Measurement-based quantum simulation of gauge theories

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@ Hirosifest, October 21, 2022. Work with Hiroki Sukeno.

- I went to Caltech as a graduate student in 2000. Hirosi was there as a faculty. In 2001 I became his student.

- Around the time, the hep-th people at Caltech and USC had joint activities.

- Hirosi, Okawa san, and I started working on a project about string field theory. We spent quite a bit of time, but eventually Okawa san wrote one paper and I wrote another paper, which was my first research paper.

- Later Hirosi and I wrote two papers on topological strings (one after I gradated).

- I found Hirosi's projects tough. To make progress I had to struggle, reading papers, talking to people, and coming up with related projects (and writing papers on them). Helped me become independent.

- He was generous about his time for discussing physics. Whenever I had something I wanted to discuss with him, he either welcomed me into his room on the spot or booked a time in the near future.

- What did I learn from Hirosi?

    - Intensity during discussion

    - Importance of interacting with people

    - Thinking in a broader context

    - Diversity in research themes

Measurement-based quantum simulation of gauge theories

Work with Hiroki Sukeno, should have appeared just now

Plan of the talk

- Motivations and background
- Hamiltonian lattice gauge theories
- Measurement-based quantum computation and resource states
- Simulation protocols
- Holography/bulk-boundary correspodence

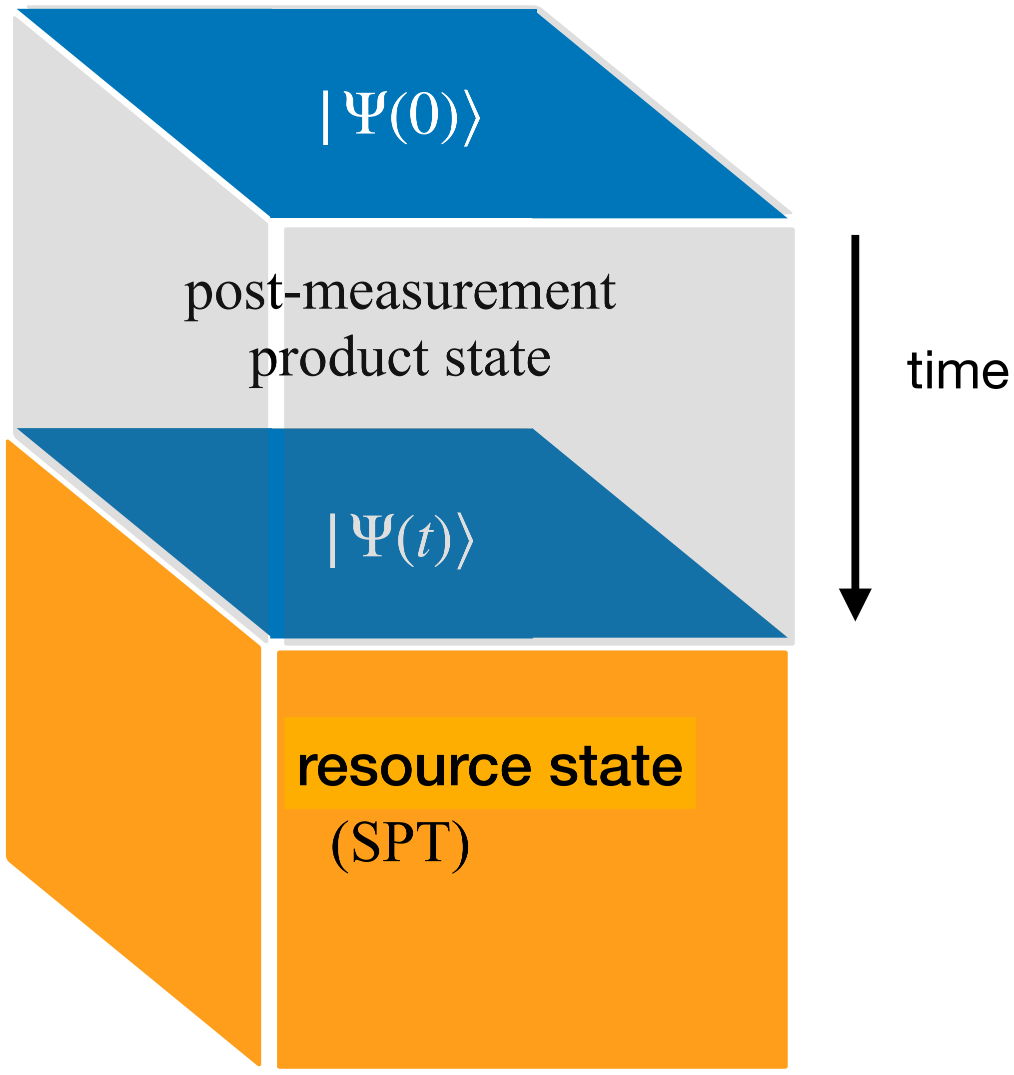
Motivations and background

- Quantum computing
  - Methods for quantum simulation of QFTs to be explored
  - New quantum simulation scheme for lattice QFTs
- Holography and quantum information
  - New type of bulk-boundary/holographic correspondence

Measurement-based simulation

Schematic picture

- Choose Hamiltonian lattice model to simulate.
- Prepare (experimentally) a **resource state** with $|\Psi(0)\rangle$ on one boundary
- Measure qubits **adaptively**.
  - Implement discrete time evolution deterministically.
  - Deal with “byproduct operators” that arise probabilistically.
- Obtain $|\Psi(t)\rangle$.

