HirosiFest@Kavli IPMU, Kashiwa, Oct.20-21, 2022

CFT Dual of dS3 and Pseudo Entropy

Tadashi Takayanagi (YITP, Kyoto)

Based on
2110.03197 [PRL129(2022)041601] and 2203.02852 [JHEP 05 (2022) 129]
with Yasuaki Hikida, Yusuke Taki (YITP) and Tatsuma Nishioka (Osaka)

2210.09457 with
Kazuki Doi, Jonathan Harper, Ali Mollabashi, and Yusuke Taki (YITP)

◆ My Ph D student period in Univ. of Tokyo (1998-2002)

➔ I learned a lot on world-sheet string theory from Ooguri-san's great works.

Boundary states in Calabi-Yau

arXiv > hep-th > arXiv:hep-th/9606112

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High Energy Physics - Theory

[Submitted on 18 Jun 1996 (v1), last revised 26 Jun 1996 (this version, v2)]

D-Branes on Calabi-Yau Spaces and Their Mirrors

Hirosi Ooguri, Yaron Oz, Zheng Yin (UC Berkeley/LBNL)

We study the boundary states of D-branes wrapped around supersymmetric cycles in a general Calabi-Yau manifold. In particular, we show how the geometric data on the cycles are encoded in the boundary states. As an application, we analyze how the mirror symmetry transforms D-branes, and we verify that it is consistent with the conjectured periodicity and the monodromy of the Ramond-Ramond field configuration on a Calabi-Yau manifold. This also enables us to study open string worldsheet instanton corrections and relate them to closed string instanton counting. The cases when the mirror symmetry is realized as T-duality are also discussed.

SL(2,R)CFT and Non-critical strings

arXiv > hep-th > arXiv:hep-th/9511164

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[Submitted on 23 Nov 1995]

Two-Dimensional Black Hole and Singularities of CY Manifolds

Hirosi Ooguri (UC Berkeley/LBL), Cumrun Vafa (Harvard University)

We study the degenerating limits of superconformal theories for compactifications on singular K3 and Calabi-Yau threefolds. We find that in both cases the degeneration involves creating an Euclidean two-dimensional black hole coupled weakly to the rest of the system. Moreover we find that the conformal theory of A_n singularities of K3 are the same as that of the symmetric fivebrane. We also find intriguing connections between ADE (1,n) non-critical strings and singular limits of superconformal theories on the corresponding ALE space.

AdS3/CFT2

arXiv > hep-th > arXiv:hep-th/0001053

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High Energy Physics - Theory

[Submitted on 10 Jan 2000 (v1), last revised 19 May 2000 (this version, v3)]

Strings in AdS_3 and the SL(2,R) WZW Model. Part 1: The Spectrum

Juan Maldacena, Hirosi Ooguri

In this paper we study the spectrum of bosonic string theory on AdS_3. We study classical solutions of the SL(2,R) WZW model, including solutions for long strings with non-zero winding number. We show that the model has a symmetry relating string configurations with different winding numbers. We then study the Hilbert space of the WZW model, including all states related by the above symmetry. This leads to a precise description of long strings. We prove a no-ghost theorem for all the representations that are involved and discuss the scattering of the long string.

◆ My post-doc period in Harvard U. (2002-2005)

OSV conjecture, etc.

Einstein Symposium,
@Alexandria, Egypt, June, 2005

arXiv > hep-th > arXiv:hep-th/0405146

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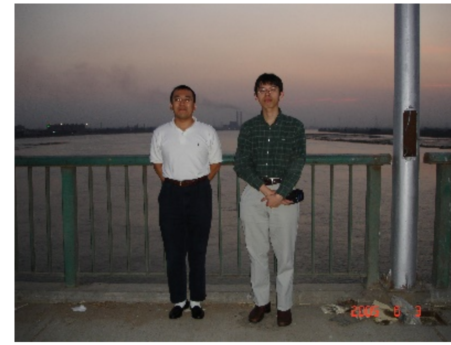
High Energy Physics - Theory

[Submitted on 17 May 2004 (v1), last revised 1 Jun 2004 (this version, v2)]

Black Hole Attractors and the Topological String

Hirosi Ooguri, Andrew Strominger, Cumrun Vafa

A simple relationship of the form Z_BH = |Z_top|^2 is conjectured, where Z_BH is a supersymmetric partition function for a four-dimensional BPS black hole in a Calabi-Yau compactification of Type II superstring theory and Z_top is a second-quantized topological string partition function evaluated at the attractor point in moduli space associated to the black hole charges. Evidence for the conjecture in a perturbation expansion about large graviphoton charge is given. The microcanonical ensemble of BPS black holes can be viewed as the Wigner function associated to the wavefunction defined by the topological string partition function.



◆ Kavli-IPMU period (2008-2012).

➔ a faculty member in string theory group led by Ooguri-san.

I am very grateful to great research environments !

◆Now, I am in YITP, Kyoto U. [2012-]

HEP/COS Joint Seminar
"Symmetry in QFT and Gravity"
@ YITP, Kyoto, April, 2022

arXiv > hep-th > arXiv:2006.13953

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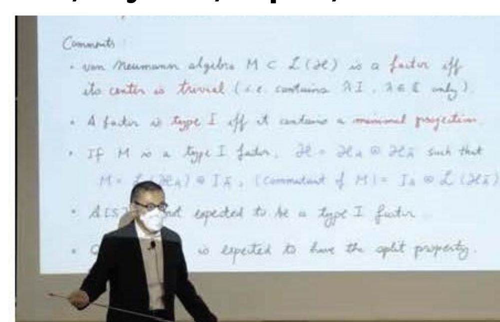
High Energy Physics - Theory

[Submitted on 24 Jun 2020]

Cobordism Conjecture in AdS

Hirosi Ooguri, Tadashi Takayanagi

McNamara and Vafa conjectured that any pair of consistent quantum gravity theories can be connected by a domain wall. We test the conjecture in the context of the AdS/CFT correspondence. There are topological constraints on existence of an interface between the corresponding conformal field theories. We discuss how to construct domain walls in AdS predicted by the conjecture when the corresponding conformal interfaces are prohibited by topological obstructions.



① Introduction: A sketch of dS/CFT

<u>Quantum Gravity in Various Spacetimes</u>

[1] Quantum gravity on **flat space** （Λ=0)

⇒ Well studied via the traditional world-sheet description in string theory.

[2] Quantum gravity on **Anti de-Sitter space** (Λ<0)

⇒ Holographic approach, i.e. AdS/CFT provides a powerful tool !

[3] Quantum gravity on **de-Sitter space** (Λ>0)

⇒ Very difficult problem ! But this is relevant for our universe.

Again, holographic approach is a major candidate to solve this problem.

➡ **Let us develop dS/CFT !** (How does time emerge from CFT ?)

