UNIVERSITY of PENNSYLVANIA

Workshop on Very Light Dark Matter 2023

Tidal Love numbers for environmental black-hole mergers

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March 29, 2023

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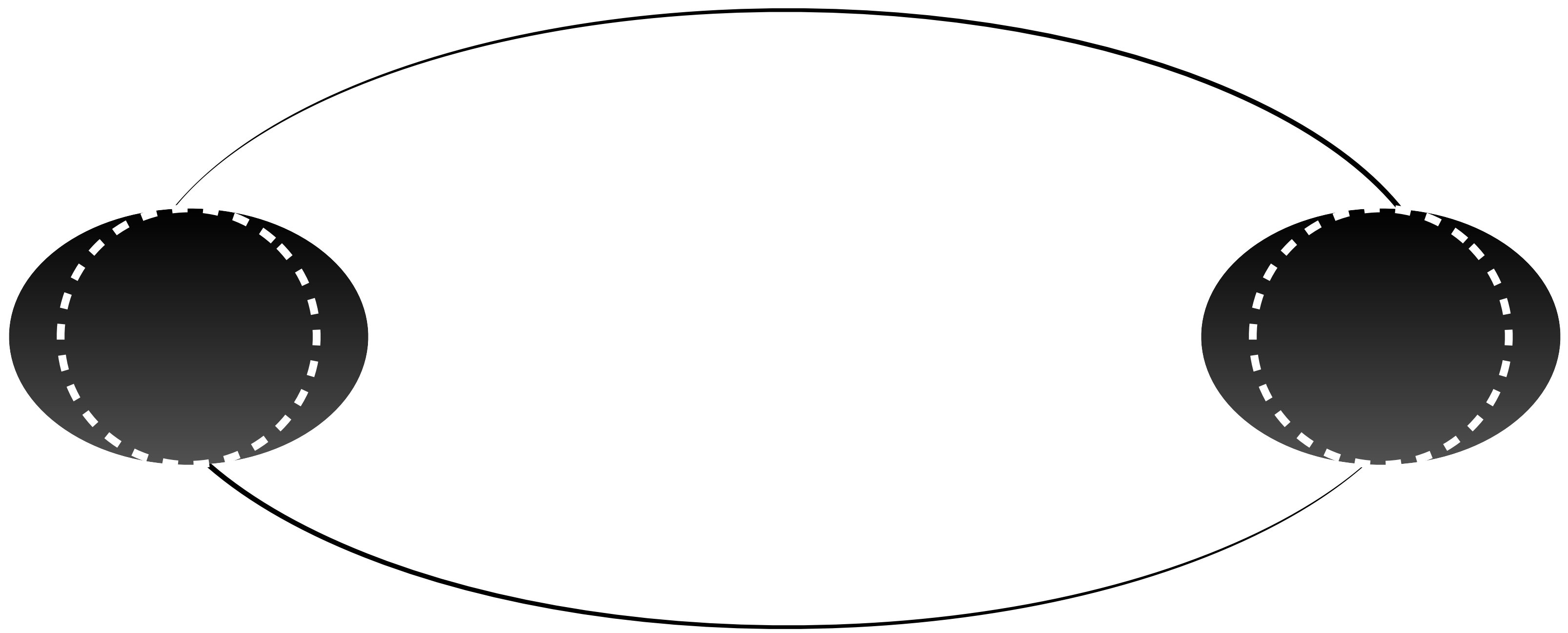
Tidal Love numbers

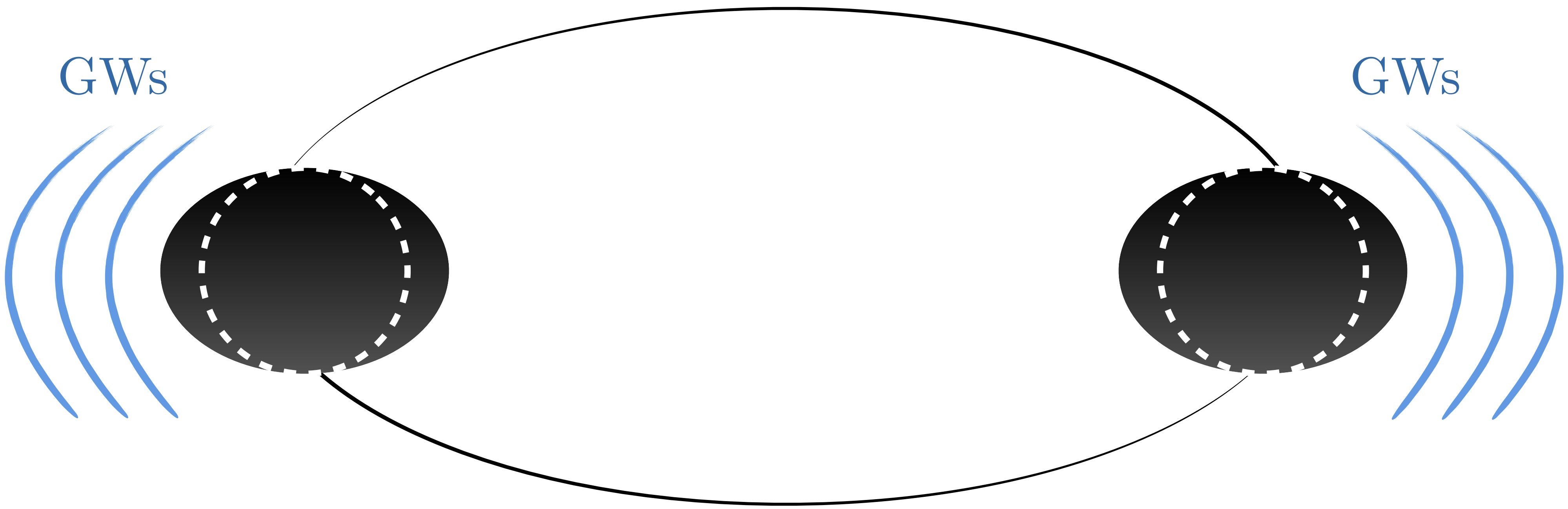
Astrophysical objects may be deformed by external tidal fields
(e.g. companion in a binary system)

