Probing ultra-light dark matter
with lensed gravitational waves

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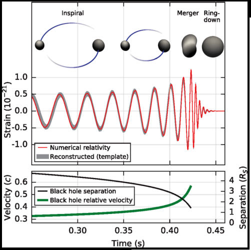
MAX-PLANCK-GESELLSCHAFT

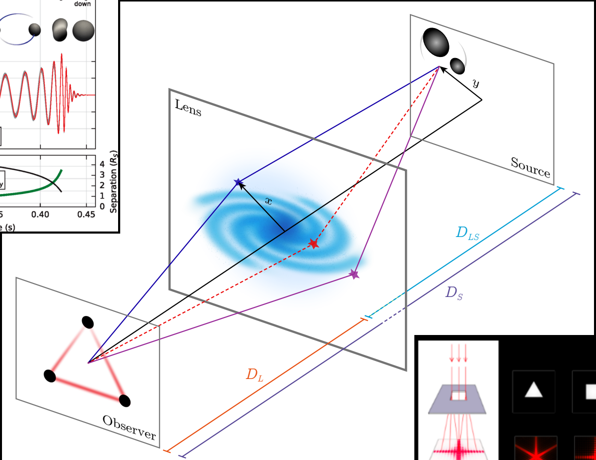
Max Planck Institute for Gravitational Physics
(Albert Einstein Institute)

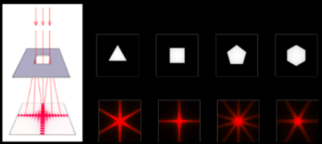
with G. Tambalo, L. Dai, HY Cheung, S. Singh & G. Brando

VLDM Workshop, March 2023

Gravitational Wave Lensing (LIGO '16)



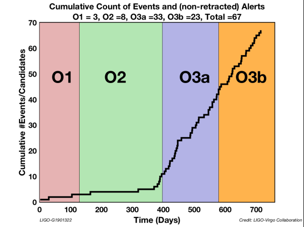


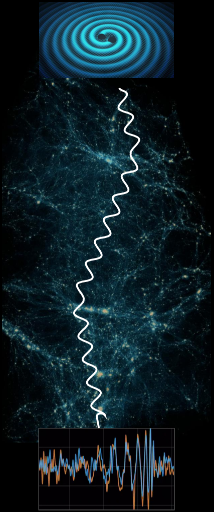


Deflection, multiple images, magnification, time delay, diffraction

Why GW lensing?

- EM lensing → Large-scale structure, dark matter...
- GWs highly complementary:
  - Coherent, low frequency → wave effects
  - Weakly coupled → universe transparent to GWs
  - Well modeled → less uncertainty
- Many GW events → lensing increasingly relevant
  (e.g. LIGO/Virgo/Kagra searches)

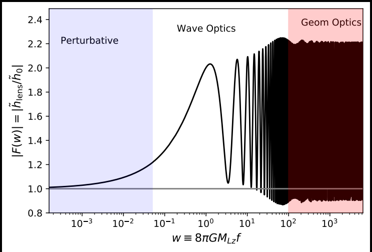




Frequency dependence (Schneider+93, Takahashi+03, Tambalo+22)

$$F(w) \equiv \frac{\tilde{h}_{\rm lens}}{\tilde{h}_{\rm flat}}$$

$$\boxed{w \equiv 8\pi G M_{Lz} f} \sim \left(\frac{M_L}{10^4 M_\odot}\right)\left(\frac{f}{\rm Hz}\right)$$



(E.g. $30+30M_\odot$ starting at 40Hz)

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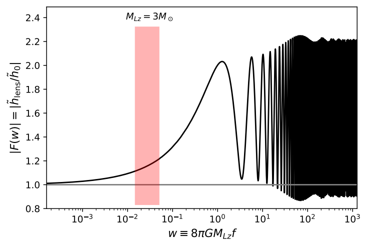
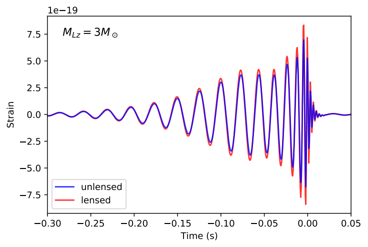
- Perturbative $(w \to 0)$

  $$F \approx 1 + A w^\alpha$$





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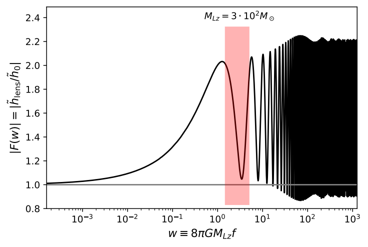
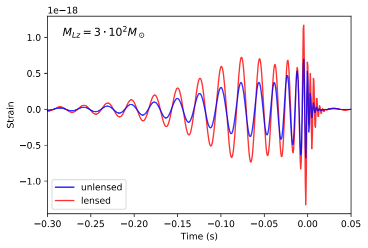
$$F = \frac{w}{2\pi i}\int d\vec{x} e^{iwT(\vec{x})}$$





