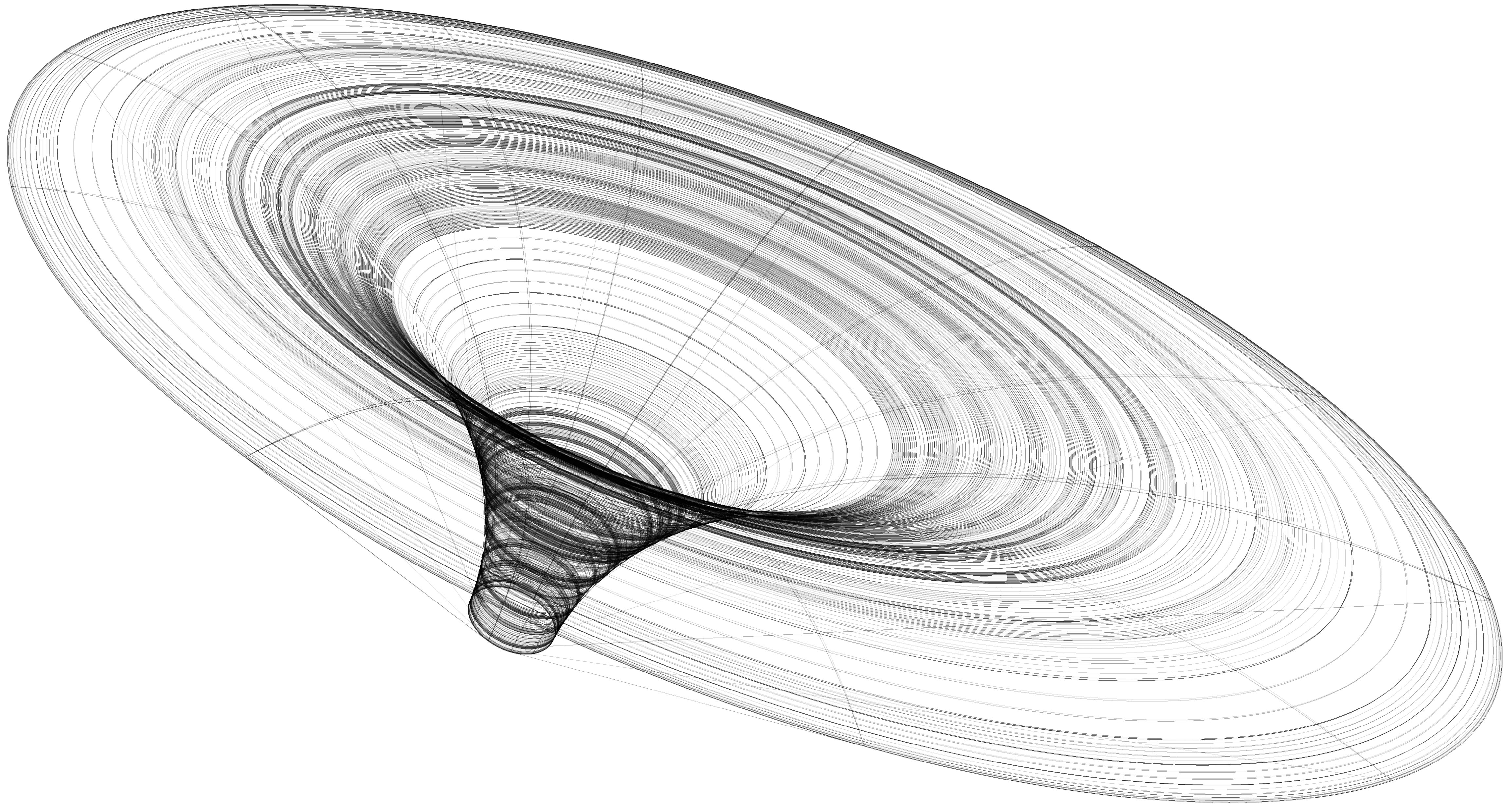


*30th March 2022 @ FY2021 "Dark Matter" Symposium*

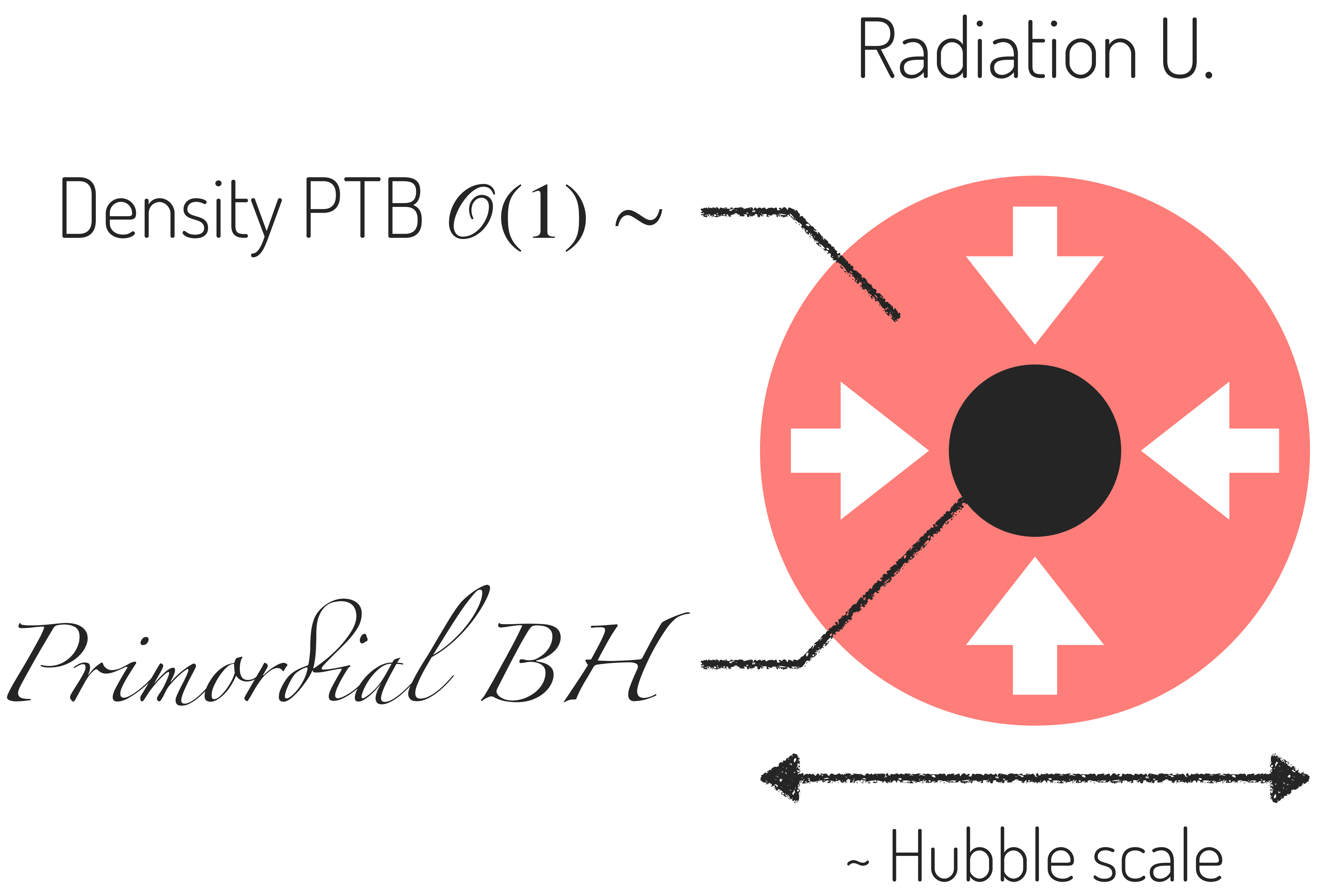
# Primordial BHs and induced GWs in light of the NG tail

Yuichiro TADA (Nagoya)

N. Kitajima, YT, S. Yokoyama, C. M. Yoo 2109.00791
A. Escrivà, YT, S. Yokoyama, C. M. Yoo 2202.01028
K. T. Abe, R. Inui, YT, S. Yokoyama in prep.

# Primordial BH

Carr & Hawking '71, '74, '75




- Dark Matter?
- LVK merger GW?
- SuperMassive BH seeds?
- OGLE lensing obj.?
- Planet 9?
- Trigger of r-process?

