DERIVING THE SIMPLEST GAUGE-STRING DUALITY

Rajesh Gopakumar

HirosiFest, Caltech, Oct. 28th, 2022

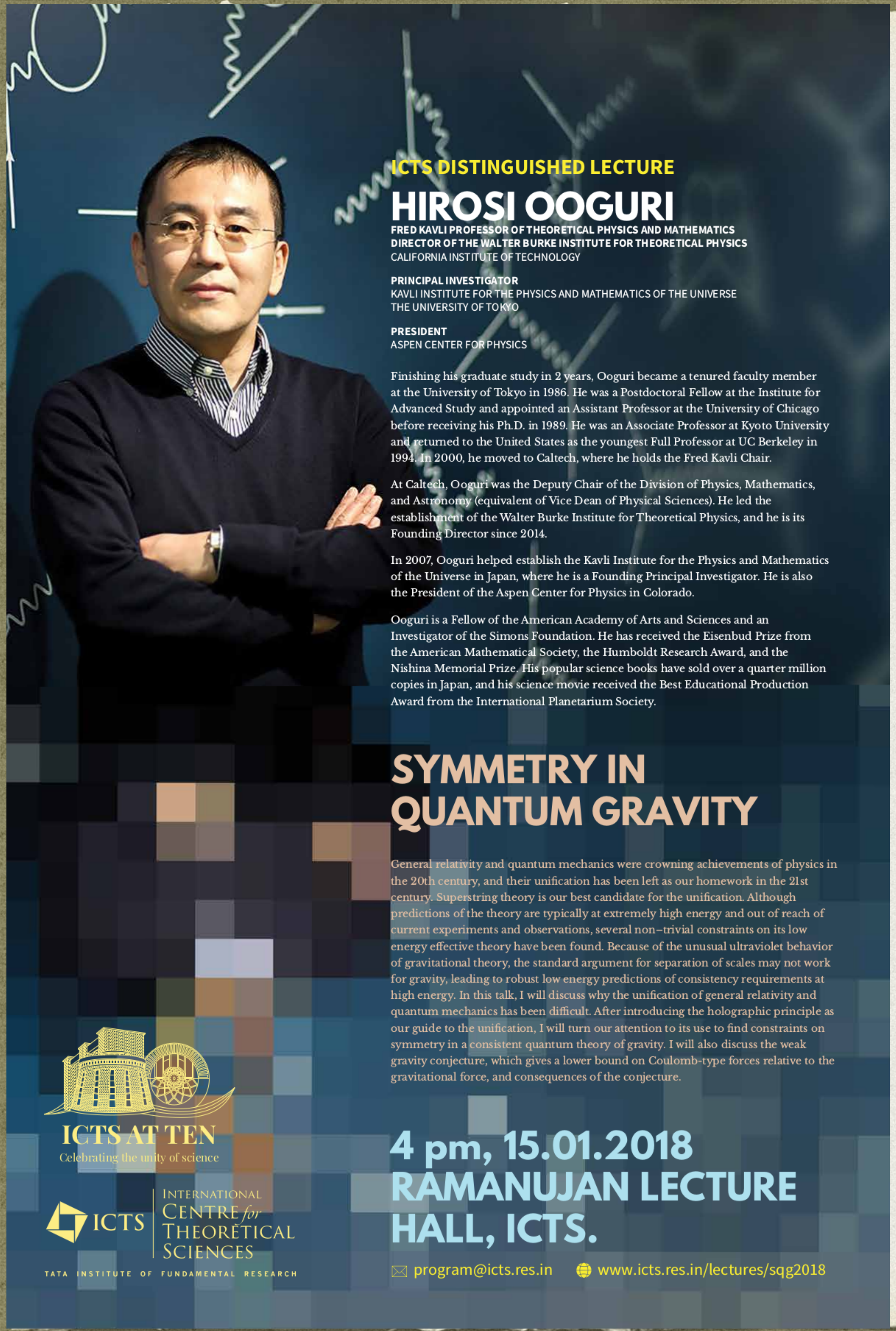
Based on work to appear w/ Edward Mazenc:

