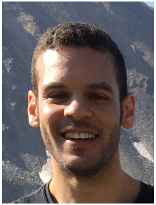
From geometry to 4D physics

HirosiFest, Caltech

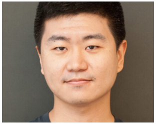
October 28, 2022

Washington (Wati) Taylor, MIT

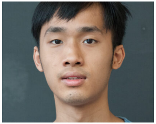
Based in part on recent and upcoming papers with:



Patrick Jefferson



Manki Kim



Shing Yan (Kobe) Li

Andrew Turner

Hirosi Ooguri

Hirosi has made important contributions
to many aspects of string theory and mathematical physics

Hirosi's ability to balance the beautiful (geometry) and the practical (physical implications, managing large systems e.g. ACP, Burke, KIPMU etc.) are an inspiration to us all.

Some themes of his work that this talk will touch on:

Calabi-Yau manifolds

Mirror symmetry

The string landscape/swampland

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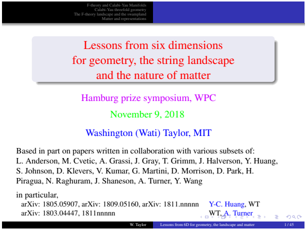

This talk is in some sense an update of that talk, now in 4D!

1. The 4D F-theory landscape

2. Middle cohomology on elliptic Calabi-Yau fourfolds
and chiral matter in 4D F-theory models

3. Standard Model constructions from direct tuning and E_6, E_7 breaking

Interlude: large U(1) charges for massless/light fields in 4D

4. Mirror symmetry and elliptic CY fourfolds

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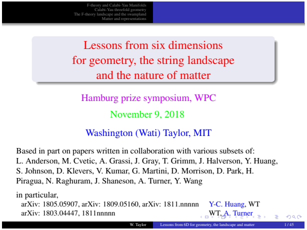
F-theory and Calabi-Yau Manifolds
Calabi-Yau threefold geometry
The F-theory landscape and the swampland
Matter and representations

Lessons from six dimensions
for geometry, the string landscape
and the nature of matter

Hamburg prize symposium, WPC

November 9, 2018

Washington (Wati) Taylor, MIT

Based in part on papers written in collaboration with various subsets of:
L. Anderson, M. Cvetic, A. Grassi, J. Gray, T. Grimm, J. Halverson, Y. Huang, S. Johnson, D. Klevers, V. Kumar, G. Martini, D. Morrison, D. Park, H. Piragua, N. Raghuram, J. Shaneson, A. Turner, Y. Wang

in particular,
arXiv: 1805.05907, arXiv: 1809.05160, arXiv: 1811.nnnnn Y-C. Huang, WT
arXiv: 1803.04447, 1811nnnnn WT, A. Turner

W. Taylor Lessons from 6D for geometry, the landscape and matter 1 / 45

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Primary research program goals:

— Global picture of 4D supersymmetric F-theory landscape (connected)

— What structures are typical? (lots of geometrically rigid gauge groups)

— What is possible/impossible? (i.e. swampland)

— Identify different Standard Model realizations
Which are most natural?
What are the phenomenological differences/predictions?

Although the landscape is vast, some structures are clearly impossible, and other structures seem strongly favored, by enormous exponential factors.

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F-theory: Nonperturbative formulation of type IIB string theory

Dictionary for geometry ↔ physics [Vafa, Morrison-Vafa]

$\sim$ compactification of IIB on compact Kähler (non-CY) space B (e.g. $\mathbb{P}^n$)
B_2 (complex surface) $\to$ 6D, $B_3 \to$ 4D.

Elliptic fibration: $\pi : X(CY) \to B$,
$\pi^{-1}(p) \cong T^2$, for general $p \in B$

Fiber singularities $\to$

Gauge group G (codimension 1 in B)

Matter (codimension 2 in B)

Defined by Weierstrass model (fiber τ = 10D IIB axiodilaton)

$$y^2 = x^3 + fx + g, \quad f, g \text{ "functions" on } B_2$$

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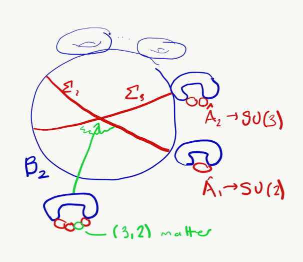

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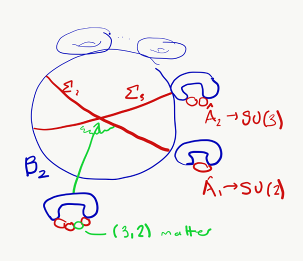

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M-theory vs. IIB description

Philosophy of this talk: take IIB description seriously

Most work on F-theory involves explicit resolution of singularities $X \to \tilde{X}$ (i.e. M-theory description). e.g. [Witten, Grimm]

Different resolutions $\to$ different details (e.g. intersection #'s)

