Evolution of binary systems accompanying axion clouds in extreme mass ratio inspirals

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based on arXiv: 2301.13213[gr-qc]

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1. Introduction
2. Setup and Formulation
3. Results
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Axion

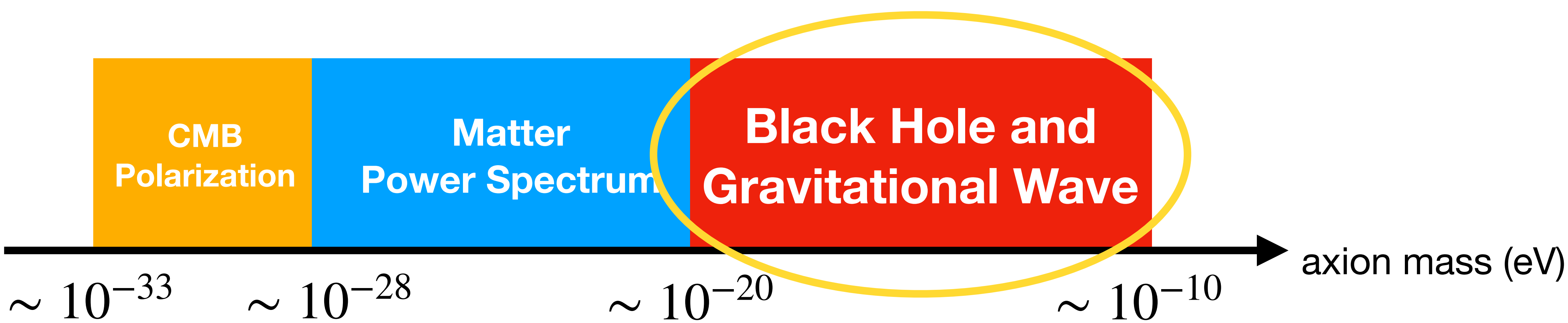
- QCD axion … Solution to the strong CP problem
- String axion … Prediction from string theory

[A. Arvanitaki et. al., 2010]

Properties：

Scalar fields with ultralight mass and weak coupling

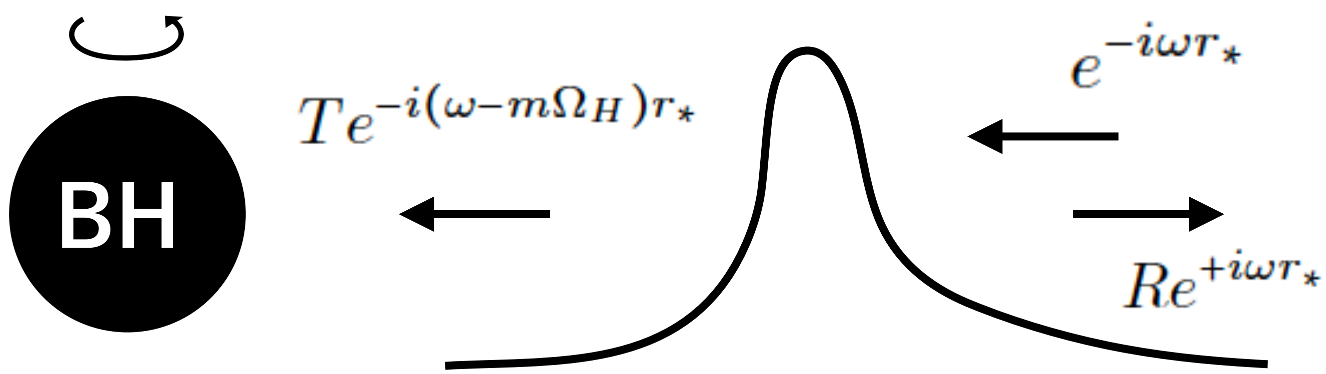
→ Exploration by cosmological and astrophysical phenomena



BH Superradiant Instability

Superradiance : Energy extraction from a BH by waves

A scalar field around a rotating BH



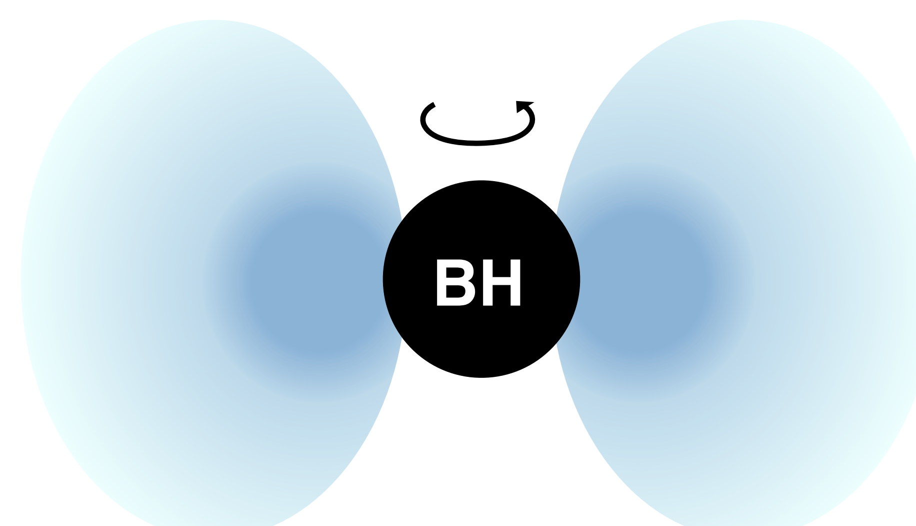
$$|R|^2 = 1 - \frac{\omega - m\Omega_H}{\omega}|T|^2$$

if $\omega < m\Omega_H$, $|R|^2 > 1$

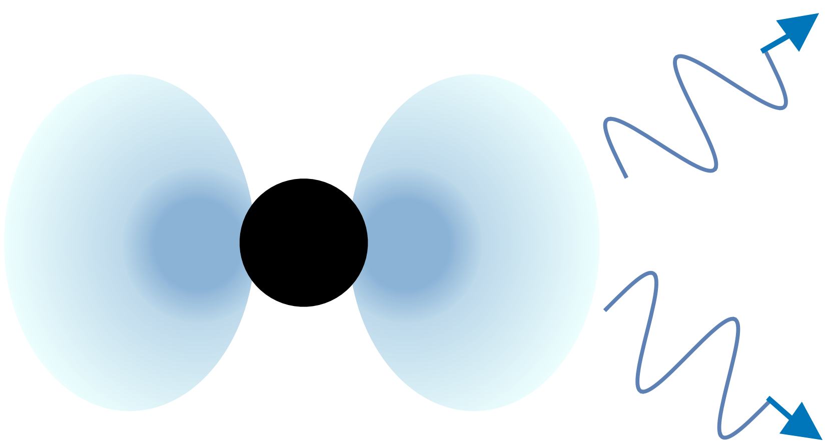
BH angular velocity : Ω_H

Gravitationally bounded axions keep growing by superradiance.

