Quantum Nucleation of Chiral Soliton Lattice

Based on : T. Higaki (Keio), KK, K. Nishimura (KEK), PRD 106 (2022) 096022, arXiv: 2207.00212 [hep-th]

also see: M. Eto & M. Nitta, JHEP09 (2022) 077, arXiv:2207.00211 [hep-th]

Kohei Kamada

(RESCEU, U Tokyo)

Workshop on Very Light Dark Matter 2023
29/03/2023, Marion Royal Kaikan, Chino

Courtesy: H. Oide

The message of this talk

1. In finite density (with “baryon” chemical potential), a layer of pion/axion domain wall is known to be stable once we apply a magnetic field, a. k. a. the Chiral Soliton Lattice.

2. We clarify the mechanism how to create such an object, through the quantum nucleation.

Courtesy: H. Oide

Chiral Soliton Lattice

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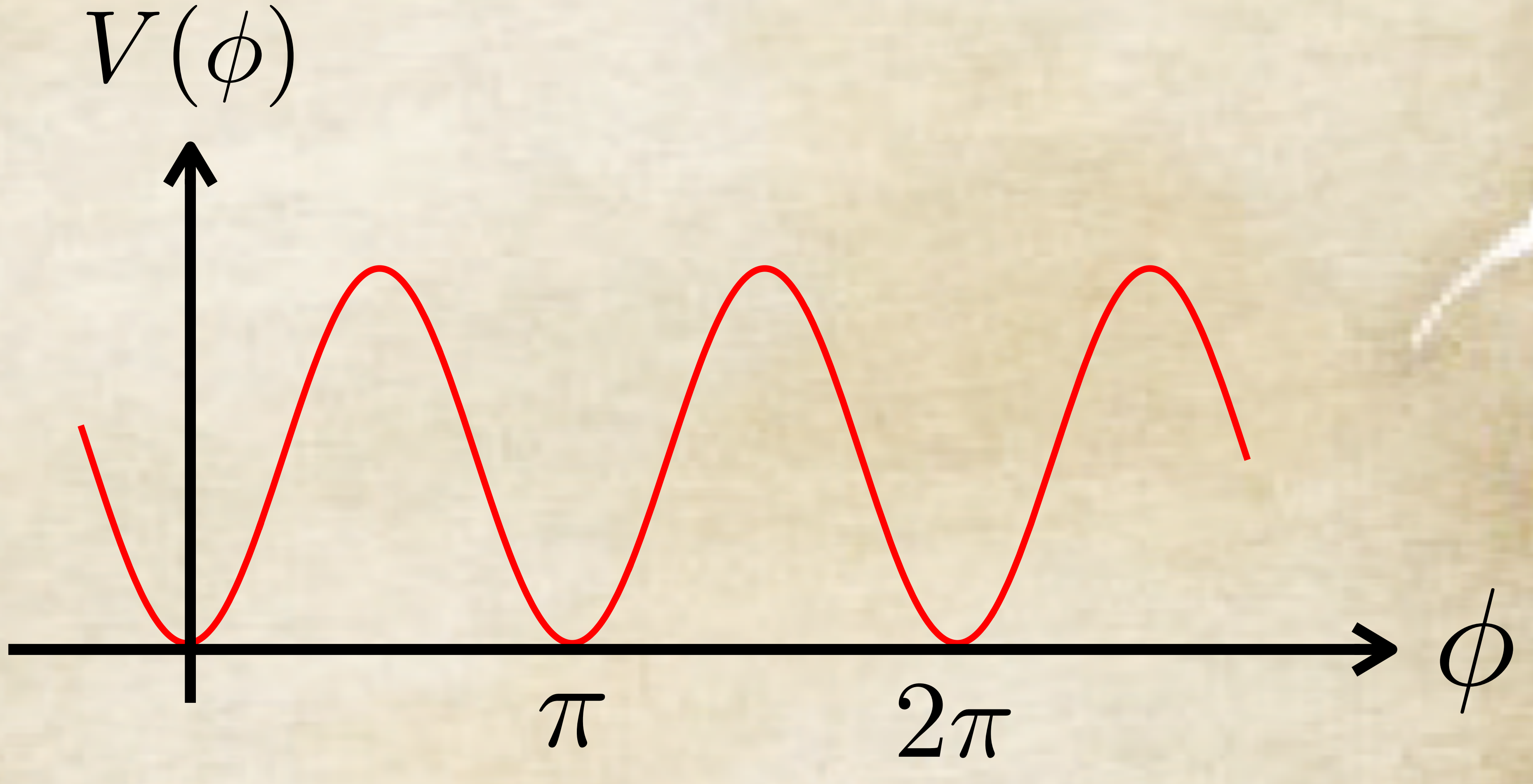
EFT of Axion-like particles/pions

$$\mathcal{L} = \frac{f^2}{2}(\partial_\mu \phi)^2 + f^2 m^2 (\cos\phi - 1)$$



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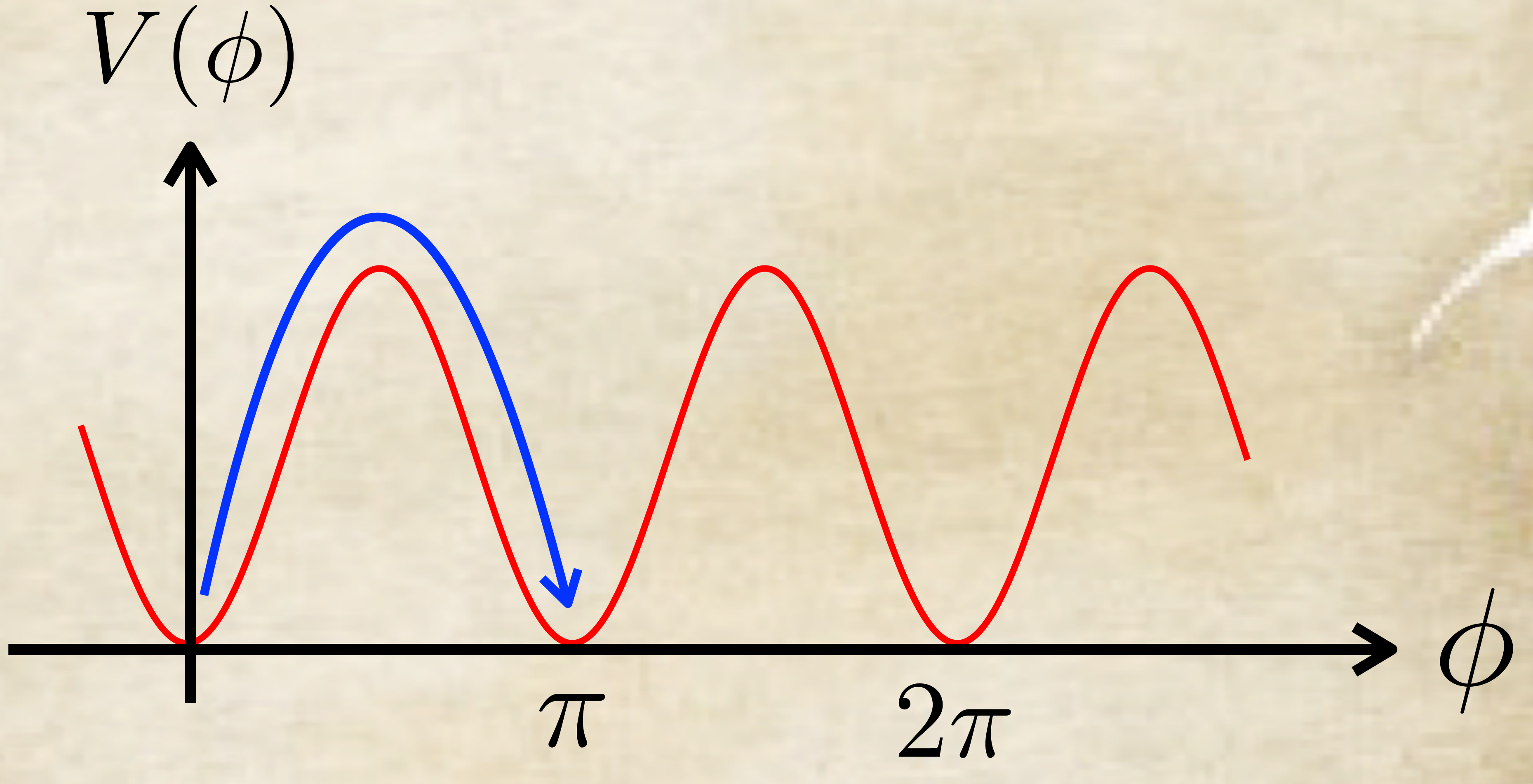
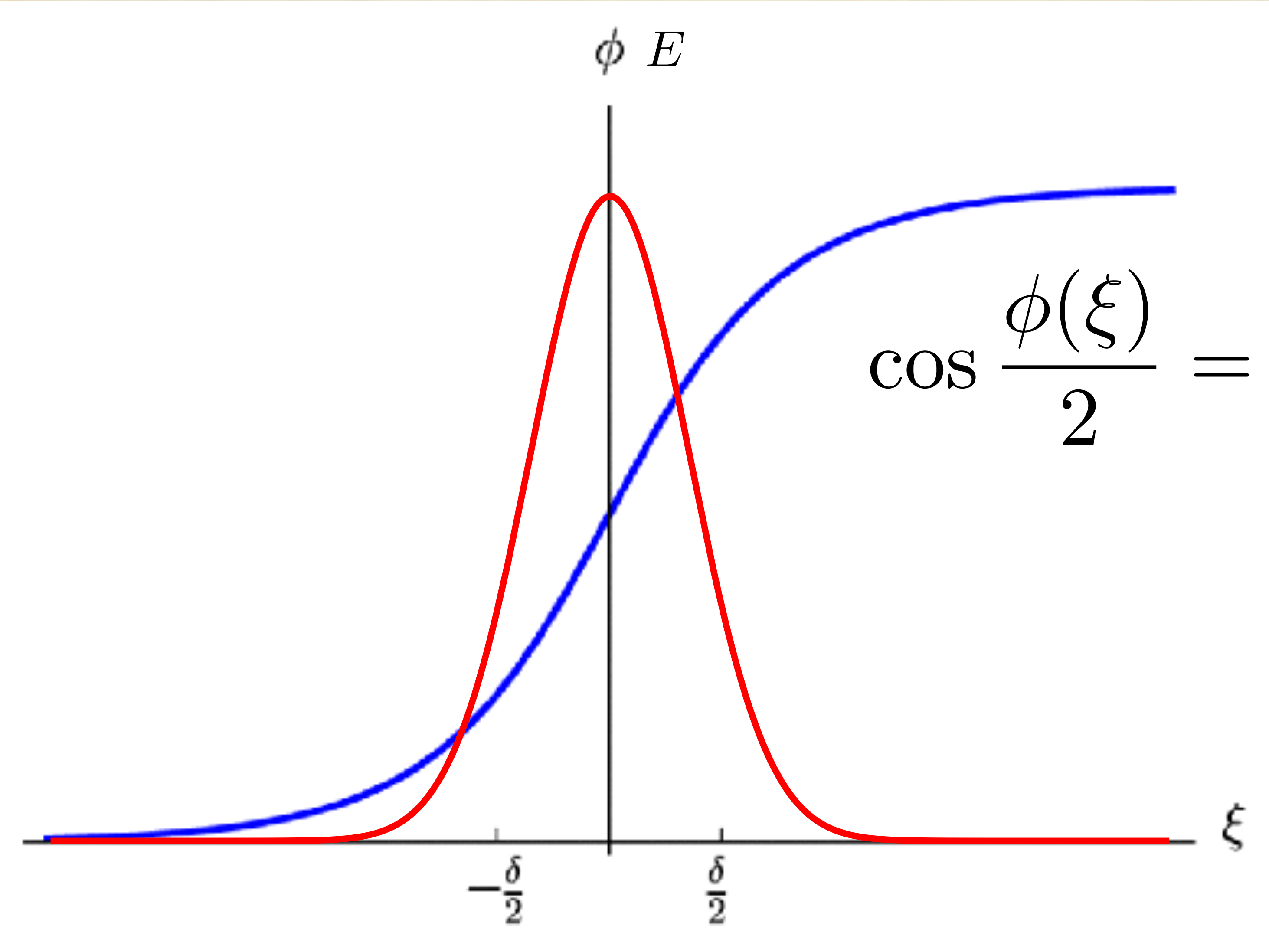


allows a domain wall solution.