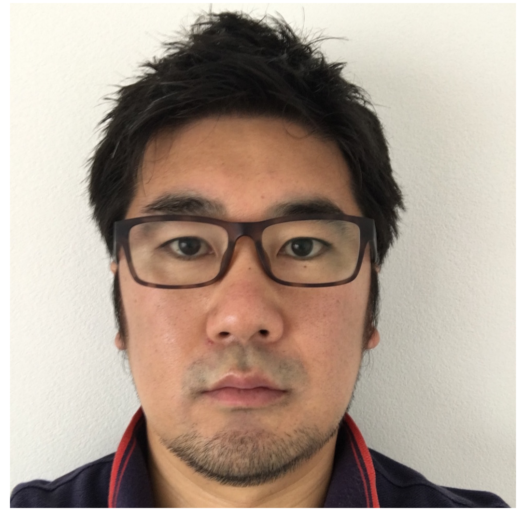
⋮

# Why abundance?

Renaux-Petel Vennin

## Large Primordial PTB

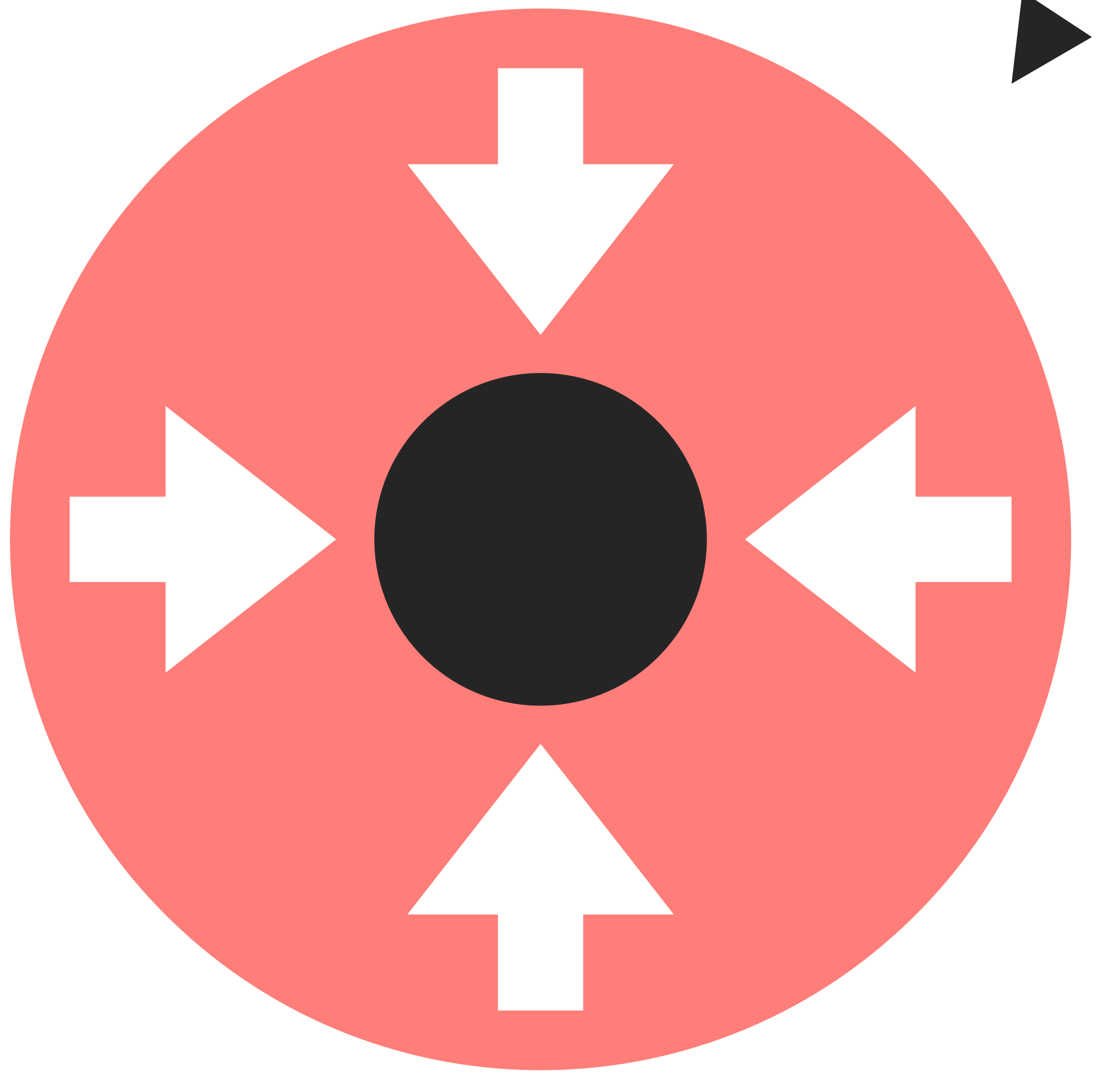
C. M. Yoo



S. Yokoyama

*Primordial BH*

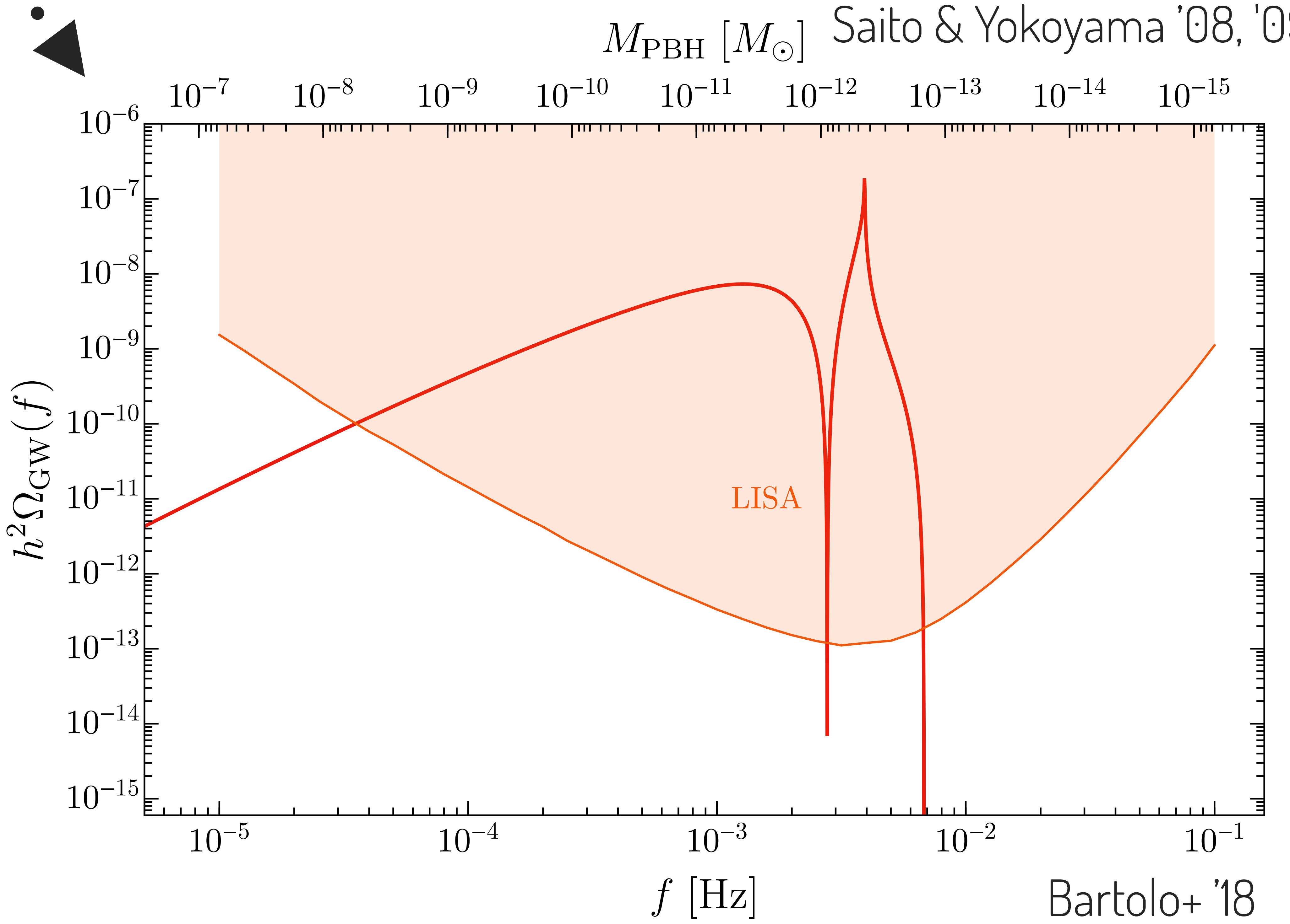
~ asteroid ( $\lesssim 10^{-10} M_\odot$)



Cross-Check

2nd-order Induced SGWB

Saito & Yokoyama '08, '09

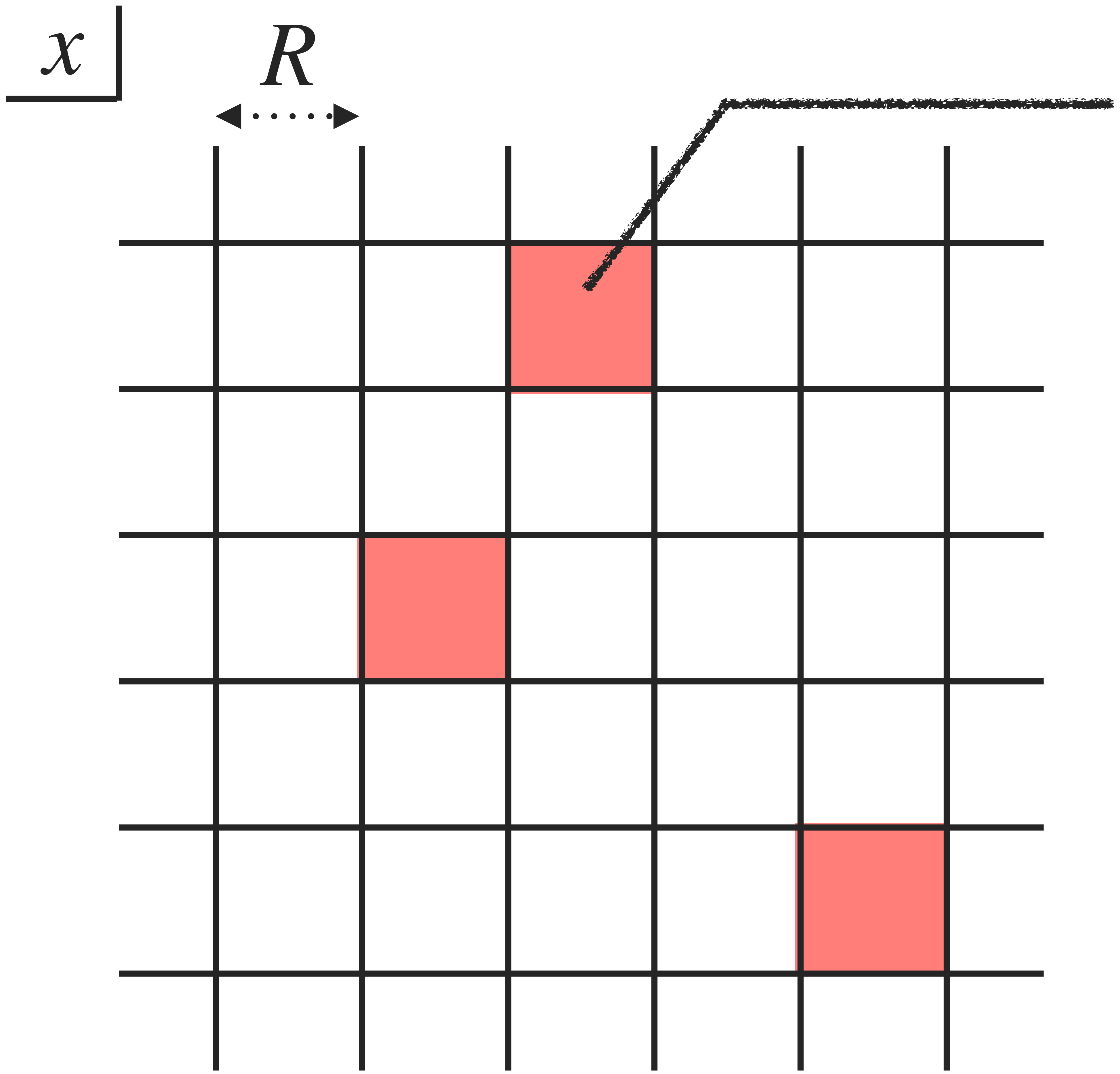



Bartolo+ '18

# Mass function

## The Simplest approach

Carr '75 (Press & Schechter'74)




$$\delta_R(\mathbf{x}) = \int \mathrm{d}^3y\, W_R(\mathbf{x}-\mathbf{y})\delta(\mathbf{y}) \gtrsim \frac{1}{3} \left( = \frac{p}{\rho} \right)$$

➡ Primordial BH !!

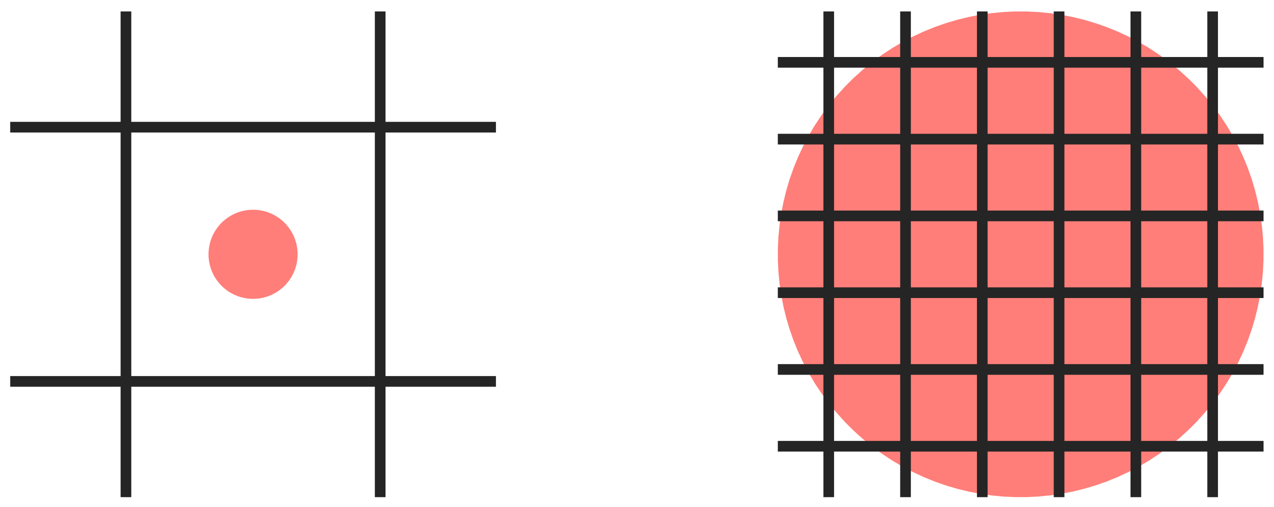
* Abundance : $\dfrac{\rho_{\mathrm{PBH}}}{\rho_{\mathrm{tot}}} = \displaystyle\int_{1/3}^{\infty} \frac{1}{\sqrt{2\pi\sigma_R^2}} \mathrm{e}^{-\delta_R^2/2\sigma_R^2}$
* Mass : $M_{\mathrm{PBH}} \sim M_H \Big|_{R=H^{-1}} = \dfrac{4\pi}{3}\rho R^3 \Big|_{R=H^{-1}}$