→ Axions form a cloud

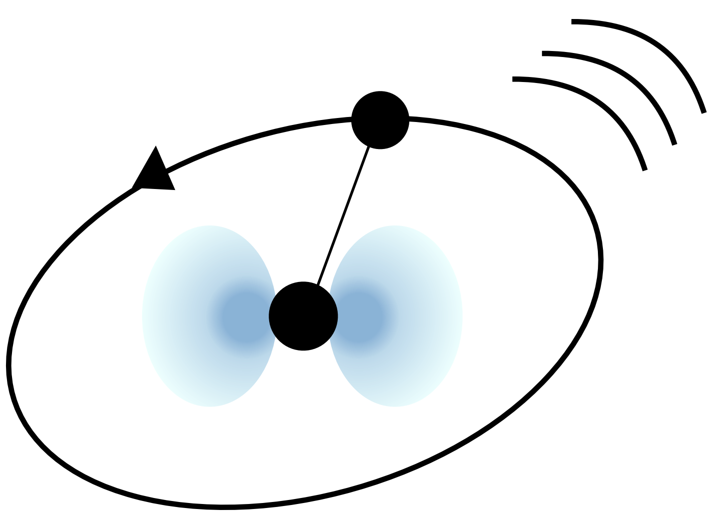


Probing Axion with GW



Continuous GW from the cloud

- Pair annihilation of axions



GW from binary coalescences

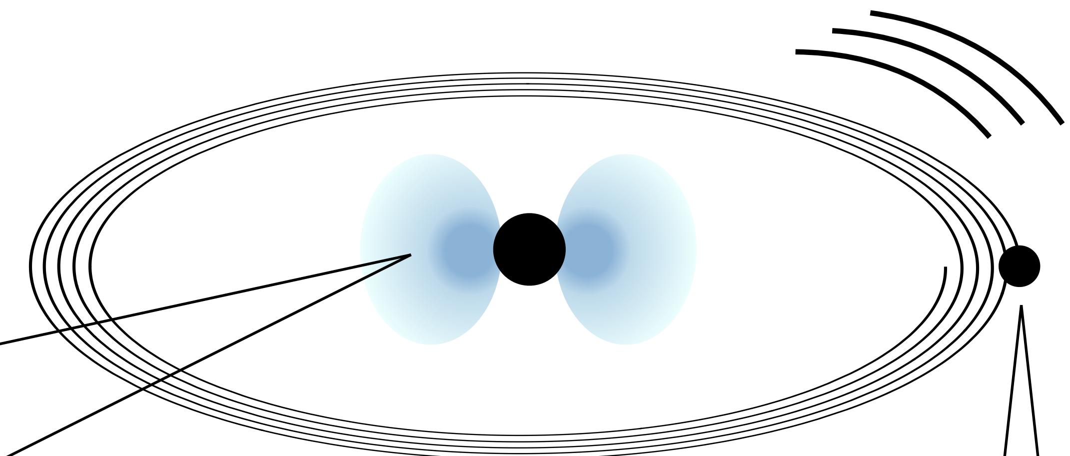
- Modification of the waveform
- Parameter of BHs

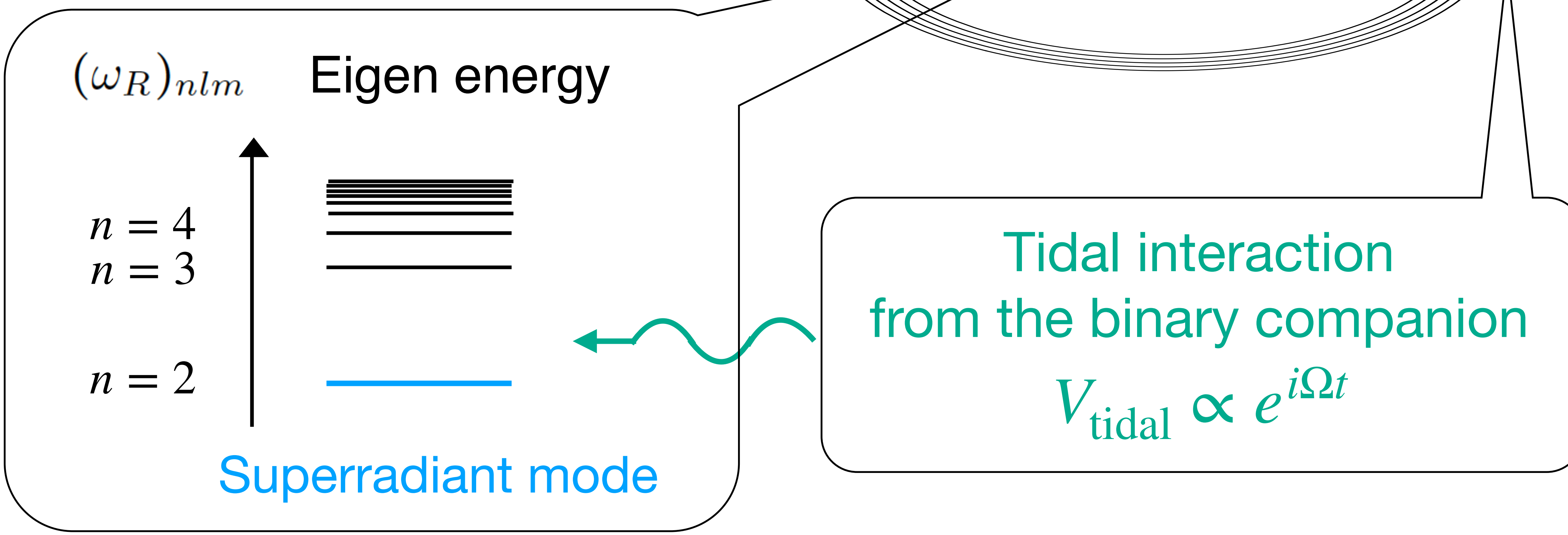
We focus on binary systems.

Resonance in Binary Systems

Bound states of axions

$$\phi = \sum_{nlm} e^{-i\omega_{nlm}t} f_{nlm}(\boldsymbol{x})$$





(Energy gap) = (Orbital frequency) ➡ **Resonance**

The evolution of the cloud and the orbit change dramatically.

[D. Baumann et. al., 2019]

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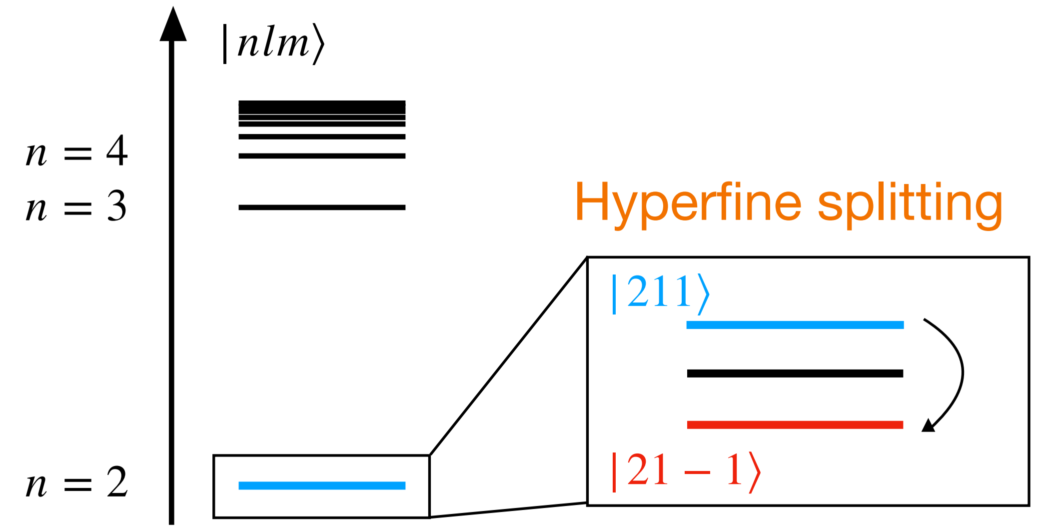
Setup

- The first resonance during inspiral

Resonance frequency

$$f_{\rm res} = 2.2\ {\rm mHz}\ \frac{1}{1+4\alpha^2}\left(\frac{\alpha}{0.1}\right)^7\left(\frac{10M_\odot}{M}\right)$$



