Motion of S2 and bounds on scalar clouds around SgrA*

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& the ***GRAVITY*** *collaboration*

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Motivation

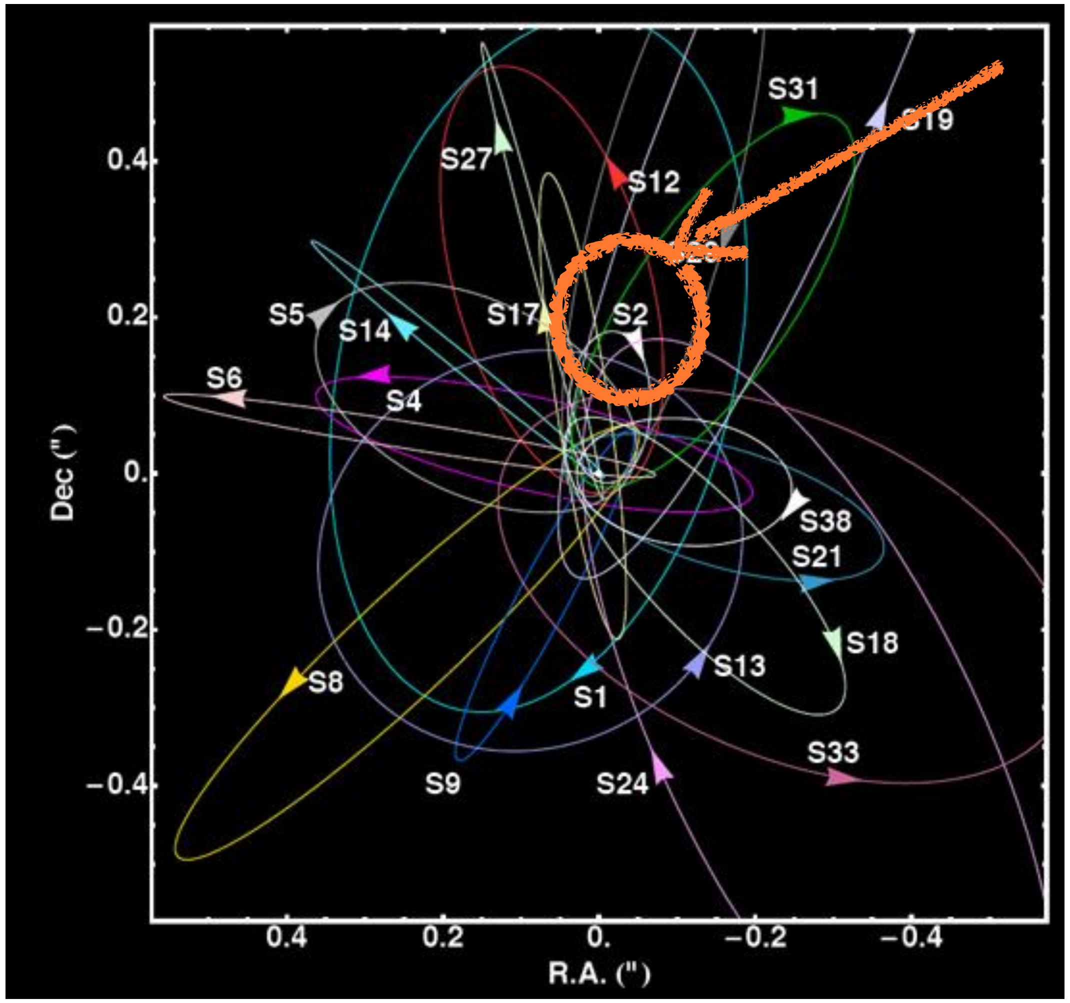
Motivation: Ultralight bosons are possible candidates for **Dark Matter** (DM).
DM may cluster around supermassive BHs (Sadeghian *et al*. 2013).

Idea: Constrain an ultralight scalar field cloud around the supermassive Black Hole (BH), ***Sagittarius A****, at the center of the Milky Way using orbital motion of S-stars.

We will focus on star S2.

Data: We have **astrometry** (positions in the sky) and **spectroscopy** (radial velocity measurements).

Several works used S-stars to obtain upper bounds on the extended mass around Sgr A*.



