Anomalies and Symmetry Fractionalization

Jaume Gomis

PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

Hirosifest, Caltech

w/ Delmastro, Hsin, Komargodski

Background

- Arrived to Pasadena in 1999, same as Hirosi and Edward
- amazing faculty, visitors, postdocs, students and staff
- Caltech provided a truly warm, open and stimulating place

- I was at Caltech from 1999-2004

- Hirosi and I wrote two papers in this period

Nonrelativistic closed string theory

Jaume Gomis (Caltech and CIT-USC), Hirosi Ooguri (Caltech and CIT-USC) (Sep, 2000)

Published in: *J.Math.Phys.* 42 (2001) 3127-3151 • e-Print: hep-th/0009181 [hep-th]

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Penrose limit of N = 1 gauge theories

Jaume Gomis (Caltech), Hirosi Ooguri (Caltech) (Feb, 2002)

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- and we have coauthored a paper since

Shortening Anomalies in Supersymmetric Theories

Jaume Gomis (Perimeter Inst. Theor. Phys.), Zohar Komargodski (Weizmann Inst.), Hirosi Ooguri (Caltech and Tokyo U., IPMU), Nathan Seiberg (Princeton
Inst. Advanced Study), Yifan Wang (Princeton U.) (Nov 9, 2016)

Published in: *JHEP* 01 (2017) 067 • e-Print: 1611.03101 [hep-th]

- We gave a microscopic worldsheet definition of Nonrelativistic String Theory:
  - unitary
  - UV complete
  - string spectrum and S-matrix is (string) Galilean invariant
  - string amplitudes localize on the moduli space of Riemann surfaces
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  - unitary
  - UV complete
  - string spectrum and S-matrix is (string) Galilean invariant
  - string amplitudes localize on the moduli space of Riemann surfaces
  - ...
- Renewed interest in Nonrelativistic String Theory:
  - associated geometry and NLSM (string-Newton Cartan structure)
  - nonrelativistic backgrounds
  - D-branes and nonrelativistic Yang-Mills
  - ...

- Hirosi has impeccable taste and aesthetics
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Thank You Hirosi!

Introduction

- Symmetries provide a powerful organizing principle
  - 0-form symmetry $G \Longleftrightarrow \exists$ topological invertible operators
  - local operators $\mathcal{O}(x)$ transform in (vector) representations of G
  - line operators $\mathcal{L}$ transform in (projective) representations of G

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- A 't Hooft anomaly for G informs the dynamics
  - couple to G connection $\Longleftrightarrow$ lay down network of topological junctions
  - 't Hooft anomaly:
    - non-invariance of Z under G cannot be cured by adding local counterterms
    - failure of network of topological junctions to consistently recombine
    - admit a topological classification $\Longrightarrow$ renormalization group invariants

In this talk

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$\Longrightarrow$ need to characterize Γ of a physical system!

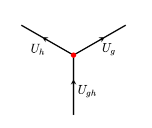
- need to fully specify the action of G on $\mathcal{L}$
- distinct ways G is realized on $\mathcal{L}$ describe different "fractionalization classes"
- fractionalization classes can result in distinct 't Hooft anomalies for G
- 't Hooft anomaly matching must be reconsidered

Symmetry Fractionalization

- one-form symmetry $\Gamma \Longleftrightarrow \exists$ topological codimension 2 invertible operators

GKSW

- topological networks for G can be enriched with such operators



- action of G on local operators $\mathcal{O}(x)$ unchanged, BUT
- action of G on line operators is modified by phases
  - enriched G junction can change projective representation carried by $\mathcal{L}$
  - consistency implies that choices are labeled by a class in $H^2_\rho(G,\Gamma)$
  - fractionalization classes related by turning on $B \in H^2_\rho(G,\Gamma)$ for Γ

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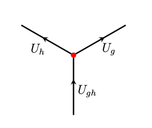

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- 't Hooft anomaly for G can depend on choice of fractionalization class if:
  - the system has a 't Hooft anomaly for Γ, or
  - there is a mixed $G - \Gamma$ 't Hooft anomaly