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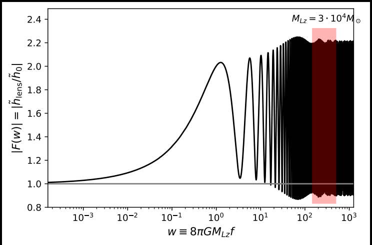
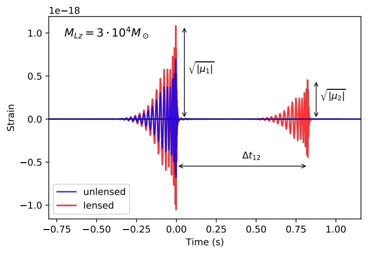
- Geometric optics $(w \to \infty)$

$$F \to \sum_I \sqrt{|\mu_I|} e^{i(wT_I + \pi n_I)}$$





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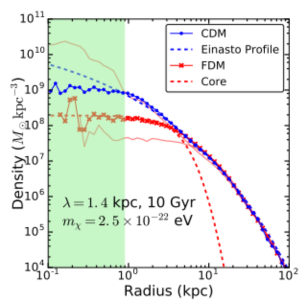
Probing a cored lens with GWs

Cored profile

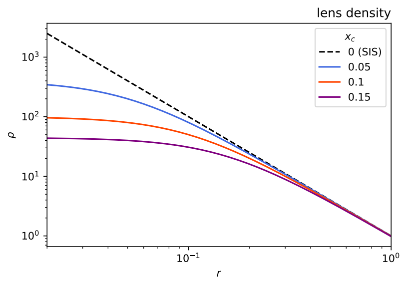
$$\rho(r) \propto \frac{1}{r^2 + r_c^2}$$

e.g. self-interacting DM,
ultra-light DM $\longrightarrow$



(Zhang+18)

Probing a cored lens with GWs



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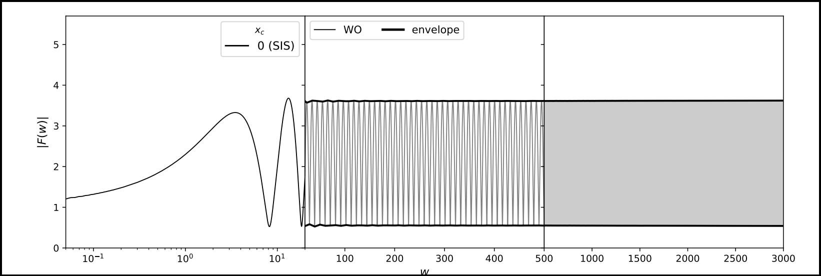

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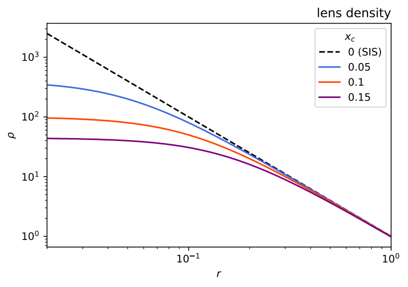

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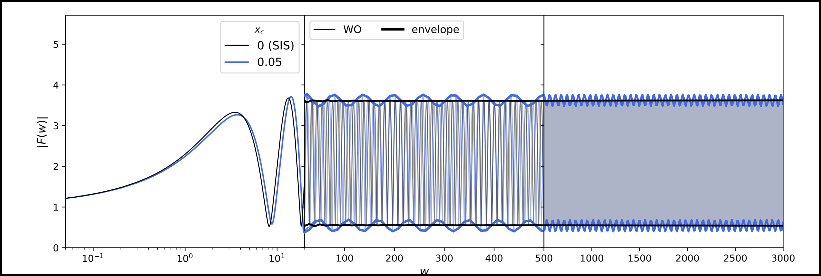

