Black holes and random matrices

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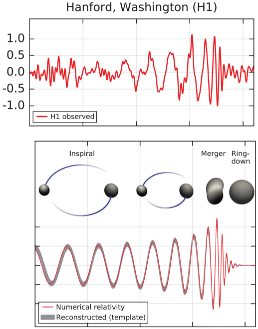
Lossy horizons

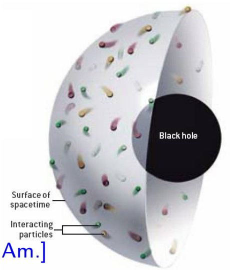
- Black hole horizons appear to display irreversible, dissipative behavior.
- Ping the black hole, look at the response:



- Oscillates and decays – quasinormal modes. Classical gravity, signal decays forever.

Black holes are ordinary quantum systems

- Central dogma: black holes, at least when viewed from the outside, are ordinary quantum systems.
- Gauge/gravity duality, AdS/CFT, holographic duality
- In certain large N quantum systems, strongly coupled quantum behavior on the "boundary" is "dual" to weakly coupled quantum gravitational behavior in the "bulk" spacetime [Maldacena].
- High energy thermal states of the boundary are dual to black holes. Don't evaporate.

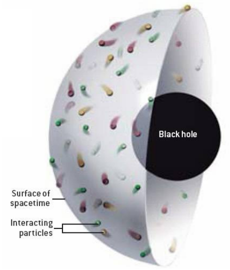


[Maldacena, Sci. Am.]

Long time behavior of correlation functions

- Boundary version of pinging the black hole: Thermal correlator $\langle O(t)O(0)\rangle$. Thermal relaxation $\leftrightarrow$ quasinormal modes [Horowitz-Hubeny].
- Boundary calculation:
$$\langle O(t)O(0)\rangle = \sum_{m,n} e^{-\beta E_m} |\langle m|O|n\rangle|^2 e^{i(E_m - E_n)t}/Z$$
- We expect that black hole energy levels are discrete (finite (Bekenstein-Hawking) entropy S) and nondegenerate (chaos).
- Then at long times $\langle O(t)O(0)\rangle$ stops decreasing. It oscillates in an erratic way and is exponentially small (in S).
- What is the bulk explanation for this? Maldacena's version of the black hole information problem.

What are the rules?



- We know the rules for the boundary.
- We know a lot about the bulk, but we don't know all the rules describing it.
- We don't know enough to calculate the spectrum E_n from the bulk point of view.
- Turn to a simple model where we can do more.

Sachdev-Ye-Kitaev model

- In recent years a simple model of black holes in 1 space and 1 time dimension has been introduced: the Sachdev-Ye-Kitaev (SYK) model.
- 0 space, 1 time boundary system – an ordinary quantum mechanical system.

$$H_{SYK} = \sum_{abcd} J_{abcd}\psi_a\psi_b\psi_c\psi_d$$

ψ_a are N Majorana fermions. J_{abcd} are independent, random, Gaussian distributed couplings. An *ensemble* of boundary QM systems.

Spectral form factor

- Correlation function

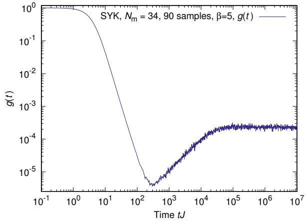
$$\langle O(t)O(0)\rangle = \sum_{m,n} e^{-\beta E_m} |\langle m|O|n\rangle|^2 e^{i(E_m-E_n)t}/Z$$

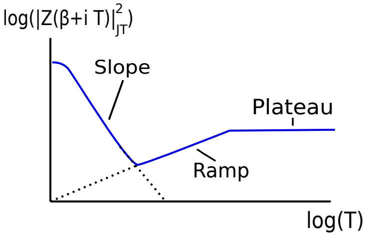
The oscillating phases are the main actors here. Use a simpler, related diagnostic, the "spectral form factor" (SFF) [Papadodimas-Raju]:

$$\mathrm{SFF}(t) = \sum_{m,n} e^{-\beta(E_m+E_n)} e^{i(E_m-E_n)t} = Z_L(\beta+it)Z_R(\beta-it)$$

- The Fourier transform of the energy differences
- Study in SYK model:
  First, do computer "experiments" ...
  [You-Ludwig-Xu,Garcia-Garcia-Verbaaschot,
  Cotler-Gur-Ari-Hanada-Polchinski-Saad-SS-Stanford-Streicher-Tezuka ...]

Ramp and plateau





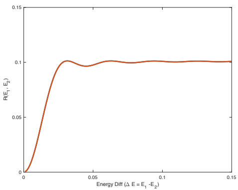
- Slope is analog of quasinormal mode decay.
- Does not continue forever. Exponentially small ramp and plateau (erratic behavior smoothed out by ensemble averaging).
- What does this pattern mean?
- A signature of random matrix statistics.

