Stochastic Properties of Ultralight Scalar Fields

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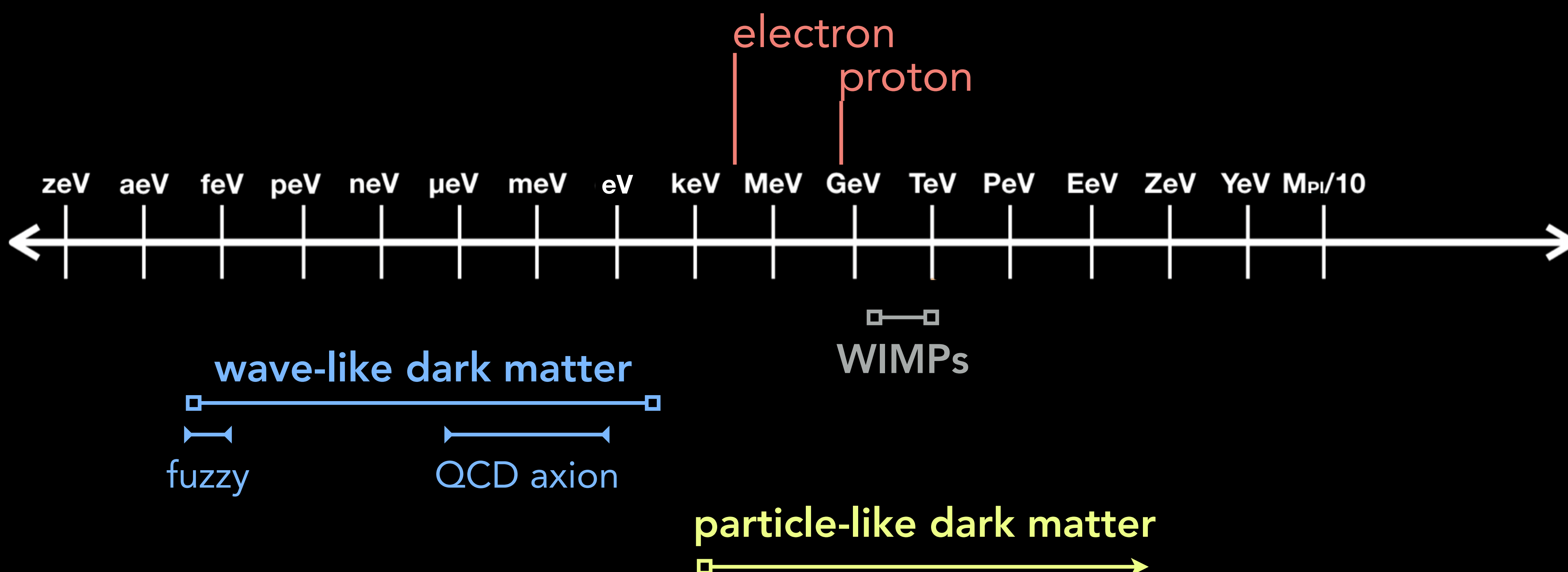
Lisanti, Moschella, Terrano PRD 104, 055037 (2021)
Lee, Lisanti, Terrano, and Romalis PRX in print (2023)