DSD I: Open-Closed-Open Triality

DSD II: B-Model

DSD III: A-Model

HIROSI & ICTS


ICTS DISTINGUISHED LECTURE
HIROSI OOGURI
FRED KAVLI PROFESSOR OF THEORETICAL PHYSICS AND MATHEMATICS
DIRECTOR OF THE WALTER BURKE INSTITUTE FOR THEORETICAL PHYSICS
CALIFORNIA INSTITUTE OF TECHNOLOGY
PRINCIPAL INVESTIGATOR
KAVLI INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE
THE UNIVERSITY OF TOKYO
PRESIDENT
ASPEN CENTER FOR PHYSICS
Finishing his graduate study in 2 years, Ooguri became a tenured faculty member at the University of Tokyo in 1986. He was a Postdoctoral Fellow at the Institute for Advanced Study and appointed an Assistant Professor at the University of Chicago before receiving his Ph.D. in 1989. He was an Associate Professor at Kyoto University and returned to the United States as the youngest Full Professor at UC Berkeley in 1994. In 2000, he moved to Caltech, where he holds the Fred Kavli Chair.
At Caltech, Ooguri was the Deputy Chair of the Division of Physics, Mathematics, and Astronomy (equivalent of Vice Dean of Physical Sciences). He led the establishment of the Walter Burke Institute for Theoretical Physics, and he is its Founding Director since 2014.
In 2007, Ooguri helped establish the Kavli Institute for the Physics and Mathematics of the Universe in Japan, where he is a Founding Principal Investigator. He is also the President of the Aspen Center for Physics in Colorado.
Ooguri is a Fellow of the American Academy of Arts and Sciences and an Investigator of the Simons Foundation. He has received the Eisenbud Prize from the American Mathematical Society, the Humboldt Research Award, and the Nishina Memorial Prize. His popular science books have sold over a quarter million copies in Japan, and his science movie received the Best Educational Production Award from the International Planetarium Society.
SYMMETRY IN QUANTUM GRAVITY
General relativity and quantum mechanics were crowning achievements of physics in the 20th century, and their unification has been left as our homework in the 21st century. Superstring theory is our best candidate for the unification. Although predictions of the theory are typically at extremely high energy and out of reach of current experiments and observations, several non–trivial constraints on its