$$\cos\frac{\phi(\xi)}{2} = \tanh(m\xi)$$

Courtesy: H. Oide

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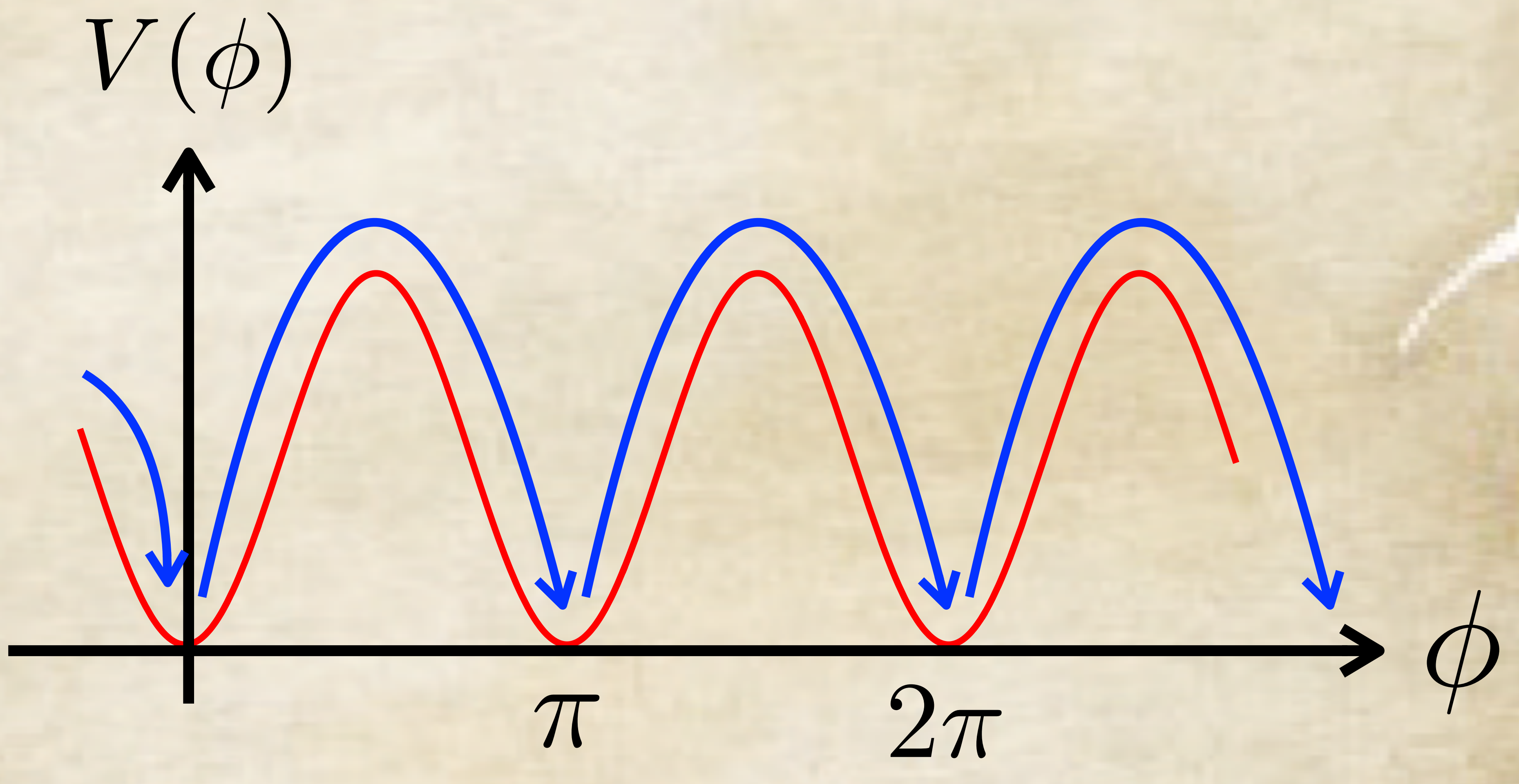
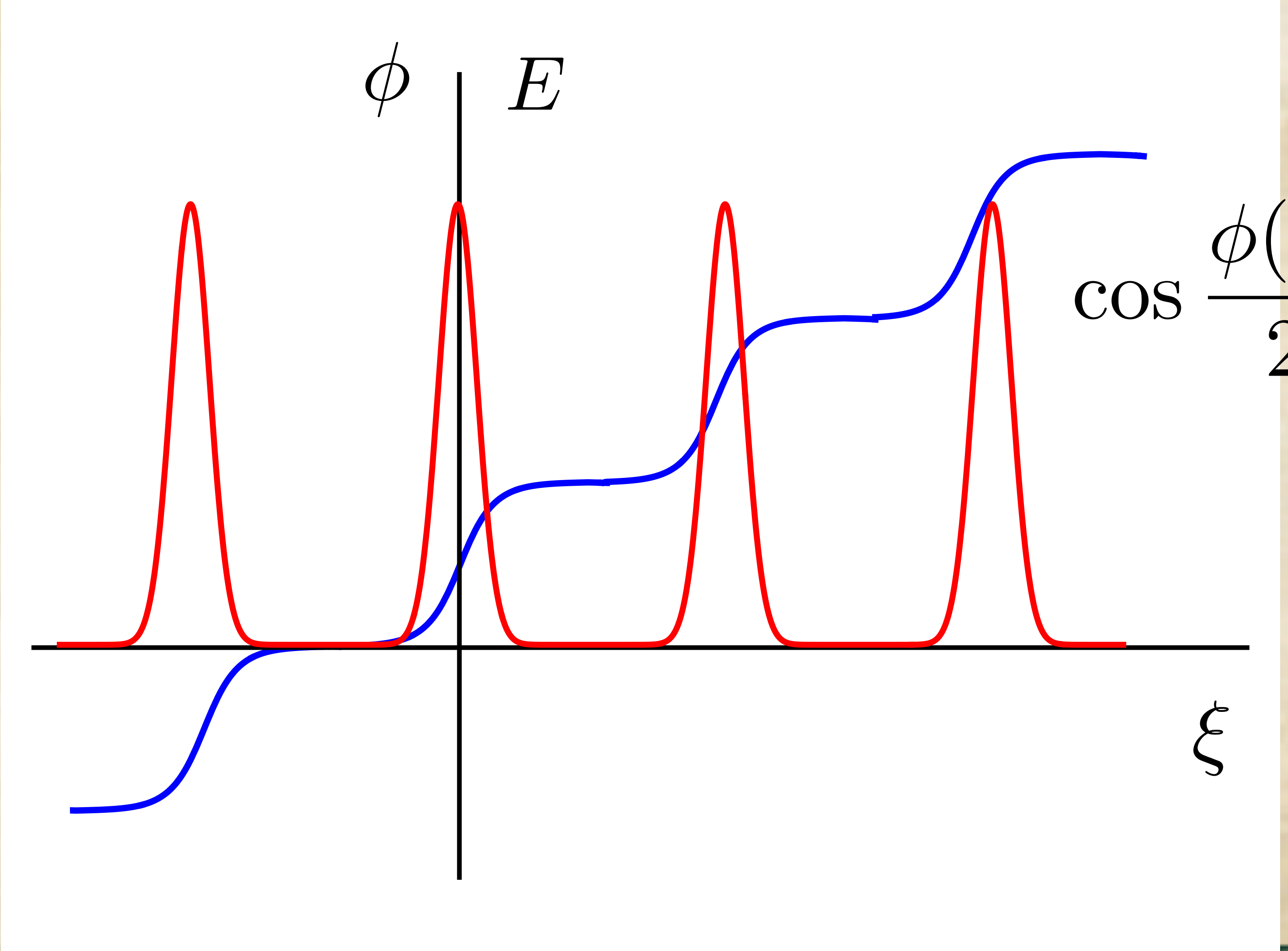


Layer of domain walls are also allowed.



$$\cos\frac{\phi(\xi)}{2} = \mathrm{sn}\left(\frac{m\xi}{k}, k\right)$$

$$(0 < k < 1)$$

k determines the distance between walls

Courtesy: H. Oide

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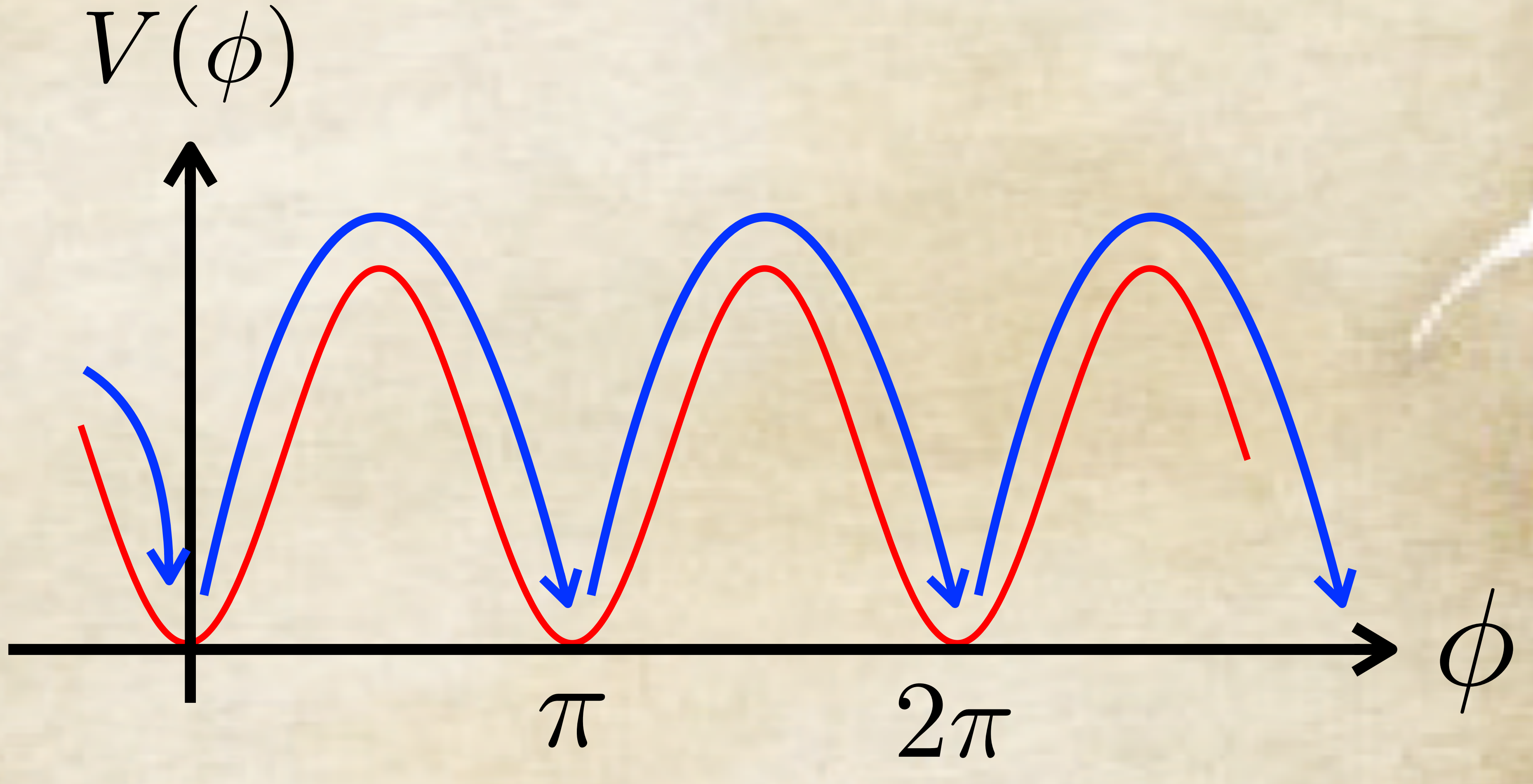

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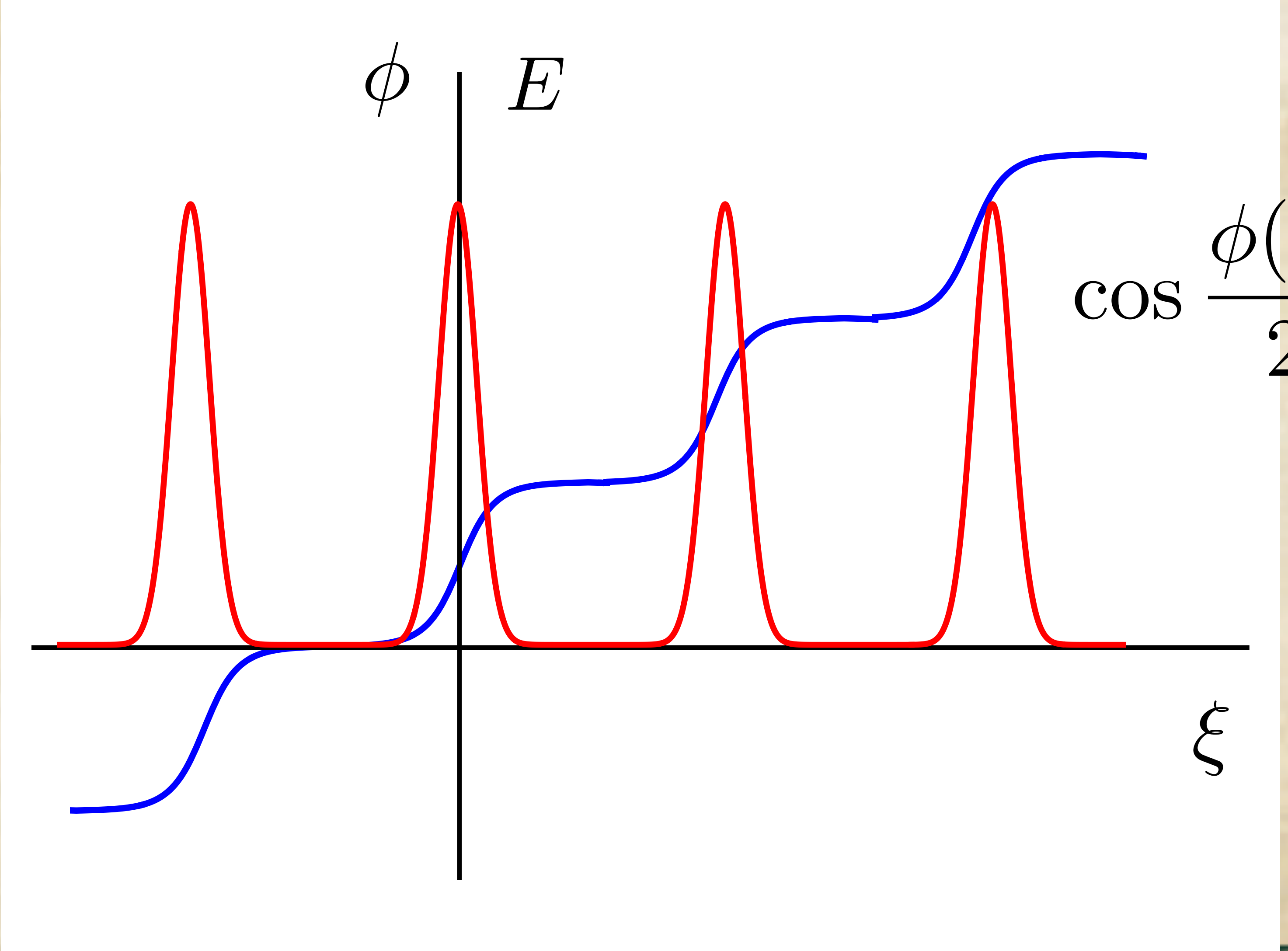

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k determines the distance between walls

Topologically stable, but higher energy than trivial field configuration.