A Sketch of dS/CFT [Strominger 2001, Witten 2001, Maldacena 2002,⋯.]

d+1 dim. Lorentzian de−Sitter spacetime

Dual

Euclidean d. dim CFT on S^d

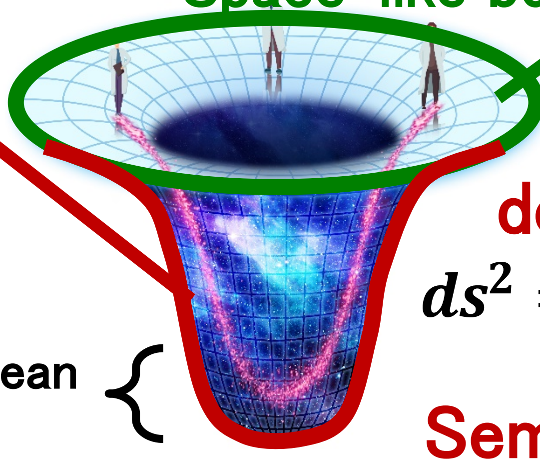
Space−like bdy

Time

Lorentzian time

Euclidean time

Time emerges from Euclidean CFT !



$t=\infty$

de Sitter

$$ds^2 = L_{dS}^2(-dt^2 + \cosh^2 t\, d\Omega^2)$$

Semi sphere

$$ds^2 = L_{dS}^2(d\theta^2 + \sin^2\theta\, d\Omega^2)$$

$$\theta = it + \frac{\pi}{2}$$

Ψ [dS gravity] = Z [CFT]

Note

A regular Euclidean holographic CFT is dual to a Euclidean AdS.
➔ The Euclidean CFT dual to a dS should be "exotic".

Ex1. Proposed gravity dual of 4 dim. Higher spin dS gravity
➔ 3 dim. Sp(N) vector model [anti-commuting scalar fields]
[Anninos-Hartman-Strominger 2011]

Ex2. "Holographic entanglement entropy" gets complex valued.
[No space-like extreme surface ending on bdy. Narayan 2015, Sato 2015,…]

➡ Examples of CFTs dual to Einstein dS gravity are still missing !

Cf. Other possibilities: choices of other holographic screens

Application of dS/dS duality and TTbar [Alishahiha-Karch-Silverstein-Tong 2004, .., Dong-Silverstein-Torroba 2018, Gorbenko-Silvestein-Torroba 2018,..]

Static patch holography [Susskind 2021] Surface/state duality [Miyaji-TT 2015]

Useful basic facts in AdS/CFT

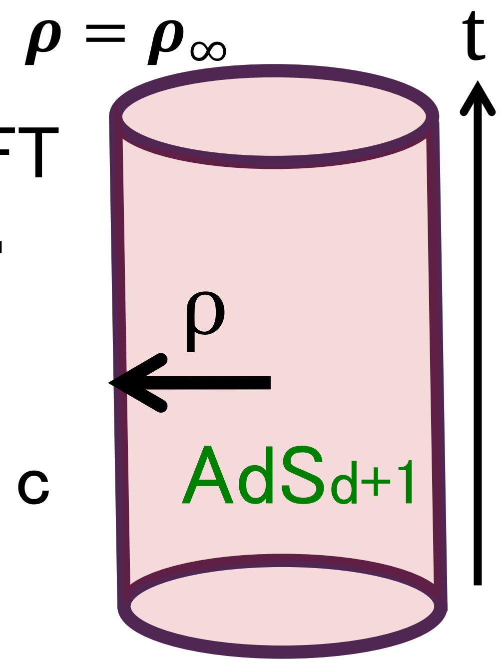
Symmetry: SO(2,d) Killing sym. of AdS_{d+1}
= Conformal sym. of Lorentzian CFT

Degrees of freedom:

Divergent volume = **UV divergence** of QFT

$e^{\rho_\infty} = \frac{1}{\varepsilon}$: UV cut off in CFT

Gravity action $\propto \frac{L_{AdS}^{d-1}}{G_N} \propto$ **Central charge** c in CFT_d



What we expect for dS/CFT

→Let us assume dS Einstein gravity and extract general expectations. [see Maldacena astro-ph/0210603,⋯]

d+1 dim. (Lorentzian) de-Sitter $ds^2 = L_{dS}^2(-dt^2 + \mathrm{Cosh}^2 t\, d\Omega^2)$

↓

S^{d+1} (Euclidean de-Sitter) $ds^2 = L_{dS}^2(d\theta^2 + \mathrm{Sin}^2\theta d\Omega^2)$

↓ $L_{AdS} = iL_{dS},\ \rho = i\theta$

Euclidean AdS (H^{d+1}) $ds^2 = L_{AdS}^2(d\rho^2 + \mathrm{Sinh}^2\rho d\Omega^2)$

Central charge:

$$c \sim \frac{L_{AdS}^{d-1}}{G_N} = i^{d-1} \cdot \frac{L_{dS}^{d-1}}{G_N}$$

We are interested in d=2 case in this talk !

➡

(i) Central charge becomes imaginary for d=even !

(ii) Central charge gets larger in classical gravity limit.

Contents

② Our construction of dS3/CFT2

(2-1) Two known facts on Chern-Simons Formulation

The Einstein gravity on 3d de Sitter space can be rewritten as the 3d CS gauge theory with the gauge group G=SU(2)×SU(2):

$$\boldsymbol{A} \qquad \boldsymbol{\bar{A}}$$

$$I_{\text{dS gravity}} = i\,(I_{CS}[A] - I_{CS}[\bar{A}]),$$

$$I_{CS}[A] = \frac{k}{4\pi}\int_{S^3} \mathrm{Tr}\left[A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right]$$

k = level

[Witten 1988,⋯ for recent analysis, refer to e.g. Castro, Sabella-Garnier , Zukowski 2020.]

$$Z_{CS(\text{dS})} = \int DAD\bar{A}\, e^{-I_{\text{dS gravity}}[A,\bar{A}]}$$

➡ Einstein gravity on S^3

$$A = e + \omega, \quad \bar{A} = e - \omega$$

$$k = i \cdot \frac{L_{dS}}{4G_N}$$

Now, we note another famous fact（“CS holography”）:

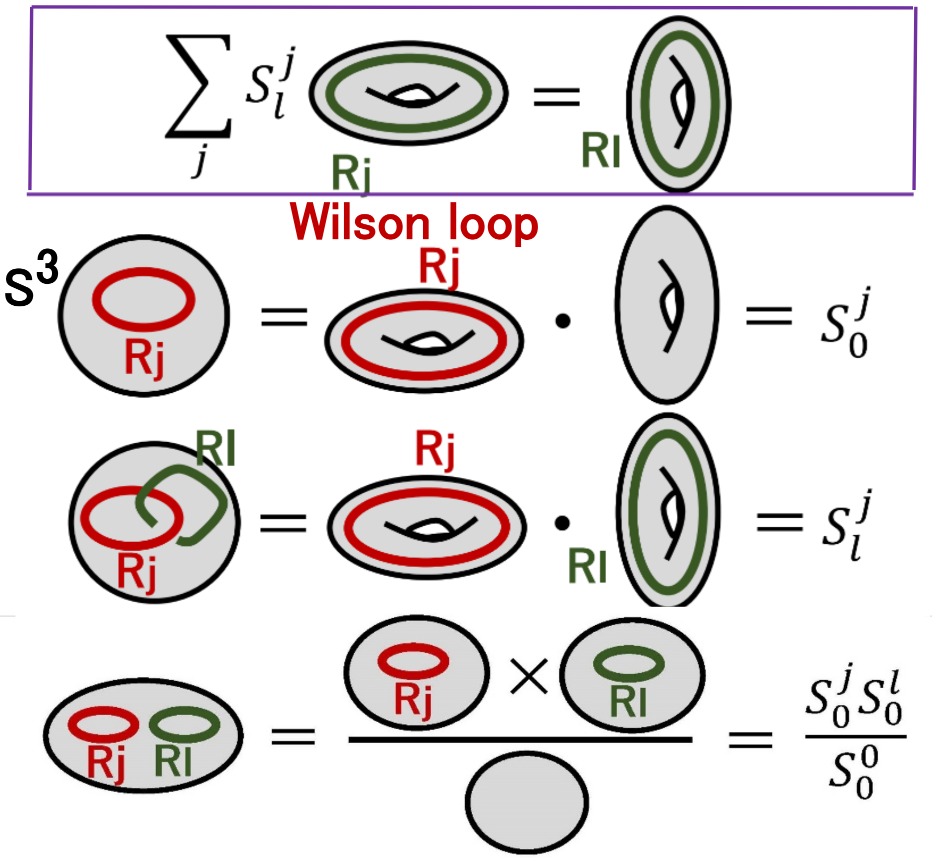
[Witten 1989]