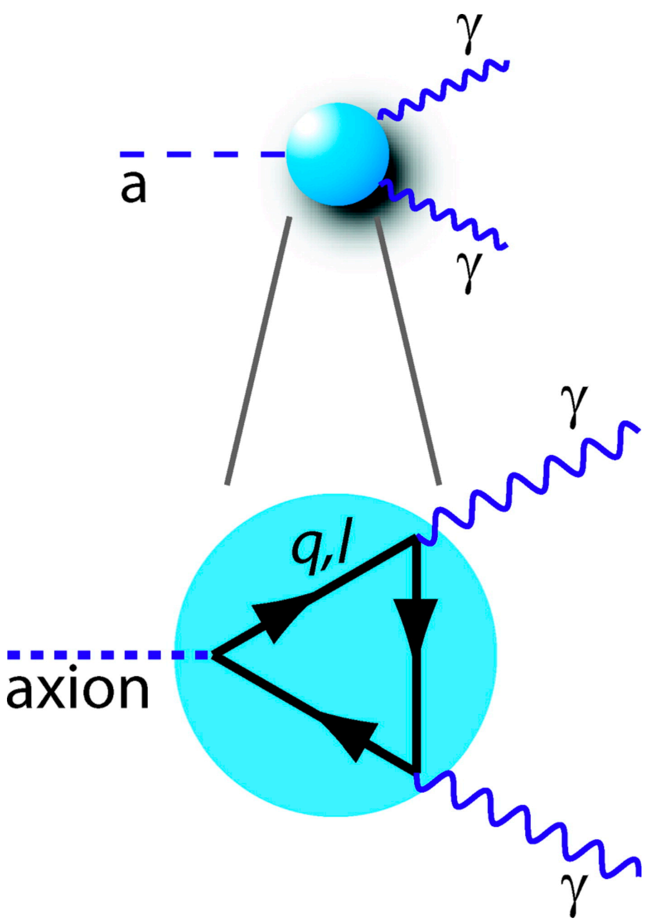
image credit: pngtree

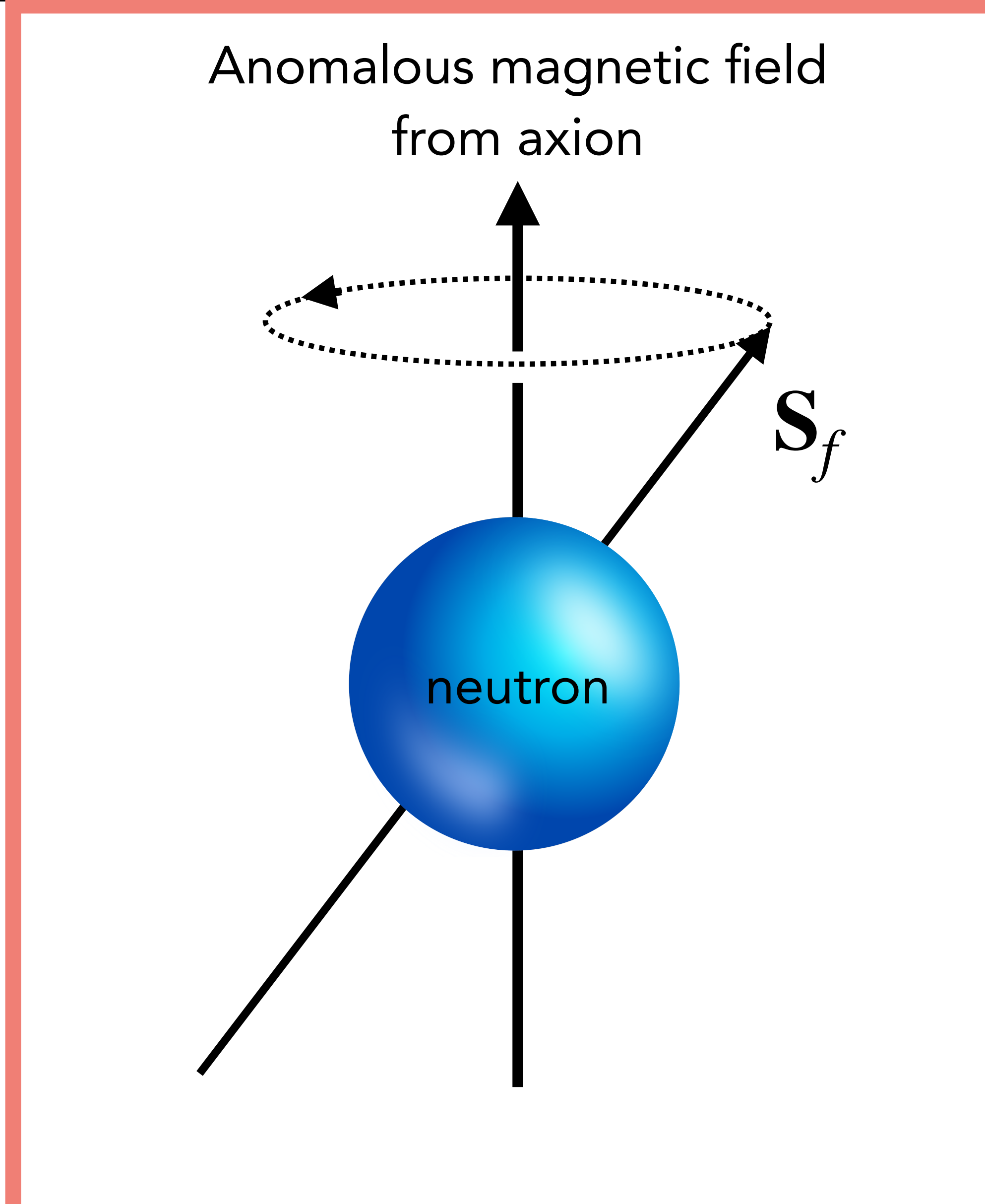
Dark Matter Mass Range

Well-motivated dark matter models cover an extensive mass range



Examples of Axion Interactions





$$\mathscr{L} \propto g_{a\gamma\gamma}\, a\, \mathbf{E} \cdot \mathbf{B}$$

$$\mathscr{L} \propto g_{aff} \nabla \mathbf{a} \cdot \mathbf{S}_f$$

Image Credit: Carosi, van Bibber and Pivovaroff (2008)