Courtesy: H. Oide

Take into account the chiral anomaly

in the vacuum;
well-known Chern-Simons term

$$\mathcal{L} = \frac{f^2}{2}(\partial_\mu \phi)^2 + f^2 m^2 (\cos\phi - 1) + c\alpha\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Not relevant for domain walls

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Not relevant for domain walls

In the medium with charged fermions:

Chiral magnetic effect (CME): $\boldsymbol{J}_V = \boldsymbol{J}_R + \boldsymbol{J}_L = \frac{e(\mu_R - \mu_L)}{2\pi^2}\boldsymbol{B}$

'80 Vilenkin, '06 Fukushima, Kharzeev, & Warringa

Chiral separation effect (CSE): $\boldsymbol{J}_5 = \boldsymbol{J}_R - \boldsymbol{J}_L = \frac{e(\mu_R + \mu_L)}{2\pi^2}\boldsymbol{B}$

'80 Vilenkin, '04 Son & Zhitnitsky

At low energy, the chiral transformation
turns to the shift of axions/pions

$$\delta\mathcal{L} = \frac{1}{2}(\partial_\mu\phi) j_5^\mu \ni \frac{\mu_B}{4\pi^2}\boldsymbol{\nabla}\phi \cdot \boldsymbol{B}$$

$(\mu_B = \mu_R + \mu_L; e = 1)$

Courtesy: H. Oide

Take into account the chiral anomaly

in the medium;
another "topological" term

$$\mathcal{L} = \frac{f^2}{2}(\partial_\mu \phi)^2 + f^2 m^2(\cos\phi - 1) + c\alpha\phi F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\mu_B}{4\pi^2}\boldsymbol{B}\cdot\boldsymbol{\nabla}\phi$$

Relevant for domain walls…?

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also can be understood the WZW term with a spurious axial U(1) gauge field

’04 Son & Zhitnitsky

Courtesy: H. Oide

In the presence of non-zero chemical potential and external magnetic field

Domain wall solution is unchanged,

$$\cos\frac{\phi(\xi)}{2} = \mathrm{sn}\left(\frac{m\xi}{k}, k\right)$$

then there are no effects from the topological term?

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Not really. Energy of the system changes.

Total energy of the system of length *L* in the *z*-direction:

$$\mathcal{E} = \frac{Vm}{2kK(k)}\left[\frac{2E(k)}{k} + \left(k - \frac{1}{k}\right)K(k) - \frac{\mu_B B}{2\pi}\right]$$

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Negative energy is induced
from the spatial gradient of the field configuration.

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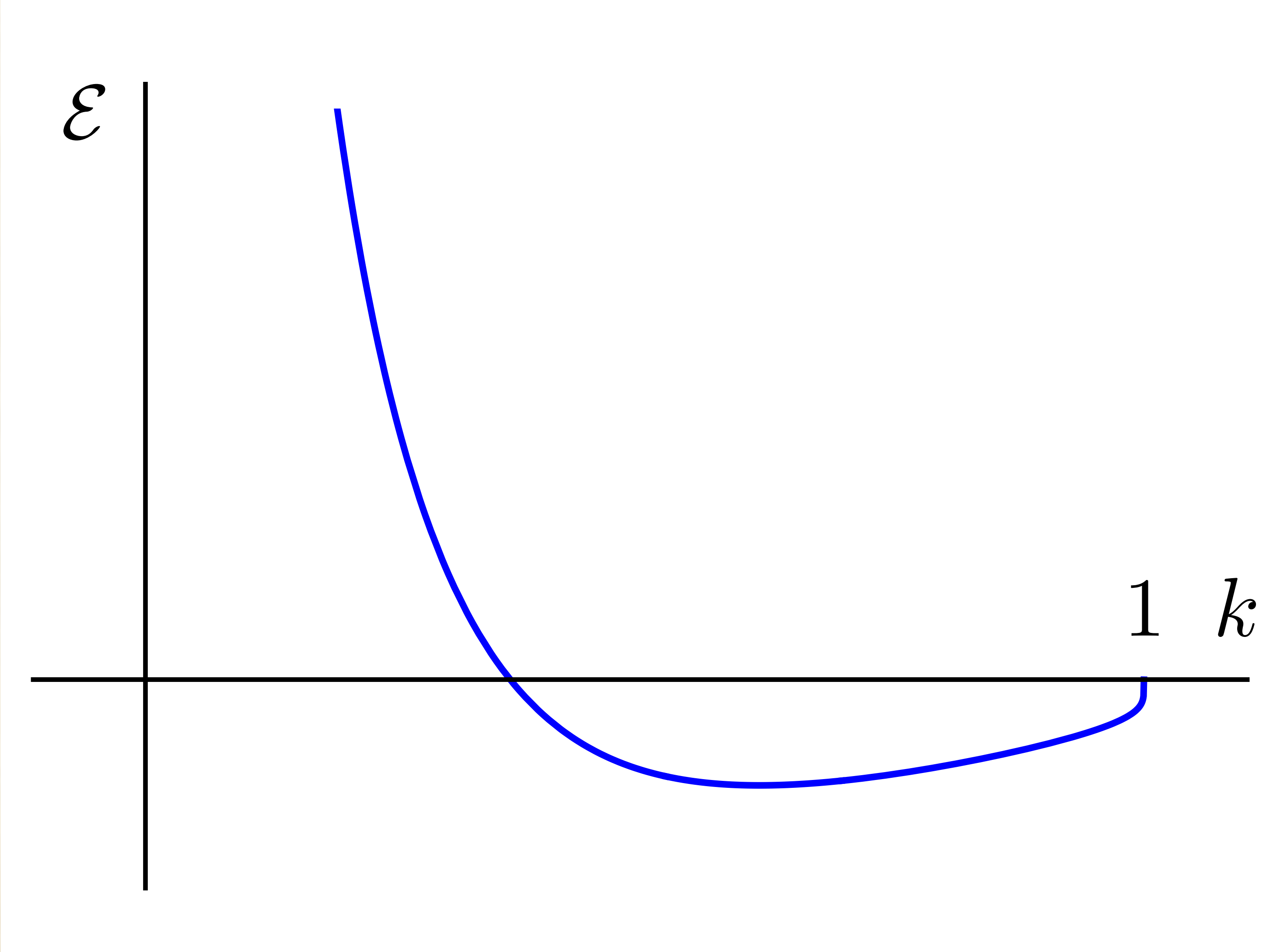

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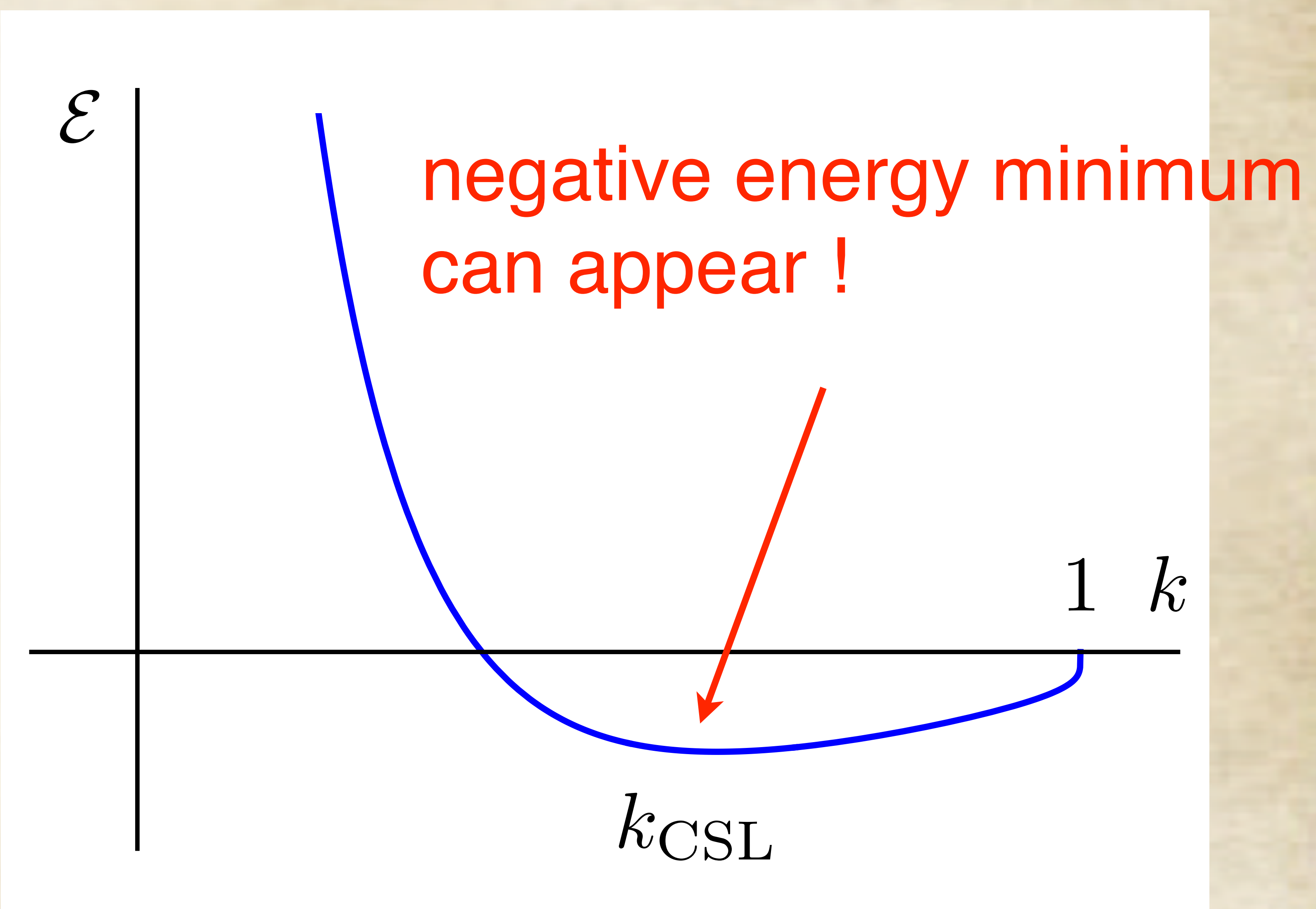


Courtesy: H. Oide

In the presence of non-zero chemical potential and external magnetic field

Domain wall solution is unchanged,

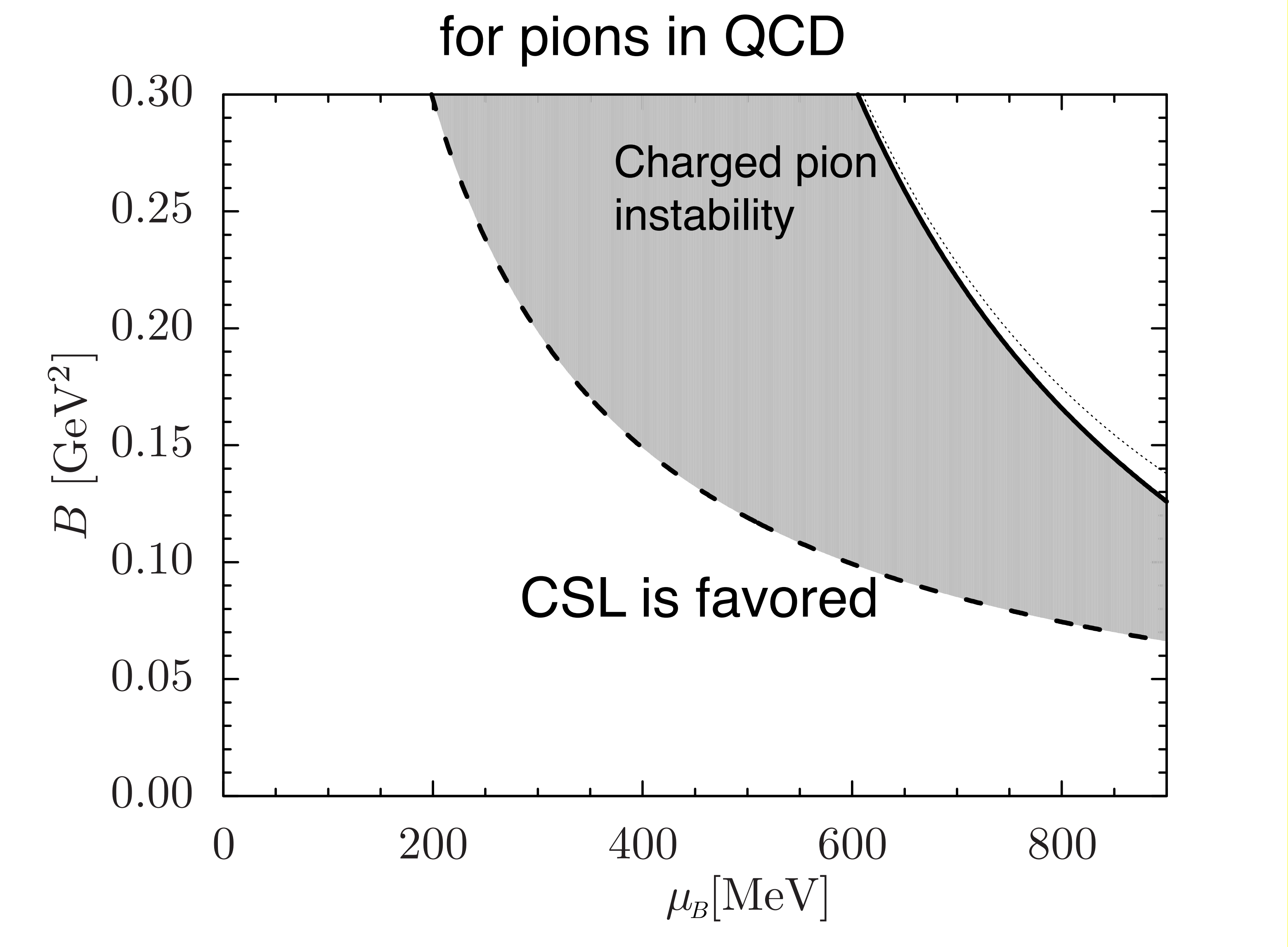
term?