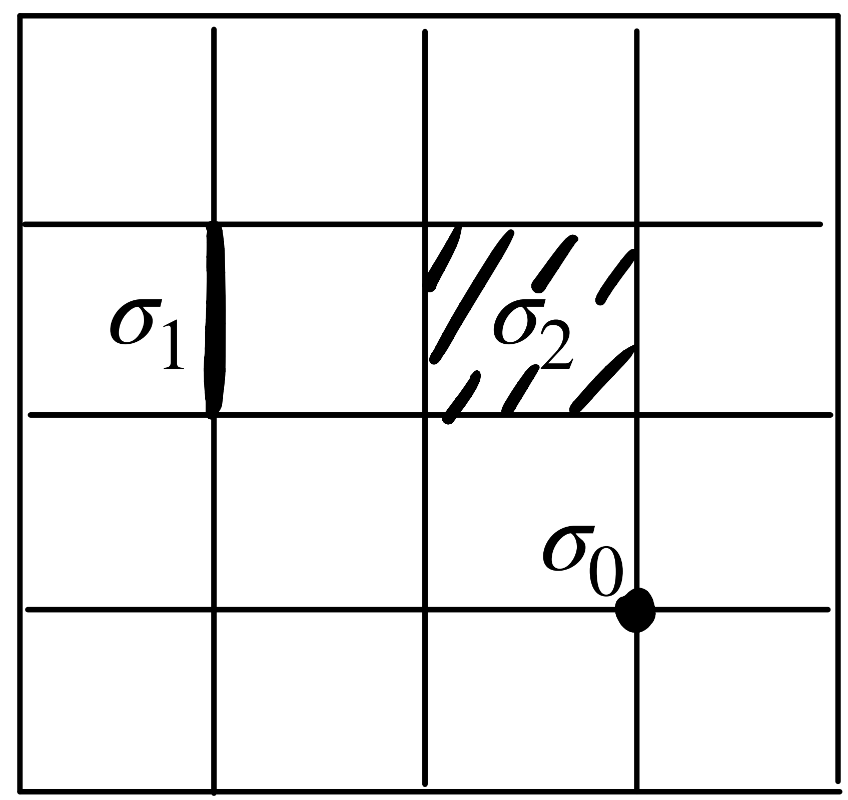


Hamiltonian lattice $\mathbb{Z}_2$ gauge theory in 2+1 dimensions

- Notations: Pauli matrices

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

- Cell complex for a square lattice.
    - 0-cells $\sigma_0 \in \Delta_0$
    - 1-cells $\sigma_1 \in \Delta_1$
    - 2-cells $\sigma_2 \in \Delta_2$
- Degrees of freedom are on 1-cells (bonds) $\sigma_1 \in \Delta_1$.



Hamiltonian lattice $\mathbb{Z}_2$ gauge theory

- Hamiltonian

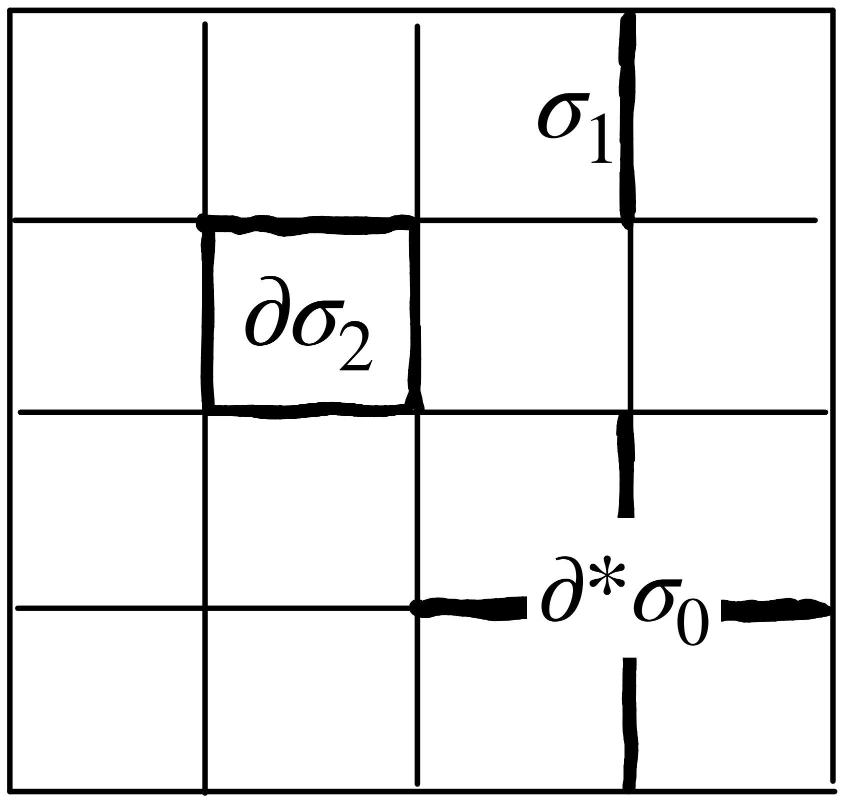
$$H = -\sum_{\sigma_1 \in \Delta_1} X(\sigma_1) - \lambda \sum_{\sigma_2 \in \Delta_2} Z(\partial\sigma_2).$$

- $$Z(\partial\sigma_2) = \prod_{\sigma_1 \subset \partial\sigma_2} Z(\sigma_1).$$

- Gauss law constraint: for any $\sigma_0 \in \Delta_0$,

$$X(\partial^*\sigma_0)\,|\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle.$$

- Generalization: $\mathbb{Z}_2$ gauge theory in 2+1 dimensions = $M_{(3,2)}$ $\Rightarrow$ **Wegner's model** $M_{(d,n)}$: $(n-2)$-form gauge theory in d dimensions.



