Quantum Gravity and Statistical Physics

Jan de Boer, Amsterdam

Hirosifest
October 27, 2022

Happy 60th birthday Hirosi!

Date: Fri, 5 Jan 1996 11:37:28 -0800 (PST)
From: OOGURI@theor3.lbl.gov
To: deboer@insti.physics.sunysb.edu
Cc: OOGURI@theor3.lbl.gov
Subject: Miller Fellowship

First email from Hirosi

Dear Jan,

Happy New Year to you.

I am writing this to encourage you to come to Berkeley. As you may have heard from Bruno Zumino and Marty Halpern, the theory group here unanimously supported your Miller Fellowship nomination and we very much hope that you will accept the offer.

Yesterday, we made 2 additional offers of regular post-doctoral positions to formal theorists as well as 1 offer to a phenomenologist, and we are looking forward to an exciting new academic year starting this fall.

If you have any question or concern, please do not hesitate to call me 510 - 486 - 4340 or send me an e-mail.

Sincerely,
Hirosi Ooguri

Some differences became immediately obvious

Hirosi’s lunch

My lunch

Overall, I had a great time in Berkeley, and we ended up writing 7 papers together on supersymmetric gauge theories, AdS3, and conformal interfaces.

But I have often wondered: who is Hirosi really?

In the spirit of statistical physics, we can try to come up with a model of Hirosi based on our superficial observations.

Before getting there, I will first point out that this philosophy can be seen to underly many interesting recent developments in chaotic systems and quantum gravity.

One nice way to think about statistical physics is that gives us the best description of a system given limited information.

Suppose for example that we want to find a state ρ such that the expectation value of the energy E is fixed while maximizing our ignorance (=entropy). So extremize

$$-\mathrm{Tr}(\rho \log \rho) + \lambda(\mathrm{Tr}(\rho H) - E)$$

Result:

$$\rho = Z^{-1} \exp(\lambda H)$$

We fix the Langrange multiplier by computing E and find the canonical ensemble with

$$\lambda = -\beta(E)$$

As another example, suppose we have a system where we know

1. The approximate spectral density
2. The approximate finite temperature two-point function of some operator A.

We can then find the classical probability distribution for the matrix elements $A_{ij} = \langle E_i | A | E_j \rangle$ by maximizing the classical entropy with infinitely many constraints

$$\int \prod_{i,J} dA_{i,j} \; -\mu[\{A_{ij}\}] \log \mu[\{A_{ij}\}] + \mu[\{A_{ij}\}] \int dt d\beta \lambda(t,\beta) \left[\langle A(0) A(t) \rangle_\beta - f(t,\beta) \right]$$

This yields a quadratic matrix model

$$\mu[\{A_{ij}\}] \sim \exp(-\sum_{i,j} c_{i,j} |A_{i,j}|^2)$$

This reproduces the random matrix part of the Eigenstate Thermalization Hypothesis (ETH):

$$\langle E_i | O_a | E_j \rangle = \delta_{ij} f_a(\bar{E}) + e^{-S(\bar{E})/2} g_a(\bar{E}, \Delta E) R^a_{ij}$$

Deutsch ’91
Srednicki ’94
Foini, Kurchan ‘19

$f_a(\bar{E})$: one point functions of simple operators
$g_a(\bar{E}, \Delta E)$: two point functions of simple operators
R^a_{ij} : Gaussian random variables

$$\langle R^a_{ij} \rangle = 0, \qquad \langle R^a_{ij} R^b_{kl} \rangle = \delta^{ab} \delta_{il} \delta_{jk}$$

Cf Jafferis, Kolchmeyer, Mukhametzhanov, Sonner ‘22

Yet another example: suppose that we know the approximate spectral density or equivalently the partition function $Z(\beta)$

We can consider a probability distribution for the Hamiltonian and extermize the classical entropy while fixing the expectation value of the partion function to be $Z(\beta)$

$$\int dH \; -\mu[H]\log\mu[H] + \mu[H]\int d\beta\lambda(\beta)\left(\mathrm{Tr}(e^{-\beta H}) - Z(\beta)\right)$$

One finds

$$\mu[H] \sim \exp\left(\int d\beta\lambda(\beta)\mathrm{Tr}(e^{-\beta H})\right) \sim \exp(-\mathrm{Tr}V(H))$$

where V is arbitrary but needs to be fixed to yield the right partition function or spectral density.