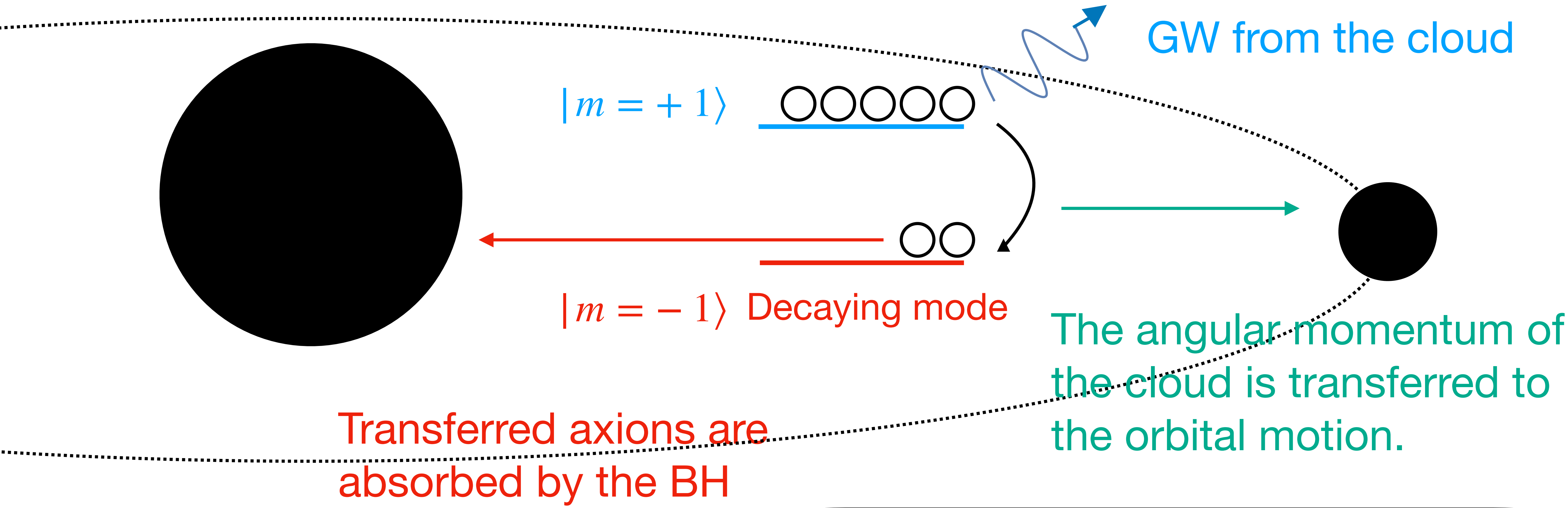
$\alpha = M\mu$ M : BH mass, μ : axion mass

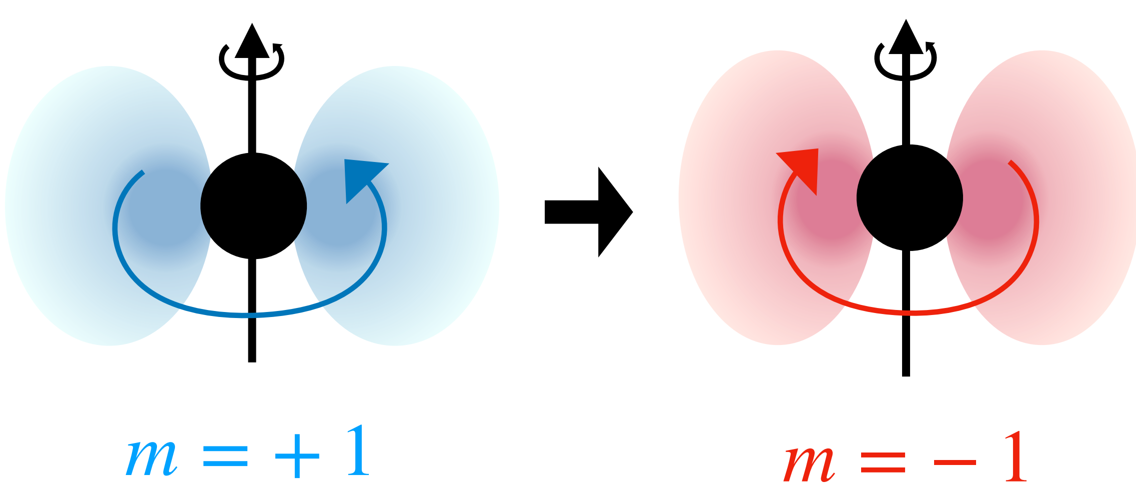
- Binary system with small mass ratio

※ We can neglect this resonance for nearly equal mass binaries.
[TT et. al., 2021]

The goal is to reveal the time evolution of the binary system accompanying axion cloud around the resonance.

Some Effects involved in Resonance





- Backreaction to the orbital motion and the central BH
- GW decay of the cloud

We include all these effects!

Formulation

We describe the evolution of the system
within the adiabatic approximation $(\omega_R \gg \omega_I)$

Variables : BH mass M angular momentum J
Orbital velocity Ω
Coefficients of each mode c_1, c_2

$$\psi = c_1(t)\varphi_1 + c_2(t)\varphi_2$$

Energy flux balance at the BH horizon

$$\frac{dM}{dt} + 2\omega_I^{(1)} M_c^{(1)} + 2\omega_I^{(2)} M_c^{(2)} = 0 \ ,$$

Angular momentum flux balance at the BH horizon

$$\frac{dJ}{dt} + \frac{2\omega_I^{(1)}}{\mu} M_c^{(1)} - \frac{2\omega_I^{(2)}}{\mu} M_c^{(2)} = 0 \ ,$$

Toal angular momentum conservation

$$\frac{d}{dt}\left(J_{\rm orb} + J + J_{\rm c}\right) + \frac{1}{\mu}\frac{dE_{\rm GW}}{dt} = -\mathcal{T}_{\rm GW}$$

Time evolution of the coefficients of each mode

$$i\frac{dc_1}{dt} = \left(-(\Omega - \Omega_{\rm res}) + i\omega_I^{(1)} - i\Gamma_{\rm GW}\right) c_1 + \eta c_2 \ ,$$

$$i\frac{dc_2}{dt} = \eta c_1 + \left((\Omega - \Omega_{\rm res}) + i\omega_I^{(2)}\right) c_2 \ ,$$

Equations for Axion Cloud

Non relativistic approx.

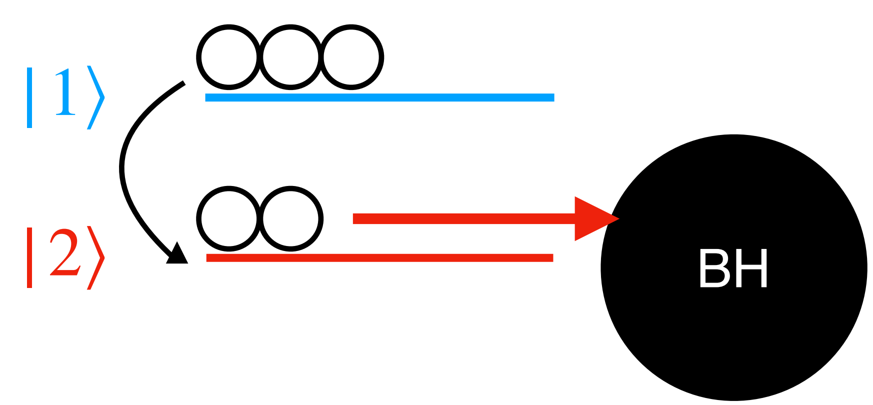
$$i\frac{\partial}{\partial t}\psi = \left(-\frac{1}{2\mu}\nabla^2 - \frac{\alpha}{r} + V_*(t)\right)\psi$$

