Bubble wall velocities in local thermal equilibrium

2109.13710

1. Motivations

SM is not complete:

- Dark Matter
- Matter-anti-matter asymmetry
- Inflation
- Strong CP problem, see, however, 2001.07152

Detection of new physics:

- Energy frontier
- Intensity frontier
- $\boxed{\text{Cosmic frontier}}$: GWs PT strength

GWs from 1st PTs : H_*, α, v_w, β ← transition rate

Hubble parameter during percolation

2. Total force on a bubble wall

Coupled system

Scalar field ϕ + plasma $f_i(p,x)$

EoM: $\Box\phi + \frac{\partial V(\phi)}{\partial\phi} + \sum_i \frac{dm_i^2(\phi)}{d\phi}\int\frac{d^3\vec{p}}{(2\pi)^3 2E_i} f_i(p,x) = 0$, $\frac{df_i}{dt} = -C[f_i]$

Can be derived more systematically from the 2PI effective action

$$\frac{\delta\Gamma_{2PI}[\phi, \Delta_i]}{\delta\phi} = 0, \qquad \frac{\delta\Gamma_{2PI}[\phi, \Delta_i]}{\delta\Delta_i} = 0$$

imaginary time → Gap equation in some limit/approximation

$$\Gamma_{2PI}[\phi, \Delta_\phi, \cdots] = S[\phi] + \frac{i}{2}\mathrm{Tr}\ln(\Delta_\phi^{-1}) + \frac{i}{2}\mathrm{Tr}(G_\phi^{-1}(\phi)\Delta_\phi) + (\cdots) + \Gamma_2[\phi, \Delta_\phi, \cdots]$$

eg: $$\left(\frac{i}{2}\mathrm{Tr}\ln(\Delta_\chi^{-1}) + \frac{i}{2}\mathrm{Tr}(G_\chi^{-1}(\phi)\Delta_\chi) \right)$$

Taking $f_i(p,x) = f_i^{eq}(p,x) + \delta f_i(p,x)$

$$\Rightarrow \quad \underbrace{\Box\phi + \frac{\partial V_{eff}(\phi,T)}{\partial\phi}}_{\text{(equilibrium)}} + \underbrace{\sum_i \frac{dm_i^2(\phi)}{d\phi}\int\frac{d^3\vec{p}}{(2\pi)^3 2E_i}\delta f_i(p,x)}_{\text{(out-of-equilibrium)}} = 0$$

(out-of-equilibrium) $\hookrightarrow$ dissipative friction

Conventional understanding: finite v_w require nonvanishing dissipative friction

not true !

$$\int dz \frac{d\phi}{dz}\left(\cancel{\Box\phi} + \frac{\partial V_{eff}(\phi,T)}{\partial\phi} + \sum_i \frac{dm_i^2(\phi)}{d\phi}\int\frac{d^3\vec{p}}{(2\pi)^3 2E_i}\delta f_i(p,x)\right) = 0$$

$$\hookrightarrow \int dz \frac{d\phi}{dz}\frac{\partial V_{eff}(\phi,T)}{\partial\phi} = \int dz\left(\frac{dV_{eff}}{dz} - \frac{\partial V_{eff}}{\partial T}\frac{dT}{dz}\right)$$

$$= \Delta V_{eff} - \int dz \frac{\partial V_{eff}}{\partial T}\frac{dT}{dz}$$

$$\Rightarrow \underbrace{\Delta V_{eff}}_{\text{driving force}} = \underbrace{\int dz \frac{\partial V_{eff}}{\partial T}\frac{dT}{dz} - \sum_i \int dz \frac{d\phi}{dz}\frac{dm_i^2(\phi)}{d\phi}\int\frac{d^3\vec{p}}{(2\pi)^3 2E_i}\delta f_i(p,x)}_{\text{backreaction force}} = 0$$

$$= \frac{F_{back}}{A}$$

Thermal eqrilibrium: $\frac{F_{back}}{A}$ not necessarily vanish !

