Recent progress on the black hole information problem

Daniel Harlow

MIT

October 28, 2022

So far I have written three papers with Ooguri-sama:

Constraints on symmetry from holography

Daniel Harlow[1] and Hirosi Ooguri[2,3]
[1] *Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*
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In this letter we show that a set of old conjectures about symmetries in quantum gravity hold within the Anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence. These conjectures are that no global symmetries are possible, that internal gauge symmetries must come with dynamical objects that transform in all irreducible representations, and that internal gauge groups must be compact. These conjectures are not obviously true from a bulk perspective, they are nontrivial consequences of the non-perturbative consistency of the correspondence. More details of and background for these arguments are presented in an accompanying paper.

Symmetries in Quantum Field Theory and Quantum Gravity

Daniel Harlow[a] and Hirosi Ooguri[b,c]
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ABSTRACT: In this paper we use the AdS/CFT correspondence to refine and then establish a set of old conjectures about symmetries in quantum gravity. We first show that any global symmetry, discrete or continuous, in a bulk quantum gravity theory with a CFT dual would lead to an inconsistency in that CFT, and thus that there are no bulk global symmetries in AdS/CFT. We then argue that any "long-range" bulk gauge symmetry leads to a global symmetry in the boundary CFT, whose consistency requires the existence of bulk dynamical objects which transform in all finite-dimensional irreducible representations of the bulk gauge group. We mostly assume that all internal symmetry groups are compact, but we also give a general condition on CFTs, which we expect to be true quite broadly, which implies this. We extend all of these results to the case of higher-form symmetries. Finally we extend a recently proposed new motivation for the weak gravity conjecture to more general gauge groups, reproducing the "convex hull condition" of Cheung and Remmen.

An essential point, which we dwell on at length, is precisely defining what we mean by gauge and global symmetries in the bulk and boundary. Quantum field theory results we meet while assembling the necessary tools include continuous global symmetries without Noether currents, new perspectives on spontaneous symmetry-breaking and 't Hooft anomalies, a new order parameter for confinement which works in the presence of fundamental quarks, a Hamiltonian lattice formulation of gauge theories with arbitrary discrete gauge groups, an extension of the Coleman-Mandula theorem to discrete symmetries, and an improved explanation of the decay $\pi^0 \to \gamma\gamma$ in the standard model of particle physics. We also describe new black hole solutions of the Einstein equation in $d+1$ dimensions with horizon topology $\mathbb{T}^p \times \mathbb{S}^{d-p-1}$.

A universal formula for the density of states in theories with finite-group symmetry

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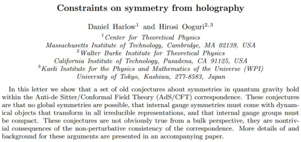

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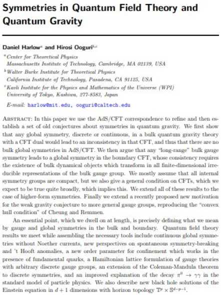

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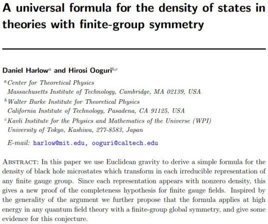

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The first two are sort of well-known, but I think the third is also nice: we argue that in any quantum field theory with a finite global symmetry G the high-energy density of states in each representation α obeys $\rho_\alpha(E) = \frac{d_\alpha^2}{|G|}\rho(E)$. Maybe testable in condensed matter systems?

Of course the best part of working with Hirosi is having fun with Hirosi!

Especially the food

Incidentally his impeccable dress extends also to active situations

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Please do not diminish our big chestnut by turning him into a little chestnut!

Happy birthday Ooguri-sama!

Introduction

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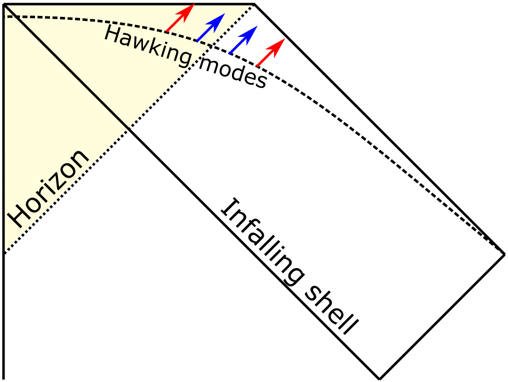
- It is too soon to say that the problem has been fully resolved, but I think it is fair to say that many of us feel a resolution is in sight.
- In this talk I will attempt to give a brief overview of what I think is the current status.