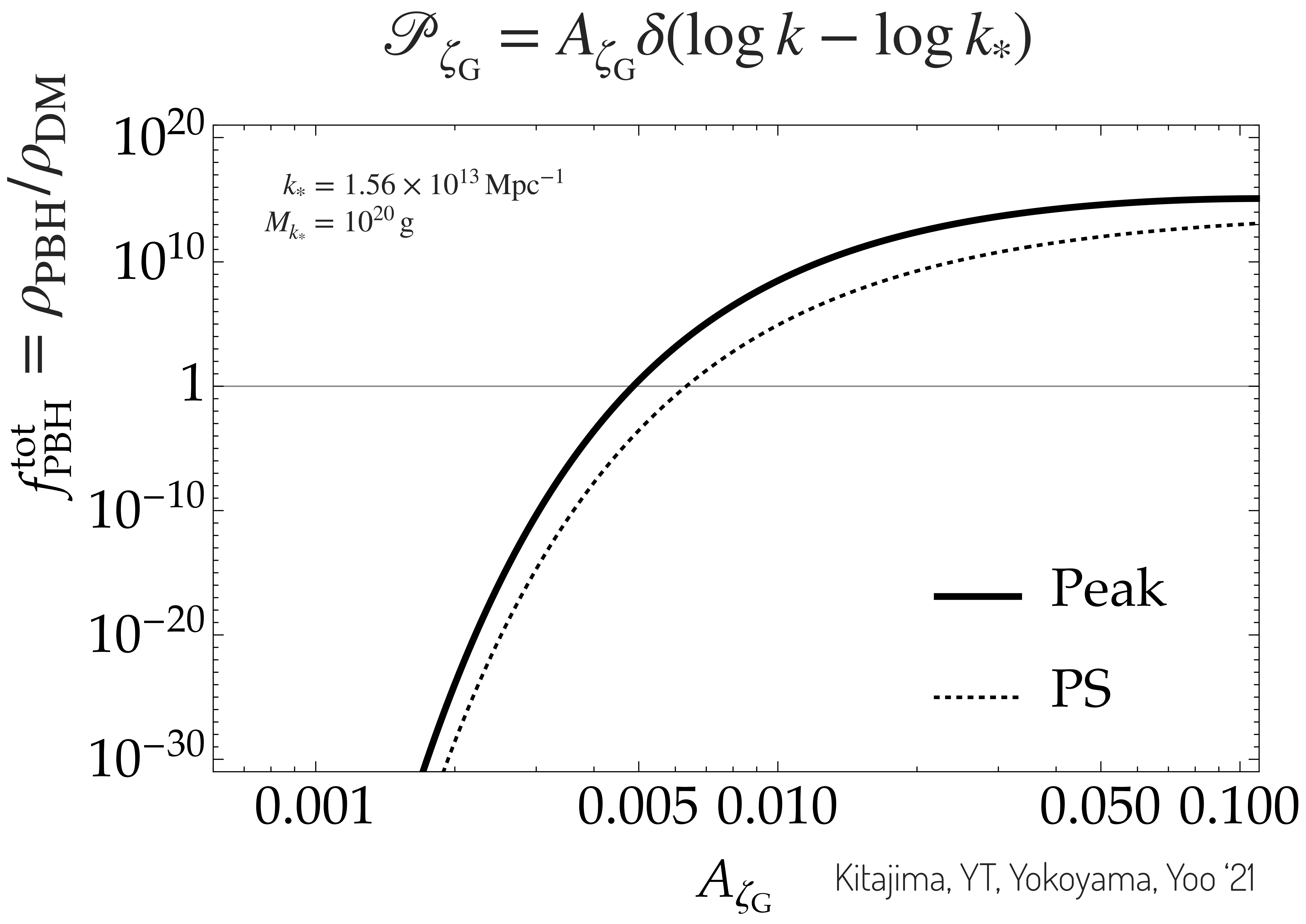
# Mass function

## The Simplest approach

Carr '75 (Press & Schechter'74)

- Always "1 / 3"?
- Which window func.?
- Is $\delta_R$ Gaussian?
- $M_{\rm PBH} \overset{?}{\sim} M_H\Big|_{R=H^{-1}}$
- Are peaks correctly counted?






Kitajima, YT, Yokoyama, Yoo '21

$\left(\text{cf. } \Omega_{\rm GW} \sim A_\zeta^2 \Omega_{\rm r}\right)$

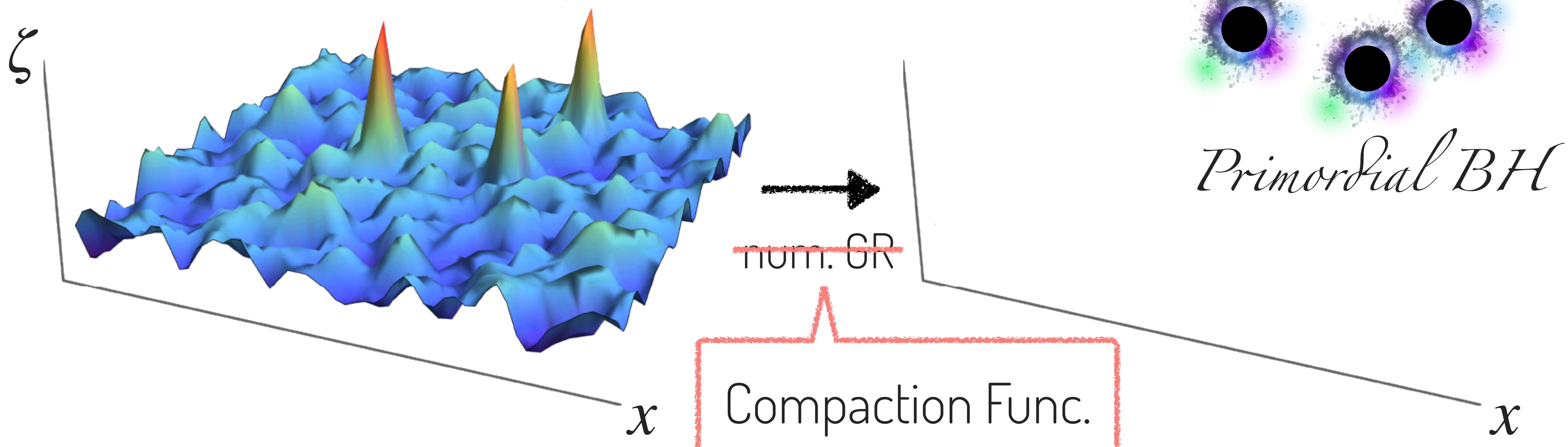
# 1st principle

Inflation theory

Precise Statistics of Prim. PTB

Peak Theory

~~simulate~~

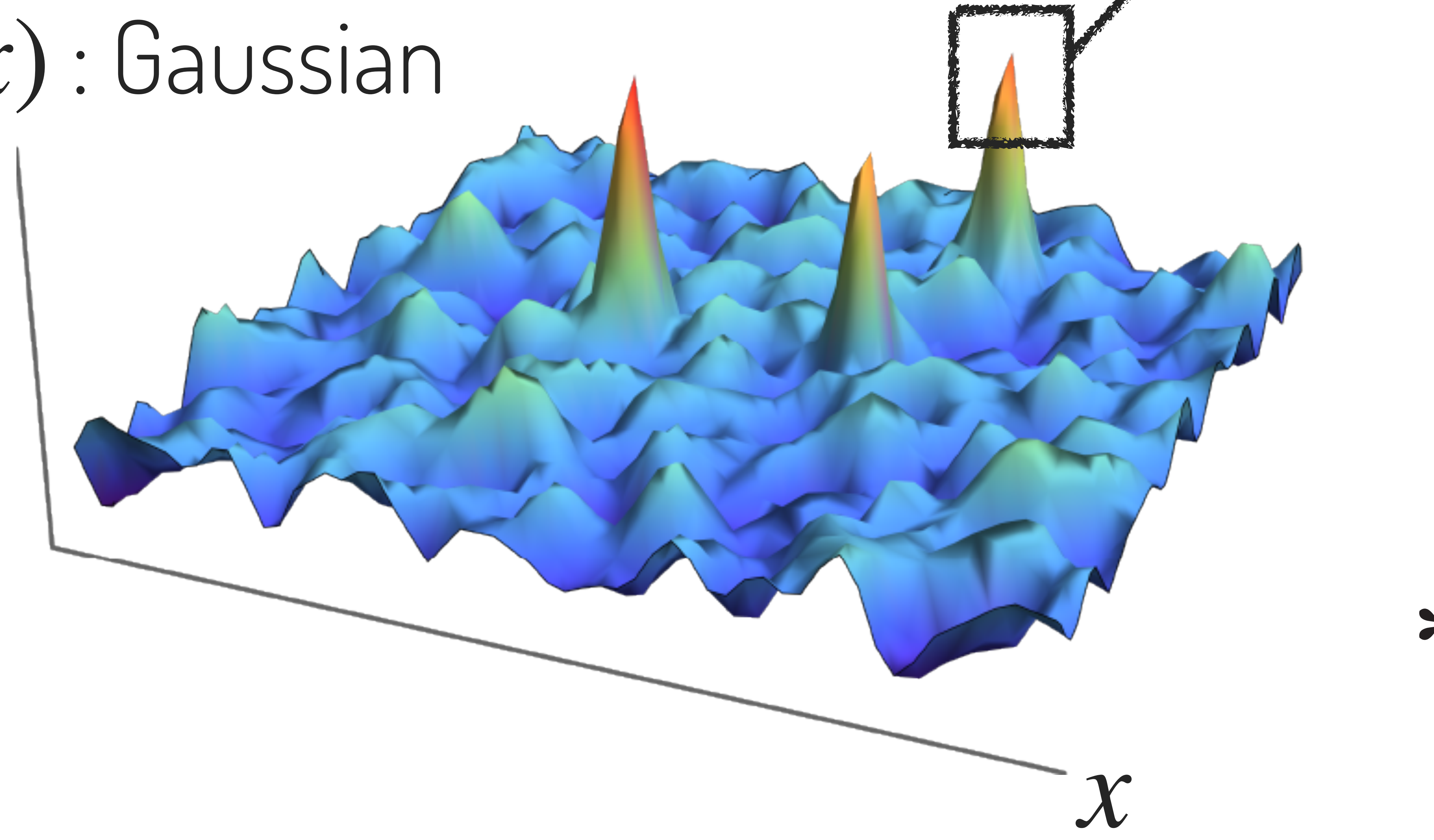



Compaction Func.