Want to identify resolution-independent structure

– Physics must be independent of resolution

– Should be captured by nonperturbative IIB description

– Other recent related work [Grassi/Halverson/Long/Shaneson/Sung, Katz/WT]

– Focus here: structure of intersection theory on singular elliptic CYs

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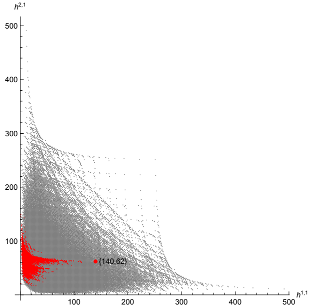
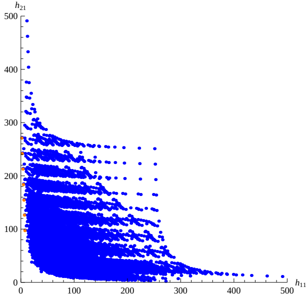
6D F-theory: summary of the global picture

• One large connected moduli space with (finitely) many branches;
Max ($h^{2,1}(X_3) = 491$) (cf. Keller/Ooguri)



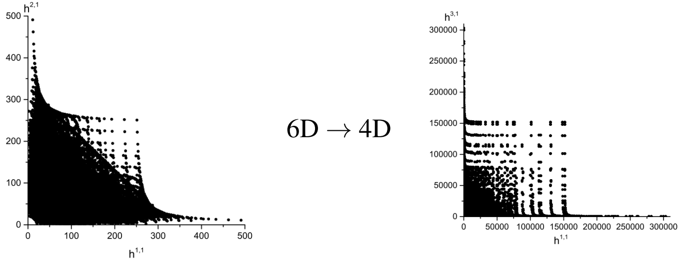


• Most known Calabi-Yau threefolds (and fourfolds) are elliptic
(Empirical results, theoretical arguments: [Huang/WT, Anderson/Gao/Gray/Lee])

• Typical CY3 has large rigid (geometrically non-Higgsable) gauge group
$G \sim E_8^5 \times F_4^6 \times (G2 \times SU(2))^{10}$ (natural dark matter scenario!)

The 4D F-theory landscape

- Global structure of the 4D landscape very similar



Global picture less complete.
e.g. 65,000 toric bases $\to\sim 10^{3000}$ ($\sim\sim 10^{50}$ w/o triangulation); current work with Wang/Yu+Raman, Kim/Li on toric and non-toric landscape

- Gauge factors $E_8, E_7, E_6, F_4, G_2, SU(2)$ ubiquitous

Biggest difference from 6D: superpotential, chiral matter controlled by G-flux, must understand $H_4(X, \mathbb{Z})$.

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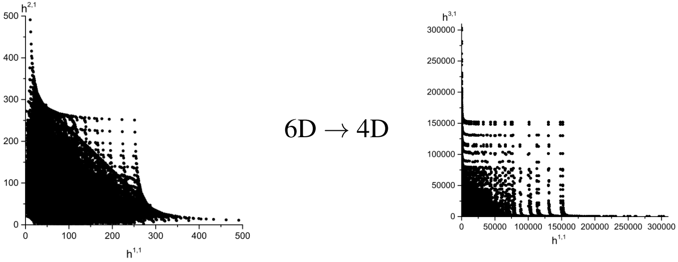

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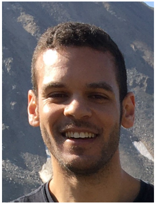
2. Middle cohomology on elliptic Calabi-Yau fourfolds and chiral matter in 4D F-theory models



Patrick Jefferson

Andrew Turner

Based on arXiv:2108.07810 by P. Jefferson, WT, A. Turner

Topology of elliptic Calabi-Yau fourfolds

Divisors (codimension one hypersurfaces) for elliptic CY fourfold

$h^{1,1}(X) = h^{1,1}(B) + \text{rk } G + 1$ (Shioda-Tate-Wazir);
Divisors organized into section D_0, pullbacks from base $D_\alpha = \pi^* D_\alpha^{(B)}$,
$D_i + D_a$ nonabelian Cartan + abelian sections

For fluxes and chiral matter, we are interested in *vertical* cohomology in H_4

$$H_{2,2}^{\text{vert}} = \text{span}_{\mathbb{Z}}(H^{1,1}(X,\mathbb{Z}) \wedge H^{1,1}(X,\mathbb{Z}))$$

Denote $S_{IJ} = D_I \cap D_J$; note, homology relations $\to$ linear dependencies

Fluxes in $H_{2,2}^{\text{vert}} \to$ chiral matter

$H^4(X)$ has orthogonal decomposition [Greene/Morrison/Plesser, Braun/Watari]

$$H^4(X,\mathbb{C}) = H_{\text{vert}}^{2,2}(X,\mathbb{C}) \oplus H_{\text{rem}}^{2,2}(X,\mathbb{C}) \oplus H_{\text{hor}}^{4}(X,\mathbb{C})\,.$$

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Chiral matter in 4D F-theory models

Flux: $G_{\mathbb{Z}} = G - \frac{c_2(X)}{2} \in H^4(X, \mathbb{Z})$ [Witten]

Satisfies various conditions (SUSY/primitivity, tadpole, ...)

