Semi-inclusive cross section in two dimensional type 0B string theory

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Congratulations to

Hirosi Ooguri

on his 60th birthday

Type 0B string theory in two dimensions

World-sheet theory contains,

1. A free scalar X^0 and its superpartner Majorana fermion ψ^0

2. A super-Liouville theory with c=27/2

3. b, c, β, γ, $\bar{b}$, $\bar{c}$, $\bar{\beta}$, $\bar{\gamma}$ ghosts

GSO projection acts simultaneously on left and right sector, leading to only NSNS and RR sectors

Physical spectrum:

A massless scalar ϕ_{NS} in NSNS sector

A massless scalar ϕ_R in the RR sector

This string theory has D-instantons and anti-D-instantons

– obtained by imposing the ZZ brane boundary condition on the Liouville field and Dirichlet boundary condition on X^0, ψ^0

– give non-perturbative contribution $\propto e^{-\pi/g_s}$ to the S-matrix

g_s: appropriately normalized closed string coupling constant

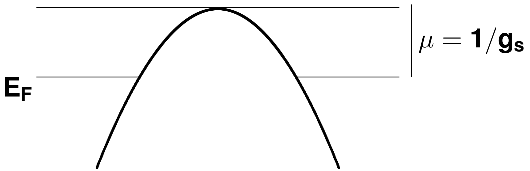
We work in the $\alpha' = 2$ convention **Balthazar, Rodriguez, Yin**

Dual matrix model description: Free non-interacting fermions ψ with single particle Hamiltonian:

$$H = \frac{p^2}{2} - \frac{q^2}{2}$$

Fermi level: Height μ below the top of the potential

Takayanagi, Toumbas; Douglas, Klebanov, Kutasov, Maldacena, Martinec, Seiberg; Balthazar, Rodriguez, Yin



$\rho = \psi^\dagger \psi$ **creates fermion hole pair excitations**

$\Rightarrow$ **massless scalars χ_L, χ_R on the left / right side of the barrier**

Das, Jevicki; Moore, Plesser, Ramgoolam

Map: $\phi_{NS} = \chi_L + \chi_R, \quad \phi_R = \chi_R - \chi_L$ **up to normalization**

Perturbative closed string amplitudes are mapped to reflection of fermion-hole pair from the barrier, ignoring tunneling

Effect of tunneling is captured by the D-instanton effects

D-instanton: Effect of fermion tunneling from right to left or hole tunneling from left to right

Anti-D-instanton: Effect of hole tunneling from right to left or fermion tunneling from left to right

$\Rightarrow$ a D-instanton will convert a fermion-hole pair on the right to a transmitted fermion on the left and a reflected hole on the right

– does not have a regular closed string description!

In the D-instanton amplitude in string theory, this effect shows up as infrared divergences in the annulus partition function

– makes the amplitudes with finite no of external closed strings vanish DeWolfe, Roiban, Spradlin, Volovich, Walcher; Balthazar, Rodriguez, Yin

The goal of this talk:

1. Define appropriate infrared finite semi-inclusive cross section induced by D-instanton

2. Compare the results on the string theory side with the matrix model side

Balthazar, Rogriguez, Yin (BRY), arXiv:2204.01747

Chakravarty, A.S., arXiv: 2207.07138

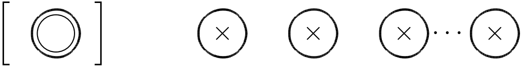
A.S., arXiV: 2012.00041, 2208.07385

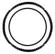
Leading order D-instanton amplitude

– can have disconnected world-sheet since individual world-sheet amplitudes do not conserve energy / momentum

– maximize the number of disks since each disk gives $1/g_s$

– can use as many annuli as we want since annuli $\sim (g_s)^0$

$\exp[-\pi/g_s]$ exp 

$\exp\Big[$  $\Big] = \exp\left[-\int_0^\infty \frac{dt}{2t}\right]$

2 t: ratio of circumference to the width of the annulus

– the exponent diverges at large and small t

Divergence at large t is associated with open string zero modes

– can be treated using open string field theory (to be discussed)

Divergence at small t is associated with IR divergence in the closed string channel

– makes the amplitude vanish

– reflects the impossibility for single D-instanton process to produce finite number of final state closed strings

Consider D-instanton anti-D-instaton induced process:

1. Overall factor $\exp[-2\pi/g_s]$ instead of $\exp[-\pi/g_s]$

2. Exponential of the annulus amplitude is replaced by,

– : anti-D-instanton boundary condition

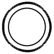
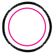
3. Each disk one point function is replaced by

$$(\sigma e^{\pm\sigma\pi P + i\sigma\omega^E x_1} - \sigma e^{\mp\pi\sigma P + i\sigma\omega^E x_2}) \qquad +,- : \chi_R, \chi_L$$

$\omega^E = -i\omega$, $\omega =$ energy, $P =$ Liouville momentum

$\sigma = 1(-1)$ for incoming (outgoing)

The expression for

exp [ +  + 2 ]

in $\alpha' = 2$ units is

$$\exp\left[\int_0^\infty \frac{dt}{2t}\left\{-2 + 2\, e^{2\pi t\left(\frac{1}{2} - \frac{1}{2}\left(\frac{x_1 - x_2}{2\pi}\right)^2\right)}\right\}\right]$$

x_1, x_2: Positions of D-instanton and anti-D-instanton in the Euclidean time direction

Note: No divergence from the small t region

However there are divergences in the large t region

1. -2 represents the effect of open string zero modes

2. For $x_1 - x_2 < 2\pi$ there are additional divergences due to open string tachyons

Strategy for dealing with large t divergence:

1. Use the identities, valid for $h_b, h_f > 0$,

$$\exp\left[\int \frac{dt}{2t}\left(e^{-2\pi t h_b} - e^{-2\pi t h_f}\right)\right] = \sqrt{\frac{h_f}{h_b}}$$

$$h_b^{-1/2} = \int \frac{d\phi_b}{\sqrt{2\pi}} e^{-\frac{1}{2} h_b \phi_b^2}, \qquad \phi_b : \text{grassmann even}$$

$$h_f = \int dp_f dq_f e^{-h_f p_f q_f}, \qquad p_f, q_f : \text{grassmann odd}$$

2. Interpret the modes ϕ_b, p_f, q_f as open string fields (D=0) and the exponent as open string field theory action in Siegel gauge

3. Modes with $h_b < 0$ are tachyonic modes and integration over them can be carried out along the steepest descent contour

4. Modes with $h_b = 0$ and $h_f = 0$ represent respectively the bosonic and fermionic zero modes

– need to be treated carefully.

In the present example, we have

1. 2 bosonic zero modes associated with the freedom of translating the instanton and the anti-instanton along the euclidean time

Remedy: Change variables from bosonic zero modes to D-instanton positions x_1, x_2, picking up the Jacobian factor.

2. 4 fermion zero modes coming from ghost - anti-ghost pair on the instanton and the anti-instanton

– result of wrongly fixing the U(1) 'gauge symmetry' on the instanton and anti-instanton

Remedy: Undo the gauge fixing by using a gauge invariant form of the path integral

This is a well tested procedure that has been verified in many cases.

1. c=1 bosonic string theory A.S.

2. c<1 bosonic string theory Eniceicu, Mahajan, Murdia, A.S.

3. Type IIB in D=10 A.S.

4. Type IIA / IIB on CY_3 Alexandrov, A.S., Stefanski

We shall skip the details and quote the result.

Result:

The annulus partition function is replaced by

$$\frac{1}{4\pi^2}\int dx_1 dx_2\,\frac{1}{(x_1-x_2)^2-4\pi^2}$$

Integration over x_1, x_2 needs to be done after taking the product with disk one point functions

The normalization agrees with the one found by BRY by comparison with the matrix model result.

Poles at $x_1 = x_2 \pm 2\pi$ needs to be treated using principal value prescription if we want to preserve unitarity. A.S.

BRY checked that the amplitudes computed this way agrees with the matrix model results.

Subtleties with unitarity:

$\mathbf{M_n}(\mathbf{a}, \mathbf{b})$: **n-instanton (or anti-instanton) contribution to** $\mathbf{S}(\mathbf{a} \to \mathbf{b})$

Consider the amplitude of a closed string χ_R of energy ω_1 to get reflected as a closed string χ_R of energy ω_2.

$$M_2(\omega_1, \omega_2) = -e^{-2\pi/g_s} \frac{1}{2\pi} \delta(\omega_1 - \omega_2) \cosh(2\pi\omega_1) \sinh(2\pi\omega_1)$$

In a unitary theory, we have

$$M_2(\omega_1, \omega_2) + M_2^*(\omega_2, \omega_1) + \sum_n M_1(\omega_1, n) M_1(\omega_2, n)^* = 0$$

If M_1 vanishes due to IR divergence, we have a contradiction!

We'll now describe the remedy BRY; A.S.