Trotterization

- Ideally we want to implement the continuous time evolution e^{-iHt} for any t. Decompose $H = H_1 + H_2$. $H_1 = -\sum_{\sigma_1 \in \Delta_1} X(\sigma_1)$ and $H_2 = -\lambda \sum_{\sigma_2 \in \Delta_2} Z(\partial \sigma_2)$ do not commute.

- In digital quantum simulation (such as by quantum circuits), we implement $e^{-iH_1 t}$ and $e^{-iH_2 t}$ separately.

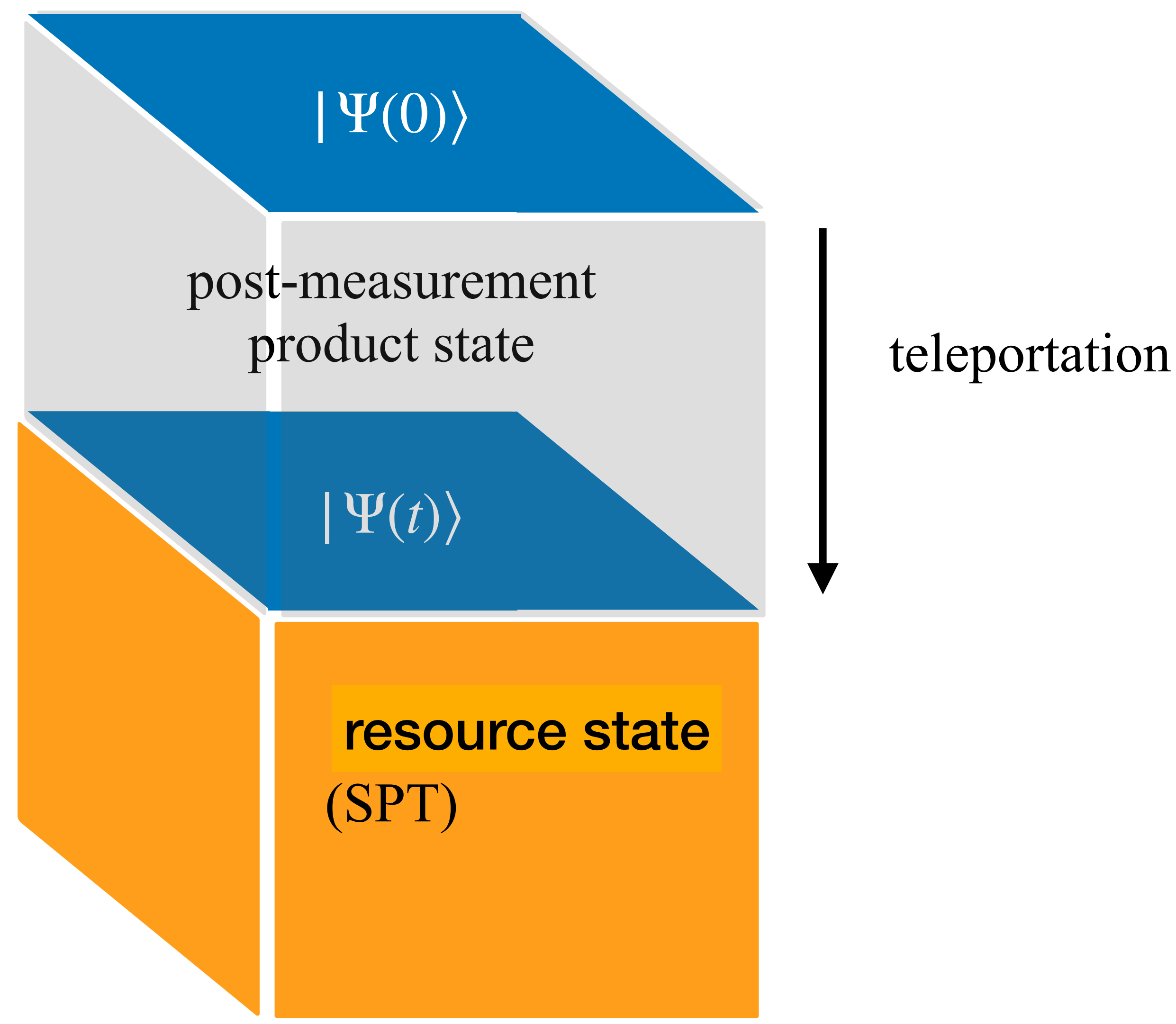
- Suzuki-Trotter approximation: $e^{-Ht} \simeq \left(e^{-iH_1 t/n} e^{-iH_2 t/n}\right)^n$.

- We want to realize $e^{-iH_1 \delta t} = \prod_{\sigma_1 \in \Delta_1} e^{i\delta t X(\sigma_1)}$ and $e^{-iH_2 \delta t} = \prod_{\sigma_2 \in \Delta_2} e^{i\lambda \delta t Z(\partial \sigma_2)}$.