$\alpha = M\mu$ M : BH mass, μ : axion mass

Tidal potentail

$$\psi = \sum_i c_i(t)\varphi_i \xrightarrow{\text{projection}} i\frac{dc_i}{dt} = \sum_j \left((\omega_j - \mu)\delta_{ij} + \int d^3x\, \varphi_i^* V_* \varphi_j\right) c_j$$

- The decay rate of the second mode is large compared to the tidal interaction
- We can effectively describe the cloud with only the particle number at the original mode



$$\frac{dn_1}{dt} = 2\omega_I^{(1)} n_1 + \frac{2\Gamma\eta^2}{\Delta^2 + \Gamma^2} n_1 - \frac{1}{M_{c,0}}\frac{dE_{\mathrm{GW}}}{dt}$$

$$n_1 \propto |c_1|^2$$

Formulation

We describe the evolution of the system
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$$\psi = c_1(t)\varphi_1 + c_2(t)\varphi_2$$

Variables : BH mass M angular momentum J
Orbital velocity Ω
Coefficients of each mode c_1, c_2 → $n_1 \propto |c_1|^2$

Energy flux balance
at the BH horizon

$$\frac{dM}{dt} + 2\omega_I^{(1)} M_c^{(1)} + 2\omega_I^{(2)} M_c^{(2)} = 0 ,$$

Angular momentum flux balance
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$$\frac{dJ}{dt} + \frac{2\omega_I^{(1)}}{\mu} M_c^{(1)} - \frac{2\omega_I^{(2)}}{\mu} M_c^{(2)} = 0 ,$$

Toal angular momentum
conservation

$$\frac{d}{dt}(J_{\rm orb} + J + J_{\rm c}) + \frac{1}{\mu}\frac{dE_{\rm GW}}{dt} = -\mathcal{T}_{\rm GW}$$

Time evolution of the
particle number of the cloud

$$\frac{dn_1}{dt} = 2\omega_I^{(1)} n_1 + \frac{2\Gamma\eta^2}{\Delta^2 + \Gamma^2} n_1 - \frac{1}{M_{\rm c,0}}\frac{dE_{\rm GW}}{dt}$$

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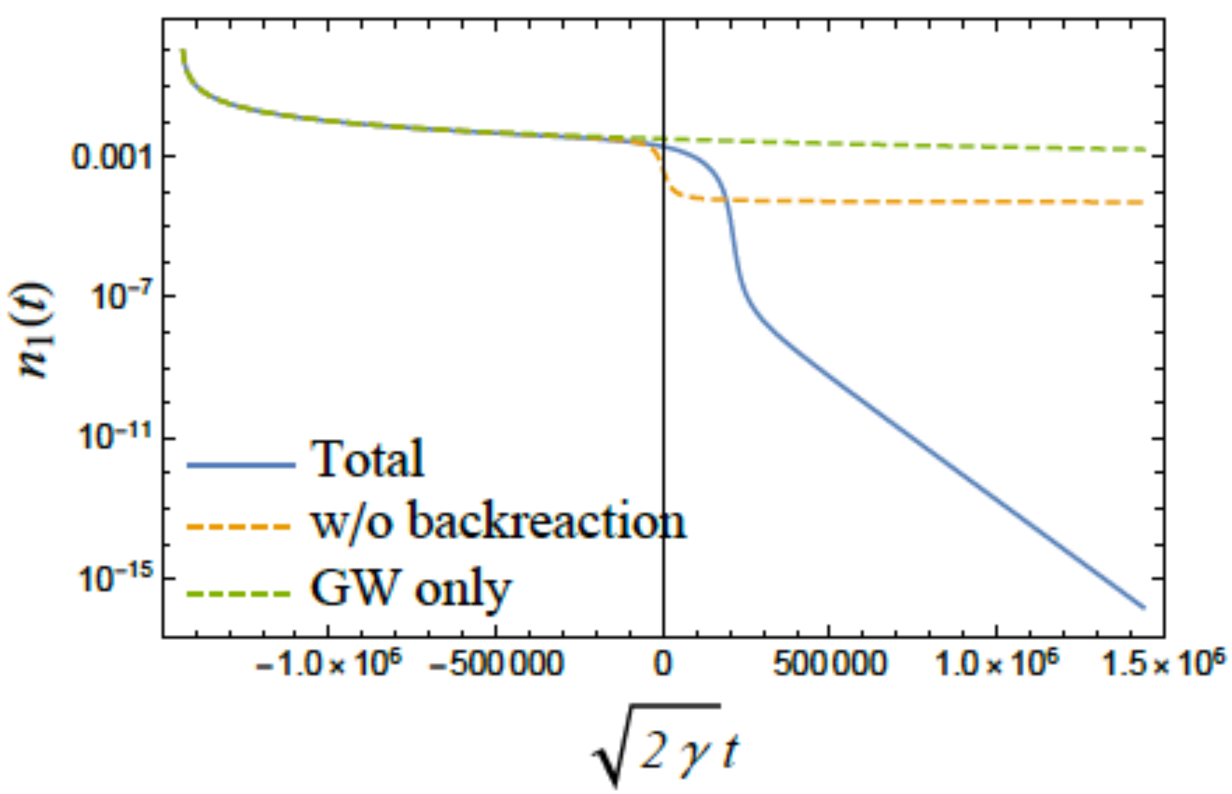
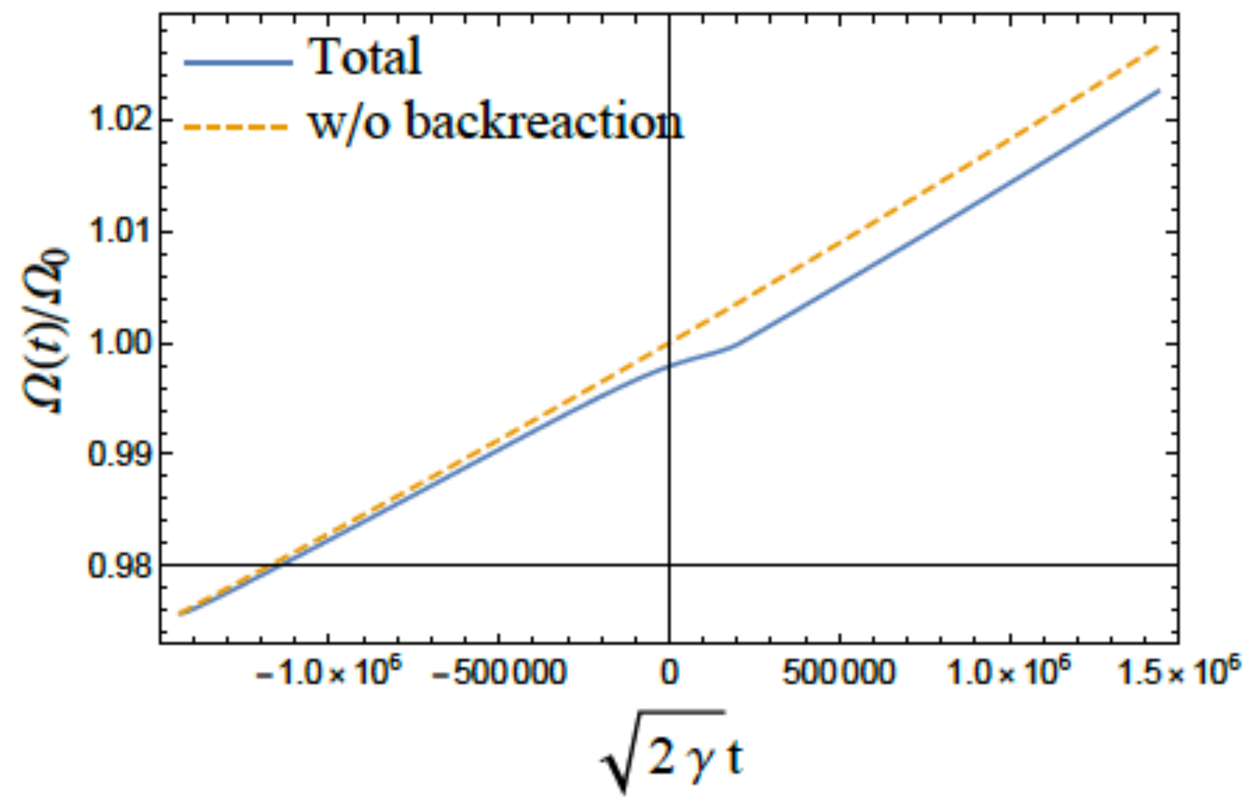
Evolution of the System