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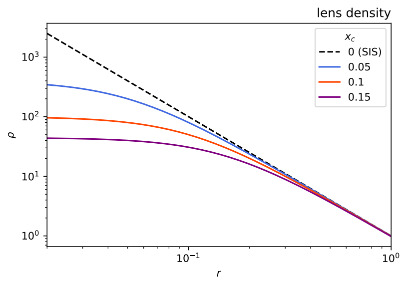

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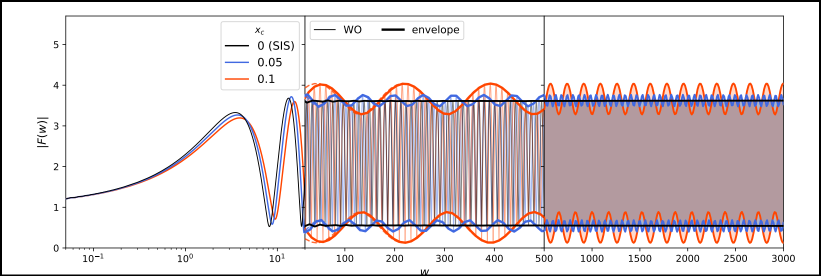

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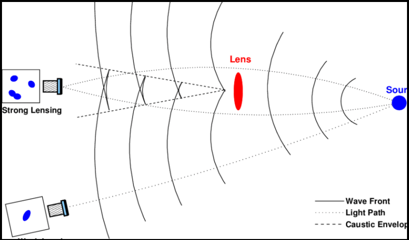

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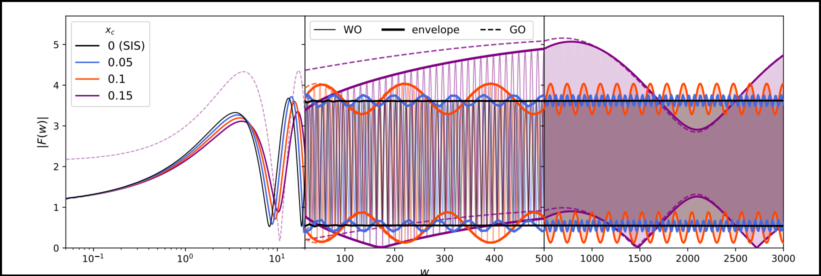
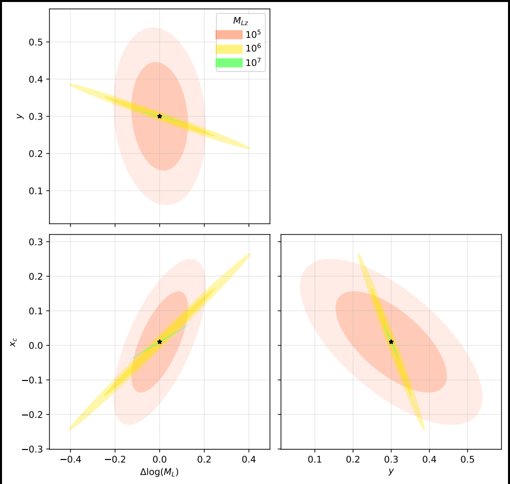


Reconstructing lens parameters (Fisher matrix)



LISA ($M_{\rm BBH} = 10^6 M_\odot$, $y = 0.3, x_c = 0.01$, fixed SNR=1000)

See also Takahashi+04, Caliskan+22

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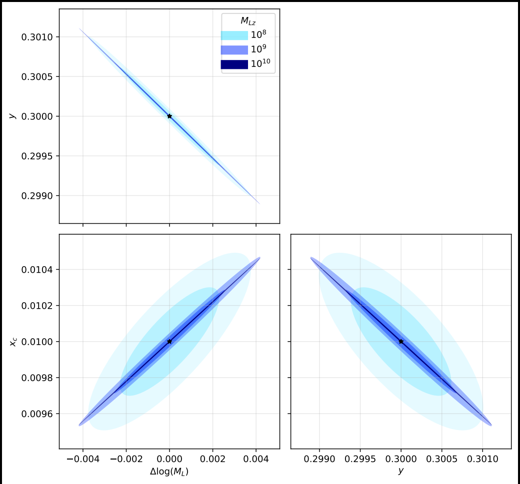