Express the original expression of $M_2(\omega_1, \omega_2)$ as

$$M_2(\omega_1,\omega_2) = e^{-2\pi/g_s} \exp\left[\int_\epsilon^\infty \frac{dt}{2t}\left\{-2+2e^{-\pi th}\right\} - \int_\epsilon^\infty \frac{dt}{t} e^{-\pi th} + \int_0^\infty \frac{dt}{t}\left\{e^{2\pi t\left(\frac{1}{2}-\frac{1}{2}\left(\frac{x_1-x_2}{2\pi}\right)^2\right)} - H(\epsilon - t)\right\}\right]$$

$$(e^{\pi P_1 + i\omega_1^E x_1} - e^{-\pi P_1 + i\omega_1^E x_2})(e^{\pi P_2 - i\omega_2^E x_2} - e^{-\pi P_2 - i\omega_2^E x_1}), \qquad \omega^E = -i\omega$$

First two lines are rewriting of the annulus partition function, with h, ϵ arbitrary positive number and H the step function

Last line is the product of disk amplitudes

For small ϵ, we have

Chakravarty, A.S.

$$\exp\left[\int_\epsilon^\infty \frac{dt}{2t}\left\{-2+2e^{-\pi th}\right\} - \int_\epsilon^\infty \frac{dt}{t} e^{-\pi th}\right] \Rightarrow \frac{1}{16\pi^4 h}\int dx_1 dx_2 \times \pi\,\epsilon\, h\, e^{\gamma_E}$$

by treating the zero modes using open string field theory.

1. For the rest, change variable to s=1/(2t)

2. Represent the s dependent integrand as result of momentum integration in closed string channel

3. Do the s integral

$$M_2(\omega_1,\omega_2) = e^{-2\pi/g_s}\,\frac{\epsilon}{16\pi^3}\,e^{\gamma_E}\int dx_1 dx_2$$
$$\exp\left[\frac{2}{\pi}\int\frac{d^2k_E}{k_E^2}\left\{e^{-i\omega^E(x_1-x_2)}\left(\cosh^2(\pi P)+\sinh^2(\pi P)\right)-e^{-\pi k_E^2/\epsilon}\right\}\right]$$
$$(e^{\pi P_1+i\omega_1^E x_1}-e^{-\pi P_1+i\omega_1^E x_2})(e^{\pi P_2-i\omega_2^E x_2}-e^{-\pi P_2-i\omega_2^E x_1}), \qquad k_E\equiv(\omega^E,P)$$

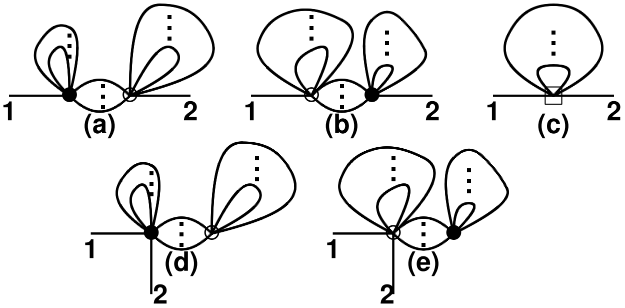
$$\int d^2k_E \equiv \int_0^\infty dP\int_{-\infty}^\infty d\omega^E$$

Note: There is no divergence from $k_E = 0$ region

We shall now put a lower cut-off η on P and manipulate the term by writing it as a sum of terms, each of which diverges as $\eta\to 0$.

$$\int d^2k_E \equiv \int_\eta^\infty dP\int_{-\infty}^\infty d\omega^E$$

By expanding the exponential, we can interpret M_2 as a sum over Feynman diagrams



Single D-instanton induced n-point vertex $\bullet$ with external closed strings of momenta $k_1 = (\omega_1, P_1), \cdots, k_n = (\omega_n, P_n)$:

$$\bullet \ : \ 2\pi\, i\, \delta\left(\sum_{i=1}^{n} \sigma_i \omega_i\right) \left(e^{-2\pi/g_s} \frac{\epsilon}{16\pi^3} e^{\gamma_E}\right)^{1/2} \prod_{i=1}^{n} e^{-\pi k_i^2/(2\epsilon)} \begin{pmatrix} \sigma_i \cosh \pi P_i \\ \sinh \pi P_i \end{pmatrix},$$

$\cosh(\pi P)$ refers to RR-sector states, $\sinh(\pi P)$ refers to NSNS sector states and σ_i takes value $+1$ if the i-th state is incoming and -1 if the i-th state is outgoing.

