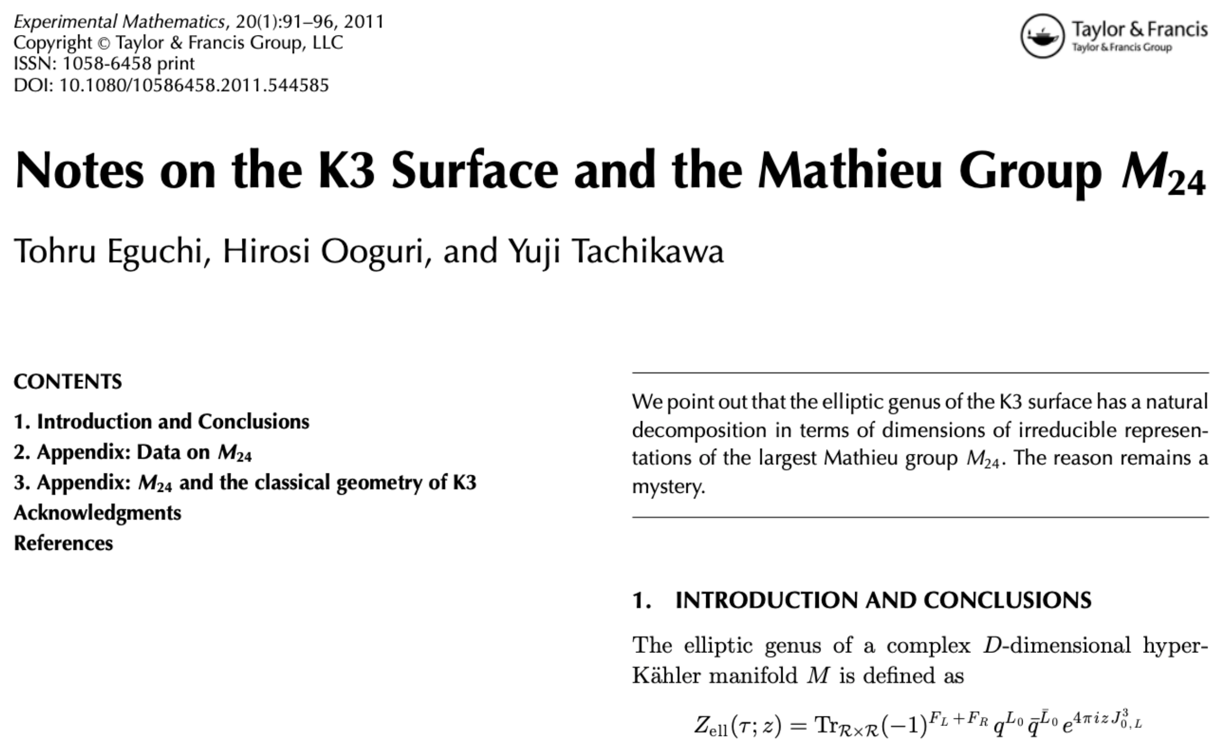
Top-down topological holography & twists on twistor space

HirosiFest, 2022

Natalie Paquette

It’s no exaggeration to say that every topic I’ve invested significant time on had its genesis in one of Hirosi’s papers

Moonshine



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Notes on the K3 Surface and the Mathieu Group M_{24}

Tohru Eguchi, Hirosi Ooguri, and Yuji Tachikawa

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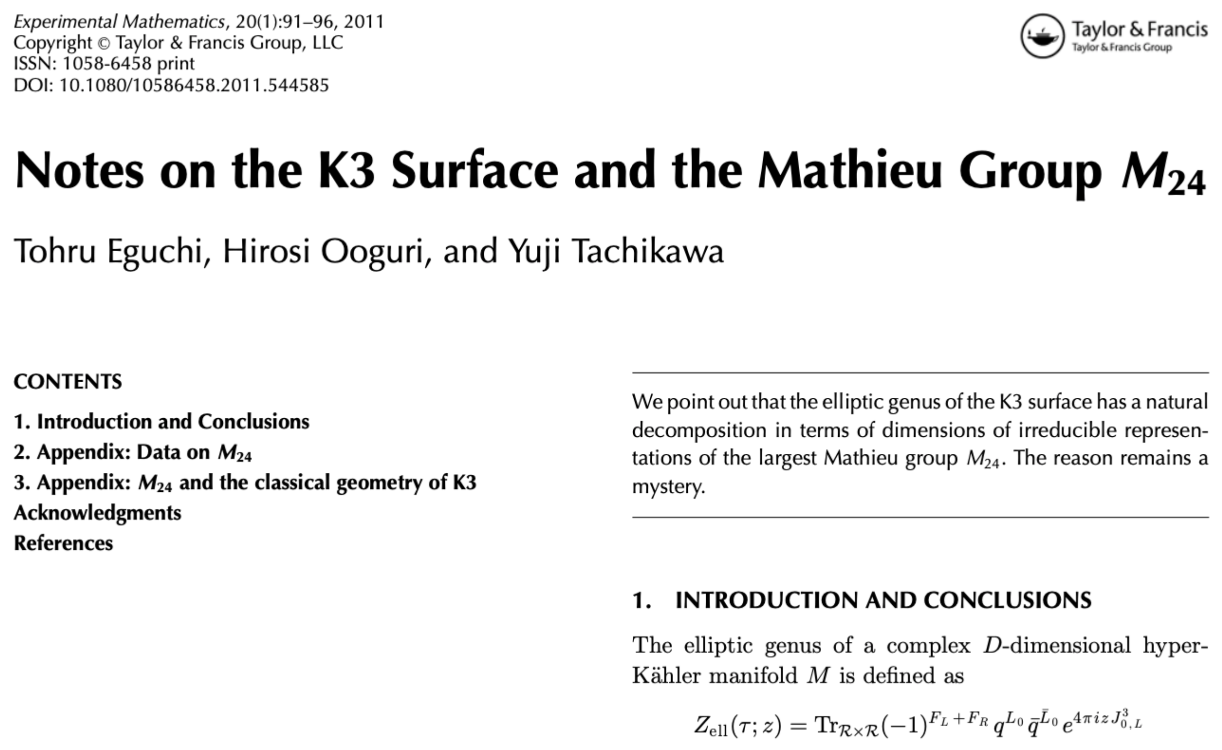

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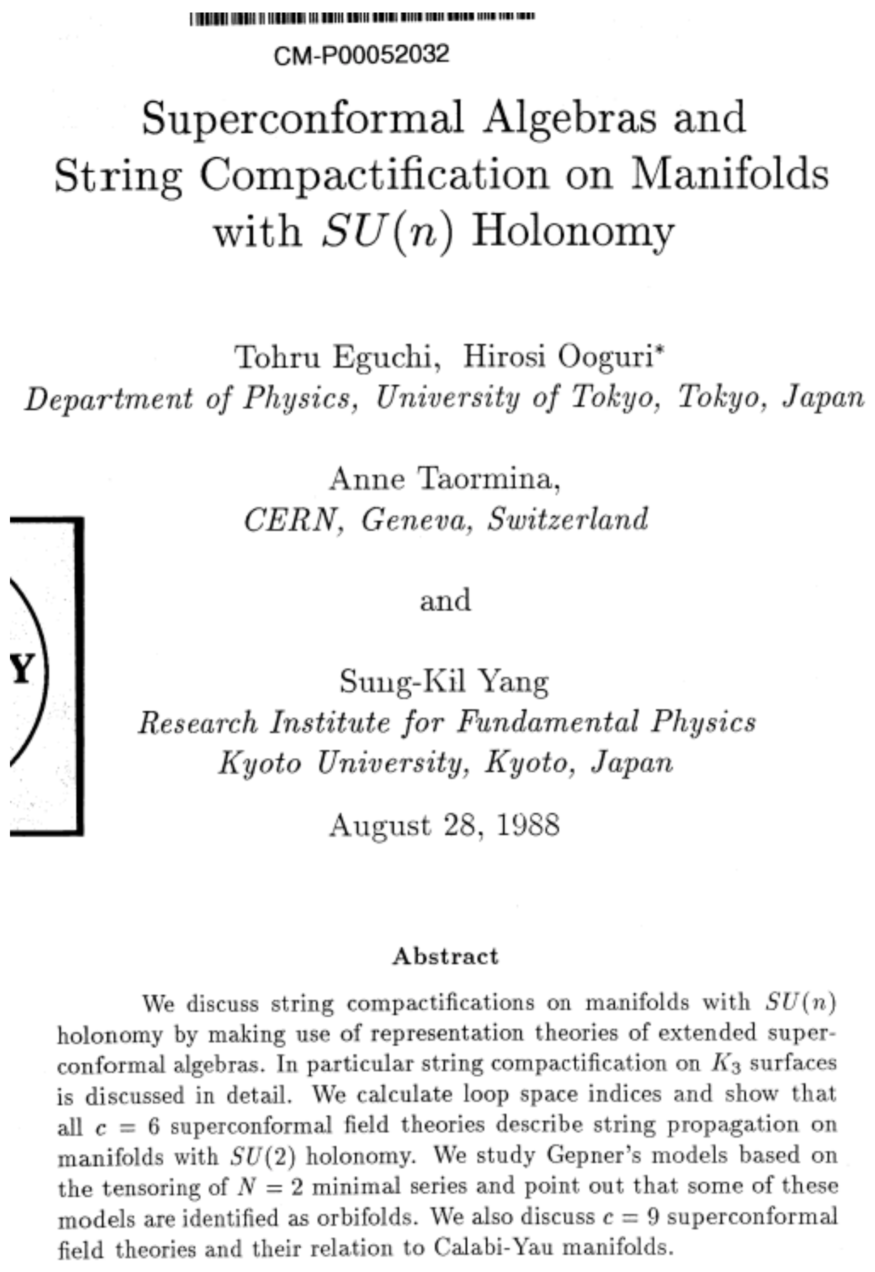

CM-P00052032

Superconformal Algebras and String Compactification on Manifolds with $SU(n)$ Holonomy

Tohru Eguchi, Hirosi Ooguri*
Department of Physics, University of Tokyo, Tokyo, Japan

Anne Taormina,
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and

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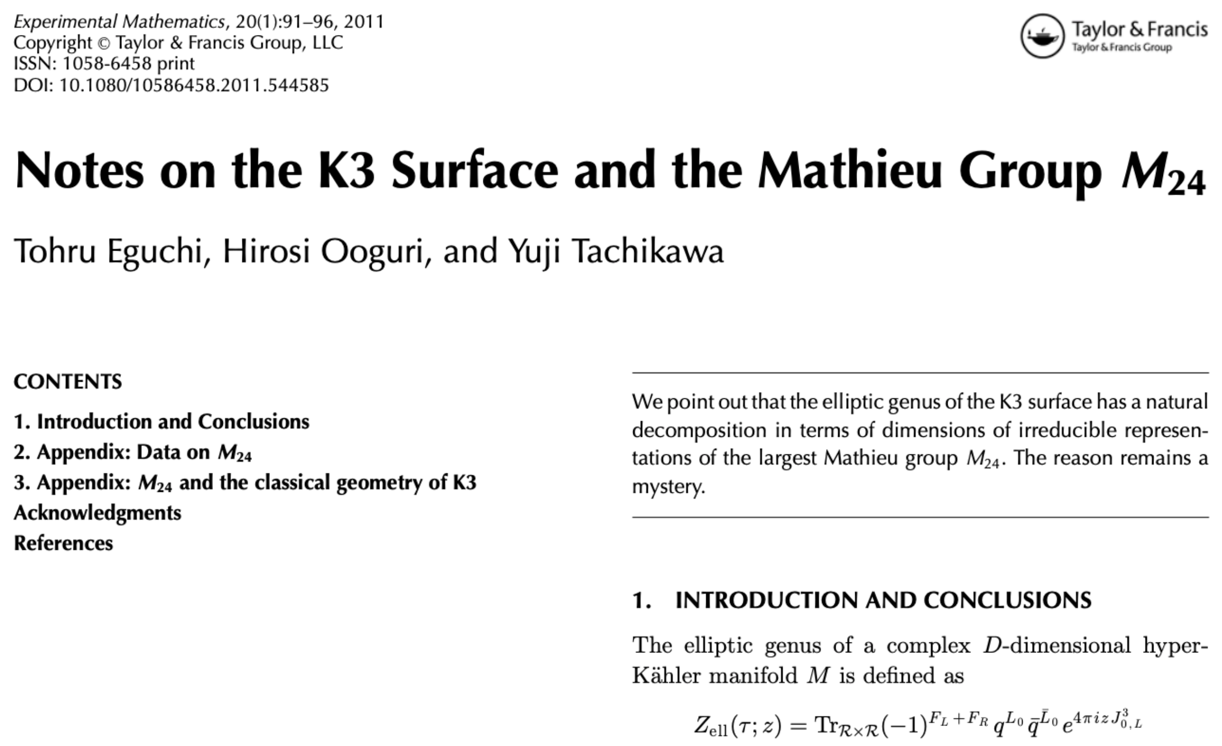

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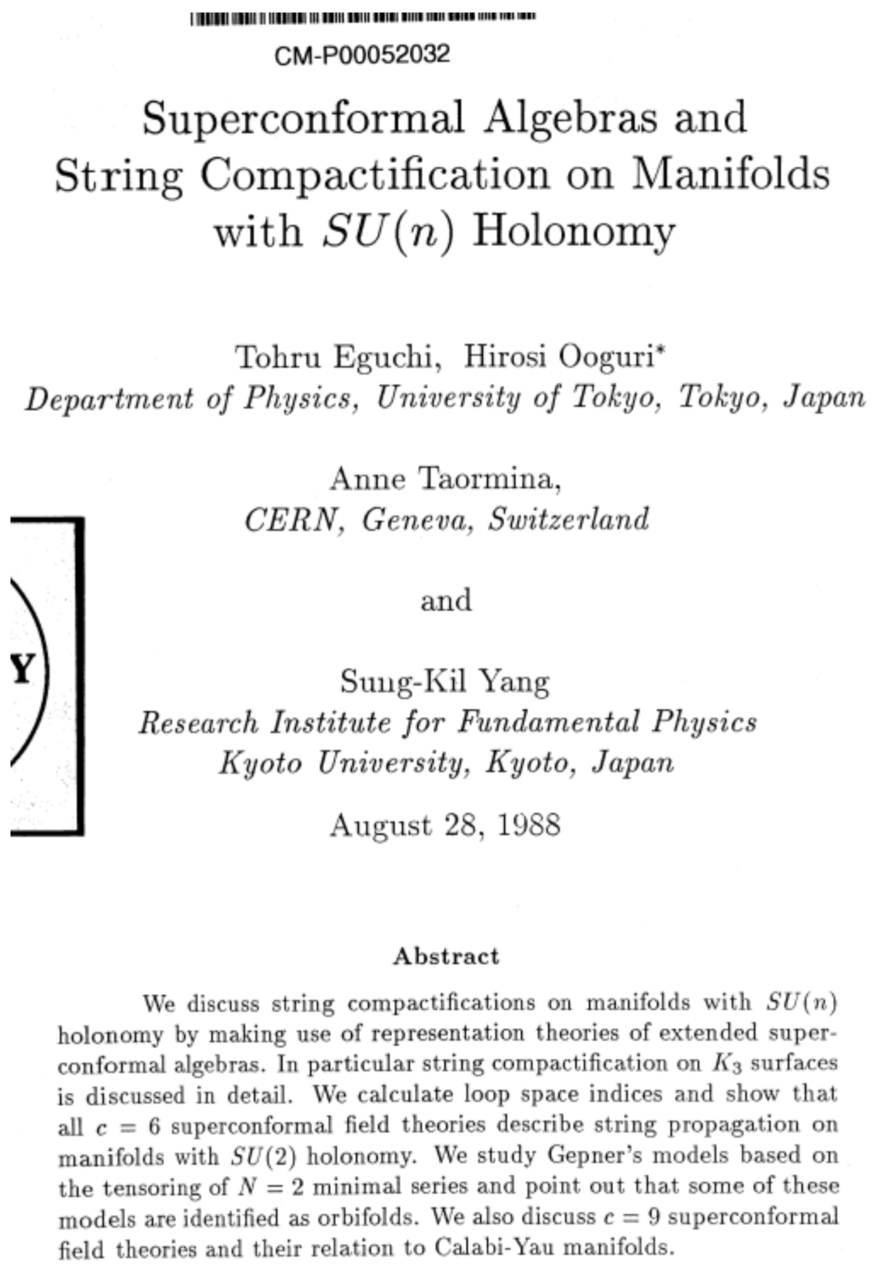