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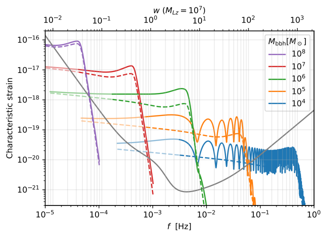

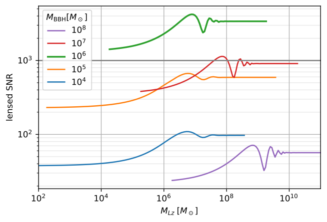

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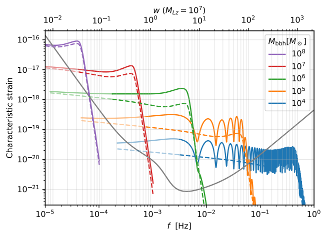
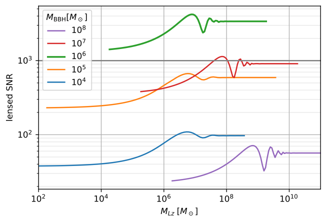
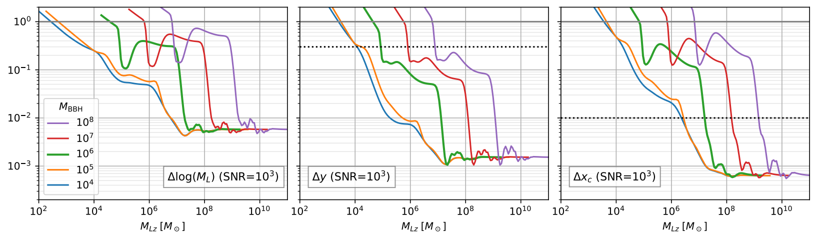


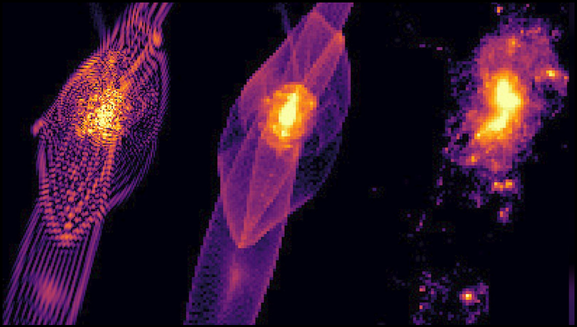




Lens mass	Impact param.	Core size
	$(y = 0.3)$	$(x_c = 0.01)$

Tests of Dark Matter

(Mocz+20, Hui+16, $\cdots$)



fuzzy DM warm DM Cold DM

"Fuzzy" DM: ultra-light axion $r_c > 0.33\text{kpc}\frac{10^9 M_\odot}{M_c}\left(\frac{10^{-22}\text{eV}}{m_\phi}\right)^2$

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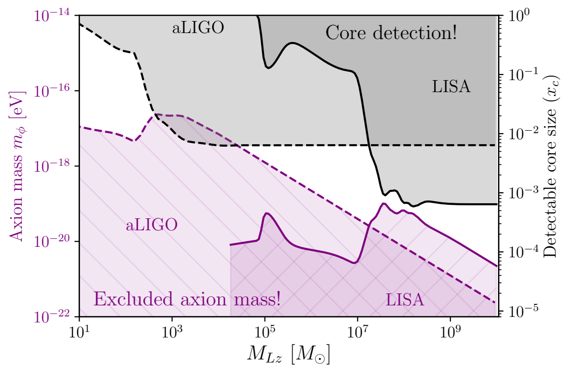

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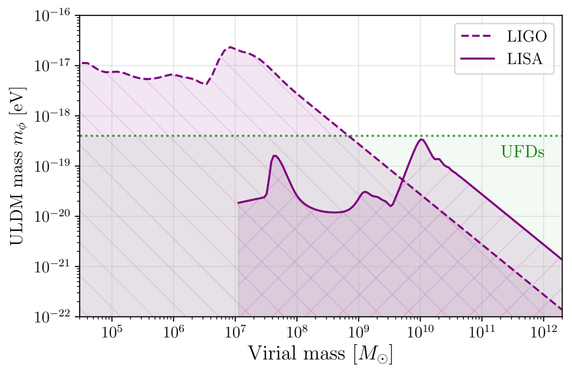