Single anti-D-instanton induced n-point vertex $\circ$ with external closed strings of momenta $k_1 = (\omega_1, P_1), \cdots, k_n = (\omega_n, P_n)$:

$$\circ \ : \ 2\pi\, i\, \delta\left(\sum_{i=1}^{n} \sigma_i \omega_i\right) \left(e^{-2\pi/g_s} \frac{\epsilon}{16\pi^3} e^{\gamma_E}\right)^{1/2} \prod_{i=1}^{n} e^{-\pi k_i^2/(2\epsilon)} \begin{pmatrix} -\sigma_i \cosh \pi P_i \\ \sinh \pi P_i \end{pmatrix}.$$

D-instanton - anti-D-instanton induced composite n-point vertex $\square$ with external closed strings of momenta $k_1 = (\omega_1, P_1), \cdots, k_n = (\omega_n, P_n)$:

$$\square \ : \ 2\pi\, i\, \delta\left(\sum_{i=1}^{n} \sigma_i \omega_i\right) e^{-2\pi/g_s} \frac{\epsilon}{16\pi^3} e^{\gamma_E}$$

$$\int dy \prod_{i=1}^{n} e^{-\pi k_i^2/(2\epsilon)} \begin{pmatrix} \sigma_i \cosh \pi P_i (e^{iy\, \sigma_i \omega_i^E/2} - e^{-iy\, \sigma_i \omega_i^E/2}) \\ \sinh \pi P_i (e^{iy\, \sigma_i \omega_i^E/2} + e^{-iy\, \sigma_i \omega_i^E/2}) \end{pmatrix}$$

$$\left[\exp\left\{\frac{2}{\pi} \int \frac{d^2 k_E}{k_E^2} \left(1 - e^{-\pi k_E^2/\epsilon}\right) \left\{e^{-i\omega^E y} (\cosh^2(\pi P) + \sinh^2(\pi P))\right\}\right\} - 1\right].$$

– correspond to real term in the effective action if we use principal value prescription for integrating through the singularity at $y \equiv x_1 - x_2 = 2\pi$.

Propagator of a closed string of momentum $\mathbf{k} = (\omega, \mathbf{P})$

$$-\frac{8\pi i}{k^2 - i\varepsilon} = \frac{8\pi i}{\omega^2 - P^2 + i\varepsilon},$$

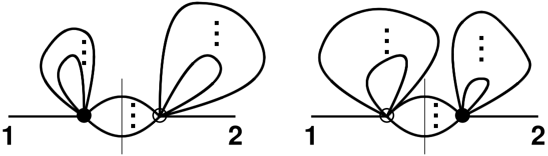
Integration measure over the internal momenta

$$d^2k/(4\pi^2) = i\, d^2k_E/(4\pi^2)$$

Once we have expressed the result in terms of sum over Feynman diagrams, unitarity is manifest

– follows from Cutkosky rules since the effective action is real.

$-(M_2 + M_2^*)$ is given by the sum of cut diagrams



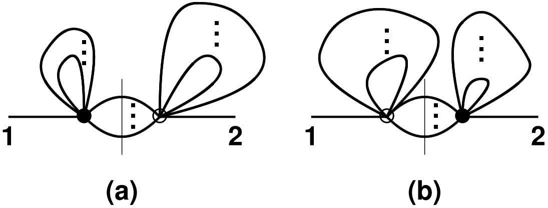
(a) (b)

This gives

$$-(M_2 + M_2^*) = M_1^\dagger M_1$$

– statement of unitarity

But M_1 is supposed to vanish due to IR divergence!



If we keep the number of cut propagators fixed and sum over all possible virtual closed strings, then the result does vanish as we take the IR cut-off η to zero

Exponentiation of the virtual loop contribution gives 0 due to IR divergence.

However if for fixed η we first sum over all possible cut propagators and then take $\eta \to 0$ limit, we get a finite result that agrees with the unitarity prediction

Once we have understood how unitarity is realized, we can compute semi-inclusive cross section

What is the cross section for producing a fixed set of closed strings in specific energy range plus anything?

– requires fixing the mometa of a finite set of cut propagators and summing over the rest

The specification of the final state closed strings can be done either in the $\phi_{\mathbf{NS}}, \phi_{\mathbf{R}}$ basis or in the $\chi_{\mathbf{L}}, \chi_{\mathbf{R}}$ basis.

We shall choose $\chi_{\mathbf{L}}, \chi_{\mathbf{R}}$ basis.