Credits to S. Gillessen, GRAVITY Coll., Max Planck Institute for Extraterrestrial Physics

Setup

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G c^{-4}} - \frac{1}{2} g^{\alpha\beta} \partial_\alpha \Psi^* \partial_\beta \Psi^* - \frac{\mu}{2} \Psi \Psi^* \right)$$

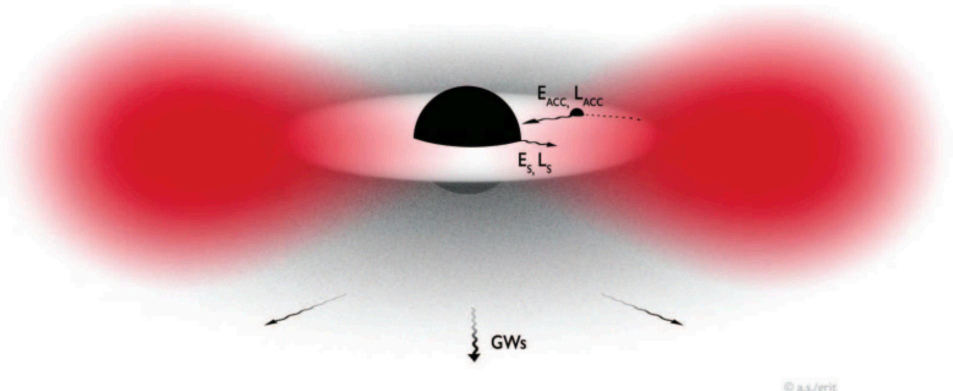
Mass coupling: $\alpha = r_g \mu = \left[\frac{GM_\bullet}{c^2} \right] \left[\frac{m_s c}{\hbar} \right]$

In the limit $\alpha \ll 1$, the fundamental mode of the field $(\ell = m = 1)$ is given by (Brito *et al.* 2015)

$$\Psi = A_0 e^{-i(\omega_R t - \varphi)}\, r\, \alpha^2 e^{-r\alpha^2/2} \sin\theta$$

where $A_0^2 = \Lambda \frac{\alpha^4}{64\,\pi}$

$$\Lambda = \frac{M_{\rm cloud}}{M_\bullet}$$



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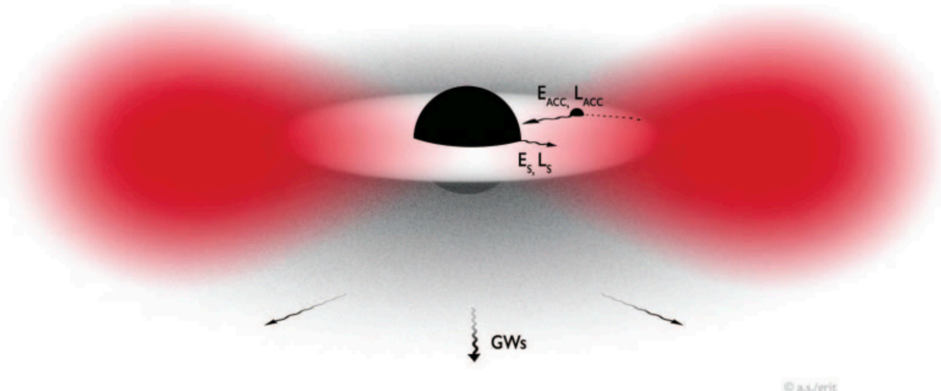
where $A_0^2 = \Lambda\frac{\alpha^4}{64\,\pi}$ $\quad \Lambda = \frac{M_{\rm cloud}}{M_\bullet}$

The energy density of the scalar field is:

$$\rho = \frac{m_s^2c^2}{\hbar^2}|\Psi|^2 + \mathcal{O}\left(c^{-4}\right)$$



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Solving $\nabla^2 U_{\rm scalar} = 4\pi\rho$ we obtain the scalar potential:

$$U_{\rm scalar} = \sum_{\ell m}\frac{4\pi}{2\ell+1}\left[q_{\ell m}(r)\frac{Y_{\ell m}(\theta,\varphi)}{r^{\ell+1}} + p_{\ell m}(r)r^\ell Y_{\ell m}(\theta,\varphi)\right] = \Lambda\left[P_1(r,\alpha) + P_2(r,\alpha)\,\cos^2\theta\right]$$

and the Lagrangian:

$$\mathcal{L} = \frac{1}{2}\left(\dot r^2 + r^2\dot\theta^2 + r^2\sin^2\theta\dot\phi\right) + \frac{M_\bullet}{r} + \Lambda\left(P_1(r,\alpha) + P_2(r,\alpha)\cos^2\theta\right)$$