# Peak theory

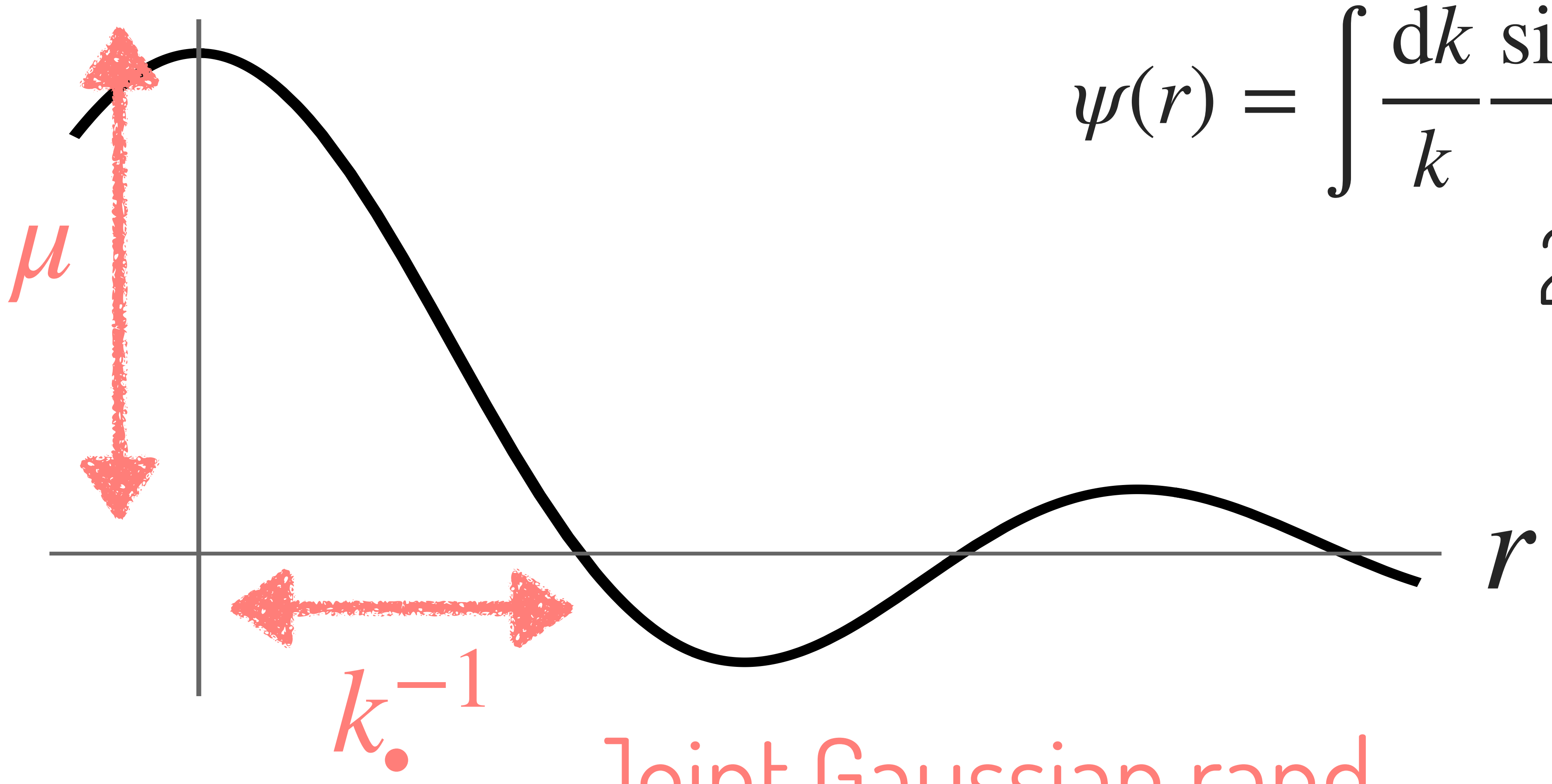
Bardeen, Bond, Kaiser, Szalay '86
Yoo, Harada, Garriga, Kohri '18
Yoo, Gong, Yokoyama '19
Yoo, Harada, Hirano, Kohri '20

$g(x)$ : Gaussian




* Typically spherical sym. profile

$$\hat{g}(r) = \hat{g}\left(\psi(r); \mu, k_\bullet\right)$$




$$\psi(r) = \int \frac{\mathrm{d}k}{k} \frac{\sin kr}{kr} \mathcal{P}_g(k)$$

2pt. func.

Joint Gaussian rand.

* Real-space # density

$$n_{\mathrm{pk}}(\mu, k_\bullet)\, \mathrm{d}\mu \mathrm{d}k_\bullet$$

# Compaction func.

Shibata & Sasaki '99
Harada, Yoo, Nakama, Koga '15

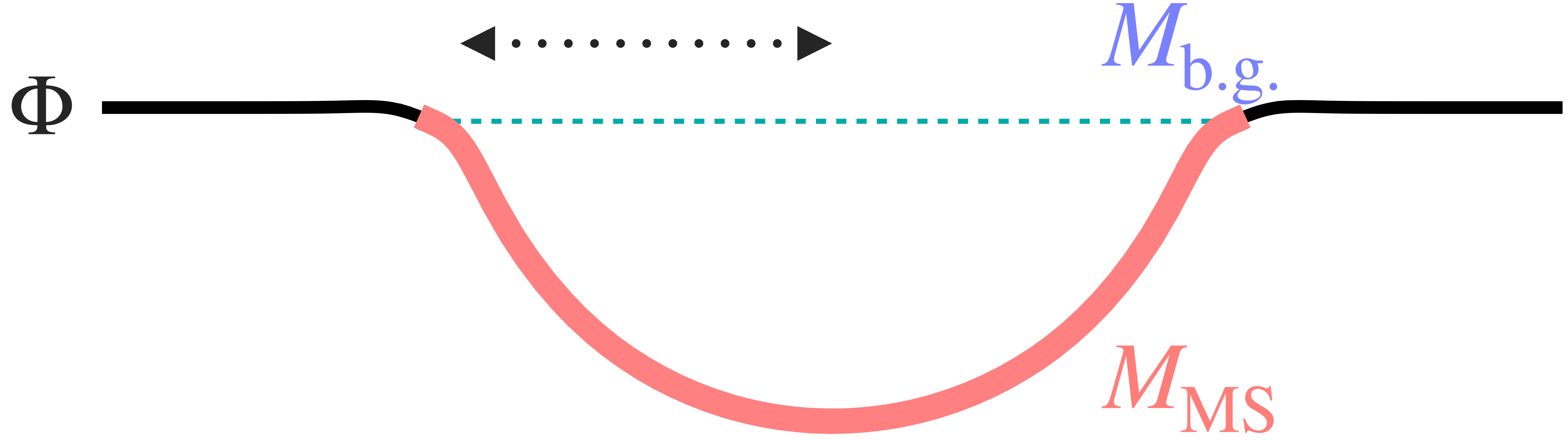
$$\delta = -\frac{8}{9}\frac{1}{a^2H^2}e^{-5\zeta/2}\Delta e^{\zeta/2}$$

$$\mathscr{C} = G\frac{M_{\mathrm{MS}} - M_{\mathrm{b.g.}}}{R} = \frac{1}{V(R)}\int_0^R \delta \times 4\pi R^2 \mathrm{d}R \bigg|_{R=H^{-1}} = \frac{2}{3}\left[1-(1+r\zeta')^2\right] \overset{?}{>} \mathscr{C}_{\mathrm{th}}$$

Coarse-grained $\delta$

conserved on superHubble

$$R = a e^{\zeta} r$$



# Compaction func.

Shibata & Sasaki '99
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$$\delta = -\frac{8}{9}\frac{1}{a^2H^2}\mathrm{e}^{-5\zeta/2}\Delta\mathrm{e}^{\zeta/2}$$

$$\mathscr{C} = G\frac{M_{\rm MS}-M_{\rm b.g.}}{R} = \frac{1}{V(R)}\int_0^R \delta \times 4\pi R^2 \mathrm{d}R\bigg|_{R=H^{-1}} = \frac{2}{3}\left[1-(1+r\zeta')^2\right] > \mathscr{C}_{\rm th}$$

Coarse-grained $\delta$

conserved on superHubble

## Coarse-grained Compaction Func. is (almost) universal index!

$$\bar{\mathscr{C}} = \frac{1}{V(R)}\int_0^R \mathscr{C}\times 4\pi R^2 > \bar{\mathscr{C}}_{\rm th} \simeq \frac{2}{5}$$

$$\left(\to \mu > \mu_{\rm th}(k_\bullet, \cdots)\right)$$

- $f_{\rm NL} > 0$, exp.-tail, ...

  Atal, Cid, Escrivà, Garriga '19
  Escrivà, Germani, Sheth '19

- fitting for $f_{\rm NL} < 0$

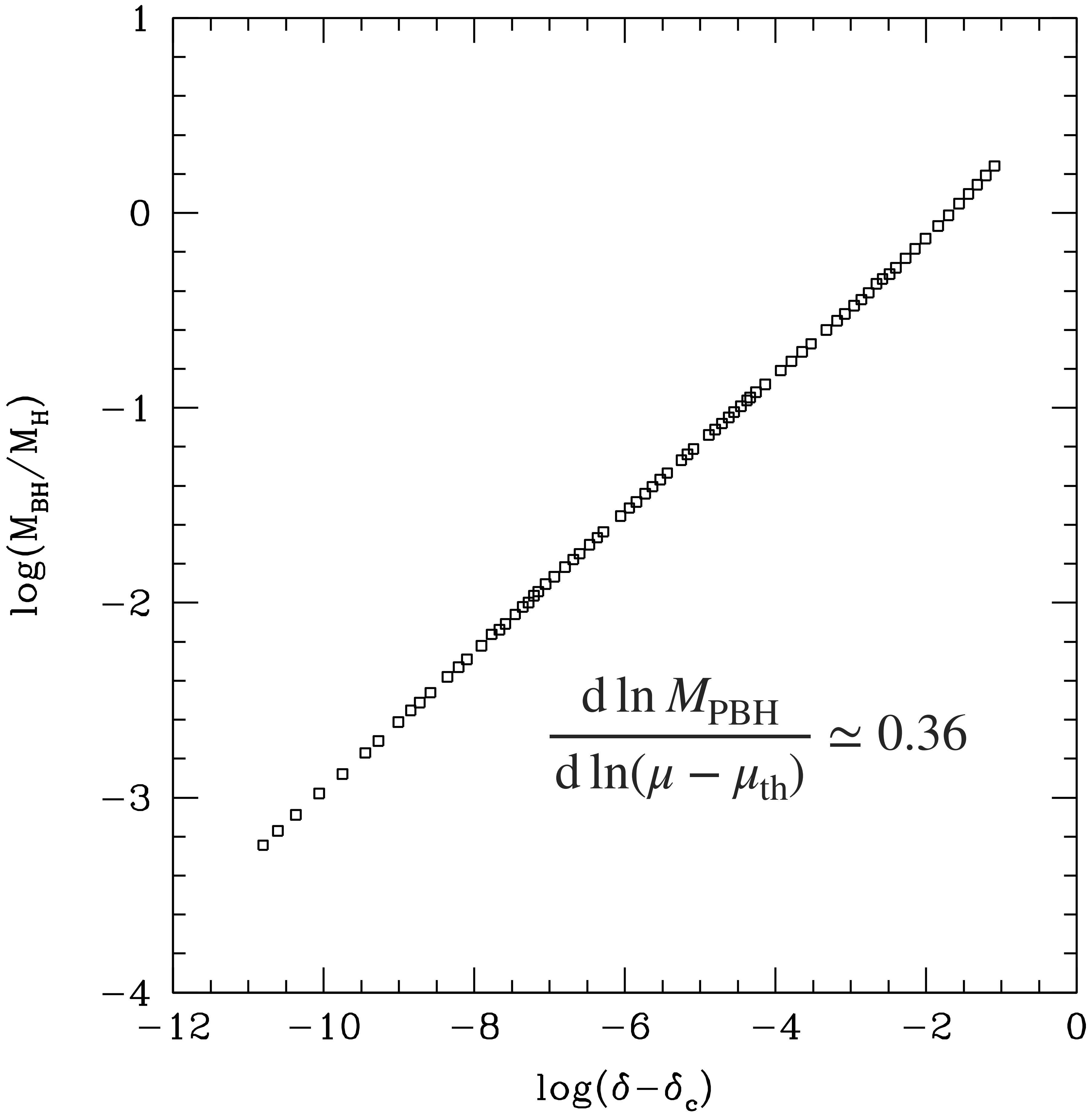
  Escrivà, YT, Yokoyama, Yoo, '22

# Mass

Choptuik '93
Niemeyer & Jedamzik '94, '97

* Scaling law

$$M_{\mathrm{PBH}} \simeq \left(\mu - \mu_{\mathrm{th}}(k_\bullet, \cdots)\right)^{0.36} M_H \Big|_{R=H^{-1}}$$