Normalized Cloud Mass



Normalized Orbital Frequency



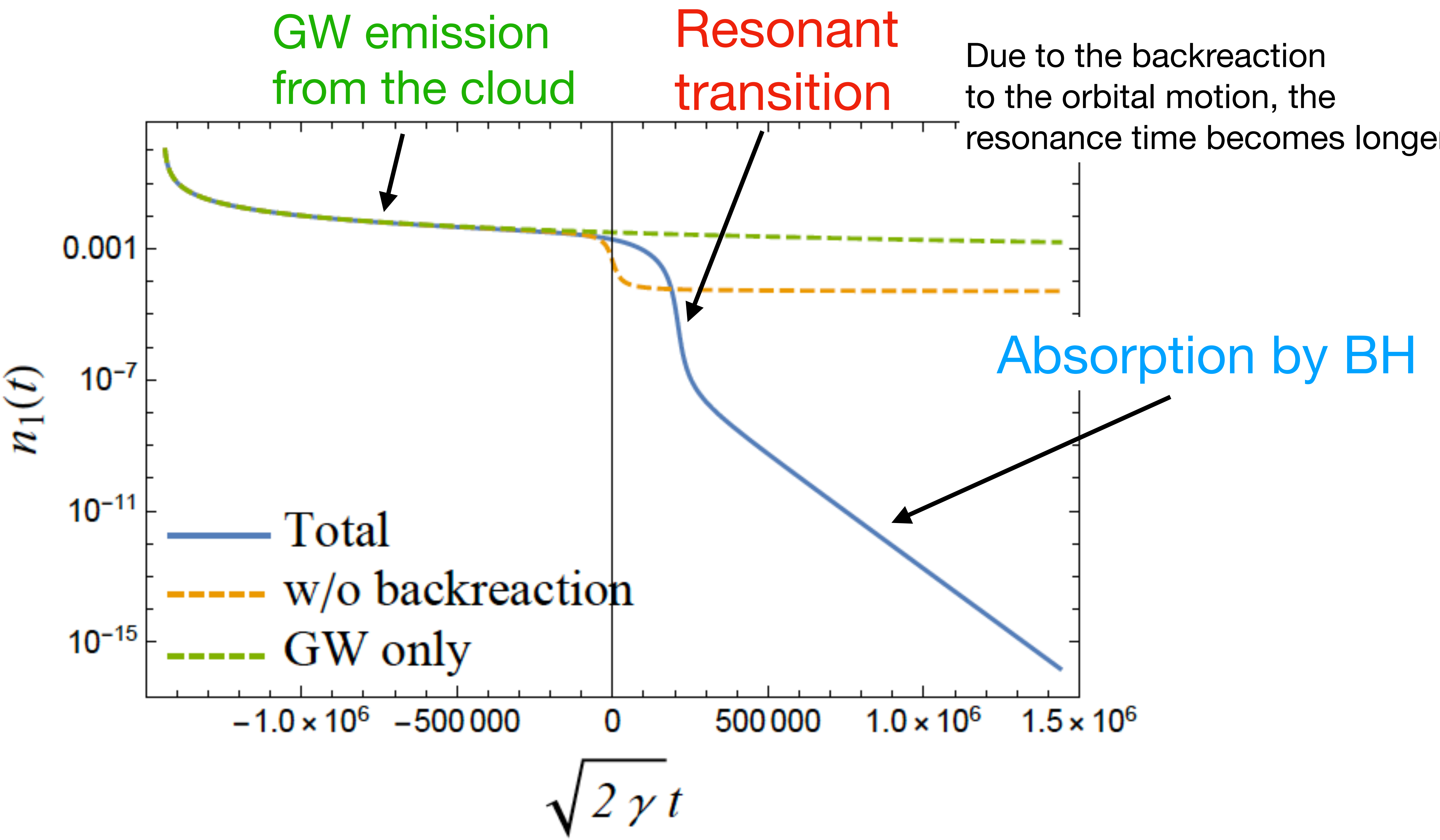
Mass ratio $M_{\rm companion}/M_{\rm BH} = 10^{-4}$

Axion mass $M_{\rm BH}\mu_{\rm a} = 0.1$

Initial cloud mass $M_{\rm c,ini} = 10^{-3} M_{\rm BH}$

- Resonance occurs at $t = 0$
- Blue solid line shows the result including all effects

Evolution of the Cloud Mass



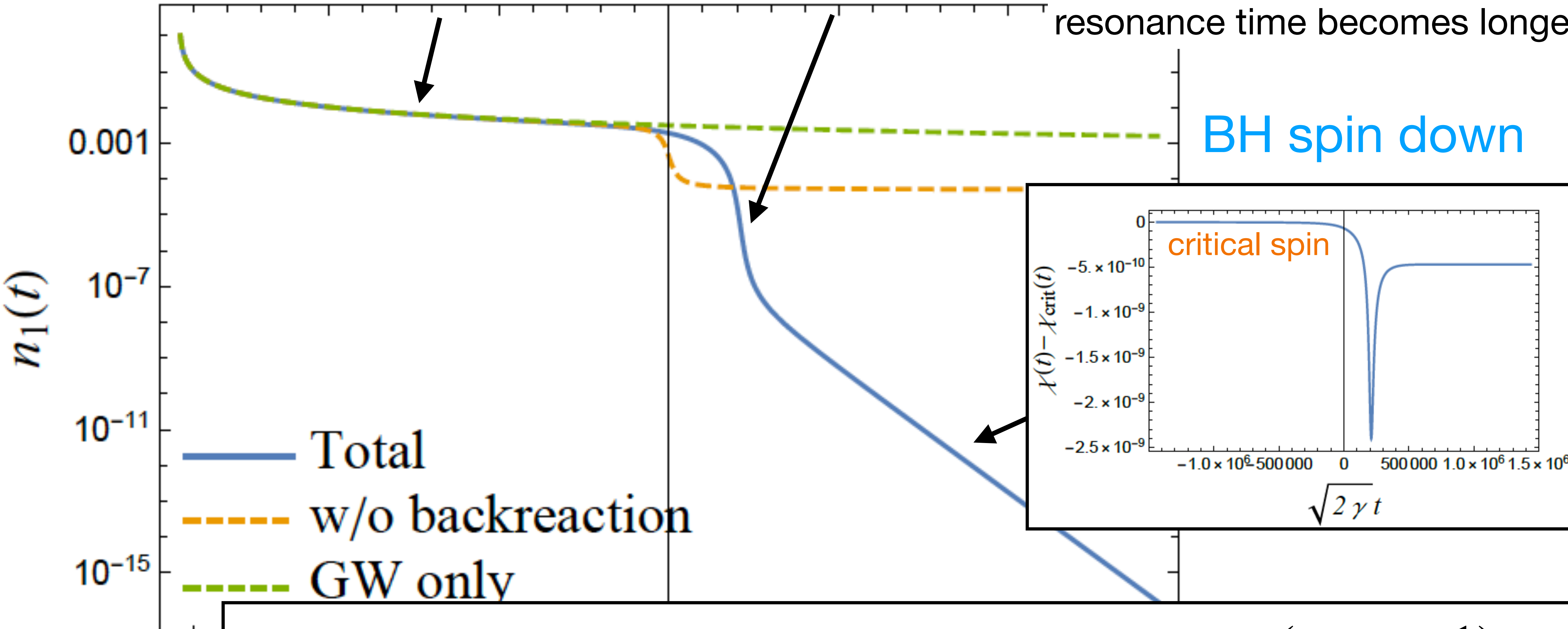
Evolution of the Cloud Mass

GW emission
from the cloud

Resonant
transition

Due to the backreaction
to the orbital motion, the
resonance time becomes longer

BH spin down



Transition destination mode has negative ang. mom.($m=-1$).