You are hopefully familiar with the basic argument:



Entanglement between interior and exterior modes causes the black hole to radiate, losing energy, but this radiation cannot carry information about the infalling shell since this is still deep inside. By the time the black hole reaches Planckian size, it doesn't have enough energy left to return this information to the exterior.

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These are all things we really would like to be true, so any resolution of the paradox will teach us something deep!

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Our challenge is thus to understand what replaces (3).

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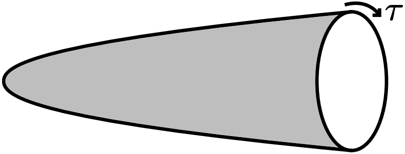

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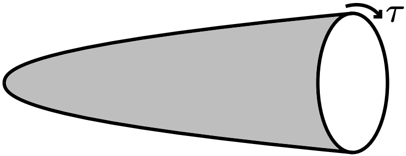

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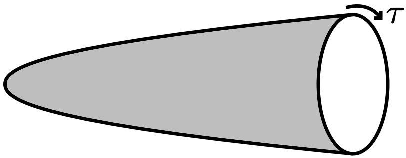

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Why then did I cite string theory and AdS/CFT to justify this formula?

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- It is interesting to think about why (and when) this works, but it isn’t my focus today.
- The reason why so many people were excited about the work of Strominger and Vafa is that it enabled us to compute black hole entropy *using the definition of entropy*, without relying on the mysterious black box of quantum gravity.

Spacetime emergence

- In understanding what might replace (3) (the validity of EFT away from singularities), it is useful to note that in AdS/CFT it not obvious that *any* version of (3) holds: the fundamental description is the boundary CFT, and the bulk spacetime is at best *emergent*.

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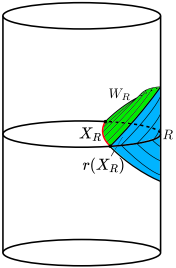
- Moreover the high-energy states qualitatively behave as one would expect for states with a big black hole in the bulk, and in some cases we can even confirm the entropy formula (e.g. using the Cardy formula).
- What is less clear is how to think about the emergence of spacetime *inside* black holes, and this has been the main topic of recent work.

Quantum extremal surfaces

A key insight into the nature of emergent spacetime in AdS/CFT is the *quantum extremal surface formula*: Ryu, Takayanagi, Hubeny, Rangamani, Faulkner, Maldacena, Lewkowycz, Engelhardt, Wall



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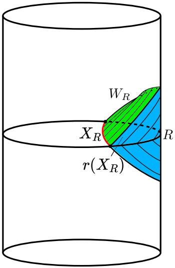

$$S(\rho_R) = \min_{X_R}\left[\operatorname*{ext}_{X_R}\left(\frac{\mathrm{Area}(X_R)+\dots}{4G}+S_{bulk}(r(X_R))\right)\right].$$

Here X_R is required to be *homologous* to R, meaning there exists a bulk region $r(X_R)$ such that $\partial r(X_R) = R - X_R$. The domain of dependence of $r(X_R)$ is called the *entanglement wedge* of R, denoted W_R.

- As with the black hole entropy formula, of which it is a generalization, there is a Euclidean gravity "derivation" of the QES formula. Lewkowycz/Maldacena
- One can also look for a more direct Hilbert space explanation a la Strominger/Vafa, and currently our best direct understanding is based on the idea that the QES formula is a generic feature of quantum error-correcting codes. Almheiri/Harlow/Dong, Pastawski/Yoshida/Harlow/Preskill, Harlow, Hayden/Nezami/Qi/Thomas/Walter

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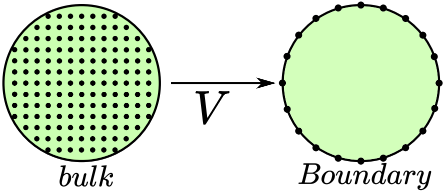
- For the latter the basic idea is that there is an approximate isometry V mapping the bulk effective field theory Hilbert space into the CFT Hilbert space, whose error-correcting properties require the QES formula to hold.