Setup

Corrections to the Newtonian model

(GRAVITY Coll. 2018, Alexander 2005)

- **Newtonian effect**: the Roemer delay due to finite value of c.
- **Relativistic effects**: the Doppler shift and the gravitational redshift.
- **1 Post Newtonian (PN) correction**

Schwarzschild precession has been detected on S2 motion at 8σ confidence level (GRAVITY Coll. 2020)

$$a_{1\mathrm{PN}} = f_{\mathrm{SP}} \frac{M_\bullet}{r^2} \left[\left(\frac{4M_\bullet}{r} - v^2 \right) \frac{\boldsymbol{r}}{r} + 4\dot{r}\boldsymbol{v} \right]$$

where $f_{\mathrm{SP}} = 1, \boldsymbol{r} = r\hat{r}, \boldsymbol{v} = \left(\dot{r}\hat{r}, r\dot{\theta}\hat{\theta}, r\dot{\phi}\sin\theta\hat{\phi} \right), v = |\boldsymbol{v}|$

Method

First step: minimize the χ^2

Effective peak position of ρ

$$R_{\rm peak} = \frac{\int_0^\infty \rho \bar{r} d\bar{r}}{\int_0^\infty \rho d\bar{r}} = \frac{3M_\bullet}{\alpha^2}$$

