Ultre ulativistic EW babbles frour simplest Higgs portel
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Wen Yin
Sumoy
(1) Whot: Bubble dynanic, Friction and teminal velocity
(a) Fort \& splunicel bubbles
(b) Friction: 3 types
(CWI NLO teminal velocity
(2) Why: stochostic GW beckgrand \& application
(C) stochertic GWV brg
(b) Scattening off the wall
(C) appliction
(3) Vetre elevistic FOPT: $\mathbb{Z}_{2}$ nol singent extensian of the SM [JHEP $10(2022) 017$ ]
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(b) Application to DM production
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Backup

- 2D Tunuling ( + unbauded potential)
- Daisy uxuatien and effective potential
- 3D reduction, camputotion of the wall velocity from firt principer, $\gamma$ vs $\gamma^{2}$ in No
(1) Bubble dynounic, friction \& terunial velocity

A FOPT Can ocean when Э $T_{c}$ st. two degenender minima separated by a bonier develop.

Tunneling deon rote of the FU

$$
\Gamma \sim T^{4} e^{-5_{3 / T}} \quad \text { Euclidean action }
$$




Friction
when plasma particles hit the wall they slow down its expansion. Full computation of these effects is quite involved, but for relativistic exparrien, $\gamma_{w} \gg 1$, they become mich Simpler.
$O(d)$ spherical symues. $\phi=\phi(r) \quad r=\sqrt{\tau^{2}+x^{2}}$


The most velvent contributions to the friction ore the following
(1) Recoil of the particles getting moss posing through the wall

$$
\Delta P_{L 0} \simeq \sum_{i} g_{i} c_{i} \frac{\Delta m_{i}^{2}}{2 u} T_{m u c}^{2} \quad g_{i}=\# d o f, \quad c_{i}=\left\{\begin{array}{l}
1 \\
\frac{B}{F}
\end{array} \quad\left[B r^{\prime}, \partial g\right]\right.
$$

(2) In presace of unixing between light and heavy particles

$$
\left.\Delta P_{L 0}^{\text {rising }}=\frac{T^{2} y^{2}}{4 p}\langle\phi\rangle^{2} \theta\left(\gamma_{w} T_{m c}-M^{2} L_{w}\right) \quad \mathcal{L} y \phi \bar{\psi} N+\text { hic. [AA et al, }, 21\right]
$$

(3) In presence of gauge boson which jain non during PT

$$
\Delta P_{N L O} \simeq \sum_{i} \frac{g_{i}}{16 \pi^{2}} g_{\text {gauge }}^{2} M_{v_{i,}} \gamma_{w} T_{\text {mc }}^{3}
$$

$$
[B H, 17]
$$

(1) + (2) Only

$$
\text { (1) }+(2)+3
$$

if $\Delta V>\Delta P_{\text {tor }}$ the bubbles will keep eccelerding until th a collision (runaway)

Vector bosons partecipating the $P T$, Equating friction ad shriving force gives terminal $\gamma_{w}$

$$
\gamma_{w, \text { mex }}=\frac{2 R_{*}}{3 R_{0}}\left(1-\frac{\Delta P_{b}}{\Delta V}\right) \sim \frac{M_{p e} T_{m c}}{\langle\phi\rangle^{2}}
$$

where $R_{*}=\frac{\left(\left.8 \pi\right|^{1 / 3} v_{w}\right.}{\beta\left(T_{\text {pac }}\right)}, R_{0} \sim 1 / T_{\text {inc }}$

$$
\gamma_{n}^{\text {tuinal }} \simeq \frac{\Delta V-\Delta P_{\text {lo }+ \text { mixing }}}{g_{g q_{x}}^{2} \mu_{v} T_{\text {mun }}^{3}}
$$

Note the horectr factor $\gamma_{\text {max }}$ con be computed equating the surface energy of the bubble to the pain in the potentid energy

$$
\begin{gathered}
4 \pi R_{*}^{2} \gamma_{\text {max }} \sigma=\frac{4}{3} \pi R_{*}^{3}\left(\Delta V-P_{10}\right) \\
\gamma_{\text {max }}=\frac{R_{*}}{3 \sigma}\left(\Delta V-P_{L_{0}}\right)
\end{gathered}
$$