For sufficiently large magnetic field,

$$B > B_{\mathrm{CSL}} = \frac{16\pi m f^2}{\mu_B}$$

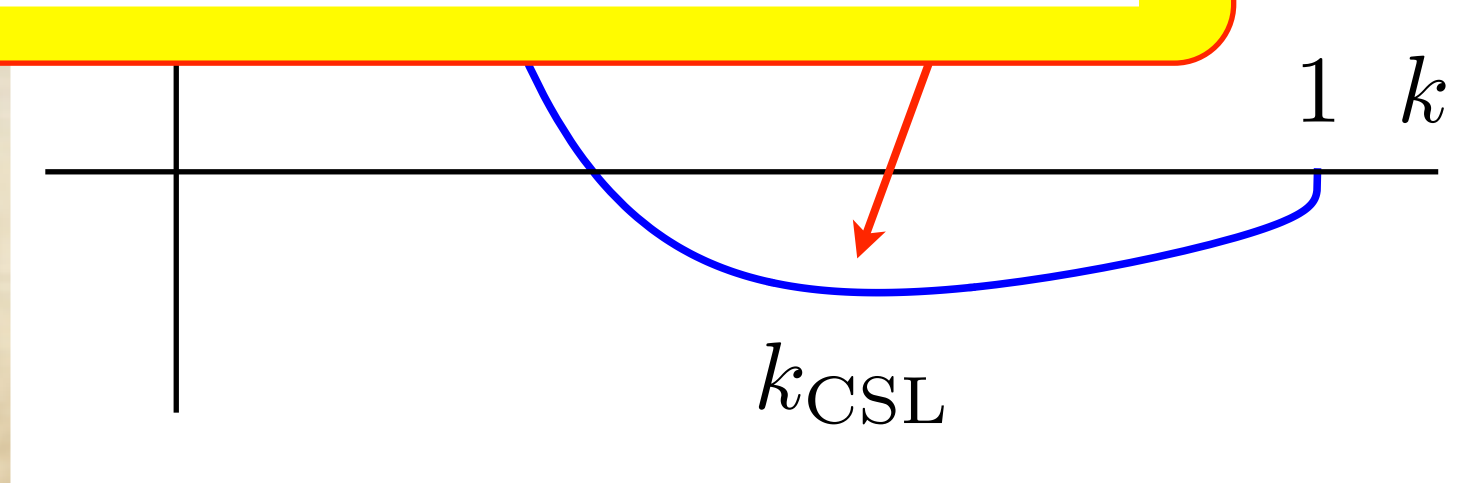
Domain wall layer is energetically favored.

=> Configuration at energy minimum k_{CSL} :
"Chiral Soliton Lattice"



'16 Brauner & Yamamoto

minimum



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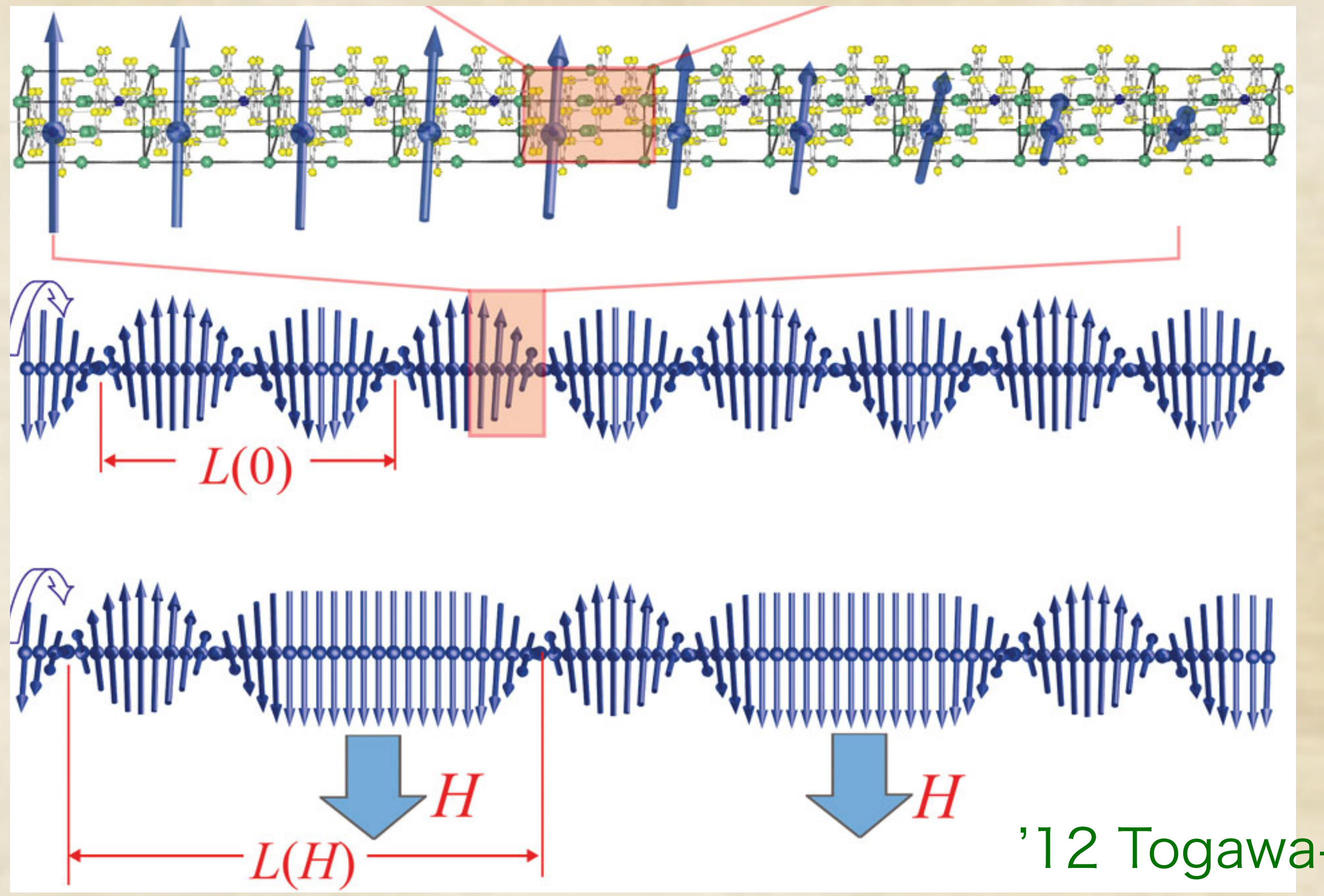
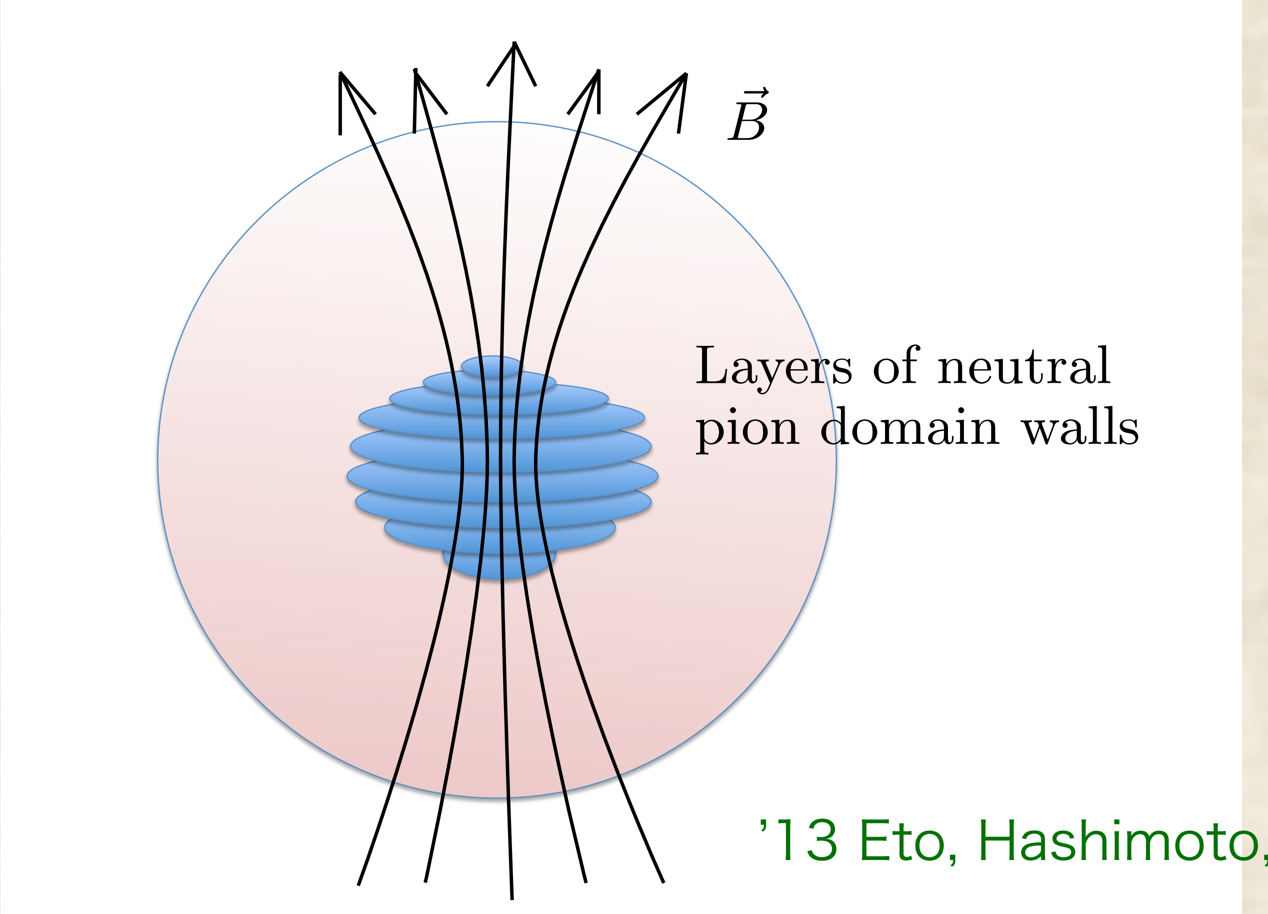
How it is interesting?