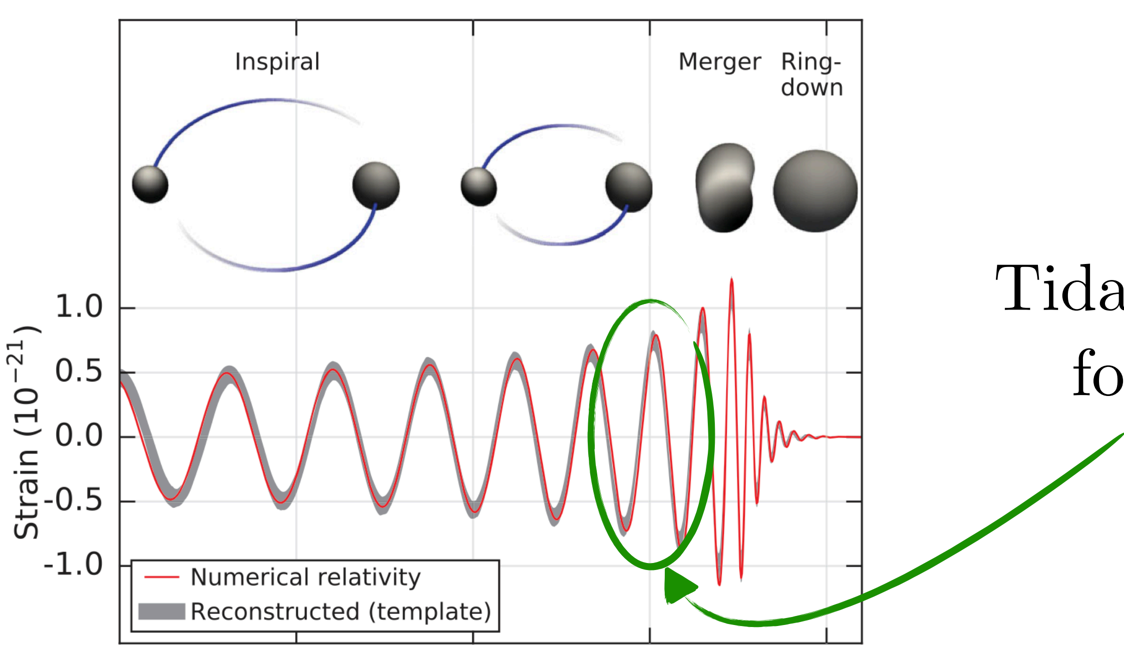


The tidal deformability of a compact object is expressed in terms of its
Tidal Love numbers, which depend on the internal properties of the object

Gravitational waves

Compact objects may assemble in binary systems,
emitting gravitational waves as the coalescence proceeds



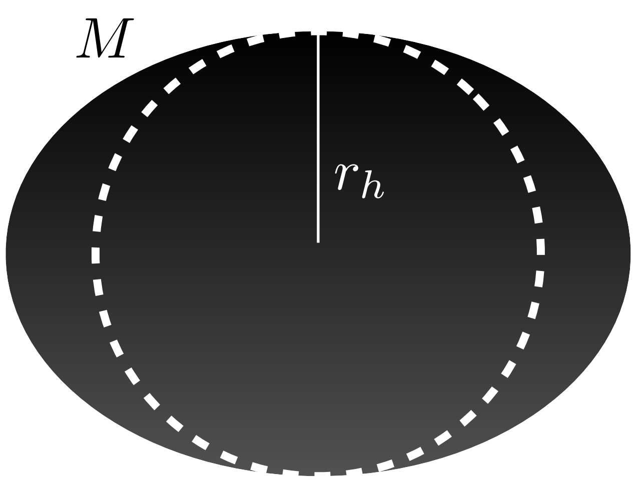


Tidal interactions affect the binary dynamics, leaving a footprint in the emitted GW signals at 5PN order

Flanagan, Hinderer (2008)

Newtonian gravity

Interactions of BHs with external tidal fields at long distances



Gravitational potential

$$U_{\rm tot} = -\frac{GM}{r} - \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} Y_{\ell m}\left[\frac{(\ell-2)!}{\ell!}\mathcal{E}_{\ell m} r^{\ell} - \frac{(2\ell-1)!!}{\ell!}\frac{I_{\ell m}}{r^{\ell+1}}\right]$$

Tidal field — Mass quadrupole

Linear response theory:

$$I_{\ell m}(\omega) = -\frac{(\ell-2)!}{(2\ell-1)!!}\lambda_{\ell m}(\omega) r_h^{2\ell+1}\mathcal{E}_{\ell m}(\omega)$$

$$U_{\rm tot} = -\frac{GM}{r} - \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} Y_{\ell m}\frac{(\ell-2)!}{\ell!}\mathcal{E}_{\ell m} r^{\ell}\left[1 + \lambda_{\ell m}(\omega)\left(\frac{r_h}{r}\right)^{2\ell+1}\right]$$

Response: $\lambda_{\ell m} \simeq k_{\ell m} + i\nu_{\ell m}(\omega - m\Omega) + \dots$

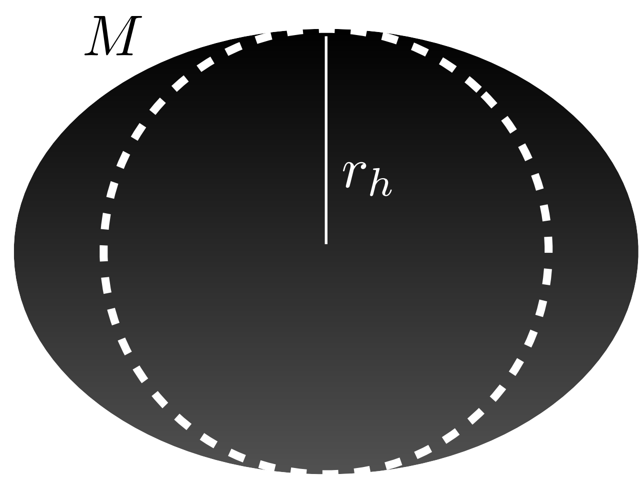
Static TLNs ($k_{\ell m}$) — Dissipative ($\nu_{\ell m}$)

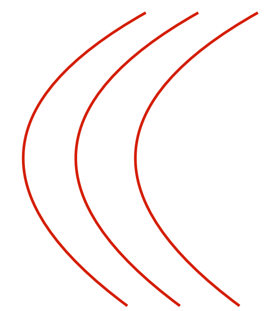
Fang, Lovelace (2005)
Damour, Nagar (2009)
Binnington, Poisson (2009)
Kol, Smolkin (2011)

General Relativity

BH perturbation theory

dynamics of massless fields propagating in a Schwarzschild background





spin-0 ϕ

spin-1 A_μ

spin-2 $h_{\mu\nu}$

Equation of motion: $\mathcal{O}_s \Psi_s = 0$

Boundary conditions for Ψ_s : $\begin{cases} \text{regular at } r_h \\ \text{normalised at } r \to \infty \end{cases}$

$$\Psi_s \propto r^{\ell+1}\left[1 + k_s^{(\ell)}\left(\frac{r}{r_h}\right)^{-2\ell-1}\right]$$

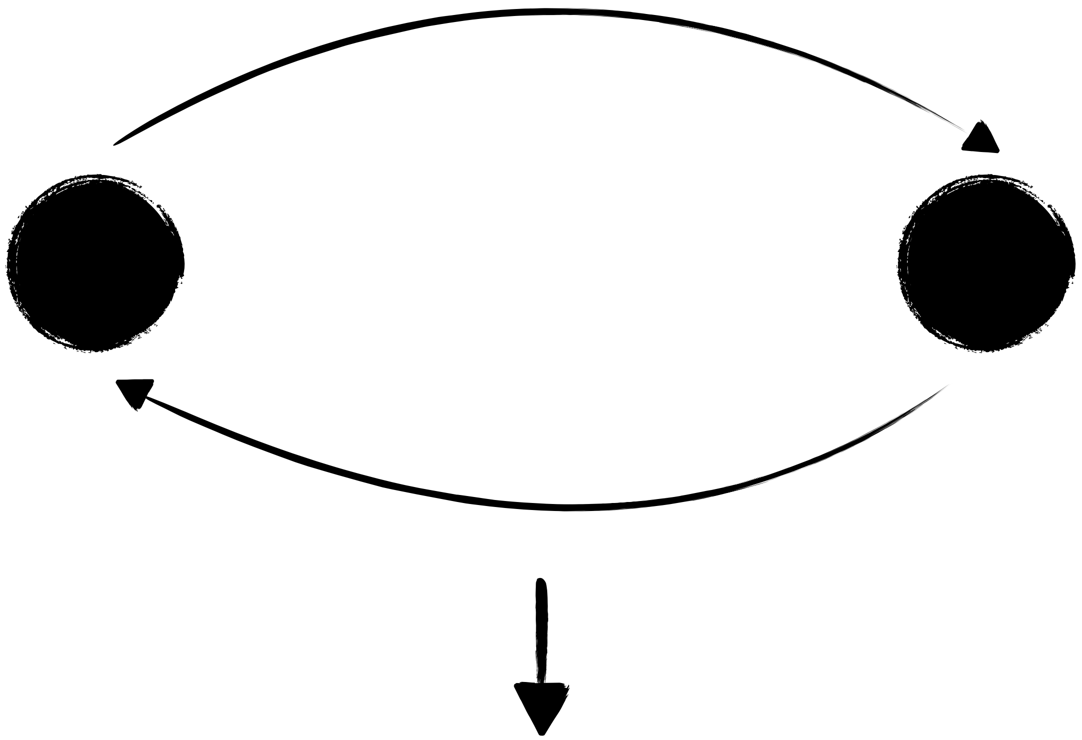
Newtonian matching:

match full GR calculation with Newtonian expansion

Black holes

The TLNs of (non-)spinning black holes in vacuum are exactly zero within Einstein gravity in 4-dimensions

Damour, Nagar (2009)
Binnington, Poisson (2009)



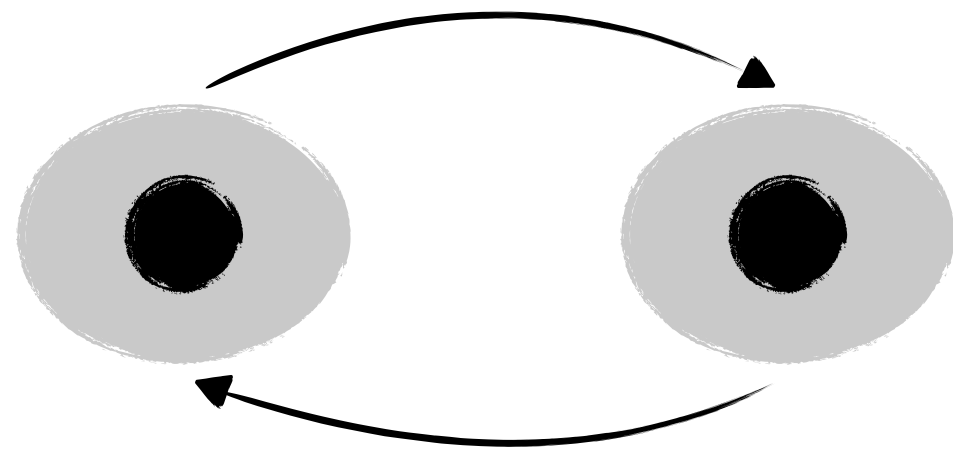
This result has been connected to special symmetries of perturbations fields around black holes

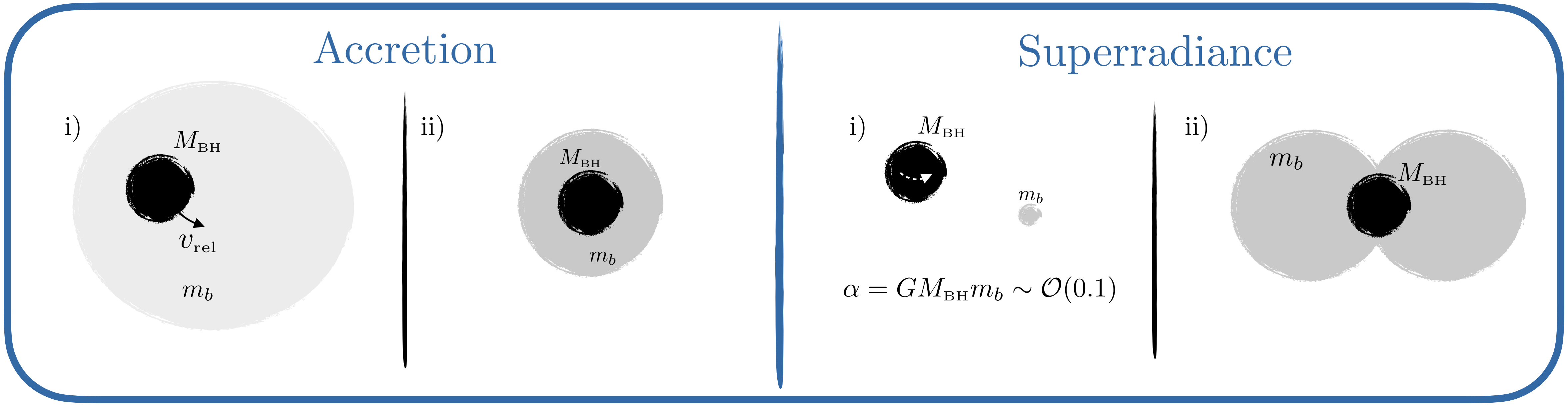
Hui, Joyce, Penco, Santoni, Solomon (2021)
Charalambous, Dubovsky, Ivanov (2021, 2022)
Ben Achour, Livine, Mukohyama, Uzan (2022)

What happens for BHs in environments?

Dressed BHs

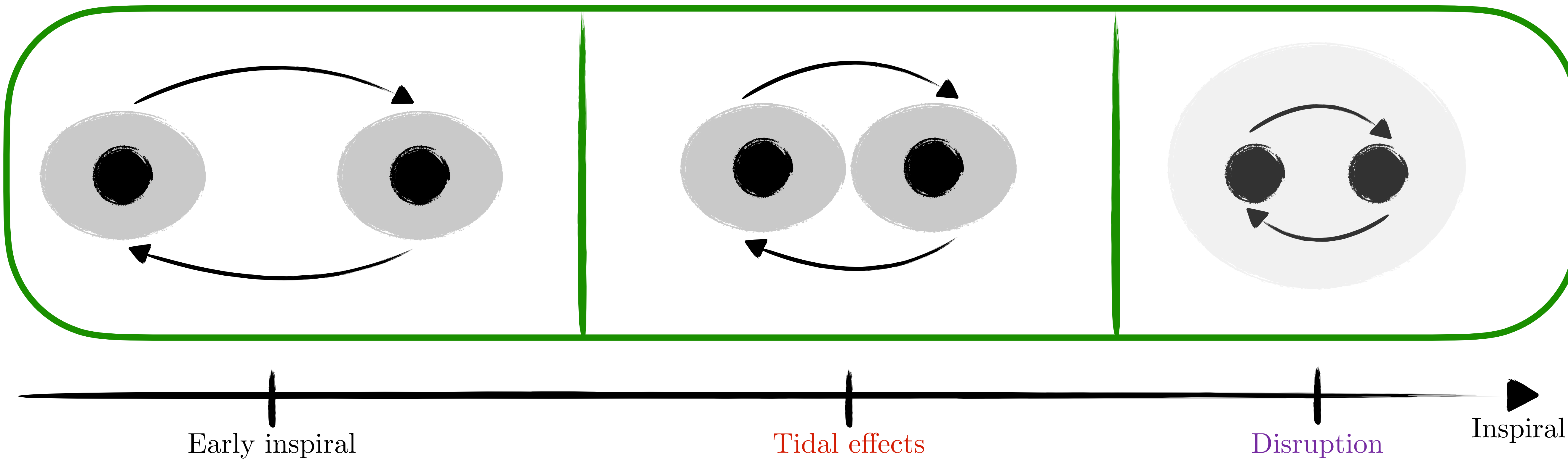
BHs surrounded by clouds of ultralight bosons can be tidally deformed