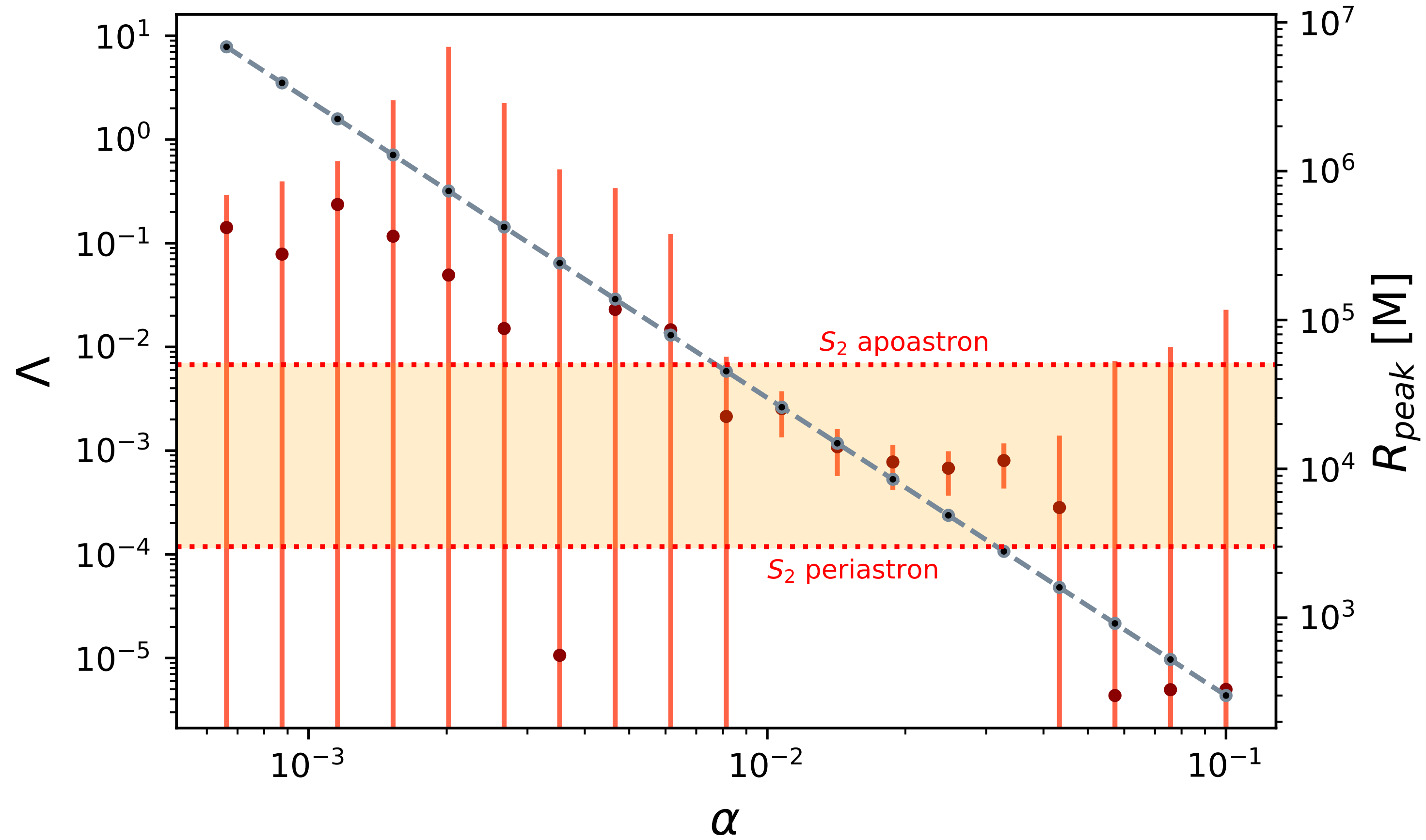
Smaller uncertainties in Λ for

$$0.01 \lesssim \alpha \lesssim 0.03$$

which (roughly) corresponds to

$$35\, M_\bullet \lesssim R_{\rm peak} \lesssim 30000\, M_\bullet$$

$$\left(3000\, M_\bullet \lesssim r_{S2} \lesssim 50000\, M_\bullet\right)$$



Method

Second step: applying Markov Chain Monte Carlo (MCMC) method using **emcee** (Foreman-Mackey *et al*. 2013) Python package

We need to *sample* $P(\theta | D) \propto P(D | \theta) P(\theta)$ for different fixed values of α

D = data set

$$\theta_i = \{e, a_{\rm sma}, \Omega, i, \omega, t_p, R_0, M_\bullet, x_0, y_0, v_{x0}, v_{y0}, v_{z0}, \Lambda\}$$

- $e, a_{\rm sma}, \Omega, i, \omega, t_p$: Keplerian elements
- $R_0, M_\bullet$: BH Mass and GC distance
- $x_0, y_0, v_{x0}, v_{y0}, v_{z0}$: Correction to NACO and RV data
- Λ: Scalar field fractional mass

$P(D | \theta)$ = Gaussian Likelihood

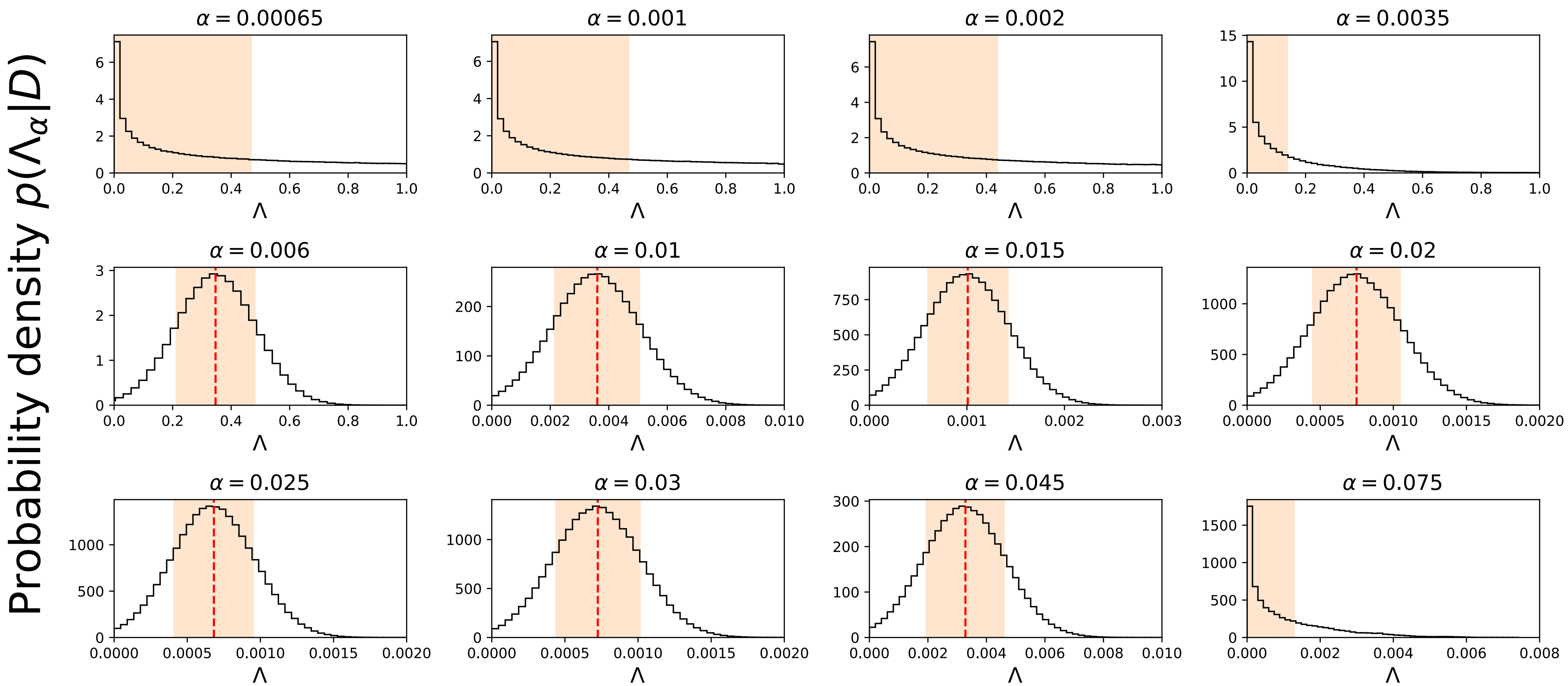
$P(\theta)$ = **Uniform** priors for physical parameters, **Gaussian** priors for $(x_0, y_0, v_{x0}, v_{y0}, v_{z0})$ (Plewa *et al*. 2015)