We can find the critical looking extrevizing the action

$$
R_{c}=\frac{2 \sigma}{\Delta V} \Rightarrow \gamma_{\text {mex }}=\frac{2 R_{x}}{3 R_{c}}\left(1-\frac{P_{1 \sigma}}{\Delta v}\right)
$$

(2) Why: stochastic GW beckgraund \& application

If bubbles collide they con produce a stochastic GW bockgrand from

1. bubble collision
2. sand waves
3. turbuluce

$\Rightarrow$ primordial GiNs could be observed soon (if they exist)


We can produce heovy states if the wall ore ultro-relativitic

$$
\begin{gathered}
P_{\psi} \simeq(\gamma T, 0,0, \gamma T) \quad P_{\phi}=\left(\mu_{\phi}, 0,0,0\right) \\
\sqrt{S} \sim M_{N} \sim \sqrt{\gamma T \mu_{\phi}} \quad>\mu_{\phi, T}
\end{gathered}
$$

Applications

1. Heavy DM production
2. FOTP $+C / C D$-vidating \& $B$-violating interaction w/ heovy states $\Rightarrow$ Baryogenasis
(3) Vetze elevistic FOPT: $\mathbb{Z}_{2}$ nol siught extension of the SM

Ain: find general features for relativistic FOPT $k$ explicit redisation of relefivistic EWPT.
Well Known that all the PT in the Sie ore $2^{\text {ned }}$ ovolen, we consider the simplest exteusion of the SM W/ FOPT $\Rightarrow S M+$ ral $z_{2}$ singlet

$$
\left(\lambda_{5}=\frac{u_{2}^{2}}{2 v_{5}^{2}}\right)
$$

$$
V_{0}(H, S)=\underbrace{-\frac{\mu_{n}^{2}}{2} H^{+} H+\lambda\left(H^{+} H\right)^{2}}_{\text {Hizgs mexicen pot. }}-\underbrace{\left(\mu_{s}, v_{s}, \lambda_{n s}\right)}_{\text {vel singlet }}+\underbrace{\frac{\mu_{s}^{2}}{4} s^{2}+\frac{\lambda_{s}}{4} S^{4}}_{\text {fre promering }}+\underbrace{\frac{\lambda_{n s}}{4} s^{2}\left(H^{+} H\right)}
$$

Fivite teup. 1foctive potertiel
It can be computed in ditthert ways
a) Frou Euclidean YI caupectifing time of $\beta=1 / T$ scuning aver Matscehara undes
b) Quoutum conectious + free energy of reativistic masnive paticles field-depenclut uns

$$
V_{C W}\left(M_{i}^{2}(|\phi|)=(-1)^{F_{i}} g_{i}\left[\frac{m_{i}^{4}(\phi)}{64 \pi^{2}}\left(\log \frac{\mu_{i}^{2}(\phi)}{m_{i}^{2}\left(v_{t}\right)}-\frac{3}{2}\right)+2 m_{i}^{2}(\phi) m_{i}^{2}\left(v_{p}\right)\right] \quad\right. \text { (t)ou-shell wu. shi. }
$$

$$
V_{T}\left(m_{i}^{2}(\phi)\right)=(-1)^{F i} \frac{g_{i}}{2 \pi^{2}} T^{4} J_{B F}\left(\frac{m_{i}^{2}(\phi)}{T^{2}}\right)
$$

(*) Note an an shall wen. schume
) Isrue w/GB sinch $H_{G_{B}}^{2}(V)=0$ $\left.\frac{d V_{\text {ef }}}{d h}\right|_{(h, s)=\left(V_{\text {Ew }}, 0\right)}$ $=\left.0 \quad \frac{d^{2} v_{V} t}{d h^{2}}\right|_{T V}=m_{h}^{2}$ $\frac{d h_{l+t}}{d h^{2}}=\omega_{n}^{2}-\sum\left(p^{2}=\mu_{n}^{2}\right)+\sum(0)$ in thus may $\mathrm{H}_{G^{+}+\infty}^{2}(T V) \longrightarrow m_{n}^{2}$ ouly iathe lof