SU(2) CS gauge theory at level k

= conformal block of SU(2) WZW model at level k

$$\sum_j S_j^l Z_j(\tau) = Z_l\left(-\frac{1}{\tau}\right) \longrightarrow S_j^l = \sqrt{\frac{2}{k+2}}\,\mathrm{Sin}\left[\frac{\pi(2j+1)(2l+1)}{k+2}\right]$$

Modular S-matrix



$$Z_{CS} = \int DAD\bar{A}\, e^{iI_{CS}[A]} W(R_j) \cdots$$

$$Z_{CS}[S^3, R_j] = S_0^j$$

$$Z_{CS}[S^3, L(R_j, R_l)] = S_l^j$$

$$Z_{CS}[S^3, R_j, R_l] = \frac{S_0^j S_0^l}{S_0^0}$$

(2-2) Our formulation of dS3/CFT2

A puzzle about dS3/CFT2

By employing the facts explained, one may suspect

3d de Sitter gravity $\stackrel{?}{=}$ SU(2)×SU(2) CS gauge theory
$\stackrel{?}{=}$ SU(2) WZW model ➡ **Is this CFT dual ?**

However, this does not seem to work because

Einstein gravity limit: $k = i \cdot \frac{L_{dS}}{4G_N} \to i\infty$

leads to $c_{SU(2)} = \frac{3k}{k+2} \to 3$. **This is not the large c limit , expected from the dS/CFT !**

Our claim

Instead, we argue that **"k→−2 limit"** realizes the dS/CFT duality:

$$k \approx -2 + \frac{4iG_N}{L_{dS}} \quad \Rightarrow \quad c_{SU(2)} = \frac{3k}{k+2} \approx i\frac{3L_{dS}}{2G_N} \equiv ic_{ds}$$

This is what we expect from dS/CFT.

Correspondense of Excitations

We identify excitations in dS with primary operators in WZW CFT:

Conformal dim. → $$\Delta_j = \frac{2j(j+1)}{k+2} = iL_{dS}E_j$$ ← Energy in dS

The spin j is continuous and can be complex valued.

We have in mind a non-rational version of SU(2) WZW CFT ~ Liouville CFT.

③ DS3 Free Energy from CFT2

Our Claim [Hikida-Nishioka-Taki-TT]

k→−2 limit of SU(2) WZW model (a 2dim. CFT) $c_{SU(2)} \to i\infty$

⇕ Dual

Einstein Gravity on a 3D de Sitter space (radius $L_{ds} \to \infty$)

One may wonder if our limit can correctly reproduce the Einstein gravity on a 3d de Sitter :

$$I_G = -\frac{1}{16\pi G_N}\int \sqrt{g}(R-2\Lambda), \qquad \left(\Lambda \equiv \frac{1}{L_{dS}^2}\right)$$

➔ Below we will compare both partition functions.

(3-1) Partition Functions with a Single Wilson loop

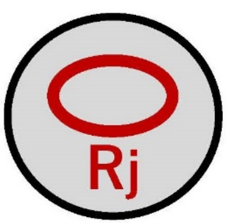
Consider partition functions with Wilson loops inserted.

Useful relation: $1-8G_N E_j = 1 - \frac{12\Delta_j}{c_{SU(2)}} \approx (2j+1)^2.$

$$S_j^l = \sqrt{\frac{2}{k+2}}\mathrm{Sin}\left[\frac{\pi(2j+1)(2l+1)}{k+2}\right] \approx e^{\frac{\pi i(2j+1)(2l+1)}{k+2}}.$$

k➔−2

CFT Prediction: partition function with (i) a single Wilson loop



$$Z_{CS(dS)}[S^3, R_j] = |S_j^0|^2 \approx e^{\frac{\pi L_{ds}}{2G_N}\sqrt{1-8G_N E_j}}$$

In particular, when E=0, we obtain the **de Sitter entropy** $\frac{\pi L_{ds}}{2G_N}$!

Gravity dual: 3 dim. de Sitter `black hole' with energy Ej

$$ds^2 = L_{dS}^2\left[(1 - 8G_N E_j - r^2)d\tau^2 + \frac{dr^2}{1-8G_N E_j - r^2} + r^2 d\phi^2\right].$$

The regularity at the horizon requires the periodicity of τ :

$$\tau \sim \tau + \frac{2\pi}{\sqrt{1-8G_N E_j}}.$$

The on-shell action for this solution is evaluated as

$$I_G = -\frac{1}{16\pi G_N}\int \sqrt{g}(R - 2\Lambda) = -\underbrace{\frac{\pi L_{dS}}{2G_N}\sqrt{1-8G_N E_j}}_{\text{Black hole entropy}}.$$