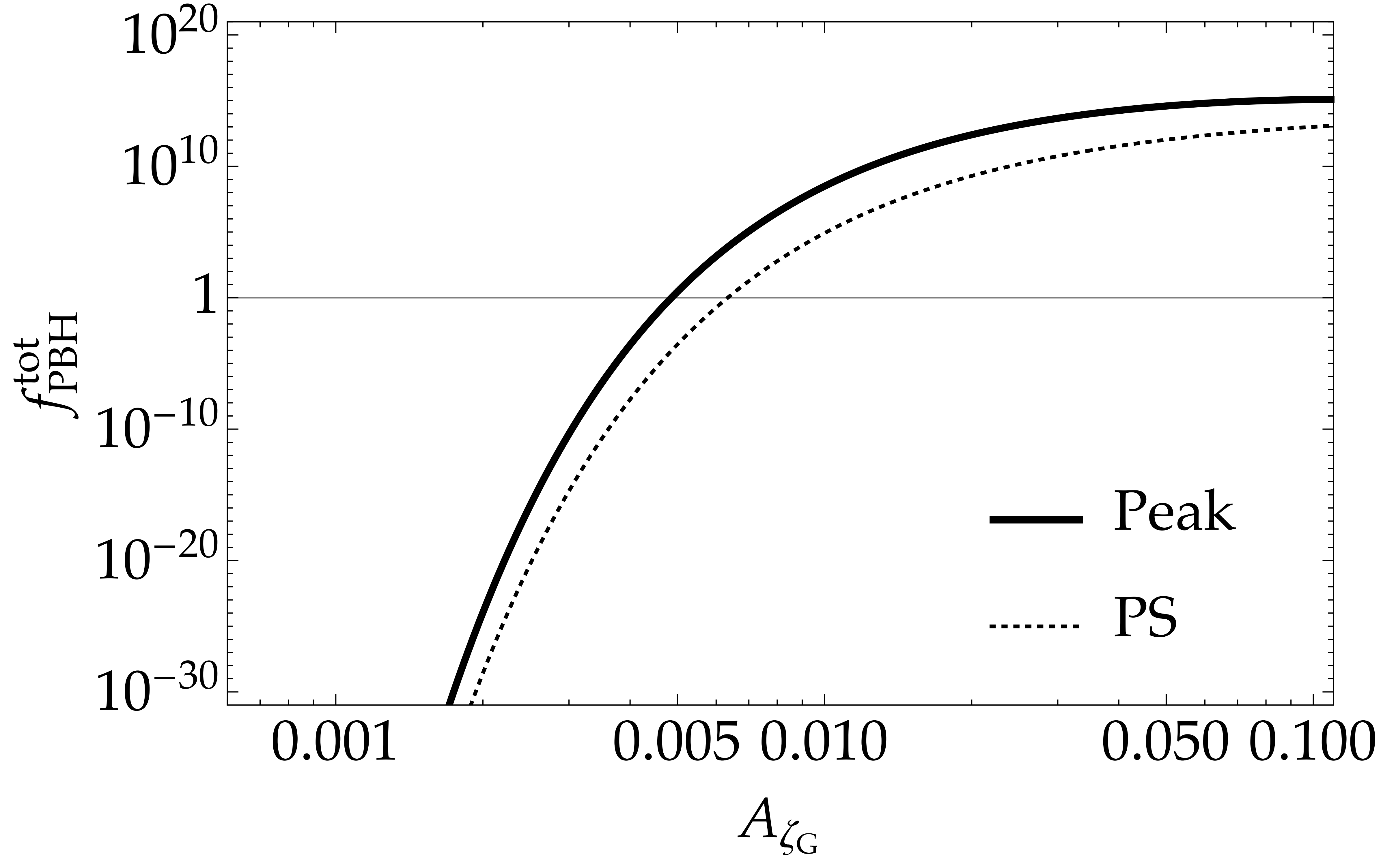



Musco, Miller, Polnarev '08

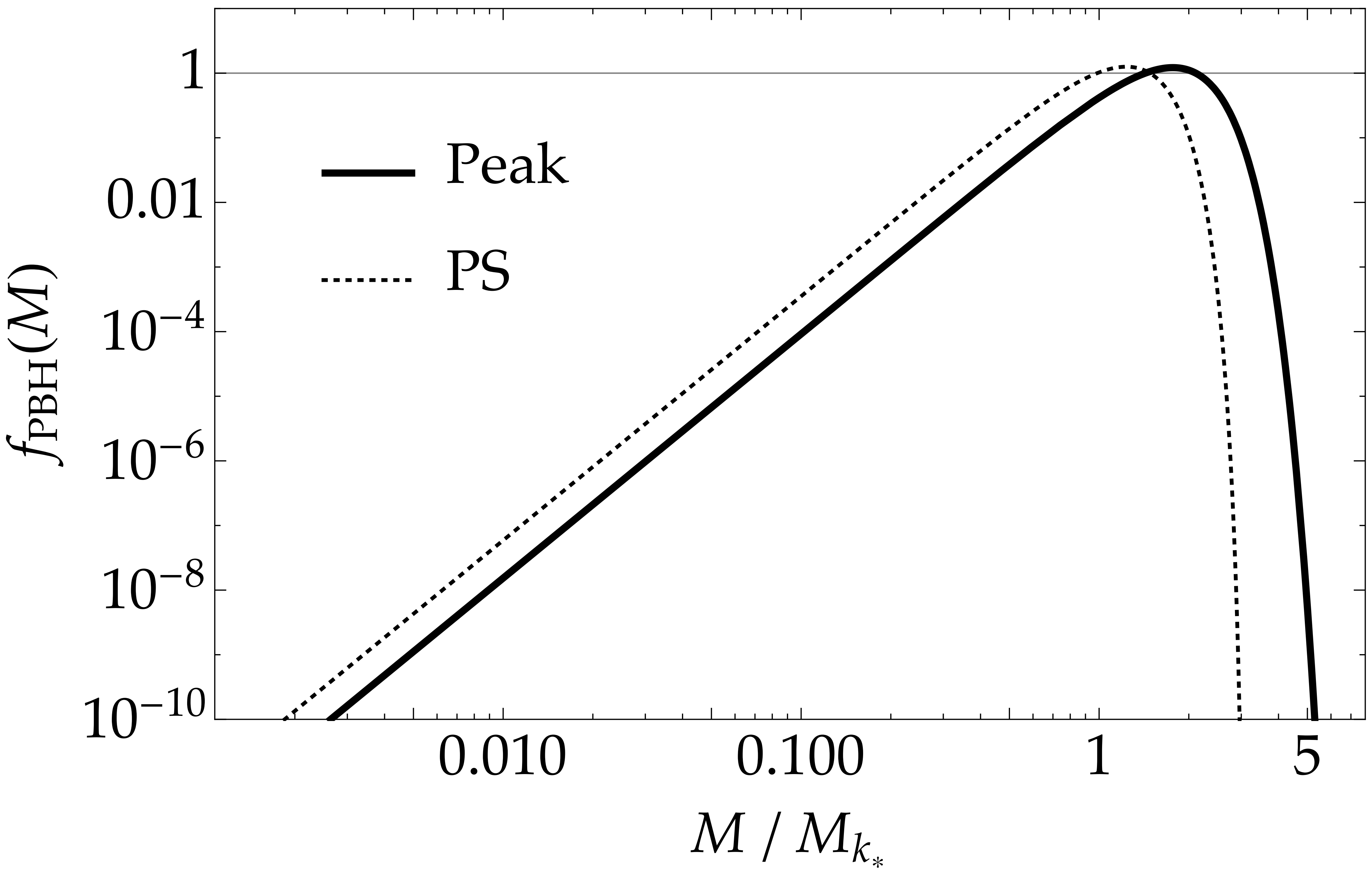
# Result (Gaussian)

Kitajima, YT, Yokoyama, Yoo '21

$$\mathscr{P}_{\zeta_G} = A_{\zeta_G}\delta(\log k - \log k_*)$$




tuned $A_{\zeta_G}$




$k_* = 1.56 \times 10^{13}\,\mathrm{Mpc}^{-1}$
$M_{k_*} = 10^{20}\,\mathrm{g}$

# Local-type NG

Yoo, Gong, Yokoyama '19

$$\zeta(\mathbf{x}) = \mathscr{F}_{\rm NG}\left(\zeta_{\rm G}(\mathbf{x})\right)$$

for example ...

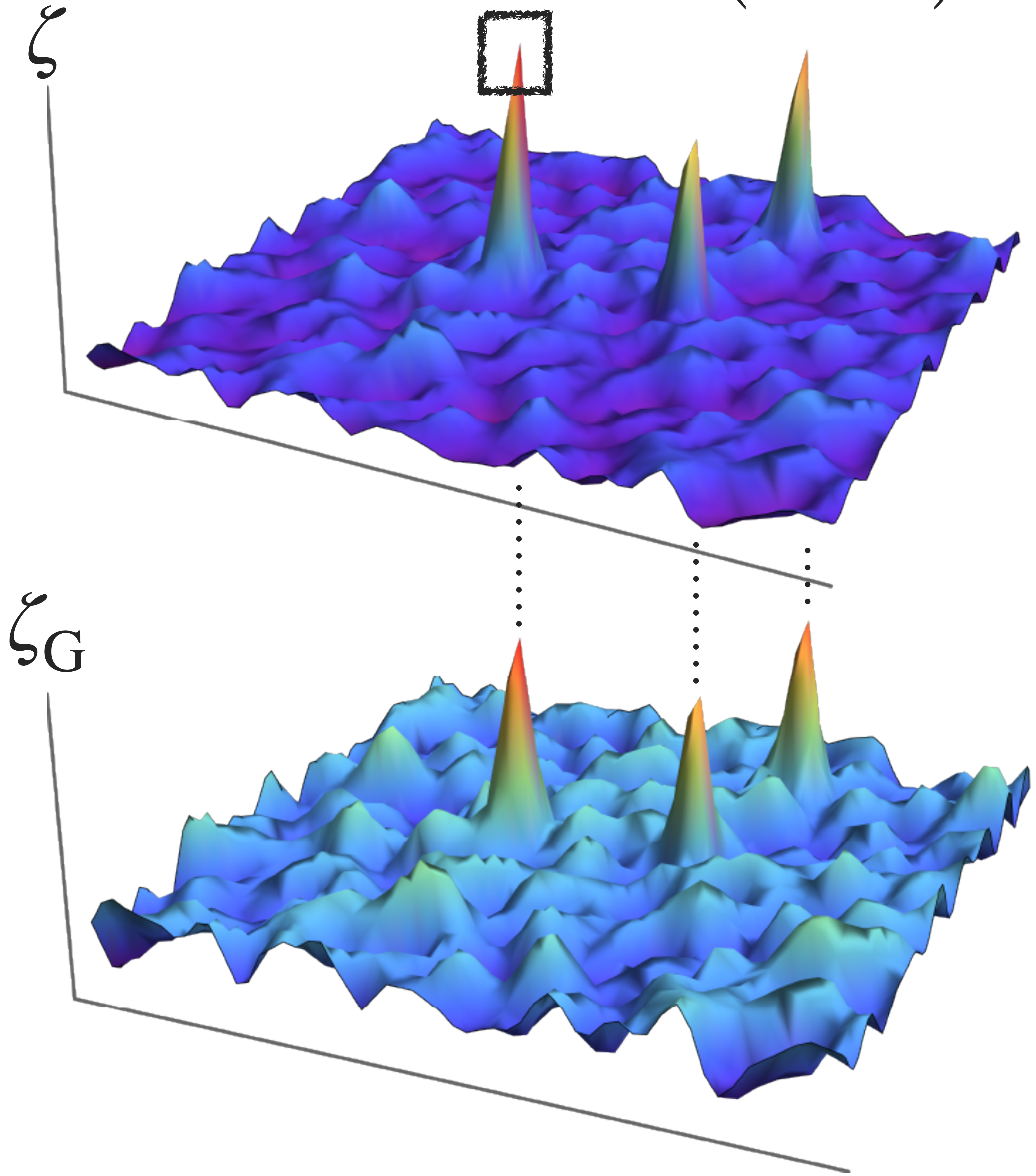
- $\zeta = \zeta_{\rm G} + \frac{3}{5} f_{\rm NL} \zeta_{\rm G}^2$
- $\zeta = -\frac{1}{3}\log\left(1 - 3\zeta_{\rm G}\right)$ : "exp-tail" in USR