The Local Dark Matter Distribution

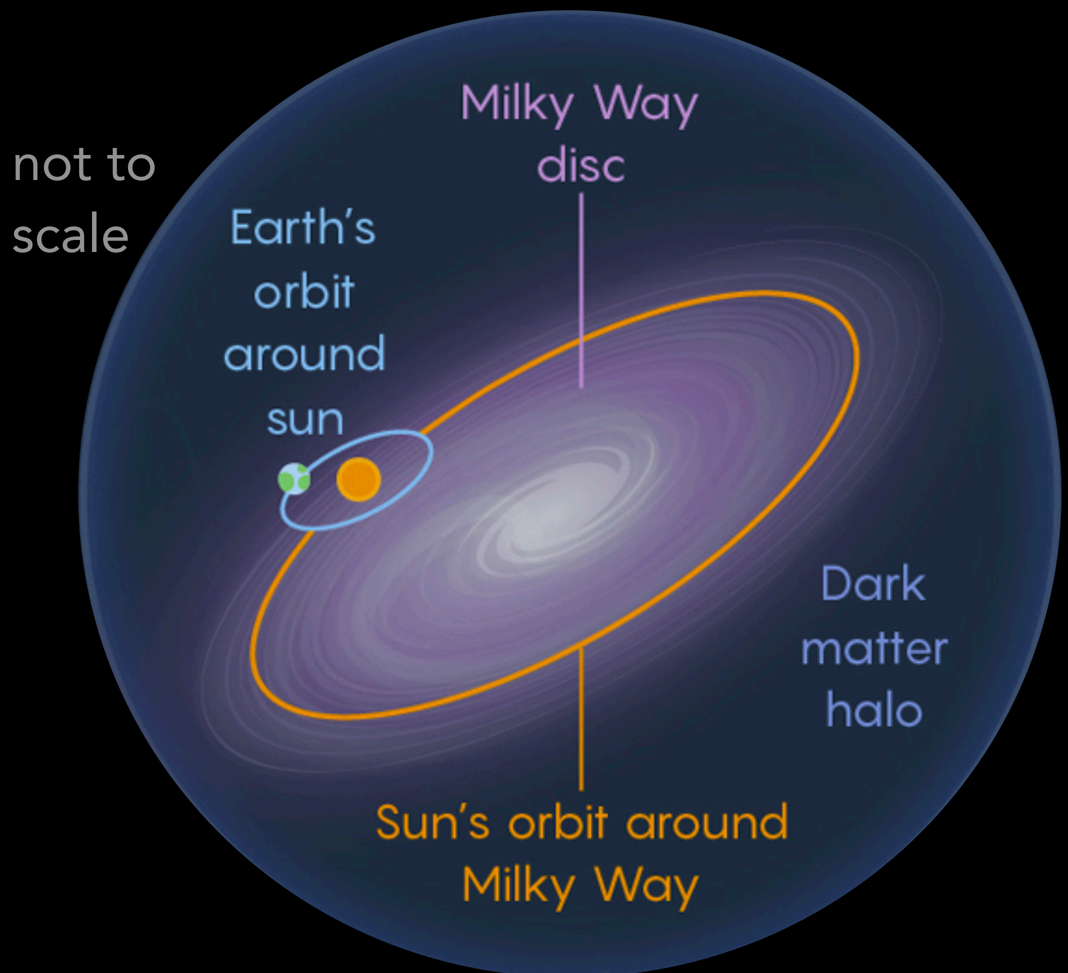


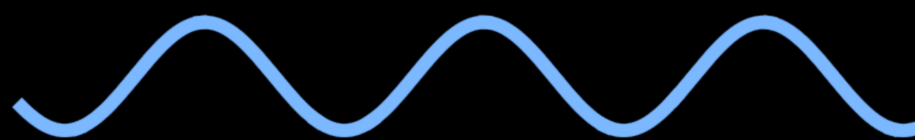
Image Credit: Lucy Reading-Ikkanda/Quanta Magazine

- Average dark matter density ~0.4 GeV/cm^3
- Average dark matter velocity ~200 km/s
- Dark matter flux modulates annually & daily
- Axions act like classical field due to high number density

$$n_\mathrm{a} \sim 10^{14} \left(\frac{10^{-6}\ \mathrm{eV}}{m_a} \right) \frac{\mathrm{axions}}{\mathrm{cm}^3}$$

The Axion Field

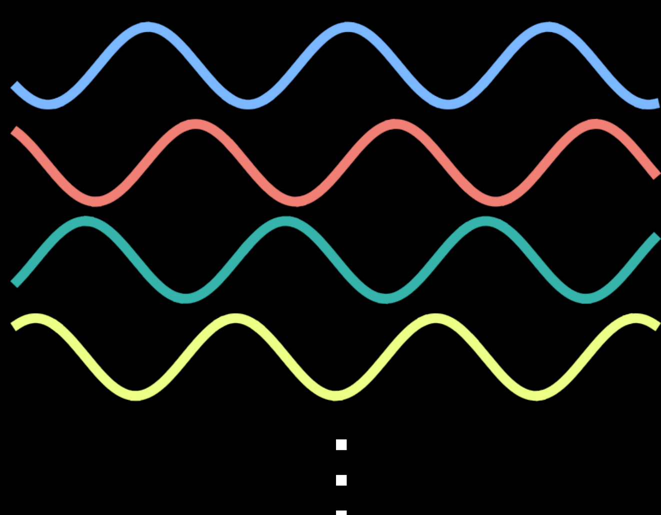
Individual axion state λ with mass m_a has random energy (E_λ) and phase (ϕ_λ)



$$a_\lambda(\mathbf{x}, t) \propto \cos\left(E_\lambda(t)\, t + \phi_\lambda \right)$$

