Stability of domain wall networks with population bias

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Based on 2211.06849 with Fuminobu Takahashi, Naoya Kitajima, and Wen Yin

Part 1
What are domain walls?

Part 2 Domain wall network stability

Part 3 Numerical simulations

Part 4 Implications for cosmic birefringence

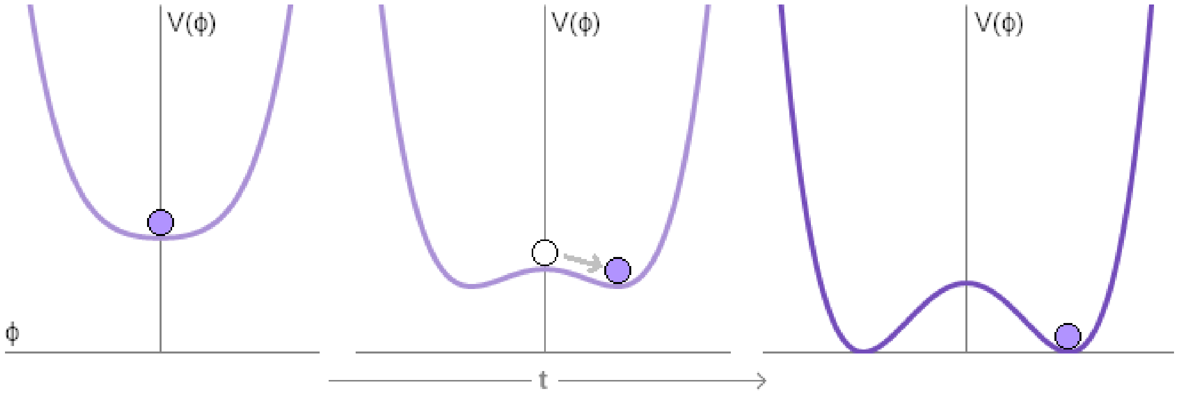
Spontaneous symmetry breaking (SSB)

Our modern understanding of physics is based on **symmetries**

KEY POINT

The state with highest symmetry ≠ The state with lowest energy

<u>An easy to visualize example of SSB:</u>



- Potential changes with time, but always has Z_2 symmetry
- The vacuum state has Z_2 symmetry at early times, but doesn't at later times → SSB!

Topological defects

KEY POINT

Topological defects are stable, high energy structures produced after a SSB

- Many types. In cosmology: monopoles (*0D*), strings (*1D*), **domain walls (DW)** (*2D*) & textures (*delocalized*).

DW dynamics:

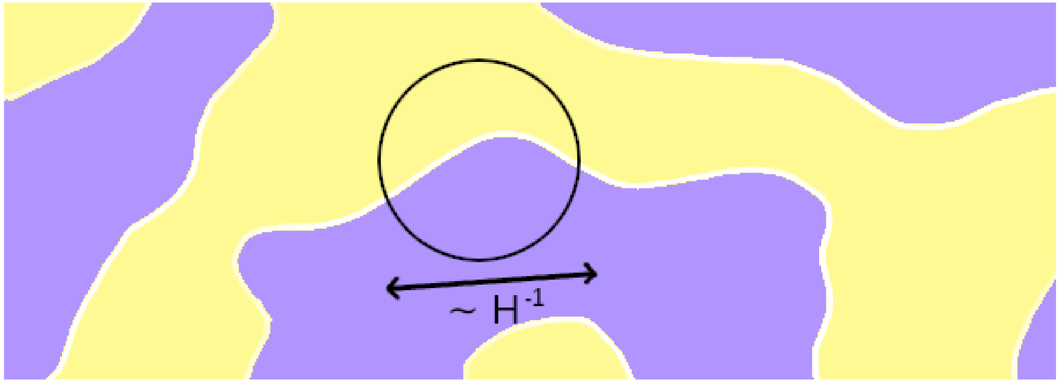
Surface tension → straighten → reach high speeds → collide with other DWs → annihilate

Scaling regime:

Within a few Hubble times, the DW network converges to a state in which there is on average one DW per horizon.

O(1) H^{-1} length

$$\rho_{DW} \sim \sigma H$$



Press, Ryden, Spergel '89, Hindmarsh '96, Garagounis and Hindmarsh '03...