Results

Second step: applying Markov Chain Monte Carlo (MCMC) method using **emcee** (Foreman-Mackey *et al*. 2013) Python package

$$\hat{\Lambda} = \arg\max \mathscr{L}(\Lambda_\alpha \,|\, \mathrm{D})$$

α	$\hat{\Lambda}$	$\log_{10} K$
0.00065	$\lesssim (0.470\,, 0.980)$	0.09
0.001	$\lesssim (0.470\,, 0.980)$	0.08
0.002	$\lesssim (0.440\,, 0.978)$	-0.06
0.0035	$\lesssim (0.140\,, 0.780)$	-10.58
0.006	0.34671 ± 0.13666	1.44
0.01	0.00361 ± 0.00147	1.29
0.015	0.00101 ± 0.00042	1.24
0.02	0.00075 ± 0.00030	1.33
0.025	0.00068 ± 0.00028	1.35
0.03	0.00073 ± 0.00029	1.33
0.045	0.00328 ± 0.00135	1.27
0.075	$\lesssim (0.0013\,, 0.0052)$	0.0001



Orange bands: 1σ confidence interval, such that $P(\Lambda_\alpha < \Lambda_1 \,|\, D) \approx 68\,\%$ of $P(\Lambda_\alpha \,|\, D)$

Bayes' factor $$K = \frac{\mathscr{L}(\hat{\Lambda}_\alpha \,|\, D)}{\mathscr{L}(\Lambda = 0 \,|\, D)}$$

According to (Kass & Raftery 1995):

$1 \leq \log_{10} K \leq 2$ evidence is **strong**

$\log_{10} K > 2$ evidence is **decisive**

Conclusions

Cloud formation process

Fluctuations of massive scalar fields can be exponentially amplified by **superradiance** (Brito *et al*. 2015). However, (Kodama & Yoshino 2012) show that for $M_\bullet \sim 4 \cdot 10^6 \, M_\odot$

$$m_s \geq 10^{-18}\,\mathrm{eV} \qquad\qquad (\alpha = 0.045,\ m_s \simeq 3 \cdot 10^{-18}\,\mathrm{eV})$$

However, we can assume DM existed by itself in the galaxy and the BH passes through it, leading to long-lived structures (Cardoso *et al*. 2022a, Cardoso *et al*. 2022b).

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To summarize…

We used the astrometry and the radial velocity measurements of S2 to constrain the fractional mass $\Lambda = M_{\mathrm{cloud}}/M$ of a scalar field cloud around Sgr A*.
Orbital range of S2 only allow us to constrain $0.01 \lesssim \alpha \lesssim 0.045$ and we found $\Lambda \lesssim 10^{-3}$ at 3σ confidence level.

