[in prep.]

Cosmological prior for the J-factor estimation of dwarf spheroidal galaxies

Shinichiro Ando, Kohei Hayashi, Shunichi Horigome

Dark matter symposium 29 March 2022

Cosmological prior for the J-factor estimation of dwarf spheroidal galaxies

- Dwarf spheroidal galaxies (dSph) play important roles for dark matter detection but their dark matter halo profiles have large uncertainties
- For the halo profile estimation of dSphs, we apply two **cosmological priors**:
 - **Satellite prior**: constraint distribution of halo parameter based on a structure formation model
 - Stellar-to-halo mass relation prior: empirical relation between stellar mass and halo mass
- The cosmological priors are useful to decrease the uncertainty in the estimation



Introduction

dSphs and DM detection

- Dwarf spheroidal galaxies (dSphs)
 - Large amount of DM
 - good candidates for the indirect detection of the WIMP DM



dSphs and DM detection

- The sensitivity of the indirect detection depends on the DM profile of dSph
 - Indirect detection: DM annihilation into SM particles (gamma-ray etc.)
 - Signal flux from dSphs:

$$\Phi(E,\Delta\Omega) = \left[\frac{C\langle\sigma\nu\rangle}{4\pi m_{\rm DM}^2} \sum_f b_f \left(\frac{dN_{\gamma}}{dE}\right)_f\right] \times J(\Delta\Omega)$$
$$J(\Delta\Omega) = \left[\int_{\Delta\Omega} d\Omega \int_{\rm l.o.s.} dl \,\rho_{\rm DM}^2(l,\Omega)\right]$$

How to estimate DM density profile

- dSph member stars move in the gravitational potential yielded by DM mass density
- Velocity of member stars is observed by spectroscopic telescope



J-factor uncertainty

- J-factor has large uncertainty
 - Limited number of dataset
 - Classical: O(100)
 - Ultrafaint: O(10)



Hayashi et al. [1603.08046]

• We use "cosmological prior" to improve the accuracy

Our analysis: Estimation with cosmological prior

Jeans analysis

- Jeans equation: kinematical equation of dSph systems
 - Assumption: sphericity

$$\boxed{\frac{1}{\nu_*(r)} \frac{\partial(\nu_*(r)\sigma_r^2(r))}{\partial r} + \frac{2\beta(r)\sigma_r^2(r)}{r}}_{\text{(stellar distribution \& velocity dispersion) ~ (inner dark matter mass)}} = -\frac{GM_{\text{DM}}(r)}{r^2}$$

• Observable: line-of-sight velocity dispersion (R-dependent)

$$\sigma_{\rm los}^2(R) = \frac{2}{\Sigma(R)} \int_R^\infty dr \left(1 - \beta(r) \frac{R^2}{r^2}\right) \frac{\nu(r) \sigma_r^2(r)}{\sqrt{1 - R^2/r^2}}$$

- Models:
 - Stellar profile: Plummer model
 - DM profile: truncated NFW model
 - Anisotropy profile: constant model

Likelihood

• Likelihood function

$$\mathscr{L}(\Theta) = \prod \mathscr{N}[v_i; v_{\rm dSph}, \sigma_{\rm los}^2(R_i) + \delta \sigma_i^2],$$

• Posterior probability

$$P(\Theta|D) = \frac{\mathscr{L}(\Theta)\pi(\Theta)}{\int d\Theta \,\mathscr{L}(\Theta)\pi(\Theta)},$$

•
$$\pi(\Theta)$$
 : prior

Priors (1/3)

- Photometry prior: for stellar distribution
 - half-light radius determined by phtometric observation