The thernal enosses in the TFD poocedune ore $\pi_{i}(T)=c_{i} T^{2}$ :

$$
\begin{aligned}
& \pi_{h}(T)=\left(\frac{3 g^{2}}{16}+\frac{g^{\prime 2}}{16}+\frac{\lambda}{2}+\frac{y_{t}^{2}}{4}+\frac{\lambda_{n s}}{24}\right) T^{2} \\
& \pi_{s}(T)=\left(\frac{\lambda_{n s}}{6}+\frac{\lambda_{s}}{4}\right) \\
& \pi_{g}^{l}=t^{2} \text { ding }\left[\frac{11}{6} g^{2}, \frac{11}{6}\left(g^{2}+g^{\prime 2}\right)\right] \quad \pi_{g}^{T}=0 \text { guse iv. } \\
& \pi_{f}(T) \xrightarrow[0]{ } \text { chival symunetuy protection }
\end{aligned}
$$

Other, nore ufined, resunatiou schemes ore


- OPtivized Partiel Dresineg [Piecewise JB/F $\alpha /$ Eop equatious]

1-step vs 2-step

- $(0,0) \longrightarrow\left(V_{E w, 0}\right)$
(well studied, and 1d)
Relativistic bubble one uulikely

$$
V_{\text {dou }}(h \rightarrow 0, S=0, T) \sim \frac{M_{f k}^{2}(T)}{2} h^{2}+\ldots
$$

FOPT ouly for $\mu^{2} e_{t}>0$

$$
T_{\text {mic }}^{\text {min }} \geqslant 100 \mathrm{GeV} \Rightarrow \gamma_{w} \leq 10
$$

- $(0,0) \longrightarrow\left(v_{t w_{1}} v_{s}\right)$

Very coustraived frou exp via Higps - Scalar uixing!


- $(0,0) \longrightarrow\left(0, v_{s}\right) \longrightarrow\left(v_{E w}, 0\right)$
$\qquad$

$$
V_{\text {eH }}(h, s, T) \stackrel{\text { high } \cdot T}{\approx}\left(-\frac{m_{n}^{2}}{4}+c_{n} T^{2}\right) h^{2}+\frac{\mu_{n}^{2}}{8 v_{E \omega}^{2}} h^{4}+\left(-\frac{m_{s}^{2}}{4}+c_{s} T^{2}\right) s^{2}+\frac{m_{s}}{8 v_{s}^{2}} s^{4}+\frac{\lambda_{s h}}{4} s^{2} h^{2}
$$

Whot we need for 2-step PT (with 1. SOPT \& 2. FopT)
(1) Covect vecuum at $T=0$ : $\mu_{s}^{2} v_{s}^{2}<\mu_{h}^{2} v_{E N}{ }^{2}$
(2) fist step $\left.(0,0) \xrightarrow{\text { SopT }}\left(0, v_{s} \neq 0\right) \quad T_{\langle s\rangle \neq 0}\right\rangle T_{\langle n\rangle} \neq 0 \quad\left[\left.\frac{d^{2} V_{e f t}}{d s^{2}}\right|_{(0,0\rangle}>0\right]$

This cen he obtained cosily playing w/ thermal enases since $c_{s}<c_{n}$.
(3) Second step $\left(0, v_{s} \neq 0\right) \xrightarrow{\text { FORT }}\left(v_{E v, 0}\right)$ : it is $1^{s+}$ order if there is a potential bonier in $h$-direction

$$
\frac{\partial^{2} V_{1 A}}{\partial h^{2}}>0 \quad T^{N_{B}}=\sqrt{\frac{\mu_{n}^{2}-\lambda_{n s} V_{s}^{2}}{u C_{n}}}
$$

dhs controls the site of the bonier, and if $\lambda_{n s}>\frac{m_{n}{ }^{2}}{v_{s}^{2}}$ then the FV $\left(O, v_{s}\right)$ is a local eninicun ever at ${ }^{v_{s}^{2}} T=$ ?
(4) Computation of $S_{3 / T}$

The Euclidean action S3/T contains all the infos we need to study the PT.

The PT does not fellow - straight live, need to develop an cole.