In the absence of additional information, our best guess for the connected two-point function is then

$$\langle Z(\beta_1)Z(\beta_2)\rangle = Z(\beta_1)Z(\beta_2) + \frac{1}{2\pi}\frac{\sqrt{\beta_1\beta_2}}{\beta_1+\beta_2} + \ldots$$

Ambjørn, Jurkiewicz, Makeenko ’90
Saad, Shenker, Sanford ‘19

One can play a similar game for other choices of data.

The general picture is one where if one e.g. inputs connected ≤k-point correlators, one gets a “matrix model” with up to k-th order interactions in the exponent.

One can apply this philosophy for example to OPE coefficients in a chaotic CFT. We cannot directly compute these when one or more of the operators is in the high-energy, chaotic regime of the theory.

This then “explains” the OPE randomness hypothesis (Belin, JdB ’20):

$$C_{LLH} \sim f(E_H) R_H$$

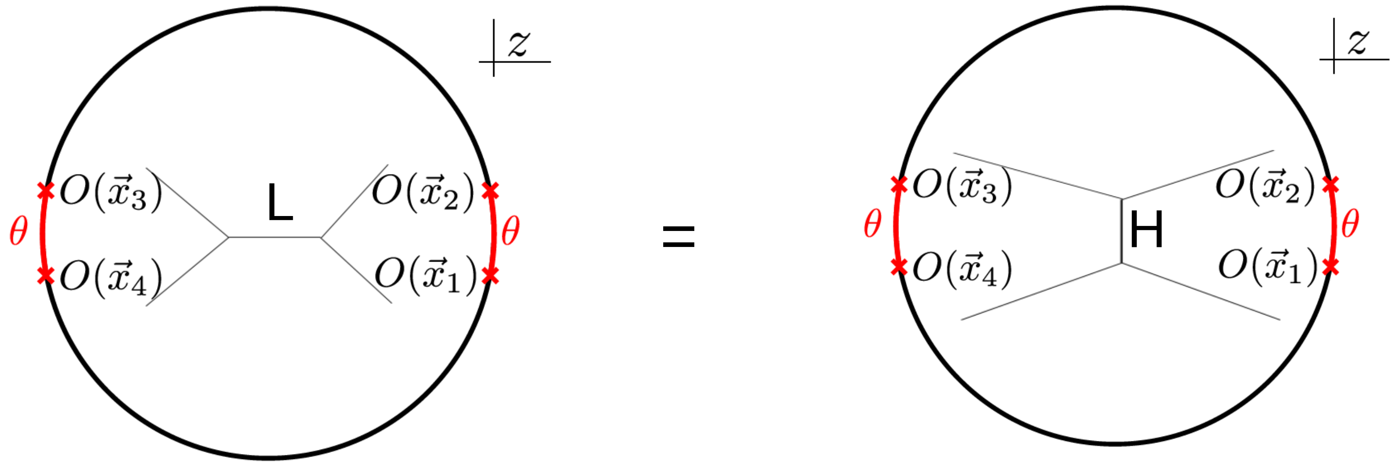
$$C_{LHH'} \sim f(\bar{E_H}, \Delta E_H) R_{HH'}$$

$$C_{HH'H''} \sim f(\bar{E_H}, \Delta E_H) R_{HH'H''}$$

Slowly varying function of arguments

- Pseudorandom
- Mean=0
- Variance=1
- Can have higher moments which are exponentially suppressed.

The form of the first one can be derived using crossing symmetry



$$\frac{1}{\theta^{4\Delta_L}} \simeq \sum_H C_{LLH}^2 \left(\cos\frac{\theta}{2}\right)^{2\Delta_H}$$

$$\overline{|C_{LLH}|^2} \sim \frac{\Delta_H^{2\Delta_L - 1}}{\Gamma(2\Delta_L)\rho(\Delta_H)}$$

Input: approximate spectral density plus

n=4, g=0 $\Rightarrow C_{LLH} \sim f(E_H) R_H$

n=2, g=1 $\Rightarrow C_{LHH'} \sim f(\bar{E}_H, \Delta E_H) R_{HH'}$

n=0, g=2 $\Rightarrow C_{HH'H''} \sim f(\bar{E}_H, \Delta E_H) R_{HH'H''}$

One can systematically add more input and get more complicated non-Gaussian “matrix” models.