Name	$\log_{10} r_e / [m pc]$		
Aquarius 2 Boötes 1 Boötes 2 CanesVenatici 1 CanesVenatici 2 Carina 2 ComaBerenices Draco Draco 2 Eridanus 2 Fornax Grus 1 Hercules Horologium 1 Hydrus 1 Leo 1	$\begin{array}{c} 2.094 \pm 0.078\\ 2.204 \pm 0.015\\ 1.523 \pm 0.068\\ 2.529 \pm 0.017\\ 1.732 \pm 0.086\\ 2.392 \pm 0.005\\ 1.870 \pm 0.045\\ 1.757 \pm 0.029\\ 2.256 \pm 0.005\\ 1.121 \pm 0.182\\ 2.196 \pm 0.003\\ 1.267 \pm 0.459\\ 2.080 \pm 0.042\\ 1.488 \pm 0.097\\ 1.727 \pm 0.030\\ 2.353 \pm 0.004\end{array}$	Leo 4 Leo T Leo 5 Pegasus 3 Pisces 2 Reticulum 2 Sagittarius Sculptor Segue 1 Segue 2 Sextans 1 Triangulum 2 Tucana 2 Tucana 3 UrsaMajor 1 UrsaMajor 2	$\begin{array}{c} 2.013 \pm 0.053 \\ 2.125 \pm 0.051 \\ 1.571 \pm 0.181 \\ 1.616 \pm 0.158 \\ 1.678 \pm 0.072 \\ 1.495 \pm 0.018 \\ 3.191 \pm 0.020 \\ 2.359 \pm 0.004 \\ 1.295 \pm 0.062 \\ 1.528 \pm 0.038 \\ 2.538 \pm 0.004 \\ 1.096 \pm 0.134 \\ 2.212 \pm 0.073 \\ 1.640 \pm 0.058 \\ 2.176 \pm 0.024 \\ 1.930 \pm 0.022 \end{array}$
Leo 2	2.217 ± 0.005	Willman 1	1.304 ± 0.045

Priors (2/3)

- Satellite prior [1803.07691, 2002.11956]
 - Accretion of subhalo: extended Press-Schechter (EPS) formalism
 - Tidal stripping effect: semi-analytical subhalo model calibrated by N-body simulation



Priors (3/3)

- The stellar-to-halo mass relation (SHMR)
 - empirical relation between the stellar and DM halo mass of galaxies: M_star = f(M_halo, z)
 - assumption: f(M_halo, z) is a monotonic function for M_halo
- We use:
 - Behroozi+(2013) [1207.6105]
 - calibrated by the Bolshoi simulation, complecate model
 - Moster+(2013) [1205.5807]
 - calibrated by the Millennium simulation, assuming simple double power law
 - Behroozi+(2019) [1806.07893]
 - Updated dataset and models, model selection based on Bayes factor
 - Moster+(2018) [1705.05373]
 - double-power law for efficiency evolution
- Prior:

$$\pi_{\rm SHMR}(\rho_s, r_s, r_t) = \frac{\mathcal{N}(M_{\rm *,obs}|M_{\rm *}(M_{\rm halo}), \sigma^2)\pi_{\rm satellite}(\rho_s, r_s, r_t)}{\int dr_s \, d\rho_s \, dr_t \, \mathcal{N}(M_{\rm *,obs}|M_{\rm *}(M_{\rm halo}), \sigma^2)\pi_{\rm satellite}(\rho_s, r_s, r_t)}$$

$$M_{\text{halo}} \leftarrow (\rho_{s,0}, r_{s,0}, r_{t,0}) \longleftrightarrow (\rho_{s,a}, r_{s,a}, z) \xrightarrow{\text{SHMR}} M_{*,a} = M_{*,0}$$

$$semi-analytic model in [2002.11956]$$



Target dSphs

• 8 classical + 26 ultrafaint dSph in [2002.11956]

Classical:

Carina Draco Fornax Leo I Leo I Sculptor Sextans Ursa Minor

UFD:

Aquarius 2 Bootes I Bootes II Canes Venatici I Canes Venatici II Carina II Coma Berenices Draco II Eridanus II Grus I Hercules Horologium I Hyrdus 1 Leo IV Leo T Leo V Pegasus III Pisces II Reticulum II Segue 1 Segue 2 Triangulum II Tucana II Tucana III Ursa Major I Ursa Major II

MCMC Analysis

- Jeans analysis
 - o 6 Parameters

- Prior choices
 - photometry only
 - photometry + satellite
 - photometry + satellite + SHMR
- Bayesian analysis to calculate posterior probability
 - MCMC tool: emcee 3.0.2

parameter	min.	max.
$\log_{10}R_e/[\text{pc}]$	1.0	3.5
$\log_{10} r_s / [pc]$	0.0	5.0
$\log_{10} ho_s / [M_{\odot} m pc^{-3}]$	-4.0	4.0
$\log_{10} r_t / [pc]$	0.0	5.0
$-\log_{10}(1-\beta_{ani})$	-1.0	1.0
$v_{\rm dSph}/[{ m kms^{-1}}]$	-1000	1000