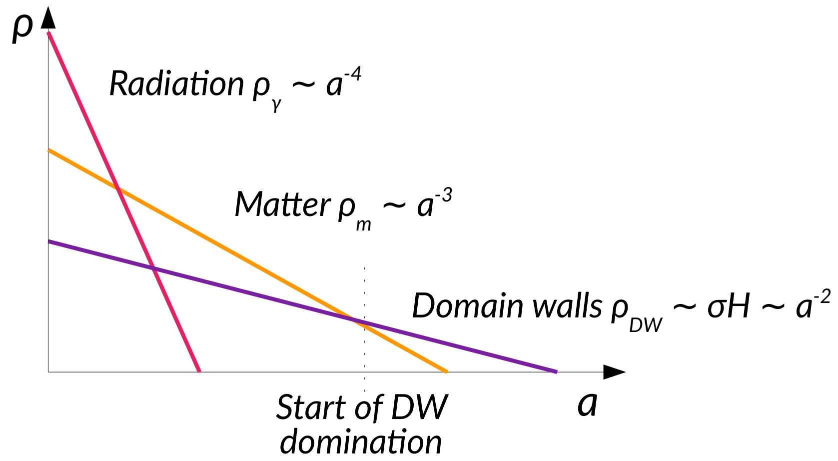
Axions & cosmological DW problem

The evidence for dark matter is astounding, but no direct detection has been made yet.
Candidate → **Axion & axion-like particles** *(light pseudoscalar fields)*

In many axion models, low energy effective lagrangian has **multiple degenerate minima.**
→ condition for domain wall formation.

KEY POINT

If axions exist, we will have DW formation at the early universe



Scaling regime DW networks will eventually **dominate the energy contents of the universe**

Zeldovich, Kobzarev, and Okun '74

Problem → **This isn't what we observe**

Motivation to study the stability of DW networks!

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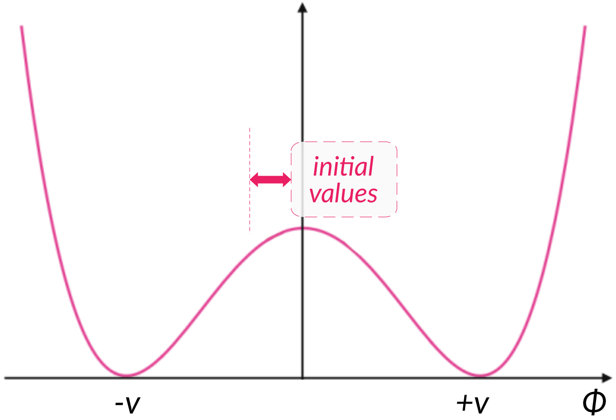
Destabilization of the DW network

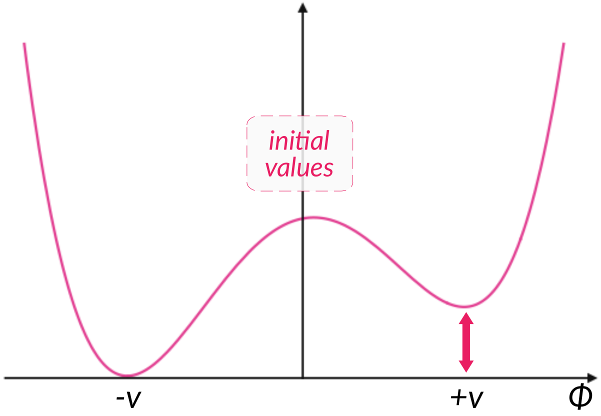
- **Biases in the initial conditions** may take place depending on the model.

- These could **destabilize the network** and thus solve the energy domination problem.

Bias on the initial distribution



Bias on the potential



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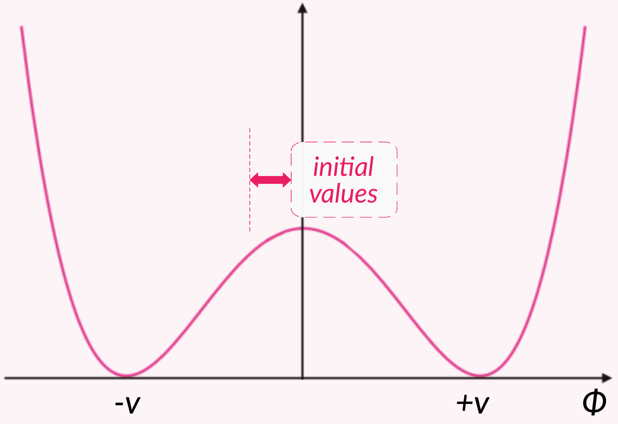


We have studied the case for **initial distribution bias.**

Bias on the potential

The potential bias case will be explained **on the next talk**!

→ Junseok Lee 3/28 14:00-14:20

Setting the initial conditions

DW networks are highly non-linear → **Numerical simulations**

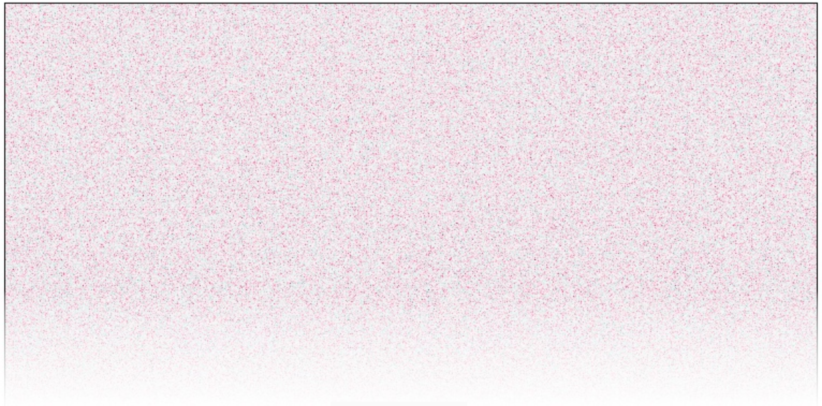
It was thought that how initial conditions are set didn't matter much as the system would always go towards the local attractor solution (scaling regime)