- Interest also for condensed matter system:
  known as Chiral Magnets. e.g., '12 Togawa+



'12 Togawa+

- Forms in the neutron star?
  Helps to amplify magnetic field?
  '13 Eto, Hashimoto, & Hatsuda



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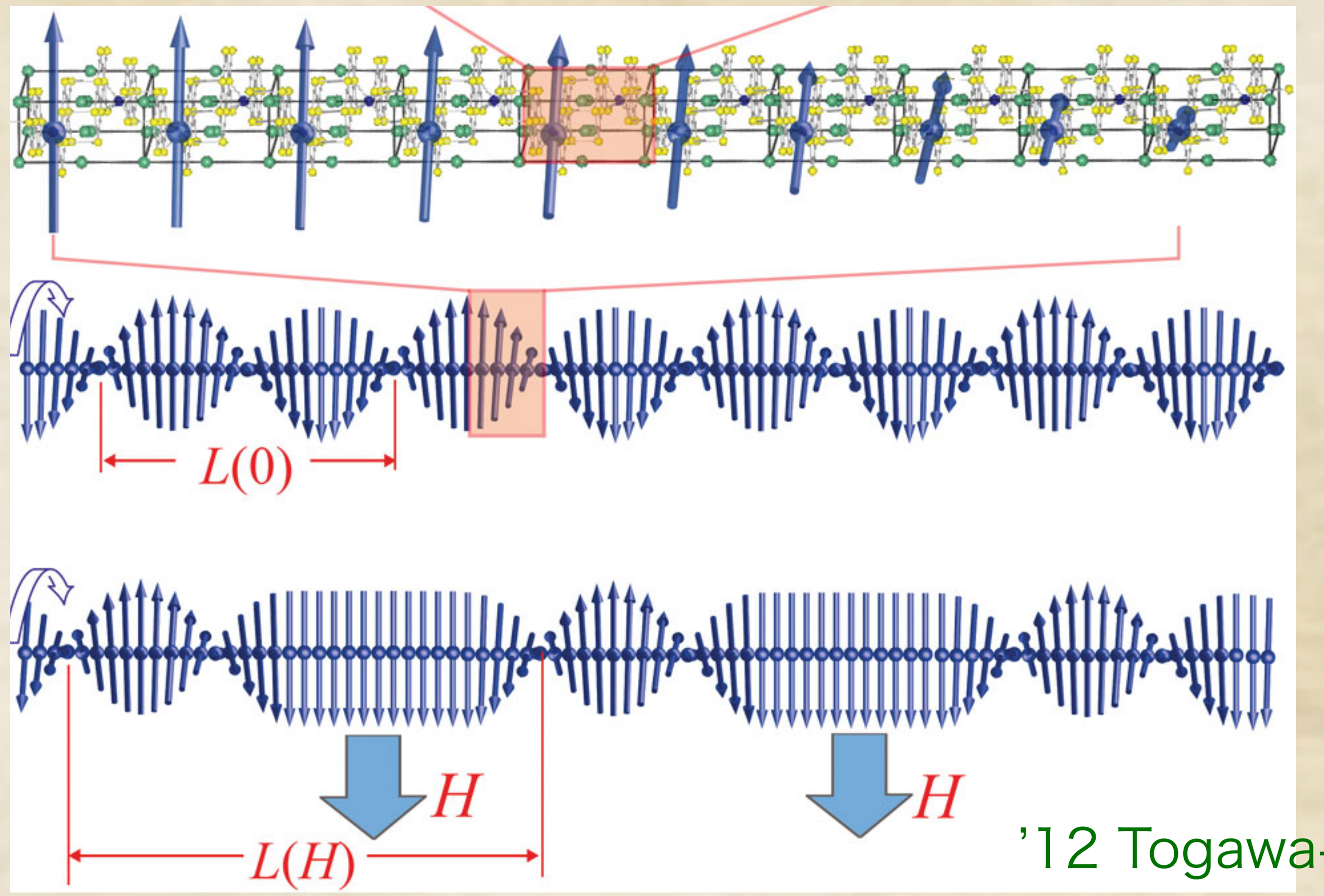

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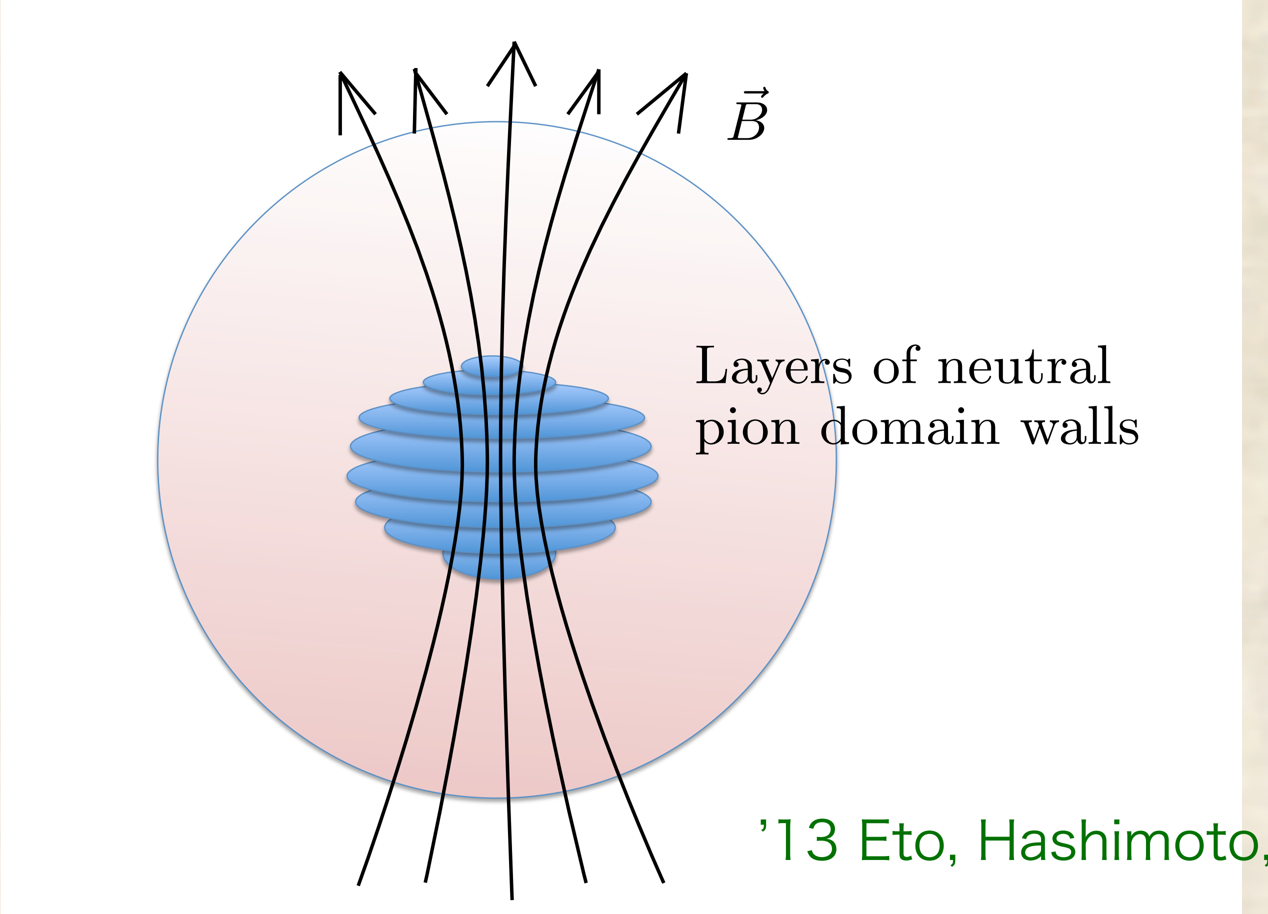

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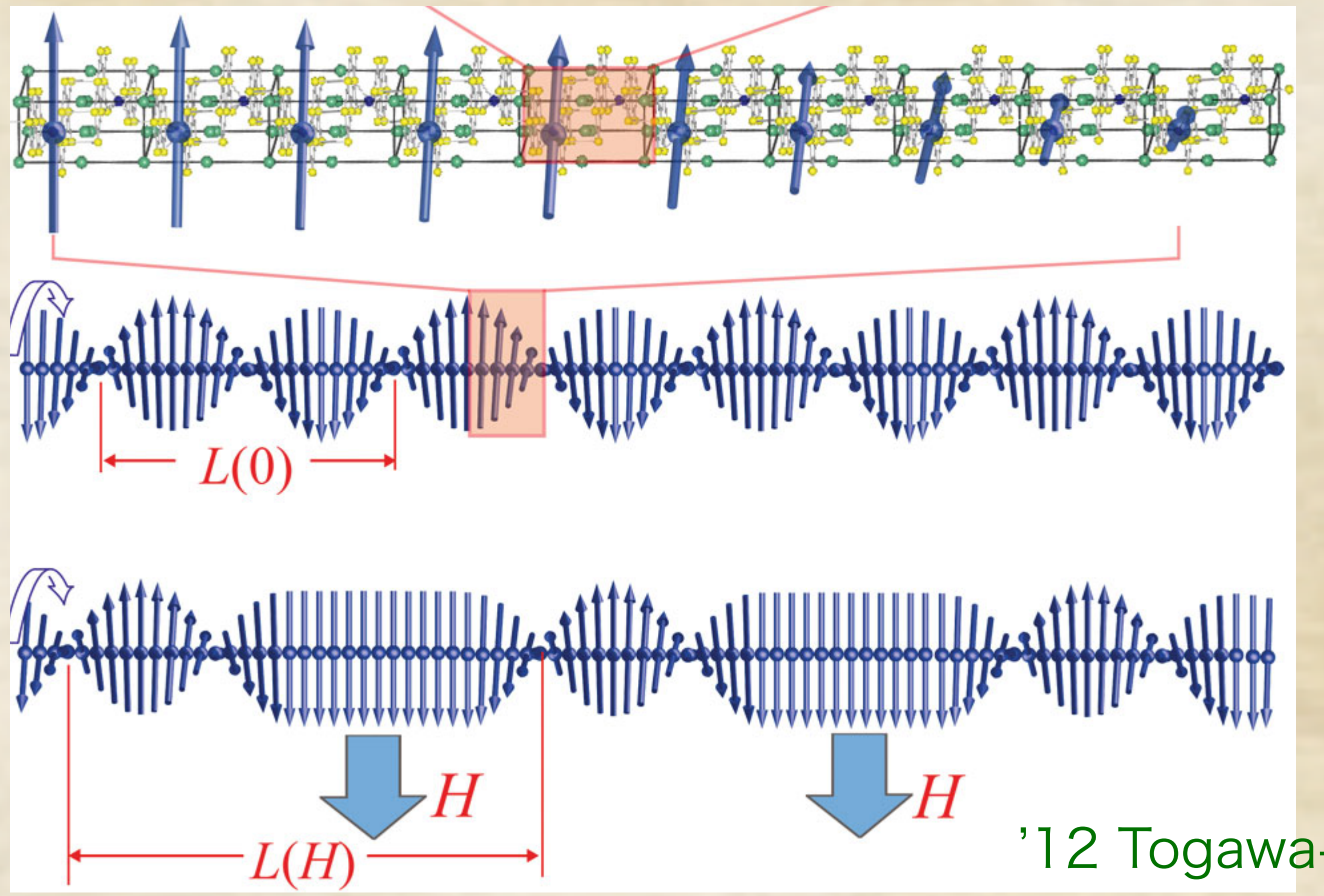

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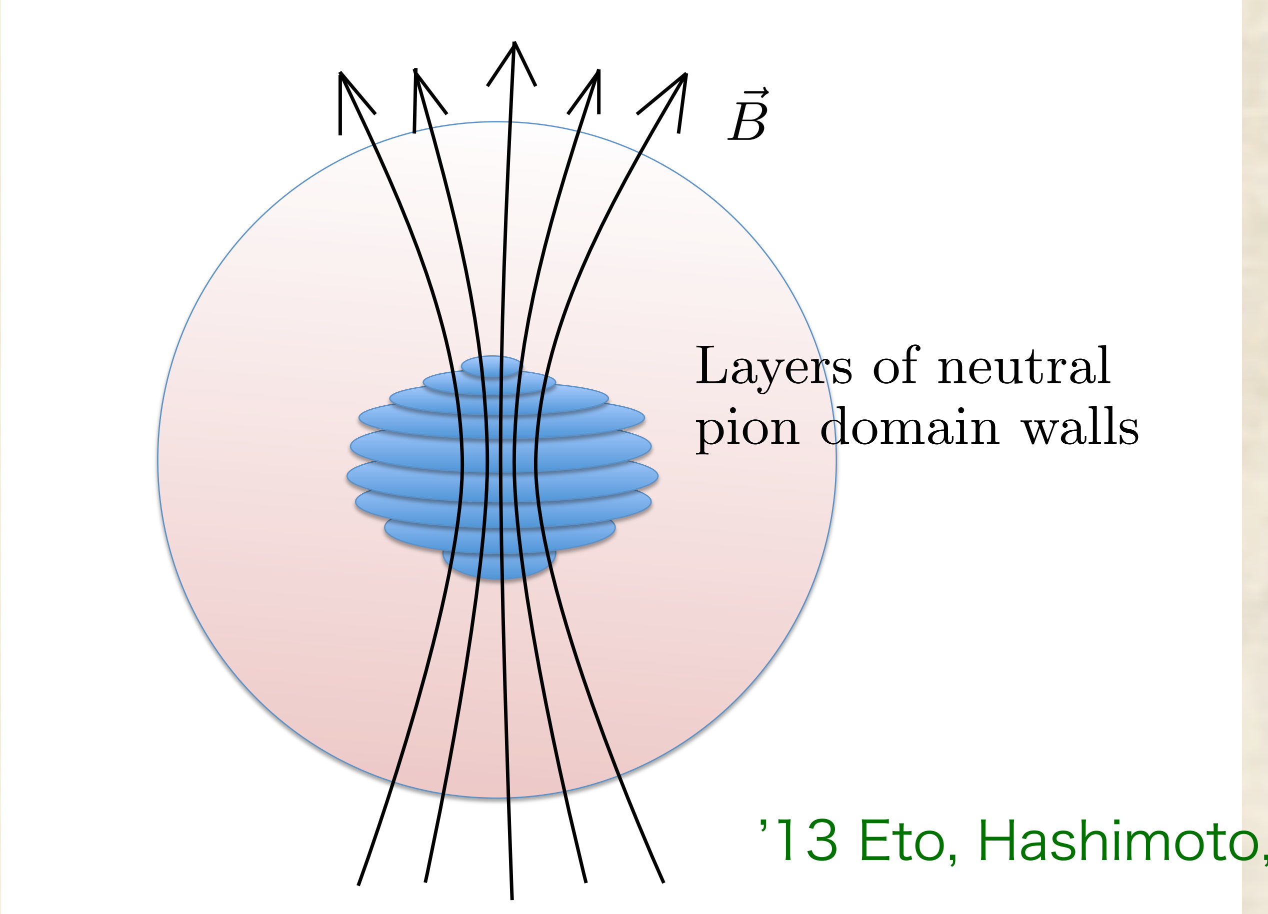

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- Can form in the early Universe?