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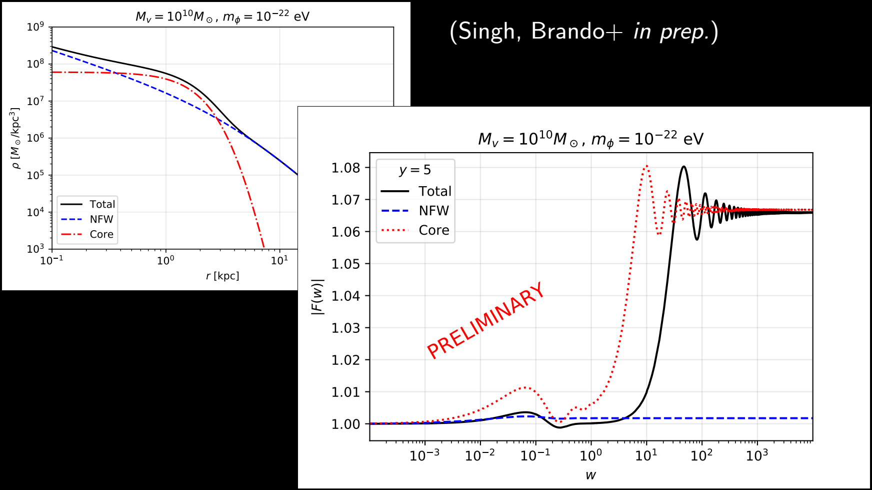
Compare w/ (Dalal, Kravtsov 22)

FDM modelling beyond cored isothermal sphere

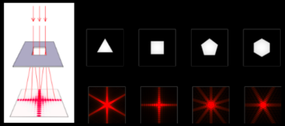
(Singh, Brando+ *in prep.*)

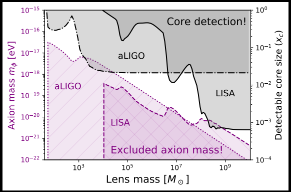


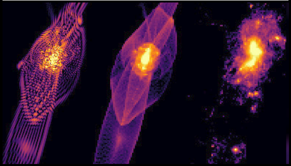
Include more realistic profile, substructure, etc...

Conclusions

- GW complement EM lensing
  - wave effects
  - probes lens properties
- Fast & accurate $F(w)$ computations
- Recover lens params
  - precision $\sim 1/\mathrm{SNR}$
  - strong parameter degeneracies
- Wave optics $\rightarrow$ additional lens info!
- Probe dark matter properties $\longrightarrow$







FDM wDM CDM

Backup Slides

Fisher matrix analysis

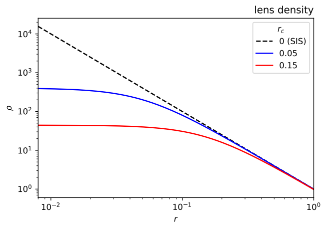
(Vallisneri '07)

$$F_{IJ} = \left(\frac{\partial h_L}{\partial \theta_I} \middle| \frac{\partial h_L}{\partial \theta_J} \right), \qquad (h|g) = 4\Re \left(\int \frac{df}{S_n(f)} \tilde{h}(f) \tilde{g}^*(f) \right)$$

- $\tilde{h}_L(w, \vec{\theta}) = F(w)\tilde{h}(w)$
- $F \to \begin{cases} \text{WO} & (w < w_{\rm cut}) \\ \text{Geom Opt.} & (w > w_{\rm cut}) \end{cases}$
- $\theta_I \in (\underbrace{\log(D_L), \phi_0}_{\text{source}}, \underbrace{\log(M_{Lz}), y, x_c}_{\text{lens}})$
- Static single detector, optimal orientation...

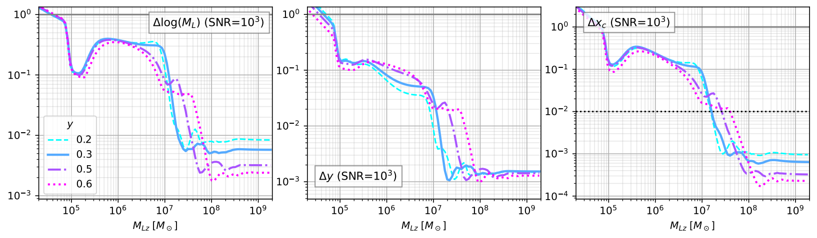
  (Caliskan+ '22 $\to$ detailed source modeling)

Lens with a core:



Vary impact parameter

LISA ($M_{\rm BBH} = 10^6 M_\odot$, fixed SNR=1000)



Lens mass — Impact param. — Core size

Vary impact parameter

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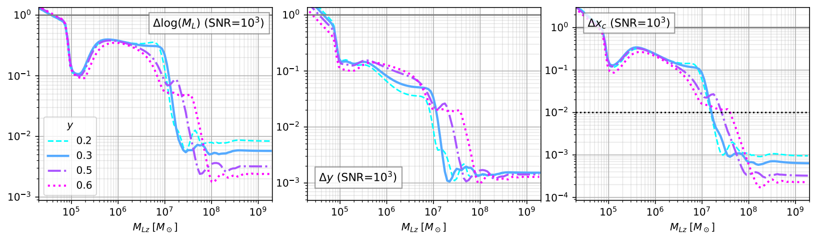
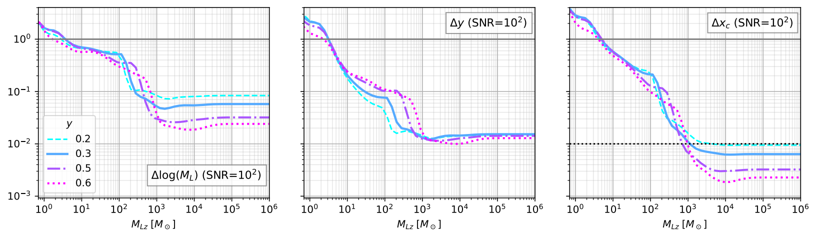


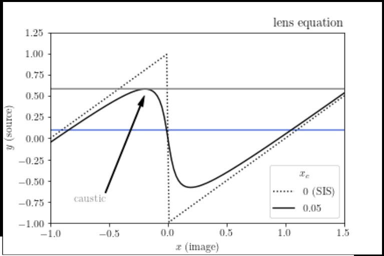
advanced LIGO ($M_{\mathrm{BBH}} = 10^6 M_\odot$, fixed SNR=100)

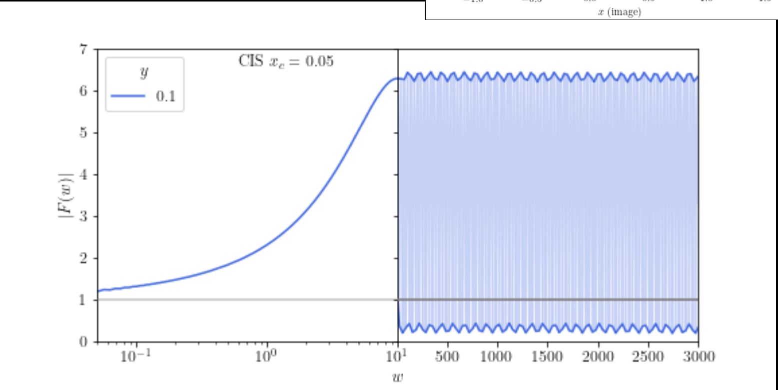


Lens mass | Impact param. | Core size

Strong vs weak lensing

Fix core size $x_c = 0.05$





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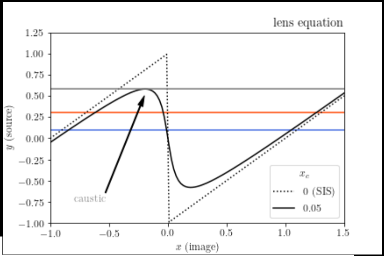

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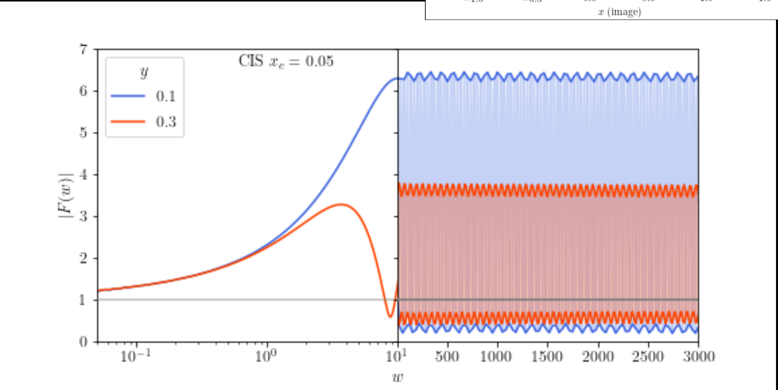