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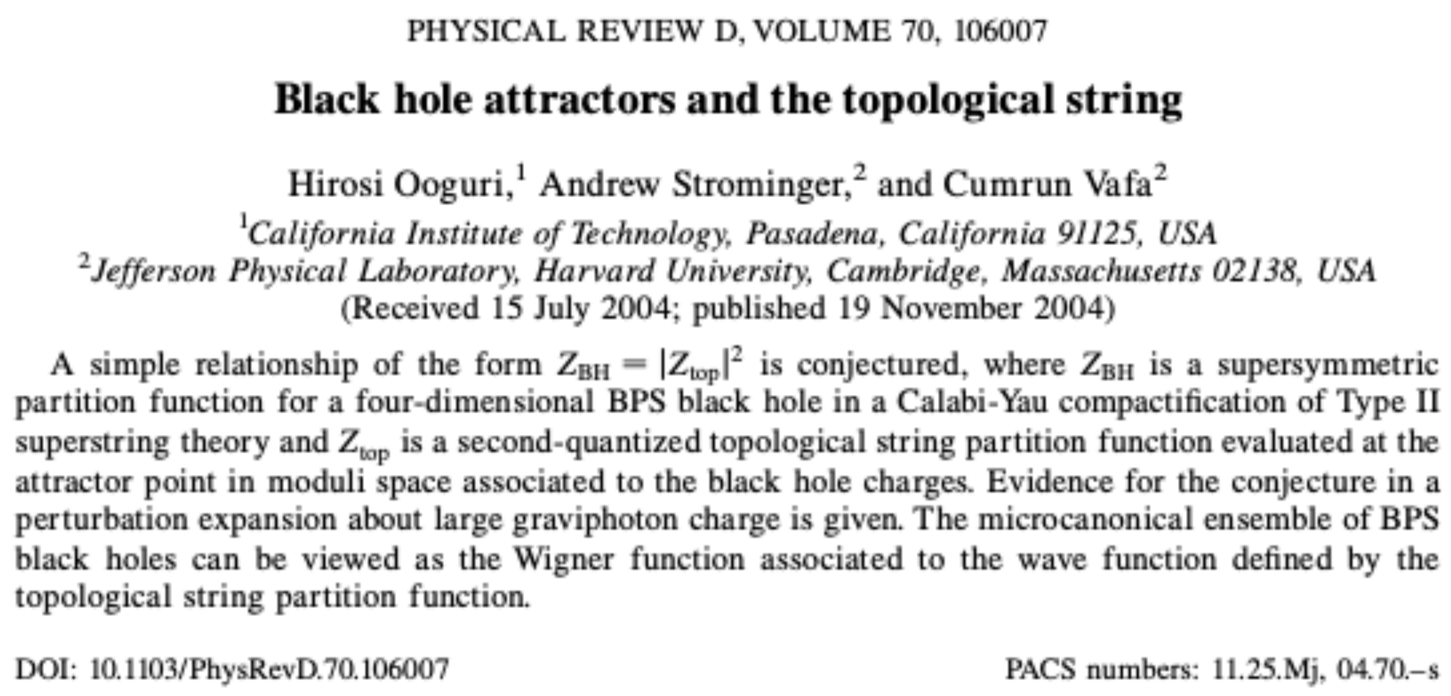

PHYSICAL REVIEW D, VOLUME 70, 106007

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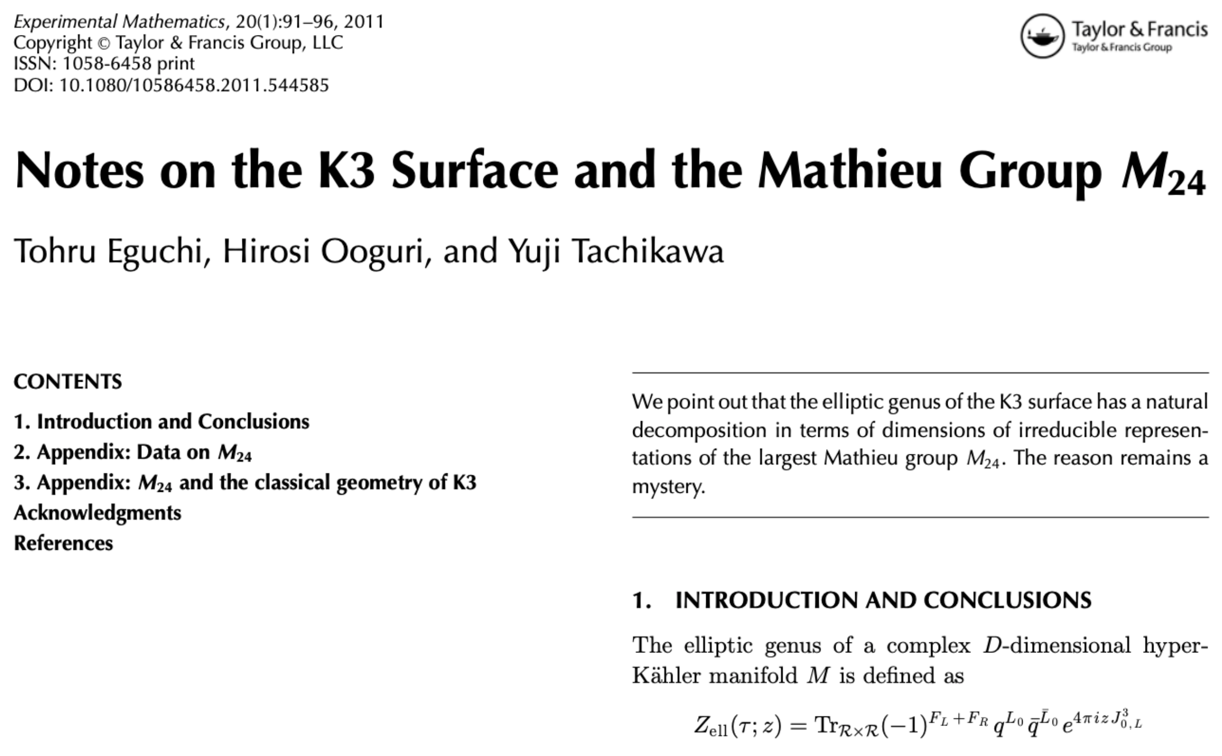

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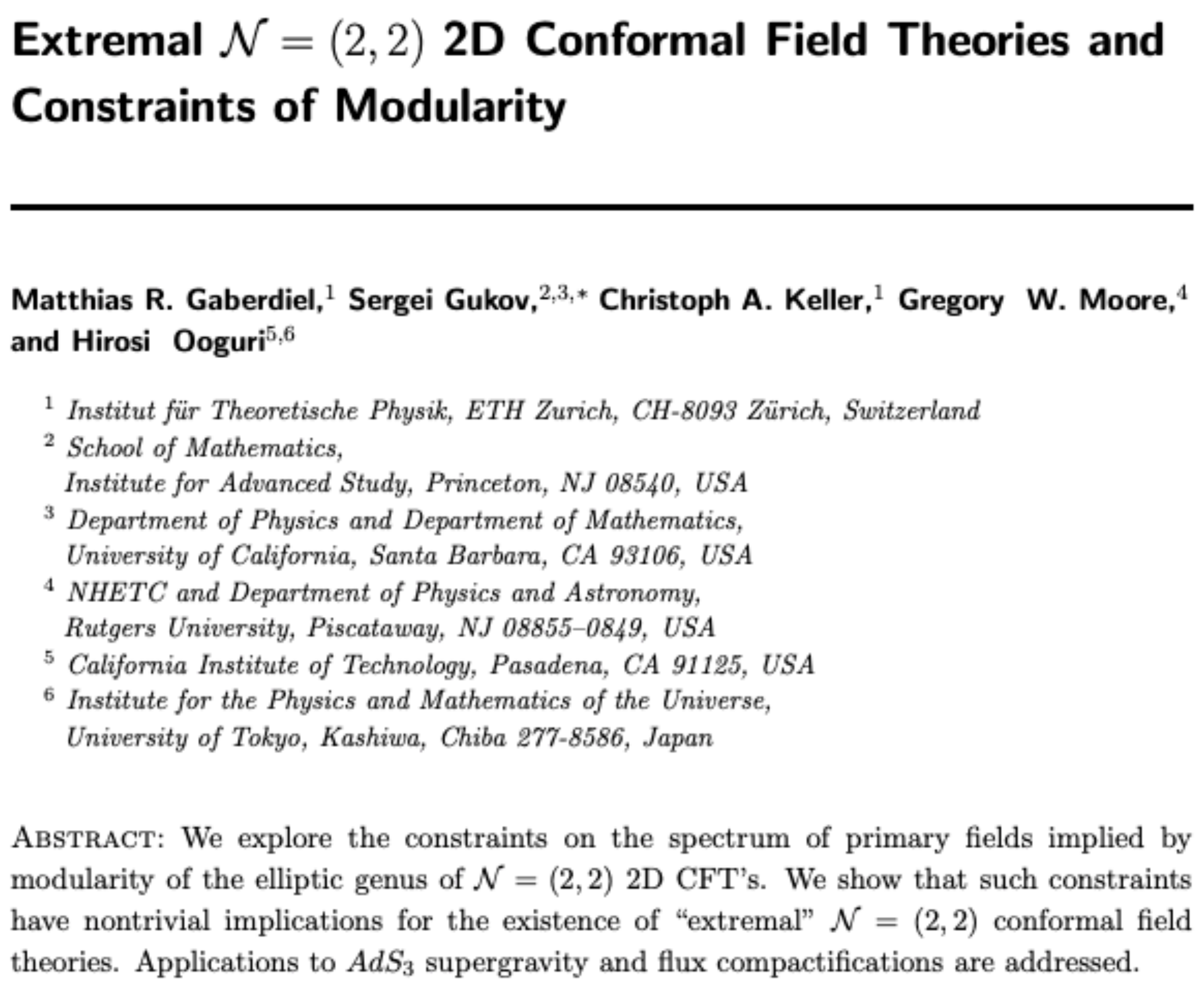

Extremal $\mathcal{N}=(2,2)$ 2D Conformal Field Theories and Constraints of Modularity

Matthias R. Gaberdiel,[1] Sergei Gukov,[2,3,*] Christoph A. Keller,[1] Gregory W. Moore,[4] and Hirosi Ooguri[5,6]

[1] *Institut für Theoretische Physik, ETH Zurich, CH-8093 Zürich, Switzerland*
[2] *School of Mathematics,*
Institute for Advanced Study, Princeton, NJ 08540, USA
[3] *Department of Physics and Department of Mathematics,*
University of California, Santa Barbara, CA 93106, USA
[4] *NHETC and Department of Physics and Astronomy,*
Rutgers University, Piscataway, NJ 08855–0849, USA
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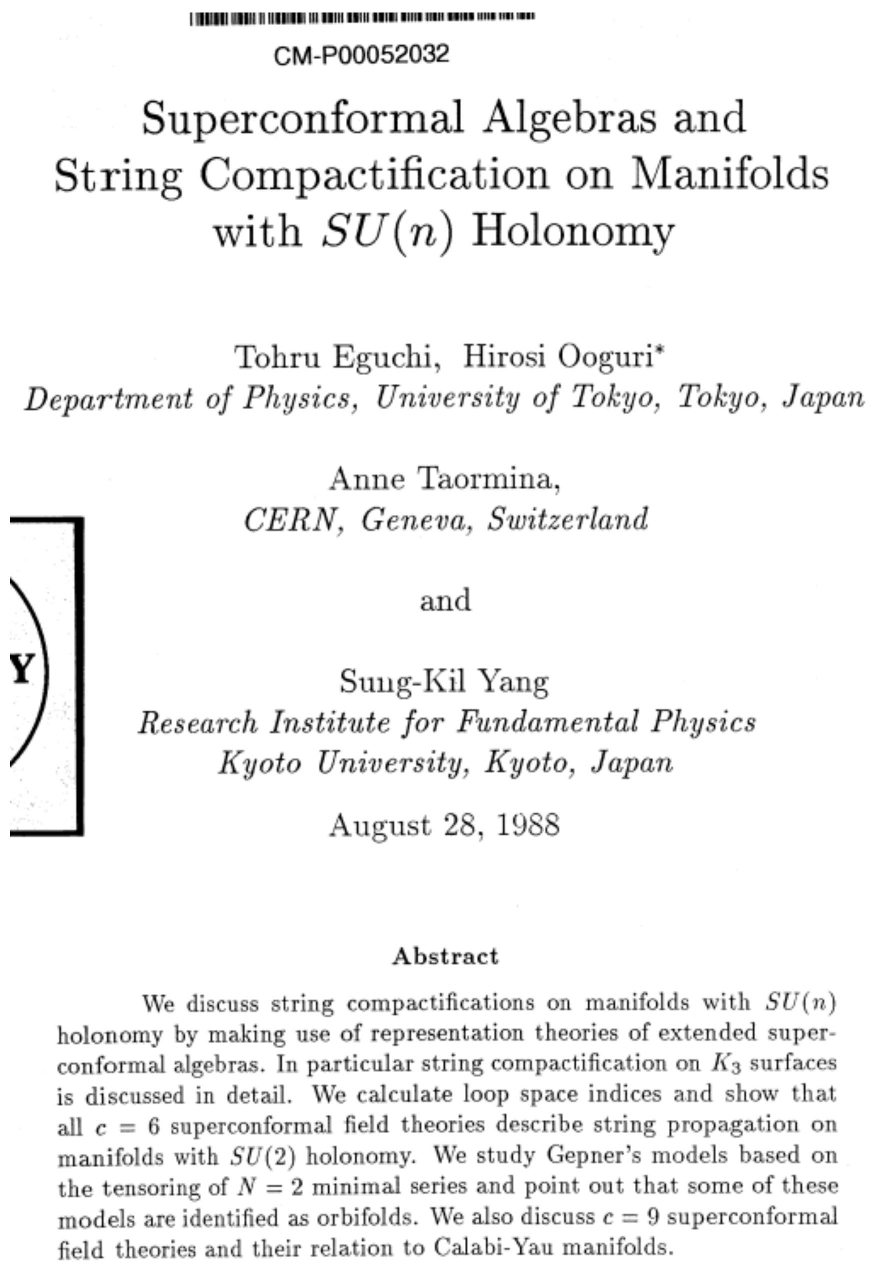