low energy effective theory have been found. Because of the unusual ultraviolet behavior of gravitational theory, the standard argument for separation of scales may not work for gravity, leading to robust low energy predictions of consistency requirements at high energy. In this talk, I will discuss why the unification of general relativity and quantum mechanics has been difficult. After introducing the holographic principle as our guide to the unification, I will turn our attention to its use to find constraints on symmetry in a consistent quantum theory of gravity. I will also discuss the weak gravity conjecture, which gives a lower bound on Coulomb-type forces relative to the gravitational force, and consequences of the conjecture.
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RAMANUJAN LECTURE HALL, ICTS.
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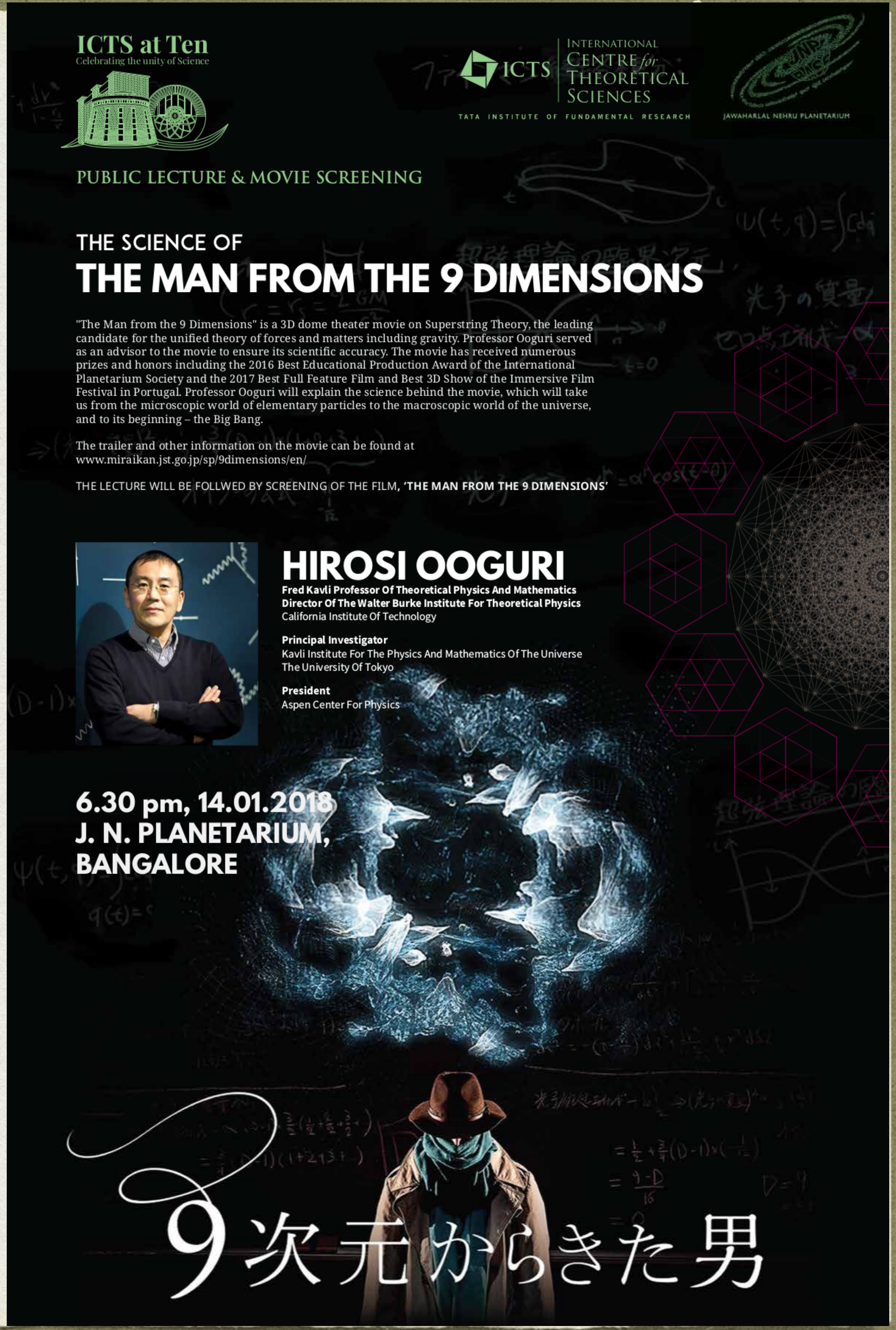
ICTS at Ten
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JAWAHARLAL NEHRU PLANETARIUM
PUBLIC LECTURE & MOVIE SCREENING
THE SCIENCE OF
THE MAN FROM THE 9 DIMENSIONS
"The Man from the 9 Dimensions" is a 3D dome theater movie on Superstring Theory, the leading candidate for the unified theory of forces and matters including gravity. Professor Ooguri served as an advisor to the movie to ensure its scientific accuracy. The movie has received numerous prizes and honors including the 2016 Best Educational Production Award of the International Planetarium Society and the 2017 Best Full Feature Film and Best 3D Show of the Immersive Film Festival in Portugal. Professor Ooguri will explain the science behind the movie, which will take us from the microscopic world of elementary particles to the macroscopic world of the universe, and to its beginning – the Big Bang.
The trailer and other information on the movie can be found at www.miraikan.jst.go.jp/sp/9dimensions/en/
THE LECTURE WILL BE FOLLWED BY SCREENING OF THE FILM, 'THE MAN FROM THE 9 DIMENSIONS'
HIROSI OOGURI
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California Institute Of Technology
Principal Investigator
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The University Of Tokyo
President
Aspen Center For Physics
6.30 pm, 14.01.2018
J. N. PLANETARIUM,
BANGALORE
9次元からきた男