- vs. radial independent analysis [2002.11956]
 - radial dependence of the likelihood break the degeneracy of the parameter



• Posterior density function

e.g. Coma Berenices

- satellite prior only
- likelihood only
- likelihood + satellite (V_50 = 10.5 km/s)
- likelihood + satellite (V_50 = 18 km/s)
- likelihood + satellite (V_50 = 10.5 km/s) + SHMR(Behroozi)
- likelihood + satellite (V_50 = 18 km/s) + SHMR(Behroozi)
- likelihood + satellite (V_50 = 10.5 km/s) + SHMR(Moster)
- likelihood + satellite (V_50 = 18 km/s) + SHMR(Moster)



• J-factor



- J-factor
 - slightly larger than those of the velocity independent analysis
 - radial dependence of the likelihood excludes too compact or faint DM halo having small J-factor
 - Note: anisotropy profile dependence in the velocity dispersion
 - SHMR priors can decrease J-factor uncertainty (upto ~50%) but model dependent
 - Test of SHMR models by using dSphs?



Summary

- dSphs play an important role of detecting dark matter by the indirect detection method, but their dark matter density are still ambiguous
- We estimate the DM density profile using velocity dependent likelihood with
 - satellite prior
 - stellar-to-halo mass relation (SHMR)
- The radial dependence of the velocity dispersion breaks the parameter degeneracy and gives more reasonable results
- SHMR priors decrease J-factor uncertainties but results have SHMR model dependence

Cosmological prior for the J-factor estimation of dwarf spheroidal galaxies

- Dwarf spheroidal galaxies (dSph) play important roles for dark matter detection but their dark matter halo profiles have large uncertainties
- For the halo profile estimation of dSphs, we apply two **cosmological priors**:
 - **Satellite prior**: constraint distribution of halo parameter based on a structure formation model
 - Stellar-to-halo mass relation prior: empirical relation between stellar mass and halo mass
- The cosmological priors are useful to decrease the uncertainty in the estimation



SHMR

- The stellar-to-halo mass relation (SHMR)
 - empirical relation between the stellar and DM halo mass of galaxies: M_star = f(M_halo, z)
 - assumption: f(M_halo, z) is a monotonic function for M_halo



J-factors (table)

