Gravitational waves at Strong Coupling with Holography

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What The Heck Happends When The Universe Boils?

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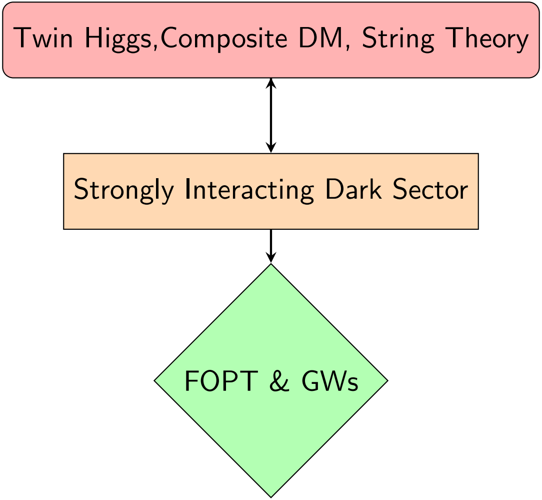
Outline

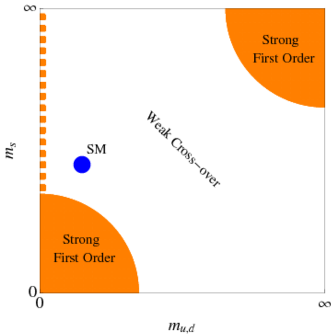
- Motivation for Pure SU(N) Yang-Mills
- Confinement PT in Pure Yang-Mills
  - Holography
  - Lattice Thermodynamics
- Gravitational wave signal
  - How it is predicted
  - The Predicted signal & Uncertainties
- Outlook

Motivation


Twin Higgs,Composite DM, String Theory
Strongly Interacting Dark Sector
FOPT & GWs

Confinement in Dark SU(N) & Strong Coupling

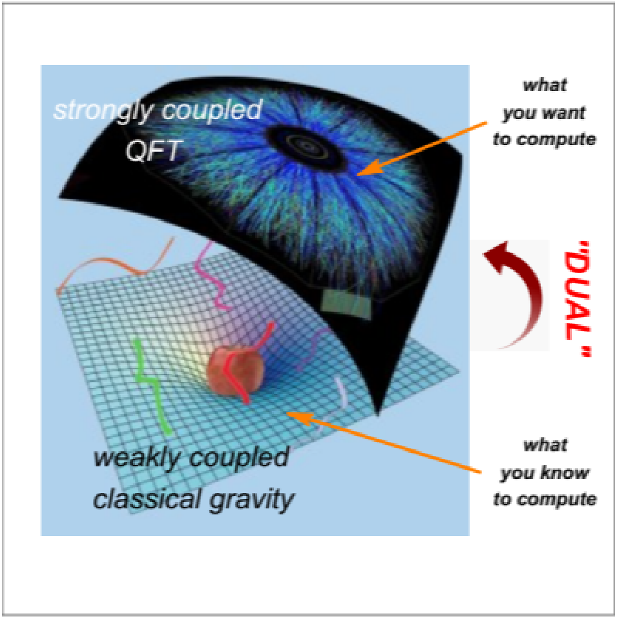
- Confinement PT first order for many choices of N_c, N_f.
- Focus on Dark SU(3)
  - "Simplest" Case
  - Lattice Data
  - Holographic Model
- Strong coupling?
  - Lattice, no Finite T real time simulations
  - EFT approach (NJL/PLM)



Schwaller 2015

AdS/CFT: Good choice of variables suitable for strong coupling!

Good choice of variables suitable for strong coupling!