Measurement-based quantum computation

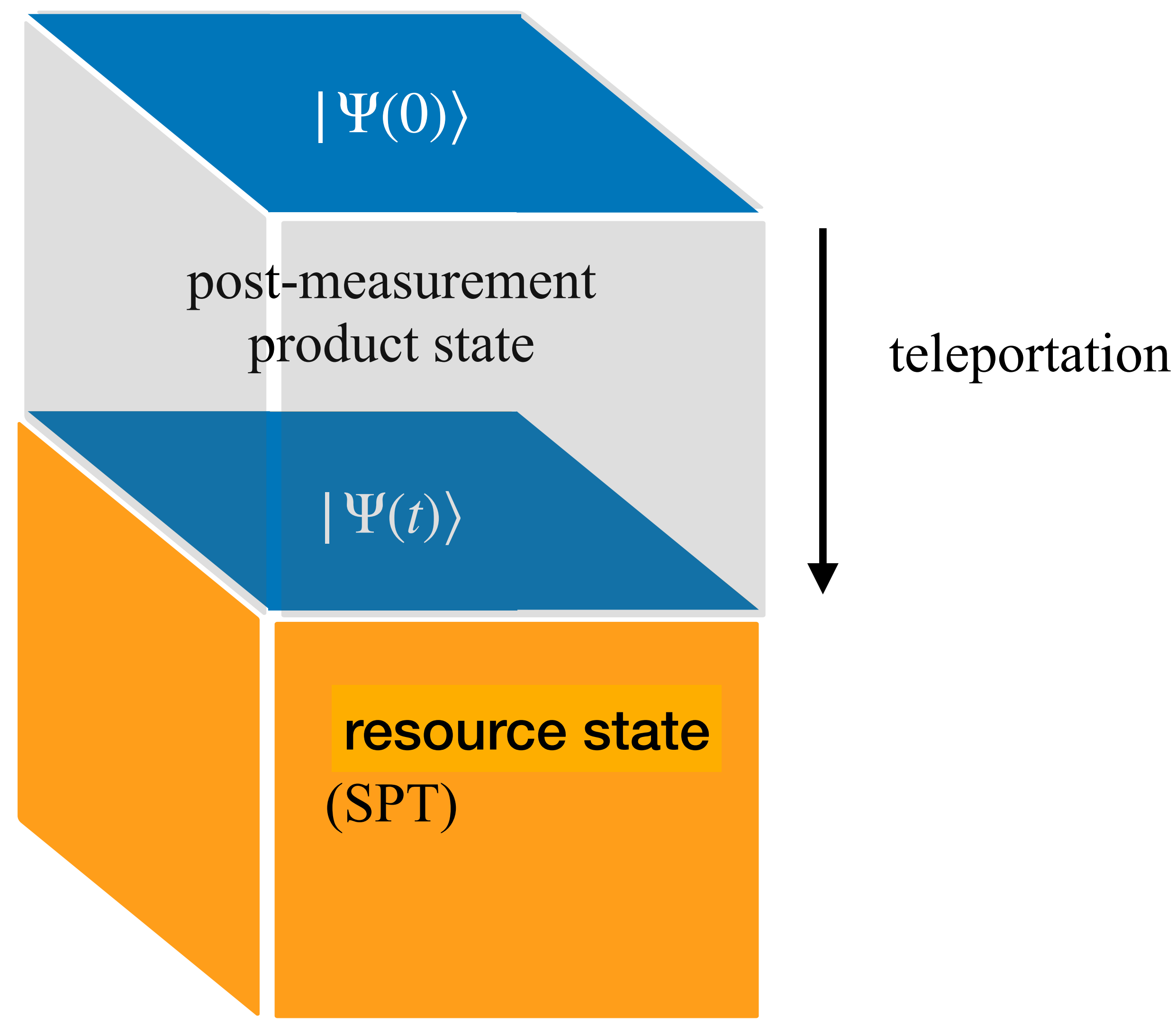
Measurement-based quantum computation (MBQC) is, like the more standard circuit-based quantum computation, is a model capable of **universal** quantum computation.

[Raussendorf-Briegel]

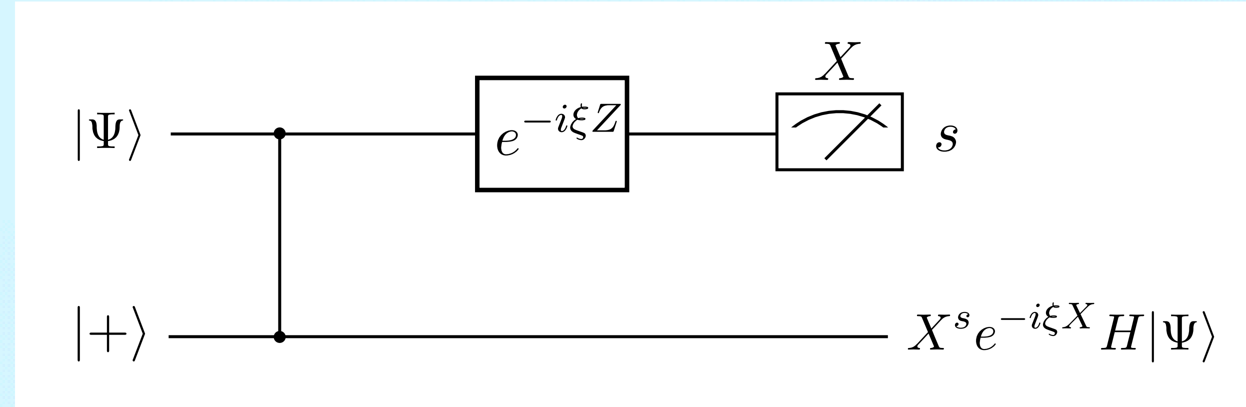


Measurement-based quantum computation

- One first prepares an entangled resource state (**cluster state**, AKLT state, …).
- Computation is driven by local measurements that induce gate teleportation.
- (Measurement-base simulation of lattice models does NOT require universality. We use a resource state tailored to simulate lattice models.)