Poincaré invariance: $\int_{S_{0\alpha}} G = 0 \,, \quad \int_{S_{\alpha\beta}} G = 0$

Gauge symmetry preserved: $\int_{S_{i\alpha}} G = 0$ (for E_6, E_7 breaking will be $\neq 0$!)

Chiral matter is determined by fluxes, primarily through vertical cycles

Chiral matter: $\chi_{\mathsf{r}} = n_{\mathsf{r}} - n_{\mathsf{r}^*} = \int_{S_{\mathsf{r}}} G \qquad$ (S_{r} a "matter surface")
[Donagi/Wijnholt, Beasley/Heckman/Vafa, Braun/Collinucci/Valandro, Marsano/Schäfer-Nameki,Krause/Mayrhofer/Weigand, Grimm/Hayashi]

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Intersection form on middle cohomology

Previous work on chiral matter in F-theory models used explicit resolutions

Our approach identifies a resolution-independent structure allowing systematic and base-independent analysis for many gauge groups

Basic idea:

M_{IJKL} intersection numbers on CY4 X generally depend on resolution.

Organize as matrix on $H_{2,2}^{\text{vert}}$: $\quad M_{(IJ)(KL)} = M_{IJKL} = S_{IJ} \cdot S_{KL}$.

We then have fluxes $\chi_R \sim \Theta_{IJ} = \int_{S_{IJ}} G = M_{(IJ)(KL)} \phi^{KL}$,
where $G = \sum_{KL} \phi_{KL} \, \mathrm{PD}(S_{IJ})$.

Removing the null space associated with trivial homology elements,

$$M \to M_{\text{red}} \quad \text{is nondegenerate}$$

Observation/conjecture: M_{red} is resolution independent up to basis
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Generalizations of resolution invariance of M_{red}

Focused here on invariance of intersection form on $H_{2,2}^{\text{vert}}$ for different resolutions of elliptic CY4's.

More generally, for a Calabi-Yau n-fold, can conjecture that there is a bilinear product that is a birational invariant

$$M^{(k,n)} : \Lambda^k \times \Lambda^{n-k} \to \mathbb{Z}\,,$$

where Λ^k is the lattice of vertical k-cycles, e.g., $S_{i_1 \cdots i_k} = D_{i_1} \cap D_{i_k}$.

Seems to hold for a number of examples, can prove in some simple classes of cases [Jefferson/Kim/WT, wip].

General proof? (M_{red} invariance natural from physics, other cases?)

Explicit form of $M_{\rm red}$

Can compute general form of $M_{\rm red}$ for various gauge groups over general bases, using systematic approach to resolution building on earlier work [Esole/Jefferson/Kang]

e.g. simple nonabelian G in basis $S_{0\alpha}, S_{\alpha\beta}, S_{i\alpha}, S_{ij}$

$$M_{\rm red} = \begin{pmatrix} D_{\alpha'}\cdot K\cdot D_\alpha & D_{\alpha'}\cdot D_\alpha\cdot D_\beta & 0 & 0 \\ D_{\alpha'}\cdot D_{\beta'}\cdot D_\alpha & 0 & 0 & * \\ 0 & 0 & -\kappa^{ij}\Sigma\cdot D_\alpha\cdot D_{\alpha'} & * \\ 0 & * & * & * \end{pmatrix}.$$

or after a (non-integral) change of basis

$$U^{\rm t} M_{\rm red} U = \begin{pmatrix} D_{\alpha'}\cdot K\cdot D_\alpha & D_{\alpha'}\cdot D_\alpha\cdot D_\beta & 0 & 0 \\ D_{\alpha'}\cdot D_{\beta'}\cdot D_\alpha & 0 & 0 & 0 \\ 0 & 0 & -\kappa^{ij}\Sigma\cdot D_\alpha\cdot D_{\alpha'} & 0 \\ 0 & 0 & 0 & \frac{M_{\rm phys}}{(\det\kappa)^2} \end{pmatrix},$$

where $M_{\rm phys}$ encodes physics of chiral matter + fluxes, $(000\chi) = M_{\rm red}\cdot[G]$.