Condition: $\frac{dT}{dz} \neq 0$

Dissipative friction force

$\rightarrow$ runaway

- Leading order v_w-independent 0903.4099

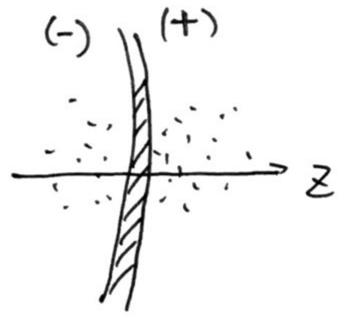
- Higher-order v_w-dependent $\longrightarrow$ subluminal speeds

1703.08215, 2007.10343 (Yikun's talk)

3. "Friction" in local thermal equilibrium & a new matching condition.

Energy-momentum conservation:

$$T_\phi^{\mu\nu} = (\partial^\mu\phi)(\partial^\nu\phi) - g^{\mu\nu}\left(\tfrac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$



$$T_f^{\mu\nu} = (\rho_f + p_f)\, u^\mu u^\nu - p_f\, g^{\mu\nu}$$

$u^\mu(z) = \gamma(z)(1, 0, 0, -v(z))$

$$\nabla_\mu T^{\mu 0} = 0 \Rightarrow \quad \omega\gamma^2 v = \text{const}$$

$$\nabla_\mu T^{\mu 3} = 0 \Rightarrow \quad \omega\gamma^2 v^2 + \tfrac{1}{2}(\partial_z\phi(z))^2 + p = \text{const}$$

Here $\quad p = p_f - V(\phi), \quad \rho = \rho_f + V(\phi)$, and $\omega = \rho_f + p_f = \rho + p$

enthalpy

Well-known matching conditions:

$$\boxed{\begin{aligned} \omega_+\gamma_+^2 v_+ &= \omega_-\gamma_-^2 v_- \\ \omega_+\gamma_+^2 v_+^2 + p_+ &= \omega_-\gamma_-^2 v_-^2 + p_- \end{aligned}}$$

recall
$\omega = Ts$
$\gamma^2 v^2 = \gamma^2 - 1$

$$p_- - p_+ = \Delta\{(\gamma^2 - 1)Ts\}$$

$$\Rightarrow \quad \frac{F_{\text{pressure}}}{A} \equiv \Delta V_{\text{eff}}, \quad \frac{F_{\text{back}}}{A} = \Delta\{(\gamma^2 - 1)Ts\}$$

(thermal equilibrium)

in general: $p_- - p_+ = \Delta V_{\text{eff}} - \Delta\delta p_f$ out-of-equilibrium effects

③

Recently, 2005.10875 proposed

$$\frac{F_{back}}{A} = (\gamma_w^2 - 1) T \Delta s \rightarrow$$ not correct

because they assumed constant temperature across the wall!

Entropy conservation:

$$\partial_\mu S^\mu \equiv \partial_\mu (s u^\mu) = 0 \Rightarrow \left.\begin{array}{l} s(z)\gamma(z)v(z) = \text{const} \\ \gamma^2 - 1 = \gamma^2 v^2 \end{array}\right\}$$

$$\Rightarrow \frac{F_{back}}{A} = \text{const} \times \Delta\{\gamma v T\}$$

$w = Ts$

Further $\left\{\begin{array}{l} w\gamma^2 v = \text{const} \\ s(z)\gamma(z)v(z) = \text{const} \end{array}\right. \Rightarrow \gamma(z) T(z) = \text{const}$

New matching condition for local equilibrium $\boxed{\gamma_+ T_+ = \gamma_- T_-}$

4. Bubble velocities in local thermal equilibrium

Bag equation of state:

$$\rho = a T^4 + \epsilon$$

$$p = \frac{1}{3} a T^4 - \epsilon$$

Generalized: $a(T, \phi)$, $\epsilon(T, \phi)$

Hydrodynamic quantities: $T(z)$, $v(z) \Rightarrow (\cancel{T_+}, T_-, v_+, v_-)$