UNWRAPPING THE GIFT....

- Proposing the Simplest Gauge-String Duality
- Verifying the Simplest Gauge-String Duality
- Deriving the Simplest Gauge String Duality
- The Big(ger) Picture

THE SIMPLEST LARGE N THEORY

- $\langle \mathrm{Tr}M^{l_1} \ldots \mathrm{Tr}M^{l_n} \rangle = \int [DM]_{N\times N}\, e^{-\frac{N}{2t}\mathrm{Tr}M^2}\, \mathrm{Tr}M^{l_1} \ldots \mathrm{Tr}M^{l_n}$

- Feynman diags. (Wick contractions) encode the combinatorics of the free large N 'Hooft expansion.

- Generalises to an interacting matrix model $\mathrm{Tr}M^2 \to \mathrm{Tr}V(M)$. [BIPZ'80][Cf. Gross-Migdal; Douglas-Shenker; Brezin-Kazakov]

- Ought to have a dual string description - without a double scaling limit [Cf. Dijkgraaf-Vafa]. A stripped-down-to-essentials version of gauge string duality - both `tensionless' (free) and away from Gaussian.

- Can we derive such a duality? Make equality of correlators manifest? (Cf. AdS_3/AdS_5 in tensionless limit) [Eberhardt-Gaberdiel-R.G.'18-19; Gaberdiel-R.G.-Knighton-Maity '20;Gaberdiel-R.G.'21].

BARE BONES GAUGE-STRING DUALITY

Gaussian Matrix Model:
$\langle \frac{1}{l_1}\mathrm{Tr}M^{l_1} \ldots \frac{1}{l_n}\mathrm{Tr}M^{l_n} \rangle_g$