AdS/CFT Crash Course

$$e^{W_{CFT}[\phi_0]} = \left\langle e^{\left(\int d^4x \phi_0(x)\mathcal{O}(x)\right)} \right\rangle = \mathcal{Z}_{string}[\phi(x,z)|_{z\to 0} = \phi_0(x)]$$

- Main interest in low energy physics in which the following limits are applicable

- $g_s \to 0$
- $\frac{l_s}{L} \to 0$
- $g_{YM}^2 N_c \to \infty$
- $N_c \to \infty$

$$\mathcal{Z}_{CFT} \simeq \mathcal{Z}_{gravity} = e^{-S_{os}[\phi_0]}$$

- Here we will employ AdS/CFT in a bottom up fashion
- AdS_5 Einstein Dilaton Gravity $\longleftrightarrow$ 4D SU(N) Yang-Mills

Improved Holographic QCD

5D Einstein dilaton gravity in AdS $\rightarrow$ Large N_c Yang-mills

$$\mathcal{S}_5 = -M_p^3 N_c^2 \int d^5x\sqrt{g}(R - \frac{4}{3}(\partial\Phi)^2 + V(\Phi)) + 2M_p^3 \int_{\partial\mathcal{M}} d^4x\sqrt{h}\mathcal{K},$$

- $V(\Phi)$ dilaton potential
- 5-D coordinate r $\Longleftrightarrow$ RG scale
- Dilaton $\lambda = e^{\Phi}$ $\Longleftrightarrow$ t'Hooft coupling $\lambda_t = N_c g_{YM}^2$
- Scale Factor $b(r)$ $\Longleftrightarrow$ Energy scale $E = E_0 b(r)$
- AdS-BH/Thermal AdS $\Longleftrightarrow$ Phases of SU(N_c)
- $g_{\mu\nu}$ perturbations $\Longleftrightarrow$ $T_{\mu\nu}$, λ fluctuations $\Longleftrightarrow$ Scalar Glueballs

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Choice of $V(\lambda)$ Based on SU(N) Pure YM Theory

UV $\lambda \to 0$

Deconfined

Asymptotic freedom!

YM β Function at two loops

$$\frac{d\lambda_t}{d\log E} = -\beta_0\lambda_t^2 - \beta_1\lambda_t^3 + \ldots$$

$$\beta_0 = \frac{22}{3(4\pi)^2}, \; \beta_1 = \frac{51}{121}\beta_0^2$$

Logarithmic Running!

IR $\lambda \to \infty$

Confinement (Order Parameter), Polyakov loop VeV at Finite T

$$W_p = Tr\left(\mathcal{P}exp\left(\int_C A_\mu dx^\mu\right)\right) = \begin{cases} \neq 0\,, \\ 0. \end{cases}$$

Mass Gap

$$m_g^2 > 0$$

Linear Glueball Spectrum

$$m_n^2 \propto n$$

$V(\lambda)$ in the UV

Geometry asymptotes to pure AdS_5 for $r \to 0$

$$b(r) = \frac{l}{r}\left(1 + \frac{4}{9}\frac{1}{\log(\Lambda r)} - \frac{4b_1}{9b_0^2}\frac{\log(-\log(\Lambda r))}{\log^2(\Lambda r)}\right)$$

$$b_0\lambda(r) = -\frac{1}{\log(\Lambda r)} + \frac{b_1}{b_0^2}\frac{\log(-\log(\Lambda r))}{\log^2(\Lambda r)}$$

Identify $b \Longleftrightarrow E$, $\lambda \Longleftrightarrow \lambda_t$ to write Holographic β Function

$$\beta = \frac{d\lambda}{d\log E} = -b_0\lambda^2 - b_1\lambda^3 + \ldots = \beta(\lambda_t) \to \begin{cases} \beta_0 = b_0\,, \\ \beta_1 = b_1\,. \end{cases}$$

$$V(\lambda) = \frac{12}{l^2}(1 + v_0\lambda + v_1\lambda^2 + \ldots),$$

$$b_0 = \frac{8}{9}v_0, \qquad b_1 = \frac{9}{4}v_1 - \frac{207}{256}v_0^2.$$

$V(\lambda)$ in the IR

Glueballs:

$$\mathcal{S}[\xi] \sim \int drd^4x\, e^{2B(r)} \left((\partial_r\xi)^2 + (\partial_i\xi)^2 + M^2(r)\xi^2\right)$$

$B(r)\, M(r)$ depends on the background and type of fluctuation.