Thermal initial conditions

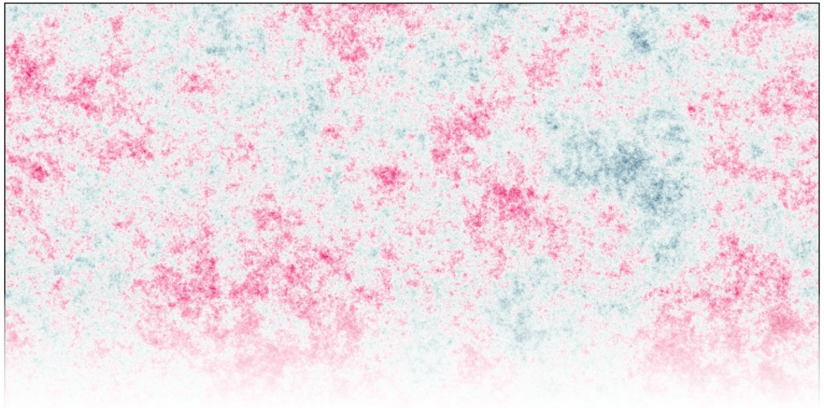
Thermal equilibrium, gaussian fluctuations.
→ Correlations at very short distances.

Inflationary initial conditions

Scale invariant fluctuations surge during inflation
→ Correlations even beyond superhorizon scales!



$100\ H^{-1}$



$100\ H^{-1}$

Much established knowledge is wrong!?

Past literature just used thermal fluctuations + some modifications to simulate post-inflationary DW formation → **this ignores super-horizon correlations**

KEY POINT

With the correct set of initial conditions, many standard results aren't true

1. System quickly forgets initial conditions

   → Information keeps entering the horizon as the universe expands

2. Scaling solution is a local attractor

   → There are large voids with no domain walls

3. Quick decay even for small biases

   → Networks are more resilient to relatively big population biases

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Lattice simulations

Gonzalez, Kitajima, Takahashi and Yin 2211.06849

- The DW dynamics don't depend much on the choice of potential.

$$\frac{\partial^2\phi(\mathbf{x},t)}{\partial t^2} + 3H\frac{\partial\phi(\mathbf{x},t)}{\partial t} - \frac{\partial_i^2\phi(\mathbf{x},t)}{a^2} + \frac{\partial V}{\partial\phi} = 0,$$

→ We choose the simple $\lambda\phi^4$ over more complex *(but realistic)* ALP potentials

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2 = \frac{\lambda}{4}v^4 - \frac{1}{2}m_0^2\phi^2 + \frac{\lambda}{4}\phi^4$$

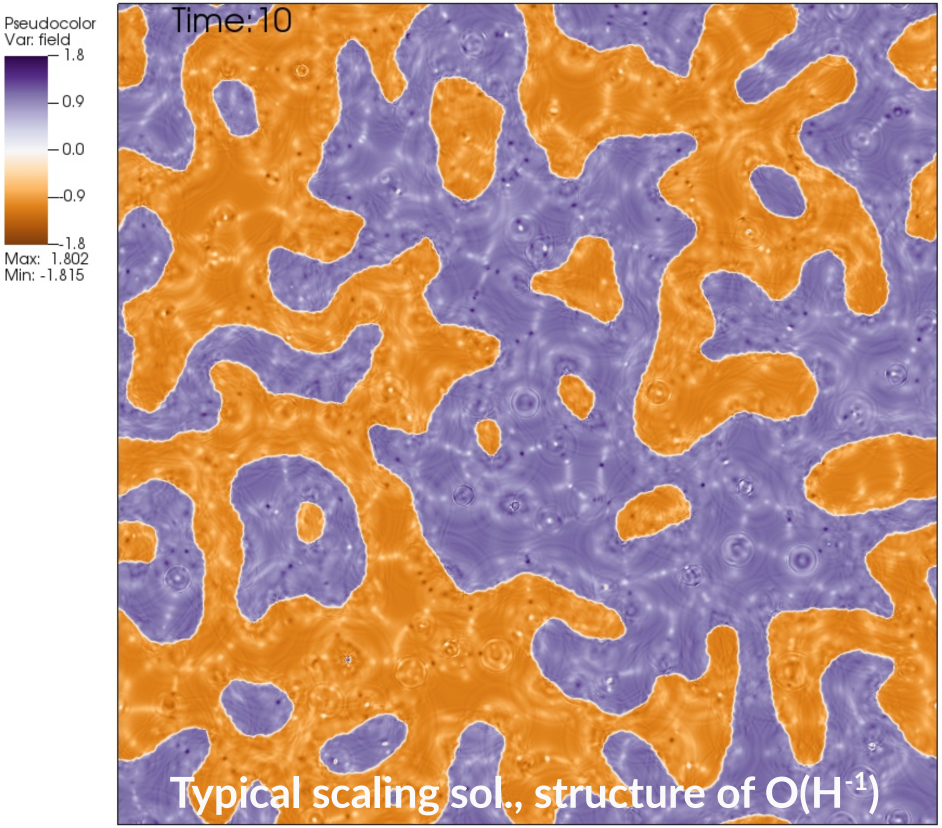
- **Conformal time**: $\tau = \sqrt{2t/m_0}$