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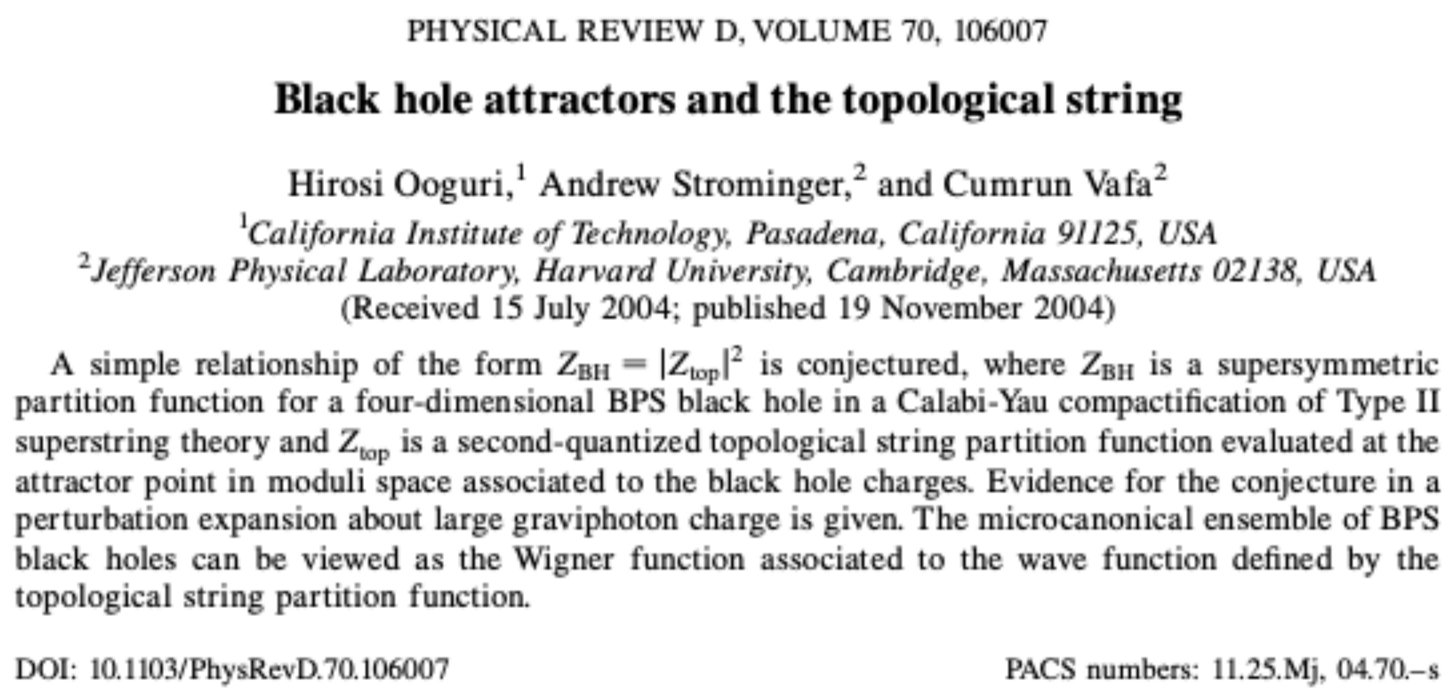

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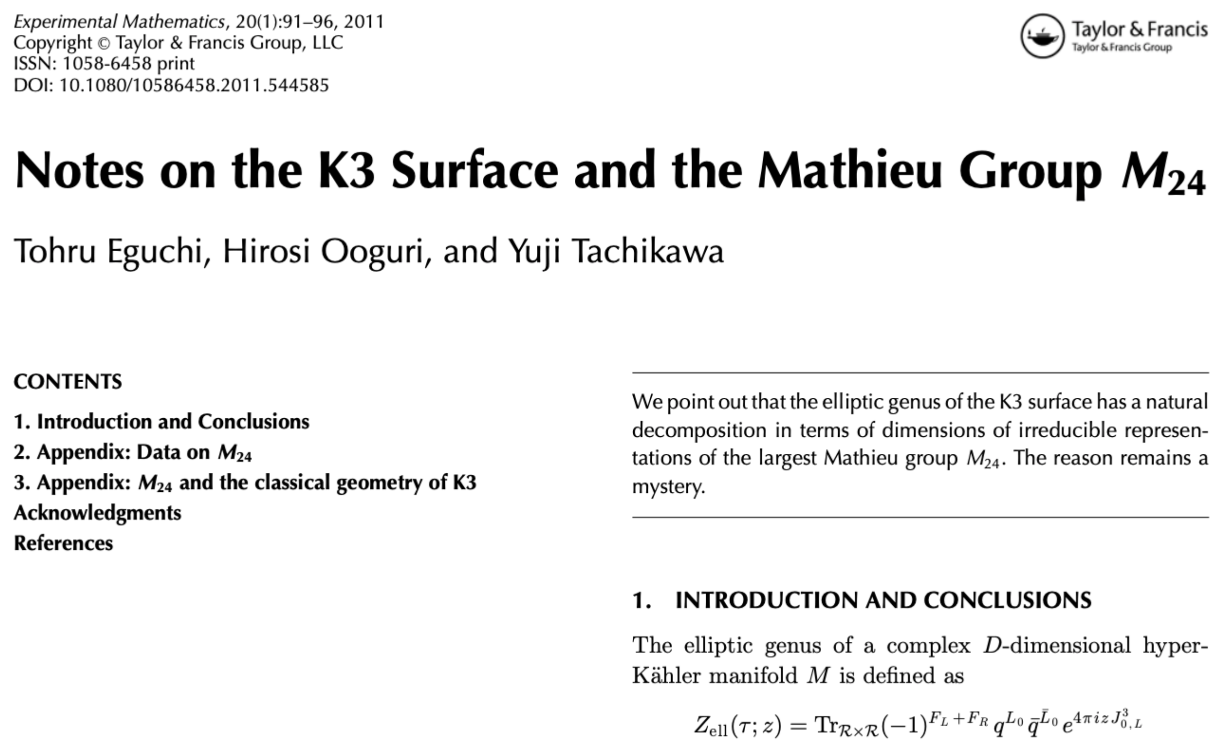

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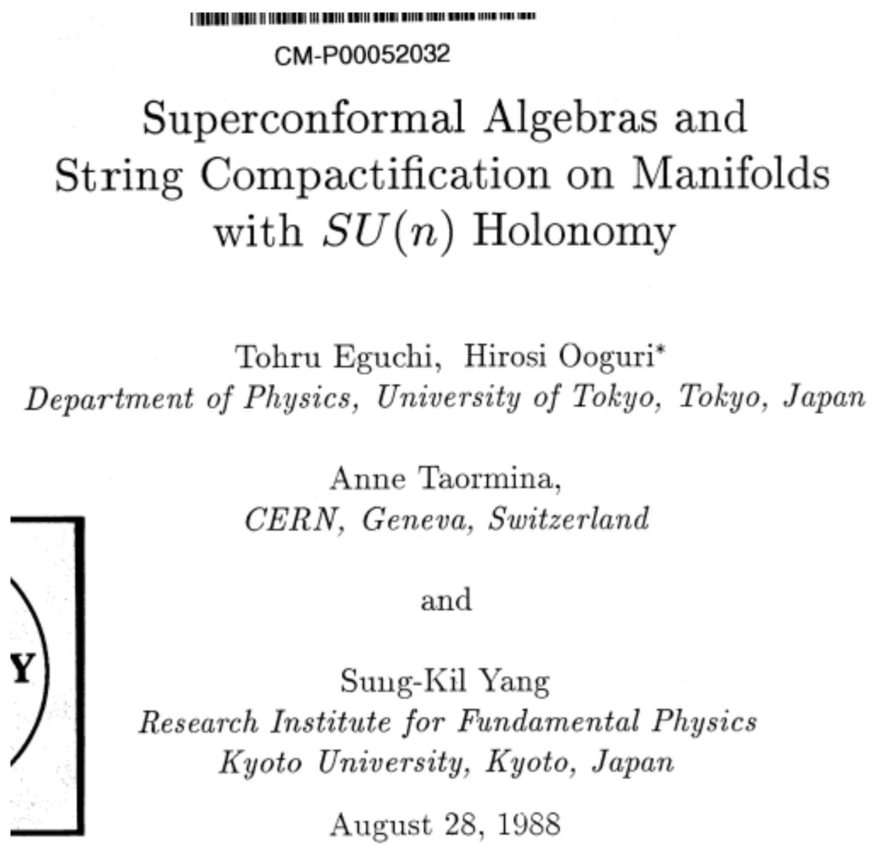

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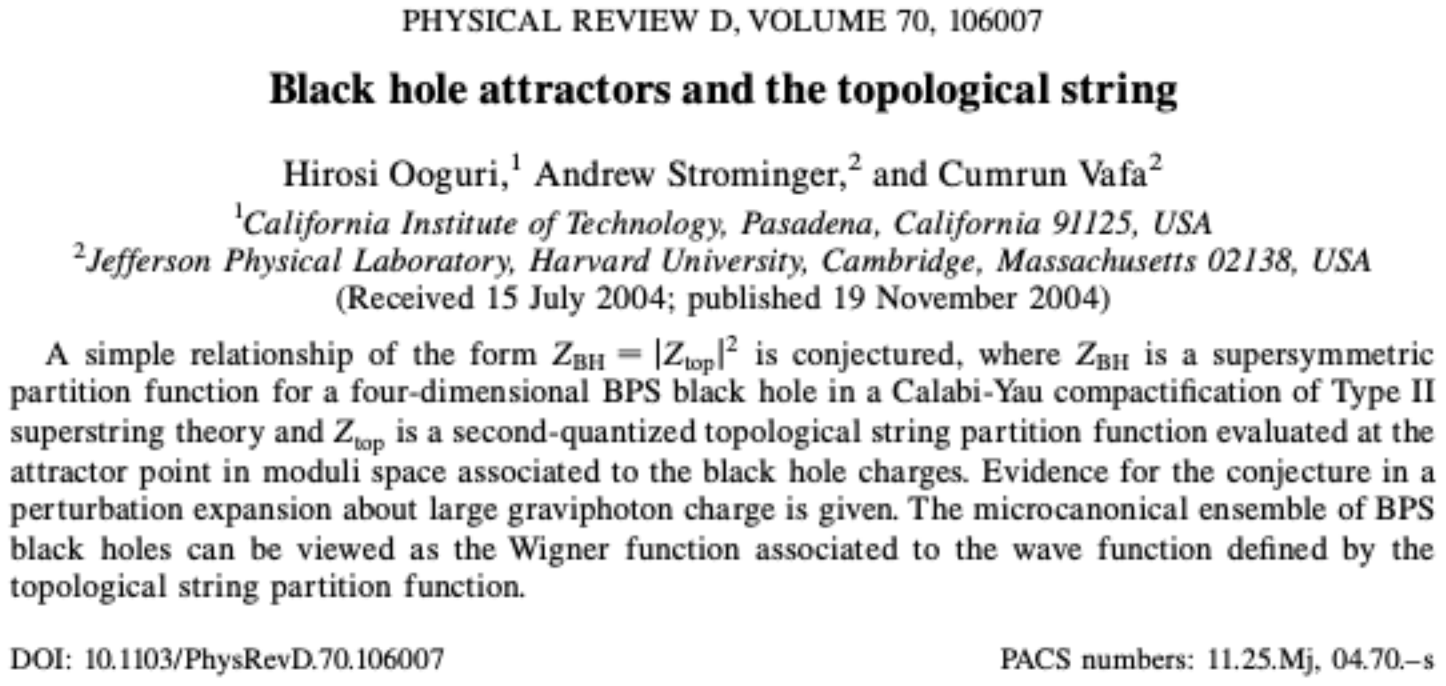

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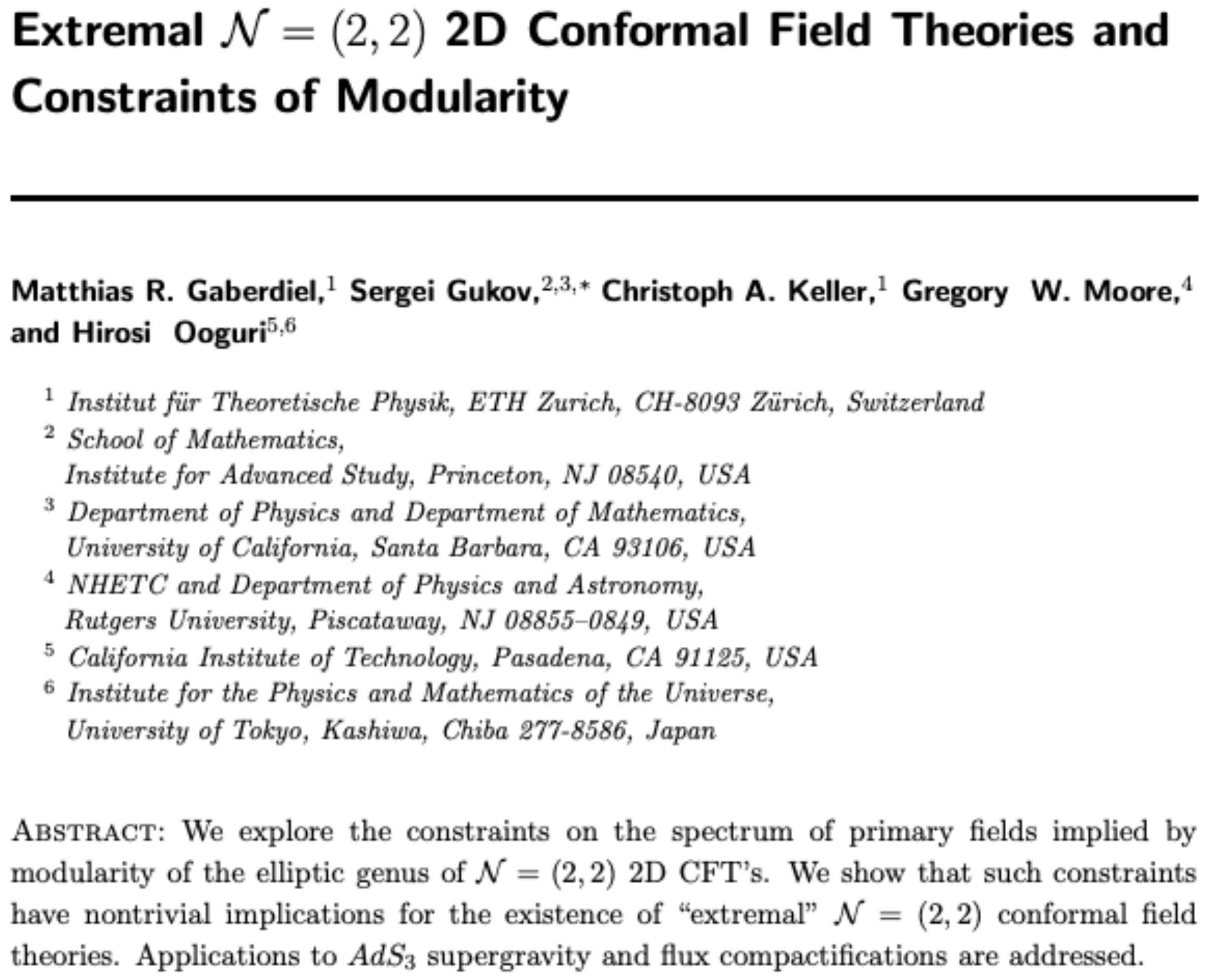