Gate teleportation



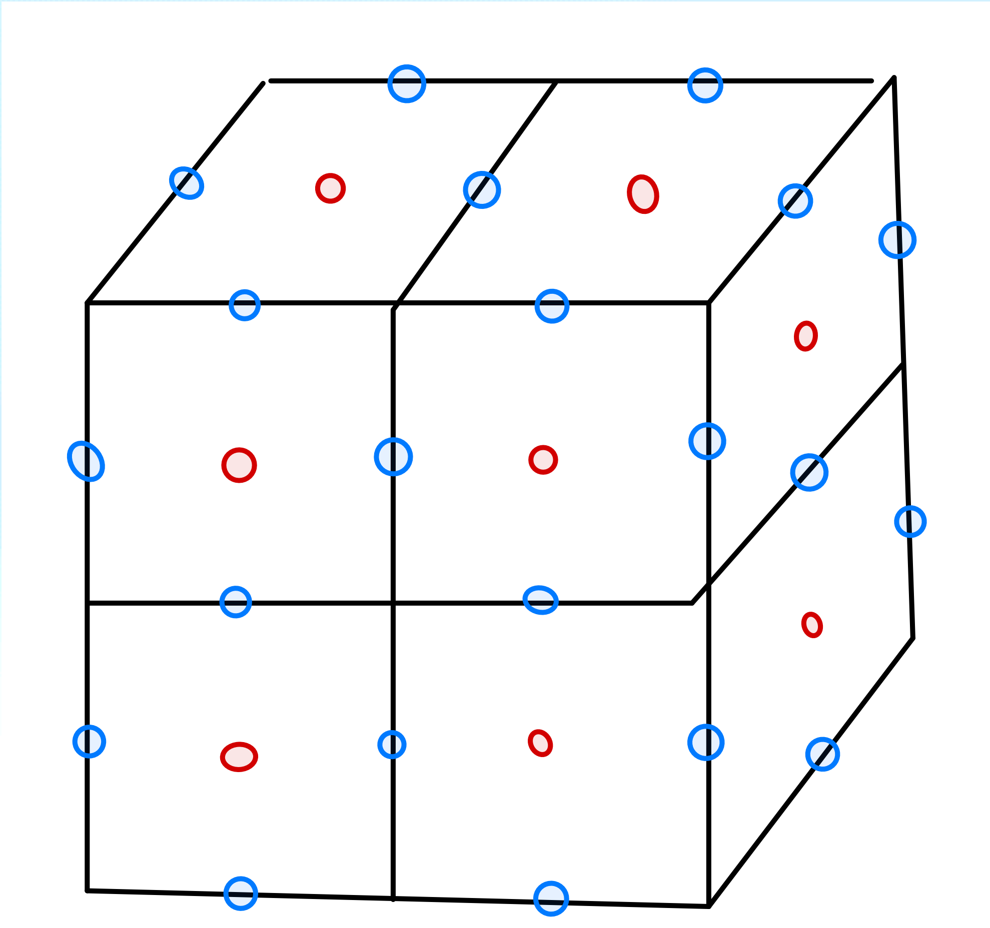
- X-eigenstate $X|\pm\rangle = \pm|\pm\rangle$
- $|\Psi\rangle$ is an arbitrary 1-qubit state
- Entangle $|\Psi\rangle$ and $|+\rangle$ by a controlled-Z gate $CZ_{ab}(=CZ_{ba})$. 
- Measure the first qubit in bases $\{e^{i\xi Z}|\pm\rangle\}$. The measurement outcome is $s=0,1$ corresponding to $\pm 1 = (-1)^s$.
- The state on the second qubit becomes $X^s e^{-i\xi X} H|\Psi\rangle$. Up to X^s and H, the unitary transformation (a part of time evolution) $e^{-i\xi X}$ is implemented. X^s is an example of a "byproduct operator".
- Different measurements give rise to Z^s as a byproduct operator. Note that $e^{-i\xi X}Z^s = Z^s e^{-(-1)^s \xi X}$. We may redefine ξ to absorb $(-1)^s$.

Resource state for $\mathbb{Z}_2$ lattice gauge theory in 2+1 dimensions

- Place one qubit on each 1-cell $\boldsymbol{\sigma}_1 \in \boldsymbol{\Delta}_1$ and 2-cell $\boldsymbol{\sigma}_2 \in \boldsymbol{\Delta}_2$.

- Entangle the neighboring 1-cells and 2-cells by controlled-Z gates.

$$|\mathrm{res}\rangle = \prod_{\boldsymbol{\sigma}_1 \subset \partial \boldsymbol{\sigma}_2} CZ_{\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2} |+\rangle^{\otimes \boldsymbol{\Delta}_1 \cup \boldsymbol{\Delta}_2}$$



- A version of three-dimensional cluster state.