- 2D lattice *(homogeneous on z axis)* with grid size of 4096^2 or 16384^2
- Both thermal and inflationary fluctuations are studied
- Bias is introduced as a **constant shift** to the initial field values
- We define the bias parameter as $b_d = \langle\phi(x)\rangle/\sigma$

DW network snapshots

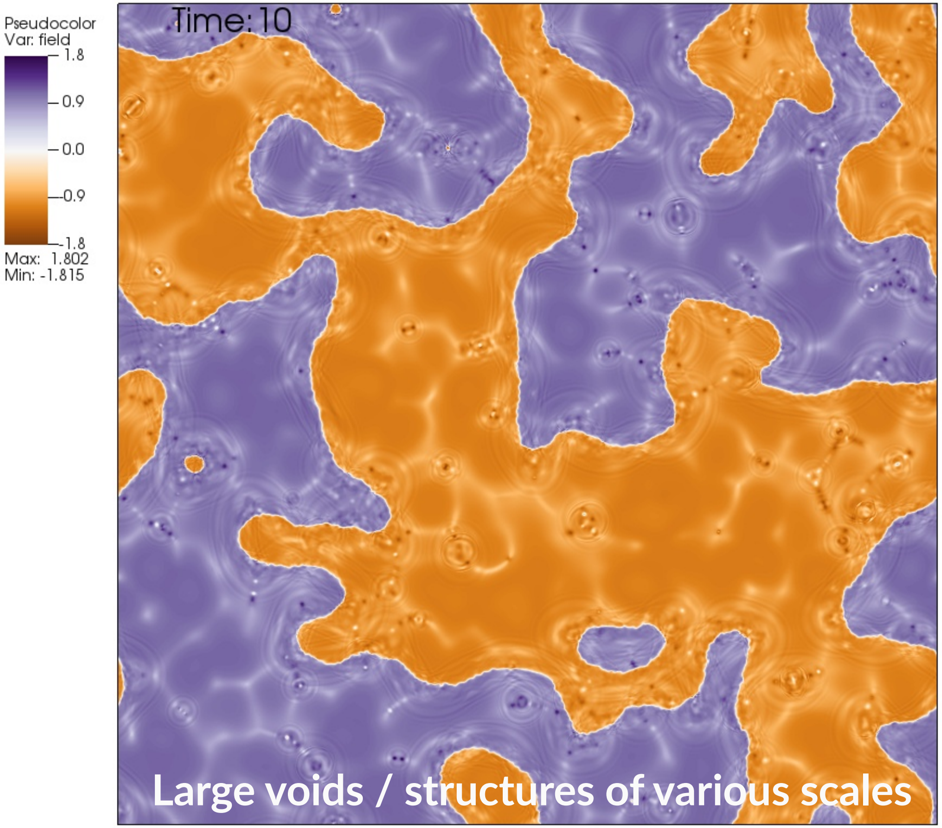
Both taken at time $\tau=10/m_0$

Thermal fluctuations



$10\ H^{-1}$

Inflationary fluctuations

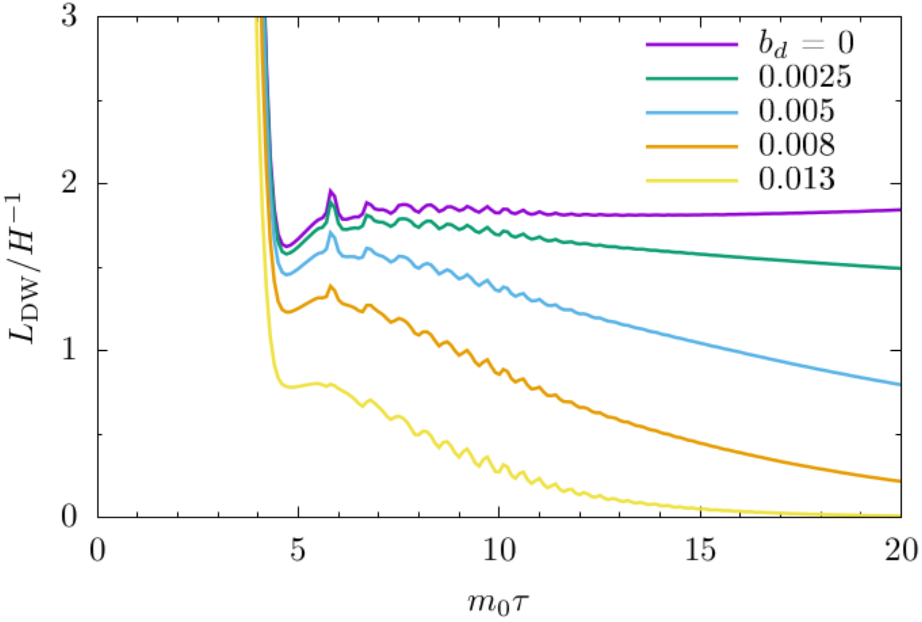


$10\ H^{-1}$

DW length per horizon

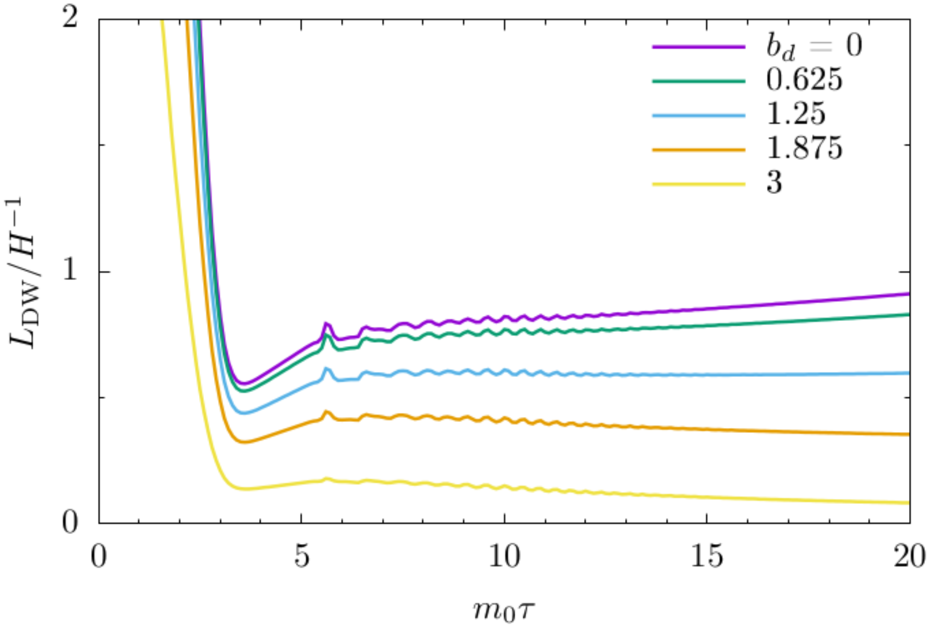
L_{DW} = Average DW length per horizon
Scaling solution $L_{DW}/H^{-1} = O(1)$
Ripples due to scalar waves between DWs

Thermal fluctuations



Fast decay for small biases

Inflationary fluctuations



Stable for relatively large biases

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Implications for cosmic birefringence

Cosmic birefringence (CB)

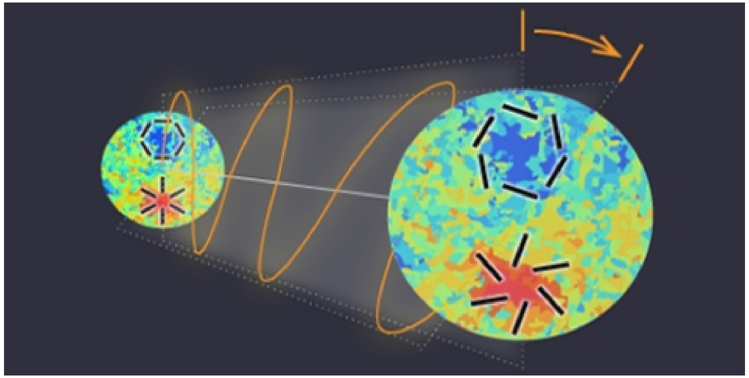
c_γ is the anomaly coefficient ≈ O(1)
α is the fine structure constant

$$\mathcal{L}_{\phi\gamma} = -c_\gamma \frac{\alpha}{4\pi}\frac{\phi}{f_\phi}F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$= -\frac{1}{4}g_{\phi\gamma\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

The axion couples to light via the **axion-photon coupling**

This **rotates** linearly polarized light:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}g_{\phi\gamma\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}, \\ &= \frac{1}{2}\left(\vec{E}^2 - \vec{B}^2\right) + g_{\phi\gamma\gamma}\phi\vec{E}\cdot\vec{B} \\ &\simeq \frac{1}{2}\left[\left(\vec{E} + \boxed{\frac{g_{\phi\gamma\gamma}\phi}{2}}\vec{B}\right)^2 - \left(\vec{B} - \frac{g_{\phi\gamma\gamma}\phi}{2}\vec{E}\right)^2\right]\end{aligned}$$



https://physics.aps.org/articles/v13/s149

Rotation of pol. light depending on the value of the axion field ϕ

$$\Phi = \frac{g_{\phi\gamma\gamma}\Delta\phi}{2} \simeq 0.42c_\gamma\left(\frac{\phi_{\rm today} - \phi_{\rm LSS}(\Omega)}{2\pi f_\phi}\right)\ \mathrm{deg}$$