Corresponding TLNs may be used to probe the environment around merging BHs

Evolution of dressed BH binaries



TLNs of dressed BHs: $$k_2^{(i)} \propto \frac{1}{(G m_i m_b)^8} \left(\frac{M_s}{m_i} \right)$$

Roche frequency: $$f_{\rm cut}^{(i)} = \frac{3\sqrt{3}}{\pi \gamma^{3/2}} (G m_i m_b)^3 \left(\frac{M_s}{m_i} \right)^{1/2} f_{\rm isco}$$

↓

After the cut-frequency, the inspiral proceeds with zero TLNs !

De Luca, Pani JCAP [2106.14428]

GW waveform with tidal effects

Gravitational waveform: $\tilde{h}(f) = C_\Omega \mathcal{A}_{\rm PN} e^{i\psi_{\rm PP}(f)+i\psi_{\rm Tidal}(f)}$

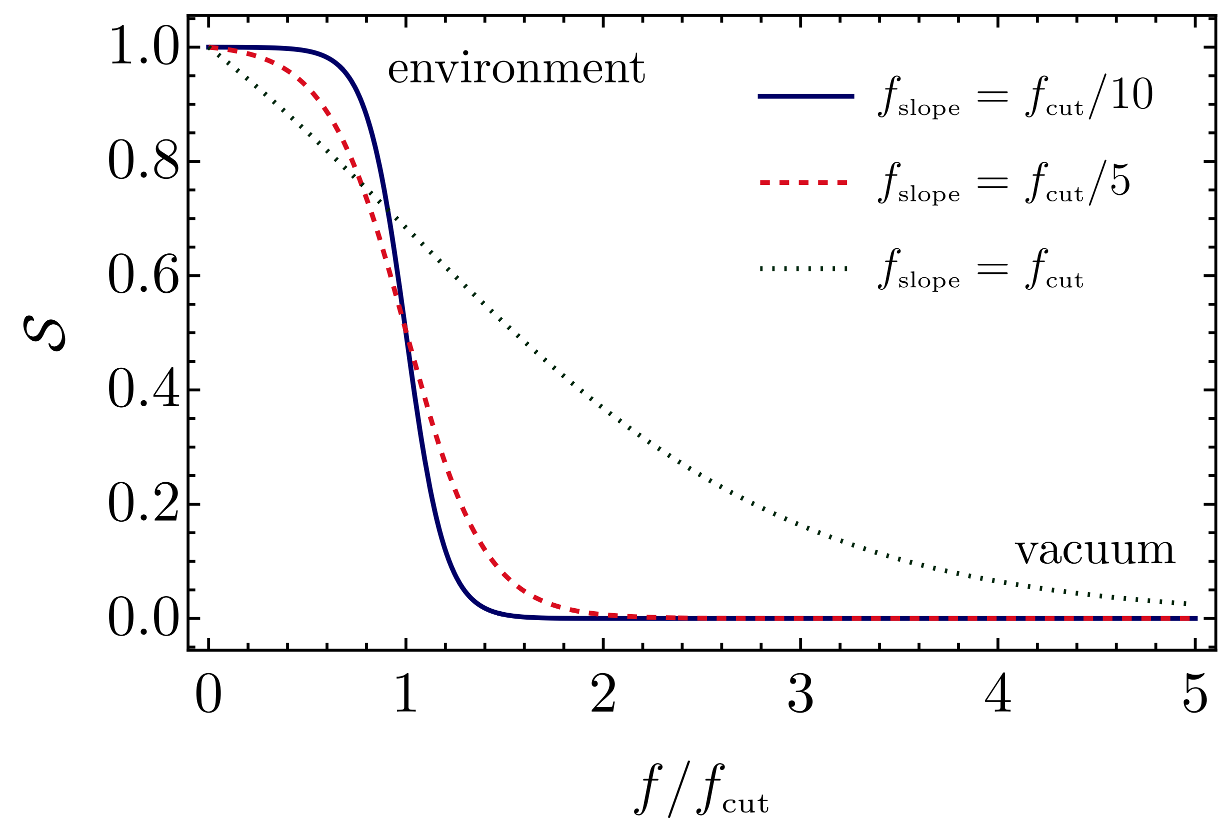
Tidal contribution:
5PN order

$$\psi_{\rm Tidal}(f) = -\frac{39}{2}\tilde{\Lambda}(\pi M f)^{5/3}$$

$$\tilde{\Lambda} = g(m_1, m_2) k_2^{(1,2)}$$

Frequency-dependent TLN

$$\tilde{\Lambda} \to \mathcal{S}(f)\cdot\tilde{\Lambda} = \left[\frac{1+e^{-f_{\rm cut}/f_{\rm slope}}}{1+e^{(f-f_{\rm cut})/f_{\rm slope}}}\right]\cdot\tilde{\Lambda}$$



How well can we measure the TLNs at GW experiments?

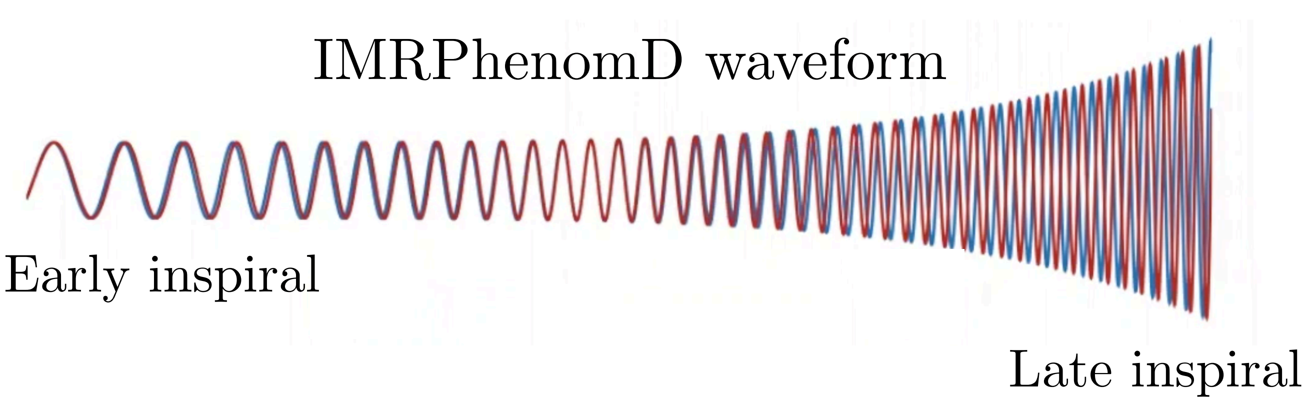
De Luca, Maselli, Pani PRD [2022.03343]