Example: SU(5) chiral matter (see also [Blumenhagen/Grimm/Jurke/Weigand, Grimm/Krause/Weigand, Marsano/Schafer-Nameki, Grimm/Hayashi])

Can compute from M_{red}

$$\Theta_{33} = \Sigma \cdot K \cdot (6K + 5\Sigma)(\phi^{33} - \phi^{35} - \phi^{44} + \phi^{45})/5\,.$$

Using matter surfaces or cnxn to 3D CS couplings ([Cvetič/Grimm/Klevers])

$$\chi_{\mathbf{5}} = -\Theta_{33} = -\chi_{\mathbf{10}}\,.$$

So we have, where generally m is an integer (exceptions e.g. if $5|K$)

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Base-independent formula for chiral multiplicities
(∼ [Cvetič/Grassi/Klevers/Piragua] w/ U(1) factors)

For example for $B = \mathbb{P}^3, \Sigma = nH, -K = 4H$,

$$\chi_{\mathbf{5}} = 4(5n - 24)m$$

Some interesting questions regarding quantization remain (see recent paper)

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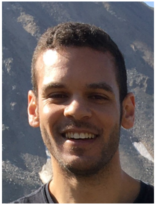
3(A). Universal tuned standard model structure in F-theory



Patrick Jefferson

Nikhil Raghuram

Andrew Turner

Based on:

arXiv:1906.11092 by WT, A. Turner

arXiv:1912.10991 by N. Raghuram, WT, A. Turner

arXiv:2210.09473 by P. Jefferson, WT, A. Turner

Universal G models

For fixed G, matter representations, a *universal G model* in F-theory is a class of Weierstrass models of full dimensionality (fixed by anomalies in 6D) that geometrically realize G

- Tate models for simple $G = SU(N), E_8, E_7, E_6, F_4, SO(N), G_2, \ldots$
- Morrison-Park model for U(1) with $q = 1, 2$

Universal Weierstrass model for G_{SM} [Raghuram/WT/Turner]

$$
\begin{aligned}
f = &-\frac{1}{48}\left[s_6^2 - 4b_1(d_0 s_5 + d_1 s_2)\right]^2 \\
&+ \frac{1}{2} b_1 d_0 \left[2b_1\left(d_0 s_1 s_8 + d_1 s_2 s_5 + d_2 s_2^2\right) - s_6(s_2 s_8 + b_1 d_1 s_1)\right], \\
g = &\frac{1}{864}\left[s_6^2 - 4b_1(d_0 s_5 + d_1 s_2)\right]^3 + \frac{1}{4} b_1^2 d_0^2 \left(s_2 s_8 - b_1 d_1 s_1\right)^2 - b_1^3 d_0^2 d_2 \left(s_2^2 s_5 - s_2 s_1 s_6 + b_1 d_0 s_1^2\right) \\
&- \frac{1}{24} b_1 d_0 \left[s_6^2 - 4b_1(d_0 s_5 + d_1 s_2)\right]\left[2b_1\left(d_0 s_1 s_8 + d_1 s_2 s_5 + d_2 s_2^2\right) - s_6(s_2 s_8 + b_1 d_1 s_1)\right].
\end{aligned}
$$

(Derived from "unHiggsing" Raghuram's U(1) $q = 1, 2, 3, 4$ model)

- Includes "F_{11}" G_{SM} models as a special case [Klevers/Mayorga Peña/Oehlmann/Piragua/Reuter, Cvetič/K/MP/O/R]

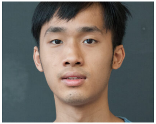
Generic matter for $G_{\rm SM}$ models

	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$	$(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$(\mathbf{1},\mathbf{1})_{1}$	$(\mathbf{3},\mathbf{1})_{-\frac{4}{3}}$	$(\mathbf{1},\mathbf{2})_{\frac{3}{2}}$	$(\mathbf{1},\mathbf{1})_{2}$
(MSSM)	1	-1	-1	-1	1	0	0	0
(exotic 1)	2	-1	-4	-2	0	1	0	1
(exotic 2)	-2	2	2	-1	0	0	1	-1

Analysis: [Jefferson/WT/Turner, recent paper]

- Generically get all three families from universal model – no constraints from geometry beyond anomaly cancellation
- Closed form formulae for chiral multiplicities χ_i
  (from complicated version of $M_{\rm red}$ with extra U(1) sections)
- Tuning two discrete parameters gives SM families
- Special case: F_{11} model, recent analysis of 10^{15} 3-generation solutions [Cvetič/Halverson/Lin/(Liu/Tian, Long)]

3(B). Standard model from E_6, E_7 breaking in F-theory



Shing Yan (Kobe) Li

Based on:

arXiv:2112.03947, 2207.14319 by S.Y. Li, WT

F-theory approaches to the standard model

There are many different ways the standard model may be realized in F-theory

	GUT	$\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
Tuned G	Tuned GUT (e.g., $\mathrm{SU}(5)$)	Direct tuned G_{SM}
Rigid G	Rigid GUT (e.g., $\mathrm{E}_6, \mathrm{E}_7$)	Rigid G_{SM}

- Previous discussion: direct tuned
- Much work: tuned GUT e.g. SU(5) [Beasley/Heckman/Vafa, Donagi-Wijnholt]

Tuned models are rare in landscape, however: require tuning many moduli, many bases will not support