Takahashi and Yin 2012.11576

Prediction of isotropic CB

- **Hint of isotropic CB from Planck data**

$$\beta = \frac{1}{4\pi}\int d\Omega\, \Phi(\Omega) = 0.36 \pm 0.11 \text{ deg}$$

from Planck 18 pol. data

Minami and Komatsu '20,
Diego-Palazuelos et al, PRL 128 '22

- **Kilobyte cosmic birefringence**

There will be O($10^{3\text{-}4}$) domains on the LSS, and the CMB polarization is either:

a) Not rotated at all *(same vacuum as us, crossed an even n. of DWs)*

b) Rotated by a fixed angle *(different vacuum from us, crossed an odd n. of DWs)*

→ Average over all angles will give a nonzero value!

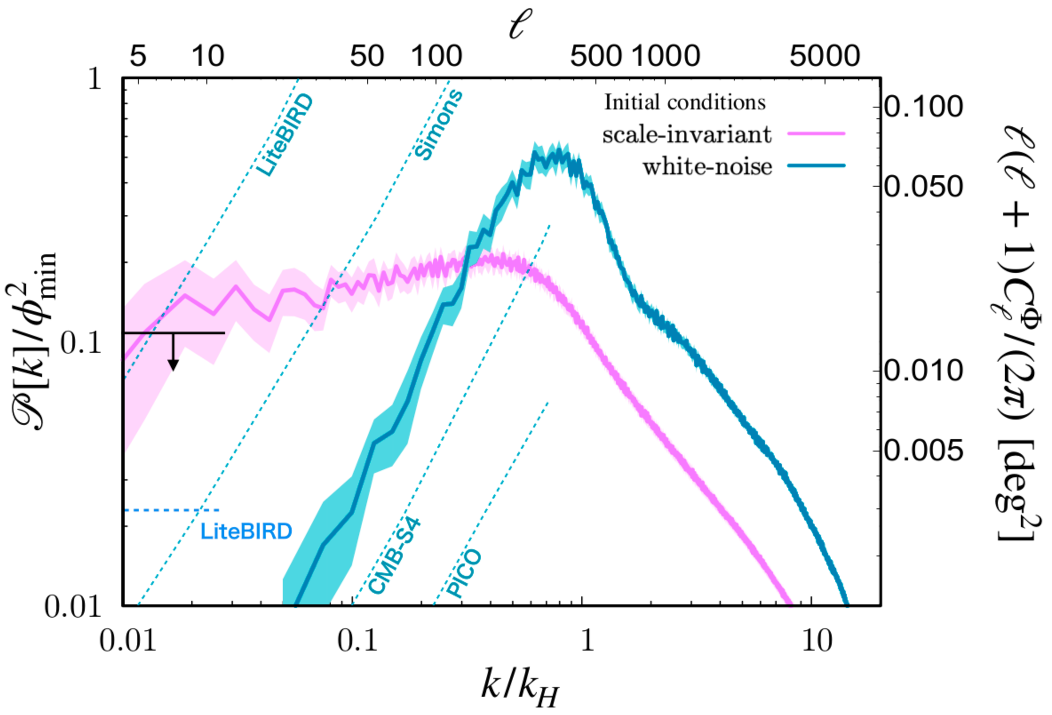
$$\beta = \frac{1}{4\pi}\int d\Omega\, \Phi(\Omega) = \frac{1}{2} c_\gamma \alpha \simeq 0.21 c_\gamma \,\text{deg}$$

Takahashi and Yin 2012.11576

The anomaly coefficient $c_\gamma \sim$ *O(1)* so this explains nicely the observed signal.

Prediction of anisotropic CB

We can make an estimate of anisotropic cosmic birefringence from the value of the power spectrum at large scales (small k), which gives a value within experimental bounds.



Gonzalez, Kitajima, Takahashi and Yin 2211.06849

- **BICEP/Keck bound**

$$\sqrt{\frac{L(L+1)C_L}{2\pi}}\left(=\frac{g_{\phi\gamma\gamma}}{2}\frac{H_{\rm inf}}{2\pi}\right) < 0.12\,\text{deg}$$

BICEP/Keck Collaboration, 2210.08038

- **Our estimate**

$$\sqrt{\frac{L(L+1)C_L}{2\pi}} \approx \sqrt{0.013\ \text{deg}^2} = 0.11\ \text{deg}$$

Within experimental bounds

Part 5
Summary

Summary

· DWs with inflationary fluctuations are **much more stable** than previously reported, due to correlations on superhorizon scales.