Algorithm
We split the 20 coupled ears

$$
\left\{\begin{array}{l}
\frac{d^{2} \stackrel{\rightharpoonup}{\phi}}{d r^{2}}+\frac{d-1}{r} \frac{d \vec{\phi}}{d r}=\vec{\nabla} V(\vec{\phi}) \\
d \vec{\phi} /\left.d r\right|_{r=0}=0 \\
\lim _{r \rightarrow \infty} \vec{\phi}(r)=F V
\end{array}\right.
$$

along the parallel and repandicular direction. In order to do this wi can guin a path, $\vec{\phi}_{g}(h, s)$, pranieflize it of $(t, f(t)) \equiv(h, s(h))$, then we can introduce the cuiviliver abscissa

$$
x(h)=\int_{h_{F v}}^{h} \sqrt{1+\left(\frac{d S\left(h^{\prime}\right)}{d h^{\prime}}\right)^{2}} d h^{\prime}
$$

then the work w/ the guened path because

$$
\left\{\begin{array}{l}
\frac{d^{2} x}{d r^{2}}+\frac{d-1}{r} \frac{d x}{d r}=\partial_{x} V\left[\phi_{g}(x)\right] \\
\frac{d^{2} \stackrel{\rightharpoonup}{g}^{g}(x)}{d x^{2}}\left(\frac{d x}{d r}\right)^{2}=\vec{\nabla}_{\perp} V\left[\vec{\phi}_{g}(x)\right]
\end{array}\right.
$$

Id twaling w overshot / under shoot method

The second eq can be seen as a coudition that the bouce Solution hos to sotisty and can be thought os a force field acting en the poth

$$
\vec{N} \equiv \frac{d \vec{\phi}_{g}(x)}{d x^{2}}\left(\frac{d x}{d r}\right)^{2}-\vec{\nabla}_{1} V\left[\vec{\phi}_{g}(x)\right]
$$

Onve we couputed the bance achoraling to om guened peth, we moolity the guened peth acconding to $\vec{N}$ and we iteratively do so until the peth is no lougen
modifical.
Features of $S_{3} / T$
In computing the Euclideser ection we found 3 diffenent behevien
(1) the bamien disoppeas © $t=0$
(2) the bavier unais eveu © $T=0$, i.e. the $\mp_{v}$ is a local unimun © $T=n$
(3) Seve as (2) but the FV @ T=0 is disploced freu its high-T velue
(1) Since the bonier disappoors © $T=0$ the PT hos to complete betac $T^{N B}$ and $S_{3} / T \rightarrow 0$, so $\Gamma \rightarrow 1$.
(2) Since the bonier remeing even (e) $T=0$ at same point the teup. don't ploy ony wole, then $S_{3} \xrightarrow[T \rightarrow 0]{ }$ const ond coure prently $S_{3} / T T_{T \rightarrow 0}{ }^{\infty}$
(3) In this cox we fand that in a certein regien of an poraneter spoce at sufficiently low temp the ectien stants to be constent
es long os $T$ is decreosing, but at save point e cancellation happens between the mixing and the CW potential froe the top and the FV shift a bit.
This shift cause a sudoleu decrease of the action, that in save cor could allow the system to moke the PT. In then core we con achieve unchotion temperatures of low of 1 GeV , leading to $Y_{w}$ up to $10^{5}$.


NoTE (Cunculletion @ T=0)
very bow T
There is a region of parameter spore when purely polyusuid potential hos co local unicicun at $\left(0, v_{s}\right)$, the IV but the effect froe the CN Contribution of the top field does the job

$$
V_{\text {CW }}^{t} \propto-\frac{3 M_{t}^{2}(h)}{8 \pi^{2}} \log \frac{M_{t}^{2}(h)}{M_{t}\left(V_{E} w\right)}
$$

Only a log contribution ceulad hove consed e shift of the vecum, nat a policumiel
there the FU Shift towards e new milium
(2) $\left(\delta v_{h}, v_{5}+\delta v_{5}\right)$.