- $\mathrm{SU}(3) \times \mathrm{SU}(2)$ can be rigid/geometrically non-Higgsable in 4D [Grassi/Halverson/Shaneson/WT]; U(1) factor difficult however to integrate

Most natural approach: rigid/non-Higgsable GUT

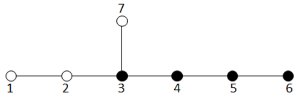
Next: breaking $E_6, E_7 \to G_{\mathrm{SM}}$ with fluxes

Breaking $E_7 \to G_{\text{SM}}$ [SY (Kobe) Li/WT, arXiv:2112.03947]

Recall

$$\Theta_{IJ} = \int_{S_{IJ}} G = M_{(IJ)(KL)} \phi^{KL} \,.$$

When $\Theta_{i\alpha} \neq 0$, breaks Cartan generator i; $\sum_i c_i \Theta_{i\alpha} = 0 \forall \alpha$ preserves U(1), etc.



Can choose fluxes to break $i = 3, 4, 5, 6$ for any geometric E_7, leaving $SU(3) \times SU(2)$

Note: this realization of $SU(3) \times SU(2)$ is unique up to E_7 automorphism

Depending on fluxes, preserve different $U(1)$ factors, different spectra

– Many $SU(3) \times SU(2) \times U(1)$ breakings, but most have exotics

Intermediate SU(5) and remainder hypercharge flux breaking

To avoid exotics, any appropriate $U(1) \to SU(5)$ enhancement!
(flux vanishes on an additional $\mathbb{P}^1$; equivalent to $\Theta_{3\alpha} = 0$)

Proceed in two steps: 1) Vertical flux breaking $E_7 \to SU(5)$,
2) Remainder flux breaking $SU(5) \to G_{\text{SM}}$
($\sim$ [Beasley/Heckman/Vafa, Donagi-Wijnholt, Blumenhagen/Grimm/Jurke/Weigand, Marsano/Saulina/Schafer-Nameki, Grimm/Krause/Weigand, ...])

Remainder flux:

$$G_4^{\text{rem}} = [D_Y|_{C_{\text{rem}}}] \,,$$

where $D_Y = 2D_1 + 4D_2 + 6D_3 + 3D_7$ generates hypercharge.

C_{rem} is a curve on Σ, homologically trivial in B. Such curves exist on some (typical?) non-toric bases [Braun/Collinucci/Valandro]

Matter content with this breaking contains only SM family

$$(\mathbf{3},\mathbf{2})_{1/6}\,, \quad (\mathbf{3},\mathbf{1})_{2/3}\,, \quad (\mathbf{3},\mathbf{1})_{-1/3}\,, \quad (\mathbf{1},\mathbf{2})_{1/2}\,, \quad (\mathbf{1},\mathbf{1})_{1}\,,$$

arising from (non-chiral) E_7 representations $\mathbf{56}$ and $\mathbf{133}$.

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Features of $E_6, E_7 \to G_{\mathrm{SM}}$ flux construction

- Explicit examples in papers ($E_6 \to$ SM details in recent paper, E_7 to appear.
- Ubiquitous/natural: construction is possible on typical bases
  coarse estimate: 18% of base threefolds have rigid E_7 [WT/Wang]
- Flux breaking of GUT E_7 without its own chiral matter
- No chiral exotics for certain breaking pattern with intermediate SU(5)
- Chiral multiplicity is naturally small from tadpole/χ.
  (3 arises very naturally)

- Does not work for E_8, but maybe from SCFT matter? [Tian/Wang]

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Another application of flux breaking: U(1) charges

Completeness conjecture: states with all gauge charges arise in a theory of quantum gravity (proven by Harlow/Ooguri in AdS)

Less clear: how large can charges of massless/light fields be?

In 2018 Hamburg talk, reported on work with Raghuram:

6D F-theory models with (implicit) massless U(1) charge $q = 21$
(other work suggested max 6, no explicit $q > 6$ known)

Work soon to appear with SY (Kobe) Li:

Flux breaking of G_2 factor in near-max base ($G = E_8^9 \times F_4^8 \times (G_2 \times SU(2))^{16}$) gives:

$$\boxed{\text{chiral massless q = 465, vectorlike (light) q = 657!}}$$

Basic idea: can break to arbitrary diagonal U(1) in Cartan, unlike w Higgsing.