	w/o SHM	IR		PB13	B13 BM13			PB19 BM		BM18	M18	
	flat	sat. 10.5	sat. ₁₈	sat.10.5	sat. ₁₈							
Aquarius2	$18.2^{+0.6}_{-0.6}$	$17.6^{+0.4}_{-0.4}$	$17.8^{+0.3}_{-0.4}$	-	-	-	-	-	-	-	-	
Bootes1	$18.2_{-0.3}^{+0.3}$	$18.0^{+0.2}_{-0.2}$	$18.1_{-0.2}^{+0.2}$	$17.9^{+0.2}_{-0.2}$	$18.0^{+0.2}_{-0.2}$	$18.1^{+0.2}_{-0.2}$	$18.1^{+0.2}_{-0.2}$	$18.1^{+0.2}_{-0.2}$	$18.1_{-0.2}^{+0.2}$	$18.3^{+0.2}_{-0.2}$	$18.3^{+0.2}_{-0.2}$	
Bootes2	$16.6^{+2.8}_{-4.9}$	$17.7_{-0.8}^{+0.7}$	$18.4_{-0.5}^{+0.5}$	$17.5_{-0.3}^{+0.3}$	$17.5_{-0.3}^{+0.3}$	$17.6_{-0.5}^{+0.5}$	$17.8_{-0.7}^{+0.7}$	$17.7_{-0.7}^{+0.6}$	$18.3_{-0.4}^{+0.4}$	$18.3_{-0.4}^{+0.4}$	$18.4_{-0.4}^{+0.4}$	
CanesVenatici1	$17.6^{+0.3}_{-0.2}$	$17.4_{-0.1}^{+0.1}$	$17.5_{-0.1}^{+0.1}$	$17.4_{-0.1}^{+0.1}$	$17.4_{-0.1}^{+0.1}$	$17.5_{-0.1}^{+0.1}$	$17.4_{-0.1}^{+0.1}$	$17.5_{-0.1}^{+0.1}$	$17.5_{-0.1}^{+0.1}$	$17.5_{-0.1}^{+0.1}$	$17.5_{-0.1}^{+0.1}$	
CanesVenatici2	$17.9^{+0.5}_{-0.5}$	$17.4_{-0.4}^{+0.4}$	$17.6_{-0.3}^{+0.3}$	$17.0^{+0.2}_{-0.2}$	$17.1_{-0.2}^{+0.2}$	$17.4_{-0.3}^{+0.3}$	$17.5_{-0.3}^{+0.3}$	$17.5_{-0.3}^{+0.3}$	$17.5_{-0.3}^{+0.3}$	$17.7_{-0.3}^{+0.3}$	$17.7_{-0.3}^{+0.3}$	
Carina2	$18.4_{-0.5}^{+0.6}$	$18.1_{-0.4}^{+0.4}$	$18.4_{-0.4}^{+0.4}$	-	-	-	-	-	-	-	-	
ComaBerenices	$19.0^{+0.3}_{-0.4}$	$18.6_{-0.3}^{+0.3}$	$18.7_{-0.3}^{+0.3}$	$18.1^{+0.2}_{-0.2}$	$18.2^{+0.2}_{-0.2}$	$18.5^{+0.2}_{-0.3}$	$18.6^{+0.2}_{-0.3}$	$18.5^{+0.3}_{-0.3}$	$18.6^{+0.2}_{-0.2}$	$18.7^{+0.3}_{-0.3}$	$18.7^{+0.3}_{-0.3}$	
Draco2	$16.8^{+2.5}_{-4.8}$	$18.3_{-0.8}^{+0.6}$	$18.9_{-0.5}^{+0.4}$	-	-	-	-	-	-	-	-	
Eridanus2	$17.3^{+0.4}_{-0.4}$	$16.9^{+0.3}_{-0.2}$	$17.0^{+0.2}_{-0.2}$	-	-	-	-	-	-	-	-	
Grus1	$17.4_{-0.9}^{+0.9}$	$17.1_{-0.5}^{+0.5}$	$17.4_{-0.4}^{+0.4}$	2	-	-	-		12	-	-	
Hercules	$17.9_{-0.4}^{+0.4}$	$17.5_{-0.3}^{+0.3}$	$17.7_{-0.3}^{+0.3}$	$17.4^{+0.2}_{-0.2}$	$17.5^{+0.2}_{-0.2}$	$17.6^{+0.3}_{-0.3}$	$17.6^{+0.3}_{-0.3}$	$17.7^{+0.3}_{-0.3}$	$17.7^{+0.3}_{-0.3}$	$17.9^{+0.2}_{-0.2}$	$18.0^{+0.2}_{-0.2}$	
Horologium1	$19.1_{-0.6}^{+0.7}$	$18.1_{-0.4}^{+0.4}$	$18.3^{+0.3}_{-0.4}$	-	-	-	-	-	-	-	-	
Hydrus1	$18.5_{-0.3}^{+0.4}$	$18.3_{-0.3}^{+0.3}$	$18.5_{-0.3}^{+0.3}$	-	-	-	-	-	-	-	-	
Leo4	$15.6^{+1.9}_{-5.1}$	$16.7^{+0.5}_{-0.6}$	$17.2^{+0.5}_{-0.5}$	$17.0^{+0.3}_{-0.3}$	$17.0^{+0.3}_{-0.3}$	$17.1^{+0.4}_{-0.4}$	$17.1^{+0.4}_{-0.4}$	$16.9^{+0.5}_{-0.5}$	$17.2^{+0.4}_{-0.5}$	$17.7^{+0.3}_{-0.3}$	$17.7^{+0.3}_{-0.3}$	
Leo5	$17.2_{-0.8}^{+0.8}$	$16.9^{+0.4}_{-0.5}$	$17.3_{-0.4}^{+0.4}$	$16.7_{-0.3}^{+0.3}$	$16.9^{+0.3}_{-0.3}$	$17.1_{-0.3}^{+0.3}$	$17.2^{+0.4}_{-0.4}$	$17.2^{+0.4}_{-0.4}$	$17.3^{+0.3}_{-0.4}$	$17.6^{+0.3}_{-0.3}$	$17.5_{-0.3}^{+0.3}$	
LeoT	$17.6_{-0.4}^{+0.4}$	$16.9_{-0.3}^{+0.3}$	$17.0_{-0.3}^{+0.3}$	$16.8_{-0.2}^{+0.2}$	$16.9^{+0.2}_{-0.2}$	$17.1_{-0.3}^{+0.3}$	$17.1_{-0.3}^{+0.2}$	$17.1_{-0.3}^{+0.3}$	$17.1_{-0.3}^{+0.3}$	$17.1_{-0.1}^{+0.1}$	$17.1^{+0.1}_{-0.1}$	
Pegasus3	$17.8^{+1.0}_{-2.1}$	$16.7^{+0.6}_{-0.7}$	$17.2_{-0.5}^{+0.4}$	-	-	-	-	-	-	-	-	
Pisces2	$17.2^{+0.9}_{-1.2}$	$16.7^{+0.6}_{-0.6}$	$17.2^{+0.5}_{-0.5}$	$16.7^{+0.3}_{-0.3}$	$16.7^{+0.3}_{-0.3}$	$16.9^{+0.4}_{-0.4}$	$16.9^{+0.5}_{-0.5}$	$16.9^{+0.5}_{-0.6}$	$17.2^{+0.4}_{-0.5}$	$17.5^{+0.3}_{-0.3}$	$17.5^{+0.3}_{-0.3}$	
Reticulum2	$19.0^{+0.4}_{-0.4}$	$18.7_{-0.3}^{+0.3}$	$18.8_{-0.3}^{+0.3}$	-	-	-	-	-	-	-	-	
Segue1	$19.7_{-0.4}^{+0.4}$	$19.2_{-0.4}^{+0.3}$	$19.3_{-0.3}^{+0.3}$	$18.2^{+0.3}_{-0.3}$	$18.4^{+0.3}_{-0.3}$	$18.9^{+0.3}_{-0.4}$	$19.2^{+0.3}_{-0.3}$	$19.0^{+0.3}_{-0.3}$	$19.2^{+0.3}_{-0.3}$	$19.2^{+0.3}_{-0.4}$	$19.2^{+0.3}_{-0.4}$	
Segue2	$18.0^{+0.7}_{-2.1}$	$18.0^{+0.5}_{-0.5}$	$18.4_{-0.4}^{+0.4}$	$17.7_{-0.3}^{+0.3}$	$17.7_{-0.3}^{+0.3}$	$17.9^{+0.4}_{-0.4}$	$18.1^{+0.4}_{-0.6}$	$18.1_{-0.5}^{+0.4}$	$18.4_{-0.4}^{+0.3}$	$18.4_{-0.3}^{+0.3}$	$18.4_{-0.3}^{+0.3}$	
Triangulum2	$14.4^{+2.9}_{-3.9}$	$17.7_{-0.9}^{+0.7}$	$18.5_{-0.5}^{+0.4}$	-	-	-	-	-	-	-	-	
Tucana2	$18.1_{-0.5}^{+0.6}$	$17.9_{-0.4}^{+0.4}$	$18.2^{+0.4}_{-0.4}$	-	-	-	-	-	-	-	-	
Tucana3	$15.7^{+1.8}_{-4.3}$	$17.5_{-0.6}^{+0.5}$	$18.1_{-0.3}^{+0.3}$		-	-	-		-	-		
UrsaMajor1	$18.7_{-0.3}^{+0.3}$	$18.3_{-0.2}^{+0.2}$	$18.3_{-0.2}^{+0.2}$	$18.0^{+0.2}_{-0.2}$	$18.1^{+0.2}_{-0.2}$	$18.3^{+0.2}_{-0.2}$	$18.3^{+0.2}_{-0.2}$	$18.3^{+0.2}_{-0.2}$	$18.3^{+0.2}_{-0.2}$	$18.4^{+0.2}_{-0.2}$	$18.4^{+0.2}_{-0.2}$	
UrsaMajor2	$19.5_{-0.4}^{+0.4}$	$19.0^{+0.3}_{-0.3}$	$19.1^{+0.2}_{-0.2}$	$18.6_{-0.2}^{+0.2}$	$18.6_{-0.2}^{+0.2}$	$18.9^{+0.3}_{-0.3}$	$19.1_{-0.3}^{+0.3}$	$19.0^{+0.2}_{-0.2}$	$19.0^{+0.3}_{-0.2}$	$19.1^{+0.3}_{-0.3}$	$19.1^{+0.3}_{-0.3}$	
Willman1	$19.5_{-0.4}^{+0.4}$	$18.8_{-0.4}^{+0.3}$	$19.0_{-0.3}^{+0.3}$	$18.0_{-0.3}^{+0.3}$	$18.1_{-0.3}^{+0.3}$	$18.6_{-0.3}^{+0.3}$	$18.9_{-0.3}^{+0.3}$	$18.7_{-0.3}^{+0.3}$	$18.8_{-0.3}^{+0.3}$	$18.9_{-0.3}^{+0.3}$	$18.9_{-0.3}^{+0.3}$	