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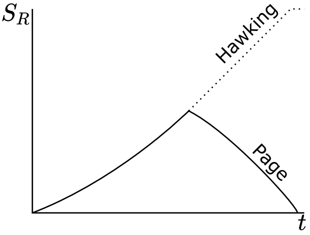

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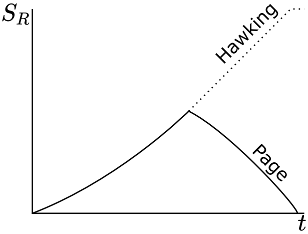
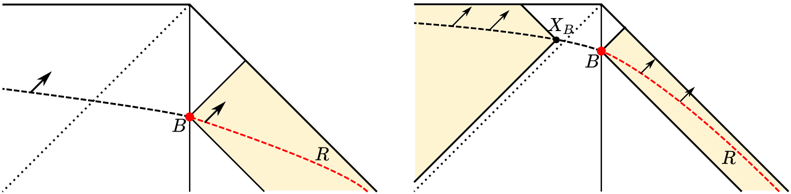


It gives a sharp test of whether or not the black hole S-matrix is unitary.

By now this calculation is well-known, so I'll just flash the picture:



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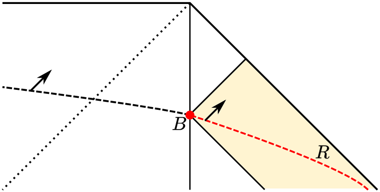

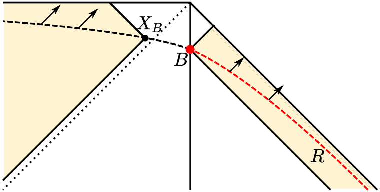

At early times the entanglement wedge of the Hawking radiation R just contains the Hawking radiation, so according to Hawking's picture of the bulk its entropy should grow with time. Eventually however it is advantageous to include an "island" in the black hole interior, which contains modes which purify the Hawking radiation entropy and whose generalized entropy also has a term proportional to the horizon area. This gives the decreasing part of the curve!

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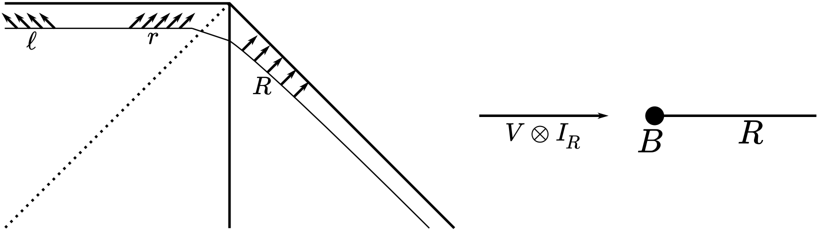
In the last few minutes I'll briefly describe some recent work which we are optimistic gives answers to these questions. Akers/Engelhardt/Harlow/Penington/Vardhan

An encoding map for the BH interior

The idea is to look for a holographic encoding map $V : \mathcal{H}_\ell \otimes \mathcal{H}_r \to \mathcal{H}_B$ on a nice slice:



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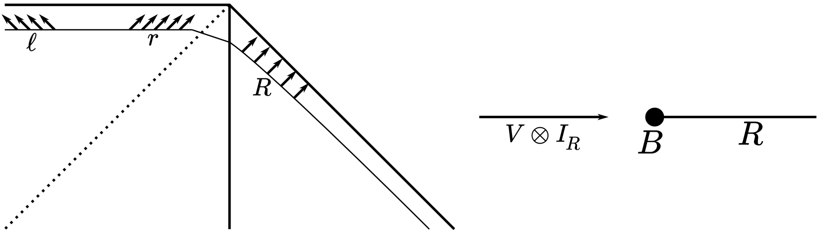

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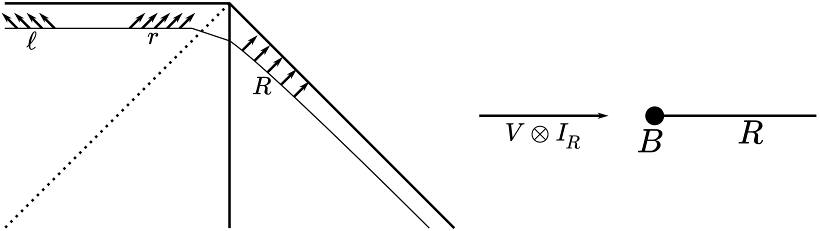