We con rec that the Minima ore reaves, so len perth to do in field spec. It is for this voson that 53 decreoses.
(5) Poraviter Scen \& result
(a) Regions of the Scou
I. SOPT: there is never a bonier separating the tus minime
II. FOPT (W/act we wolls)
III. Vetre uelativistic FOPT ( $\gamma_{w} \gg$ inveasing $\lambda_{n s}$ a fixed $v_{s}$ )
IV. No $\mathbb{Y T}$ : the system venains stuck in the FV and wever uicleate.

$$
\left[M_{S}=125 \mathrm{GeV}\right] \quad M_{s}\left(v_{E W}, 0\right)[\mathrm{GeV}]
$$

$$
\begin{array}{lllllll}
35.6 & 65.5 & 85.5 & 102 . & 116 . & 128 . & 139
\end{array}
$$



NOTE supa five twed urion bxtwen $T^{N B}=0$ unve and "NO PT" Culve $\delta \lambda_{n s} \sim o\left(10^{-4}\right), 10^{-2}$ smaller thou the full regien
(b) Hexiual mess produced $\quad M_{\text {max }} \simeq \sqrt{\gamma_{\omega} T_{\text {muc }} V_{E w}}$

Works for
$M_{s}\left(v_{E W}, 0\right)[\mathrm{GeV}]$

$$
M_{s}\left(v_{E W}, 0\right)[\mathrm{GeV}]
$$

$$
\text { oofoll }{ }_{\text {ond }}^{10^{6}}
$$


(b) Applicatien to DM productiec \& EN Banjo genesis
 particles diming ploma - bubble wall callision can be wolited

1. Fenmiaric trewsition $\quad$ diut $\supset-Y h \bar{N} g-H_{N} \bar{N} N$ where $q_{i}$ light, $N:$ heory, $h=\tilde{h}+v$

$$
P(q \rightarrow N) \simeq \frac{\left.\frac{Y^{2} v^{2}}{H_{N}^{2}} \quad \theta\left(\gamma_{w} T_{m c}-M_{N}^{2} L_{N}\right) \quad \theta\left(P_{t}-M_{N}^{2} l_{w}\right)\right)}{}
$$

2. Scalor transition $\quad$ Liut $\partial-\frac{\ln \phi}{2} \phi^{2} h^{2}+\frac{1}{2} \mu_{\phi}^{2} \phi^{2} \quad h=\tilde{h}+v$

$$
P(h \rightarrow \phi \phi) \approx \frac{1}{2 \pi^{2}} \frac{\lambda_{\phi}^{2} \sigma^{2}}{H_{\phi}^{2}} \theta\left(\gamma_{\omega} T_{u m c}-M_{\phi}^{2} l_{\omega}\right)
$$

Now we will opply verlt for bubble velocity to bM model building
(2) Solar DH caupled w/ Higgo

We arsume heoy scalon $\phi$ coupled to $S M$ vie Higgs portse

$$
\alpha_{D r}=\frac{1}{2}(\partial \phi)^{2}-\frac{1}{2} \mu_{\phi}^{2} \phi^{2}-\frac{\lambda \phi h}{2} h^{2} \phi^{2}
$$

wher the SM fild cen be stelalited imposing a $\mathbb{Z}_{2}^{\dagger}$ sgumer. After the Higzs trountion the abudace of massive $\phi, x_{\phi}^{\text {Be }}$, is giveln by emissien of $2 \phi$

$$
n_{p}^{B E} \approx \frac{2}{\gamma_{w} v_{w}} \int \frac{d^{3} p}{(2 \pi)^{3}} \frac{p_{z}}{p_{0}} f_{h}\left(p, T_{m c}\right) \cdot P(h \rightarrow \phi t) \quad \text { [woll et frome] }
$$

This strougly depends on the deunity of the Hipgs field avoliable at the ruclestiou temperatine, $f_{n}(p, T u c)$. The relvent porenater will be the ratio

$$
\left.\frac{1}{T} \sqrt{\frac{d^{2} V}{d h^{2}}}\right|_{F V}=\frac{\mu_{H}^{\text {folk }}}{T}
$$

Since os soon os $m_{s}^{+} \kappa>1$ we have the usual Boltamome suppression

$$
\int \frac{d^{\prime} P}{(2 \pi)^{3}} f_{n}\left(P, T_{m c}\right) \simeq \begin{cases}\frac{3(3)}{\pi^{2}} T_{m c}^{3} & \\ \left(\frac{\mu_{i}^{f} T_{m c}}{2 \pi}\right)^{3 / 2} & \mu_{++}^{+}<T \\ -\mu_{\# 1}^{f} / T_{m c} & \mu_{+1}^{f}>T\end{cases}
$$

