

Searching for a new type of Lie algebra (Suresh Govindarajan)

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The generating functions (and refinements thereof) of the degeneracies of quarter BPS states in four-dimensional $N=4$ supersymmetric theories that arise from type II compactifications on $K3 \times T^2$ and its asymmetric CHL orbifolds are genus-two Siegel modular forms. In some of the cases, the walls of marginal stability across which two-centred BPS states decay into single centered ones lead to rank-three Lorentzian root lattices with Weyl vector. In all but three examples, the square-roots of generating functions, are the Weyl-Kac-Borcherds denominator formula for some Borcherds-Kac-Moody (BKM) Lie superalgebra. Rank-three Lorentzian lattices with Weyl vectors have been classified long ago by Nikulin. The three examples that do not have an algebraic interpretation have Weyl vectors of hyperbolic type. Gritsenko and Nikulin have a no-go theorem that states that such Lorentzian lattices are not related to any BKM Lie superalgebra.

The dyon generating functions lead to potential denominator formulae for a new kind of Lie superalgebra. We study these denominator formulae in terms of an affine $sl(2)$ subalgebra and a Borcherds extension of the affine $sl(2)$ subalgebra. We discuss our studies on the decomposition of the potential denominator formula in terms of the characters of both sub-algebras. An important result is the appearance of fermionic roots with unusual behaviour. We are able to characterise the multiplicity of various imaginary simple roots in terms of vector valued modular forms for which we can give closed formulae in some cases.