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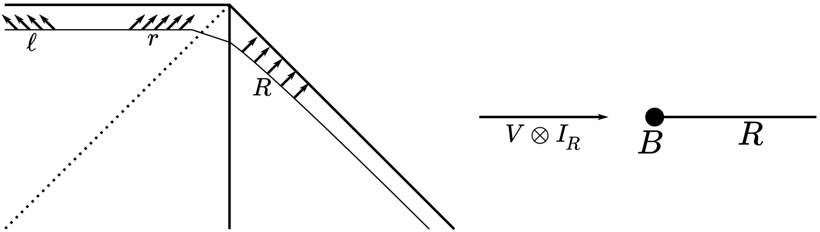

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We argue this is the key distinction between Page and Hawking!

In more detail the essence of our proposal is the following:

There is a large set of "null states" in the Hilbert space of effective field theory inside a black hole, each of which is annihilated by the holographic map to the fundamental degrees of freedom. This however cannot be detected by any observer who does not perform an operation of exponential complexity.

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In the previous language we advocate the following replacement:

~~(3) EFT valid wherever there is not a large energy density/curvature.~~

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Indeed we can construct models where appropriate analogous of (1), (2), and (3*) are all manifestly true. They are thus compatible, and so if we are willing to accept (3*) then the information problem is resolved in these models.

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- Sub-exponential interior observables can be (non-linearly but unambiguously) reconstructed in the fundamental description, in agreement with the effective description up to $O(e^{-\gamma S_{BH}})$.

A static model

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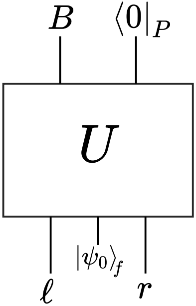
$V = \sqrt{|P|}$ 

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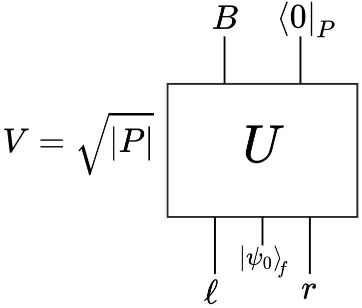

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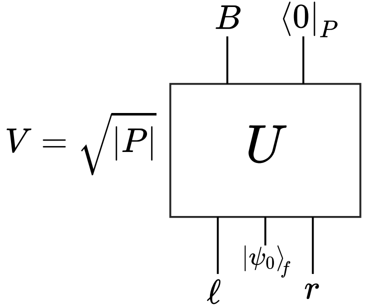

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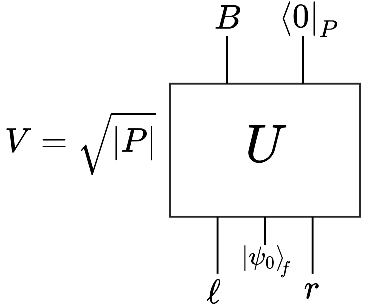

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The claimed results follow from standard unitary integration technology.

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  ("Evolve then encode" equals "encode then evolve")
- We would like to model the full dynamical process proposed by Hawking: collapse some matter to form a black hole, and then watch it evaporate and see if the process is unitary.

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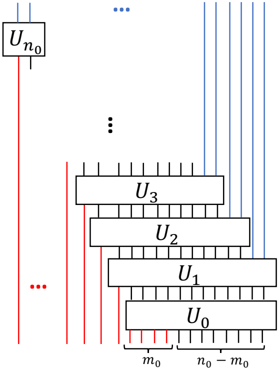

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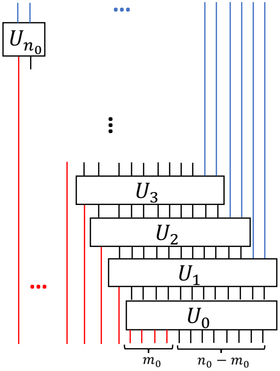

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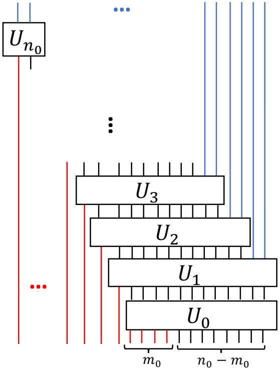

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- At each time t we then act with a random unitary U_{t+1} which absorbs one ingoing qudit from the reservoir and radiates two outgoing qudits.
- This model clearly has a finite BH entropy and a unitary S-matrix.