Original mode ($m=+1$) also becomes decaying mode

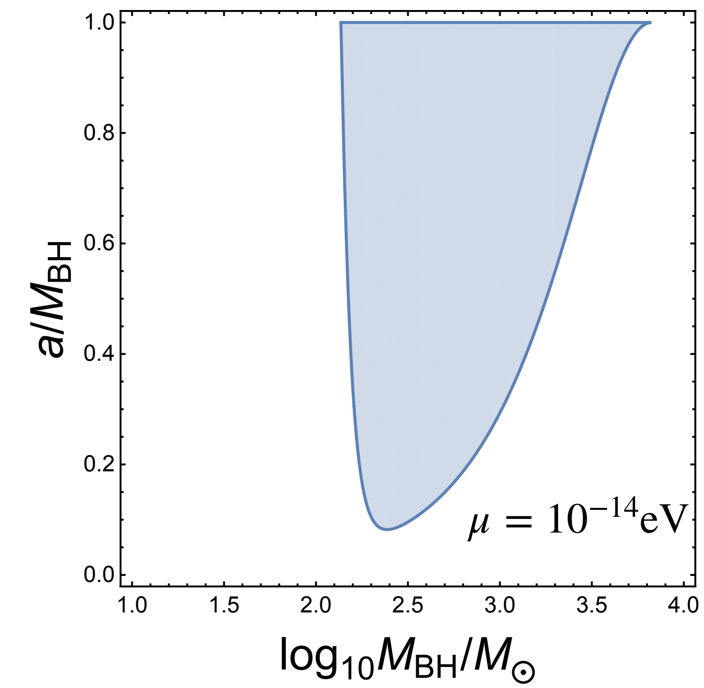
$\omega < m\Omega_H$: Grow, $\omega > m\Omega_H$: Decay

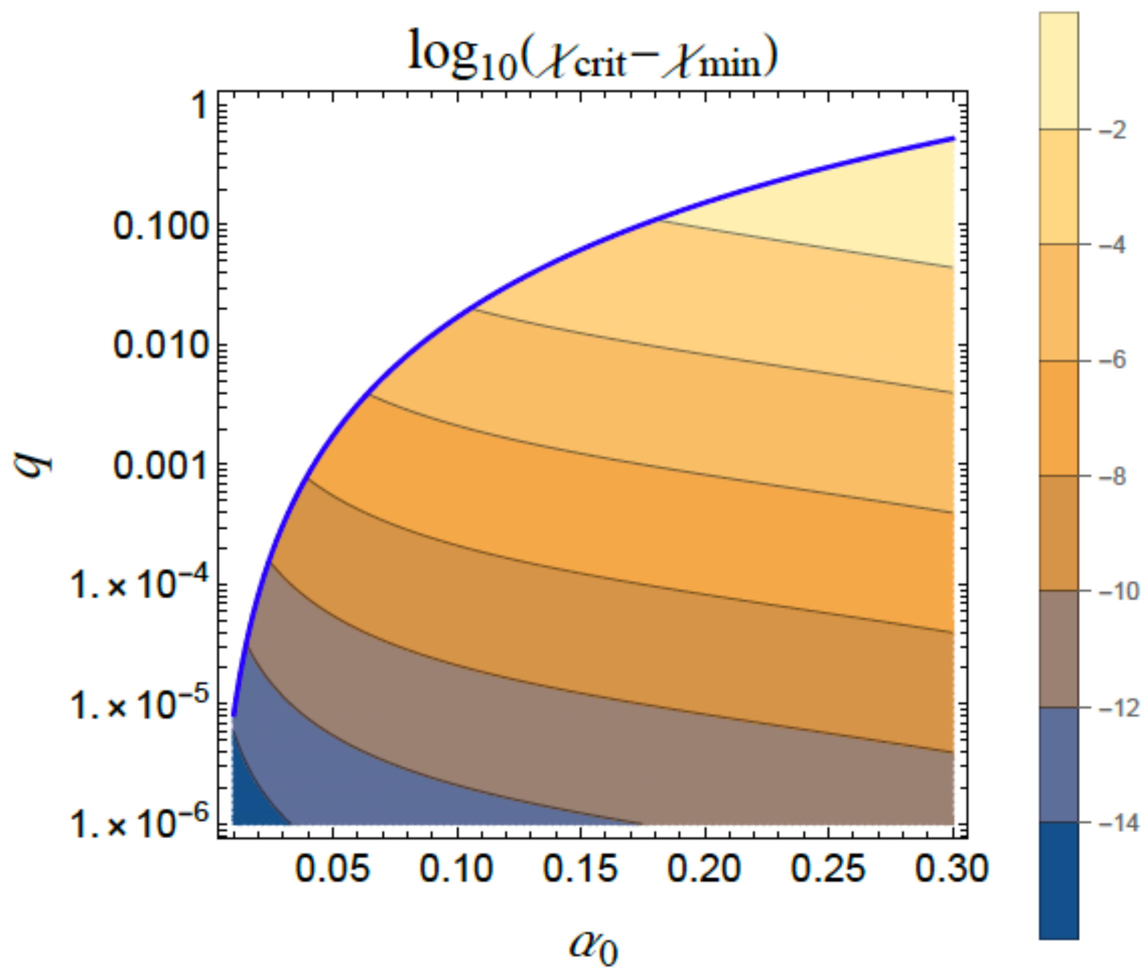
BH spin down

Axion superradiance forbids the existence of rapidly rotating BHs.

Further spin down due to the resonance change this constraints?

e.g.) forbidden region





$d\chi/dt = 0$ at $\chi = \chi_{min}$

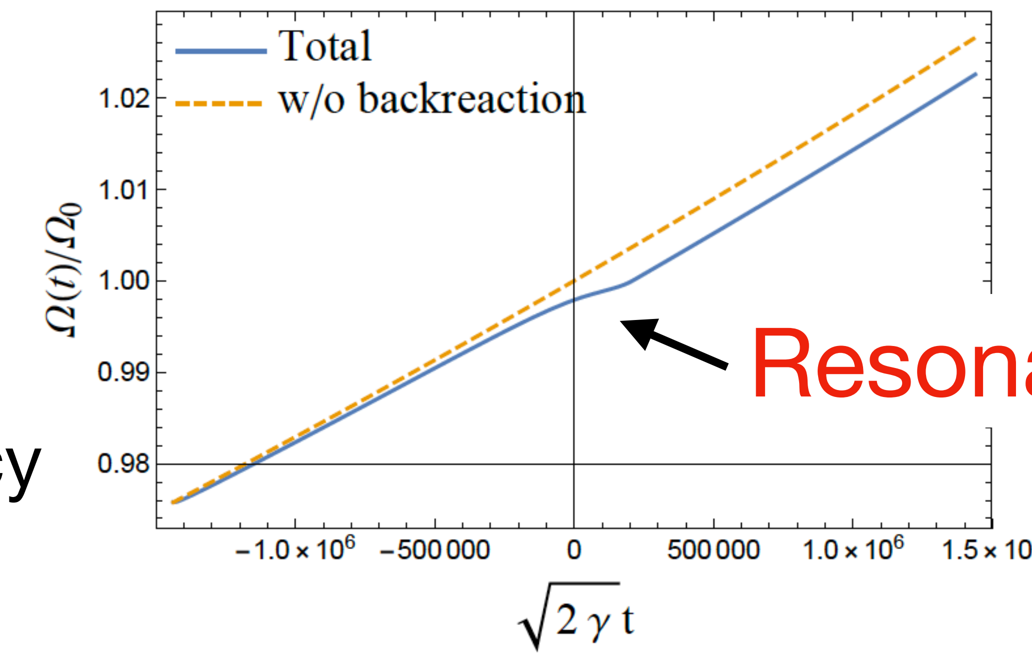
Deviation form the critical spin is very small