Incoming state: A single $\chi_{\mathbf{R}}$

Result: For infinitesimal $\delta e_i, \delta e_i'$

$$
\begin{aligned}
& \sum_n{}' M_1(\omega_1, n)\, M_1(\omega_2, n)^* \\
= \quad & e^{-2\pi/g_s} \left\{\prod_{i=1}^{r} \frac{\delta e_i}{e_i}\right\} \left\{\prod_{i=1}^{\ell} \frac{\delta e_i'}{e_i'}\right\} \delta(\omega_1 - \omega_2)\, \frac{1}{\pi} \sinh\left(2\pi\left(\omega_1 - \sum_{i=1}^{r} e_i - \sum_{i=1}^{\ell} e_i'\right)\right) \\
& \times \cosh\left(2\pi\left(\omega_1 + \sum_{i=1}^{\ell} e_i' - \sum_{i=1}^{r} e_i\right)\right)
\end{aligned}
$$

$\sum_n{}'$ on the left hand side denotes sum over all final states that contain

– r right sector closed string states of energies in the range $(e_1, e_1 + \delta e_1), \cdots, (e_r, e_r + \delta e_r)$

– ℓ left sector closed string states of energies in the range $(e_1', e_1' + \delta e_1'), \cdots, (e_\ell', e_\ell' + \delta e_\ell')$

– and any number of other closed string states.

Note: For $\ell \geq 1$ this is the leading contribution.

Computation in the matrix model:

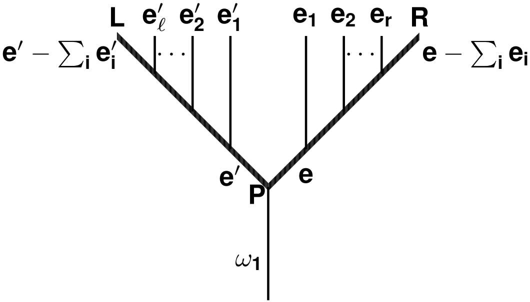
Sum over ‘any number of other states’ can be done in any basis

Free fermions and holes provide a convenient basis of states in the matrix model. DeWolfe, Roiban, Spradlin, Volovich, Walcher

So we can sum over final states containing a given set of closed strings and an arbitrary number of fermions and holes on either side of the potential

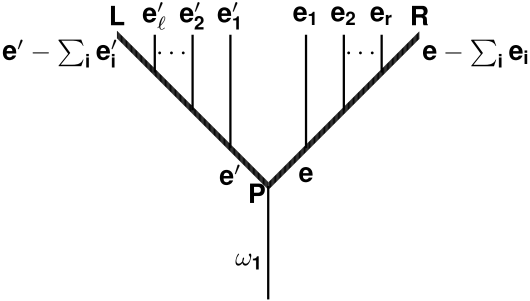
– can be computed Moore, Plesser, Ramgoolam

Pictorial representation



Thick lines: Fermion / hole, **Thin lines: closed strings**

At the leading order, the relevant term is the transmission coefficient of the fermion / hole



$$\sum_n{}' M_1(\omega_1, n) M_1(\omega_2, n)^* = \left\{ \prod_{i=1}^{r} \frac{\delta e_i}{e_i} \right\} \left\{ \prod_{i=1}^{\ell} \frac{\delta e'_i}{e'_i} \right\} 2\pi\delta(\omega_1 - \omega_2)$$

$$\int_{\sum e'_i}^{\omega_1 - \sum e_i} \frac{de'}{2\pi} \left[\left| e^{-\pi(\mu - 2e')} \right|^2 + \left| e^{-\pi(\mu + 2e')} \right|^2 \right]$$

– agrees with the string theory result.

Note: No IR divergence issues even at the intermediate steps!

The matrix model avoids the IR divergences by representing 'any number of other states' by fermion hole pair states.

In string theory the role of single fermions / holes is played by the rolling tachyon configuration on unstable D0-branes. A.S.

McGreevy, Verlinde; Takayanagi, Toumbas; Douglas, Klebanov, Kutasov, Maldacena, Martinec, Seiberg

This suggests that if in string theory we could calculate amplitudes with such final states, we could avoid the IR divergences even at the intermediate stages.

Speculation:

Rolling tachyon solution on unstable D-branes also exists in critical string theory A.S.

– could provide a new basis of states that may be more useful than the usual closed string states for some computation

For example, the free fermion basis is known to be better suited for computing entanglement entropy in the matrix model

Das; Hartnoll, Mazenc; Das, Jevicki, Zheng

It will be worth exploring if the basis of rolling tachyon states can provide a more useful basis for computing entanglement entropy in string theory.