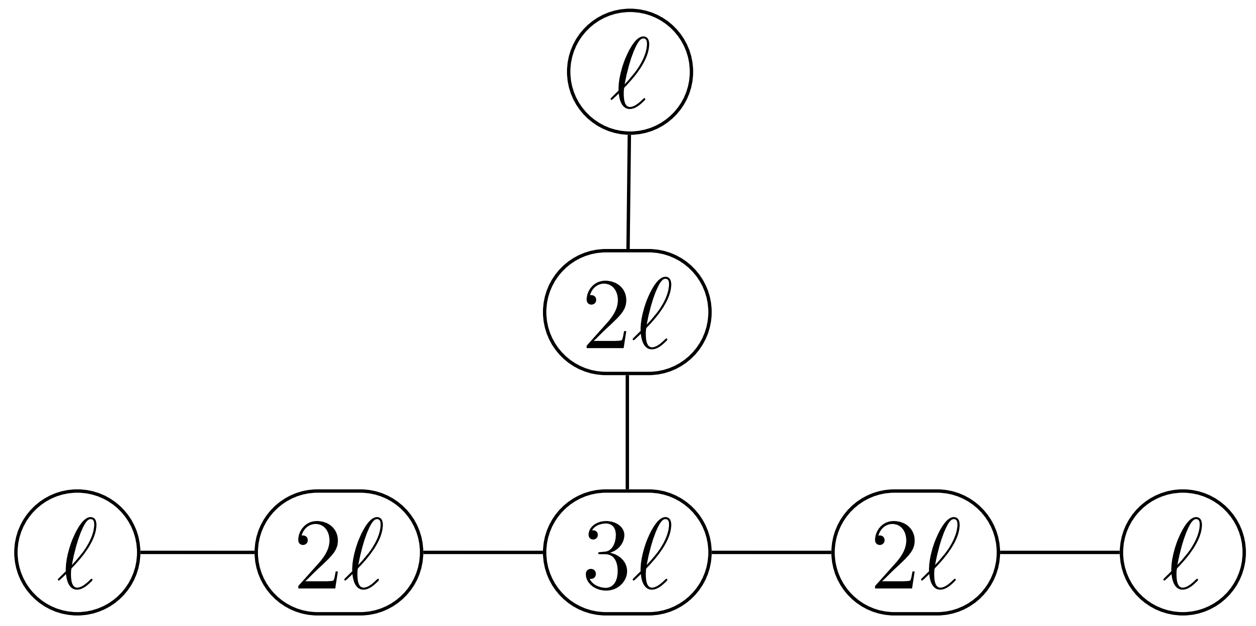
- When $G = SU(\alpha_\Gamma \ell)$, we recover **Lagrangian affine quiver** gauge theory obtained via ℓ D3-branes probing ALE singularity $\mathbb{C}^2/\Gamma$.

$$\mathcal{D}_p(SU(p\ell)) = \boxed{SU(p\ell)} - SU((p-1)\ell) - \cdots - SU(\ell)$$

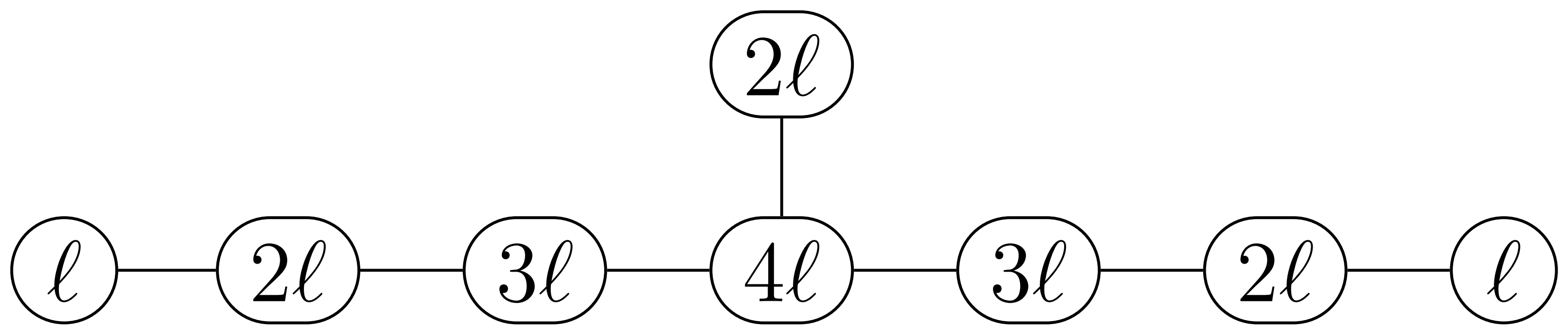
- The **holographic dual** for $\hat{\Gamma}(G)$ theories have been known for ages: It is dual to the type IIB theory on $\mathrm{AdS}_5 \times S^5/\Gamma$ with ℓ unit of 5-form flux through S^5/Γ.