Atal+ '19, Ezquiaga+ '19, Biagetti+ '21

$$= \zeta_{\rm G} + \frac{3}{5} \times \frac{5}{2} \zeta_{\rm G}^2 + \cdots$$

$f_{\rm NL}^{\rm USR}$

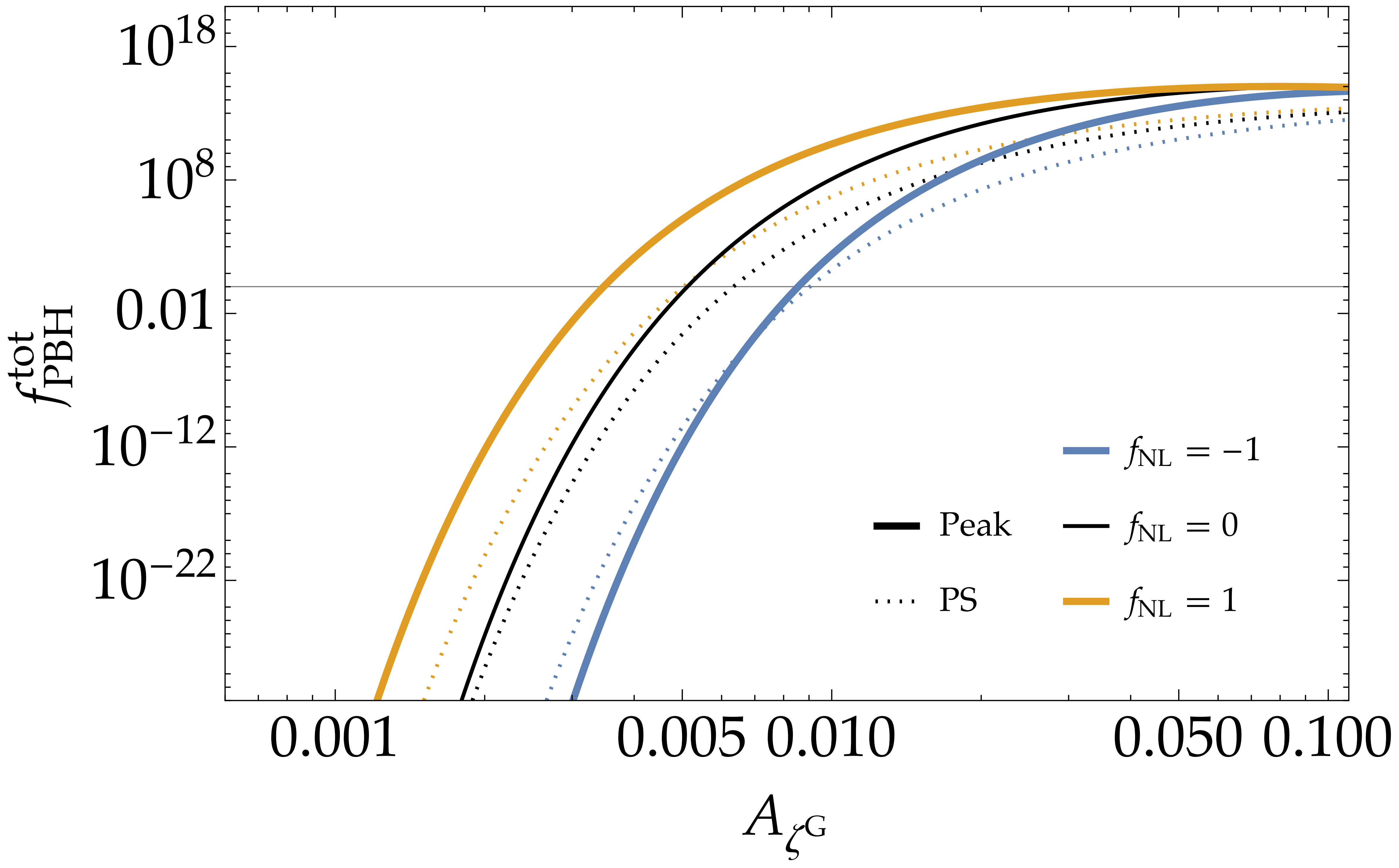
$$\hat{\zeta}(r) = \mathscr{F}_{\rm NG}\left(\hat{\zeta}_{\rm G}(r)\right)$$