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Thank you for your attention!

Back-up slides

Setup

Corrections to the Newtonian model

(GRAVITY 2018, Alexander 2005)

- **Newtonian effect**: the Roemer delay due to finite value of c

Roemer equation: $$t_{\text{obs}} - t_{\text{em}} + \frac{z_{\text{obs}}(t_{\text{em}})}{c} = 0$$

1st order expansion around t_{obs} : $$t_{\text{em}} \simeq t_{\text{obs}} + \frac{z_{\text{obs}}(t_{\text{obs}})}{c - v_{z_{\text{obs}}}(t_{\text{obs}})}$$

On average on S2 orbit
$\Delta t = t_{\text{em}} - t_{\text{obs}} \approx 8 \text{ days}$

Setup

Corrections to the Newtonian model

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- **Newtonian effect**: the Roemer delay due to finite value of c

- **Relativistic effects**: the Doppler shift and the gravitational redshift $(G = c = 1)$ must be included when S2 reaches periastron with total space velocity $\beta \sim 10^{-2}$.

Doppler: $$z_D = \frac{1 + \beta \cos\theta}{\sqrt{1 - \beta^2}} - 1$$

Gravitational redshift: $$z_{\rm grav} = \frac{1}{\sqrt{1 - 2M_\bullet / r_{\rm em}}} - 1$$

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where $f_{\mathrm{SP}} = 1, \boldsymbol{r} = r\hat{r}, \boldsymbol{v} = \left(\dot{r}\hat{r}, r\dot{\theta}\hat{\theta}, r\dot{\phi}\sin\theta\hat{\phi} \right), v = |\boldsymbol{v}|$

The equations of motion

From Euler-Lagrange equations: $$\frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{q}}\right) - \frac{\partial \mathscr{L}}{\partial q} = 0$$

$$\longrightarrow \begin{cases} \ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\dot{\phi}^2 + \dfrac{1}{r^2} - \Lambda\left(P_1'(r) + P_2'(r)\cos 2\theta\right) = 0 \\ 2r\dot{r}\sin^2\theta\dot{\phi} + 2r^2\cos\theta\sin\theta\dot{\theta}\dot{\phi} + r^2\sin^2\theta\ddot{\phi} = 0 \\ 2r\dot{r}\dot{\theta} + r^2\ddot{\theta} - r^2\cos 2\theta\dot{\theta}\dot{\phi}^2 + 2\Lambda P_2(r)\sin 2\theta\dot{\theta} = 0 \end{cases}$$

That we numerically integrate using an adaptive Runge-Kutta of order 4(5) and initial conditions given by the solution of Kepler's two body problem.

$$r(t_0) = \frac{1 - e^2}{1 + e\cos(\phi(t_0))}$$

$$\phi(t_0) = 2\arctan\left(\sqrt{\frac{1+e}{1-e}}\tan\frac{\mathscr{E}(t_0)}{2}\right)$$

$$\dot{r}(t_0) = \frac{2\pi e\sin(\mathscr{E}(t_0))}{1 - e\cos(\mathscr{E}(t_0))}$$

$$\dot{\phi}(t_0) = \frac{2\pi(1-e)}{(e\cos(\mathscr{E}(t_0)) - 1)^2}\sqrt{\frac{1+e}{1-e}}$$

with

Kepler's equation

$$\mathscr{E} - e\sin\mathscr{E} - \mathscr{M} = 0$$

$$\mathscr{M} = \frac{2\pi}{P}\left(t_0 - t_p\right)$$

How emcee works

(Foreman-Mackey et al. 2013, Goodman & Weare 2010)