- Abundant evidence that black holes are highly chaotic quantum systems.
- The energy levels of quantum chaotic systems are widely believed to have the same statistical properties as those of *random matrices*. [Wigner; Bohigas-Giannoni-Schmit...]
- Each entry of the "Hamiltonian" is independently drawn from a Gaussian distribution.
- A remarkable example of universality. Independent of dimension, locality. Only weakly dependent on symmetry.

Density-density correlations

- Random matrix formula for density-density correlations, the Sine kernel formula. [Dyson; Gaudin; Mehta]

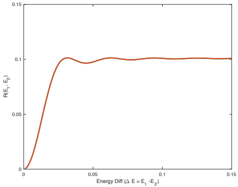
$$\langle \rho(E)\rho(E') \rangle \sim e^{2S(\bar{E})} - \frac{1}{2(\pi(E-E'))^2}(1-\cos(2\pi e^{S(\bar{E})}(E-E')))$$

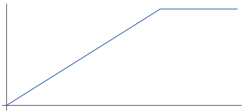


(Here $e^{S(\bar{E})}$ is the local density of states.)

- Long-range spectral rigidity, short-range level repulsion

Ramp and plateau from Sine kernel





- Fourier transform of Sine kernel formula gives ramp and plateau.
- Ramp is signature of long-range spectral rigidity.
- Sharp transition to plateau is signature of short-range level repulsion.
- Not only does the SFF signal stop decreasing, but it displays a distinctive, universal pattern late-time pattern.
- What is the bulk, gravitational explanation for this pattern?

Collective fields in SYK

- SYK gives us a tool.
- Rewrite the dynamics using collective (Hubbard-Stratonovich) fields, G, Σ (e.g., Landau-Ginzburg description of superconducting pairs).

$$G(t, t') = \frac{1}{N} \sum_{a=1}^{N} \psi_a(t)\psi_a(t').$$

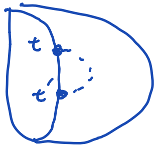
- Schematically,

$$\int d\psi_a \exp(-I_{\text{fermion}}(\psi)) \rightarrow \int dG d\Sigma \exp(-N \; I_{\text{coll}}(G, \Sigma))$$

- I_{coll} especially simple after averaging over J's.
- An *exact* rewrite.
- Collective fields connect to a gravitational description.

SYK and JT gravity

- At low energies the collective field description is related to a certain kind of 2D gravity called Jackiw-Teitelboim gravity [Jensen, Maldacena-Stanford-Yang, Engelsoy-Mertens-Verlinde].
- Concretely, $G(t, t')$ is given by the correlator of a quantum field on a 2D geometry:



- The collective field integral

$$\int dG d\Sigma \exp(-N \; I_{\text{coll}}(G, \Sigma))$$

is a proxy for a complete bulk description. In this simple model we know all the rules.

SFF in JT

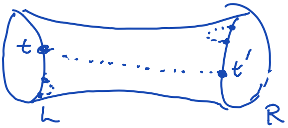
- The SFF:

$$\mathrm{SFF}(t) = \sum_{m,n} e^{-\beta(E_m+E_n)} e^{i(E_m-E_n)t} = Z_L(\beta+it)Z_R(\beta-it)$$

- L and R "replicas" of SYK. Collective fields

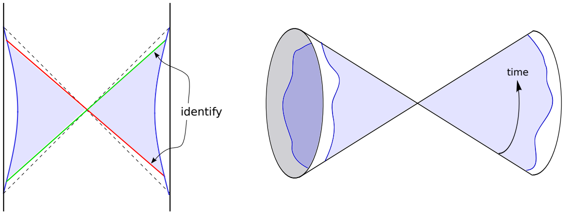
$$G_{LL} = \psi^L\psi^L,\ G_{RR} = \psi^R\psi^R,\ G_{LR} = \psi^L\psi^R$$

- The ramp comes from a nonzero saddle point for G_{LR}, linking the L and R replicas.



- A spacetime "wormhole" connecting the L and R boundary systems [Saad-SS-Stanford].
- (The plateau is more subtle, involving more complicated topologies. See [Saad-Stanford-Yang-Yao]).

The double cone



- The double cone [Saad-SS-Stanford].
- Represents a one dimensional universe (a line) evolving in time.
- A spacetime wormhole.
- Spatial wormholes, like Einstein-Rosen bridges, correspond to the one dimensional slice.

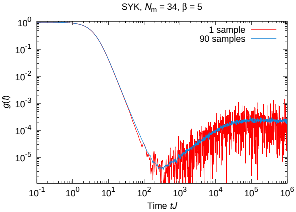
Wormholes

- Candidate wormhole configurations have been found in higher dimensions [Cotler-Jensen]. This is consistent with the universal character of random matrix statistics.
- Closely related wormhole configurations have recently been used to explain a host of subtle quantum phenomena in black holes, at least in simple models:
  - Long-time behavior of correlation functions [Saad].
  - ETH and OPE statistics [Belin-de Boer, Pollack-Rozali-Sully-Wakeham]
  - The Page curve for radiation from evaporating black holes [Almheiri-Hartman-Maldacena-Shaghoulian-Tajdini; Penington-SS-Stanford-Yang].
  - Firewall formation [Stanford-Yang].
  - ...
- But their existence raises a number of puzzling questions.