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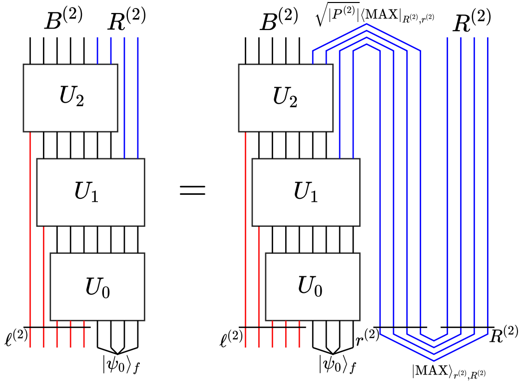
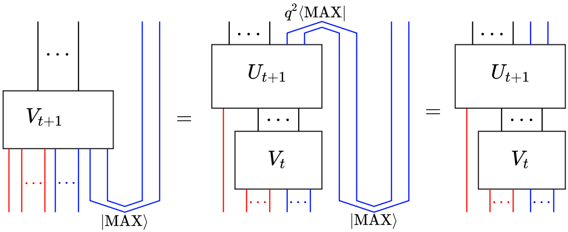


By "bending around" the outgoing modes using post-selection, we can re-interpret that fundamental dynamics as a non-isometric holographic map $V \otimes I_R$ acting on a (maximally-entangled) Hawking state in the effective description!

This map is easily shown to be equivariant:



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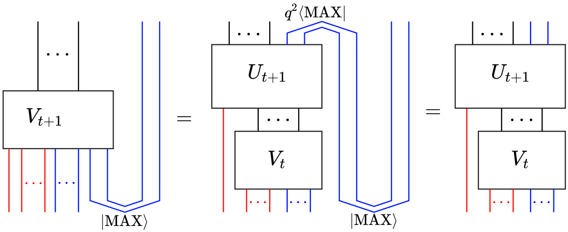

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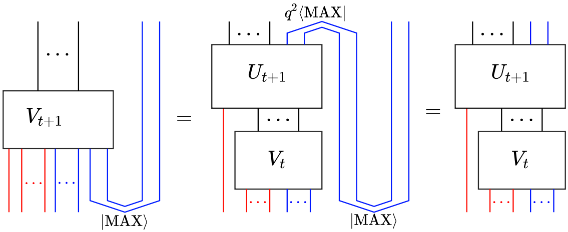

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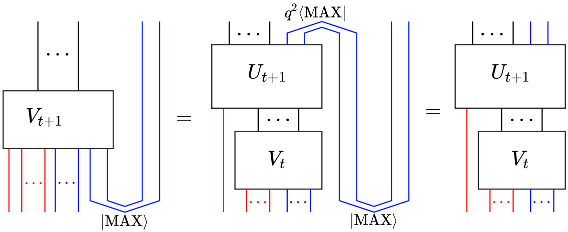

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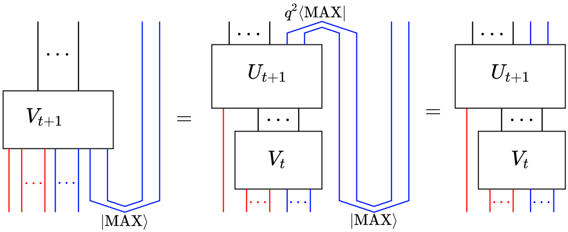

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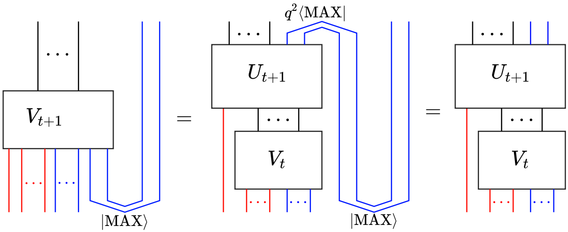

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The Hayden-Preskill scrambling argument can also be checked in a more refined version of the model where the U_t are less random.

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It does *not* use Euclidean gravity in any way, which I view as a major advantage (no black box), but it does give results which agree with it where appropriate (i.e. the QES formula).

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I am optimistic that the encoding map V could be constructed fairly uniquely in AdS/CFT from the "extrapolate" dictionary, but this would also require an assumption that the horizon is smooth for quick-collapse states. Ideally we could instead derive this from some non-perturbative version of string theory.