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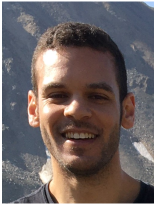
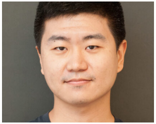
3. Mirror symmetry in 4D F-theory models


Yu-Chien Huang


Patrick Jefferson


Manki Kim


Paul Oehlmann

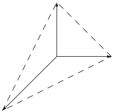
Based on:

arXiv:1811.04947 by Y-C Huang, WT

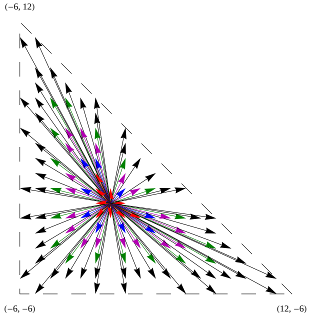
arXiv:22mm.nnnnn? by P. Oehlmann, WT

arXiv:22mm.nnnnn? by P. Jefferson, M. Kim, WT

Mirror symmetry factorizes (base $\times$ fiber) for many toric hypersurfaces!
[Huang/WT] Example: generic elliptic fibration on $\mathbb{P}^2$ (2, 272)



Hodge numbers (2, 272)



Hodge numbers (272, 2)
(toric rays: $\vec{w} \cdot \vec{v} \geq -6$, $\forall \vec{v} \in \Sigma_B$, $\vec{w}$ primitive)

$h^{1,1}(B) = 1$

$G = 1$

$h^{1,1}(X) = h^{1,1}(B) + \text{rk}\, G + 1 = 2$

$h^{2,1}(X) = 301 - 29h^{1,1}(B) - \text{dim}M_{nh} = 272$

$h^{1,1}(B) = 106 + 3 = 109, G = E_8{}^9 \times F_4{}^9 \times (G_2 \times SU(2))^{18}$

$h^{1,1}(X) = h^{1,1}(B) + \text{rk}\, G + 1 = 272$

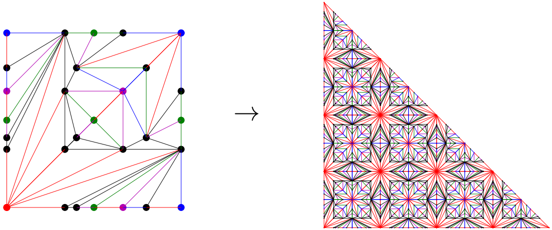
$h^{2,1}(X) = 301 - 29h^{1,1}(B) + \dim G - \text{dim}M_{nh} = 2$

4D Example: $B = \mathbb{P}^3$ standard stacking ($F = \mathbb{P}^{2,3,1} = F_{10}$)

Rays in $\tilde{B}$: primitive lattice points in tetrahedron:
w/vertices (-6, -6, -6), (18, -6, -6), (-6, 18, -6), (-6, -6, 18)

$G = E_8^{34} \times F_4^{96} \times G_2^{256} \times SU(2)^{384}$

- (Exponentially) many triangulations; construction from (projected) tiling



[Jefferson/Kim/WT, wip]

- Note: common endpoint from random blow-up sequence [WT/Wang]

Combining factorization of mirror symmetry on CY fourfolds with structure of $M_{\rm red}$ allows computation of full unimodular intersection form on $H_4(X, \mathbb{Z})$ [Jefferson/Kim/WT work in progress]

Example: $B = \mathbb{P}^3$

$h^{1,1}(X) = 2, h^{3,1}(X) = 3878$

Mirror symmetry: $h^{1,1}(Y) \leftrightarrow h^{3,1}(X)$

With 2306 toric rays and 22 E_8 factors with (4, 6) loci blown up non-torically,

$$\begin{aligned} h^{1,1}(Y) &= h^{1,1}(\tilde{B}) + {\rm rk}\tilde{G} + 1 \\ &= 2303 + 22 + (34 \times 8 + 96 \times 4 + 256 \times 2 + 384) + 1 \\ &= 3878 \end{aligned}$$

Expect that full intersection form on $H_4(X, \mathbb{Z})$ includes

$$M_{\rm red}(X, \mathbb{Z}) \oplus M_{\rm red}(Y, \mathbb{Z})$$

since $H^{\rm vert}_{2,2} \leftrightarrow H^{\rm hor}_{2,2}$ [Braun/Watari], here $H^{\rm rem}_{2,2} = 0$.

Expect $M_{\rm red}(X, \mathbb{Z}) \oplus M_{\rm red}(Y, \mathbb{Z})$ is unimodular or has unimodular overlattice

In the example $X = \mathbb{P}^3$, there is no gauge group so

$$M_{\rm red}(X, \mathbb{Z}) = \left(\begin{array}{cc} K \cdot D_{\alpha'} \cdot D_\alpha & D_\alpha \cdot D_\beta \cdot D'_\alpha \\ D_\alpha \cdot D'_\alpha \cdot D'_\beta & 0 \end{array} \right) = \left(\begin{array}{cc} -4 & 1 \\ 1 & 0 \end{array} \right)$$

This is unimodular so expect $M_{\rm red}(Y, \mathbb{Z})$ also unimodular.

$$M_{\rm red}(Y, \mathbb{Z}) \sim \begin{pmatrix} D_{\alpha'} \cdot K \cdot D_\alpha & D_{\alpha'} \cdot D_\alpha \cdot D_\beta & 0 & 0 \\ D_{\alpha'} \cdot D_{\beta'} \cdot D_\alpha & 0 & 0 & * \\ 0 & 0 & -\kappa^{ij} \Sigma \cdot D_\alpha \cdot D_{\alpha'} & * \\ 0 & * & * & * \end{pmatrix}.$$

Upper left 2×2 unimodular by Poincare duality, toric curves span Chow ring

E_8 factors unimodular, $F_4 \to$ overlattice/extra surfaces, $G_2 \times SU(2)$ extra surfaces. Extra surfaces also from (crepant/non-crepant) singularities.