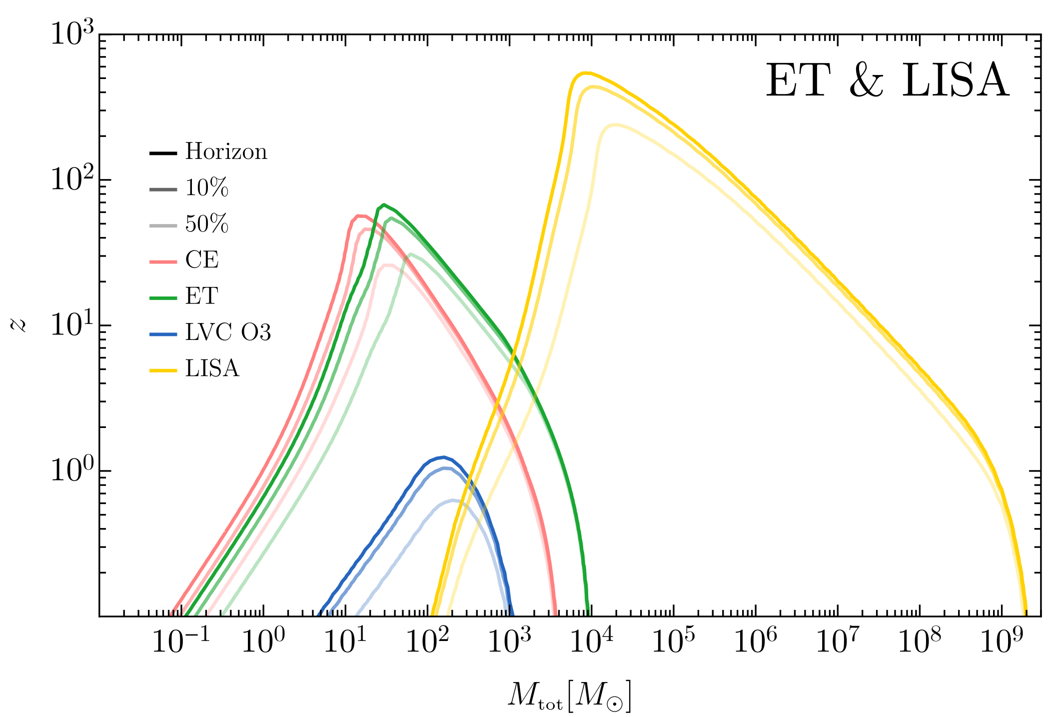
Fisher analysis

Model parameters: $\vec{\theta} = \{\mathcal{M}, \eta, \chi_s, \chi_a, t_c, \phi_c, d_L, \theta, \phi, \psi, \iota, \tilde{\Lambda}, f_{\rm cut}, f_{\rm slope}\}$

Forecast tidal deformability using GW measurements

Fisher analysis: $\Gamma_{ij} = \left\langle \frac{\partial h}{\partial \theta_i} \middle| \frac{\partial h}{\partial \theta_j} \right\rangle_{\vec{\theta}=\hat{\vec{\theta}}} \xrightarrow{\mathbf{\Sigma} = \mathbf{\Gamma}^{-1}} \sigma_i = \Sigma_{ii}^{1/2}$



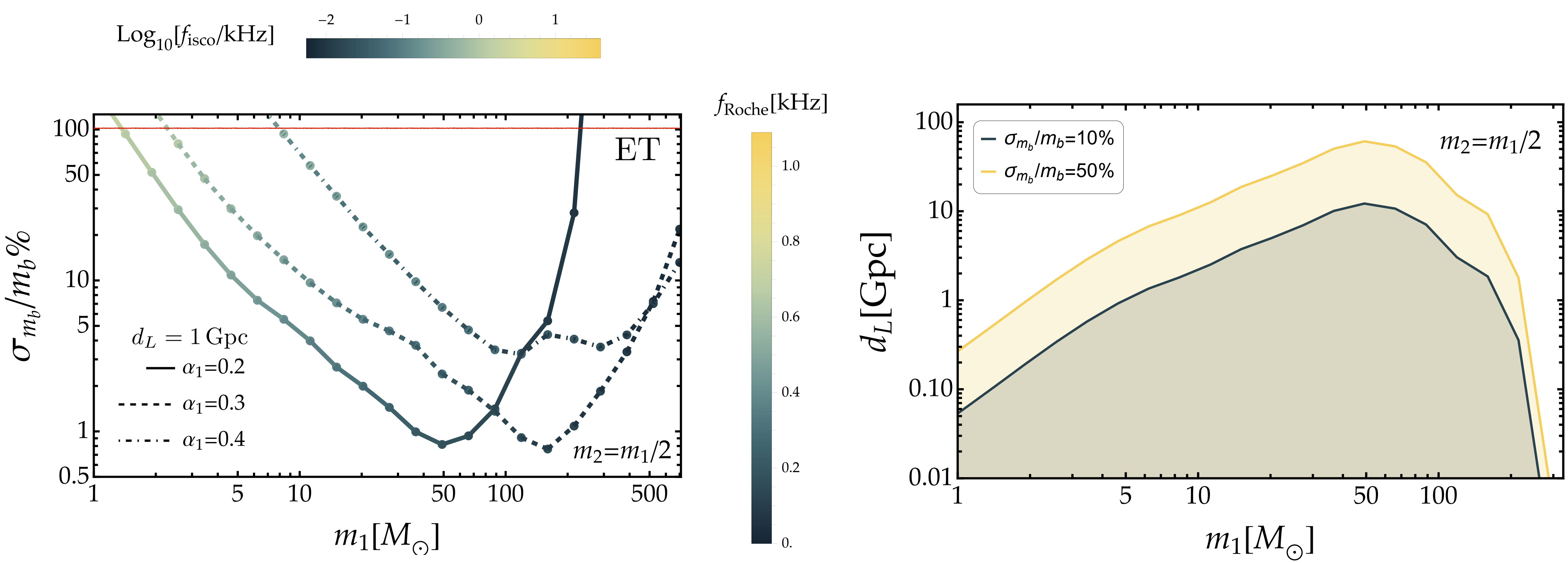


Bounds on dressed BHs

Forecasts on frequency-dependent effective TLN:
case of BHs dressed by ultralight bosons