related to T_{nuc} treated as given

$$w_+ \gamma_+^2 v_+ = w_- \gamma_-^2 v_- \qquad (1)$$

$$w_+ \gamma_+^2 v_+ + p_+ = w_- \gamma_-^2 v_-^2 + p_- \qquad (2)$$

$$\gamma_+ T_+ = \gamma_- T_- \qquad (3)$$

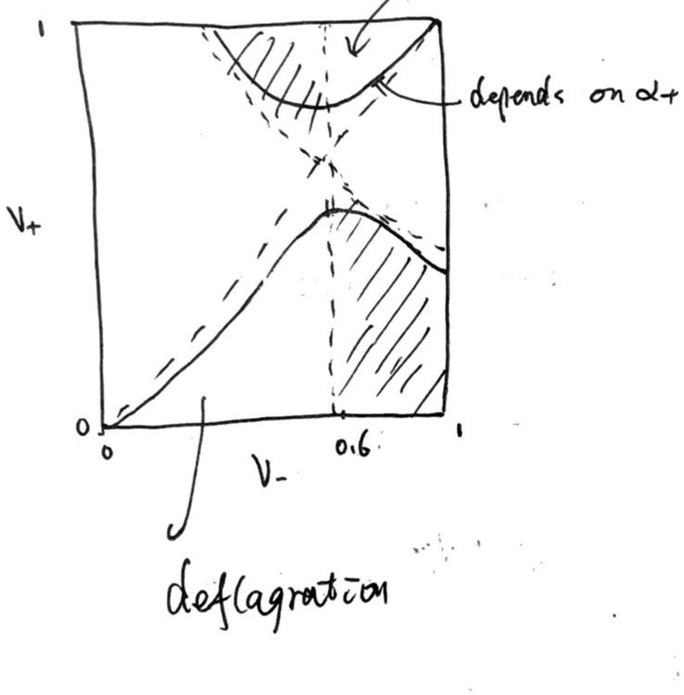
Need assumption on $b \equiv \frac{a_-}{a_+}$ model-dependent

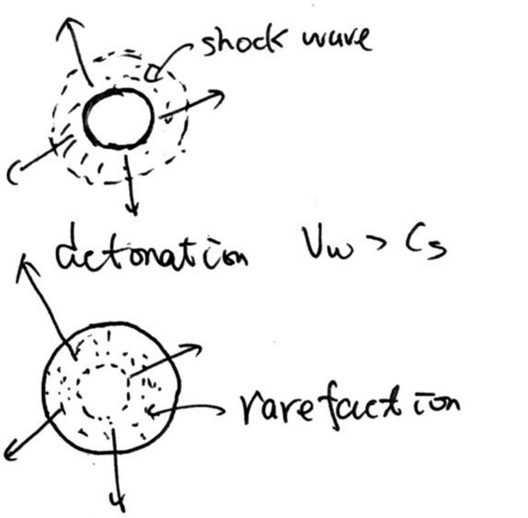
(1) & (2):

$$v_+ v_- = \frac{1-(1-3\alpha_+)r}{3-3(1+\alpha_+)r}$$

$$\frac{v_+}{v_-} = \frac{3+(1-3\alpha_+)r}{1+3(1+\alpha_+)r}, \qquad \alpha_+ \equiv \frac{\Delta\epsilon}{a_+ T_+^4}, \quad r \equiv \frac{a_+ T_+^4}{a_- T_-^4}$$

$$\Rightarrow v_+ = \frac{1}{6(1+\alpha_+)v_-}\left[1+3v_-^2 \pm \sqrt{1+6(6\alpha_+^2 4\alpha_+ - 1)v_-^2 + 9v_-^4}\right]$$

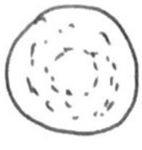




We can also obtain

$$v_- = \frac{1}{6v_+}\left[1-3\alpha_+ + 3(1+\alpha_+)v_+^2 \pm \sqrt{[1-3\alpha_+ + 3(1+\alpha_+)v_+^2]^2 - 12 v_+^2}\right]$$