Rather rich story but unimodular structure appears to arise for a general class of bases. Gluing from $G \to G$, or $[G, E_8]$ ("E_8 rule" [Berglund/Mayr])

Conclusions

• New general approach to understanding resolution-independent intersection form on $H_{2,2}^{\text{vert}}$, key for understanding flux compactifications and chiral matter

• General formulae for chiral matter including for universal G_{SM} model; in all cases independent families of chiral matter only constrained by anomalies

• New approach to realizing Standard Model gauge group and chiral matter with 3 generations and no exotics from flux breaking of $E_6, E_7 \to SU(5) \to (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$

• Structure of intersection form M_{red} allows computation of full integer intersection form on $H_{2,2}(X, \mathbb{Z})$ using mirror symmetry

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HAPPY BIRTHDAY, HIROSI!!!

[clipart library]

Thanks so much for all your contributions,
both scientifically and to building the high-energy community!

MANY HAPPY RETURNS

Following slides are all extra, with technical details, not part of the main talk.

[Extra slides:] *Generic* matter for fixed group G: [WT/Turner]

• Matter in highest dimensional branch of (geometric) moduli space; same in 6D, 4D (least tuning)

• Matches simplest singularities in F-theory

• e.g. $SU(N)$: $\{\square, \begin{smallmatrix}\square\\\square\end{smallmatrix}, \text{adjoint}\}$

$SU(3) \times SU(2) \times U(1)$: Standard Model matter not generic (e.g. no $(3, 2)_{q \neq 0}$)

$G_{\text{SM}} = (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$: SM matter + several exotics generic

For given G, generic matter typical, anything else fine-tuned

e.g. $SU(N)$ $\square\square$, SU(2) $\square\square\square$ possible "exotic" matter in F-theory [Klevers/Morrison/Raghuram/WT]

[Extra slides] A simple example of $E_7 \to SU(5)$ (chiral multiplicity for SU(5) only)

We consider the base B a $\mathbb{P}^1$ bundle over Hirzebruch $\mathbb{F}_1$,
Σ an $\mathbb{F}_1$ section with normal bundle $N_\Sigma = -8S - 7F$
$(S, F$ generate divisors of B with $S \cdot S = -1, S \cdot F = 1, F \cdot F = 0)$

$\Rightarrow$ rigid E_7 factor on Σ

To solve the flux constraints in the Kähler cone we need:
$0 > \phi_{iS}/\phi_{iF} = n_S/n_F \neq \infty$ identical for all i

We then have:
$\chi_{(\mathbf{3},\mathbf{2})_{1/6}} = 7n_S + 4n_F, \quad (\phi_{1S}, \phi_{2S}, \phi_{3S}, \phi_{4S}, \phi_{7S}) = (2, 4, 6, 5, 3)n_S \ (+S \to F)$

From $\chi(X) = 51096, h^{2,2}(X) = 34076 g\chi(X)/24$, a random flux typically has most entries 0 and small nonzero values.

Minimal solution:

$n_S = -n_F = \pm 1 \Rightarrow$ Number of generations is ± 3

While this is just one example, others have other values; this local structure is

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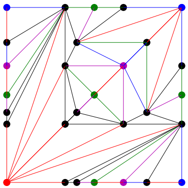
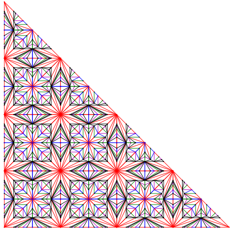
Example computation: $h^{\text{vert}}_{2,2}(Y)$

Counting independent contributions from $S_{0\alpha}, S_{\alpha\beta}, S_{i\alpha}$,

$|S_{0\alpha}| = |S_{\alpha\beta}| = h^{1,1}(\tilde{B}) = 2325 \rightarrow 4650$



$\rightarrow$

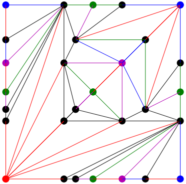


$|S_{i\alpha}| = \sum_i \text{rk}\, \tilde{G}_i(h^{1,1}(\Sigma_i)) = 8 \times (30 \times 22 + 4 \times 16) + 4 \times (32 \times 14 + 64 \times 2) + 2(128 \times 4 + 128 \times 2) + (384 \times 2) \rightarrow 10400$

From mirror symmetry, $H^{\text{vert}}_{2,2}(Y) = 15562 = 10400 + 4650 + 512$

Remaining 512 surfaces: 256 from $G_2 \times SU(2)$ clusters, 256 from F_4 factors with codimension 3 (4, 6) loci