$$k_2^{(i)} \propto \frac{1}{(Gm_i m_b)^8}$$

$$f_{\rm cut}^{(i)} \propto (Gm_i m_b)^3$$

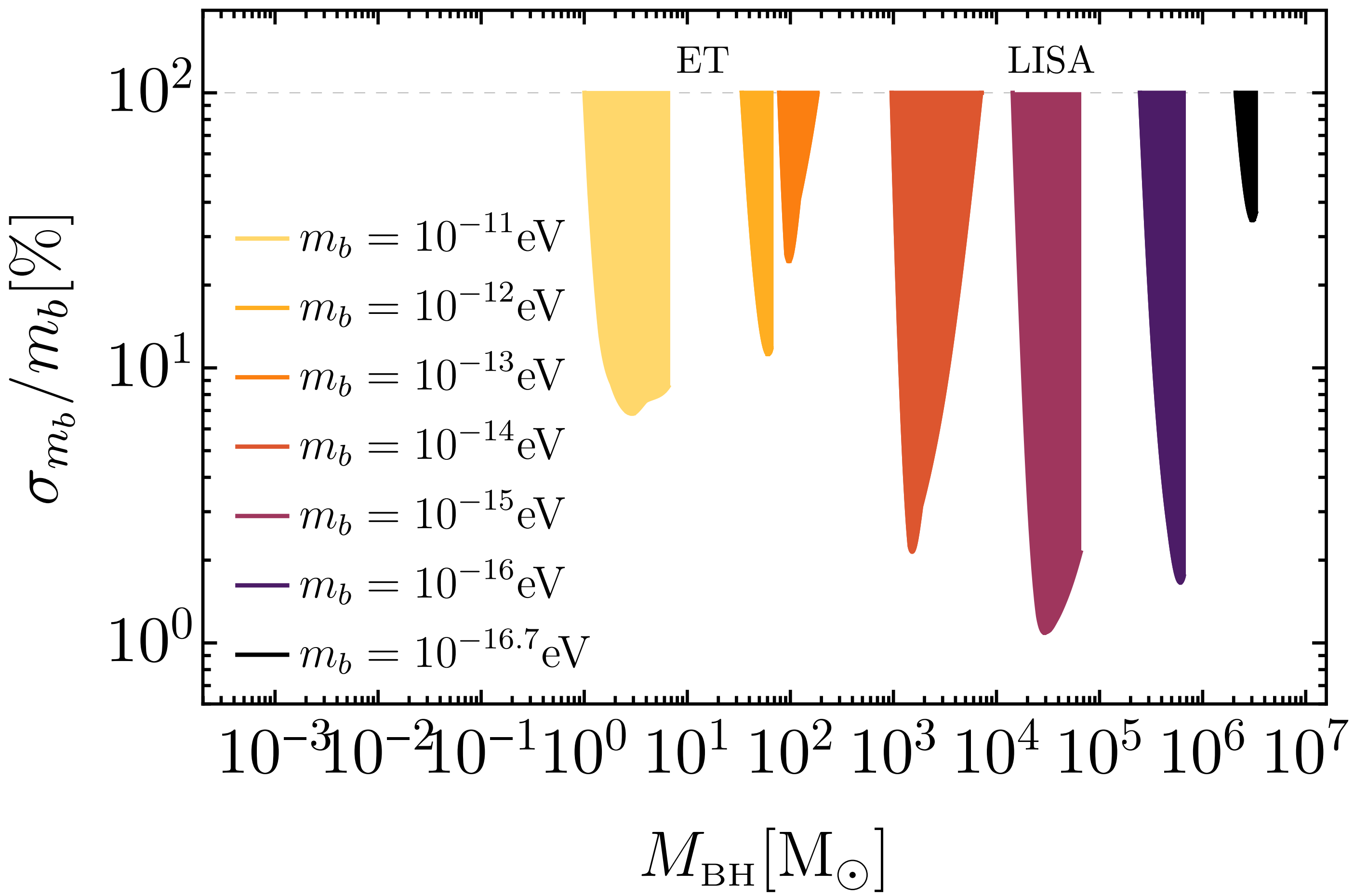


- Smaller number of waveform parameters, since m_b determines both TLN and cut-off
- Boson mass can be measured with accuracy of few percent with distances of Gpc

De Luca, Maselli, Pani PRD [2022.03343]

Bounds on ultralight bosons

Forecast tidal deformability using GW measurements at ET or LISA



- ET and LISA together could probe the ultralight range $10^{-17} \lesssim m_b/\text{eV} \lesssim 10^{-11}$
- Tidal interactions can be used to probe the range around 10^{-14} eV, filling the gap from other superradiance-driven constraints

De Luca, Pani JCAP [2106.14428]

Conclusions

- Tidal Love numbers provide deep insights on the nature of compact objects, and play a role in gravitational wave physics

- Tidal Love numbers of black holes are found to be exactly zero in the vacuum within Einstein gravity

- LISA and ET can probe tidal effects from BHs dressed by scalar condensates in the combined mass range $(10^{-17} - 10^{-11})$eV

Thank you!

Backup slides

Metric of dressed BHs

Spherically symmetric isolated body within scalar field environment

Schwarzschild metric: $$ds^2 = -f\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2$$

Klein-Gordon equation: $$\Box\Phi = m_b^2\Phi$$

$\downarrow$ $\alpha \ll 1$

scalar field condensate

$$\Phi(r) = \Phi_0\left(\frac{4\alpha^2}{\ell+1}\frac{r}{r_{\mathrm{BH}}}\right)^{\ell}\exp\left(-\frac{2\alpha^2}{\ell+1}\frac{r}{r_{\mathrm{BH}}}\right)$$

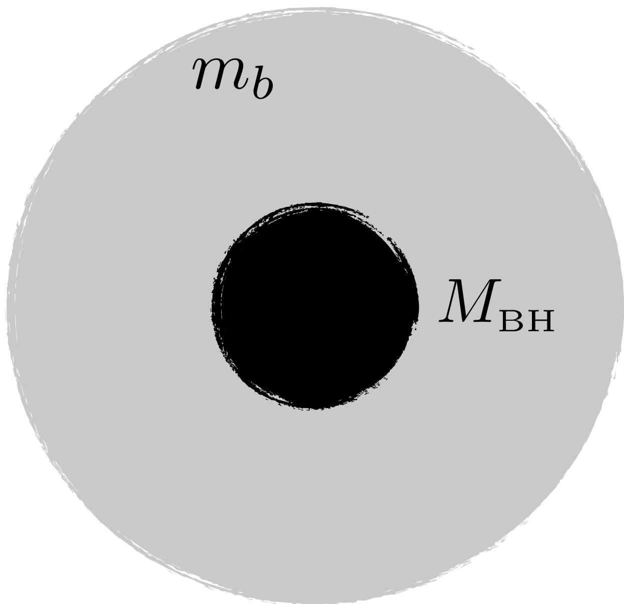
Detweiler (1980)

dress-driving process

$\downarrow$

What is backreaction of the condensate on the BH metric?

Metric of dressed BHs: accretion



Spherical accretion flow of a scalar field Φ over a naked BH

BH metric with cloud backreaction:

$$\mathrm{d}s^2 = -Fe^{2\delta\lambda(v,r)}\mathrm{d}v^2 + 2e^{\delta\lambda(v,r)}\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2$$

Perturbative expansion in scalar field:

Babichev, Dokuchaev, Eroshenko (2012)
Bamber, Tattersall, Clough, Ferreira (2021)

$$\delta M(v,r) = 8\pi(2M_{\rm BH}w)^2|\Phi_0|^2 v + \int_{r_{\rm BH}}^{r} 4\pi r'^2 (f|\partial_r\Phi|^2 + \mu^2|\Phi|^2)\mathrm{d}r'$$