$$E_\lambda(t) \approx m_a + \frac{1}{2} m_a v_\lambda(t)^2$$

Total axion field is the sum over all individual states



$a_1(t) \propto \cos\left(E_1\, t + \phi_1\right)$

$a_2(t) \propto \cos\left(E_2\, t + \phi_2\right)$

$a_3(t) \propto \cos\left(E_3\, t + \phi_3\right)$

$a_4(t) \propto \cos\left(E_4\, t + \phi_4\right)$

$$\text{Total Axion Wave} = \sum_{\lambda=1}^{n_a} a_\lambda(\mathbf{x}, t)$$

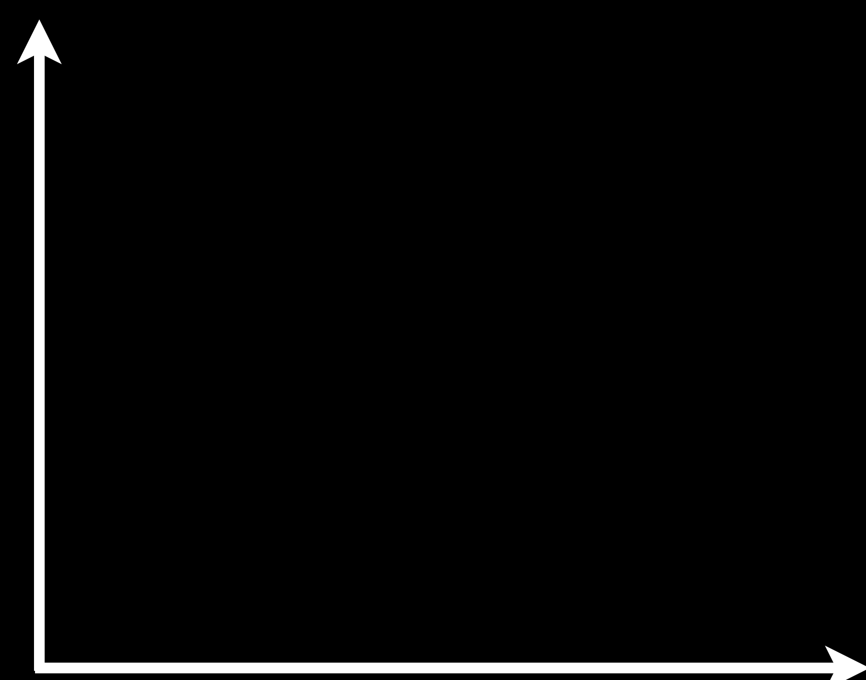
Axion Coherence

Assume axions virialized in Milky Way with velocity dispersion $\sigma_v \sim 10^{-3}$

Coherence time depends on frequency dispersion across axion states

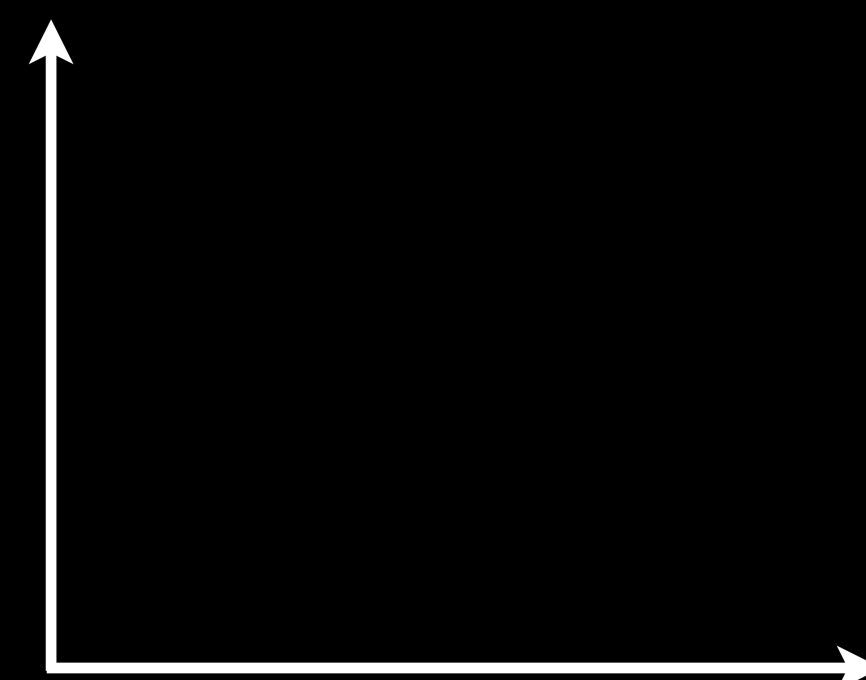
$$\tau_c \sim \frac{1}{\delta f} \sim \frac{1}{m_a \sigma_v^2}$$