Example: extra surfaces from $G_2 \times SU(2)$ clusters



$$\rightarrow \begin{pmatrix} K D_\alpha^2 & D_\alpha^3 & 0 & 0 & 0 & 0 \\ D_\alpha^3 & 0 & 0 & 0 & 0 & -2 D_\alpha^2 \Sigma_2 \\ 0 & 0 & -6 D_\alpha^2 \Sigma_1 & 3 D_\alpha^2 \Sigma_1 & 0 & -2 D_\alpha \Sigma_1 \Sigma_2 \\ 0 & 0 & 3 D_\alpha^2 \Sigma_1 & -2 D_\alpha^2 \Sigma_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 D_\alpha^2 \Sigma_2 & D_\alpha (4K + 3\Sigma_1 - 2\Sigma_2) \Sigma_2 \\ 0 & -2 D_\alpha^2 \Sigma_2 & -2 D_\alpha \Sigma_1 \Sigma_2 & 0 & D_\alpha (4K + 3\Sigma_1 - 2\Sigma_2) \Sigma_2 & -\Sigma_2 \left(8K^2 + 5\Sigma_1^2 + \Sigma_1 (13K - 3\Sigma_2) + 2\Sigma_2 (-2K + \Sigma_2)\right) \end{pmatrix}$$

$G_2, SU(2)$ factors on e.g. local Hirzebruch F_{12}, F_6 surfaces for case on LHS

Can compute explicitly . . . [Work in progress],
expect det $= K^2$, necessary for overlattice

Unimodular structure: overlattices

Some components of $H^{2,2}_{\text{vert}}(Y)$ not immediately unimodular: need overlattice

Example: F_4 inverse killing form

$$\kappa(F_4) = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -2 & 0 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & -2 & 4 \end{bmatrix}$$

This is not unimodular: $\det \kappa(F_4) = 4$

But adding an additional lattice vector
$(0, 0, 1/2, 1/2) = (S_{\Sigma 3} \cap S_{\Sigma 3} + S_{\Sigma 4} \cap S_{\Sigma 4})/2 \to$ unimodular!

Gives proper quantization for integral lattice.

Presence of extra vectors guaranteed by unimodularity of $H_4(X, \mathbb{Z})$
Confirmation from other approaches–work in progress.

Computation of full $H_4(X, \mathbb{Z})$: further issues

– For $B = \mathbb{P}^3$ example, $H_4(X, \mathbb{Z}) = M_{\text{red}}(X, \mathbb{Z}) \oplus M_{\text{red}}(Y, \mathbb{Z})$, both terms must be unimodular

– More generally $M_{\text{red}}(X, \mathbb{Z})$ not unimodular,
from non-Higgsable + tuned G blocks
Expect complement has G or $[E_8, G]$ (observed in toric duals)

– Also, generally nontrivial $H^{\text{rem}}_{2,2}$, need to compute intersection form on this

Mirror symmetry factorizes (base $\times$ fiber) for many toric hypersurfaces!
[Huang/WT]

Toric hypersurface associated with reflexive polytope ∇; mirror dual Δ.

Elliptic if $\nabla_2 \subset \nabla$ is reflexive 2-polytope.

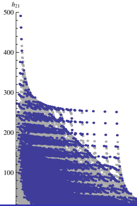
If $F = \nabla_2 \subset \nabla$ is a slice and $\tilde{F} = \Delta_2 \subset \Delta$ is also a slice
$\Rightarrow$ Mirror symmetry factorizes

Simplest cases: Standard stacking on $\mathbb{P}^{2,3,1} \leftrightarrow$ Tate form Weierstrass model

Mirror of generic elliptic fibration over B = ef over $\tilde{B}$ (may be tuned):

$$B \to \tilde{B} \sim \Sigma(-6K_B), \nabla_2 = \Delta_2 = \mathbb{P}^{2,3,1}$$

(65k examples in KS database)



Factorized mirror symmetry: more general structures [Oehlmann/WT, to appear]

- Also works for "tuned" Tate models $\leftrightarrow$ reduction on Δ

- Works for other fibers, bundle structures

e.g. $B = \mathbb{P}^2, F = F_2$; base stacked over vertex: $H = (4, 94)$
$\tilde{B} \sim -2K_B, \tilde{F} = F_{15}$; $H = (94, 4)$

$B = \mathbb{P}^2$

(mirror symmetry of fibers:[Klevers/Mayorga Pena/Oehlmann/Piragua/Reuter])

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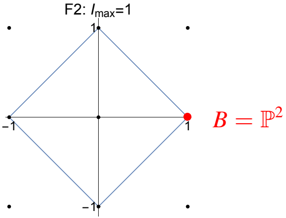

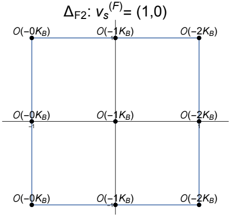

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