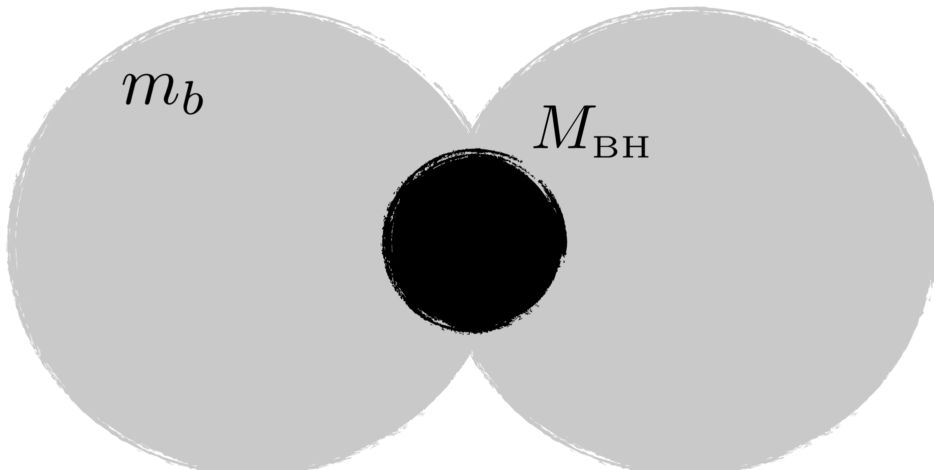
$$\delta\lambda(r) = -2\int_{r_{\rm BH}}^{r} 4\pi r'|\partial_r\Phi|^2\mathrm{d}r'.$$

Backreaction depends on the BH properties and on the profile of the scalar field condensate $\Phi_{\ell=0}(r)$

Metric of dressed BHs: superradiance

Ultralight bosonic fields can trigger the formation of nonaxisymmetric condensates around spinning BHs

$w_R < m\Omega$



Brito, Cardoso, Pani (2015)

Dominant dipolar mode $(\ell = m = 1)$

$$\Phi \propto \Phi(r)\cos(\varphi - w_R t)\sin\theta$$

Backreaction on the metric:
$G_{ab} = 8\pi T_{ab}(\Phi^2_{\ell=1})$

- $\ell = 0$ ⟶ Treated as above for spherical perturbations
- $\ell = 2$ ⟶ Neglected (conservative estimate for TLN)

We neglect the spin of the final BH (subleading contribution to the TLN)

Pani, Gualtieri, Maselli, Ferrari (2015)

TLNs of dressed BHs

Solve EOMs of tidal perturbations on the metric with cloud backreaction, and match solutions to asymptotic expansions

Asymptotic expansion of fields:
- tensor field (s=2)
- axial vector field (s=1)
- scalar field (s=0)

$$g_{tt} = 1 + \frac{2M}{r} + \sum_{l\geq 2}\left(\frac{2}{r^{l+1}}\left[\sqrt{\frac{4\pi}{2l+1}}M_l Y^{l0} + (l' < l\,\text{pole})\right] - \frac{2}{l(l-1)}r^l\left[\mathcal{E}_l Y^{l0} + (l' < l\,\text{pole})\right]\right)$$

$$A_\varphi = \sum_{l\geq 1}\left(\frac{2}{r^l}\left[\sqrt{\frac{4\pi}{2l+1}}J_l S^{l0}_\varphi + (l' < l\,\text{pole})\right] - \frac{2r^{l+1}}{l\,(l-1)}\left[\mathfrak{B}_l S^{l0}_\varphi + (l' < l\,\text{pole})\right]\right)$$

$$\phi = \phi_0 + \sum_{l\geq 1}\left(\frac{1}{r^{l+1}}\left[\sqrt{\frac{4\pi}{2l+1}}\phi_l Y^{l0} + (l' < l\,\text{pole})\right] - \frac{1}{l(l-1)}r^l\left[\mathcal{E}^{\rm S}_l + (l' < l\,\text{pole})\right]\right)$$

→

TLNs:

$$k_l^{(s=2)} \equiv -\frac{1}{2}\frac{l(l-1)}{M^{2l+1}}\sqrt{\frac{4\pi}{2l+1}}\frac{M_l}{\mathcal{E}_l}$$

$$k_l^{(s=1)} \equiv -\frac{1}{2}\frac{l(l-1)}{M^{2l+1}}\sqrt{\frac{4\pi}{2l+1}}\frac{J_l}{\mathfrak{B}_l}$$

$$k_l^{(s=0)} \equiv -\frac{1}{2}\frac{l(l-1)}{M^{2l+1}}\sqrt{\frac{4\pi}{2l+1}}\frac{\phi_l}{\mathcal{E}^{\rm S}_l}$$

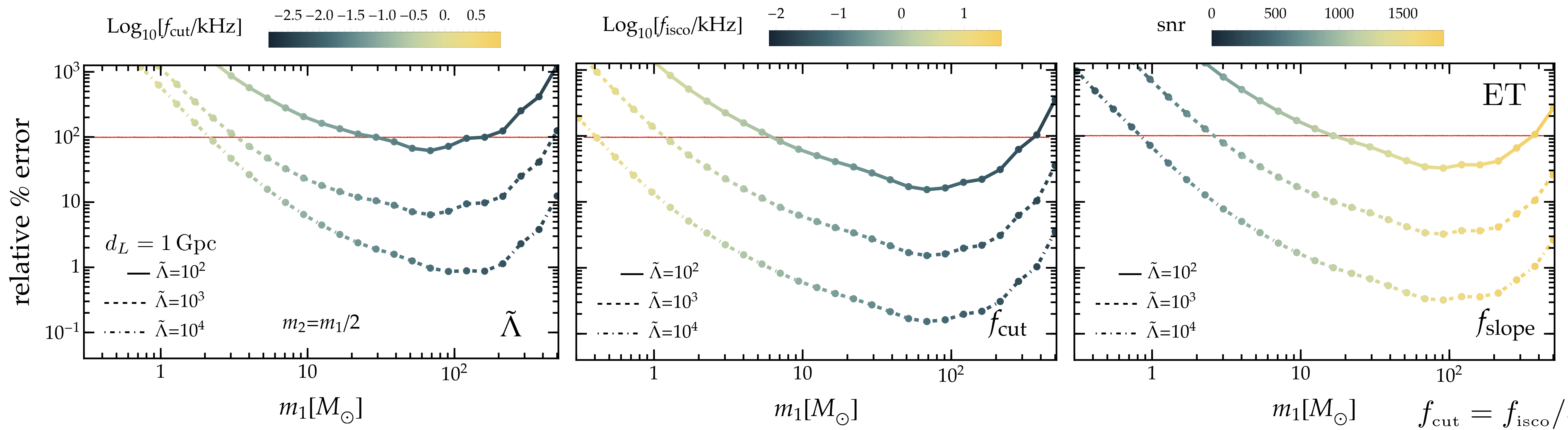
↓

TLNs of dressed BHs: $$k_2 \propto \frac{1}{(GM_{\rm BH}m_b)^8}$$

Baumann, Chia, Porto, Stout (2019)
De Luca, Pani JCAP [2106.14428]