Here we hove

$$
f_{n}\left(p_{1} T_{m c}\right)_{E_{h}=\sqrt{p_{z}^{2}+p_{L}^{2}}}=\left\{\exp \left[r_{w}\left(\frac{E_{n}-v_{w} p_{t}^{h}}{T_{m c}}\right)\right]-1\right\}^{-1} \simeq \exp \left[-r_{w}\left(\frac{E_{n}-v_{w} p_{t}^{h}}{T_{m c}}\right)\right]
$$

and the presence of the $\theta[$.$] that changes a bit the things$

$$
\begin{aligned}
& \int \frac{d P}{(2 \pi)^{3}} \frac{P_{z}}{P_{0}} f_{n}\left(P_{1}, T_{\text {inc }}\right) \theta\left(P_{7}-M_{\phi}^{2} / v\right)=\int \frac{d P_{t}}{2 \pi} \frac{d P_{t}^{2}}{4 \pi} \cdot e^{-\frac{Y_{w}}{\frac{\sqrt{P_{1}^{2}+P_{7}^{2}}-v_{w} P_{z}^{h}}{T_{w u c}}} \theta\left(P_{7}-M_{\phi}^{2} / v\right)} \\
& =\int \frac{d P_{t}}{2 \pi} \frac{2}{4 \pi}\left(1+\frac{P_{7} r_{w}}{T_{\omega c}}\right) \cdot \frac{T_{m c}^{2} c}{\gamma_{\omega}^{2}} e^{-r_{w}\left(1-v_{w}\right) P_{t}} T_{m c} \quad \theta\left(P_{7}-\mu_{\phi / v}^{2}\right) \\
& =\frac{1}{4 \pi^{2}} \cdot \frac{T_{\mu c}^{2}}{\gamma_{\omega}^{2}} e^{-\frac{\gamma}{\tau} \frac{M^{2}}{v}\left(1-v_{v}\right)}\left[\frac{M^{2}}{V\left(1-v_{\omega}\right)}+\frac{T}{\gamma_{\omega}} \frac{2-v_{\omega}}{\left(1-v_{\omega}\right)^{2}}\right]
\end{aligned}
$$

$$
\begin{aligned}
& \gamma_{\omega}\left(1-v_{\omega}\right)=\gamma_{\omega}-\sqrt{V_{\omega}^{2}-1} \longrightarrow \frac{1}{2 \gamma_{\omega}} \quad 1-v_{\omega}=1-\sqrt{1-1 / r_{\omega}^{2}} \simeq \frac{1}{2 \gamma_{\omega}^{2}} \\
& =\frac{T_{\omega}^{2} c}{4 \pi^{2} r_{\omega}^{2}}\left[\frac{\mu^{2}}{V} \cdot 2 r_{\omega}^{2}+T \cdot 2 r \cdot 2 r_{\omega}^{2}\right] e^{-\frac{\mu^{2}}{T} 2 \gamma_{\omega v}} \\
& \left\{\left.M_{B E}^{k}=\frac{2}{\gamma_{w} V_{w}} \frac{1}{2\left\langle\pi^{2}\right.} \frac{\lambda_{\phi_{b}}^{2} \sigma^{2}}{H_{\phi}^{2}}[\ldots] \right\rvert\,=\gamma_{w} \cdot T_{w M c}^{3} \exp \left[-\frac{N^{2}}{2 \gamma_{w} T_{m c}}\right]+O\left(\gamma_{w}^{0}\right)\right.
\end{aligned}
$$