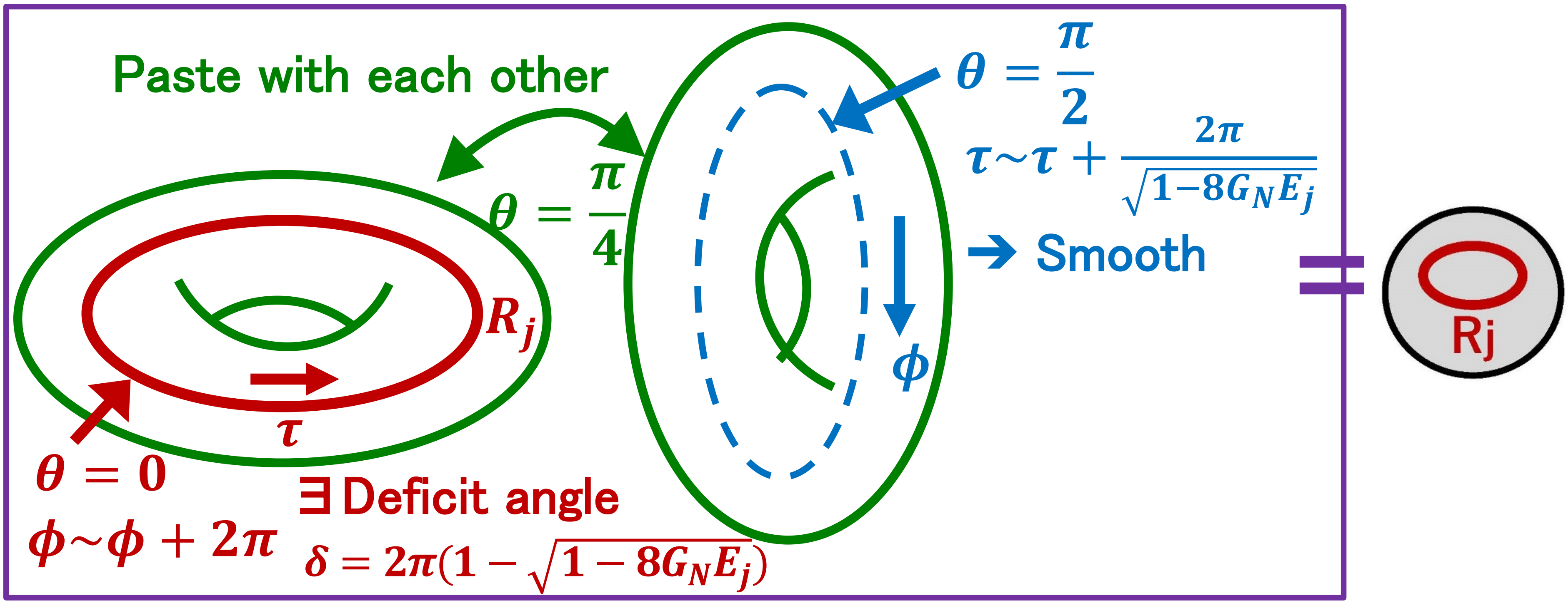
This reproduces the CS result: $Z_{CS(dS)}\left[S^3, R_j\right] = e^{-I_G} = e^{S_{\mathrm{BH}}}.$

Comment on BH Geometry

$$ds^2 = L_{dS}^2\left[\left(1 - 8G_N E_j - r^2\right)d\tau^2 + \frac{dr^2}{1-8G_N E_j - r^2} + r^2 d\phi^2\right].$$

$$r = \sqrt{1 - 8G_N E_j}\,\mathrm{Sin}\theta \qquad (0 \le \theta \le \pi/2)$$

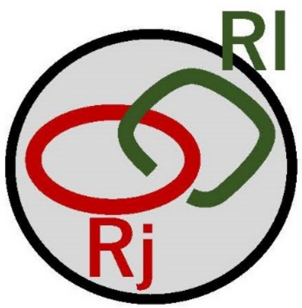
$$ds^2 = L_{dS}^2\left[d\theta^2 + \left(1 - 8G_N E_j\right)\left(\mathrm{Cos}^2\theta d\tau^2 + \mathrm{Sin}^2\theta d\phi^2\right)\right].$$



(3-2) Partition Functions with Two Wilson loops

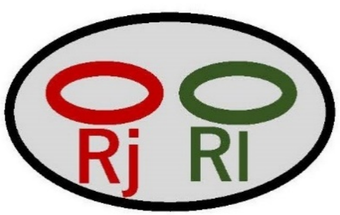
CFT prediction

Partition function with (ii) Two Linked Wilson loop



$$Z_{CS(dS)}[S^3, L(R_j, R_l)] = \left|S_j^l\right|^2 \approx e^{\frac{\pi L_{ds}}{2G_N}\sqrt{1-8G_N E_j}\sqrt{1-8G_N E_l}}$$

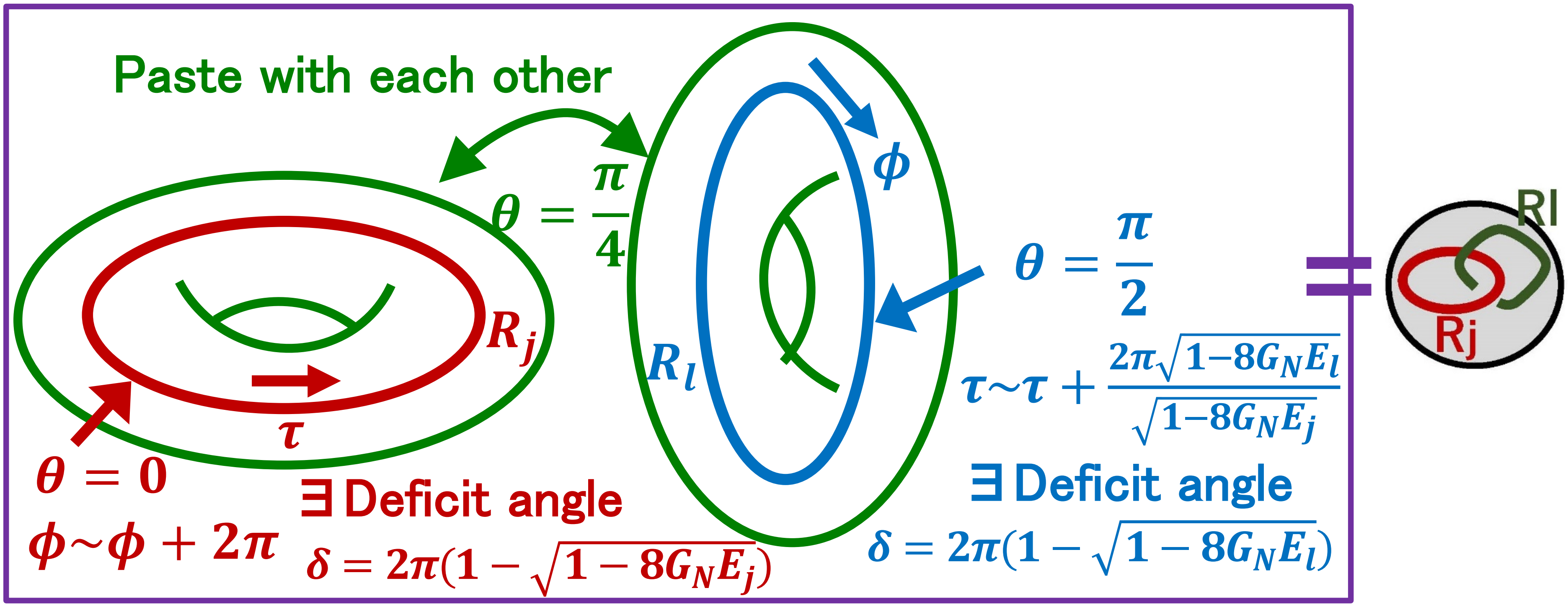
Partition function with (iii) Un-linked Wilson loop



$$Z_{CS(dS)}[S^3, R_j, R_l] = \left|\frac{S_0^j S_0^l}{S_0^0}\right|^2 \approx e^{\frac{\pi L_{ds}}{2G_N}\left(\sqrt{1-8G_N E_j}+\sqrt{1-8G_N E_l}-1\right)}$$

Gravity dual of (ii): Linked Wilson loops

$$ds^2 = L_{dS}^2\left[d\theta^2 + \left(1 - 8G_N E_j\right)\left(\mathrm{Cos}^2\theta\, d\tau^2 + \mathrm{Sin}^2\theta\, d\phi^2\right)\right].$$



$$I_G = -\frac{\pi L_{ds}}{2G_N}\sqrt{1 - 8G_N E_j}\sqrt{1 - 8G_N E_l}.$$

Agree with the CS result !