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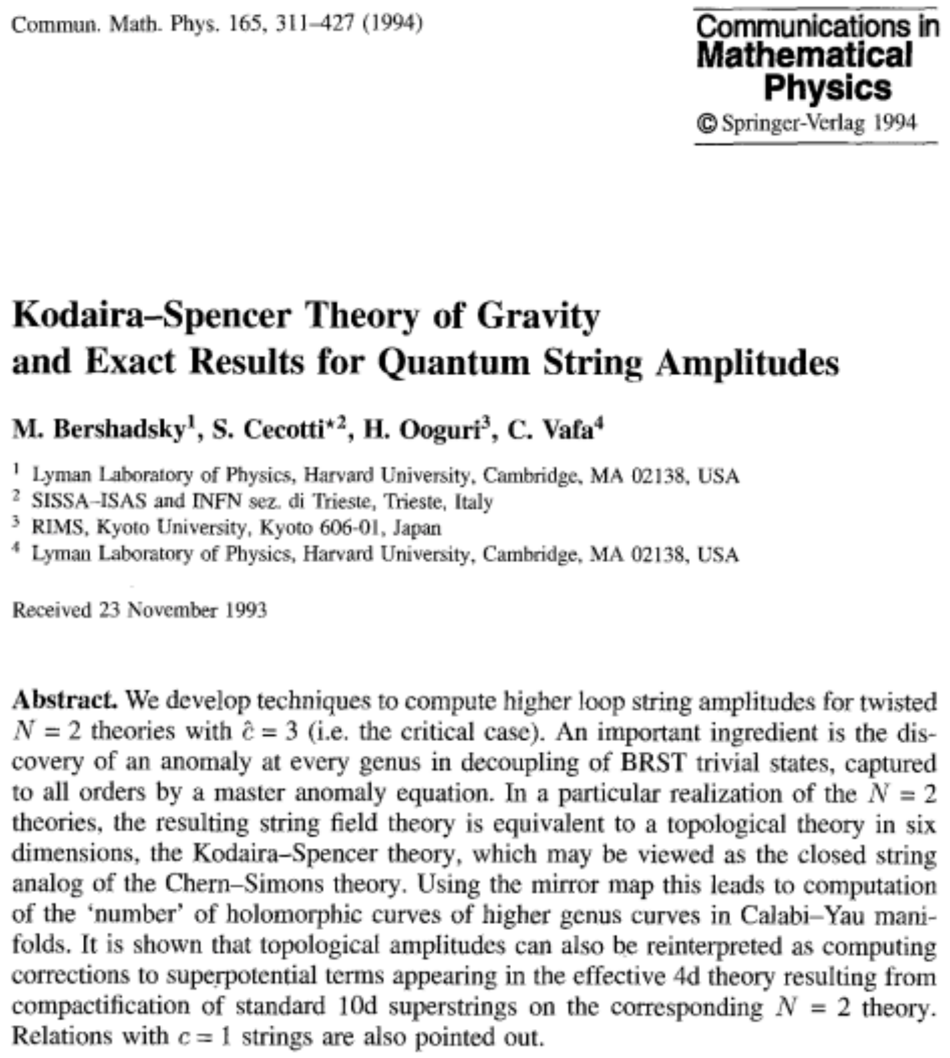

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Communications in
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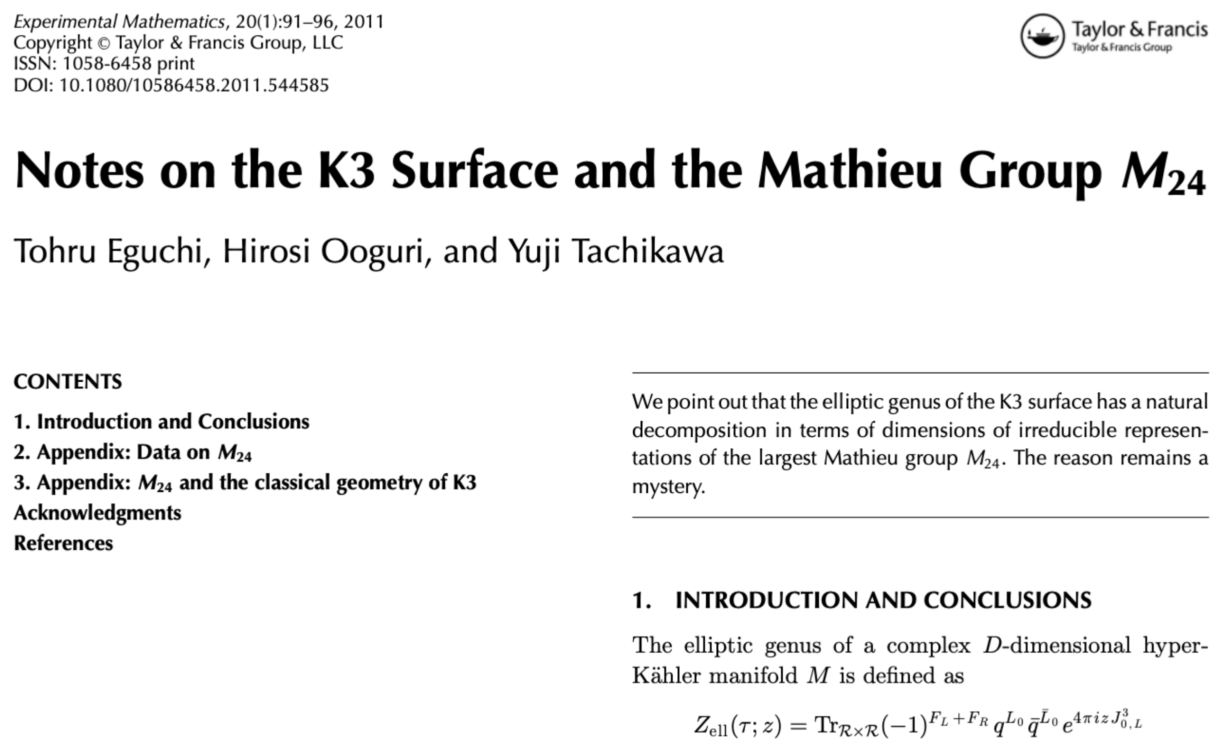
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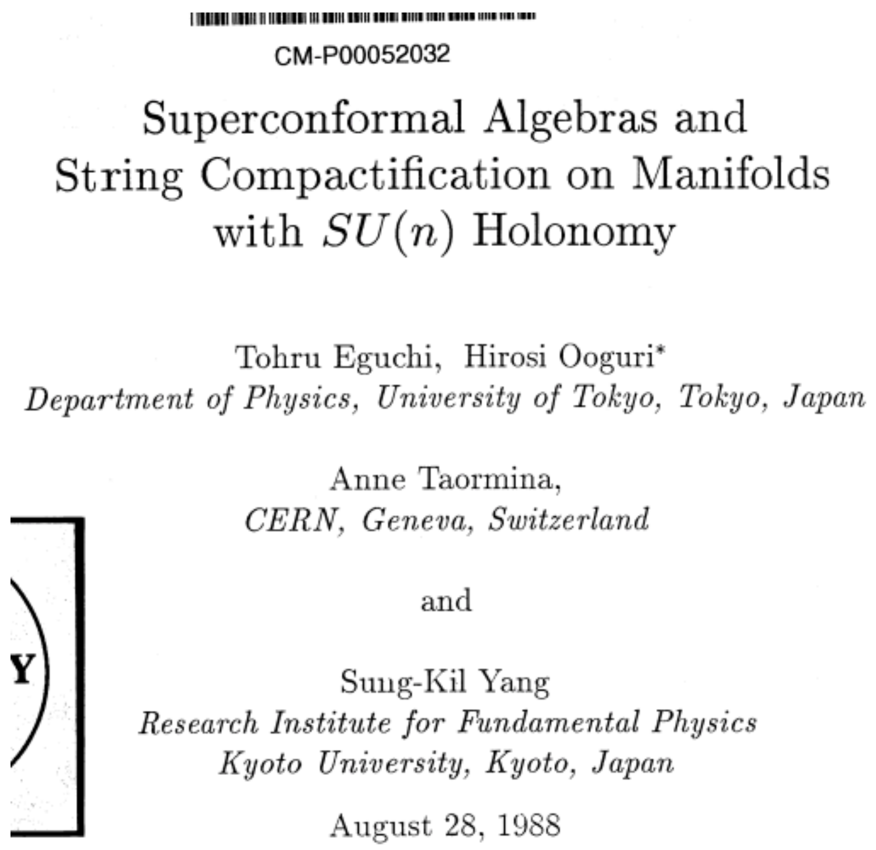
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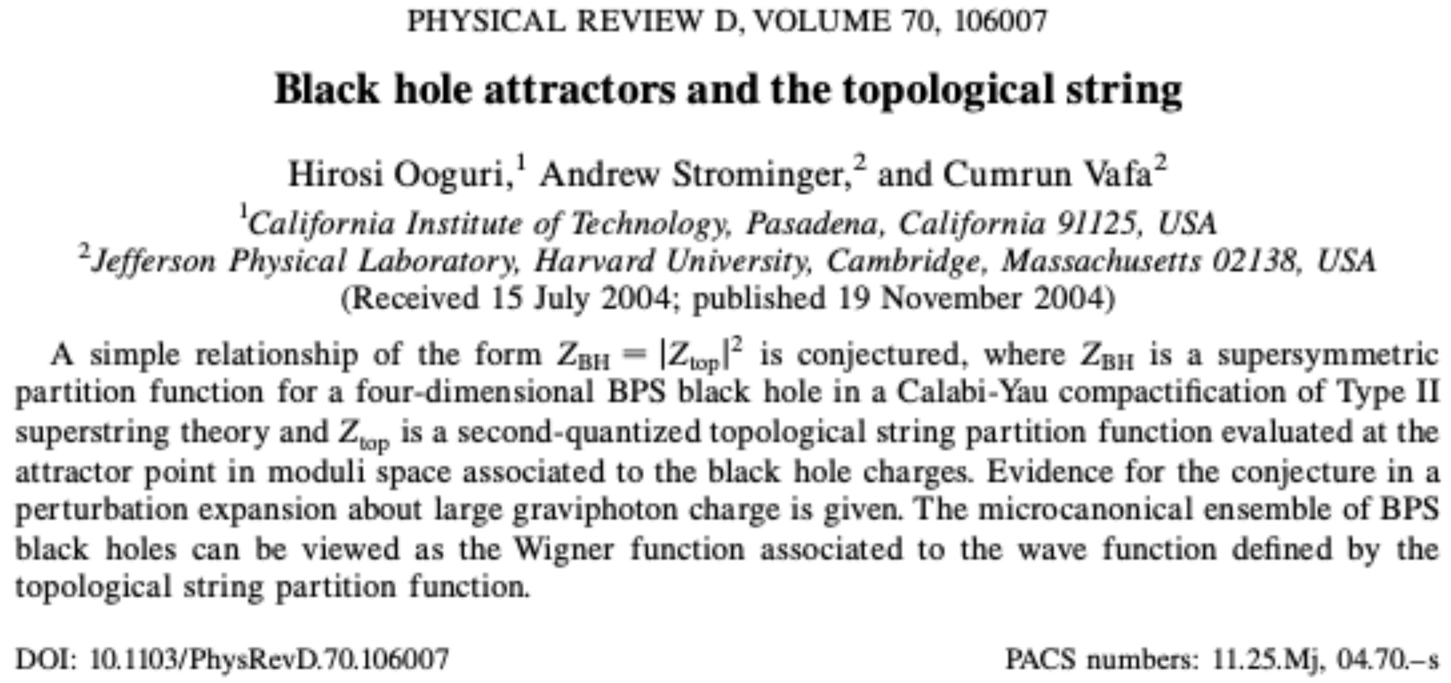
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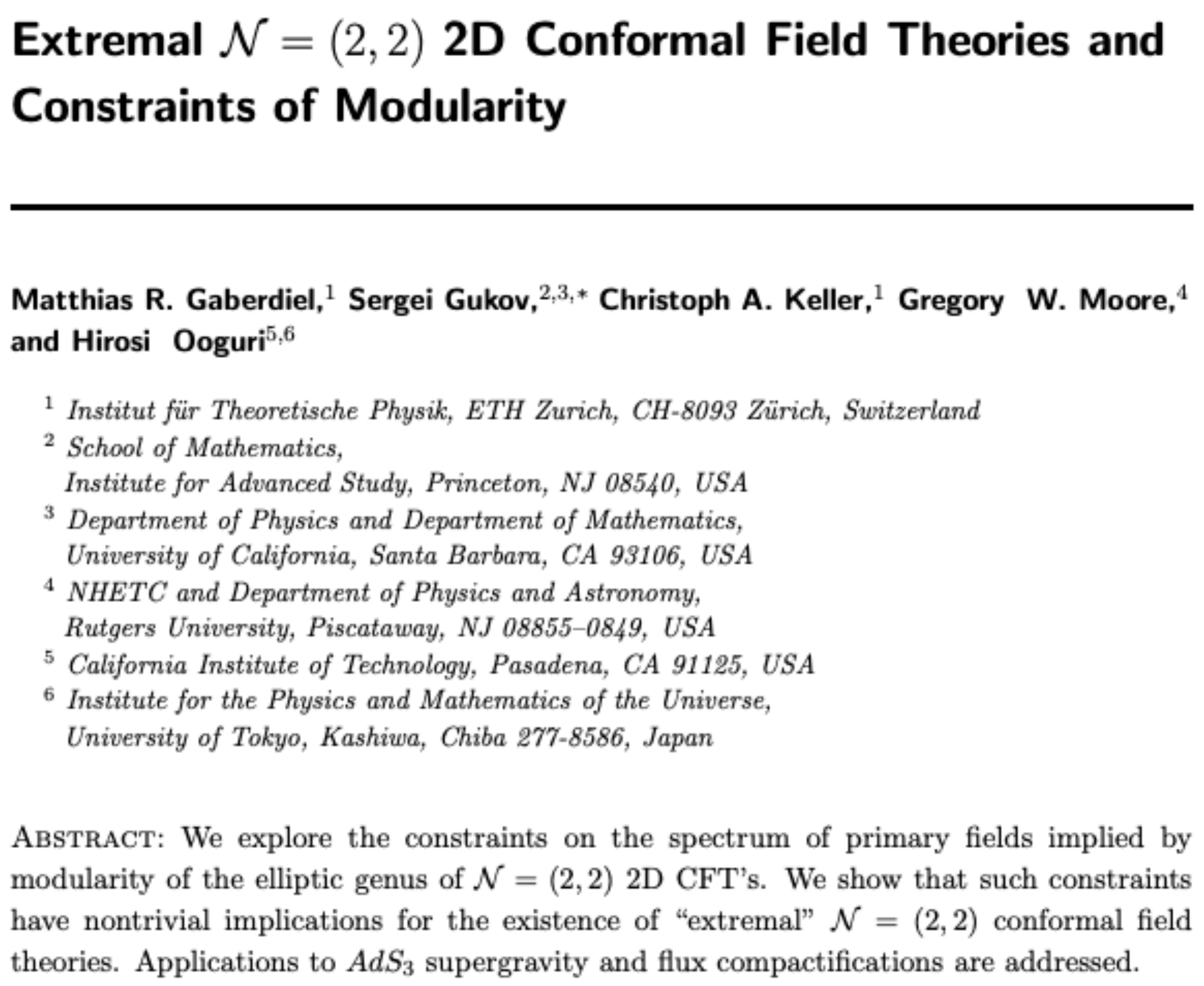
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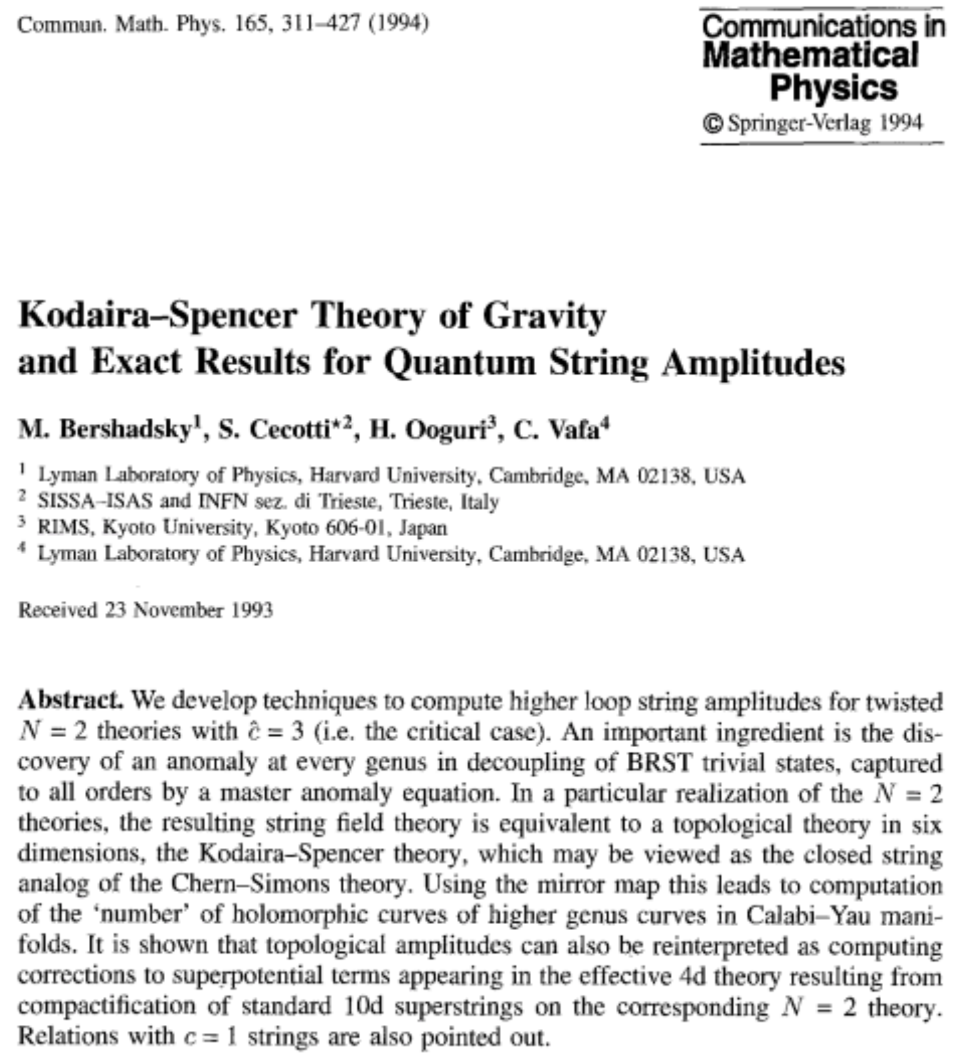
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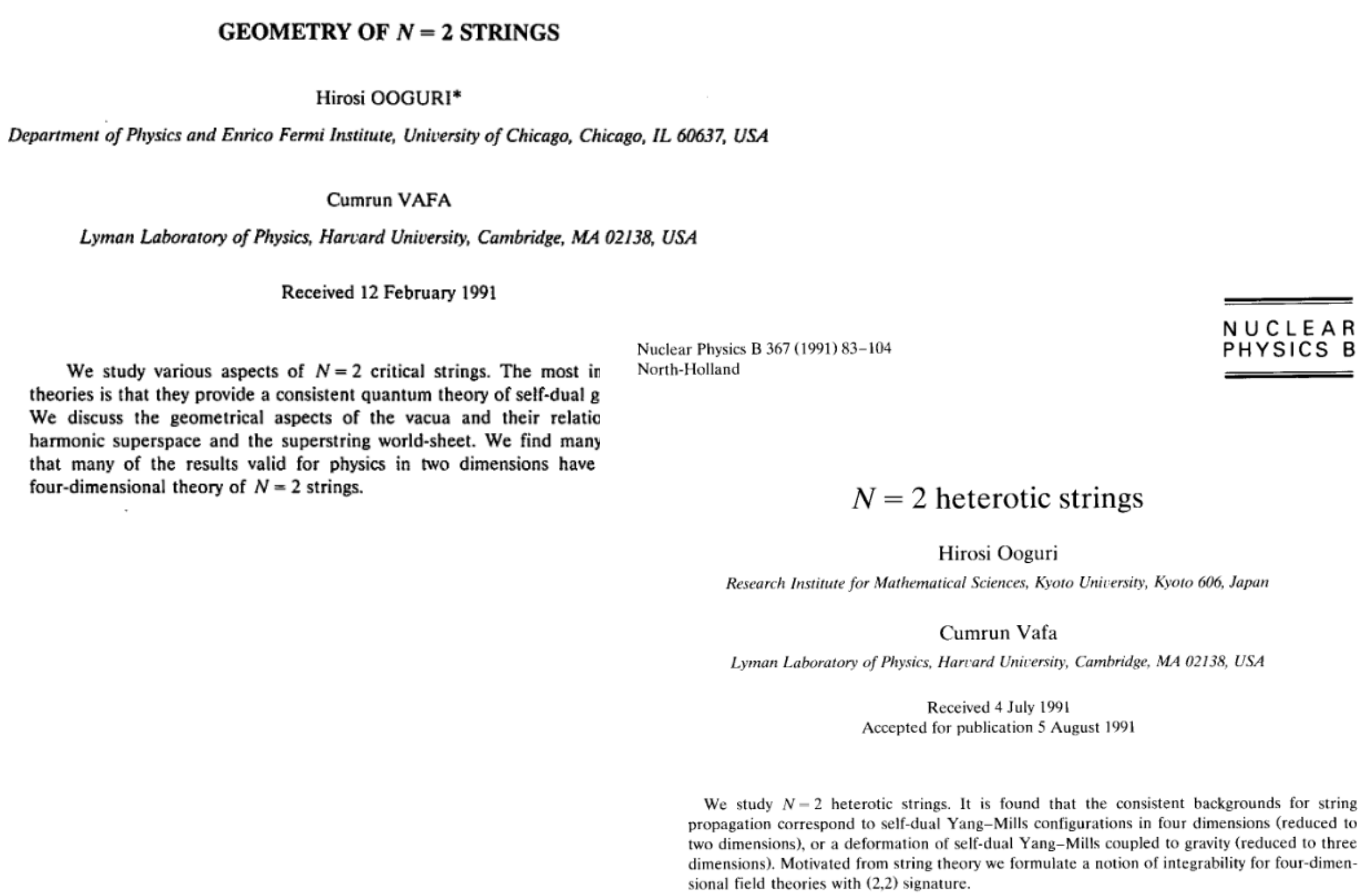
4d integrability, twistor space, self-duality, & strings