	W/o SHM	D	SHMP	SHMD		
	flat	sat.	sat.	sat.		
Carina	$17.9^{+0.1}_{-0.1}$	$17.9^{+0.1}_{-0.1}$	$17.9^{+0.1}_{-0.1}$	$17.9^{+0.1}_{-0.1}$		
Draco	$18.9^{+0.1}_{-0.1}$	$18.9^{+0.1}_{-0.1}$	$18.9^{+0.1}_{-0.1}$	$18.8^{+0.1}_{-0.1}$		
Fornax	$17.9^{+0.2}_{-0.1}$	$18.0^{+0.1}_{-0.1}$	$18.0^{+0.1}_{-0.1}$	$18.0^{+0.1}_{-0.1}$		
Leo1	$17.8^{+0.2}_{-0.1}$	$17.7^{+0.1}_{-0.1}$	$17.7^{+0.1}_{-0.1}$	$17.7^{+0.1}_{-0.1}$		
Leo2	$17.8^{+0.2}_{-0.2}$	$17.7^{+0.2}_{-0.1}$	$17.7^{+0.2}_{-0.1}$	$17.6^{+0.1}_{-0.1}$		
Sculptor	$18.6^{+0.1}_{-0.0}$	$18.5_{-0.0}^{+0.0}$	$18.6_{-0.0}^{+0.0}$	$18.5_{-0.0}^{+0.0}$		
Sextans1	$18.1_{-0.2}^{+0.4}$	$18.1_{-0.1}^{+0.1}$	$18.1_{-0.1}^{+0.1}$	$18.1_{-0.1}^{+0.1}$		
UrsaMinor	$18.5_{-0.1}^{+0.1}$	$18.5_{-0.1}^{+0.1}$	$18.5_{-0.1}^{+0.1}$	$18.5_{-0.1}^{+0.1}$		