$\sigma_v \sim 0$



frequency

$\sigma_v \gtrsim 0$

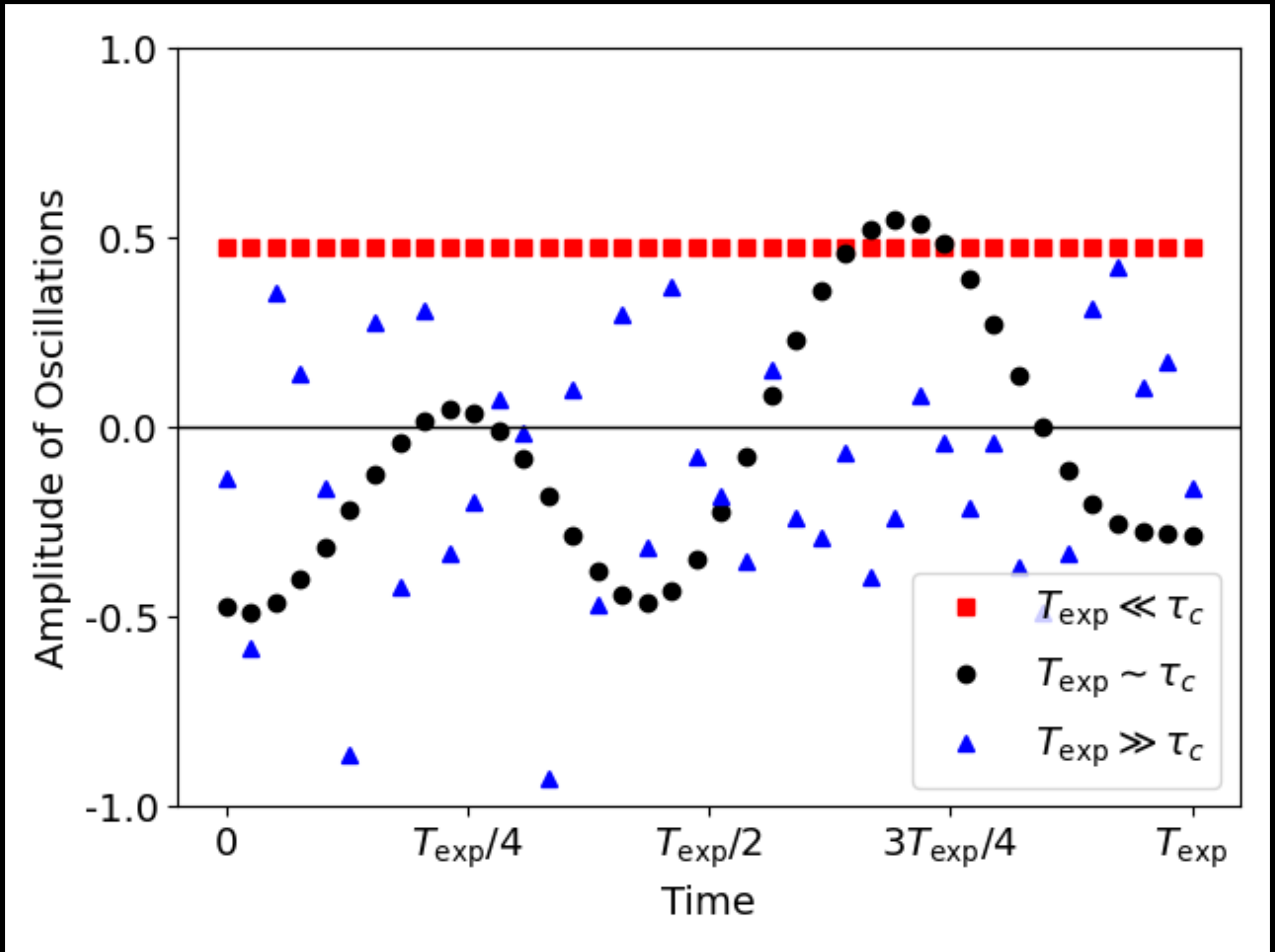


frequency

Coherence Timescale

Amplitude and phase of axion oscillations vary stochastically when $\tau \gtrsim \tau_c$

$$\tau_c \sim 10\,\mathrm{days}\left(\frac{m_a}{10^{-15}\,\mathrm{eV}}\right)^{-1}\left(\frac{v}{10^{-3}}\right)^{-2}$$

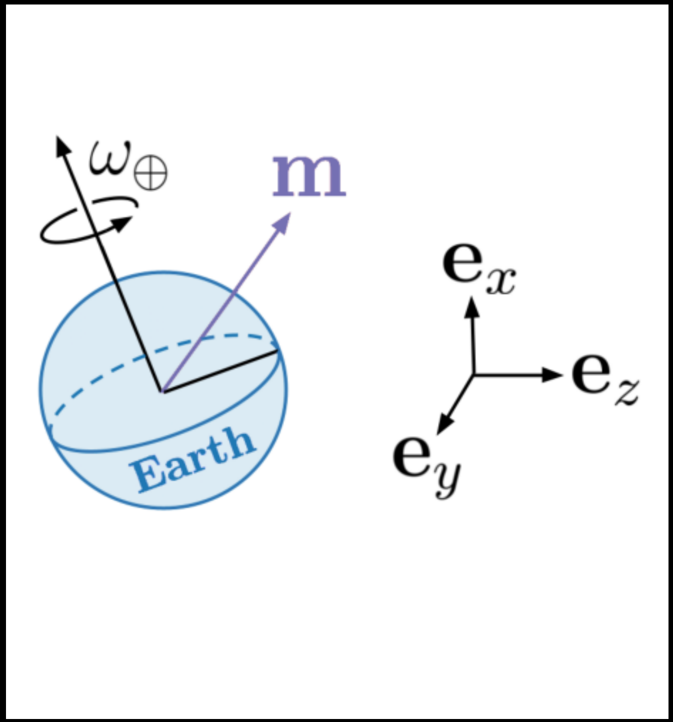


Derevianko [1605.009717]; Foster et al. [1711.10489]

Axion Field Gradient

Axion-fermion coupling is proportional to the total gradient of the field

$$\nabla \mathbf{a}(t) \propto \sum_{\lambda} \mathbf{v}_{\lambda}(t)\, \cos\left(E_{\lambda}(t)\, t + \phi_{\lambda} \right)$$