GEOMETRY OF $N = 2$ STRINGS

Hirosi OOGURI*
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Cumrun VAFA
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theories is that they provide a consistent quantum theory of self-dual g
We discuss the geometrical aspects of the vacua and their relatio
harmonic superspace and the superstring world-sheet. We find many
that many of the results valid for physics in two dimensions have
four-dimensional theory of $N = 2$ strings.

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NUCLEAR PHYSICS B

$N = 2$ heterotic strings

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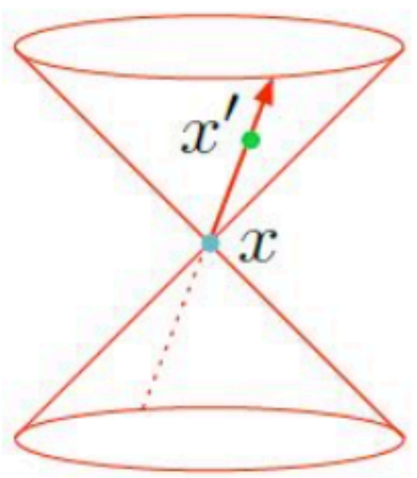
We study $N = 2$ heterotic strings. It is found that the consistent backgrounds for string propagation correspond to self-dual Yang–Mills configurations in four dimensions (reduced to two dimensions), or a deformation of self-dual Yang–Mills coupled to gravity (reduced to three dimensions). Motivated from string theory we formulate a notion of integrability for four-dimensional field theories with (2,2) signature.

Therefore the only suitable birthday present for Hirosi is to present the answer to the Mathieu moonshine observation he made, which was the beginning of my career in physics…

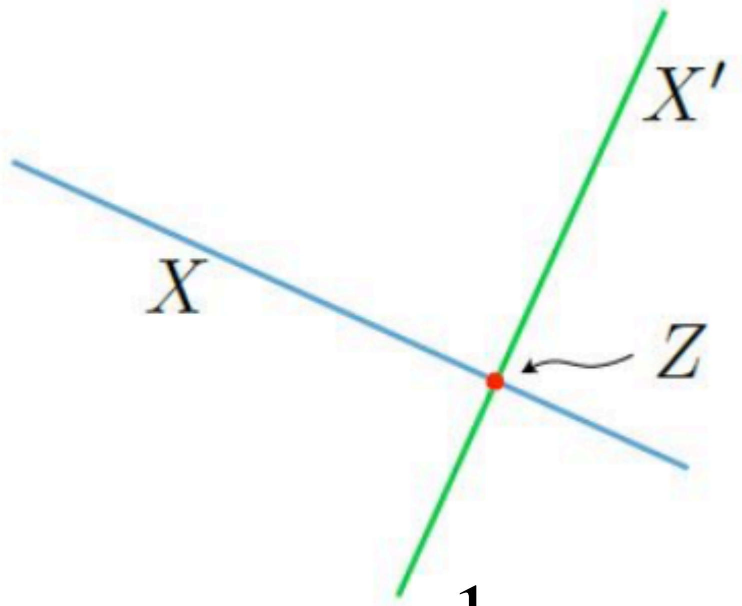
…but I'm afraid this is the best I can do at the moment

In Vino [moonshine], Veritas

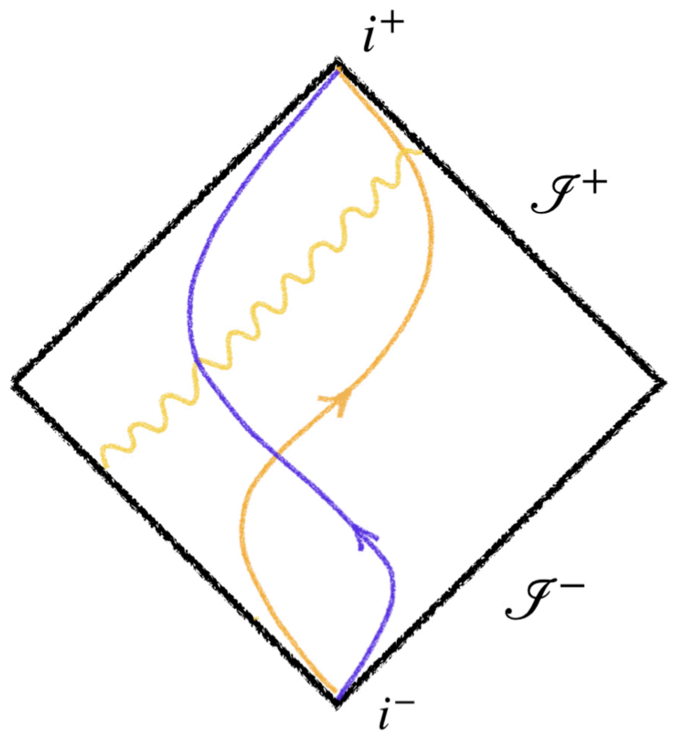
Today I’d like to discuss work on connections between 4d physics and 2d chiral algebras



$x \in \mathbb{C}^4$



$\mathbb{CP}^1_x \in \mathbb{PT}$

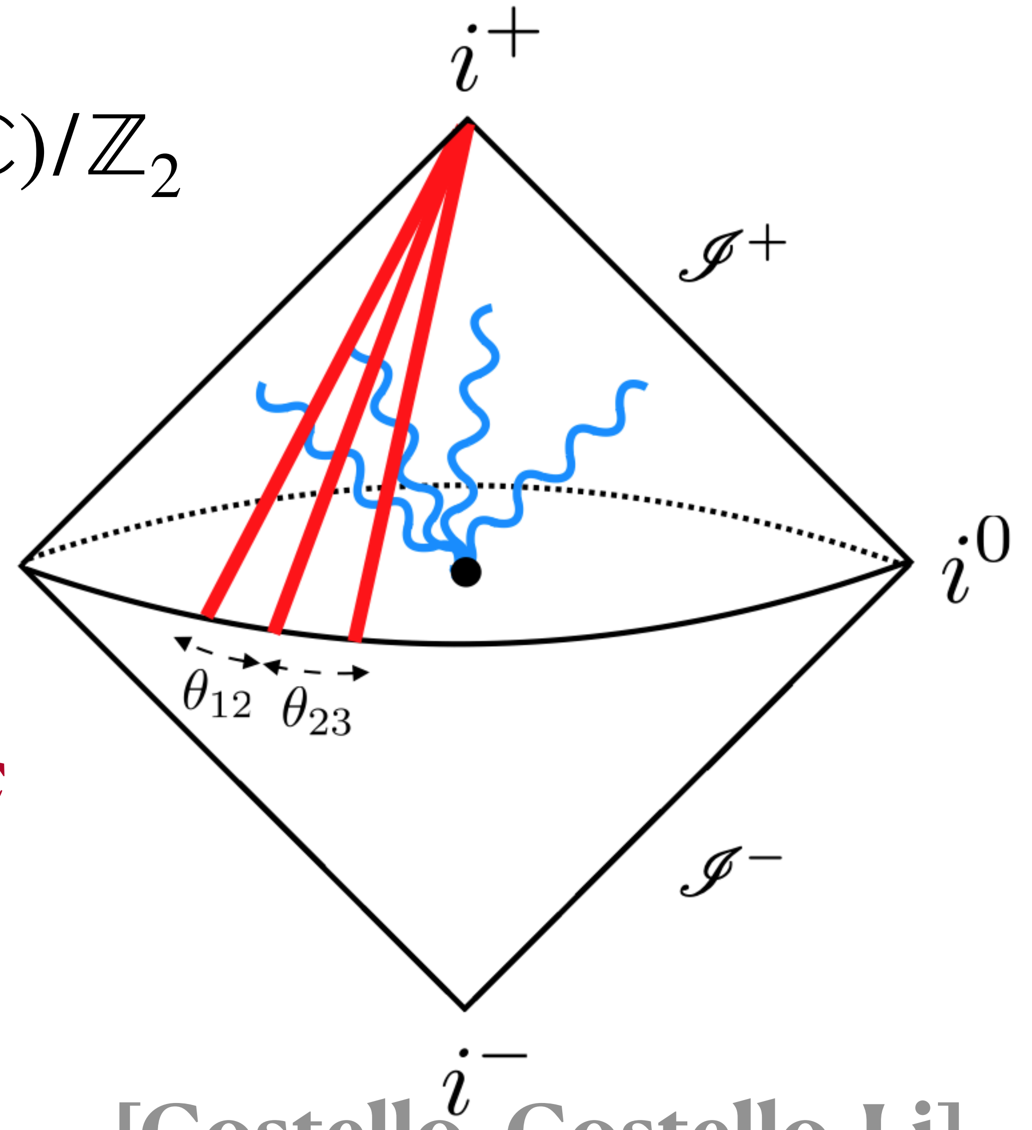


$SO^+(1,3) \simeq SL(2,\mathbb{C})/\mathbb{Z}_2$

[Guevara-Himwich-Pate-Strominger]

[Strominger]

chiral algebras of asymptotic symmetries at tree-level



[Costello, Costello-Li]

In work with Costello (2204.05301, 2201.02595), we showed that if a 4d theory admits a lift to a local holomorphic theory on twistor space, a chiral algebra can also control collinear singularities in its scattering amplitudes at loop-level

$$\begin{tikzcd}
& \mathbb{PT} = \mathcal{O}(1) \oplus \mathcal{O}(1) \simeq \mathbb{R}^4 \times \mathbb{CP}^1 \arrow[d] \\
z \in \mathbb{CP}^1 & \mathbb{CP}^1
\end{tikzcd}$$

Failures of associativity in the chiral algebra at the quantum level are tied to gauge anomalies in twistor space
The 4d theory isn’t inconsistent: this is like an obstruction to integrability

We focused on self-dual Yang-Mills, coupled to an axion with a quartic kinetic term
Similar considerations apply to self-dual gravity, or to SD SU(Nc) YM w/ Nf=Nc flavors.