Anyway we need to think how they form!

Courtesy: H. Oide

How Chiral Soliton Lattice forms

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How to model the formation?

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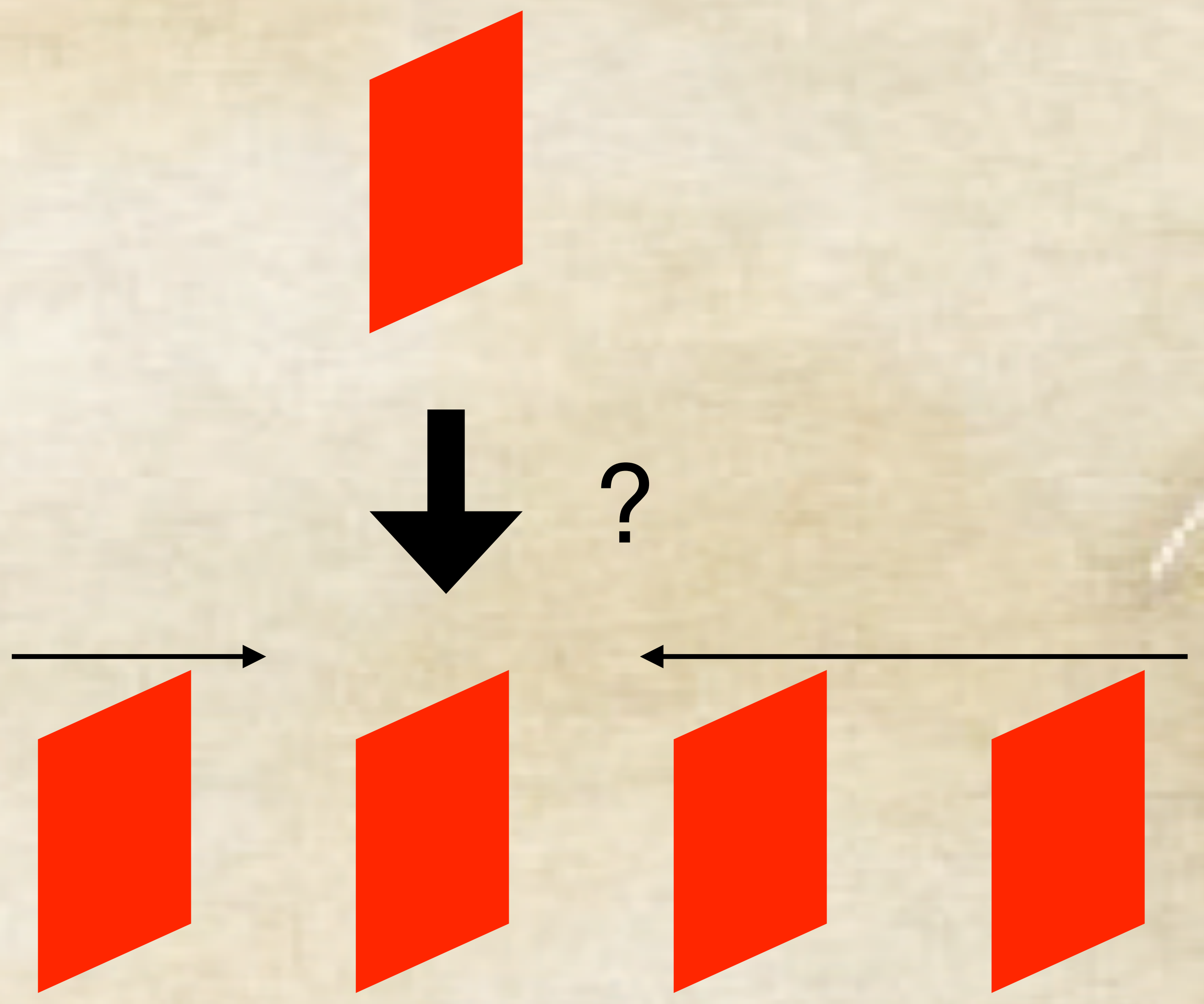
- Classical move from the infinity would be unlikely.



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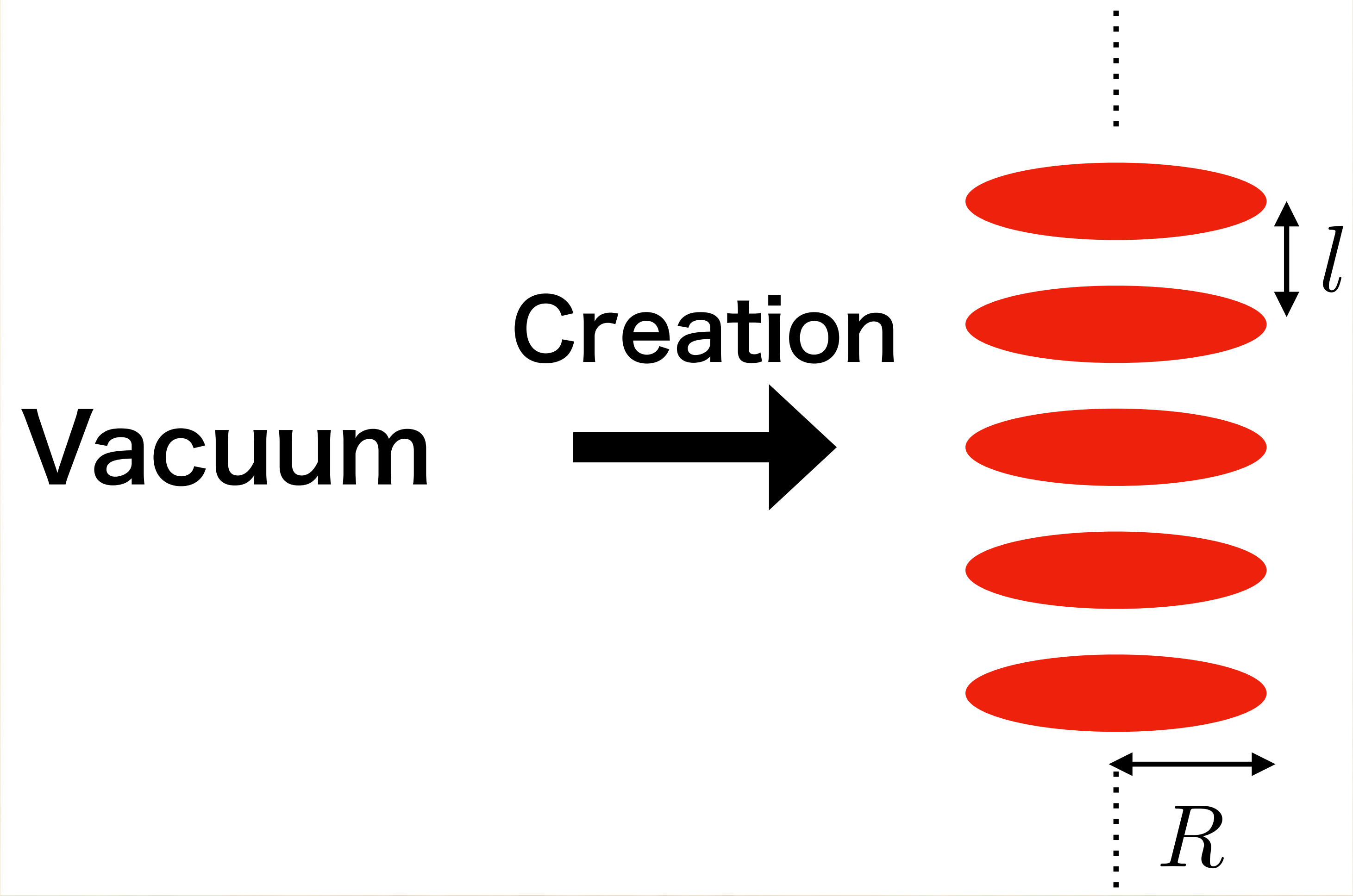
- Quantum tunneling of the layer of disk?



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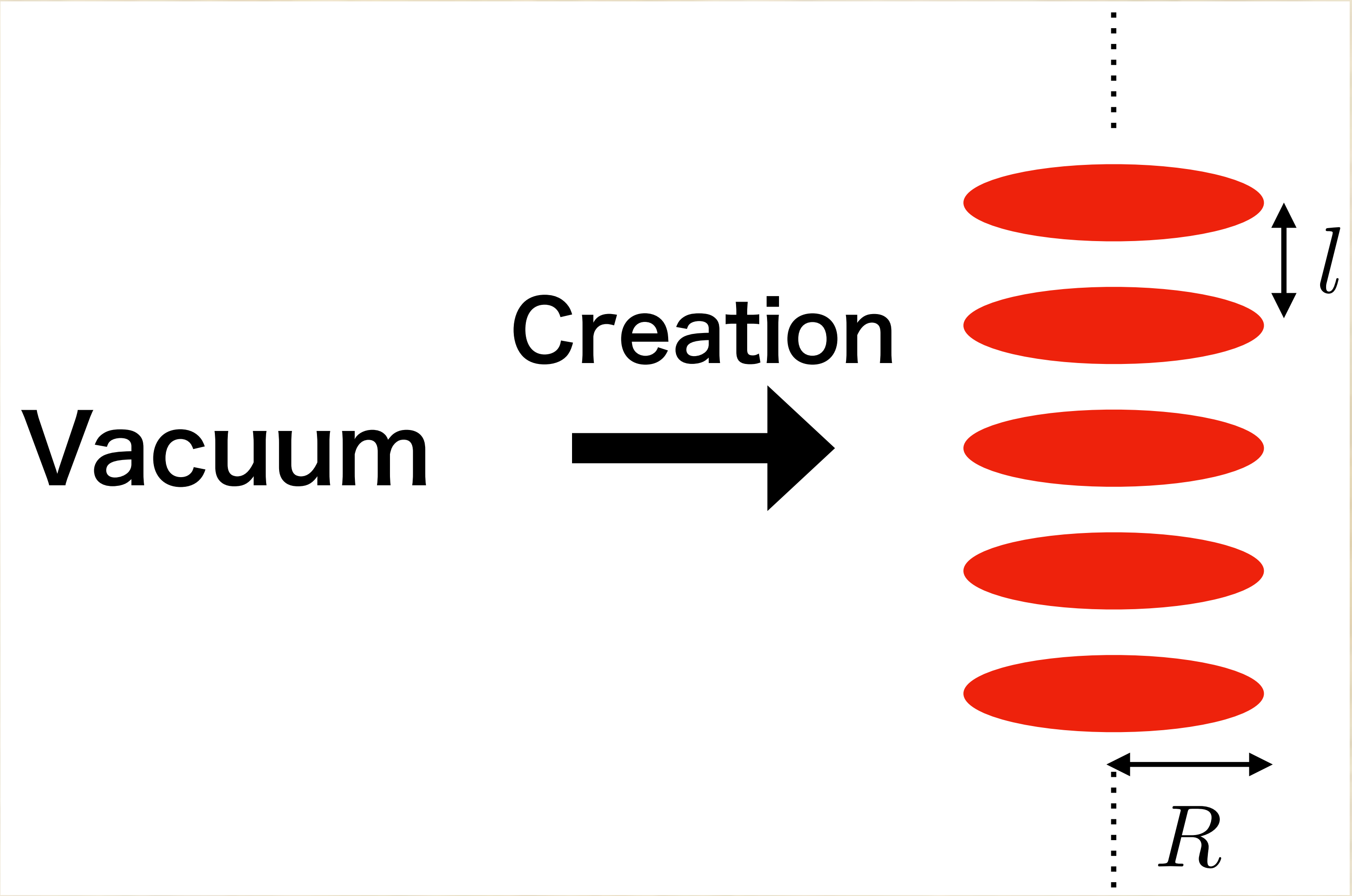

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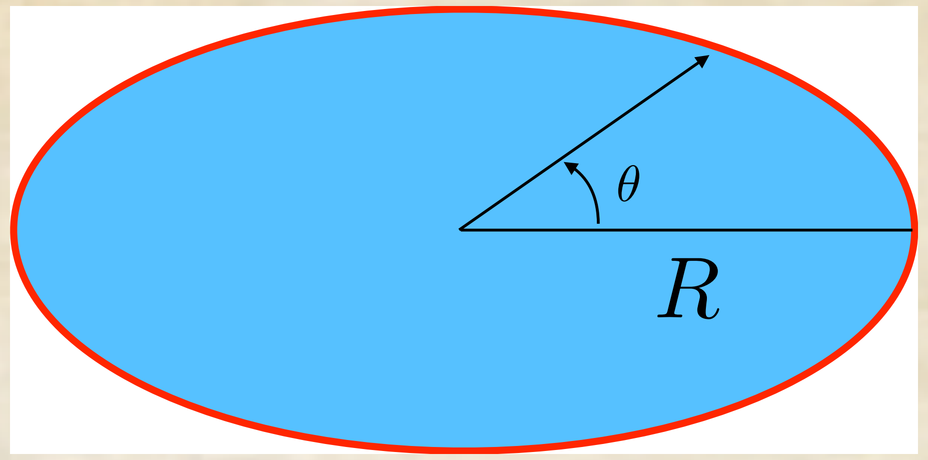
model it as the quantum mechanics of a disk with a Nambu-Goto-like action.