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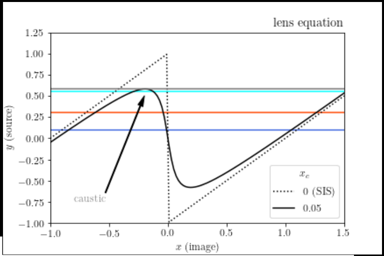

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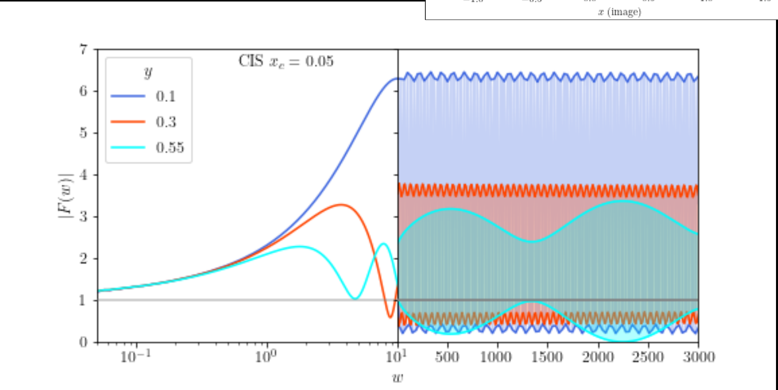

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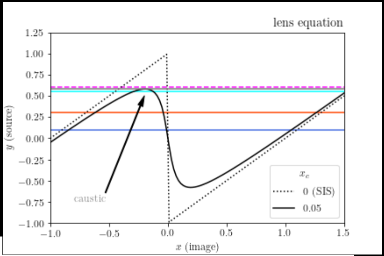

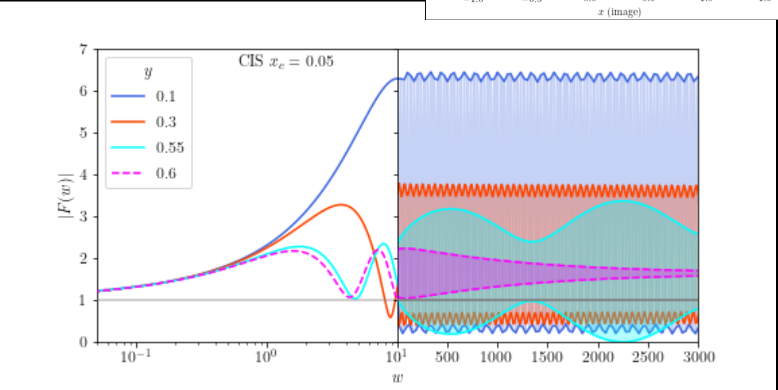

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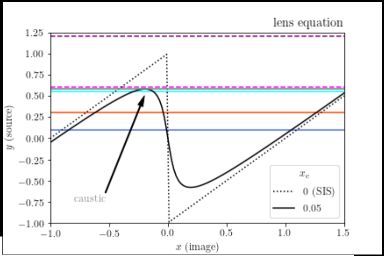
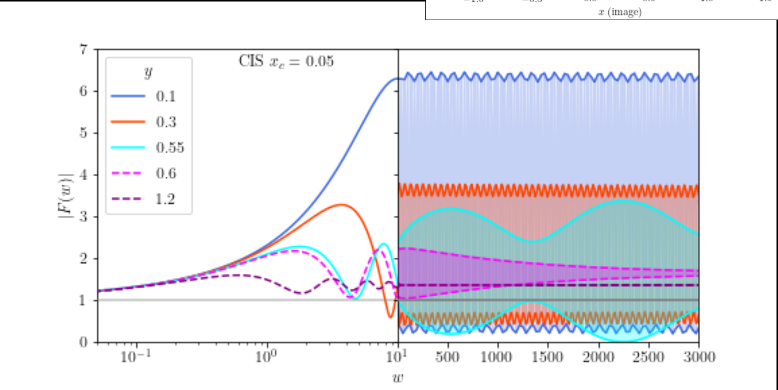