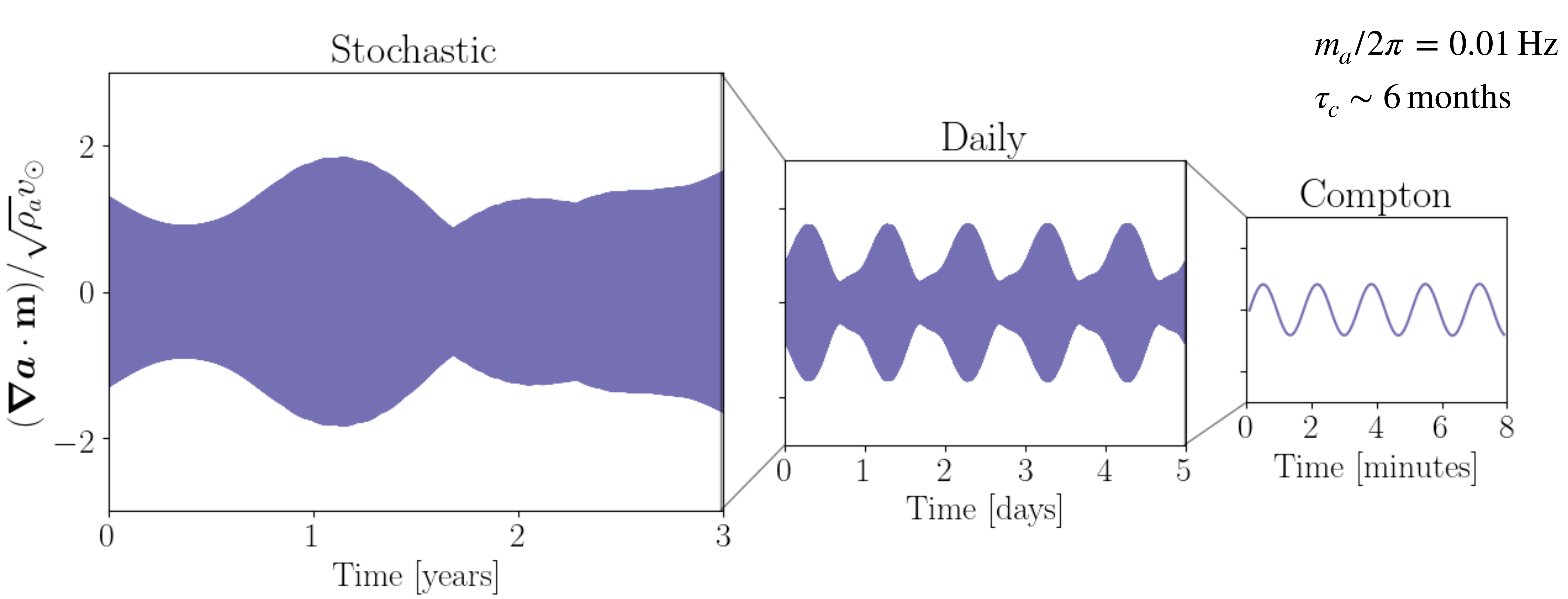


Experiments measure projection of $\nabla \mathbf{a}(t)$ onto measurement axis $\mathbf{m}(t)$

$$S(t) = g_{\mathrm{eff}}\, \nabla \mathbf{a}(t) \cdot \mathbf{m}(t)$$

Yields daily modulation

Relevant Timescales



Moschella, Lisanti, and Terrano [2107.10260]

The Coherent Limit

$$\tau_c \gg T_{\mathrm{exp}}$$

Axion Field in Coherent Limit

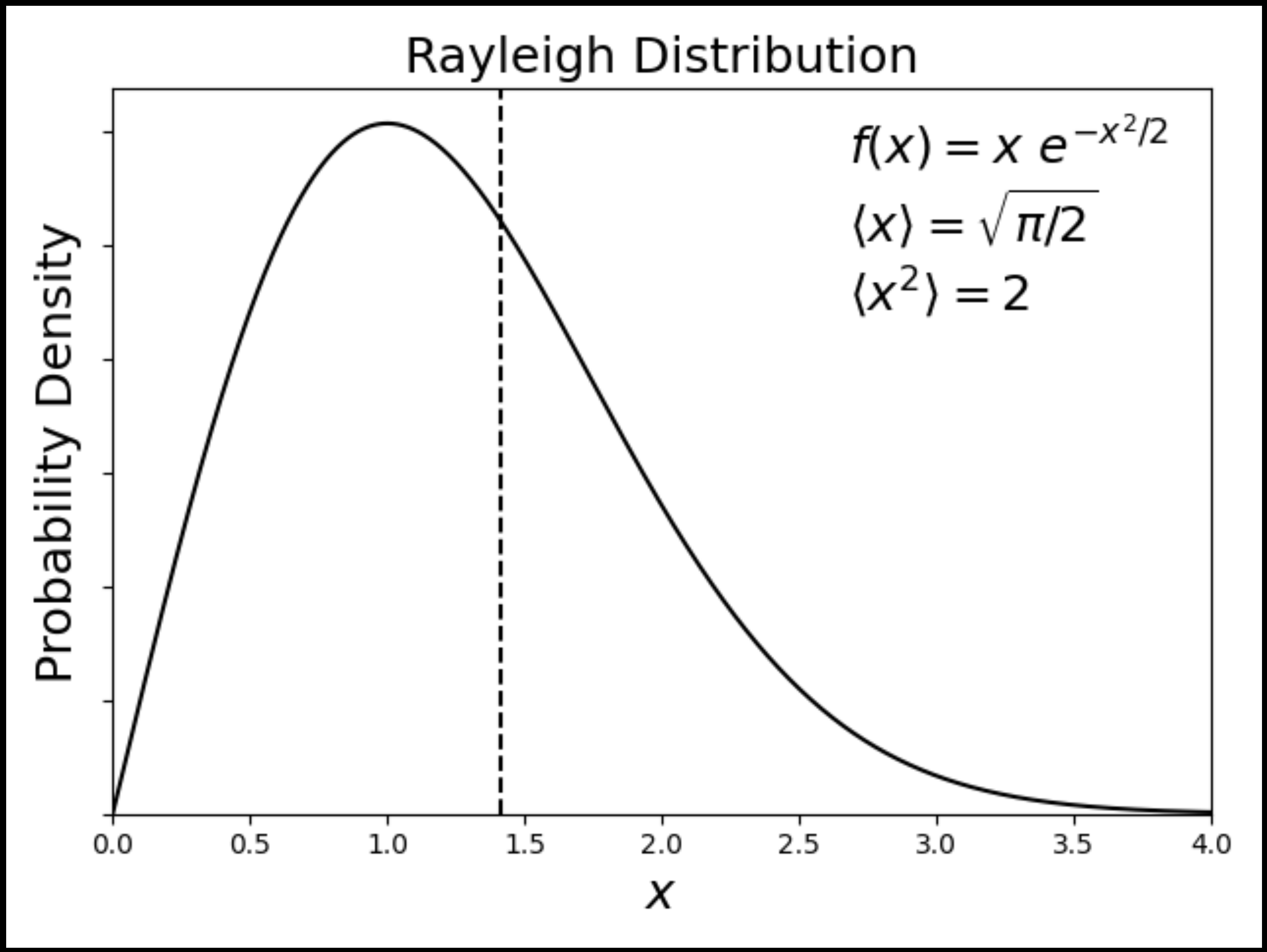
$$a(t) = \sum_\lambda a_\lambda(t) \sim \sum_\lambda \cos\left(m_a t + \phi_\lambda \right)$$