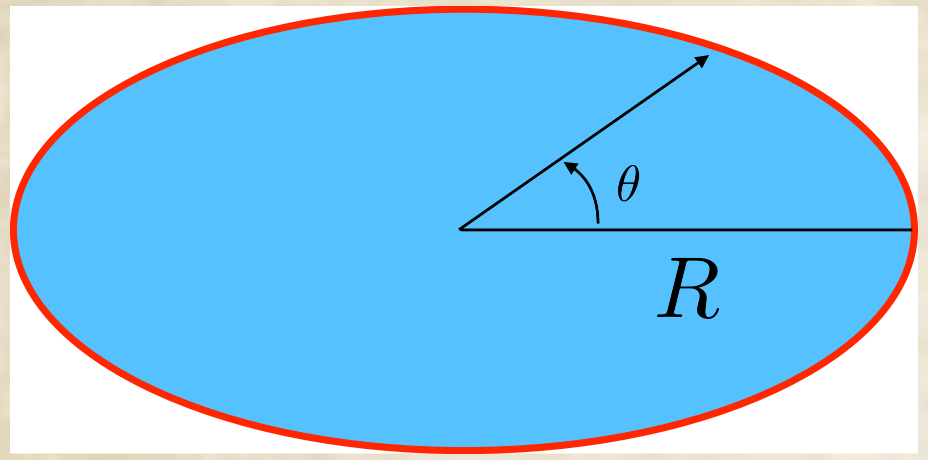
'91 Basu, Guth & Vilenkin, '20 Ai & Drewes



= disk wall + string loop on the edge

Courtesy: H. Oide

Effective action for disk



= disk wall + string loop on the edge

$$S_{\rm tot} = S_{\rm wall} + S_{\rm string} = -\pi \int d\tau \left(2TR\sqrt{1-\dot{R}^2} + \sigma R^2 \right)$$

$\sigma = 8mf^2 - \dfrac{\mu B}{2\pi}$ for one disk

< 0

$$T \sim 2\pi \times 2f^2 \times \ln \frac{R_{\rm c}}{r_{\rm c}}$$

Courtesy: H. Oide

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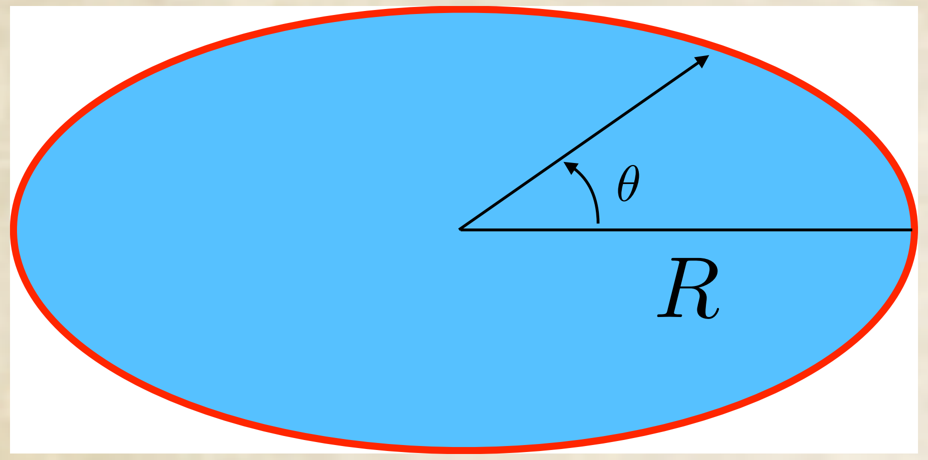

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Euclidean action

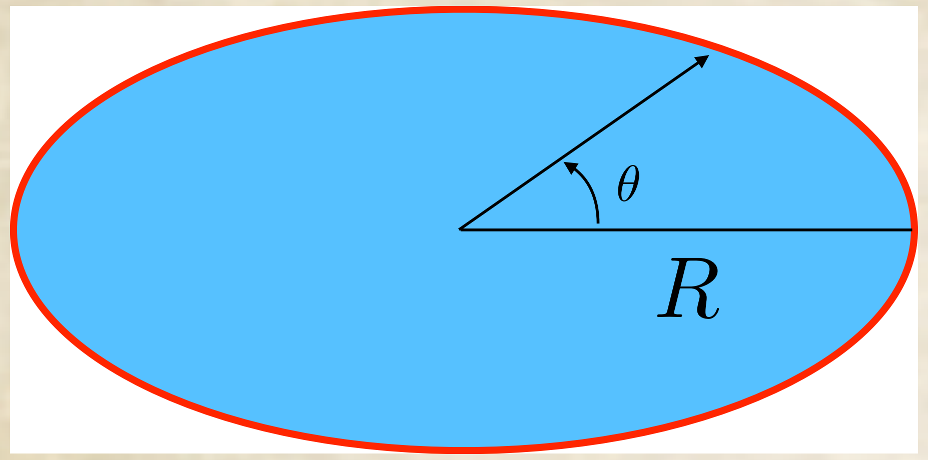
$$S_{\rm E}[R] = \pi \int d\tau_{\rm E} \left(2TR\sqrt{1+\left(\frac{dR}{d\tau_{\rm E}}\right)^2} + \sigma R^2 \right)$$

with bounce solution

$$\left(\frac{dR}{d\tau_{\rm E}}\right)^2 - 2V(R) = 0, \qquad V(R) = \frac{1}{2}\left[\frac{R^2}{\left(\epsilon - \frac{\sigma R^2}{2T}\right)^2} - 1\right], \qquad \epsilon = \frac{E}{2\pi T} \to 0$$

Courtesy: H. Oide

Effective action for disk



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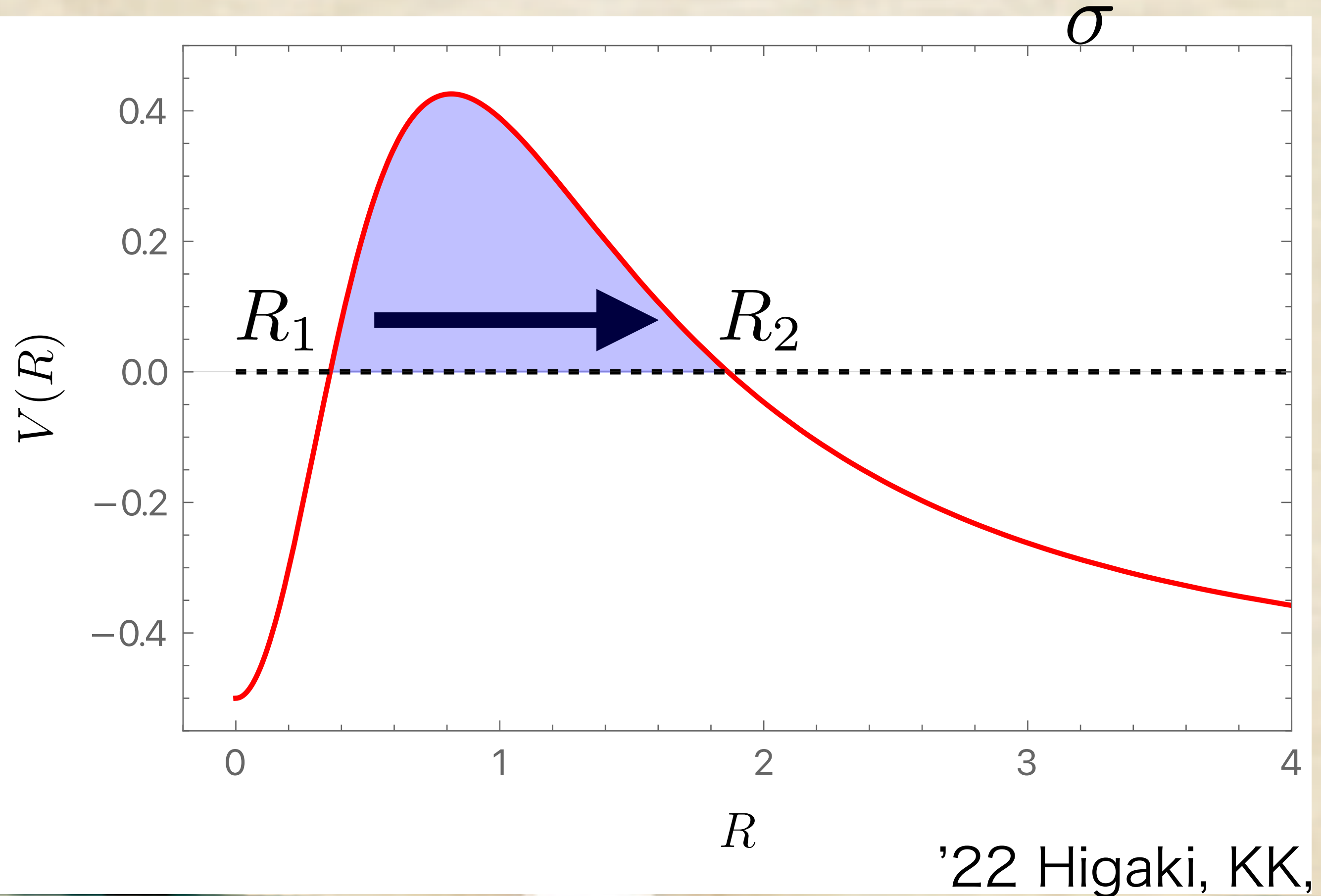
$$S_{\text{E}}[R] = \pi \int d\tau_{\text{E}} \left(2TR\sqrt{1 + \left(\frac{dR}{d\tau_{\text{E}}}\right)^2} + \sigma R^2 \right)$$

understood as the tunneling

$$R_1 \to R_2 \Rightarrow 0 \to -\frac{2T}{\sigma}$$

with bounce solution

$$\left(\frac{dR}{d\tau_{\text{E}}}\right)^2 - 2V(R) = 0, \quad V(R) = \frac{1}{2}\left[\frac{R^2}{\left(\epsilon - \frac{\sigma R^2}{2T}\right)^2} - 1\right], \quad \epsilon = \frac{E}{2\pi T} \to 0$$



’22 Higaki, KK, Nishimura

Courtesy: H. Oide

Transition rate

Bounce action: $\mathcal{B} = 2 \times 2\pi T \int_0^{R_2} dR \sqrt{R^2 - (\sigma R^2/2T)^2} = \frac{16\pi T^3}{3\sigma^2}$

Coleman's formula: $P \simeq \mathcal{A} e^{-\mathcal{B}} = \mathcal{A} e^{-16\pi T^3/3\sigma^2}$ $\qquad \mathcal{A} \simeq \left(\frac{\mathcal{B}}{2\pi}\right)^2 \frac{1}{R_2^4}$

'77 Coleman; Callan & Coleman

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Exponential suppression factor (bounce action) is less the 1 for

$$B > B_c = \frac{16\pi m f^2}{\mu} \left(\frac{(\ln(R_c/r_c))^{3/2}}{\sqrt{3}} \frac{4\pi^2 f}{m} + 1 \right) (> B_{\rm CSL})$$

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For CSL, we shall just replace σ

$$\tilde{\sigma} = \mathcal{E}_{\mathrm{CSL}} = 4 m f^2 \left[\frac{2E(k)}{k} + \left(k - \frac{1}{k} \right) K(k) \right] - \frac{\mu B}{2\pi}$$

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$$\tilde{\sigma} = \mathcal{E}_{\rm CSL} = 4mf^2 \left[\frac{2E(k)}{k} + \left(k - \frac{1}{k}\right) K(k) \right] - \frac{\mu B}{2\pi}$$

smaller amplitude of tension
=> larger bounce action

Courtesy: H. Oide

Transition rate

Bounce actic

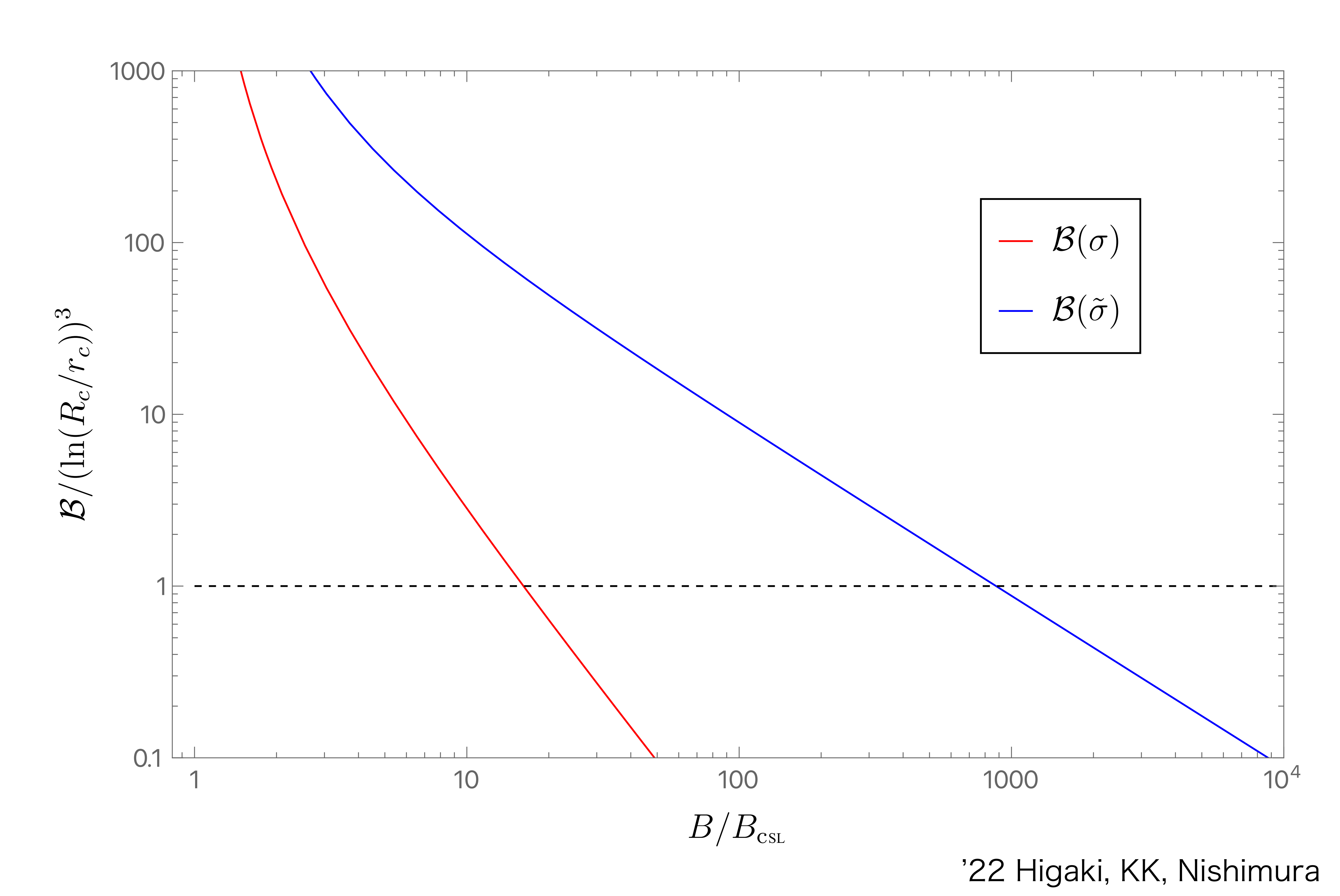
Coleman's fc

'77 Coleman

Exponential s

for

For CSL, we



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smaller amplitude of tension
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Courtesy: H. Oide