→ The network not decaying could have an **impact in cosmology**

· Stable Axion DW networks **explain the hint for isotropic CB** and gives a prediction **within experimental bounds for anisotropic CB**.

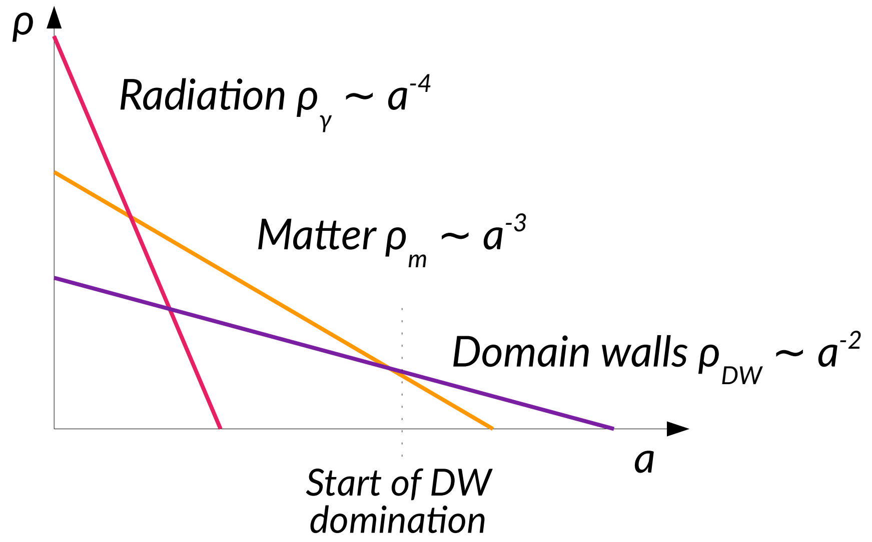
Questions & comments

Axions

Common lagrangian in axion models:

- Complex scalar field $\phi = fe^{i\theta}$, Z_N sym. $\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - \frac{1}{4}\lambda(\phi^\dagger \phi - \eta^2)^2 + 2(m^2\eta^2/N^2)(\cos N\theta - 1)$
- Low energy effective lagrangian $\mathcal{L}_\theta = \eta^2(\partial_\mu \theta)^2 + 2(m^2\eta^2/N^2)(\cos N\theta - 1)$
- Multiple degenerate minima! minima at $\theta = 2\pi n/N$ with $n = 0, 1, ..., N - 1$

Cosmological DW problem



A conservative upper limit on the DW density from current CMB observations would be

$$P_{DW} < 10^{-5} H_0^{\,2} M_p^{\,2} \quad \Rightarrow \quad \boxed{\sigma < (MeV)^3}$$

Avelino, Sousa '15

Depending on the mass of the scalar field that generated the DW network, **we may need the network to decay** to avoid this scenario!

About their correlations

The information about how correlations are distributed along different scales is captured in the **power spectrum**.

During inflation, massless/light scalars acquire almost scale invariant fluctuations:
(no additional bias is considered yet)

Linde and Lyth ‘90, Nagasawa and Yokoyama ‘92

Power spectrum

$$\langle \phi(\mathbf{k})\phi(\mathbf{k}')\rangle = (2\pi)^d \delta^{(d)}(\mathbf{k}+\mathbf{k}')P(k) \qquad d = 2 \text{ or } 3$$

Reduced power spectrum

INFLATIONARY

$$\mathscr{P}(k) \simeq \left(\frac{H_{\rm inf}}{2\pi}\right)^2$$

Constant!

with $\mathscr{P}(k) = \dfrac{k^d}{2\pi^{d-1}} P(k)$

Meanwhile, for thermal fluctuations,

THERMAL

$$\mathscr{P}(k) \sim k^2$$

What we knew

KEY POINT

The DW network quickly disappears even for small biases

This was first noted by D. Coulson, Z. Lalak and B. Ovrut in 1996, but many other papers have studied the phenomenon since then.

Larsson, Sarkar and White ‘97, Leite and Martins ‘14, Correia, Leite and Martins ‘18 + *many others*