Confinement: Area Law Wilson Loop

$$TE(L) = \mathcal{S}_{NG}[X^\mu_{min}(\sigma,\tau)]$$

$$S_{NG} = \frac{1}{2\pi l_s^2}\int_C d\tau d\sigma det\left(\sqrt{-g_S}\right)$$

$$V(\lambda) \sim \lambda^{\frac{4}{3}}\left(\log(\lambda)\right)^{\frac{1}{2}}$$

The potential $V(\lambda)$ and It's Parameters

UV Asymptotics

$$V(\lambda) = \frac{12}{l^2}(1 + v_0\lambda + v_1\lambda^2 + ...)$$

IR Asymptotics

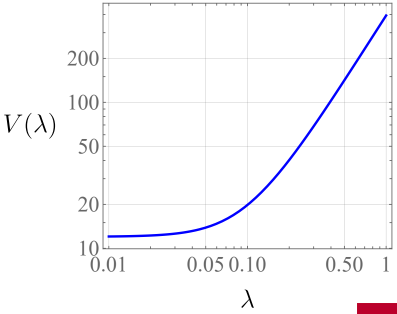
$$V(\lambda) \sim \lambda^{\frac{4}{3}} (\log(\lambda))^{\frac{1}{2}}$$

V_0 V_2 YM beta function 2-loops

V_1, V_3 Phenomenological parameters

V_1 Free gas asymptotics, V_3 Latent heat

$$V1 = 170, \qquad V3 = 14.$$



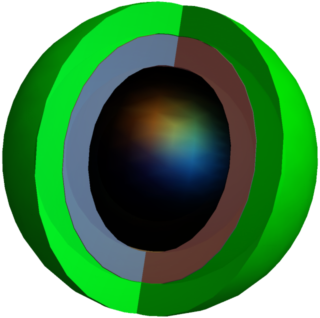
$$V(\lambda) = \frac{12}{l_{AdS}^2}\left(1 + V_0\lambda + V_1\lambda^{\frac{4}{3}}(\log[1 + V_2\lambda^{\frac{4}{3}} + V_3\lambda^2]^{\frac{1}{2}})\right)$$

Finite Temperature Solutions I

Thermal Gas AdS $\rightarrow$ Confined Phase

$$ds^2_{TH} = b_0^2(r)\left(dr^2 - dt^2 + dx^m dx_m\right).$$

Schwarzchild-AdS Black Hole metric $\rightarrow$ Deconfined phase



$$ds^2_{BH} = b^2(r)\left(\frac{dr^2}{f(r)} - f(r)dt^2 + dx^m dx_m\right).$$

UV Asymptotics for the TG_{AdS} and BH_{AdS} are equal

Finite Temperature Solutions II

$$\mathcal{S}_5 = -M_p^3 N_c^2 \int d^5x \sqrt{g}\left(R - \frac{4}{3}(\partial\Phi)^2 + V(\Phi)\right) + 2M_p^3 \int_{\partial\mathcal{M}} d^4x \sqrt{h}\mathcal{K},$$

Operator & Field Correspondence

$$\mathcal{Z}_{CFT} \simeq \mathcal{Z}_{gravity} = e^{-S_{os}[\phi_0]}$$

4D	5D
Finite T "QFT" YM Wick rotation of Time Direction	$T_h = \frac{\lvert\dot{f}(r_h)\rvert}{4\pi} = T_{AdS}$ Regular BH Solution
QGP in Thermal Eq	Area Law BH Horizon
Entropy S	$S = 4\pi M_p^3 N_c^2 V_c b(r_h)^3$
Free energy	

$$\beta\mathcal{F} = (\mathcal{S}_{BH}^{\epsilon} - \mathcal{S}_{TG}^{\epsilon})/V_3$$

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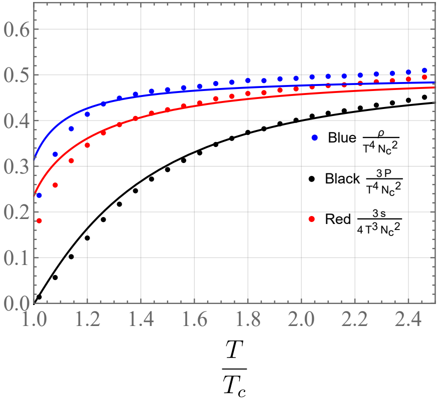
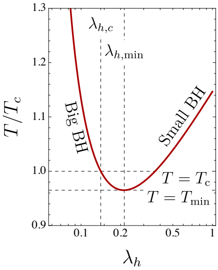
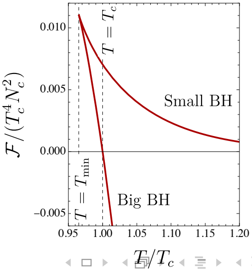
Thermodynamics