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  - there is a mixed $G - \Gamma$ 't Hooft anomaly
- 't Hooft anomalies captured by an inflow anomaly polynomial
  - let A, B be background fields for G and Γ, then 't Hooft anomalies:

$$\int f(A) + \int g(B, A)$$

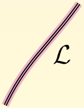
  - change of fractionalization class $B \to B + A^*\eta$, where $\eta \in H^2_\rho(G, \Gamma)$
  - this can change the 't Hooft anomaly for G

$$\int f(A) \to \int f(A) + \int g(A^*\eta, A)$$

  - 't Hooft anomaly matching across all fractionalization classes

Fractionalization Class Engineering

- add massive particles such that:
  1. break the Γ symmetry
  2. transform in a projective representation of G
  3. 0-form symmetry, acting on local operators $\mathcal{O}(x)$, remains G
- consequences of UV modification:
  - massive particles do not modify the theory at low energies, BUT
  - fixes the action of G on $\mathcal{L}$ in IR, thought as the worldline of heavy particle



  - unambiguously determines the value of 't Hooft anomalies for G
- consistent with UV decoupling since the massive particles change the symmetry structure in a discontinuous fashion

Fractionalizing by twisting G with G_{gauge}

- induce change of fractionalization class:
  - twist action of G with gauge transformations of G_{gauge} that do not modify the G symmetry algebra on elementary fields

$$G = \mathbb{Z}_4^{\mathsf{T}} : \text{consider } U \in G_{\text{gauge}} \text{ such that } (\mathsf{T}U)^2 = (-1)^F$$

  - Twisting G with a G_{gauge} gauge transformation

$$a \to a + \mathfrak{u} w_1 \,, \qquad\qquad U = e^{i\mathfrak{u}}$$

implies that line operators charged under Γ transform precisely as if a background $B \in H^2_\rho(G, \Gamma)$ had been turned on. That is, it implements a change of fractionalization class

$$\mathcal{L} \to \exp\left(\frac{2\pi i z}{|\Gamma|} \int_\Sigma \frac{dw_1}{2}\right) \mathcal{L}$$

and shifts $B \to B + \frac{dw_1}{2}$

't Hooft anomaly matching across fractionalization classes

- $2+1d$ $SU(2N)_0$ QCD with adjoint quark

J.G., Komargoski, Seiberg

$$\boxed{SU(2N) \text{ adjoint QCD } \overset{\text{IR}}{\Longrightarrow} \quad U(N)_{N,2N} \text{ Chern-Simons} + \lambda}$$

- $G = \mathbb{Z}_4^{\mathsf{T}}$, with $\mathsf{T}^2 = (-1)^F$: $\mathbb{Z}_4^{\mathsf{T}}$ anomaly classified by $\nu \in \mathbb{Z}_{16}$ Kitaev, Witten, ...

$$\nu = n_+ - n_- \mod 16$$

where $\mathsf{T}\psi_\pm = \pm\gamma^0\psi_\pm$

- $\Gamma = \mathbb{Z}_{2N}$: $\exists$ a Γ 't Hooft anomaly, roughly $\pm\frac{\pi}{2}\int B \cup B$

- Fractionalization classes $H^2(\mathbb{Z}_4^{\mathsf{T}}, \mathbb{Z}_{2N}) = \mathbb{Z}_2$. Induce $\mathbb{Z}_4^{\mathsf{T}}$ anomaly jumps

$$\delta\nu = \begin{cases} -4 & N \text{ even} \\ +4 & N \text{ odd} \end{cases}$$

- We can calculate ν for the different fractionalization classes

  1. $\mathsf{T}\psi_{ij} = \gamma^0\psi_{ij} \Longrightarrow \nu_{\mathsf{T}} = (2N)^2 - 1 \mod 16 = \begin{cases} -1 & N \text{ even} \\ +3 & N \text{ odd} \end{cases}$
  2. change fractionalization class with $U \propto \mathrm{diag}(-1, 1, \ldots, 1) \in SU(2N)$ such that $(\mathsf{T}U)^2 = (-1)^F$

$$\nu_{\mathsf{T}U} = 4N^2 - 8N + 3 \mod 16 = \begin{cases} +3 & N \text{ even} \\ -1 & N \text{ odd} \end{cases}$$

$$\Longrightarrow \quad \delta\nu = \begin{cases} -4 & N \text{ even} \\ +4 & N \text{ odd} \end{cases}$$

- Nontrivial 't Hooft anomaly matching across all fractionalization classes with IR theory $U(N)_{N,2N}$ Chern-Simons $+ \lambda$

- All QCD theories for which nonperturbative dynamics has been put forward

Conclusions

- specifying the action of G requires giving additional data

- distinct choices can give rise to different 't Hooft anomalies for G if there is a Γ or $G - \Gamma$ mixed anomaly

- Physical way to change fractionalization class

- informs the implementation of 't Hooft anomaly matching

Happy Birthday Hirosi!