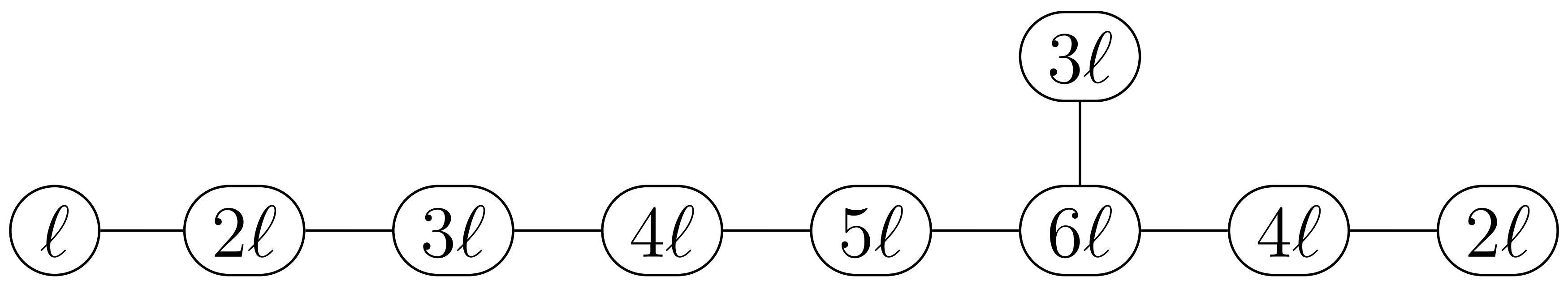
$\hat{D}_4(SU(2\ell))$



$\hat{E}_6(SU(3\ell))$



$\hat{E}_7(SU(4\ell))$



$\hat{E}_8(SU(6\ell))$

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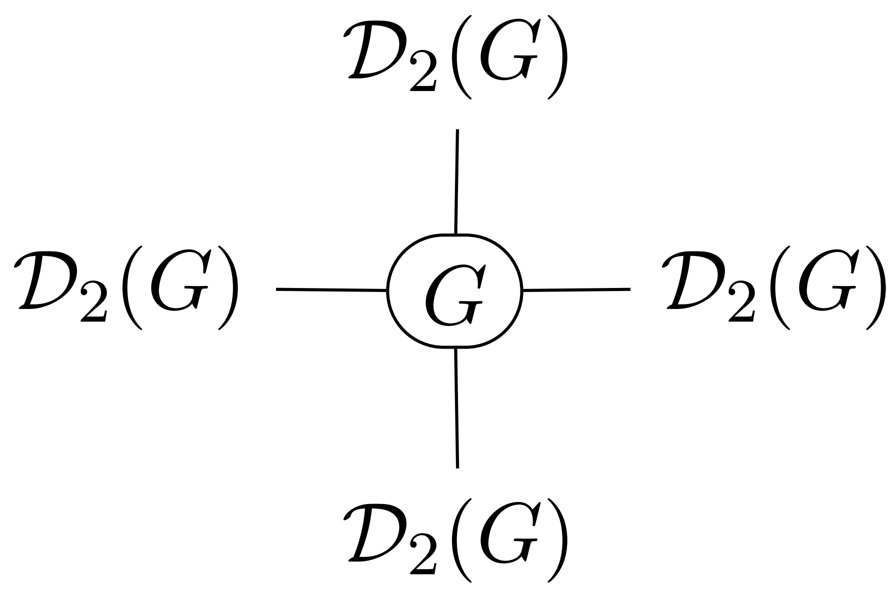