④ Relation to Higher Spin Holography

We can extend the previous duality to that in higher spin gravity.

➔ **hs[λ]**: gauge theory of Spin 2, 3, ⋯ λ fields.

[For higher spin gravity on dS3, refer to Anninos–Denef–Law–Sun 2020]

For this, consider SU(N) CS gauge theory at level k, related to $SU(N)_k$ WZW model and take the limit:

$$k \approx -N + i\frac{N(N^2-1)}{c_{dS}}$$

This leads to

$$c_{SU(N)} = \frac{k(N^2-1)}{k+N} \approx i c_{dS} \gg 1$$

➔ **dS/CFT**

In this limit, the conformal dimension looks like

$$\Delta_\lambda = \frac{(\lambda, \lambda+2\rho)}{k+N} \approx -i\frac{c_{dS}}{12} \cdot \frac{(\lambda, \lambda+2\rho)}{(\rho, \rho)}$$

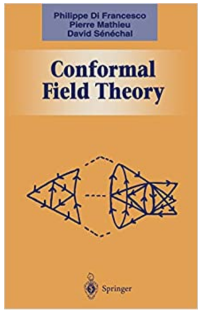
λ: Weight vector of a rep.

ρ: Weyl vector

Partition function in SU(N) CS theory with two linked Wilson loops

$$Z_{CS(dS)}\left[S^3, L(R_\lambda, R_\mu)\right] = \left|S_\lambda^\mu\right|^2 \approx e^{\frac{\pi C_{ds}}{3}\frac{(\lambda+\rho,\mu+\rho)}{(\rho,\rho)}}$$

Refer to e.g.



Dual higher spin gravity calculation

$$A = (h\, b^2\, \bar{h})^{-1}\, \mathrm{d}(h\, b^2\, \bar{h})\ , \quad \bar{A}=0$$

with parameters:

$$b = \prod_{i=1}^{N} \exp\left[\rho_i\, e_{i,i}\right] \qquad \left(\rho_i \equiv \frac{N+1}{2} - i\right)\ ,$$

$$h = \prod_{i=1}^{\lfloor \frac{N}{2} \rfloor} \exp\left[-(e_{2i-1,2i} - e_{2i-1,2i})\, (n_i\, \phi + \tilde{n}_i\, \tau)\right]\ ,$$

$$\bar{h} = \prod_{i=1}^{\lfloor \frac{N}{2} \rfloor} \exp\left[(e_{2i-1,2i} - e_{2i-1,2i})\, (n_i\, \phi - \tilde{n}_i\, \tau)\right]\ .$$

Here $e_{i,j}$ are $N \times N$ matrices with elements $(e_{i,j})_k{}^l = \delta_{ik}\, \delta_j{}^l$.

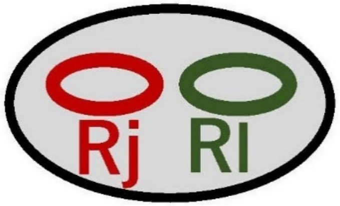
The on-shell action for the gauge configuration can be evaluated as

$$I_{\mathrm{CSG}} = -\frac{\pi}{G_N} \frac{\sum_{i=1}^{\lfloor \frac{N}{2} \rfloor} n_i \tilde{n}_i}{(\rho,\rho)}\ ,$$

$$I_G = -\frac{\pi C_{ds}}{3}\frac{(\lambda+\rho, \mu+\rho)}{(\rho,\rho)}$$

Perfectly matching !

This analysis also explains **the result for (iii) unlinked two loops**:



$$Z_{CS(dS)}\left[S^3, R_j\,, R_l\right] = \left|\frac{S_0^j S_0^l}{S_0^0}\right|^2$$

$$\approx e^{\frac{\pi L_{ds}}{2G_N}\left(\sqrt{1-8G_N E_j}+\sqrt{1-8G_N E_l}-1\right)}$$

by setting $\lambda = \lambda_j + \lambda_l, \quad \mu = 0$.

Perfect match !

Indeed, we find

$$I_G = -\frac{\pi C_{ds}}{3}\frac{(\lambda_j+\lambda_l+\rho,\rho)}{(\rho,\rho)}$$

$$= -\frac{\pi C_{ds}}{3}\cdot\frac{(\lambda_j+\rho,\rho)+(\lambda_l+\rho,\rho)-(\rho,\rho)}{(\rho,\rho)}.$$

Interpretation from Higher Spin Holography

W_N − minimal model

$$\frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}}$$

= [Gaberdiel−Gopakumar 2010]

3D Higher Spin Gravity hs[λ]

Asymptotic sym.=W∞[λ]

Dominant in our limit k⟶−N

$$N \to \infty, k \to \infty, \lambda \equiv \frac{N}{N+k} = \text{fixed}$$

Triality:

$$(N', \lambda') = (\lambda, N)$$

$N \to \infty$

[Gaberdiel−Gopakumar 2012]

Our limit

$$k \to -N$$

=

$$\lambda' = N$$
$$N' \to -i\infty$$

Classical spin N gravity

For N=2, Einstein gravity on dS3 !

⑤ Emergent Time from Pseudo Entropy

So far our argument has been mainly for gravity on S^3 , rather than that on dS_3, missing the emergence of Lorentzian time.

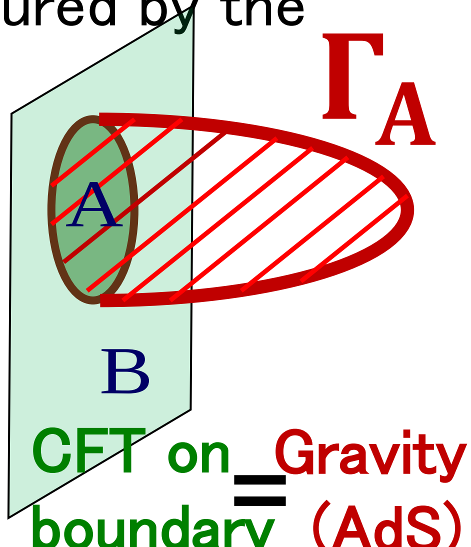
➔ To try this problem, we remember that in AdS/CFT, the holographic space emerges from quantum entanglement.