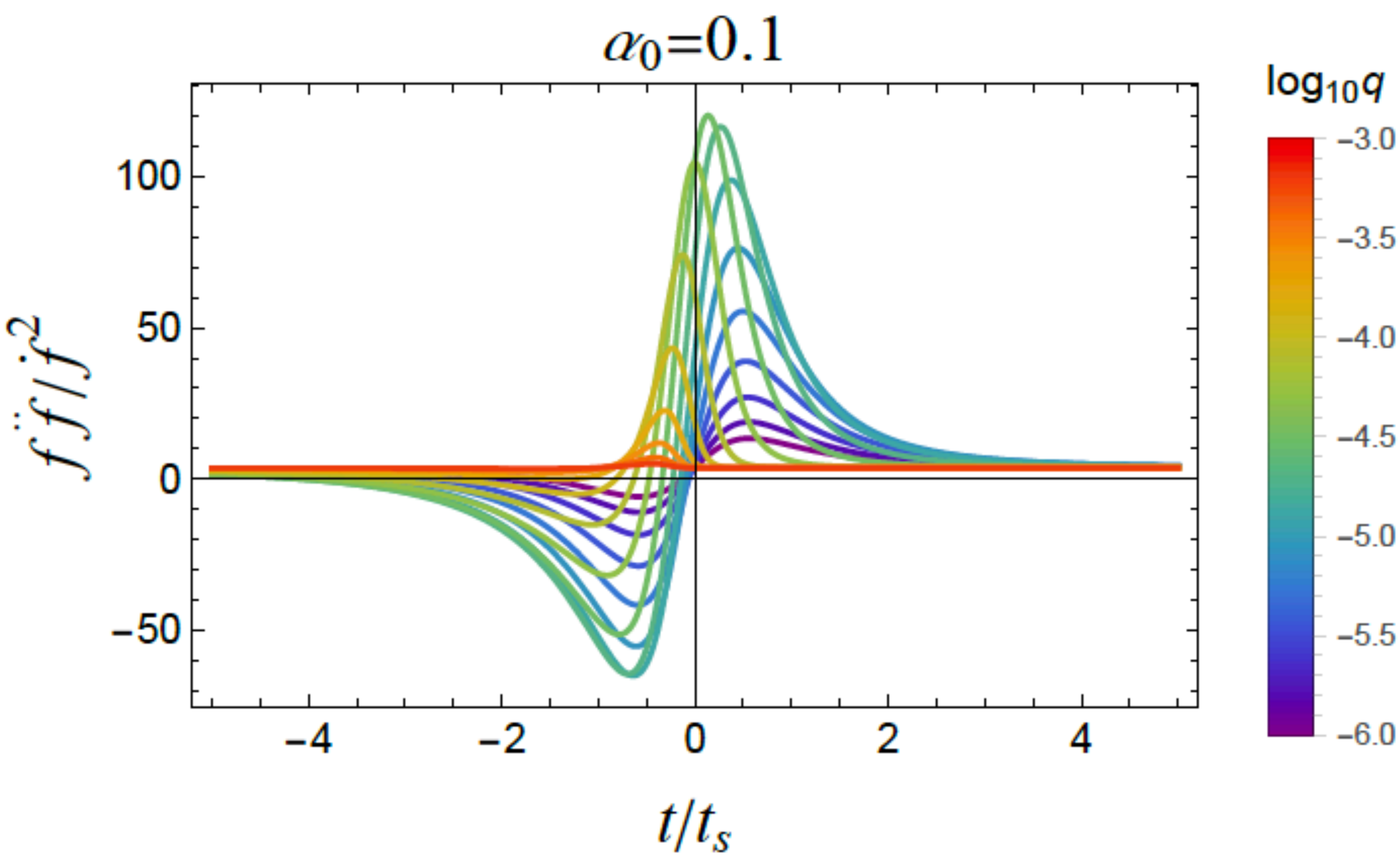
GW Frequency Evolution

Angular momentum of the cloud
is transferred to the orbital motion.

⟶ $\dot{f}, \ddot{f}$?

It is difficult to resolve the degeneracy
with chirp mass and mass ratio.





Mass ratio

Indicator $\dfrac{f\ddot{f}}{\dot{f}^2}$

If there are no clouds,
$f\ddot{f}/\dot{f}^2 = 11/3.$

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Summary

- We have investigated the evolution of binary systems accompanying axion clouds around the resonance.
- We present a formulation that allows this problem to be solved for a wide parameter region.
- A slight BH spin-down can have a significant effect on the fate of the cloud, but are negligible in BH spin measurement.
- The modification of the GW frequency evolution can be a signature of the presence of the cloud.