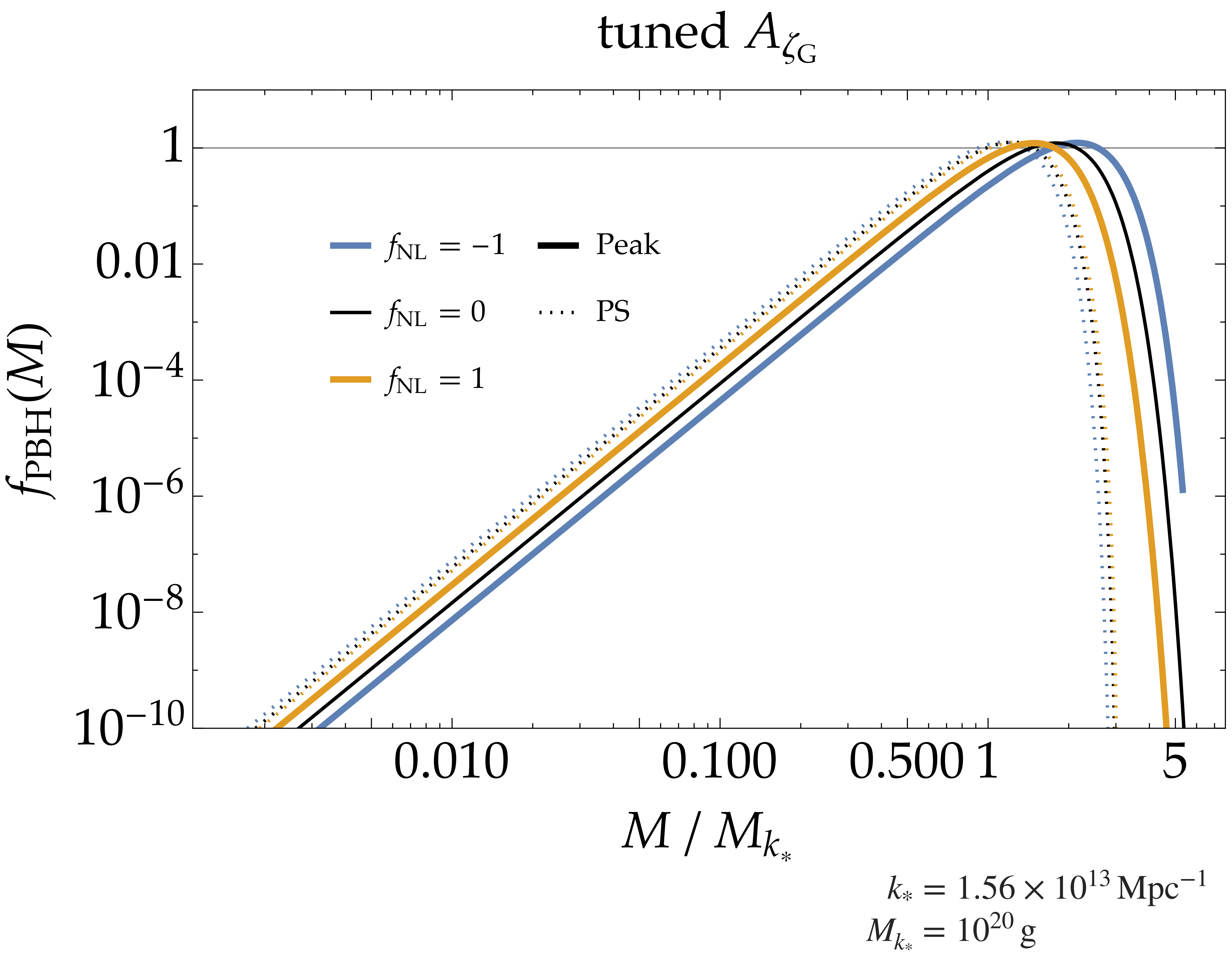


# $f_{\rm NL}$

Escrivà, YT, Yokoyama, Yoo '22

$$\mathcal{P}_{\zeta_{\rm G}} = A_{\zeta_{\rm G}} \delta(\log k - \log k_*)$$

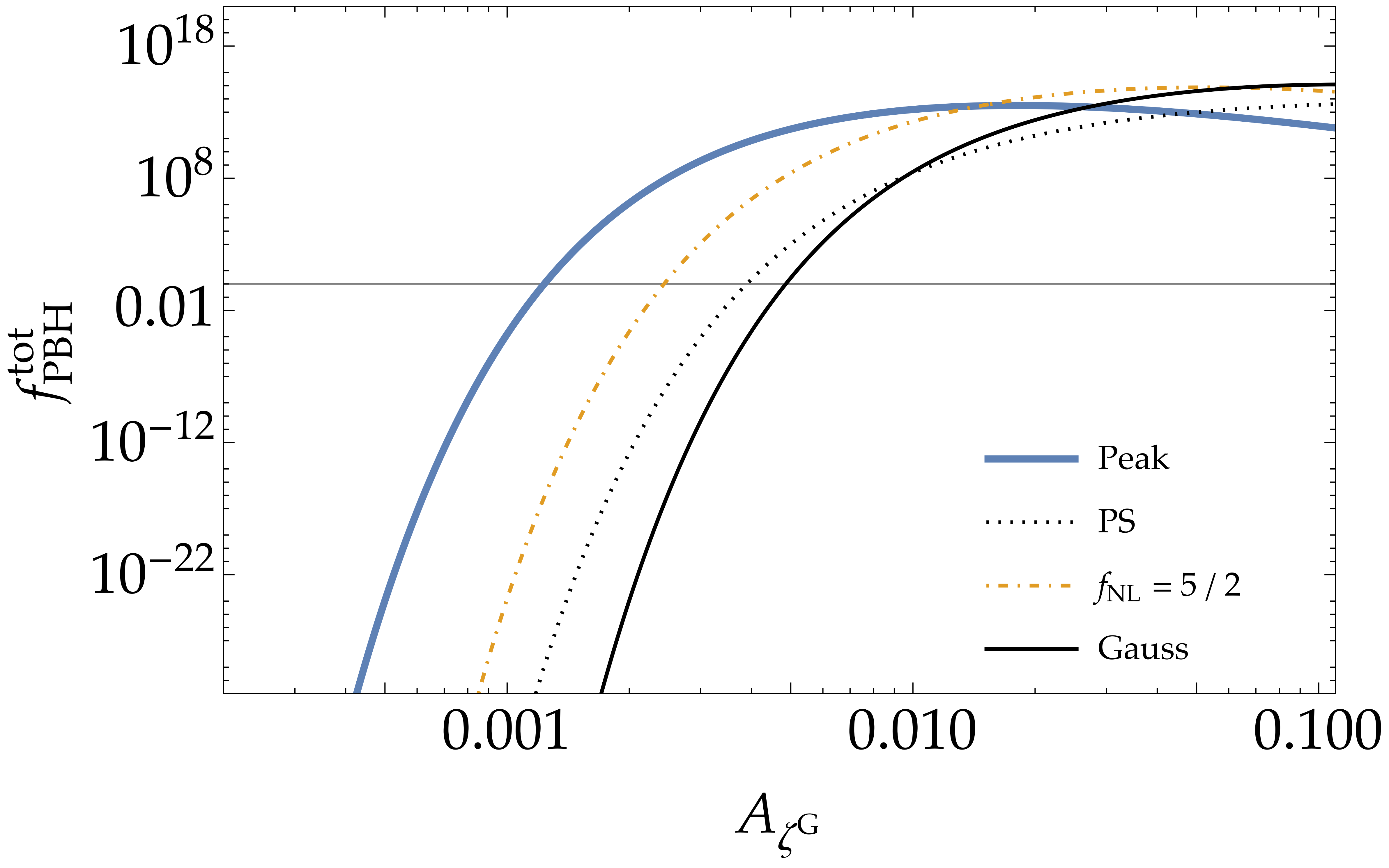


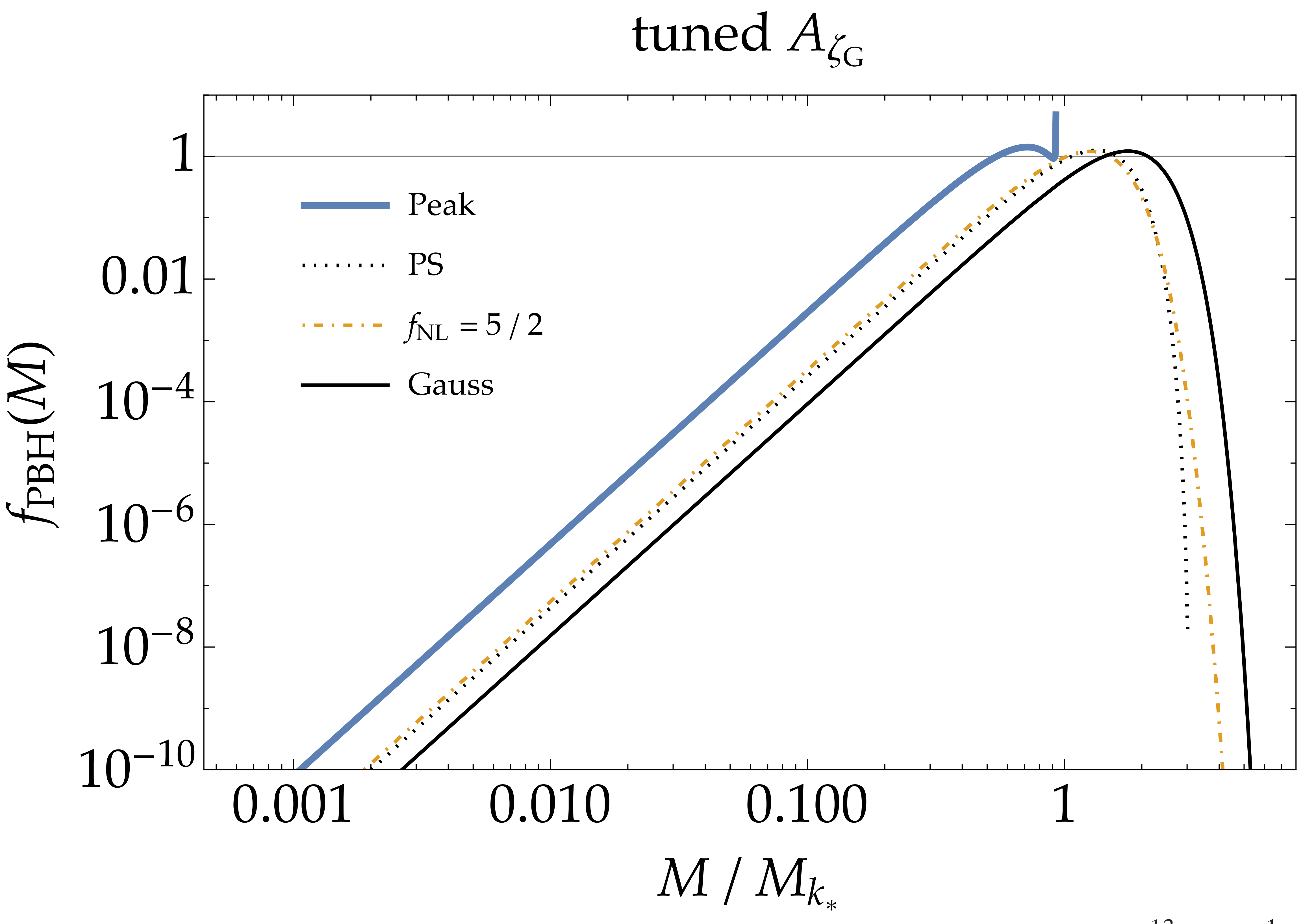



# Exp.-tail

Kitajima, YT, Yokoyama, Yoo '21

$$\zeta = -\frac{1}{3}\log\left(1 - 3\zeta_{\mathrm{G}}\right)$$



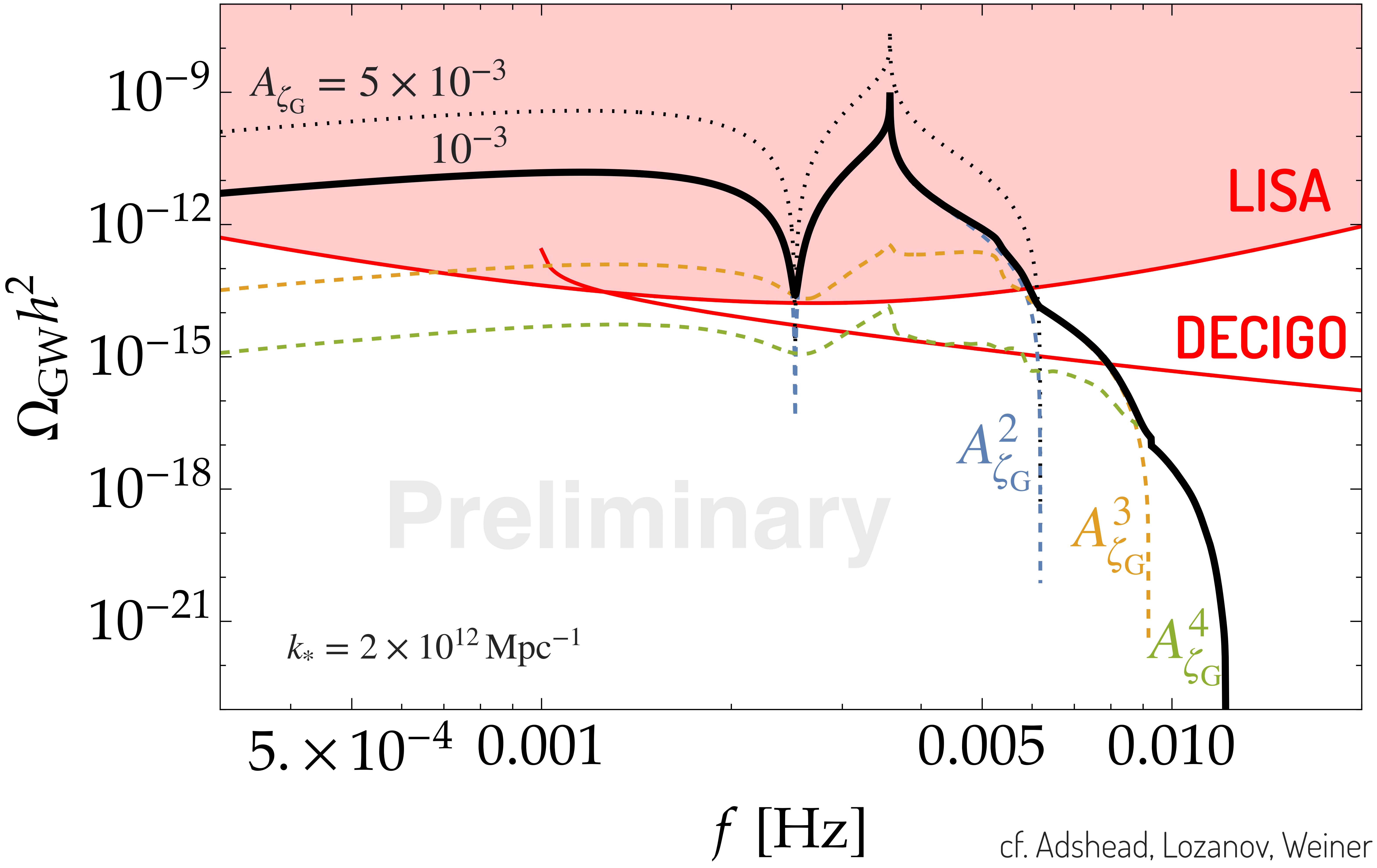




$k_* = 1.56 \times 10^{13}\,\mathrm{Mpc}^{-1}$
$M_{k_*} = 10^{20}\,\mathrm{g}$

# Induced GW

Abe, Inui, YT, Yokoyama in prep.

$$\zeta = -\frac{1}{3}\log\left(1-3\zeta_G\right) = \zeta_G + \frac{3}{2}\zeta_G^2 + 3\zeta_G^3 + \cdots$$




cf. Adshead, Lozanov, Weiner '21

# Conclusions

- Peak theory for PBH
  - compatible w/ Mass Scaling Law & NG of $\zeta$
- Exp-tail
  - significantly amplify PBH abundance
  - hard-cut on mass function
- Induced GW
  - still detectable by LISA
  - NG feature on UV tail?

# Appendix

$$\hat{g}(r) = \mu\left[\frac{1}{1-\gamma^2}\left(\psi(r)+\frac{1}{3}R_\bullet^2\Delta\psi(r)\right) - k_\bullet^2\frac{1}{\gamma(1-\gamma^2)}\frac{\sigma_0}{\sigma_2}\left(\gamma^2\psi(r)+\frac{1}{3}R_\bullet^2\Delta\psi(r)\right)\right]$$

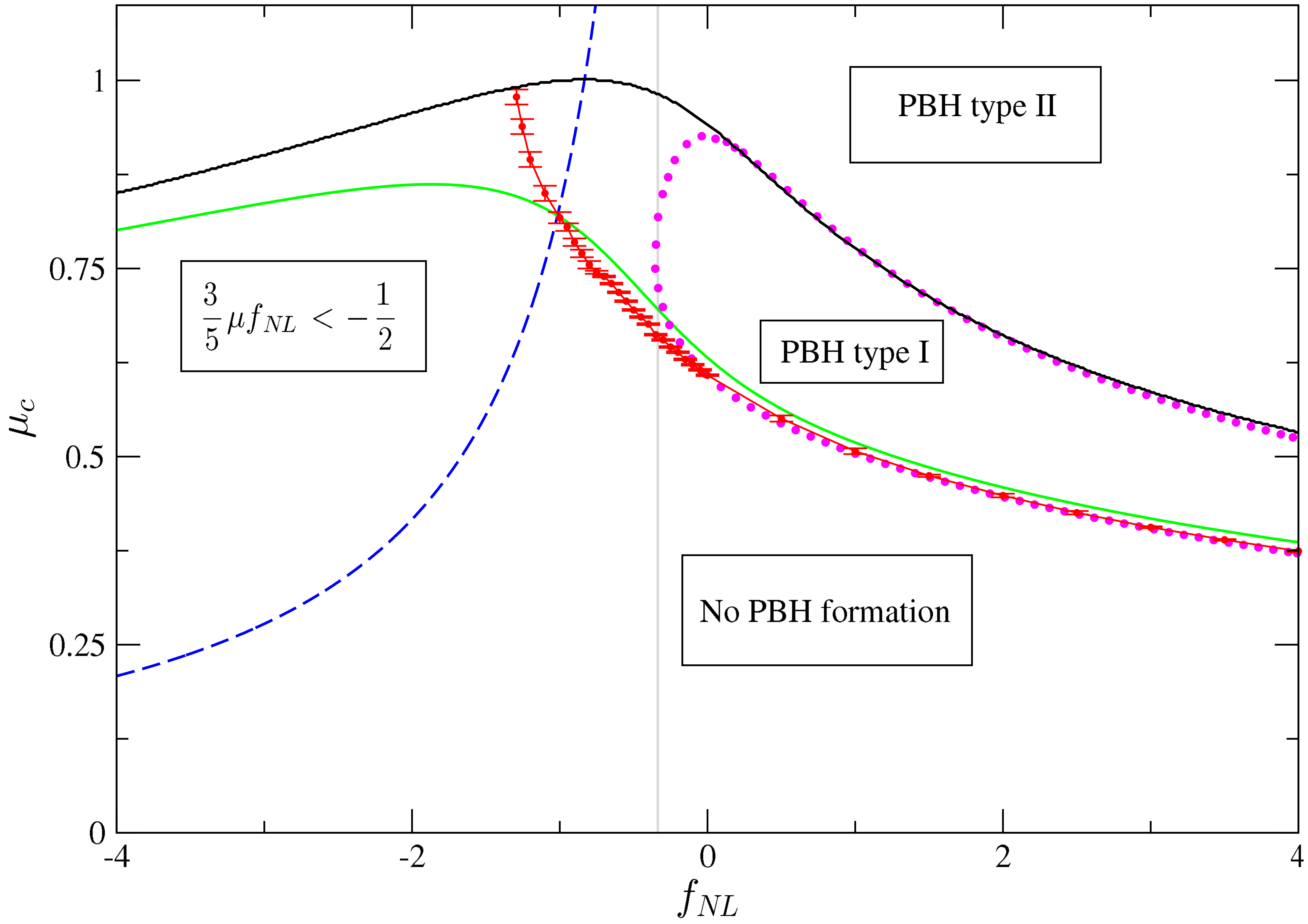
$$\sigma_n^2 = \int\frac{\mathrm{d}k}{k}k^{2n}\mathscr{P}_g(k), \quad \gamma = \frac{\langle k^2\rangle}{\sqrt{\langle k^4\rangle}}, \quad R_\bullet = \sqrt{\frac{3\langle k^2\rangle}{\langle k^4\rangle}} \qquad \psi(r) = \frac{1}{\sigma_0^2}\int\frac{\mathrm{d}k}{k}\frac{\sin kr}{kr}\mathscr{P}_g(k)$$