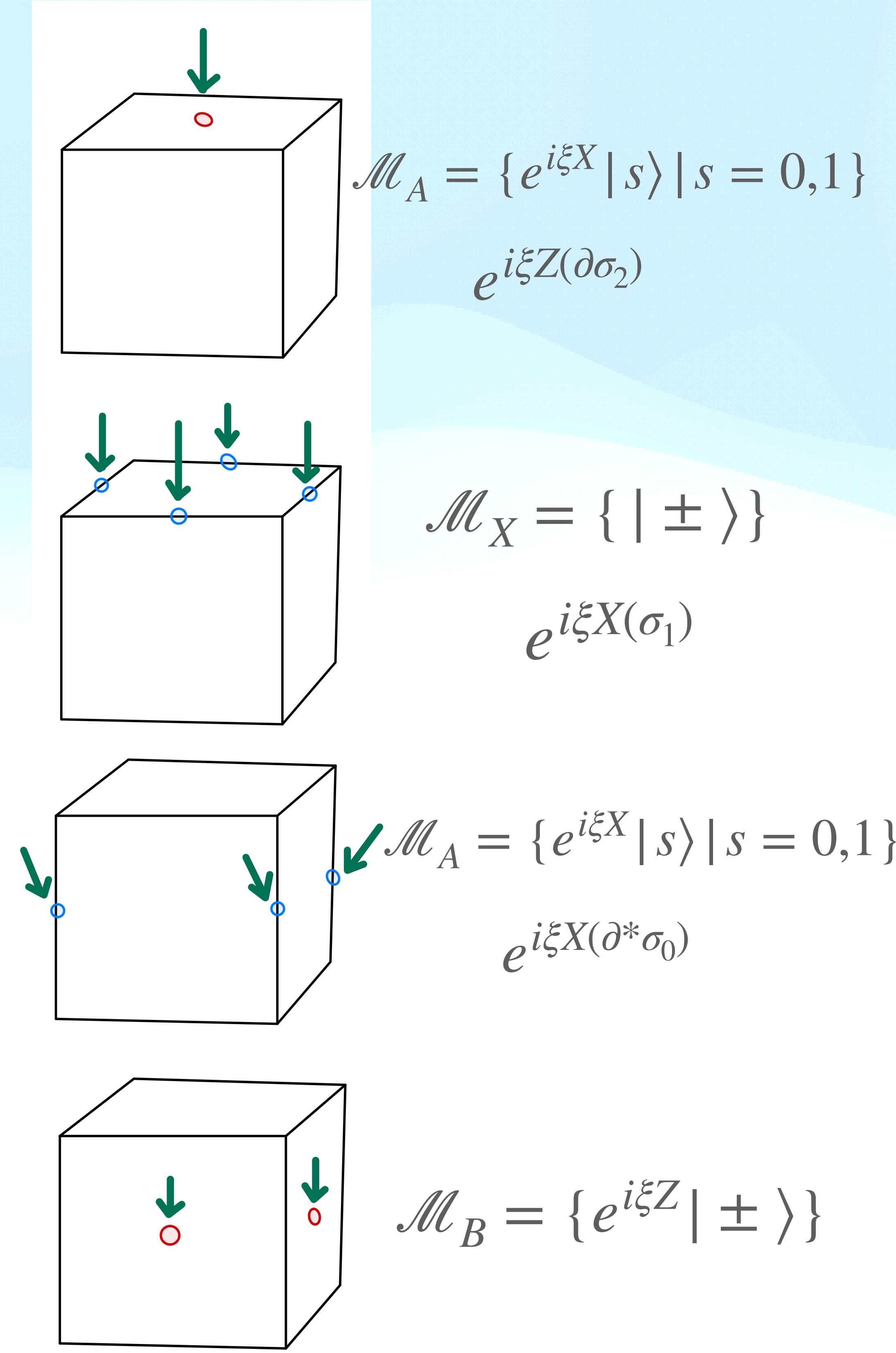
- Stabilizers
$K(\boldsymbol{\sigma}_2) = X(\boldsymbol{\sigma}_2)X(\partial\boldsymbol{\sigma}_2)$ and
$K(\boldsymbol{\sigma}_1) = X(\boldsymbol{\sigma}_1)X(\partial^*\boldsymbol{\sigma}_1)$.

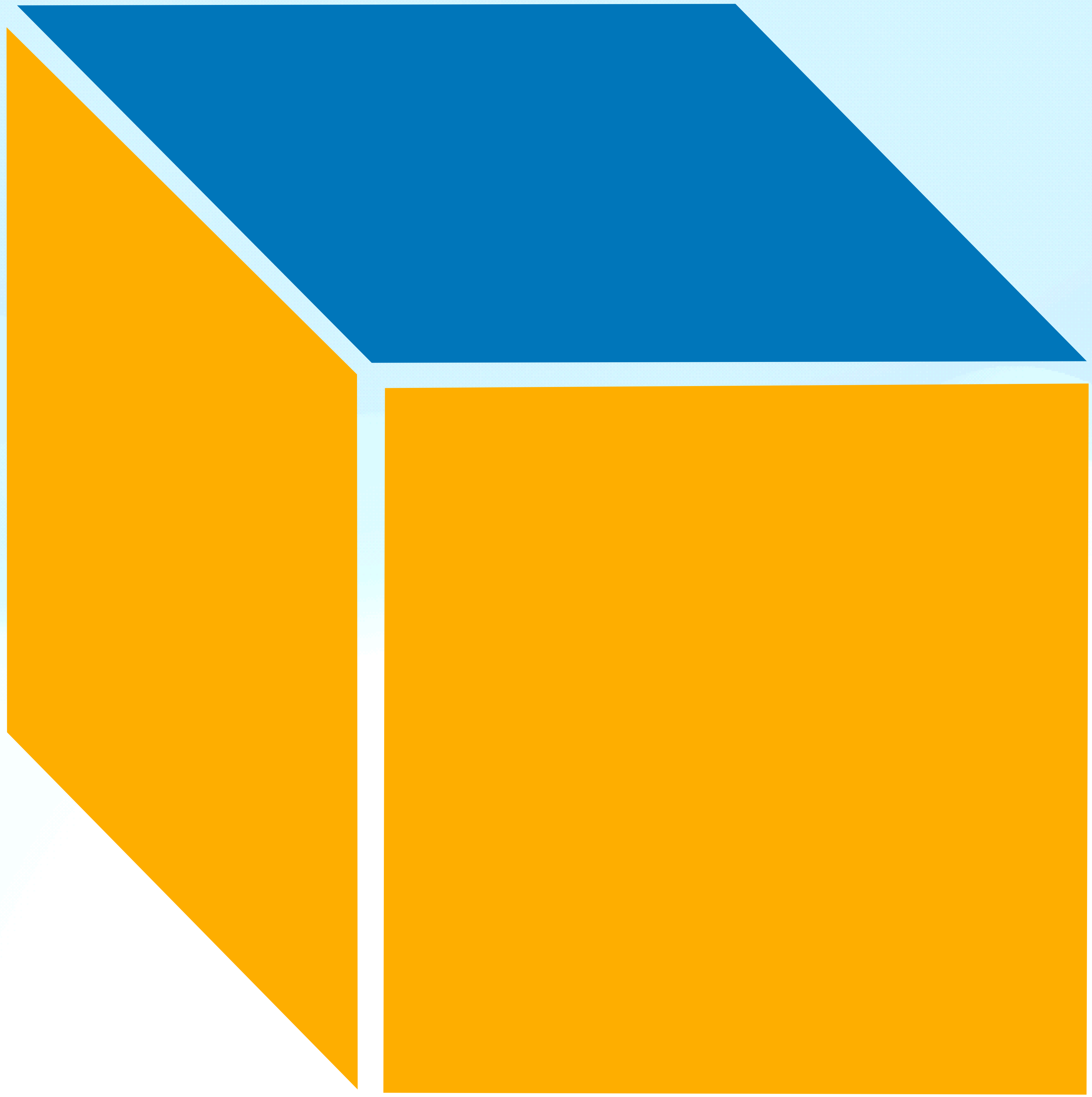
$$K(\boldsymbol{\sigma}_1)|\mathrm{res}\rangle = K(\boldsymbol{\sigma}_2)|\mathrm{res}\rangle = |\mathrm{res}\rangle$$