After rd shifting to today the stable produced adbundence tokes the form

$$
\begin{aligned}
& \Omega_{B E, \phi}^{\text {today }} h^{2}=\frac{\rho_{\phi}}{\rho_{c} / h^{2}} \cdot\left(\frac{a\left(T_{\text {uh }}\right)}{a\left(T_{0}\right)}\right)^{3}=\frac{\mu_{\phi} \mu_{\phi}^{B E}}{\rho_{c} / h^{2}} \frac{g_{*, S}^{0} T_{0}^{3}}{g_{*,}\left(T_{\text {th }}\right) T_{w h}^{3}}
\end{aligned}
$$

$$
\begin{aligned}
& P_{c}=1.054 \cdot 10^{-5} \mathrm{~h}^{2} \mathrm{GeV} \mathrm{~cm}{ }^{-3}, g_{*, 5}^{\text {trades }} \simeq 3.94, T_{0} \simeq 0,24 \mathrm{meV}
\end{aligned}
$$

This expression has to be supplemented w/ Fo contu-butien

due \% the brief stage of inflation dining the PT.
Then

$$
\Omega_{\phi, \text { tot }}^{\text {today }}=\Omega_{\phi, B E}^{\text {today }} h^{2}+\Omega_{\phi, \neq 5}^{\text {today } h^{2}} \simeq 0.1
$$

Features of the plot

1. Inside the isocoutouns under-producel outside the isocantoun over-poduced.
2. Upper curve: or production dominated by BE. Stephen due
to the fact that $\exp \left[-\mu_{\phi}^{2} / 2 \alpha_{\omega} V_{\text {en }} T_{u c}\right\rceil \ll 1$ in all the ugion
3. Lower curve: DM production docuninated by Fo $^{2}$
4. Vertical line comecting the two:
 thenual production ster reheating.
$\Rightarrow$ So in geneal the ucodel predictslage over production of DM in BE unlen Belttmoun supperion plays a cole.
Note Cu s be DM) (No!)
Suppose very precise $\mathbb{Z}_{2}$, after PT T~hobev, the singlet is in thermal ap, thu e frees Fo
(b) Siught partol sme

DH coupling w/ siuglet pertel

$$
\mathcal{L}_{\text {iut }}>, \frac{\lambda_{s \phi}}{2} \phi^{2} s^{2}+\frac{1}{2} \mu_{\phi}^{2} \phi^{2} \quad P\left(s-\phi^{2}\right) \approx\left(\frac{\lambda_{s}+v_{s}}{\mu_{\phi}}\right)^{2} \frac{1}{2 k \pi}, \theta\left(r_{u} T_{m e}-M_{\phi}^{2} L_{w}^{s}\right)
$$

Hew diftenkece wnt. pevions cose is that the singlet in the IV is mosive, then Boltzeneun sypromion ploys en impertent vole. This mokes a shitt to the lett in the plot.
(C) Fenmen medietod DM

Here we counider a moshlen Femiaric BM patide in the symm. phoal, so by definition it doer not suttren from Boltzaoun sypressiou.

$$
\mathcal{L}=\mathcal{L}_{S H}+Y_{4} \bar{L} H N+M_{N} \bar{N} N+Y_{O M} \bar{N} \chi \phi
$$

L.H: lepton, Higgs doublet

N: vector-like rentral fermian (singlet under SM) (heovy)
x: Semien $\left.\begin{array}{ll}\phi: S c a l a r\end{array}\right\} x_{2}$ oold $\}$ sector

$$
P^{\text {teen }}(L \rightarrow N)=\frac{Y_{N}^{2} V_{E N}^{2}}{M_{N}^{2}} \theta\left(\gamma_{u} T_{\text {unc }}-M_{N}^{2} L_{v}\right)
$$

then unstable heovy $N$ acunlate betriad the wall as

$$
\begin{aligned}
M_{N}^{B E} & \simeq \frac{Y_{*}^{2} v_{E w}^{2}}{M_{N}^{2}} \cdot \frac{1}{\gamma_{u} V_{w}} \int \frac{d^{3} P}{(2 \pi)^{3}} \frac{P_{t}}{P_{0}} \times f_{L}\left(p_{1} T_{u c}\right) \theta\left(P_{t}-M_{N}^{2} / V_{E N}\right) \\
& \simeq \frac{Y_{*}^{2} v_{E w}^{2}}{2 \pi^{2} M_{N}^{2}} T_{w c}^{3} e^{-\frac{M_{N}{ }^{2} / 2 r_{w} v_{E a} T_{m i c}}{}+O\left(1 / r_{w}\right)}
\end{aligned}
$$