$$n_{\mathrm{pk}}(\mu, k_\bullet)\,\mathrm{d}\mu\,\mathrm{d}k_\bullet = \left[\frac{1}{V_\Omega}\int_\Omega \mathrm{d}^3x \sum_{\nabla g(\mathbf{x}_\mathrm{p})=0}\delta^{(3)}(\mathbf{x}-\mathbf{x}_\mathrm{p})\delta(\mu-\mu(\mathbf{x}_p))\delta(k_\bullet - k_\bullet(\mathbf{x}_\mathrm{p}))\right]\mathrm{d}\mu\,\mathrm{d}k_\bullet$$

$$= \frac{2\times 3^{3/2}}{(2\pi)^{3/2}}\mu k_\bullet\frac{\sigma_2^2}{\sigma_0\sigma_1}f\left(\frac{\mu k_\bullet^2}{\sigma_2}\right)P_1\left(\frac{\mu}{\sigma_0}, \frac{\mu k_\bullet^2}{\sigma_2}\right)\mathrm{d}\mu\,\mathrm{d}k_\bullet$$

$$f(\xi) = \frac{1}{2}\xi(\xi^2-3)\left(\mathrm{erf}\left[\frac{1}{2}\sqrt{\frac{5}{2}}\xi\right]+\mathrm{erf}\left[\sqrt{\frac{5}{2}}\xi\right]\right)+\sqrt{\frac{2}{5\pi}}\left\{\left(\frac{8}{5}+\frac{31}{4}\xi^2\right)\exp\left[-\frac{5}{8}\xi^2\right]+\left(-\frac{8}{5}+\frac{1}{2}\xi^2\right)\exp\left[-\frac{5}{2}\xi^2\right]\right\} \qquad P_1(\nu,\xi) = \frac{1}{2\pi\sqrt{1-\gamma^2}}\exp\left[-\frac{1}{2}\left(\nu^2+\frac{(\xi-\gamma\nu)^2}{1-\gamma^2}\right)\right]$$

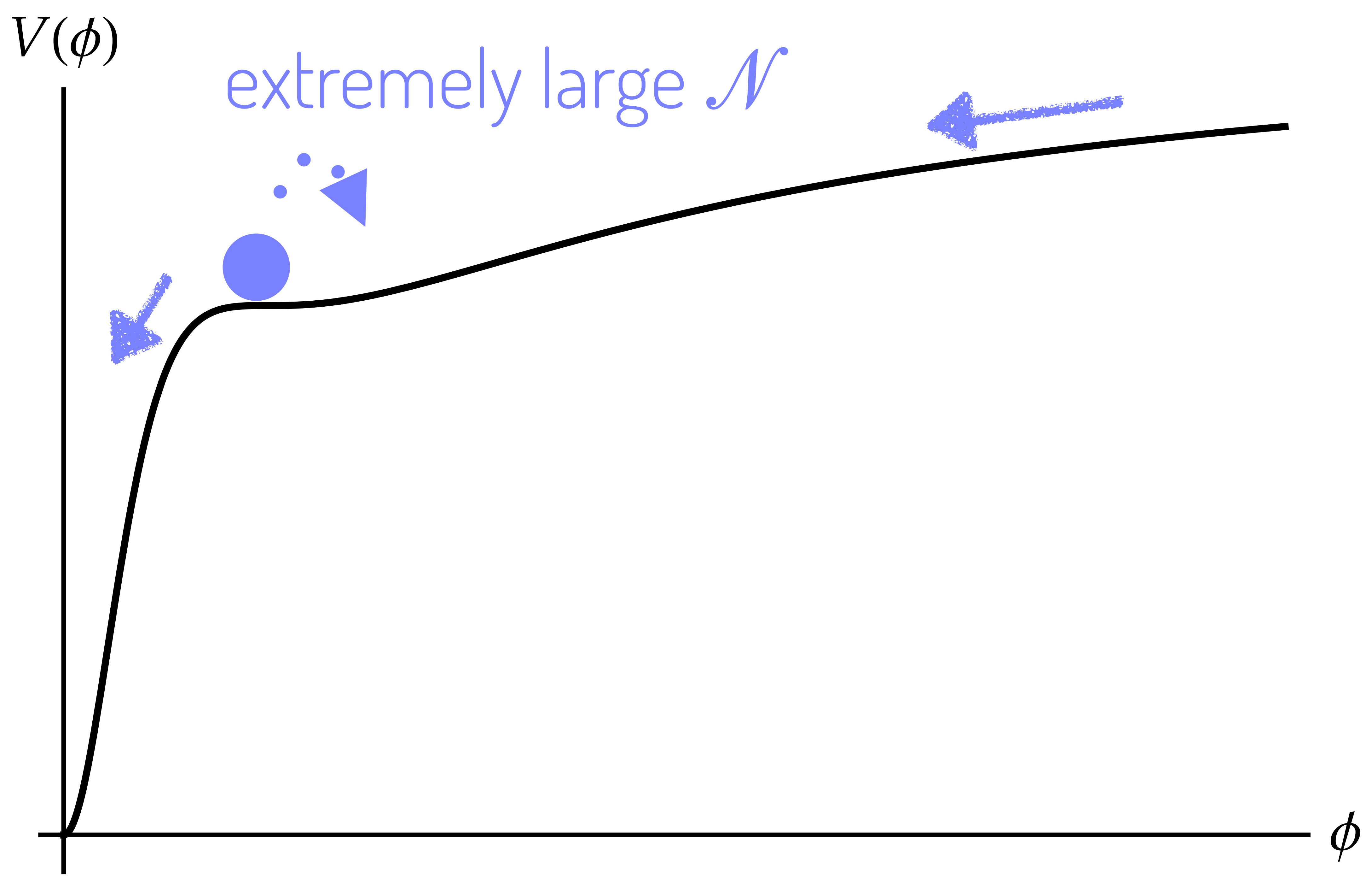
# Appendix

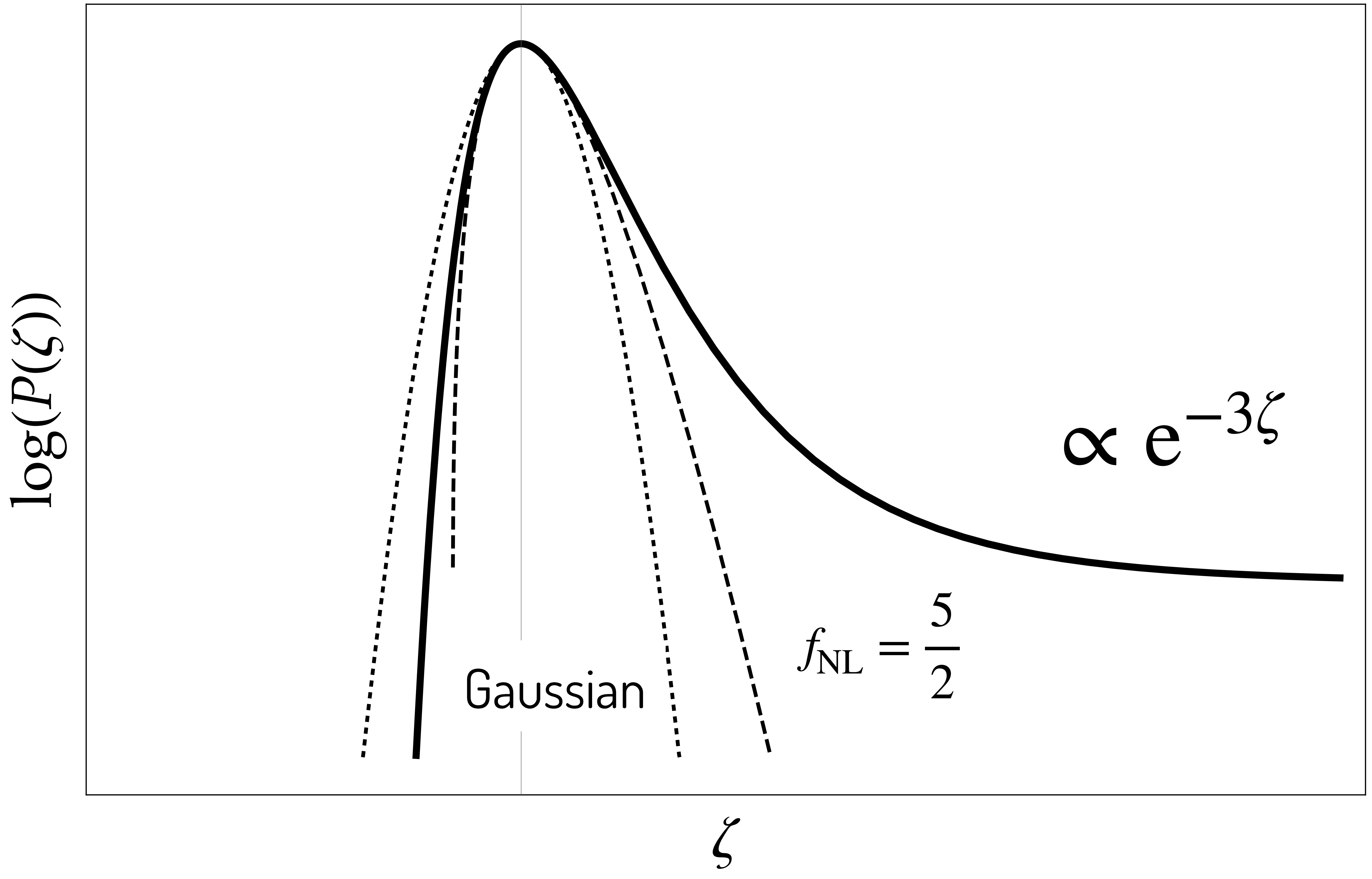


PBH type II
PBH type I
No PBH formation
$\frac{3}{5}\mu f_{NL} < -\frac{1}{2}$
$\mu_c$
$f_{NL}$
1
0.75
0.5
0.25
0
-4
-2
0
2
4

# Appendix

$$\zeta = -\frac{1}{3}\log\left(1 - 3\zeta_G\right)$$



# Appendix

$$\zeta = -\frac{1}{3}\log\left(1 - 3\zeta_{\mathrm{G}}\right)$$