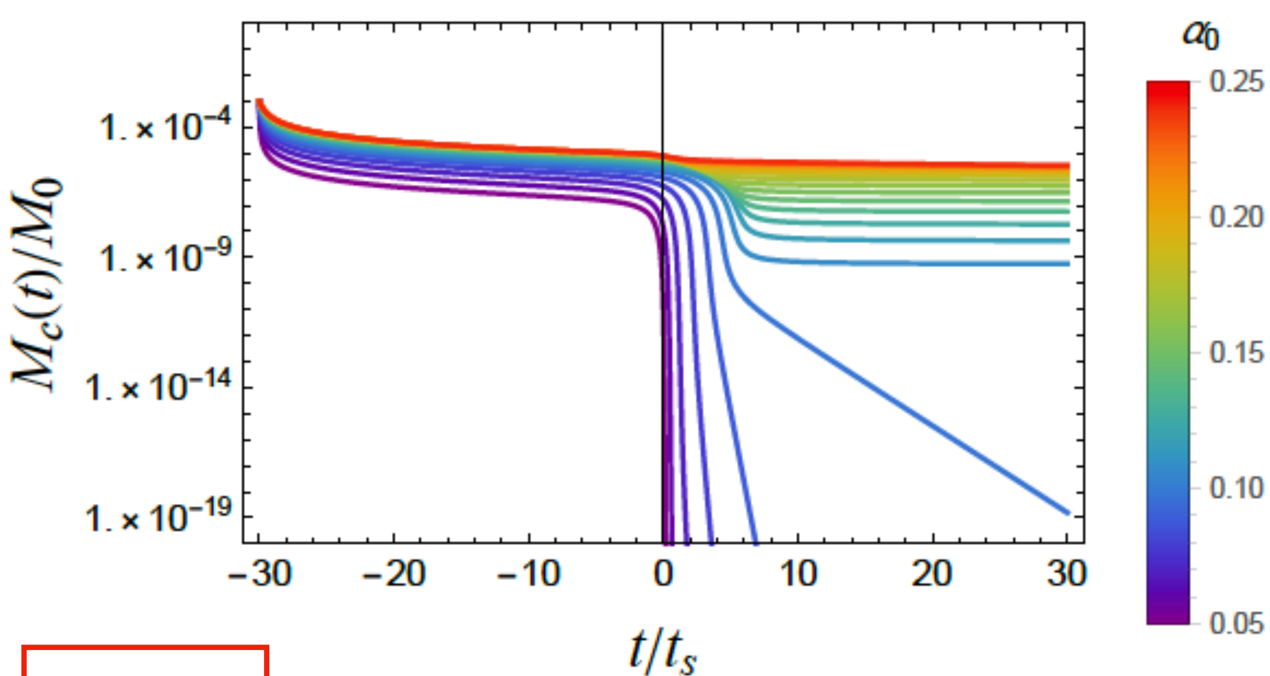
Back up

Back up

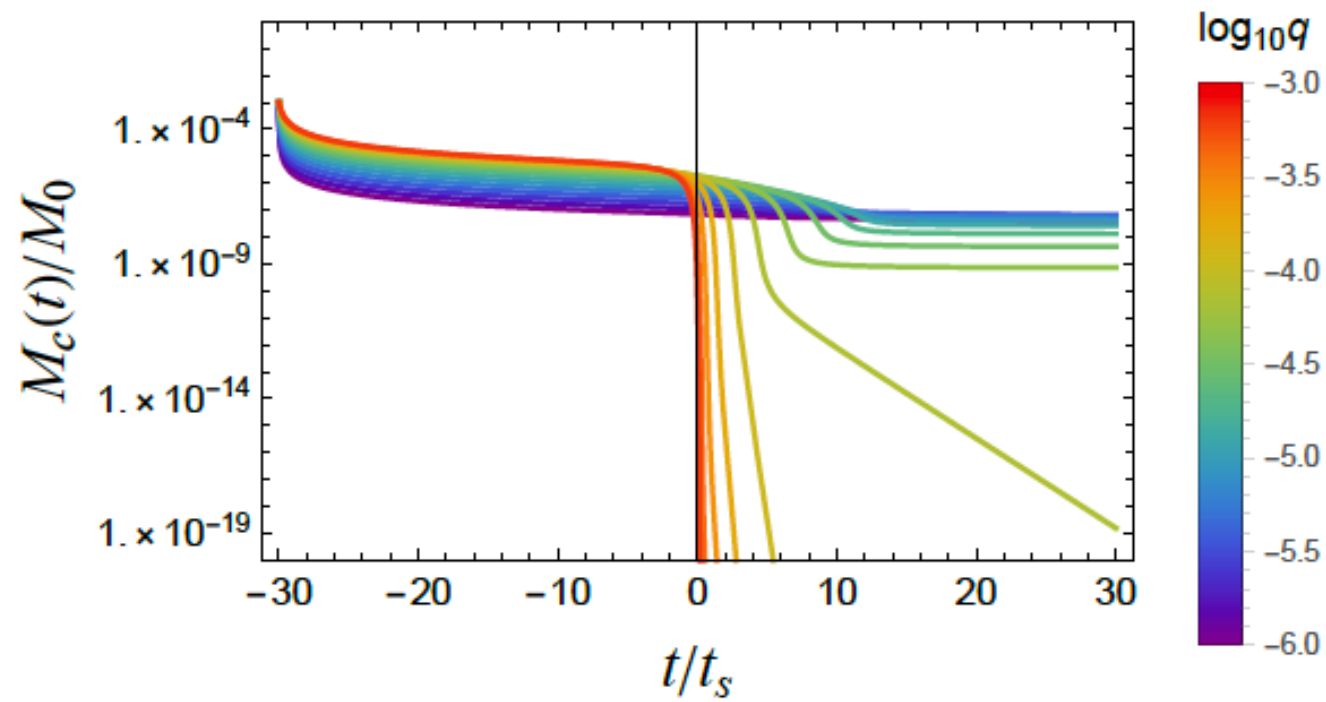
Parameter Dependence

Cloud

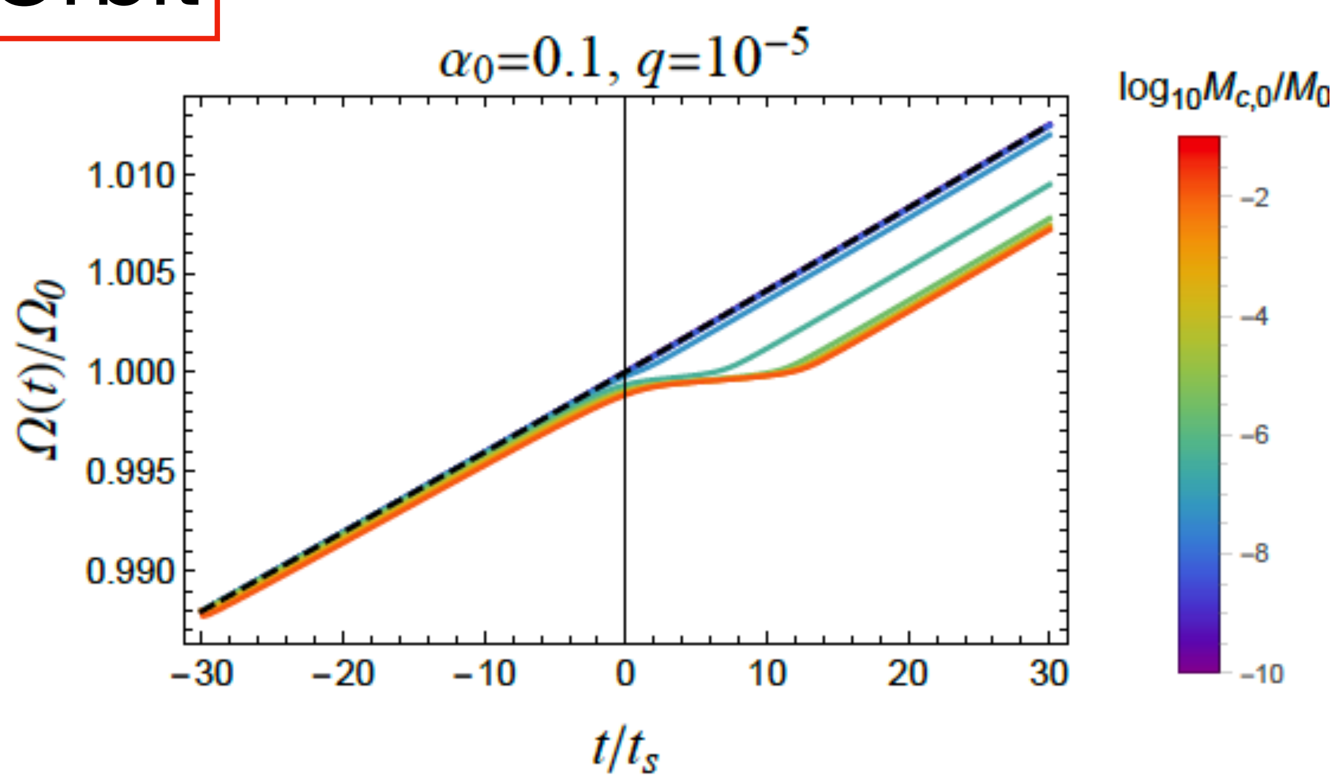
Axion mass



Mass ratio



Orbit



$$M_{\rm c,GW} = \frac{M^2}{\tau_{\rm bin}}\frac{\alpha^{-14}}{C}$$

$$\simeq 8.9\times10^{-4} M_0 \frac{q}{(1+q)^{1/3}}\left(\frac{\alpha_0}{0.1}\right)^{14/3}$$