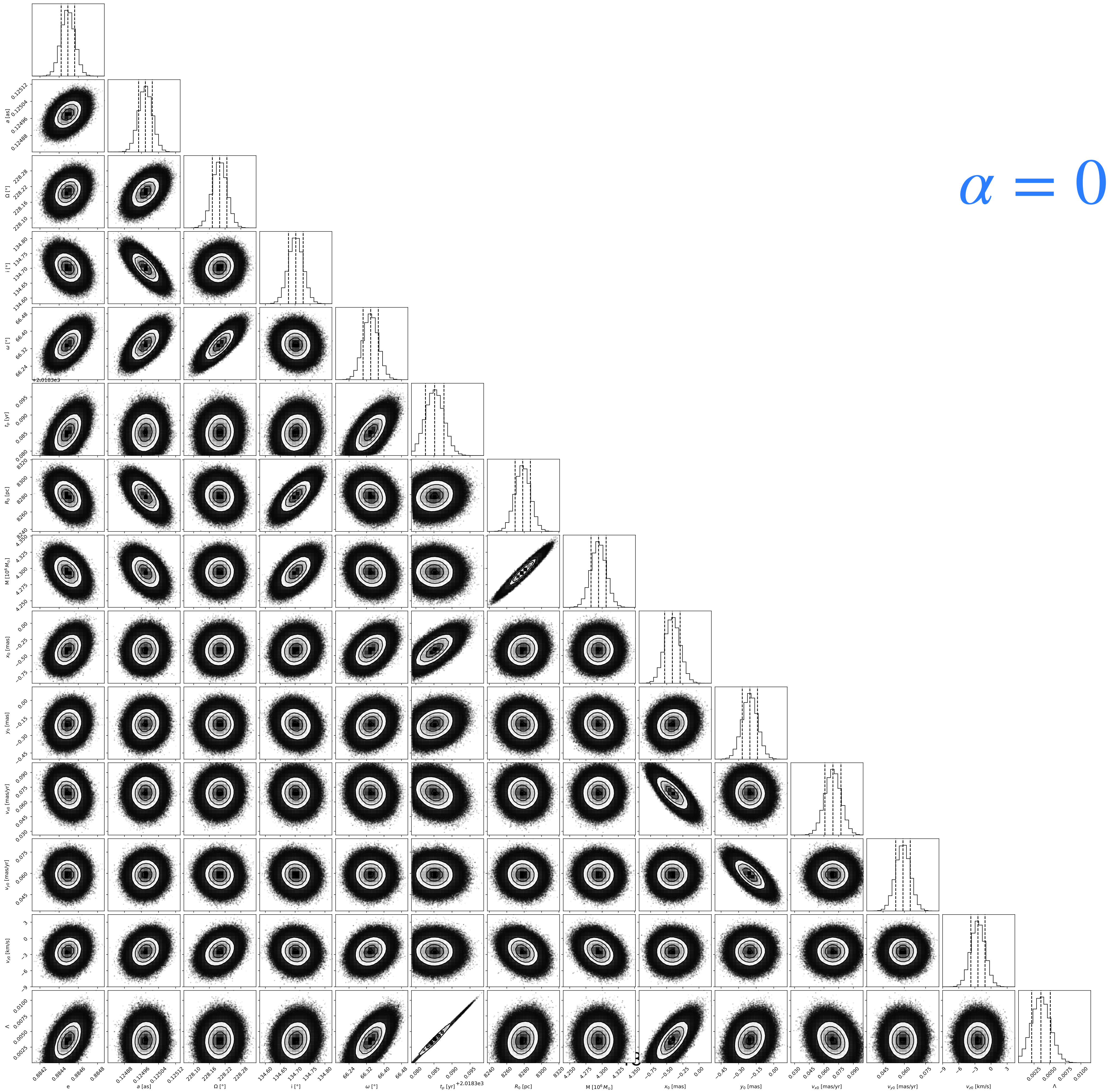
- **Step 1**. It generates K walkers around any initial value of the parameters θ_i^0 from $\mathcal{N}(\theta_i^0, \sigma)$ ($\sigma = 10^{-5}$);

- **Step 2.** To update the position of a walker at $X_k(t)$, a walker X_j is randomly extracted from the complementary ensemble $S_{[k]} = \left\{ X_j, \forall j \neq k \right\}$ and the new position is generated as $Y = X_j + Z\left[X_k(t) - X_j\right]$, where Z is drawn from $g(Z = z)$ defined as:

$$g(z) \propto \begin{cases} \frac{1}{\sqrt{z}} & \text{if } z \in \left[\frac{1}{a}, a\right] \\ 0 & \text{otherwise} \end{cases}$$

- **Step 3**. It computes $q = \min\left(1, Z^{N-1}\frac{p(Y)}{p\left(X_k(t)\right)}\right)$, where N is the number of parameters, for each walker.