(5-1) Holographic Entanglement Entropy (HEE) in AdS/CFT

In AdS/CFT, quantum entanglement can be measured by the area of minimal surface.

$$S_A = \min_{\Gamma_A}\left[\frac{\mathrm{Area}(\Gamma_A)}{4G_N}\right]$$

[Ryu-TT 06, Hubeny-Rangamani-TT 07]



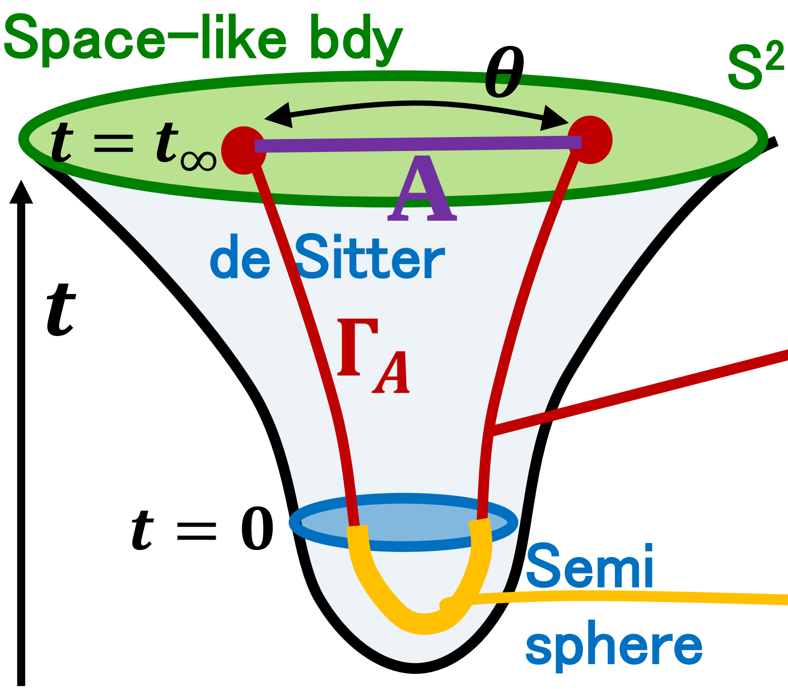
(5-2) Holographic Pseudo Entropy in dS3/CFT2

If we naively apply the HEE in AdS/CFT to dS/CFT, we obtain

$$S_A = \frac{\mathrm{L}(\Gamma_A)}{4G_N} = i\frac{c_{ds}}{3}\log\left(\frac{2}{\epsilon}\mathrm{Sin}\frac{\theta}{2}\right) + \underbrace{\frac{c_{ds}}{6}\pi}_{S_{dS}/2}.$$

SdS/2

$$ds^2 = L_{ds}^2(-dt^2 + \mathrm{Cosh}^2 t\,(d\theta^2 + Sin^2\theta d\varphi^2))$$



$$\mathrm{L}(\Gamma_A) = \underbrace{2it_\infty + i\log(\mathrm{Sin}^2\frac{\theta}{2})}_{} + \pi$$

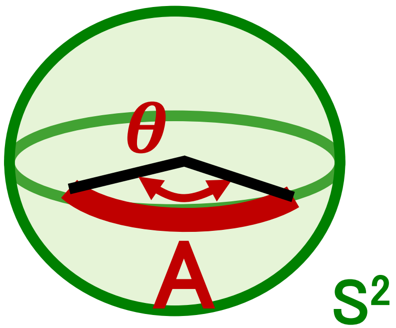
Length of time-like geodesics
➔imaginary value

Space-like geodesic (Semi circle)

This nicely reproduces the 2d CFT result as follows:

$$S_A = \frac{c_{CFT}}{6} \log\left[\frac{\mathrm{Sin}^2 \frac{\theta}{2}}{\tilde{\varepsilon}^2}\right], \quad \text{by setting}$$

$$c_{CFT} = i c_{dS} \text{ and } \tilde{\varepsilon} = i\varepsilon = i e^{-t_\infty}.$$



However, one may wonder why the EE is complex valued.

We argue it is more properly considered as the pseudo entropy.

[Doi-Harper-Mollabashi-Taki-TT 2022]

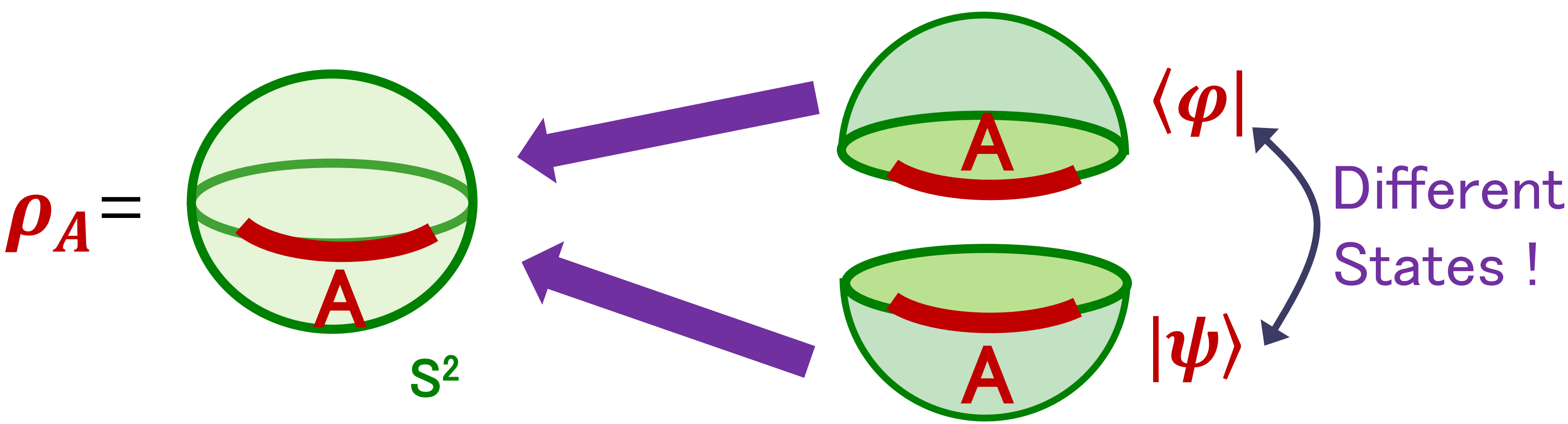
This is because the reduced density matrix ρ_A **is not Hermitian** in the CFT dual to dS, as it is not unitary.

➔For the dual 2d CFT on Σ with metric $h_{ab} = e^{2\phi}\delta_{ab}$, we have

$$Z_{CFT}(S^2) \approx e^{-I_{CFT}[\phi]}, \quad I_{CFT}[\phi] = i\frac{c_{ds}}{24\pi}\int d^2x[(\partial_a \phi)^2 + e^{2\phi}].$$

Complex valued ! ➔ $\rho_A \neq \rho_A^\dagger$

In other words,



Pseudo Entropy [Nakata–Taki–Tamaoka–Wei–TT, 2020]

Transition Matrix: $\mathcal{T}^{\psi|\varphi} := \dfrac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}$ $\left(\mathcal{T}_A^{\psi|\varphi} := \mathrm{Tr}_{\bar{A}}\mathcal{T}^{\psi|\varphi}\right)$

Initial State ($|\psi\rangle$)   Final State ($\langle\varphi|$)

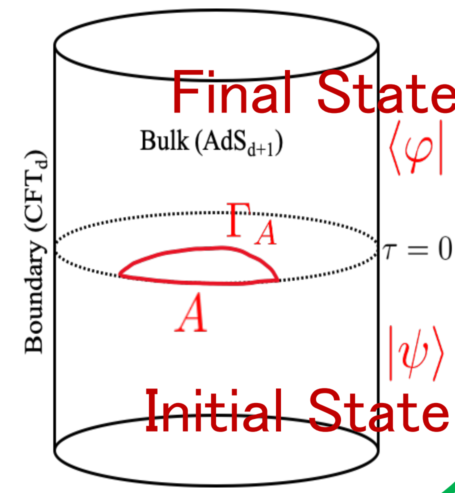
Reduced Transition Matrix⇒**Not Hermitian** in general !