Measurement pattern = simulation protocol

- Trotterized time evolution is implemented by the measurement pattern and adaptive choices of the measurement angles ξ to absorb minus signs $(-1)^s$.

- Main result of the paper. The resource state reflects the spacetime structure of the simulated gauge theory.





SPT order of the resource state

- Often, the computational power of a resource state can be attributed to the symmetry-protected topological (SPT) order. Examples: AKLT state and 1d cluster state protected by $\mathbb{Z}_2 \times \mathbb{Z}_2$.

- Claim: the natural resource state (qubits on n- and $(n-1)$-cells) for simulating Wegner's model $M_{(d,n)}$ is protected by global $\mathbb{Z}_2$ $(n-1)$- and $\mathbb{Z}_2$ $(d-n)$-form symmetries.

- For the $\mathbb{Z}_2$ gauge theory in $2+1$ dimensions $M_{(3,2)}$, they are both one-form symmetries generated by membrane (surface) operators $\prod_{\boldsymbol{\sigma}_2 \subset z_2} X(\boldsymbol{\sigma}_2)$ with 2-cycle z_2 ($\partial z_2 = 0$) and $\prod_{\boldsymbol{\sigma}_1 \subset z_2^*} X(\boldsymbol{\sigma}_1)$ with dual 2-cycle z_2^* ($\partial^* z_2^* = 0$).

SPT order of the resource state

- The SPT order of the resource state for $M_{(d,n)}$ can be demonstrated by showing that "gauging" the symmetries of the resource state and the product state give rise to distinct topological orders. [Levin-Gu, Yoshida]

- Other evidence for the SPT order includes
  - appearance of a projective representation on the boundary
  - appearance of a projective representation in the tensor network representation of the resource state

Generalizations/extensions in the paper

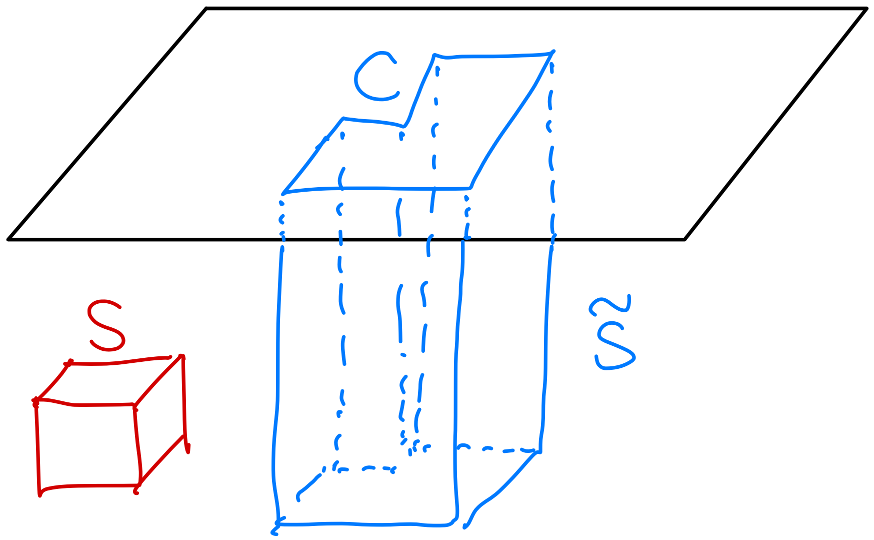
- In the paper we generalize the measurement-based simulation to
  - $(n-2)$-form gauge symmetry in d dimensions (Wegner's generalized Ising model)
  - $\mathbb{Z}_N$ and $\mathbb{R}$ (non-compact $U(1)$) gauge groups
  - Kitaev Majorana chain
- Enforcing Gauss law constraint against noise by error correction
- Imaginary time evolution
- Partition function in terms of the resource state

New type of holography

- Holography for 1d AKLT state has been discussed. [Miyake]
- The simulated theory lives on the codimension-1 boundary of the resource state.
- The states of the simulated theory are in the Hilbert space of the edge modes of the SPT state. They are degenerate.
- Symmetries of the bulk and the boundary are related.
- Unlike other holographic relations (AdS, dS, Minkowski, …), the direction transverse to the boundary is the **real time** direction for the simulated theory, and a spatial direction for the bulk resource state.