Confinement PT in AdS/CFT

Confinement of Pure YM $\Longleftrightarrow$ Hawking Page PT in D+1 AdS

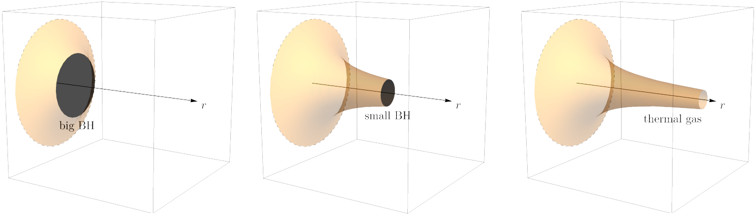
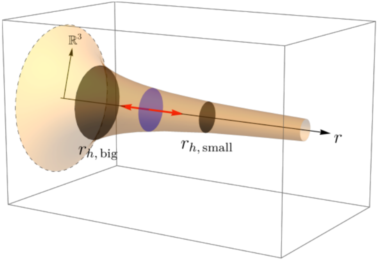


Figure: Thanks To Enrico!!

Effective Action for Tunneling



Thanks To Enrico!

Interpolate between the BBH and TG SBH acts as Instanton

Order parameter of choice (λ_h, r_h).

Incorporate Non-Equilibrium Effect in Gluon Plasma by

Violating $T \neq T_h$

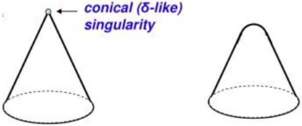
Conical Singularity

Regularize conical deformation and calculate its Free energy Contribution!

Conical Singularity

Expand around the horizon position r_h

$$f(r) \approx \dot{f}_h(r - r_h), \qquad b(r) \approx b_h$$



From Saharian 07

Define the new variables

$$y = \frac{2b_h}{(\dot{f}_h)^{1/2}}\sqrt{r - r_h} \qquad \text{with } y > 0\,,$$

$$\varphi = 2\pi T t \qquad \text{with } 0 < \varphi < 2\pi\,,$$

The metric is

$$ds^2 = dy^2 + y^2 \left(\frac{\dot{f}_h}{4\pi T}\right)^2 d\varphi^2$$

Regularize action with Spherical Cap

$$\mathcal{S}^{BH}_{\text{cone}} = -M_p^3 N_c^2 \int d^5x\sqrt{g}[\mathcal{R} - \frac{4}{3}(\partial\phi)^2 + V(\phi)]$$

Effective Potential & Kinetic Term

Stationary Points $\Longleftrightarrow$ Regular Solutions

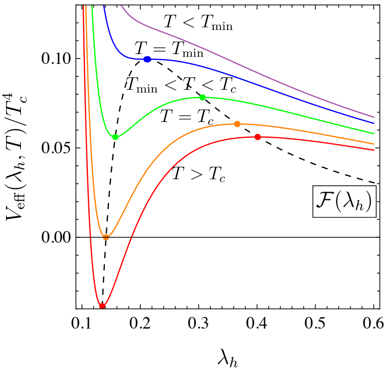
$T > T_c$ (Big BH Stable)

$T < T_{min}$ No BH Solution $\rightarrow$ No Deconfined Phase

Kinetic Term Normalization

$$c\frac{N_c^2}{16\pi^2}$$

We vary $c \rightarrow \frac{1}{3} - 3$, Moderate dependence on GW spectrum



$$V_{\rm eff}(\lambda_h, T) = \mathcal{F}(\lambda_h) - 4\pi M_p^3 N_c^2 b(\lambda_h)^3 \left(1 - \frac{T_h}{T}\right) .$$

Bubble Nucleation rate from effective Action

Thermal Tunneling effective action $\mathcal{O}(3)$ symmetric bounce

$$\mathcal{S}_{eff} = \frac{4\pi}{T} \int d\rho\, \rho^2 \left[c\frac{N_c^2}{16\pi^2} (\partial_\rho \lambda_h(\rho))^2 + V_{\text{eff}}(\lambda_h(\rho), T) \right]$$