- **Step 4**. It randomly extracts a variable $r \sim U\left[0, 1\right]$. If $r \leq q$ then the move is ***accepted*** and $X_k(t+1) \rightarrow Y$. If $r > q$ the move is ***rejected*** and $X_k(t+1) \rightarrow X_k(t)$.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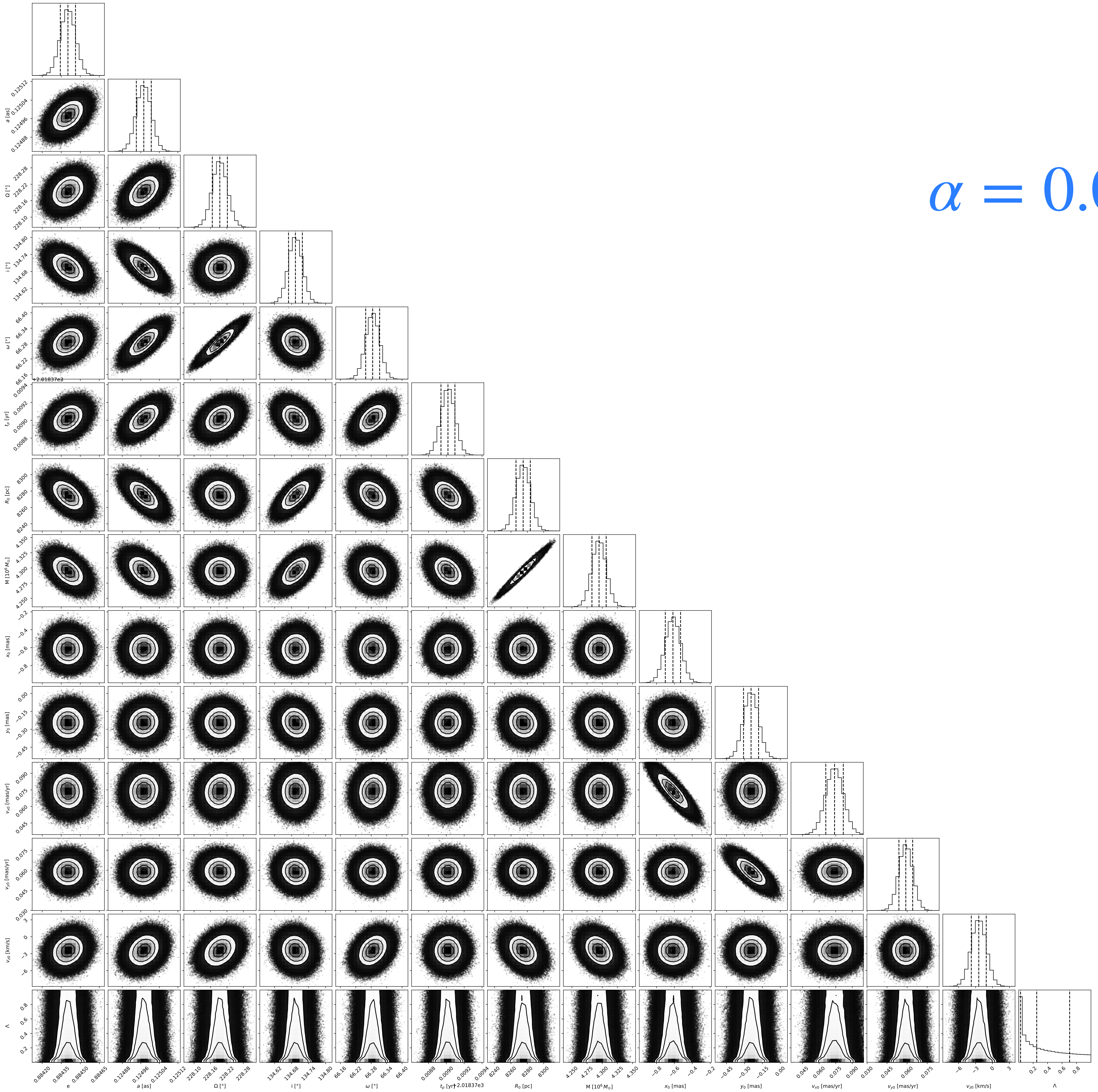
Λ

$\alpha = 0.01$



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v_{y0} [mas/yr]
v_{z0} [km/s]
Λ

$\alpha = 0.001$