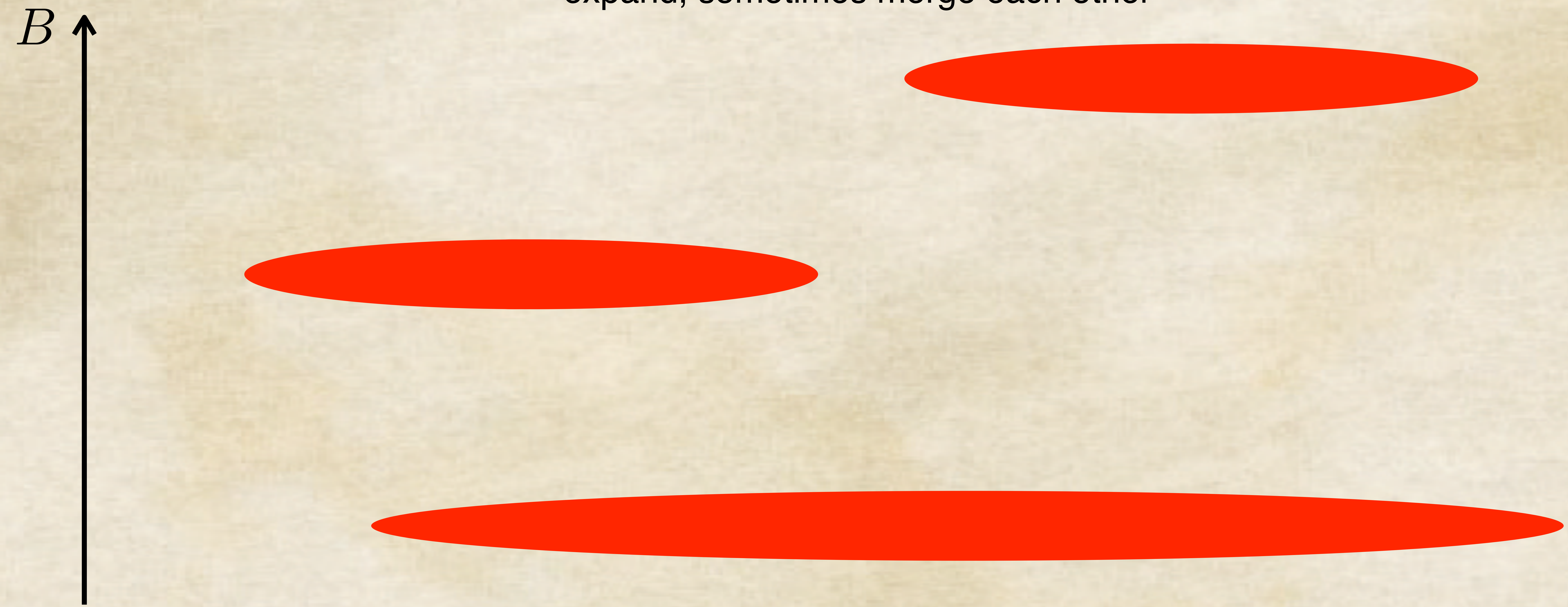
Whole picture?

single wall individually form

Courtesy: H. Oide

Whole picture?

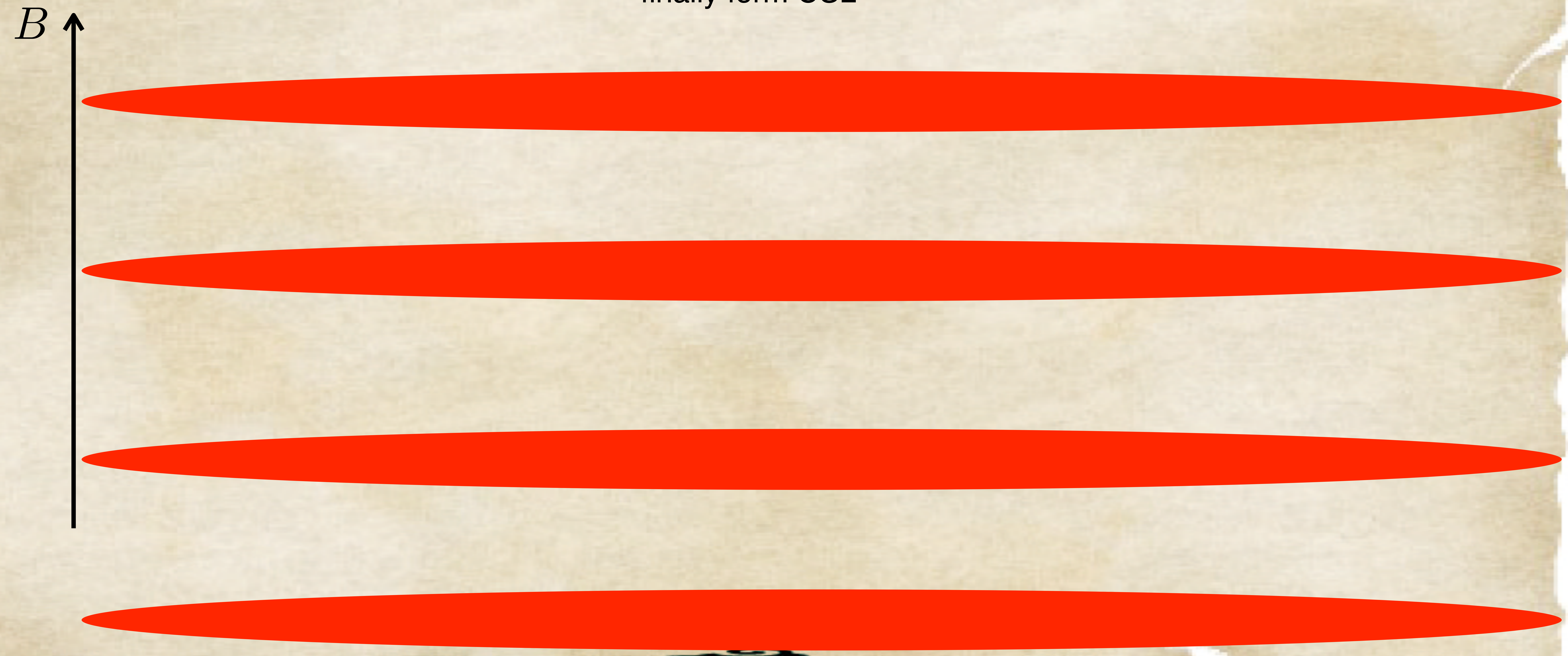
expand, sometimes merge each other



Courtesy: H. Oide

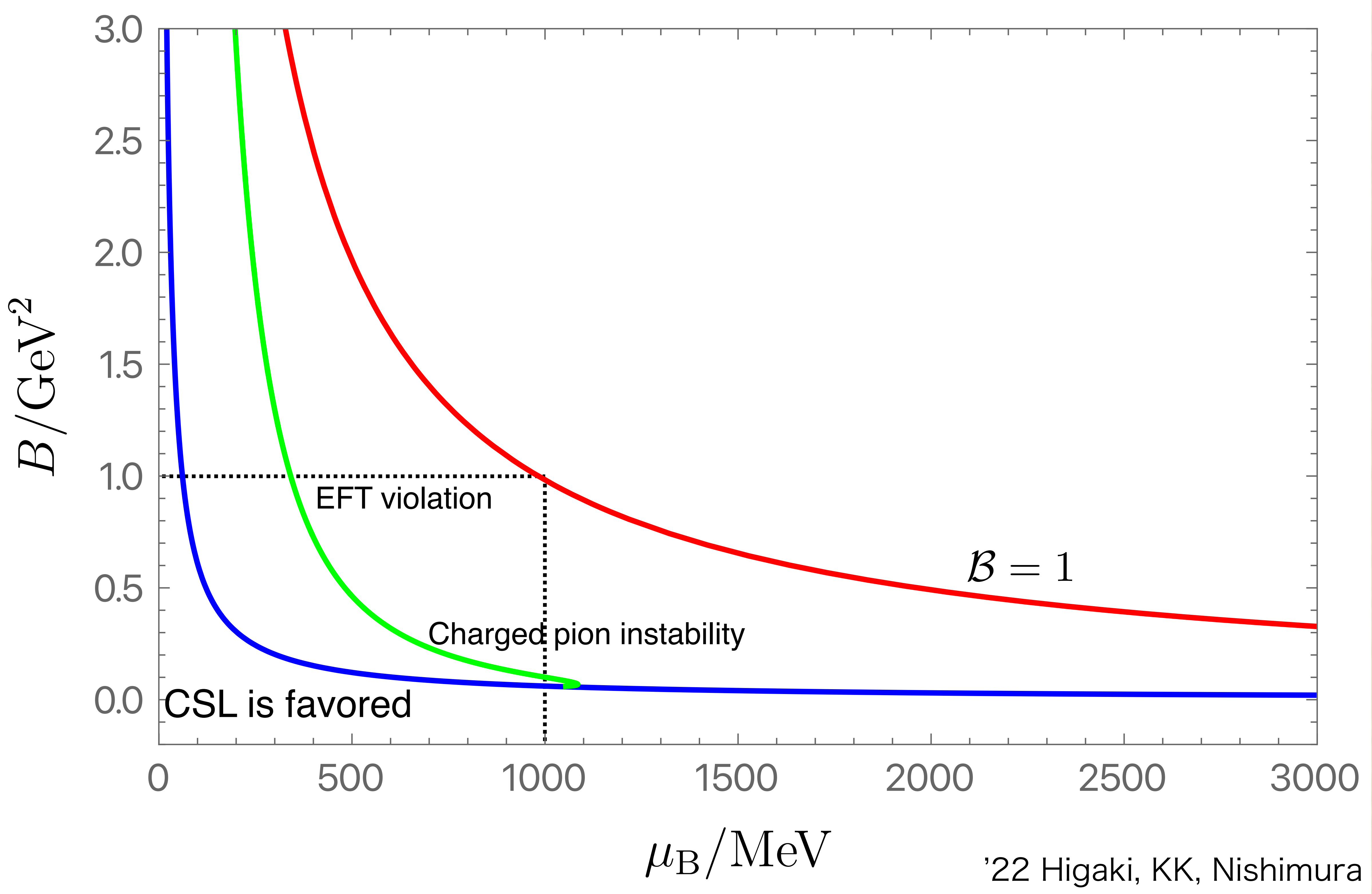
Whole picture?

finally form CSL



Courtesy: H. Oide

Concrete examples: QCD pion



It is difficult for CSL to form without exponential suppression

Courtesy: H. Oide

Concrete examples: ALP

- also difficult to setup the situation where $\mathcal{B} < 1$

$$B_{\mathcal{B}=1} \simeq \frac{64\pi^2 f^3}{\mu_B}$$

after EWSB $\mu_B \ll B^{1/2} \lesssim T$ => for $f \gg T$ inevitably $\mathcal{B} \gg 1$

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before EWSB, it can be $\mu_B \sim B^{1/2} \lesssim T$

with $f \sim T$ we might have $\mathcal{B} \sim 1$

Unfortunately, we cannot expect for the effect on cosmic birefringence in CMB.

Courtesy: H. Oide

Summary

Courtesy: H. Oide

- Chiral anomaly induces a nontrivial topological interaction between axion/pion and magnetic field at finite density (chemical potential).
- It makes the domain wall layer energetically favorable, which is called as "Chiral Soliton Lattice".
- Its formation is described by the nucleation of a domain wall disk.
- The formation rate is relatively suppressed.
- Implication of light axions? Indeed, thin-wall approximation becomes bad.

Courtesy: H. Oide