For example genus three provides a 4th order moment for $C_{HHH.}$ More generally:

$$\langle C^k_{LHH} \rangle \sim e^{-(k-1)S}$$

Foini, Kurchan ‘19

$$\langle C^k_{HHH} \rangle \sim e^{-\frac{5k-4}{4}S}$$

$$\langle C^k_{HHH} \rangle \sim e^{-\frac{9k-6}{8}S}$$

Belin, JdB, Liska ‘21

$$\langle C^3_{LLH} C_{HHH} \rangle_{\text{all}} \sim e^{-3S}$$

$$\langle C^3_{LLH} C_{HHH} \rangle_{d=2\,\text{quasi-prim}} \sim e^{-3S}$$

$$\langle C^3_{LLH} C_{HHH} \rangle_{d=2\,\text{prim}} \sim e^{-\frac{9}{4}S}$$

Anous, Belin, JdB, Liska ‘21

Back to gravity:

All these inputs are also available in semi-classical gravitational theories in AdS.

The approximate spectral density follows from the black hole entropy. Similarly, approximate higher genus partition functions and correlators of simple operators can be computed.

The fact that gravity has access to coarse grained high-energy information makes it somewhat different from standard low-energy effective field theories.

Above, everything was based on things that can be computed in AdS with one CFT boundary.

The matrix model structure then predicts connected correlators when there is more than one boundary (i.e. wormholes).

There are then two possibilities:

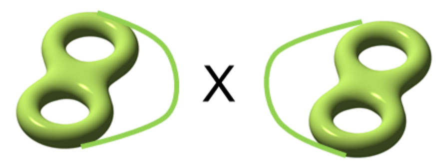
-the predictions agree with gravity computations and one finds no further refinement of the marix model, or

-the predictions do not agree with gravity computations and one needs to add the wormhole computation as additional input to refine the matrix model.

Example: the square of the high-temperature genus two partition function (dominated by high-energy states)

$$Z_{g=2\times g=2} = \left\langle \left(\sum_{i,j,k} C_{ijk} C^*_{ijk} e^{-3\beta\Delta} \right) \left(\sum_{l,m,n} C_{lmn} C^*_{lmn} e^{-3\beta\Delta} \right) \right\rangle$$

$$Z_{g=2\times g=2} = \left\langle \left(\sum_{i,j,k} C_{ijk} C^*_{ijk} e^{-3\beta\Delta} \right) \left(\sum_{l,m,n} C_{lmn} C^*_{lmn} e^{-3\beta\Delta} \right) \right\rangle$$



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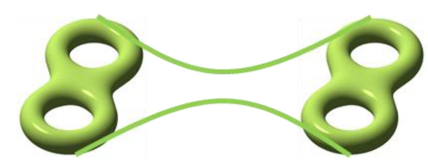

In this example, the wormhole exists (Maldacena, Maoz ‘04) and agrees with the above prediction from the “matrix model”.

Belin, JdB ‘20

The same pattern persists in many more examples (Chandra, Collier, Hartman, Maloney ‘22).

Conjecture: one-sided gravitational information plus principles of statistical physics are sufficient to predict all gravitational computations that involve multiple boundaries.

Belin, JdB, Liska ‘21

Semi-classical gravity is the theory of the statistics of the high-energy, chaotic sector of the theory.

So far everything involved classical statistics. The canonical ensemble was quantum statistics. Is there something that combines both? Extremize

$$\int d\rho \ -\mu[\rho]\log\mu[\rho] + \mu[\rho]S[\rho] + \mu[\rho](\sum \lambda_i F_i[\rho])$$

This has various interesting properties, stay tuned

JdB, Liska, Post, Sasieta, WIP
Arav, Chapman, JdB, WIP

We can also use some of the above ingredients to define an interesting notion of "approximate CFT".

Belin, JdB, Jafferis, Nayak, Sonner, to appear

Let’s now try to model Hirosi. We need some input:

Facebook:

$$
\begin{aligned}
\text{Hirosi} \; &= \int dN \;\; \exp\Bigg(\\
&- 82.3(\text{impeccable behavior} - N(\text{random events}))^2 \\
&- 12.8(\text{excellent science} - N(\text{papers, discussions}))^2 \\
&- 17.1(\text{happy} - N(\text{good food}))^2 \\
&- 3.8(\text{not happy} - N(\text{bad food}))^2 \\
&- 7.3(\text{outreach} + N(\text{distractions}) - B)^2 \\
&- 21.9(\text{administrative roles} - N(\text{requests}))^2 \\
&- 14(\text{happy} - N(\text{culture}))^2 \\
&- 0.02(\text{say what you think} - N(\text{inappropriate behavior}))^2 \\
&+ \ldots \Bigg)
\end{aligned}
$$

Happy 60th birthday Hirosi!