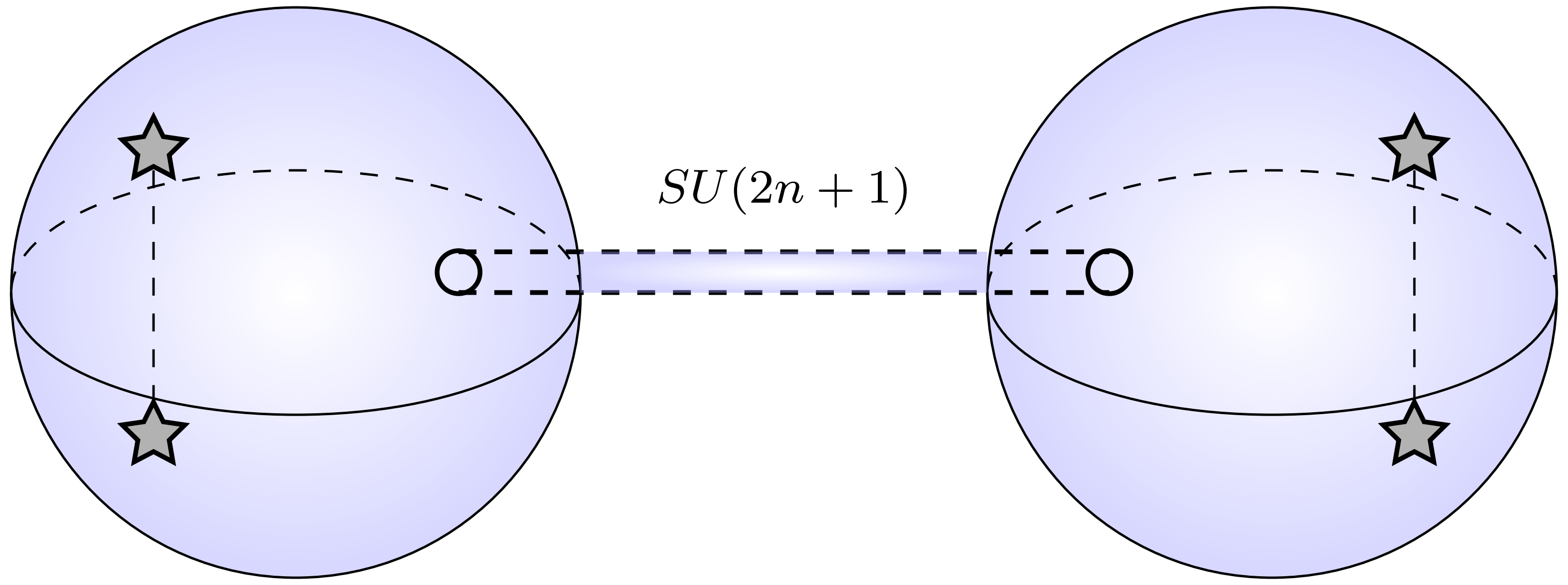

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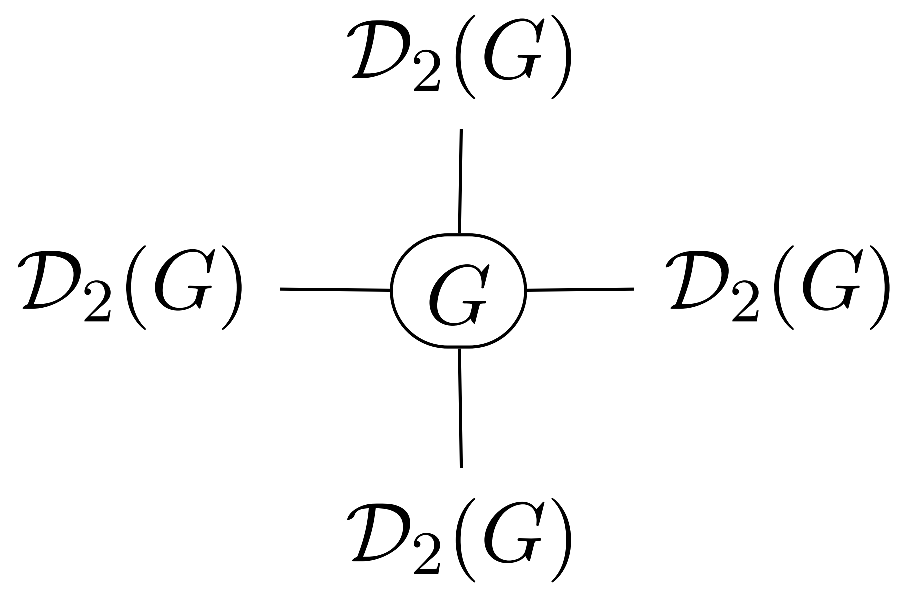