Frequency-dependent TLN

Forecasts on frequency-dependent effective TLN



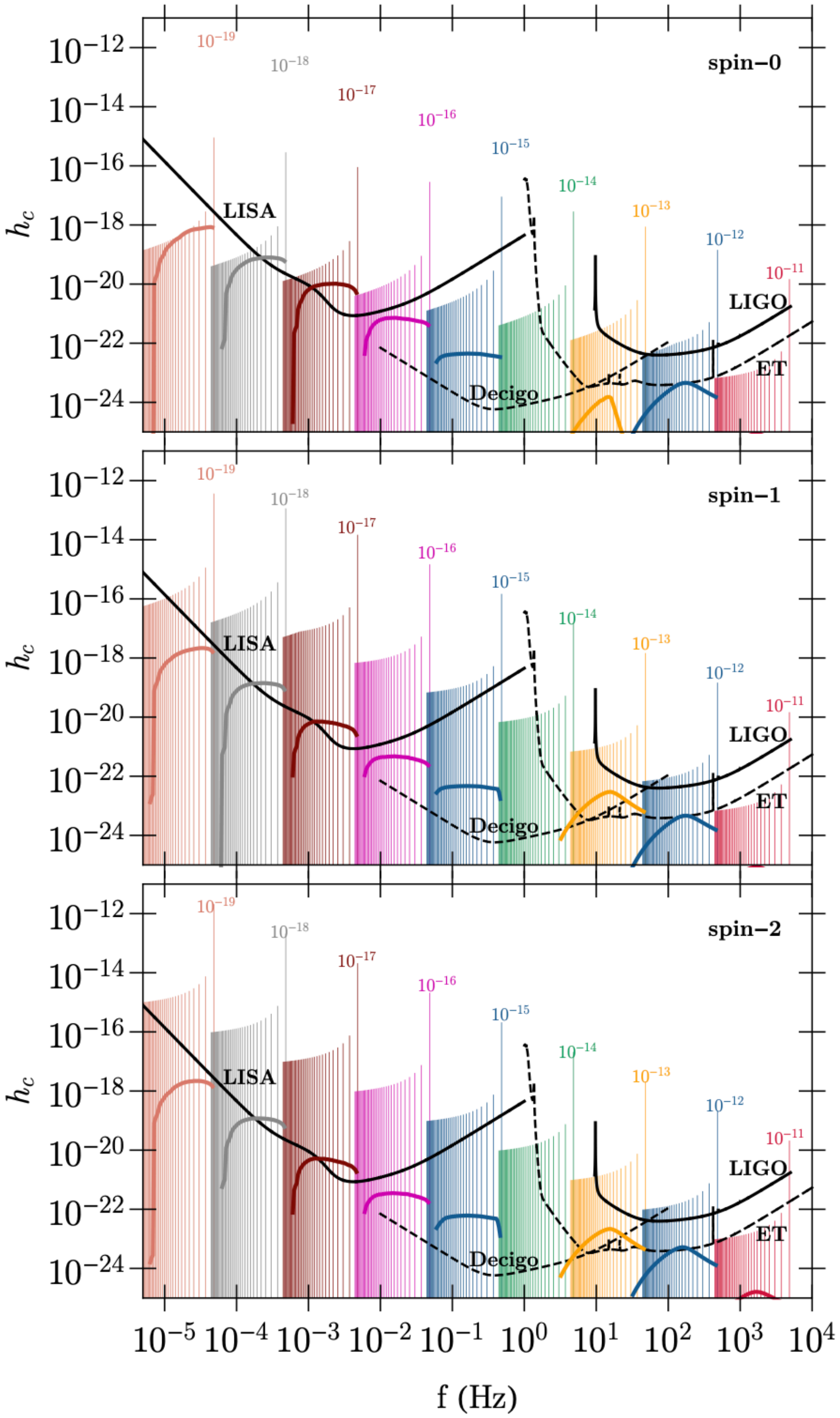
$f_{\text{cut}} = f_{\text{isco}}/5$

$f_{\text{slope}} = f_{\text{cut}}/5$

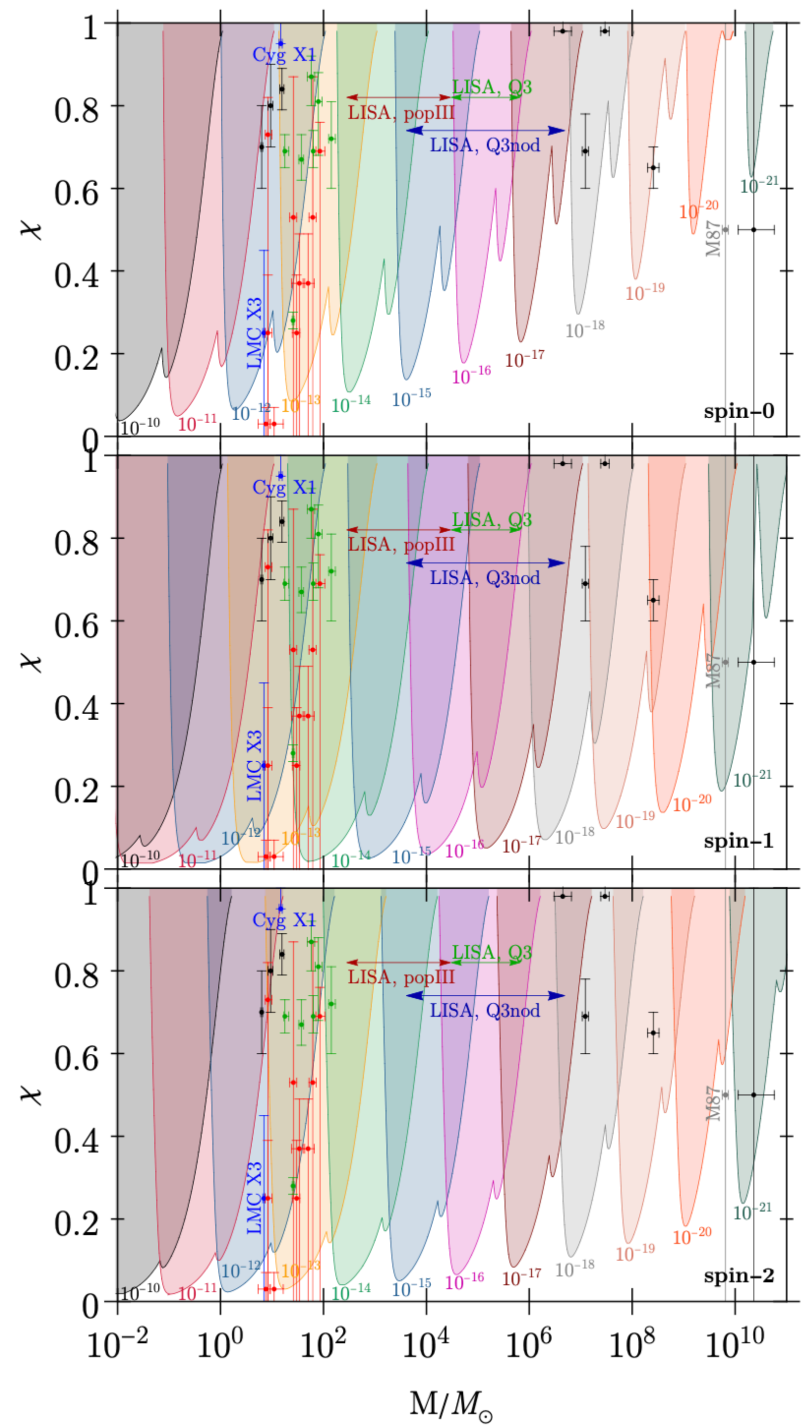
- All tidal parameters can be measured with high accuracy
- Potential of ET in measuring the transition point where the TLN vanishes
- Multiband analyses between ET & LISA for $M_{\text{BH}} \simeq 10^2 M_\odot$

De Luca, Maselli, Pani PRD [2022.03343]

Bounds on ultralight bosons



Continuous GW signals



Regge plane: exclusion region

EM and GW observations can potentially constrain ultralight scalar fields $10^{-22} \lesssim m_b/\text{eV} \lesssim 10^{-10}$

Brito, Cardoso, Pani "Superradiance" Lect. Notes Phys. 971 (2020)