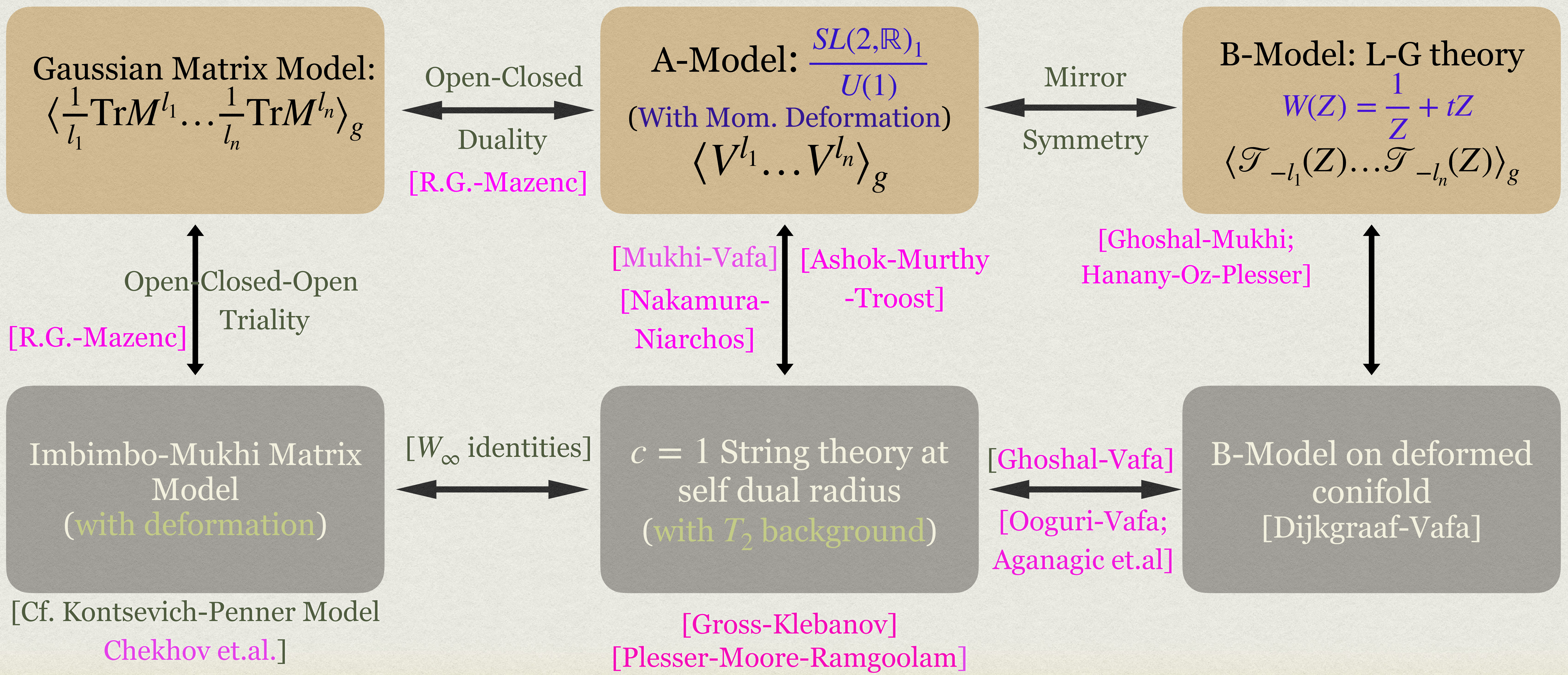
⟷ Open-Closed Duality

A-Model Kazama-Suzuki on $\frac{SL(2,\mathbb{R})_1}{U(1)}$
(with mom. background)
$\langle V^{l_1} \ldots V^{l_n} \rangle_g$

⟷ Mirror Symmetry [Hori-Kapustin]

B-Model: L-G theory
$W(Z) = \frac{1}{Z} + tZ$
$\langle \mathcal{T}_{-l_1}(Z) \ldots \mathcal{T}_{-l_n}(Z) \rangle_g$

THE SIMPLEST DUALITY CHAIN COMPLEX



VERIFYING THE SIMPLEST DUALITY

Gaussian Matrix Model:
$\langle \frac{1}{l_1}\mathrm{Tr}M^{l_1}\ldots\frac{1}{l_n}\mathrm{Tr}M^{l_n}\rangle_g$

Open-Closed-Open Triality

Exact equality of n-point correlators for all genus g.

Imbimbo-Mukhi Matrix Model (with deformation)

$$Z(t,\bar{t}) = \frac{1}{Z_N}\int [DK][DM]_{N\times N}\, e^{\frac{N}{t_2}\mathrm{Tr_N}[V_p(K)-K(M-Y)]} \prod_{a=1}^{Q} det_N(x_a - M)$$

$$= \frac{1}{Z_Q}\int [DA][DB]_{Q\times Q}\, e^{-\frac{N}{t_2}\mathrm{Tr_Q}[V_p(A)+A(B-X)]} \prod_{i=1}^{N} det_Q(y_i - A)$$

Using Hubbard-Stratonovich trick with fermions. [cf. Maldacena-Moore-Seiberg-Shih; Aganagic-Dijkgraaf-Klemm-Marino-Vafa]

For the Gaussian case:

(\`Compact Branes' - ZZ Branes)

$$Z_G(t,\bar{t}) = \frac{1}{Z_N}\int [DM]_N\, e^{-N\mathrm{Tr_N}[\frac{1}{t_2}M^2+\sum_k \bar{t}_k M^k]} \qquad [N\bar{t}_k = \frac{1}{k}Tr_Q X^{-k}]$$

(\`Noncompact Branes' - FZZT Branes)

$$= \frac{Z_{penner}}{Z_Q}\int [DA]_{Q\times Q}\, e^{-\frac{N}{t_2}\mathrm{Tr_Q}[A^2-AX]-(N+Q)\mathrm{Tr}\ln A} = Z_{IM}(t_2\neq 0,\bar{t}_k)$$

Exchanges graphs with dual graphs - “V-type” ↔ “F-type” duality [R. G.-Jo'burg workshop'11; Jiang-Komatsu-Vescovi].

VERIFYING THE SIMPLEST DUALITY

Gaussian Matrix Model: $\langle \frac{1}{l_1}\mathrm{Tr}M^{l_1}\dots\frac{1}{l_n}\mathrm{Tr}M^{l_n}\rangle_g$

Open-Closed-Open Triality

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$\mathrm{Tr}K^2$

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VERIFYING THE SIMPLEST DUALITY

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Open-Closed-Open Triality

Exact equality of n-point correlators for all genus g.