$$\int_{\mathbb{PT}} \mathrm{Tr}(\mathscr{B}\mathscr{F}^{(0,2)}(\mathscr{A})) \mapsto \int_{\mathbb{R}^4} \mathrm{Tr}(BF(A)\_)$$

[Costello, Costello-Li]

$\mathscr{B} \in \Omega^{3,1}(\mathbb{PT}, \mathfrak{g})$
$\mathscr{A} \in \Omega^{0,1}(\mathbb{PT}, \mathfrak{g})$

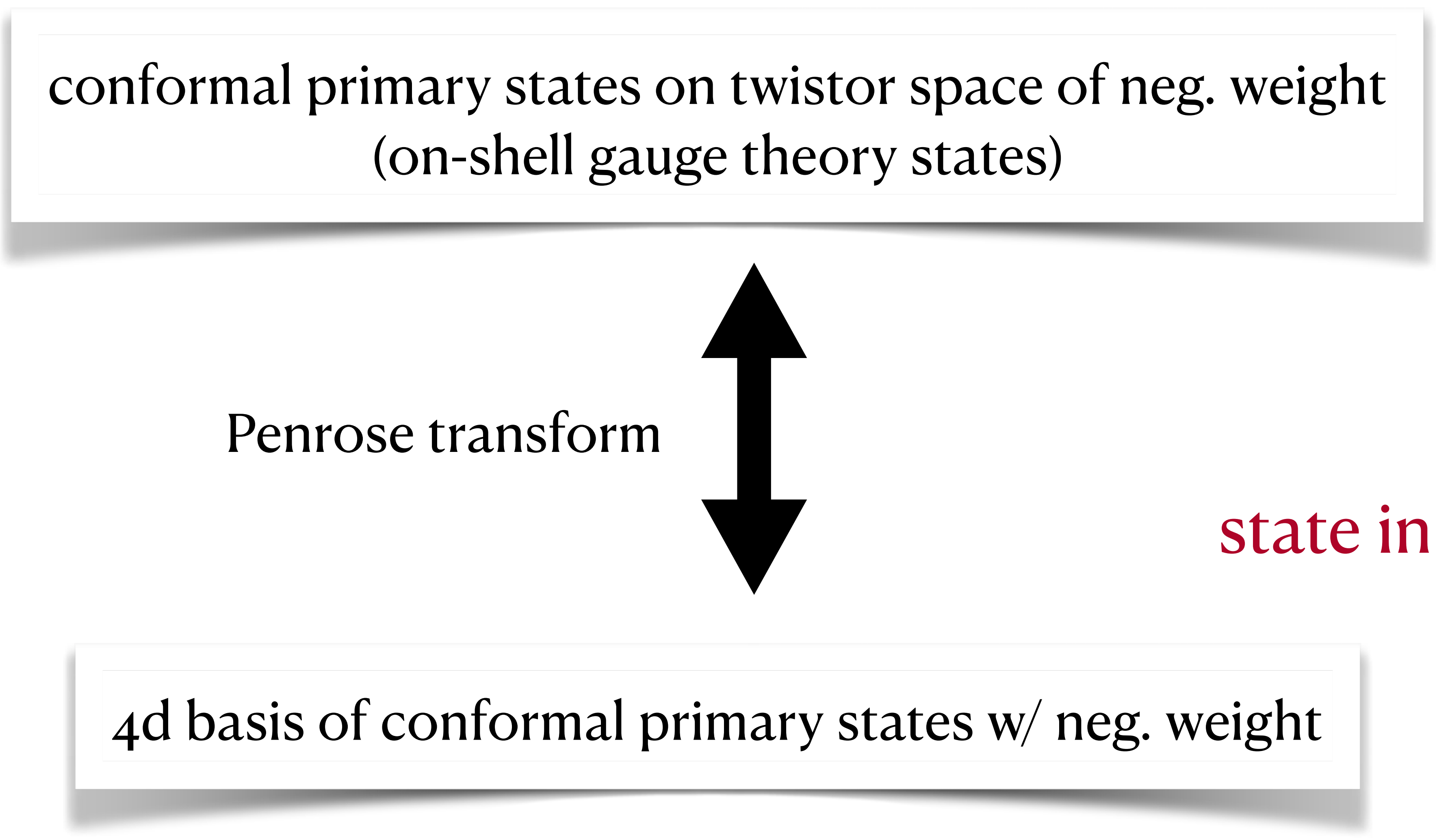
$B \in \Omega^2_-(\mathbb{R}^4, \mathfrak{g})$

$\mathfrak{g} = su(2), su(3), so(8), e_{6,7,8}$

$$\frac{1}{2}\int(\partial^{-1}\eta)(\bar{\partial}\eta) + k\hat{\lambda}_g \int \eta \mathrm{tr}(\mathscr{A}\partial\mathscr{A}) \mapsto \frac{1}{2}\int(\Delta\rho)^2 + k'\hat{\lambda}_g \int \rho(F \wedge F)$$

6d: free “closed string” (BCOV) sector

The associated chiral algebra (conformally soft modes on celestial sphere, governing collinear singularities) can be obtained from Koszul duality approaches on twistor space [see my Strings '22 talk]


conformal primary states on twistor space of neg. weight
(on-shell gauge theory states)

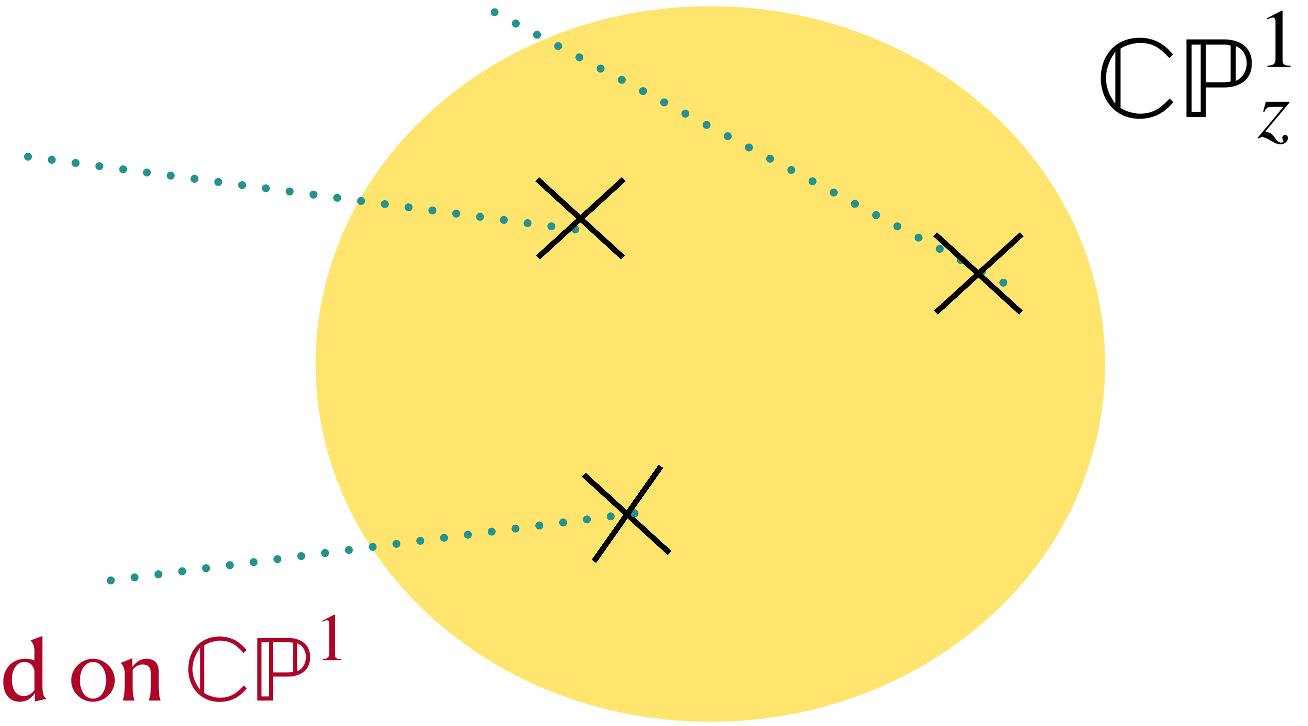
Penrose transform

4d basis of conformal primary states w/ neg. weight

[Pasterski-Shao-Strominger]

$$J[r,s](z_i) \leftrightarrow \mathscr{A} = \delta_{z=z_i}(\tilde{\lambda}^{\dot{1}})^r(\tilde{\lambda}^{\dot{2}})^s$$

state in vacuum module = on-shell background field localized on $\mathbb{CP}^1$



it is a very large, non-unitary algebra

Generator	Spin	Weight	$SU(2)_+$ representation	Field	Dimension
$J[m,n]$, $m,n \geq 0$	$1-(m+n)/2$	$(m-n)/2$	$(m+n)/2$	A	$-m-n$
$\widetilde{J}[m,n]$, $m,n \geq 0$	$-1-(m+n)/2$	$(m-n)/2$	$(m+n)/2$	B	$-m-n-2$
$E[m,n]$, $m+n>0$	$-(m+n)/2$	$(m-n)/2$	$(m+n)/2$	ρ	$-m-n$
$F[m,n]$, $m,n \geq 0$	$-(m+n)/2$	$(m-n)/2$	$(m+n)/2$	ρ	$-m-n-2$

Table 1: The generators of our 2d chiral algebra and their quantum numbers. Dimension refers to the charge under scaling of $\mathbb{R}^4$.

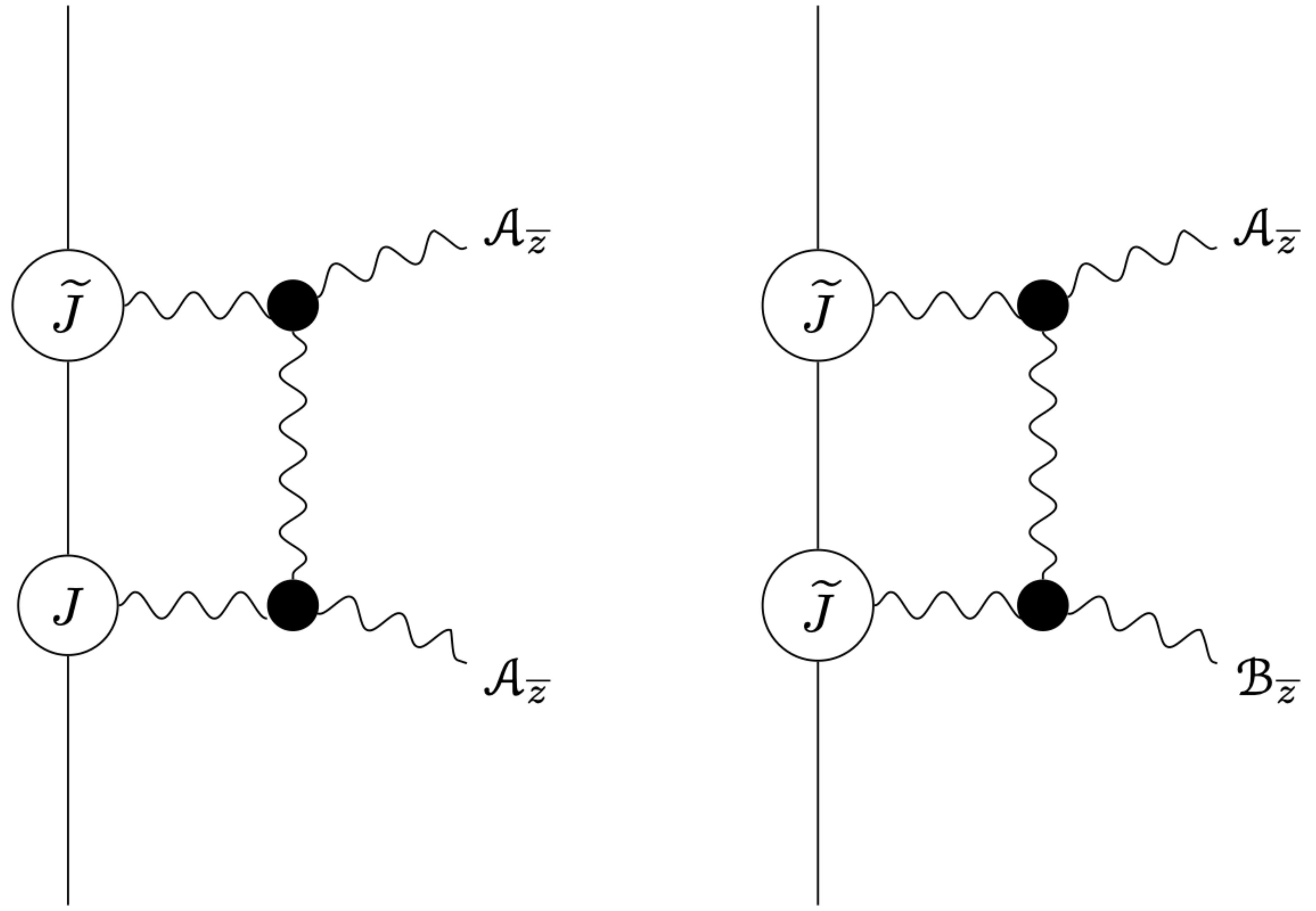
There is a prescription for obtaining the OPEs, not just classically
but including possible deformations

$$PExp \sum_{r,s\geq 0} \int_{\mathbb{CP}^1_z} (\partial^r_{\tilde{\lambda}^{\dot{1}}} \partial^s_{\tilde{\lambda}^{\dot{2}}} \mathscr{B}^a_{\bar{z}}) \tilde{J}_a[r,s](z)$$

$$PExp \sum_{r,s\geq 0} \int_{\mathbb{CP}^1_z} (\partial^r_{\tilde{\lambda}^{\dot{1}}} \partial^s_{\tilde{\lambda}^{\dot{2}}} \mathscr{A}^a_{\bar{z}}) J_a[r,s](z)$$

gauge inv’t couplings to arbitrary defect
↔ Hom from Koszul dual algebra into defect algebra

Quantum deformations:



OPEs among currents on defect by imposing gauge (BV-BRST) invariance

→ universal or “Koszul dual” algebra
Tree level: recover current algebra for gauge symmetry

BRST variation of all diagrams at given loop order must be zero

A more bottom-up perspective: the OPEs are related to 4d collinear singularities, which are known in detail in Yang-Mills.