$$\sim \cos\left(m_a t\right)\left(\sum_\lambda \cos\phi_\lambda\right) - \sin\left(m_a t\right)\left(\sum_\lambda \sin\phi_\lambda\right)$$

$$\sim \frac{1}{\sqrt{2}} X \cos\left(m_a t\right) - \frac{1}{\sqrt{2}} Y \sin\left(m_a t\right)$$

$$\sim \frac{1}{\sqrt{2}} \alpha \cos\left(m_a t + \phi \right)$$

Axion Field in Coherent Limit

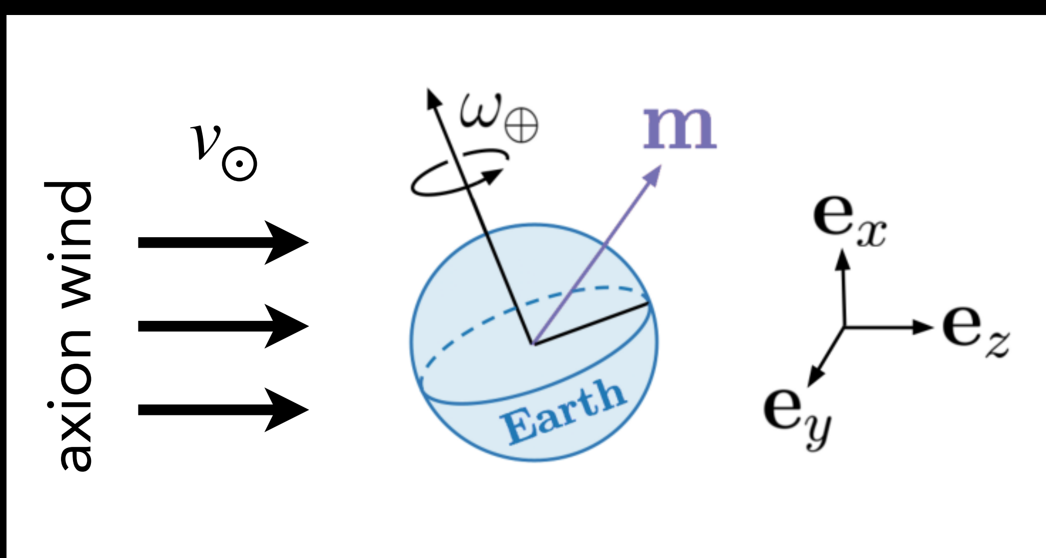


Axion Gradient Field

Experimental signal for gradient field depends on 6 Gaussian random variables

3 Rayleigh-distributed amplitudes

3 uniformly-distributed phases



$$S_{\text{3D, stoch.}}(t) \propto g_{\text{eff}} \sqrt{\sigma_v^2 + v_\odot^2}\, \alpha_z \cos\left(m_a t + \phi_z\right)\, \mathrm{m}_z(t) + g_{\text{eff}}\, \sigma_v\, \alpha_y \cos\left(m_a t + \phi_y\right)\, \mathrm{m}_y(t) + g_{\text{eff}}\, \sigma_v\, \alpha_x \cos\left(m_a t + \phi_x\right)\, \mathrm{m}_x(t)$$