Imbimbo-Mukhi Matrix Model (with deformation)

$[W_\infty$ identities$]$

$c = 1$ String theory at self dual radius (with T_2 background and $\mu = -iN$)

$$Z_G(t,\bar{t}) = \frac{1}{Z_N}\int [DM]_N \, e^{-N\mathrm{Tr_N}[\frac{1}{t_2}M^2 + \sum_k \bar{t}_k M^k]} \qquad [N\bar{t}_k = \frac{1}{k}Tr_Q X^{-k}]$$

$$[\mu = -iN]$$

$$\propto \frac{Z_{penner}}{Z_Q}\int [DA]_{Q\times Q} \, e^{i\mu \mathrm{Tr_Q}AX - (i\mu+Q)\mathrm{Tr}\ln A - i\mu t_2 \mathrm{Tr_Q}A^2} = Z_{IM}(t_2 \neq 0, \bar{t}_k)$$

Captures the S-Matrix of c=1 string theory of -ve momentum Tachyons in the presence of T_2 background

[Moore-Plesser-Ramgoolam]

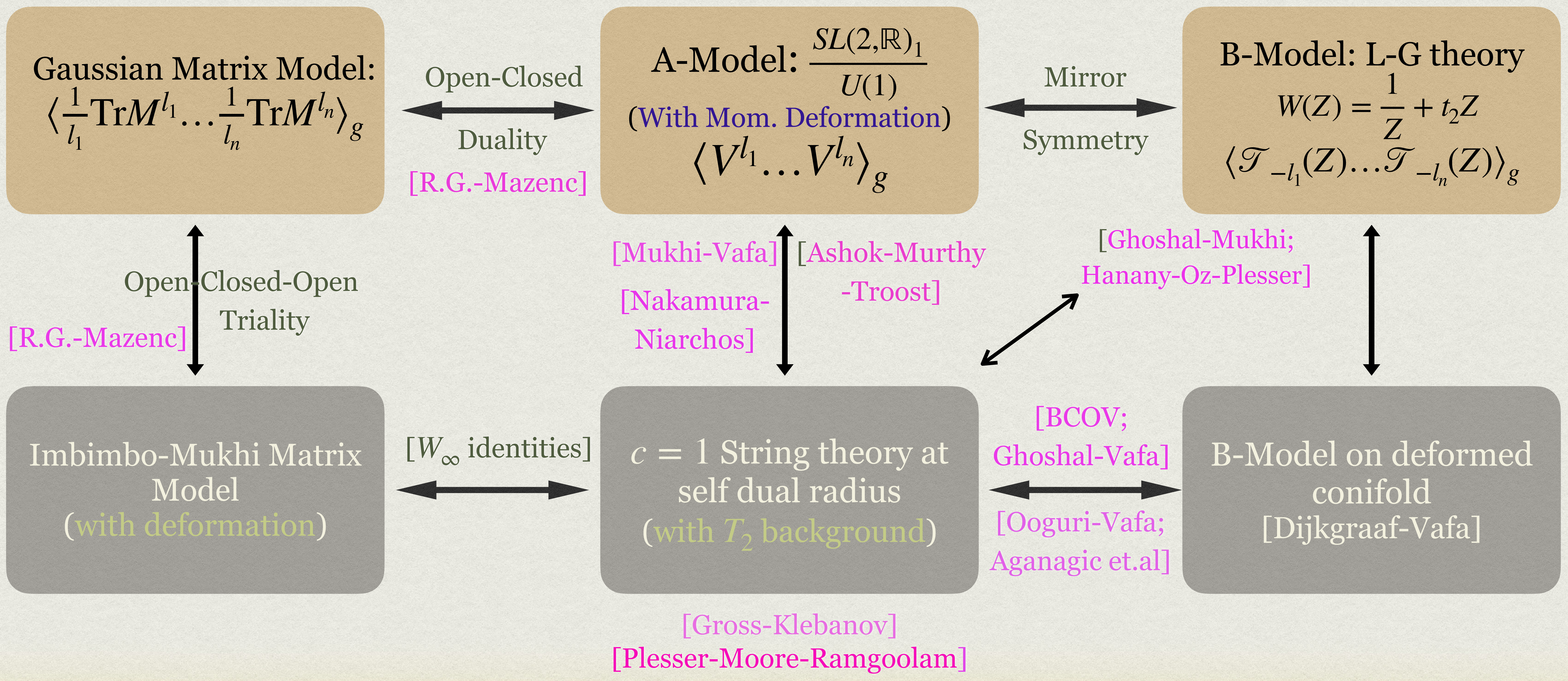
Exact expression for all genus in an expansion in $\mu^{2-2g} = (-1)^g N^{2-2g}$

Sanity checks: Gaussian 1-pt. function $\langle \mathrm{Tr}_N M^{2l}\rangle = \langle\langle T_{-2l}\rangle\rangle \propto \langle (T_2)^l T_{-2l}\rangle$ for all genus

[R.G.-Mukhi ('95) - unpublished]

3 and 4 pt. functions at genus zero.