Compute the associator (say at tree or 1-loop order) and see if it vanishes!

$$J^a[r,s](0)J^b[t,u](z) \sim \frac{1}{z} f^{ab}_c J^c[r+t,s+u](0)$$
$$J^a[r,s](0)\widetilde{J}^b[t,u](z) \sim \frac{1}{z} f^{ab}_c \widetilde{J}^c[r+t,s+u](0)$$

Tree-level

Includes level-0 Kac-Moody algebra for Maps$(\mathbb{C}^2, \mathfrak{g})$

[Guevara-Himwich-Pate-Strominger]

$$J^a[r,s](0)E[t,u](z) \sim \frac{1}{z}\frac{(ts-ur)}{t+u}\widetilde{J}^a[t+r-1,s+u-1](0)$$
$$J^a[r,s](0)F[t,u](z) \sim -\frac{1}{z}\partial_z\widetilde{J}^a[r+t,s+u](0) - \frac{1}{z^2}(1+\frac{r+s}{t+u+2})\widetilde{J}^a[r+t,s+u](0)$$
$$\begin{aligned} J^a[r,s](0)J^b[t,u](z) \sim & \frac{1}{z}K^{ab}(ru-st)F[r+t-1,s+u-1](0) \\ & -\frac{1}{z}K^{ab}(t+u)\partial_z E[r+t,s+u](0) - \frac{1}{z^2}K^{ab}(r+s+t+u)E[r+t,s+u](0). \end{aligned}$$

Failure of associativity in pure SDYM theory in one-loop

Axion field necessary for its restoration

[Bern-Dixon-Kosower]

$$\mathrm{Split}^{[1]}_+(a^+,b^+) = -\frac{N_c}{96\pi^2}\frac{[ab]}{\langle ab\rangle^2}$$

$$\begin{aligned} & J_a[1,0](0)J_b[0,1](z) \\ & = -\frac{1}{2\pi i z} C K^{fe}(f^c_{ae}f^d_{bf} + f^d_{ae}f^c_{bf}) : J_c[0,0]\widetilde{J}_d[0,0] : \\ & \quad + \frac{1}{2\pi i z}\tfrac{1}{2} D f^c_{ab}\partial_z \widetilde{J}_c(0) + \frac{1}{2\pi i z^2} D f^c_{ab}\widetilde{J}_c(0). \end{aligned}$$

Quantum deformation

C, D are known & fixed by ***anomaly coefficient in 6d***

For self-dual YM, no further collinear singularities at higher loops.
Costello and I further showed that a 4d theory with a twistorial uplift has form factors which are isomorphic to chiral correlators of the 2d theory.
In the case of Yang-Mills, this leads to some cute 2d expressions for certain 4d amplitudes

$$P^{\alpha\dot\alpha} =: \lambda^\alpha \tilde\lambda^{\dot\alpha}$$

$$\lambda^\alpha \equiv (1,z)$$

2d chiral algebra	**4d theory**
conf. primary generators	conf. primary states (boost eigenbasis)
OPEs	collinear limits
conformal blocks (cf. CS/WZW)	local operators
correlation functions	form factors

$$\langle tr(B^2)\,|\,\tilde J^a(z_1)\tilde J^b(z_2)J^c(z_3)\rangle = \frac{z_{12}^3}{z_{13}z_{23}} f^{abc}$$

$$\langle ij\rangle = z_i - z_j$$

Momentum eigenbasis:

$$J(\tilde\lambda, z) = \sum_{r,s} \omega^{r+s} \frac{(\tilde\lambda^{\dot 1})^r(\tilde\lambda^{\dot 2})^s}{r!s!} J[r,s](z)$$

$$\langle tr(B^2)\,|\,J^{a_1}(z_1)\dots\tilde J^{a_2}(z_i)\dots\tilde J^{a_j}(z_j)\dots J^{a_n}(z_n)\rangle = \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\dots\langle n1\rangle} tr(t^{a_1}\dots t^{a_n}) + \text{ permutations}$$

SDYM + axion isn't the only 4d theory with a nice twistorial uplift…
WZW4 with G = SO(8) + quartic "Kahler scalar" field
Costello-Li show this follows from a topological string analogue of Green-Schwarz mechanism for type I string

Recall [Donaldson, Nair, Losev-Moore-Nekrasov-Shatashvili]:

$g : M \to SO(8)$

$$\mathscr{L} = \frac{N}{8\pi^2}\int_M \partial\bar{\partial}K \wedge \mathrm{tr}(g^{-1}\partial g \wedge g^{-1}\bar{\partial} g) - \frac{N}{24\pi^2}\int_{M\times[0,1]} \partial\bar{\partial}K \wedge \mathrm{tr}(\tilde{g}^{-1}d\tilde{g})^3$$

$N \in \mathbb{Z}_+$

5d analogue of WZ term

4d analogue of KM level

Classically, a gauge-fixed formulation of SDYM w/ $A = -\bar{\partial}g g^{-1}$

``Closed string/gravitational" sector is the theory of a scalar controlling perturbations of the Kahler potential
(See Costello's Strings 2021 talk)

e.o.m.: $R(K+\rho) = 0$

$K \mapsto K + \rho$

Again, a special, integrable 4d theory. But I claim this is the seed for a nice toy model of holography in asymptotically flat spacetimes! Work w/ Costello & Sharma, 2208.14233 + to appear
Should be a relation to N=2 heterotic string of Ooguri & Vafa

Let's use the origin of the 6d anomaly cancellation from topological type I open+closed string theory on twistor space

Add **additional** N D1-branes on top of $\mathbb{CP}^1$ and ``backreact'', study resulting open/closed duality a la topological strings or AdS/CFT

$\mu^\alpha = 0$

In type I Kodaira-Spencer theory, Hirosi & collaborators taught us that this is a deformation of complex structures

$$Z_0 = \underset{\mu^1}{\mathscr{O}(1)} \oplus \underset{\mu^2}{\mathscr{O}(1)} \to \underset{z}{\mathbb{CP}^1}$$

$$\bar\partial V + \frac{1}{2}[V,V] = (2\pi)^2 N\,\bar\delta^2(\mu)\; z^2\,\frac{\partial}{\partial z}$$

$$z \mapsto \frac{1}{z} \implies \mu^\alpha \mapsto \frac{\mu^\alpha}{z}$$

$$\bar\partial = \mathrm{d}\bar z\,\frac{\partial}{\partial \bar z} + \mathrm{d}\bar\mu \cdot \frac{\partial}{\partial \bar\mu}$$

$$\boxed{\begin{gathered} V = N\,\frac{\bar\mu^{\bar 1}\,\mathrm{d}\bar\mu^{\bar 2} - \bar\mu^{\bar 2}\,\mathrm{d}\bar\mu^{\bar 1}}{||\mu||^4}\;z^2\,\frac{\partial}{\partial z} \\ \mathcal{L}_V\Omega_0 = 0 \quad \text{away from } \mu^\alpha = 0 \end{gathered}}$$

$$\Omega_0 = \frac{\mathrm{d}z\;\mathrm{d}\mu^1\,\mathrm{d}\mu^2}{z^2}$$

(This is in the spirit of the twisted holography program initiated by Costello, Gaiotto, Li, and myself)

$$u^\alpha = (u^1, u^2) \in \mathbb{C}^2\,, \qquad ||u||^2 = |u^1|^2 + |u^2|^2$$

4d: **Burns metric**

$$ds^2 = ||du||^2 + \frac{|u^1du^2 - u^2du^1|}{||du||^4}$$

Note: Burns space is **asymptotically flat**

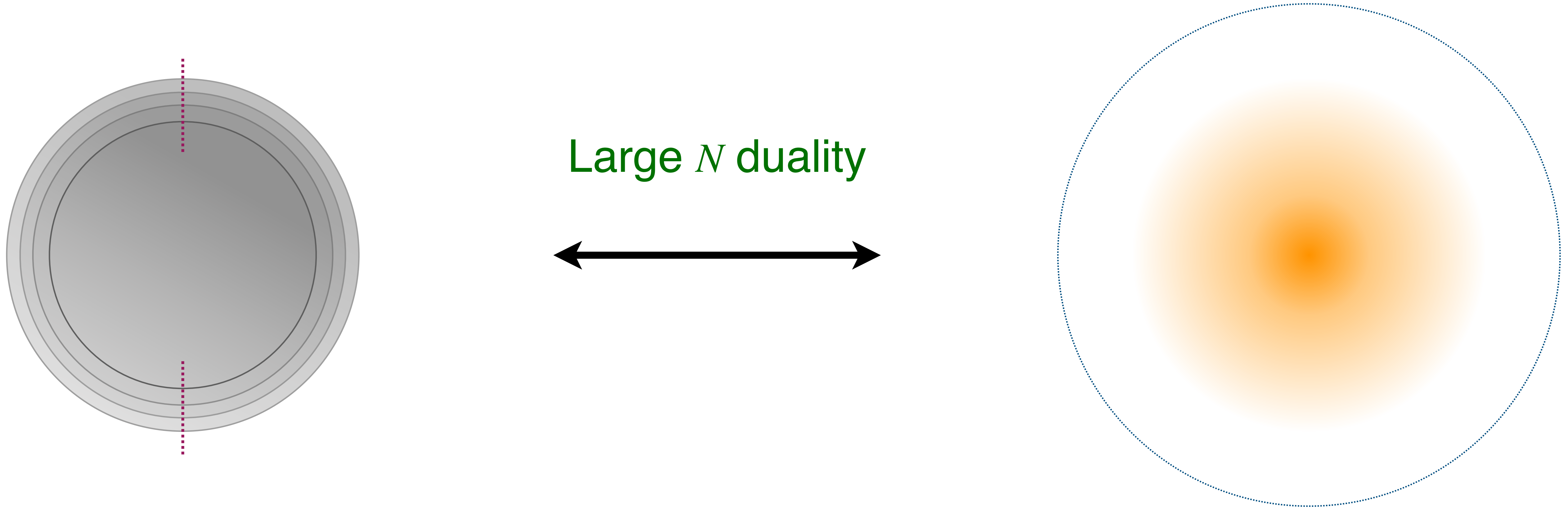
Moreover, worldvolume theories on D1-branes in the topological string are well known.
Proceeding a little more carefully and putting the pieces together ultimately gives us:

$\mathbb{CP}^1$

N D1 branes in twistor space

$\mathscr{B}_N$

Burns space
4d, Euclidean signature, asymptotically flat



$\beta\gamma$ system with
Sp(N) gauge symmetry
+ SO(8) flavour
+ defects at $0, \infty$

WZW4 for G=SO(8)
+
self-dual scalar-flat gravity

Here: dual chiral algebra understood at **finite N**
In principle, exact description of collinear limits of 4d theory at finite coupling, from 2d chiral algebra at finite N

To start, we have checked 2 & 3-pt funs in this proposed duality when $N \to \infty$
For example, matching states between part of chiral algebra to the WZW4 sector in the bulk:

Open string-sector states in chiral algebra

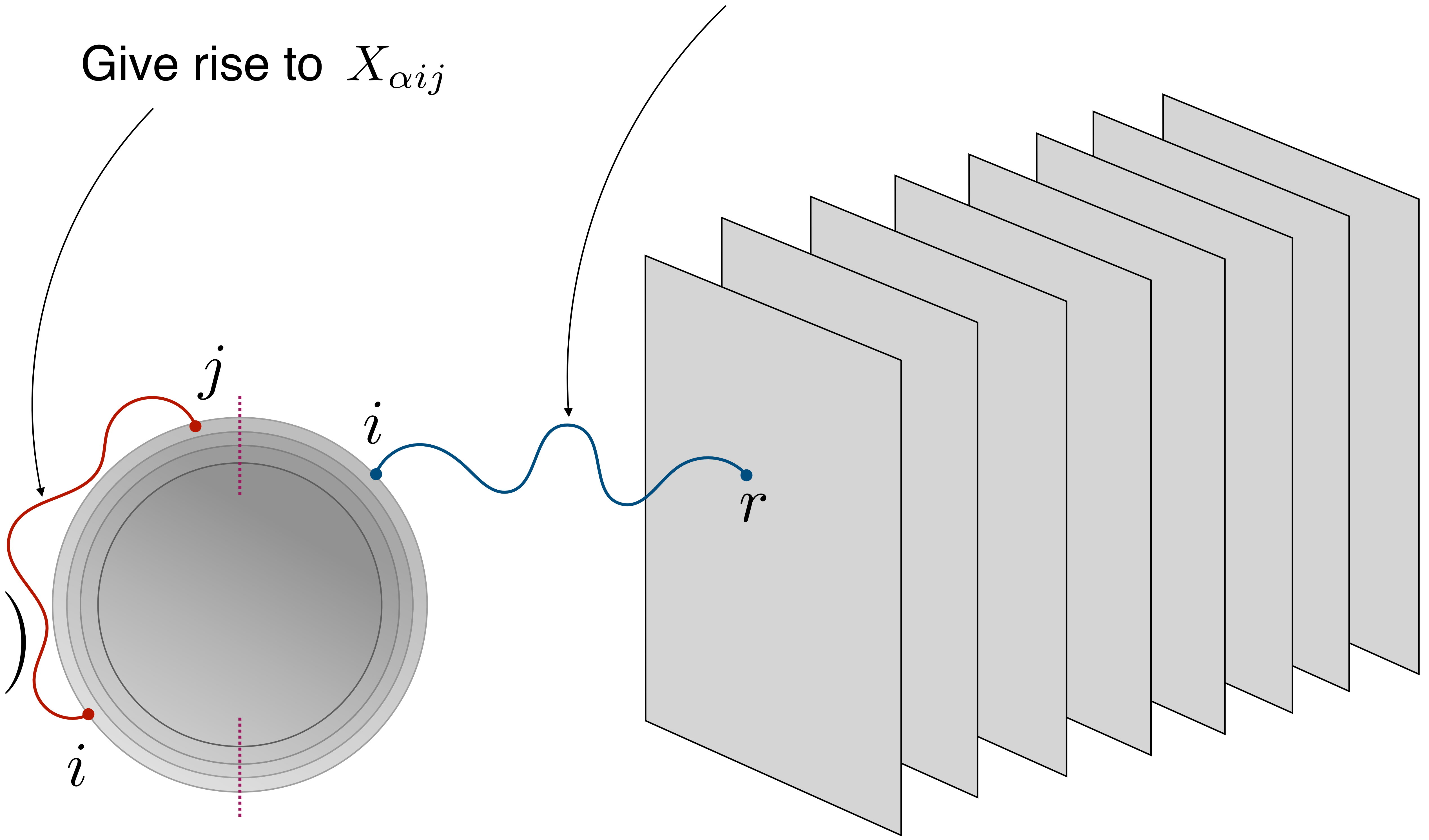
$$X_{\alpha ij} \in \mathbb{C}^2 \otimes {\bigwedge}^2_{\text{t.f.}} \mathbb{C}^{2N}$$

$$I_{ri} \in \mathbb{C}^8 \otimes \mathbb{C}^{2N}$$

+ ghosts for BRST reduction

$$I_{im}(z_1) I_{jn}(z_2) \sim \frac{\delta_{ij}\omega_{mn}}{z_{12}}$$

$$X_{\alpha mn}(z_1) X_{\beta rs}(z_2) \sim \frac{\epsilon_{\alpha\beta}}{z_{12}} \left(\omega_{m[r|}\omega_{n|s]} - \frac{\omega_{mn}\omega_{rs}}{2N} \right)$$


Give rise to $X_{\alpha ij}$

Give rise to I_{ri}

States in WZW4 on Burns space

$$g = \exp(\phi\, \mathfrak{t})$$

linearized field eqn

$$\triangle \phi = 0$$

closed form
sol'n for "momentum eigenstates"

(Recovers $e^{ik\cdot x}$ when $N = 0$)

N D1 branes at $\mu^\alpha = 0$ 4 space-filling ``D5'' branes (+ O-plane)

Chiral worldvolume actions follow from Witten's prescriptions. [Witten '95] [Costello, Gaiotto '18]

- Dictionary between soft modes of states and symmetry currents in the dual CFT

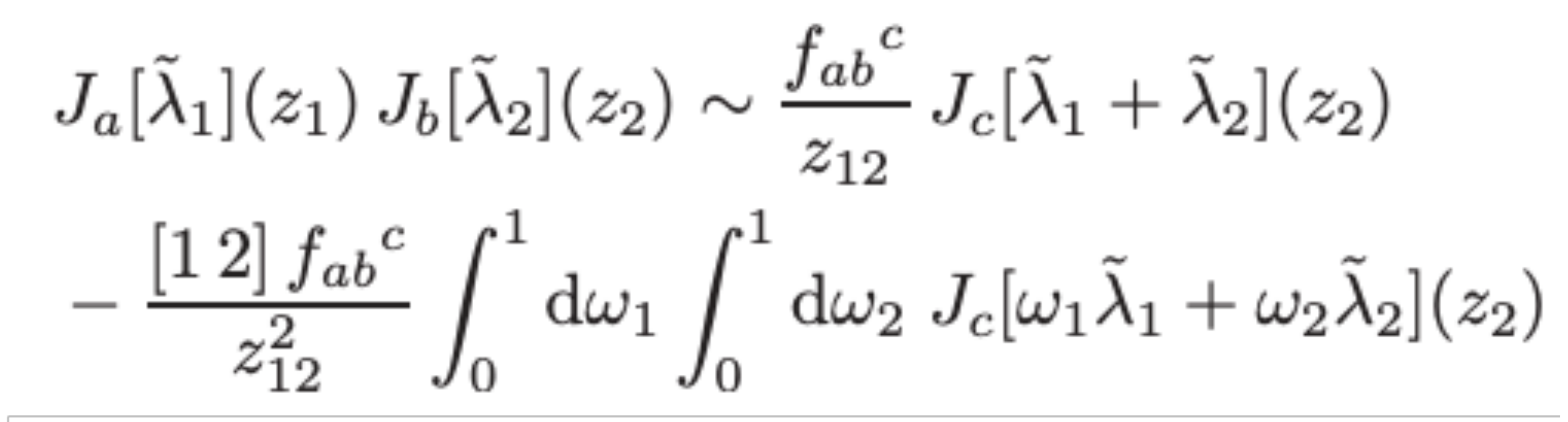
$$\tilde{\lambda}_\alpha = \omega(1, \tilde{z})$$