Symmetries in bulk and boundary

- Symmetries on the boundary
  - Gauge (0-form) symmetry ⇒ Gauss law constraint $X(\partial^* \sigma_0) = 1$ ⇒ Shape invariance of the loop operator $X(C)$ with $\partial^* C = 0$ ⇒ $\mathbb{Z}_2$ 1-form symmetry
- Symmetries in the bulk resource state
  - Stabilizers ⇒ Invariance under surface operators $X(S)$ with $\partial S = 0$ and $X(\tilde{S})$ with $\partial^* \tilde{S} = 0$ (or $= C$).



Toward experimental realization

- The measurement-based approach requires only simple interactions (such as Ising interactions) between qubits because interactions are only used to create the resource state.
- Since the resource state includes the time direction, the measurement-based approach requires more qubits than the circuit-based approach.
- Possible experimental platforms:
    - Optical lattices formed by cold atoms
    - Cluster states (for continuous variables) created by photons

Measurement-based simulation

(Recap)

- Choose Hamiltonian lattice model to simulate.
- Prepare (experimentally) a **resource state** with $|\Psi(0)\rangle$ on one boundary
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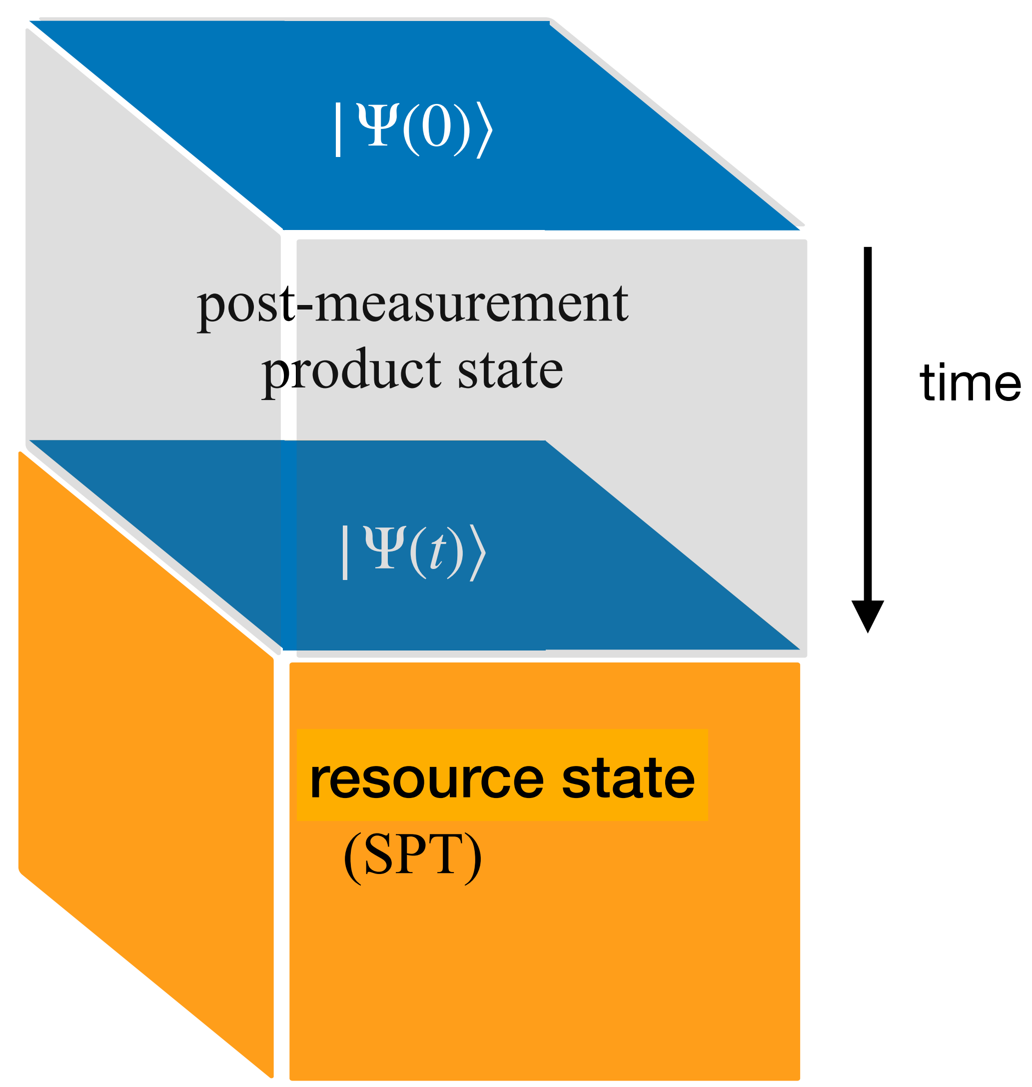

Summary

- Proposed a measurement-based quantum simulation scheme for abelian lattice (possibly higher-form) gauge theories and the Kitaev Majorana chain.
- Defined resource states for the lattice gauge theories.
- Demonstrated that the resource states are have (higher-form) SPT orders.

Future directions

- Non-abelian gauge groups.
- More general fermions.
- Characterization of the higher-form SPT order by an analog of group cohomology or cobordism?
- Relate SPT order to computational power.
- Experimental realizations.
- Quantum simulation on cloud quantum computers with mid-circuit measurement capabilities.

Thank you for the support over the years

Happy 60th birthday!