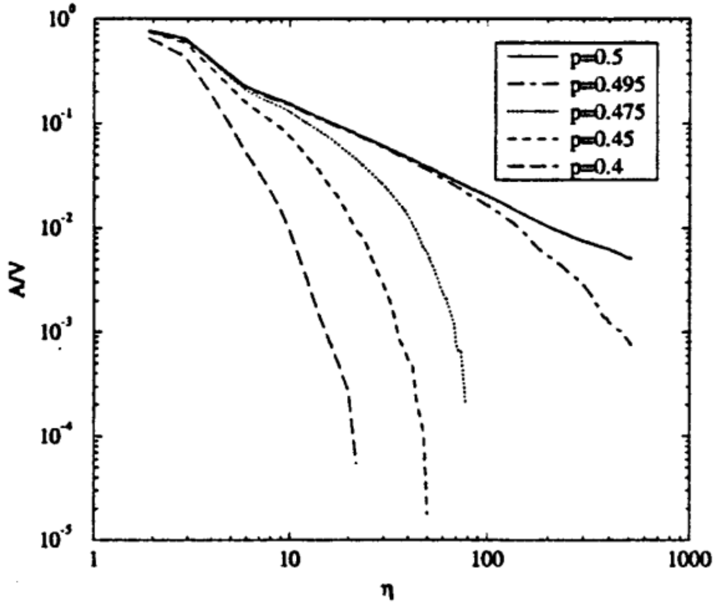
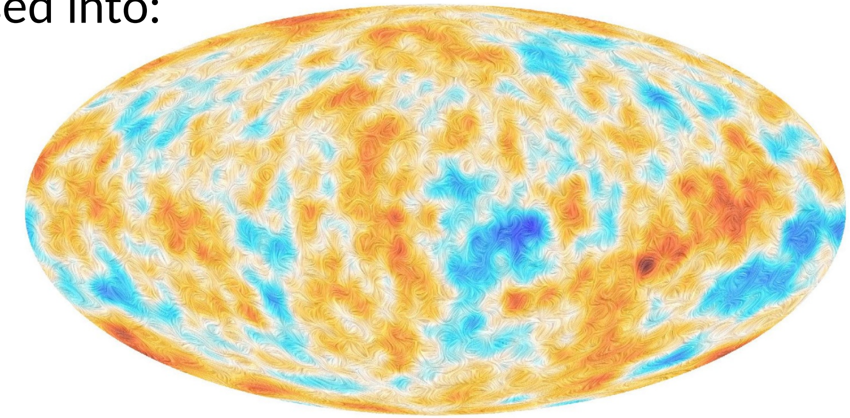


FIG. 7. Evolution of the comoving area, A, of domain walls per volume, V, with conformal time, η, in two dimensional runs. For each bias, p, we show the evolution of one realization.

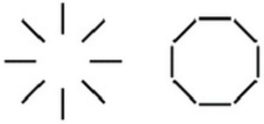
Coulson, Lalak and Ovrut ‘96

How to measure cosmic birefringence

The CMB polarization anisotropy map can be decomposed into:

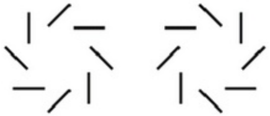


E-modes



- Parity even
- Produced by scalar pert.
- Dominates the CMB

B-modes



- Parity odd
- Produced by tensor pert.

- The power spectra of E- and B-modes are invariant under parity transformations, but the **cross-correlation power spectrum EB** isn't.

- CB shows as nonzero $C_\ell^{EB,o} = \frac{1}{2}\sin(4\beta)(C_\ell^{EE} - C_\ell^{BB})$

Lue, Wang, Kamionkowski '98,
Minami, Komatsu '20

- There's a **hint for isotropic CB**, but only a bound for anisotropic CB yet.

Planck data on CB

- **Isotropic CB**

$$\beta = \frac{1}{4\pi}\int d\Omega\,\Phi(\Omega) = 0.36 \pm 0.11\ \mathrm{deg}$$

from Planck 18 pol. data

Minami and Komatsu ‘20,
Diego-Palazuelos et al, PRL 128 ‘22

Based on a new method that uses both the CMB and Galactic foreground to distinguish between CB (β) and detector orientation miscalibration (α).

Minami et al, PTEP ‘19, Minami PTEP ‘20, Minami and Komatsu ‘20

- **Anisotropic CB**

$$\sqrt{\frac{L(L+1)C_L}{2\pi}}\ \Bigg(\underbrace{= \frac{g_{\phi\gamma\gamma}}{2}\frac{H_{\rm inf}}{2\pi}}_{\text{For massless axion}}\Bigg) < 0.12\,\mathrm{deg}$$

(95% CL)

BICEP/Keck Collaboration, 2210.08038

Domain walls enter the picture

If we take the hint of isotropic CB and substitute it in the ALP CB prediction:

$$\beta = \frac{1}{4\pi}\int d\Omega\, \Phi(\Omega) = 0.36 \pm 0.11 \text{ deg}$$

$$\Phi = \frac{g_{\phi\gamma\gamma}\Delta\phi}{2} \simeq 0.42 c_\gamma \left(\frac{\phi_{\text{today}} - \phi_{\text{LSS}}(\Omega)}{2\pi f_\phi}\right) \text{ deg}$$

$$\boxed{\Delta\phi = \mathcal{O}(\pi f_\phi)}$$

Is how much, on average, the ALP must have moved after recombination

There will be $O(10^{3\text{-}4})$ domains on the LSS, and the CMB polarization is either:

- **Not rotated at all** (same vacuum as us, crossed an even n. of DWs)
- **Rotated by a fixed angle** (different vacuum from us, crossed an odd n. of DWs)

This was called **kilobyte cosmic birefringence**.

Takahashi and Yin 2012.11576