Pseudo Entropy: $S(\mathcal{T}_A^{\psi|\varphi}) = -\mathrm{Tr}\left[\mathcal{T}_A^{\psi|\varphi}\log\mathcal{T}_A^{\psi|\varphi}\right]$

→ In general, complex valued !

Holographic Pseudo Entropy

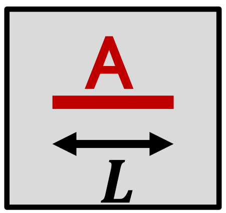
$$S(\mathcal{T}_A^{\psi|\varphi}) = \min_{\Gamma_A}\frac{\mathrm{Area}(\Gamma_A)}{4G_N}$$



(5-3) Time-like Entanglement Entropy in AdS/CFT

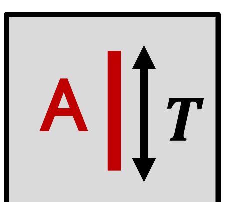
As an instructive example of pseudo entropy, consider a time-like version of entanglement entropy by rotating the subsystem A into a time-like one:

CFT on an infinite line



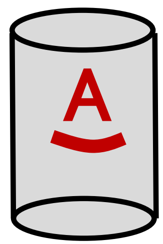
$$S_A = \frac{c_{CFT}}{3}\log\left[\frac{L}{\varepsilon}\right]$$

$$L \to iT$$



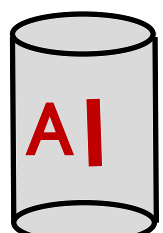
$$S_A = \frac{c_{CFT}}{3}\log\left[\frac{T}{\varepsilon}\right] + \frac{\pi}{6} i c_{CFT}$$

CFT on a circle (2π periodic)



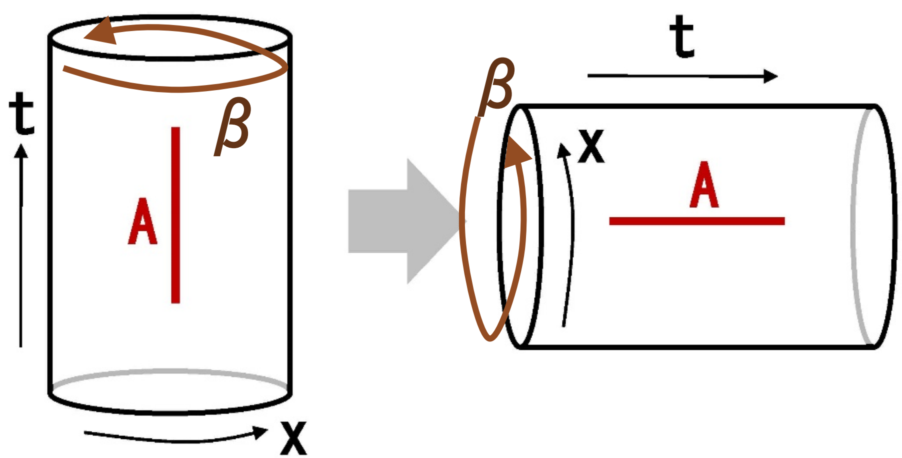
$$S_A = \frac{c_{CFT}}{3}\log\left[\frac{2}{\varepsilon} Sin\left(\frac{L}{2}\right)\right]$$

$$L \to iT$$



$$S_A = \frac{c_{CFT}}{3}\log\left[\frac{2}{\varepsilon} Sin\left(\frac{T}{2}\right)\right] + \frac{\pi}{6} i c_{CFT}$$

What does the time-like EE compute ?



Consider 2d CFT on a cylinder.
If we regard t as a space coordinate and x as a Euclidean time, then then the Hamiltonian looks like

$$H_{time-like} = iH_{CFT}$$

If we trace out a part of t-axis, the reduced density matrix reads

$$\rho_A = Tr_B[e^{i\beta H_{CFT}}]$$ ➡ Non-Hermitian !

We can interpret this as pseudo entropy by doubling Hilbert space:

$$|\psi_{TFD}\rangle \propto \sum_n e^{i\beta E_n/2}|n\rangle_1\,|n\rangle_2$$

$$|\varphi_{TFD}\rangle \propto \sum_n e^{-i\beta E_n/2}|n\rangle_1\,|n\rangle_2$$

➡

$$\rho_A = Tr_B\left[\frac{|\psi_{TFD}\rangle\langle\varphi_{TFD}|}{\langle\varphi_{TFD}|\psi_{TFD}\rangle}\right]$$

Indeed, we can obtain the time-like EE from the finite temp. EE by setting $\beta \to -i\beta$ and $\varepsilon \to i\varepsilon$ as follows:

EE at finite temp. :
$$S_A = \frac{C_{CFT}}{3} \log\left[\frac{\beta}{\pi\varepsilon} Sinh\left(\frac{\pi L}{\beta}\right)\right]$$

$$\beta \to 2\pi i$$
$$\mathrm{L} \to T$$
$$\varepsilon \to i\varepsilon$$

Time-like EE :
$$S_A = \frac{C_{CFT}}{3} \log\left[\frac{2}{\varepsilon} Sin\left(\frac{T}{2}\right)\right] + \frac{\pi}{6} i C_{CFT}$$

AdS dual of time-like EE

[1] Zero temp. and infinite line

$$S_A = \frac{c_{CFT}}{3}\log\left[\frac{T}{\varepsilon}\right] + \frac{\pi}{6} i c_{CFT}$$

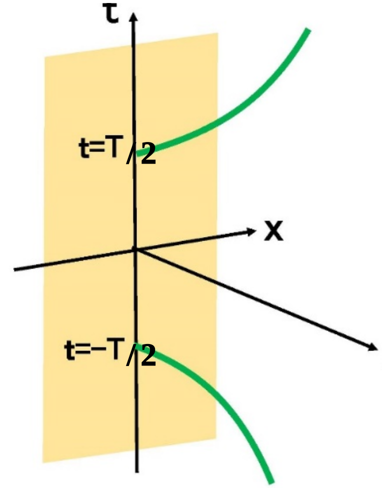
[2] Zero temp. and finite size

$$S_A = \frac{c_{CFT}}{3}\log\left[\frac{2}{\varepsilon} Sin\left(\frac{T}{2}\right)\right] + \frac{\pi}{6} i c_{CFT}$$

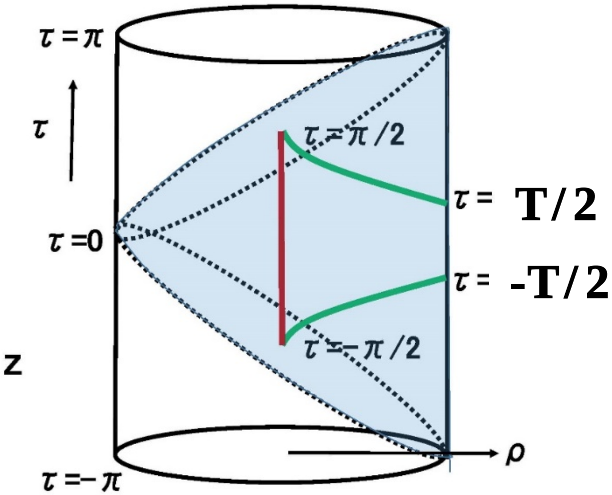
[3] Finite temp.