Soft expansion
$$\phi(\omega, z, \tilde{z}) = \frac{1}{z} \sum_{p=0}^{\infty} (\mathrm{i}\,\omega)^p \sum_{k+l=p} \frac{\tilde{z}^l}{k!\,l!}\, \phi[k,l](z)$$

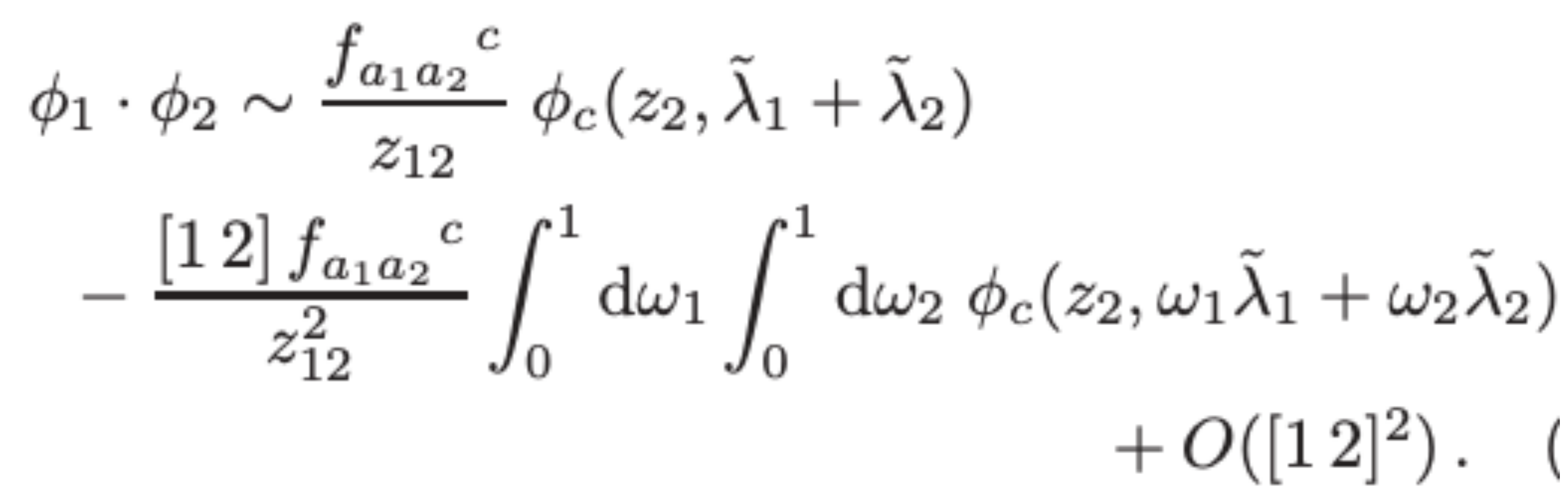
Dictionary
$$\frac{1}{z}\,\phi[k,l](z)\;\mathrm{t}_{rs} \longleftrightarrow \langle I_r, X_1^{(k}X_2^{l)} I_s\rangle(z)$$

Examples of soft modes
$$\phi[0,0] = 1\,, \qquad \phi[1,0] = u^1 - z\,\bar{u}^{\bar{2}}\,, \qquad \phi[0,1] = u^2 + z\,\bar{u}^{\bar{1}}$$
$$\phi[1,1] = \phi[0,1]\phi[1,0] + \frac{Nz}{2}\,\frac{|u^1|^2 - |u^2|^2}{|u^1|^2 + |u^2|^2}$$

- chiral algebra OPE computations easily done in planar limit by Wick contractions
- In bulk, Euclidean amplitudes computed via on-shell effective action, as in standard AdS/CFT computations


$$J_a[\tilde{\lambda}_1](z_1)\, J_b[\tilde{\lambda}_2](z_2) \sim \frac{f_{ab}{}^c}{z_{12}}\, J_c[\tilde{\lambda}_1 + \tilde{\lambda}_2](z_2)$$
$$- \frac{[1\,2]\, f_{ab}{}^c}{z_{12}^2} \int_0^1 \mathrm{d}\omega_1 \int_0^1 \mathrm{d}\omega_2\; J_c[\omega_1\tilde{\lambda}_1 + \omega_2\tilde{\lambda}_2](z_2)$$

$$\updownarrow$$


$$\phi_1 \cdot \phi_2 \sim \frac{f_{a_1a_2}{}^c}{z_{12}}\, \phi_c(z_2, \tilde{\lambda}_1 + \tilde{\lambda}_2)$$
$$- \frac{[1\,2]\, f_{a_1a_2}{}^c}{z_{12}^2} \int_0^1 \mathrm{d}\omega_1 \int_0^1 \mathrm{d}\omega_2\; \phi_c(z_2, \omega_1\tilde{\lambda}_1 + \omega_2\tilde{\lambda}_2)$$
$$+ O([1\,2]^2)\,. \quad ($$

Our toy top-down example of asymptotically-flat holography has passed standard checks.
Next: Go beyond the planar limit, study states of dim’n $\mathcal{O}(N)$, $\mathcal{O}(N^2)$, fully flesh out embedding in physical string, etc.

This gives a concrete toy-model of a ``celestial holography”-type correspondence,
analogous to supersymmetric sectors of AdS/CFT

Unfortunately, I only know how to build associative chiral algebras using these methods for theories that are integrable/self-dual in 4d. Relatedly, the closed-string sector of the bulk theory, which only captures Kahler potential fluctations, means our gravitational sector is rather poor.

Perhaps the first step towards 4d asymptotically flat holography in more physically interesting setups would be to find a chiral algebra dual for self-dual Einstein gravity (perhaps coupled to SDYM).

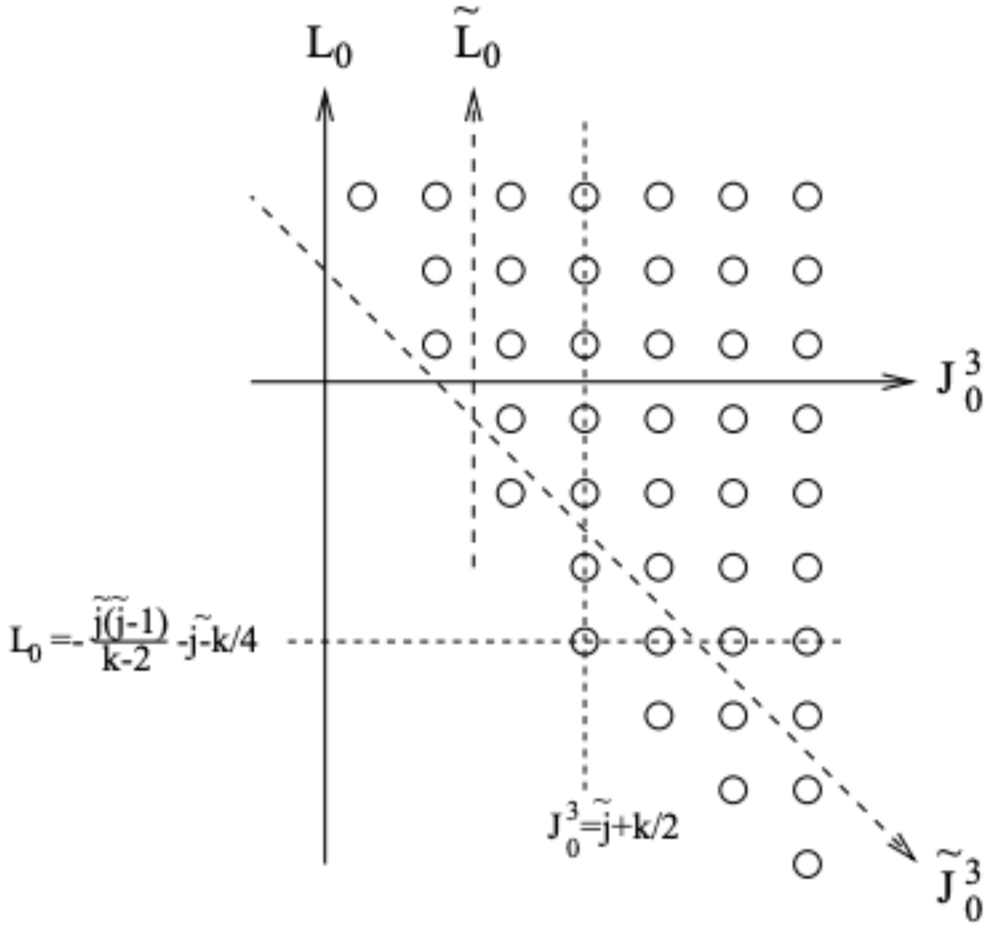
We know how to cancel the twistorial anomaly there, thanks to work of Bittleston-Sharma-Skinner.

Note that Burns space can be viewed as an Einstein-Maxwell instanton…

non-unitarity, operator product associativity, integrability, etc. are all connected,
insight for how to move beyond the twisted realm?

Cheers/kanpai, and a very happy birthday, Hirosi!

$$\lambda^2 S(A+A_0(x),x|t,\bar{t}) = \int_M A'\frac{1}{\partial}\left(\bar{\partial}A'_0 + \frac{1}{2}\partial(x+A_0)\wedge(x+A_0))'\right) + \frac{1}{2}\int_M A'_0\frac{1}{\partial}\bar{\partial}A'_0 + \frac{1}{6}\int_M ((x+A_0)\wedge(x+A_0))'(x+A_0)' + \frac{1}{2}\int_M A'\frac{1}{\partial}\left[\bar{\partial}A' + \partial((x+A_0)\wedge A)'\right] + \frac{1}{6}\int_M (A\wedge A)'A'$$



$90 = 45 + \overline{45}$

n	1	2	3	4	5	6	7	8	9	…
A_n	45	231	770	2277	5796	13915	30843	65550	132825	…

$$\sum_{q_I\in\Lambda_{el}} \Omega(p^I,q_I)e^{-\pi\phi^I q_I} \overset{?}{=} \left|\Psi_{\text{top}}(p^I+i\phi^I,2^8)\right|^2$$

I look forward to being newly inspired by all your papers to come

To start, we have checked 2 & 3-pt funs in this proposed duality when $N \to \infty$
For example, matching states between part of chiral algebra to the WZW4 sector in the bulk:

$$g = \exp(\phi\, \mathfrak{t})$$

$$\triangle \phi = 0$$

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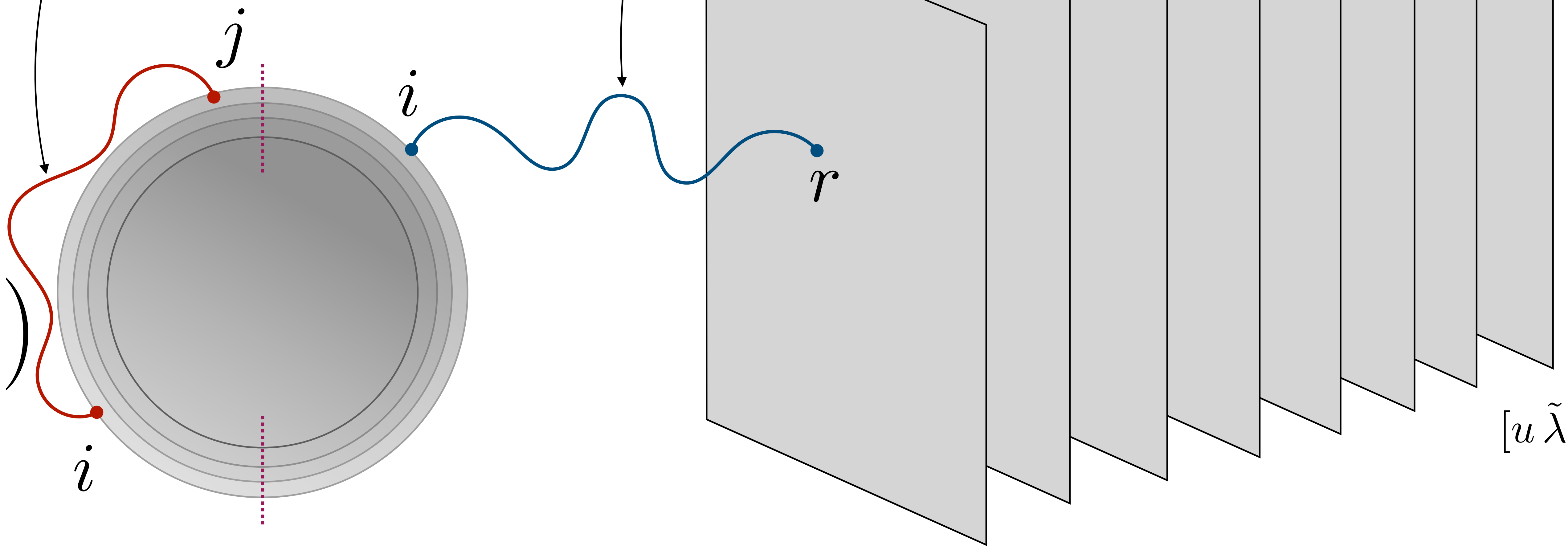
$$X_{\alpha mn}(z_1) X_{\beta rs}(z_2) \sim \frac{\epsilon_{\alpha\beta}}{z_{12}} \left(\omega_{m[r|}\omega_{n|s]} - \frac{\omega_{mn}\omega_{rs}}{2N} \right)$$

Give rise to $X_{\alpha ij}$

Give rise to I_{ri}



“Momentum eigenstates”

$$\phi = \frac{1}{z}\, \mathrm{e}^{\frac{\mathrm{i}}{2}([u\,\tilde\lambda] + z[\hat u\,\tilde\lambda])} \left(\cos\frac{\psi}{2} + \frac{\boxed{\mathrm{i}([u\,\tilde\lambda] + z[\hat u\,\tilde\lambda])}}{\psi} \sin\frac{\psi}{2} \right)$$

$$\downarrow$$

$$k \cdot x$$

$$\xrightarrow{N \to 0} \quad \frac{1}{z}\, \mathrm{e}^{\mathrm{i}k\cdot x}$$

$$\psi = \sqrt{([u\,\tilde\lambda] + z[\hat u\,\tilde\lambda])^2 + \frac{4N}{||u||^2}[u\,\tilde\lambda][\hat u\,\tilde\lambda]}$$

$$[u\,\tilde\lambda] = \tilde\lambda_1\, u^1 + \tilde\lambda_2\, u^2\,, \quad [\hat u\,\tilde\lambda] = \tilde\lambda_2\, \bar u^{\bar 1} - \tilde\lambda_1\, \bar u^{\bar 2}$$

N D1 branes at $\mu^\alpha = 0$ 4 space-filling ``D5” branes (+ O-plane)

Chiral worldvolume actions follow from Witten’s prescriptions. [Witten ’95]
[Costello, Gaiotto ’18]