J-factors





Difference of Jeans analyses

• [2002.11956]: velocity dispersion averaged over total system

$$\sigma_{\rm los}^2 = \frac{4\pi G}{3} \int_0^\infty dr \ r\nu_\star(r) M(r),$$

• This work: radial dependent velocity dispersion calculated by the spherical Jeans equation

$$\sigma_{\rm l.o.s.}^2(R) = \frac{2}{\Sigma_1(R)} \int_R^\infty {\rm d}r \left(1 - \beta_{\rm ani} \frac{R^2}{r^2}\right) \frac{\nu_1(r) \sigma_r^2(r)}{\sqrt{1 - R^2/r^2}} \; , \label{eq:sigma_loss}$$

Models

• Plummer model

$$\begin{split} \nu(r) &= \frac{3}{4\pi R_e^3} \left(1 + \left(\frac{r}{R_e}\right)^2 \right)^{-5/2}, \\ \Sigma(R) &= \frac{1}{\pi} \left(1 + \frac{R^2}{R_e^2} \right)^{-2}, \end{split}$$

- Truncated NFW model
 - Outermost halo is striped by tidal force

$$\rho(r) = \begin{cases} \rho_s \left(\frac{r}{r_s}\right)^{-1} \left(1 + \frac{r}{r_s}\right)^{-2} & (0 \le r \le r_t) \\ 0 & (r_t < r) \end{cases}, \\ M(r) = \begin{cases} 4\pi \rho_s r_s^3 \left(\log\left(1 + \frac{r}{r_s}\right) - \frac{r}{r_t + r_s}\right) & (0 \le r \le r_t) \\ 4\pi \rho_s r_s^3 \left(\log\left(1 + \frac{r_t}{r_s}\right) - \frac{r_t}{r_t + r_s}\right) & (r_t < r), \end{cases}$$

dSphs and DM detection

- Dwarf spheroidal galaxies (dSphs)
 - inner DM halo profile gives constraints on DM self-interaction



Sommerfeld effect

- Sommerfeld effect:
 - nonrelativistic effect of scattering