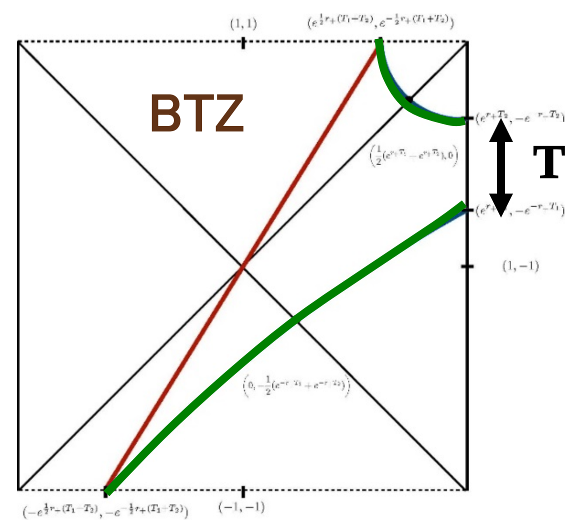
$$S_A = \frac{c_{CFT}}{3}\log\left[\frac{\beta}{\pi\varepsilon} Sinh\left(\frac{\pi T}{\beta}\right)\right] + \frac{\pi}{6} i c_{CFT}$$

Poincare AdS



Global AdS





Real part of Pseudo entropy ➔Space-like geodesic
Imaginary part of Pseudo entropy ➔ Time-like geodesic

Relation between Pseudo entropy in dS and time-like EE

[Time-like EE]

$$S_A = \frac{c_{CFT}}{3}\log\left[\frac{2}{\varepsilon}\mathrm{Sin}\left(\frac{T}{2}\right)\right] + \frac{i\pi}{6}c_{CFT}$$

Space-like Time-like

$$c_{CFT} \to i c_{ds}$$

$$T \to \theta$$

$$\varepsilon \to -\varepsilon$$

[PE in dS/CFT]

$$S_A = i\frac{c_{ds}}{3}\log\left(\frac{2}{\epsilon}\mathrm{Sin}\frac{\theta}{2}\right) + \frac{c_{dS}}{6}\pi.$$

Time-like Space-like

⑥ Conclusions

In this talk, we proposed a CFT dual of 3D de Sitter space and discussed its holographic pseudo entropy.

Our Proposal 1

2d CFT: k→−N limit of SU(N) WZW × [MCFT]

⇕

Classical Spin N Gravity on a 3D de Sitter space (radius $L_{ds} \to \infty$)

➔ Partition functions and Entanglement entropy are reproduced.

Our Proposal 2

Imaginary (or Real) part of pseudo entropy

⬇

Emergent time (or space) in holography

Future problems

◆More one Lorentzian dynamics ?

➔ Explore more on non-unitary CFTs …

◆Details of k=−N limit e.g. spectrum ?

◆Quantum corrections ?

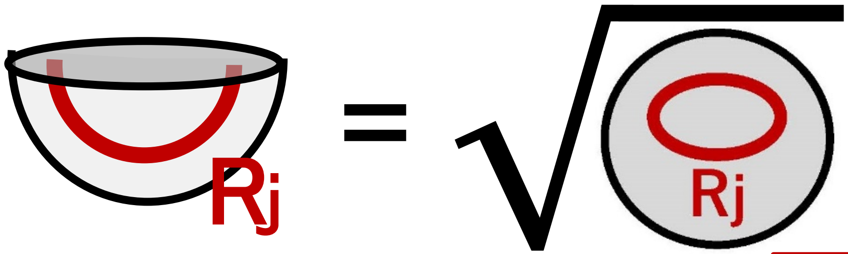
◆Quantum Information theoretic understanding of PE ?

◆Dynamics of Emergent Time ?

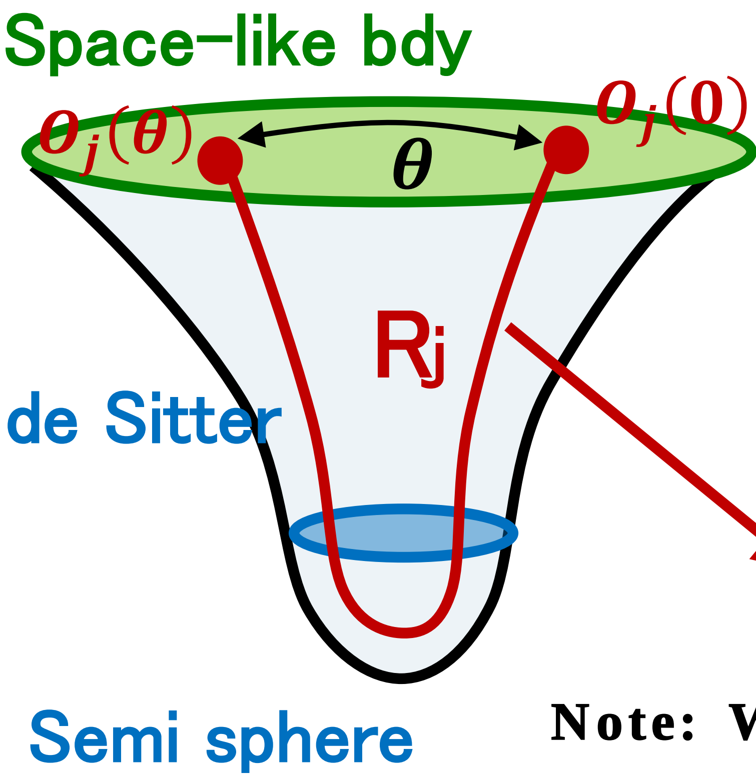
➔ Origins of Einstein equation in dS.

Happy birthday, Ooguri-san　!

Appendix: Two Point Functions



$$= e^{\frac{\pi L_{ds}}{4G_N}\left(\sqrt{1-8G_N E_j}-1\right)} \approx e^{\pi i \Delta_j}$$



$$\langle O_j(\theta) O_j(0) \rangle$$

$$\approx e^{-L_{ds}\cdot E_j \cdot L[\theta]} = e^{\pi i \Delta_j} \cdot \left(\frac{\epsilon^2}{\mathrm{Sin}^2 \frac{\theta}{2}} \right)^{\Delta_j}$$

$$\mathrm{L}[\theta] = \pi + 2it_\infty + i\log\left(\mathrm{Sin}^2 \frac{\theta}{2}\right)$$

Note: We can regard this as a CFT 2-pt function with an imaginary UV cut off $i\varepsilon = i\, e^{-t_\infty}$.