THE SIMPLEST DUALITY WEB



DERIVING THE SIMPLEST DUALITY

Topological Recursion Relations (TRR)

Gaussian Matrix Model:
$N_{g,n} = \langle : \frac{1}{l_1}\mathrm{Tr}M^{l_1} : \ldots : \frac{1}{l_n}\mathrm{Tr}M^{l_n} : \rangle_g$

Counting Belyi Maps; Discrete Volumes of Moduli Space

B-Model: L-G theory
$W(Z) = \frac{1}{Z} + t_2 Z$
$\langle \mathcal{T}_{-l_1}(Z) \ldots \mathcal{T}_{-l_n}(Z) \rangle_g$

[Eynard-Orantin; Eynard]

[Bauer-Itzykson; R.G.; R.G-Pius; de Mello Koch-Ramgoolam; Razamat]

A-Model: $\frac{SL(2,\mathbb{R})_1}{U(1)}$
(With Mom. Deformation)
$\langle V^{l_1} \ldots V^{l_n} \rangle_g$

- Spectral curve $\mathcal{S}$ of MM: $X(z) = \frac{1}{z} + t_2 z;\ Y(z) = z$.
- Two branchpoints/critical points: $dX(z) = 0$.
- Machinery of TRR (with Bergman kernel $B(z, z')$) for

$$W_{g,n}^{(\mathcal{S})}(z_1, \ldots z_n) = \langle \prod_{i=1}^{n} \mathrm{Tr}\left(\frac{dX(z_i)}{X(z_i) - M} \right) \rangle$$ generates the $N_{g,n}$.

- Expressed in terms of integrals over moduli space.

- Correlators $N_{g,n}$ combinatorially account for special holomorphic Belyi maps.
- Covering maps $\Sigma_{g,n} \to \mathbb{P}^1$ of degree $\ell = \sum_i l_i$ with exactly three branch points $(0,1,\infty)$
- Branching profile $[l_i], \ldots, [l_n]$ at ∞; $[2]^{\frac{l}{2}}$ branching at 1.
- Via integer length Strebel differentials counts lattice points on moduli spaces [Mulase-Penkava; Norbury-Scott].

DERIVING THE B-MODEL

Gaussian Matrix Model:

$N_{g,n} = \langle : \frac{1}{l_1}\mathrm{Tr}M^{l_1} : \ldots : \frac{1}{l_n}\mathrm{Tr}M^{l_n} : \rangle_g$

Topological Recursion Relns. → Intersection Numbers

B-Model: L-G theory

$W(Z) = \frac{1}{Z} + t_2 Z$

$\langle \mathcal{T}_{-l_1}(Z)\ldots\mathcal{T}_{-l_n}(Z)\rangle_g$

Mumford Kappa class

- $N_{g,n}$ (or $W_{g,n}$) are integrals over moduli space generalising Kontsevich's intersection numbers [Eynard]:

$$N_{g,n} = \langle \Lambda(\mathcal{S}) \prod_{i=1}^{n} \mathcal{O}_{l_i} \rangle_{\mathcal{M}^{(2)}_{g,n}} \qquad [\mathrm{Tr}M^l \leftrightarrow \mathcal{O}_l = \sum_{d=0}^{\infty} C_{k,d}\psi^d] \qquad [\Lambda(\mathcal{S}) = e^{\sum_{k=0}\tilde{s}_k\kappa_k} e^{\sum_\delta \sum_{k,l} l_{\delta *}\hat{B}_{k,l}\psi_k\psi_l}]$$

- $\mathcal{M}^{(2)}_{g,n}$: two copies of moduli space associated to the two branch points $dX = 0$ of spectral curve.
- View as moduli space of constant maps to the two critical points of the LG superpotential $dW(z) = 0$.
- 2d Top. Gravity w/ LG matter after integrating out the matter fields. Alternatively view solutions of TRR as LG matter integrals (after integrating out 2d gravity) [Cf. Losev]
- Also connection to Kodaira-Spencer theory on spectral curve: TRR as Schwinger-Dyson equations [Dijkgraaf-Vafa; Post-v.d-Heijden-E. Verlinde].