them the abundance of $x, \phi$ after the trousitisien is supposed by

$$
\begin{aligned}
M_{\phi} \simeq M_{X} & \simeq \frac{Y_{D M}^{2}}{Y_{D n}^{2}+Y_{d}^{2}} M_{N}^{B E} \\
& \simeq \frac{Y_{D M}^{2}}{Y_{D D}^{2}+Y_{d}^{2}} \frac{Y_{*}^{2} v_{E w}^{2}}{2 \pi^{2} M_{N}^{2}} T_{u c}^{3} e^{-M_{N}^{2} / 2 \Gamma_{w} V_{E Q} T_{m C}}
\end{aligned}
$$

and the final relic adbundeen redshiffed to today roods

Fa the freeze-out we hove $\phi \phi \rightarrow L H L H$ by neglecting co-amili elation. The coss section is highly phon space supposed

$$
\sigma_{\phi \phi \rightarrow(L H)^{*} L H} \sim \frac{M_{x}^{2}\left(y_{M M} y_{K}\right)^{L}}{\left(1 G \pi^{2}\right) 4 \pi \mu_{N}^{4}}
$$

then the adburdace today is

$$
\Omega_{\phi F_{0}}^{\text {today }} h^{2}=10^{3}\left(\frac{T_{\mu c}}{T_{\text {Th }}}\right)^{3} \frac{M_{N}^{4} \mid M_{x}^{2}}{\left(6 T_{e V}\right)^{2}} \frac{10}{\left(\left.Y_{\text {pM }} Y_{k}\right|^{4}\right.}
$$

The tool density is the sum of the two. This scenario leads to the over production of $D M$ arlen $M_{\phi}, M_{x} \leq 10 \mathrm{GcV}$.

Note Valid only for heovy SM candidates which do hot go beck to equilibrime of ter the PT. Otherwise we need to tore (Tunc/Tunl' out from the esticuete.

Let us how ivvertigate the regive $w \mid M_{\phi} \simeq M_{x}$, precisely $\left|H_{\phi}-M_{x}\right|<M_{\phi} / 2$ 。 when the co-amibilation takes place. In this cox we have

$$
\sigma_{\phi x \rightarrow A \bar{L}}-\frac{\left(Y_{D M} Y_{\psi}\right)^{2}}{k \pi M_{N}^{2}}
$$



Therefore we hove

$$
\Omega_{\phi_{1}=0, c_{0}}^{\text {today }_{0}} h^{2} \sim 0.1\left(\frac{T_{\text {Inc }}}{T_{\text {eel }}}\right)^{3} \cdot \frac{M_{N}^{2}}{(\text { lotev })^{2}} \frac{1}{\left(Y_{\text {pry }} Y_{k}\right)^{2}}
$$

Suing all of this we find that become possible to reproduce the observed SM abundance.
BE tends to over produce the DM and the relic estudance freon BE conn be reproduced of $\exp \left[-M_{n}^{2} / 2 \gamma_{n} V_{E w} T_{\text {Inc }}\right]$ stats playing e vol in Suppretring DM utic dewnity.

NOTE

then for the abundoun

$$
\Omega h^{2} \simeq \frac{M_{x} Y(\infty) S_{0}}{\rho_{c} / h^{2}} \sim \frac{1}{\langle\sigma\rangle}
$$

since $Y(\infty) \sim g_{x}^{-1 / 2} \frac{7_{t \cdot 0}}{H_{p e} M_{x}} \cdot \frac{1}{(\sigma v)}$.
(7) Couclusious

1. Fint explicit valisetion of ultre ulativistic FOPT for Ew Boryogeuexis \& PM production
2. Singlet extenion of $S M$ W/ 2 step PT

$$
(0,0) \xrightarrow{\text { SOPT }}\left(0, v_{s}\right) \xrightarrow{\text { Fopt }}\left(v_{E \omega, 0}\right)
$$

3. Tur os low as $1-2 \mathrm{GeV}$
4. Mecharism most Aticient for $M_{s}\left(v_{i u, 0}\right) \sim 70-100 G \cdot V$ It will be probed by HL-LHC