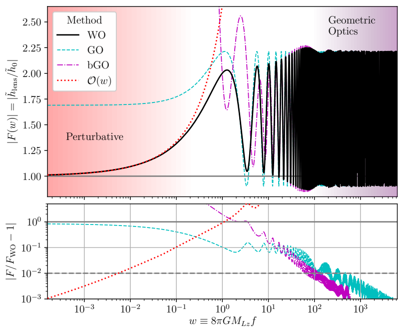
Computing amplification factor I

$$F = \frac{w}{2\pi i} \underbrace{\int d^2x e^{iwT(\vec{x},\vec{y})}}_{I(w)} \xrightarrow{w \to 0} 1 + Aw^{\alpha}$$

$$\xrightarrow{w \to \infty} \sum_I \sqrt{|\mu_I|} e^{i(wT_I + \pi n_I)}$$

1) beyond Geometric Optics:

$$\sum_I \sqrt{|\mu_I|} \left(1 + i\frac{\Delta_I}{w}\right) e^{i(wT_I + \pi n_I)}$$



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1) beyond Geometric Optics:

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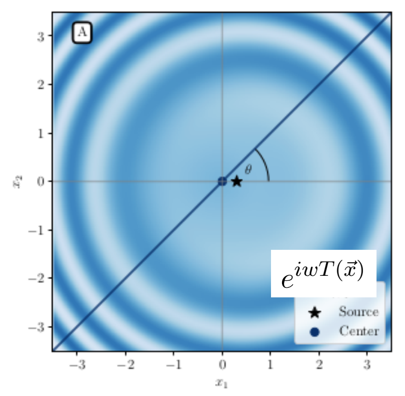
2) $\mathbb{C}$-deformation:

$$\vec{x} \to (r,\theta) \to (z(\lambda),\theta)$$

(Feldbrugge+, Tambalo, MZ+)



Computing amplification factor I

$$F = \frac{w}{2\pi i} \underbrace{\int d^2 x e^{iwT(\vec{x},\vec{y})}}_{I(w)} \quad \xrightarrow{w \to 0} \quad 1 + Aw^\alpha$$

$$\xrightarrow{w \to \infty} \quad \sum_I \sqrt{|\mu_I|} e^{i(wT_I + \pi n_I)}$$

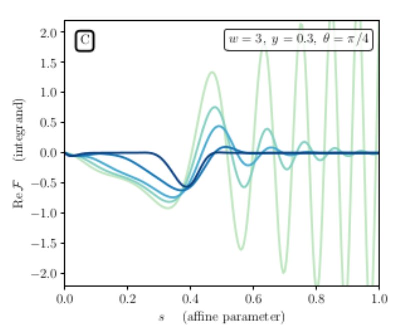
1) beyond Geometric Optics:

$$\sum_I \sqrt{|\mu_I|} \left(1 + i \frac{\Delta_I}{w}\right) e^{i(wT_I + \pi n_I)}$$

2) $\mathbb{C}$-deformation:

$$\vec{x} \to (r, \theta) \to (z(\lambda), \theta)$$

(Feldbrugge+, Tambalo, MZ+)



Computing amplification factor II

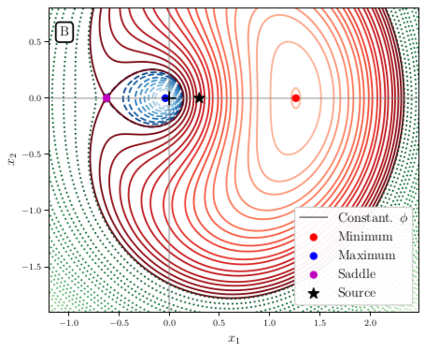
$$F = \frac{w}{2\pi i} \underbrace{\int d^2x e^{iwT(\vec{x},\vec{y})}}_{I(w)} \xrightarrow{w\to 0} 1 + Aw^\alpha$$

$$\xrightarrow{w\to\infty} \sum_I \sqrt{|\mu_I|} e^{i(wT_I + \pi n_I)}$$

Contour flow

$$\tilde{I}(\tau) = \int dw e^{-iw\tau} I(w)$$
$$= \int d^2x \delta(\tau - T(\vec{x}))$$

(Ulmer+ , Diego+, Tambalo, MZ+)



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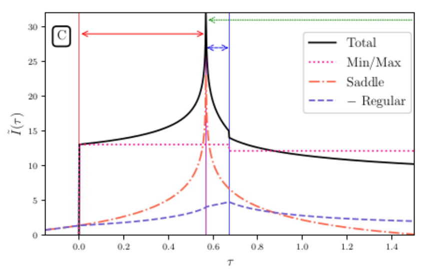

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