Gauge/gravity duality without averaging

- Standard gauge/gravity duals, like $AdS_5 \leftrightarrow$ Super-Yang-Mills$_4$, are not averaged. One specific boundary quantum system. We don't know the rules here.
- A toy model for non-averaged behavior: a single member of an ensemble.
- SYK with one choice of J's.

Single sample noise



- The ramp and plateau are *noisy*. Noise is of order the signal. Expected from RMT universality. This is what a regular AdS/CFT should produce.
- Can recover the wormhole signal – the smooth ramp – without an ensemble, by averaging over time.
- But the bulk description must contain something else to account for the noise.

Factorization puzzle

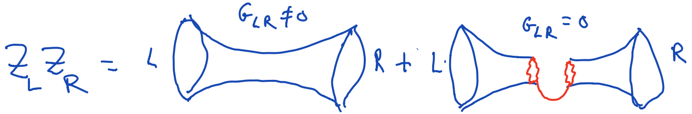
- A sharp puzzle if the boundary Hamiltonian is not averaged over.
- From the boundary point of view Z_{LR} is exactly equal to $Z_L \cdot Z_R$, since the L and R systems are decoupled. Z_{LR} *factorizes*.
- But from the bulk point of view Z_{LR} contains a LR wormhole. But Z_L and Z_R do not. So superficially Z_{LR} does not factorize.
- If wormholes are actually then the bulk description must contain something else to restore factorization.
- It seems very plausible that this "something else" is also the source of the noise.
- If we average over an ensemble of Hamiltonians then $\langle Z_{LR} \rangle \neq \langle Z_L \rangle \langle Z_R \rangle$ in general.
- And in fact the connection between wormholes and fluctuating couplings was pointed out long ago [Coleman; Giddings-Strominger].

SYK with fixed couplings

- How to study this?
- Examine the collective field description of SYK with *fixed* couplings (J's) [Saad-SS-Stanford-Yao].
- $$\int dG d\Sigma \exp(-N \; I_{\mathrm{coll}}(G, \Sigma, J))$$
- Same kind of collective field description, with a much more complicated weighting factor that depends explicitly on the random couplings J.
- Can only do a precise analysis in a toy model of the toy model – SYK with one time point (!)
  $\psi(t) \to \psi$. An ordinary integral over N Grassman variables.
- Can analyze the collective field integral with precision, and make a plausible guess for how full SYK works.

Half-wormholes

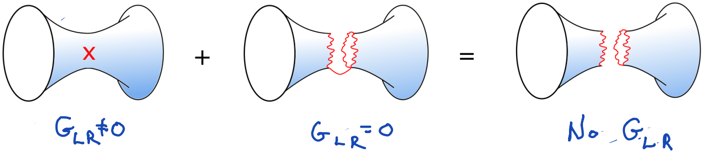
- With fixed couplings there is another saddle point of the collective field integral, in addition to the wormhole.
- This saddle point has $G_{LL}(t, t')$ and $G_{RR}(t, t')$ set to their values in the wormhole saddle, but $G_{LR}(t, t')$ set to zero.



- The contribution of the wormhole saddle is essentially independent of the choice of J's.
- But the half-wormhole contribution is very sensitive. It is the source of the noise.
- On averaging, over J's, or over a time window, the half-wormhole contribution vanishes, leaving the wormhole signal.
- Geometrical picture in JT [Blommaert-Mertens-Verscheide; Saad-SS-Yao; Blommaert-Iliesiu-Kruthoff]

Factorization

- There are multiple possible collective field (bulk) descriptions. With fixed J's there is no clear preference between them.
- For example, we can introduce G_{LR} and look for saddle points when integrating over it.
- Or we could just integrate it out first, eliminating it. This leaves a "bulk" description without wormholes. They've been integrated out.
- In this description Z_{LR} manifestly factorizes.
- The noisy ramp is described by half-wormholes in the L and R systems separately.



- The wormhole and half-wormhole contributions on the LHS combine to restore factorization. Their contributions are of the same magnitude.

Questions

- Do these structures have parallels in higher dimensional "less toylike" models of quantum gravity, like standard AdS/CFT systems?
- The idea of multiple bulk descriptions and of extra ingredients needed for factorization echo some ideas that have emerged in other simple model gravitational systems [cf. Marolf-Maxfield, Jafferis-Schneider, Eberhardt, Mukhametzhanov, Benini-Copetti-De Pietra...].
- We haven't yet made contact with with the bulk degrees of freedom that we think are important in such systems – strings, branes etc. – especially when they are interacting in a complicated, chaotic, way. (cf. the "fuzzball" program [Mathur, ...])
- We need another simple, tractable model, in the spirit of SYK, to help us move closer to understanding the rules of quantum gravity.

Happy Birthday Hirosi!