Nucleation Rate for Thermal Tunneling

$$\Gamma = T^4 \left(\frac{\mathcal{S}_B}{2\pi} \right)^{3/2} e^{-\mathcal{S}_B} .$$

Compare the Bubble nucleation rate to the Hubble expansion

$$\frac{\Gamma}{H^4} \sim 1$$

Percolation: $\mathcal{P}(true) \simeq \mathcal{P}(false)$ (End of PT! GW Emission)

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Gravitational waves Parameters & Source

$v_w \sim ?$ (Free Parameter)

PT Strength (Energy Release)

$$\alpha = \frac{4}{3}\frac{\Delta\theta}{\Delta w} = \frac{1}{3}\frac{\Delta\rho - 3\Delta p}{\Delta w} = 0.34$$

Inverse PT Rate (Source Duration)

$$\frac{\beta}{H} = T\left(\frac{d\mathcal{S}_B}{dT}\right)\Bigg|_{T=T_p} \approx 10^5 \, ,$$

$$T_p \approx T_{nuc} \sim 0.99\, T_c$$

Sound waves $\rightarrow$ (Signal Suppression) $\tau_{SW} H \sim 10^{-4}$

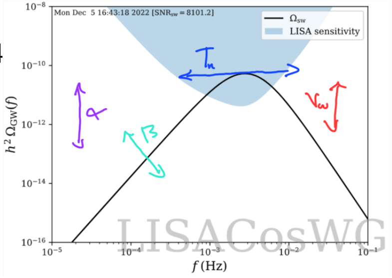
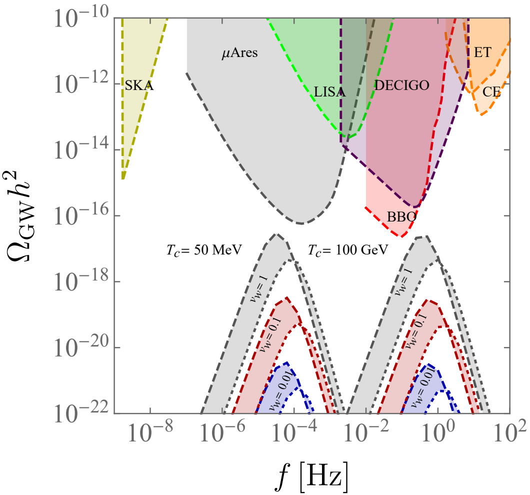


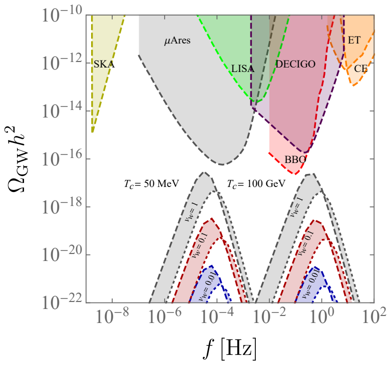
Figure: Thanks to PTPlot

Gravitational Wave Spectra SU(3) "The Money Plot"



GW spectra for SU(3) at different critical temperatures

What to learn from this



GW spectra for SU(3) at different critical temperatures

- $\mathcal{O}(1)$ Agreement with previous works of Halversson 2012.04071, Sannino 2012.11614 and Russell 1505.07109
- Uncertainties
  - Wall velocity
  - Kinetic term
  - Lattice Fit
- Increase N_c
  - Lattice data SU(8), $\Omega_{GW} h^2 \simeq \mathcal{O}(10^{-16})$
  - $N_c \approx 10^3$ LISA Sensitivity, Bigazzi 2011.08757
  - $N_c >> 10^3$ Nucleation fully suppressed, Bea 2112.15478

Summary

- GW Signal from Dark SU(3) Confinement
  - Resemble "Real" QCD-Like theory (Using Holography)
  - Robust Prediction's (Minimal Amount of Parameters, No Tuning!)
- Uncertainties
  - Wall velocity v_w (Started working)
  - Accurate normalization of the Kinetic Term (Ongoing Work)
- Outlook Wall Velocity, Large/Larger N_c (Ongoing Work), Include Flavor

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