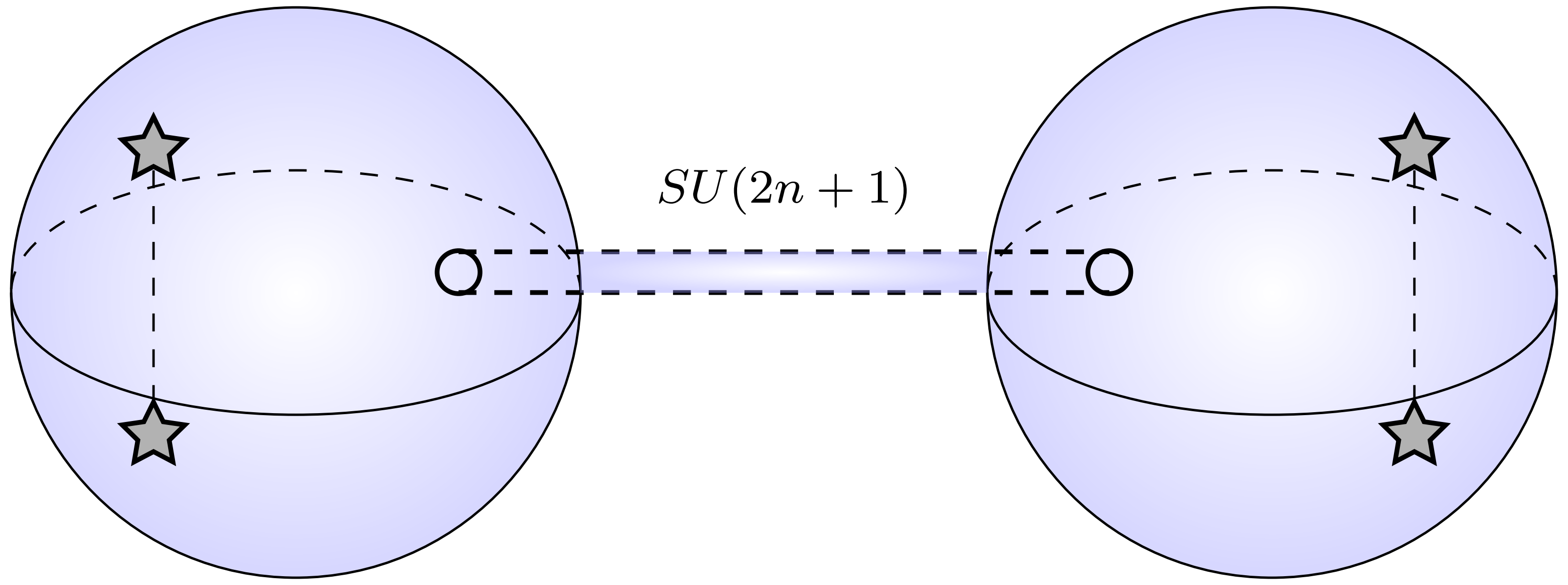

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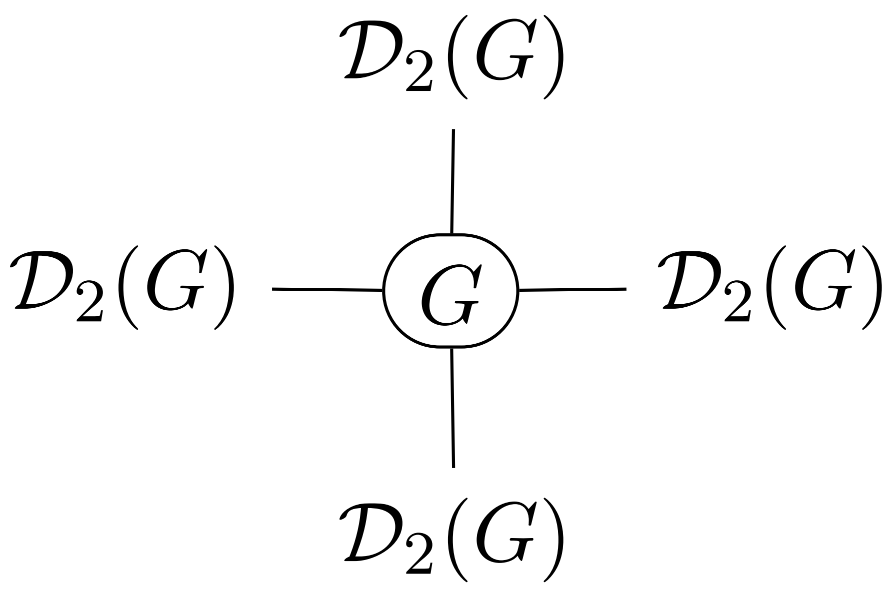

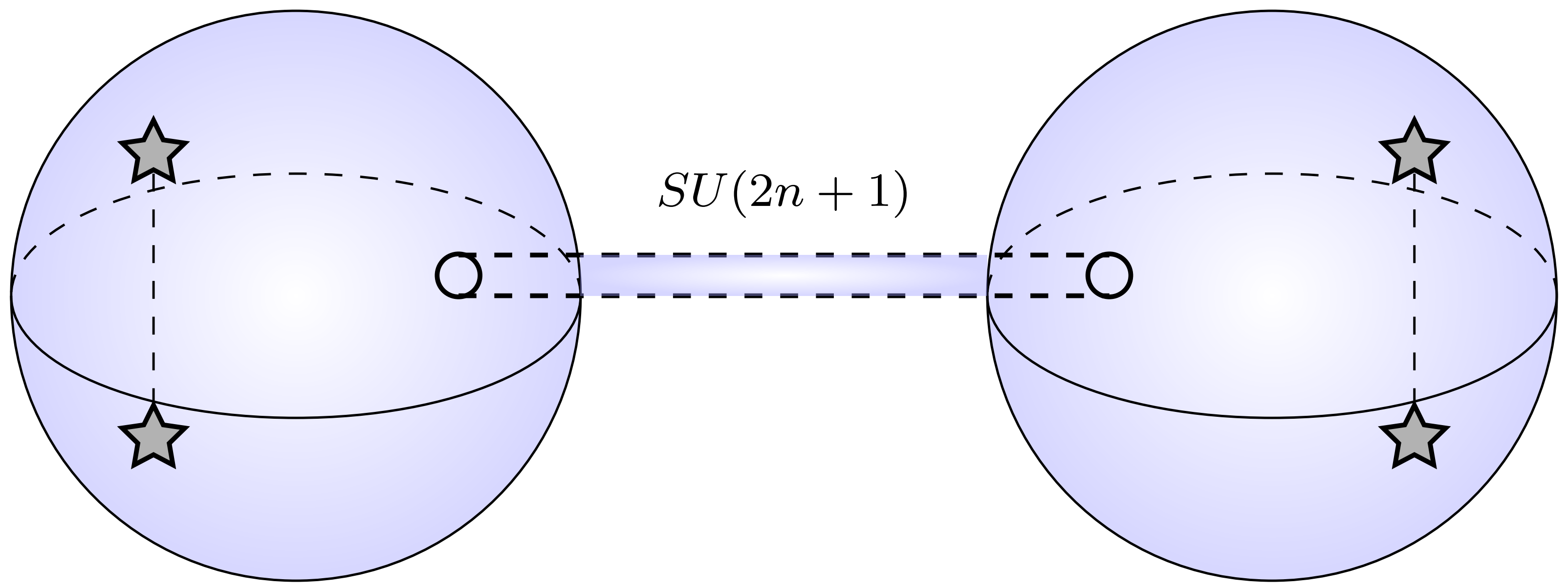

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- Any good reason for $a = c$ from gravity perspective?

Conclusion

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Why?

Congratulations Hirosi!