Detonations



$$v_+ = v_w$$

$$T_+ = T_{nuc}$$

$$\alpha_+ = \frac{\Delta\epsilon}{a(T_{nuc})\,T_{nuc}^4} \equiv \alpha_N$$

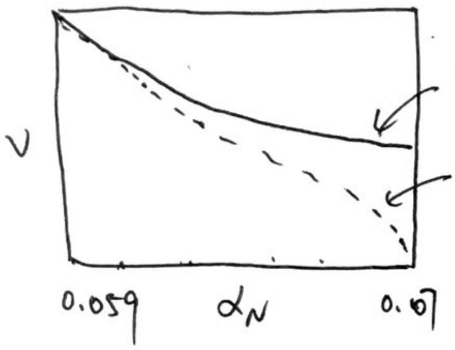
From ② & ③

$$\Rightarrow v_- = v_w \, \frac{\left(\frac{\gamma_w}{\gamma_-}\right)^4 b + 3(1+\alpha_N)}{3\left(\frac{\gamma_w}{\gamma_-}\right)^4 + (1-3\alpha_N)}\,, \qquad b \simeq 0.85 \text{ for SM}$$

$$\Delta f = (p_- - p_+)\big|_{T_{nuc}}$$

$$= \frac{T_{nuc}^4}{3}(a_- - a_+) - (\epsilon_- - \epsilon_+) \geq 0$$

$$\Rightarrow \alpha_N \geq \frac{1-b}{3} = 0.05$$



$[\alpha_{min} \simeq 0.059,\ \alpha_{max} \simeq 0.07]$

Deflagrations:

$$v_- = v_w$$

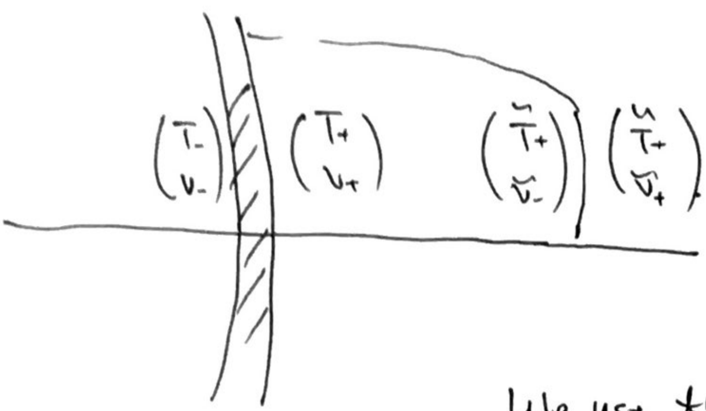
Now we need to relate (T_+, α_+) to (T_{nuc}, α_N)

$$\alpha_+ = \frac{\Delta\epsilon}{a_+ T_+^4} = \frac{\Delta\epsilon}{a_{nuc} T_{nuc}^4} \cdot \frac{a_{nuc} T_{nuc}^4}{a_+ T_+^4} \equiv \alpha_N \frac{\gamma^4}{\tilde{b}}$$

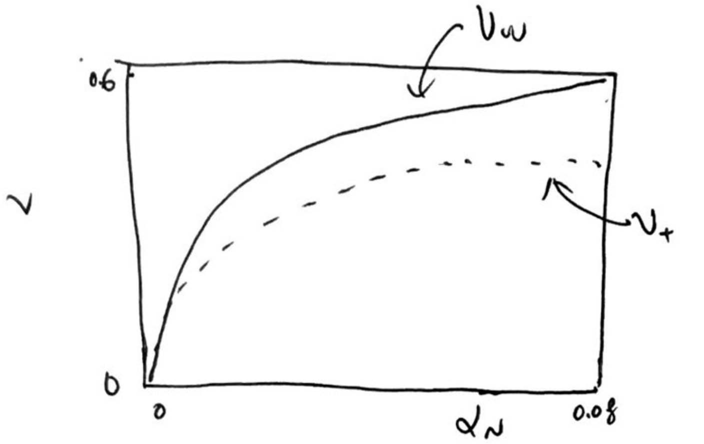
$$\tilde{b} = \frac{a_+}{a_{nuc}} \simeq 1\,, \qquad \gamma = \frac{T_{nuc}}{T_+}$$

- need to solve the fluid velocity and temperature profile away from the bubble wall

$\tilde{\alpha}_+ = 0$



We use the planar-wall approximation and the matching conditions at the shock-wave front



[$\alpha_{min} \simeq 0.0500061$, $\alpha'_{max} \simeq 0.0789645$]

Combining the analyses for detonations & deflagrations

$\Rightarrow$ Steady state for $\alpha_N < \alpha_{crit} = \alpha'_{max}$

Run away for $\alpha_N > \alpha_{crit}$