DERIVING THE A-MODEL

Gaussian Matrix Model:

$N_{g,n} = \langle : \frac{1}{l_1}\mathrm{Tr}M^{l_1} : \ldots : \frac{1}{l_n}\mathrm{Tr}M^{l_n} : \rangle_g$

Belyi Branched Coverings → Localisation as in AdS_3

A-Model: $\frac{SL(2,\mathbb{R})_1}{U(1)}$ (With Mom. Deformation)

$\langle V^{l_1} \ldots V^{l_n} \rangle_g$

- Localisation to special points on moduli space also seen in tensionless limit of AdS_3 strings [Eberhardt-Gaberdiel-R.G.].
- Followed from Ward Identities of $k=1,\ sl(2,\mathbb{R})$ worldsheet theory for spectrally flowed vertex operators.
- Contributions only from points on moduli space which admit holomorphic covering maps to $\mathbb{P}^1$ with specified branching.
- Here also SUSY $sl(2,\mathbb{R})_1$ theory. Physical vertex operators V_l are in the $D^{(l)}_{j=\frac{1}{2}}$ repn [Ashok-Murthy-Troost].
- Hence same WI apply but no spacetime position dependence. Thus branching $[l_1]\ldots[l_n]$ at ∞: compactified cigar end.
- $T_2 = e^{i\sqrt{2}X}$ background → \`clean Belyi maps' - with simple $[2]^{\frac{l}{2}}$-branching at 1 and Liouville wall interactions at 0.

THE BMN LIMIT

- In both B-Model and A-model pictures one recovers the observables of pure 2d gravity in a BMN-like limit. Zoom into the edge of the eigenvalue distribution.

[Cf. Okounkov;Okounkov-Pandharipande]

- B-Model: $\lim_{l_i \to \infty} \langle \frac{1}{l_1} \mathrm{Tr} M^{l_1} \ldots \frac{1}{l_n} \mathrm{Tr} M^{l_n} \rangle_g \propto \sum_{d_1 + \ldots d_n = d_{g,n}} \langle \prod_{i=1}^{n} \psi_i^{d_i} \rangle_{\mathcal{M}_{g,n}} \prod_i v_i^{d_i + \frac{1}{2}}$. $[l_i = \ell v_i]$

- A-Model: The discrete volumes $N_{g,n}$ of lattice points on $\mathcal{M}_{g,n}$ goes over in the large l_i limit to the continuum Kontsevich volumes. [Norbury]

- Similar continuum approach as in large twist limit of symmetric product CFTs dual to tensionless limit of AdS_3. [Gaberdiel-R.G.-Knighton-Maity]

THE BIGGER PICTURE

Gaussian Matrix Model ($\frac{1}{2}$-BPS sector) $\subset$ Chiral Algebra Sector $\subset$ $\mathcal{N} = 4$ Super Yang-Mills

Simplest Gauge-String Duality

Twisted Holography [Costello-Gaiotto; Gaiotto-Budzik; Costello-Paquette]

AdS/CFT Holography

B-Model with $W(Z) = \frac{1}{Z} + tZ$ $\leftrightarrow$ Deformed Conifold $\subset$ B-Model on $SL(2,\mathbb{C}) \approx AdS_3 \times S^3$ $\subset$ IIB Strings on $AdS_5 \times S^5$

[Make the relation to the deformed conifold precise?]

[What is the relation to tensionless AdS_3/CFT_2?]

[Embed the matrix duality in tensionless AdS_5/CFT_4 ?]

Happy to Take Questions....

....and hopefully Hirosi will be happy with the answers.

HAPPY BIRTHDAY, HIROSI AND
MANY MORE YEARS OF GREAT RESEARCH!