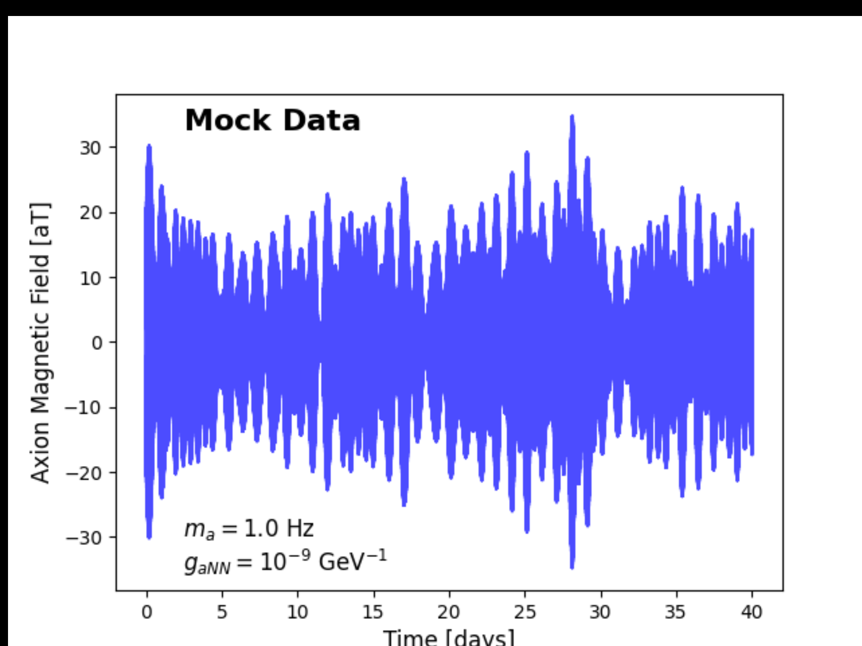
Signal Injection Tests

Signal injection tests on mock data crucial for verifying analysis pipeline

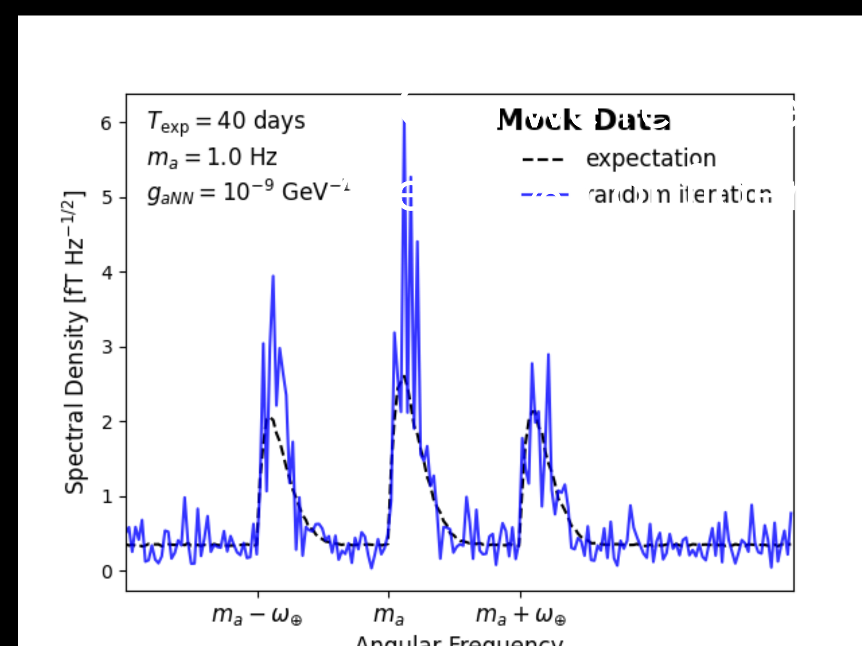
Mock Data → Likelihood Analysis → Signal Recovery

jected signal?

limits robust?

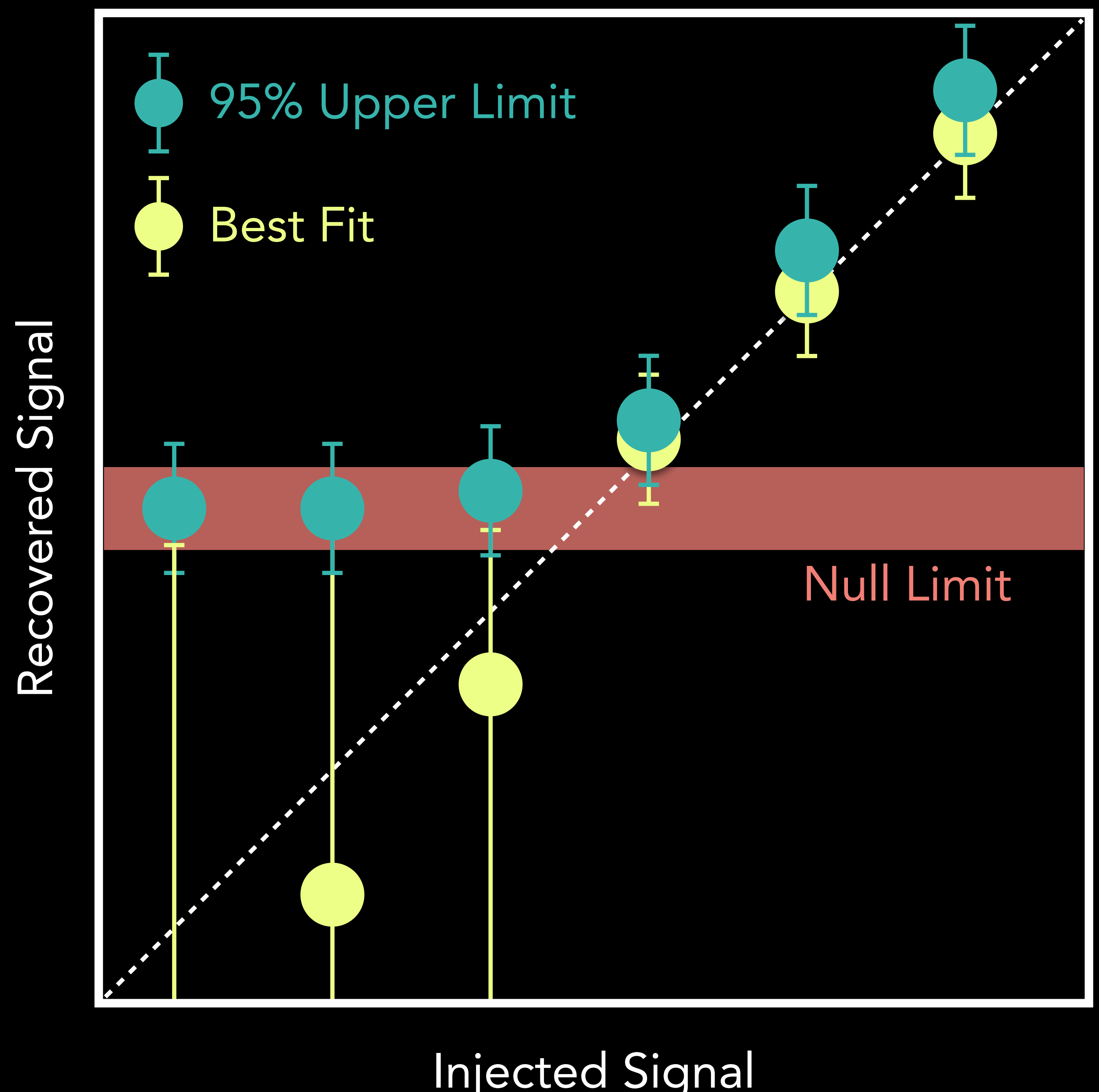


(no white noise)



(injected white noise at spectral density ~0.3 fT/Hz$^{1/2}$)

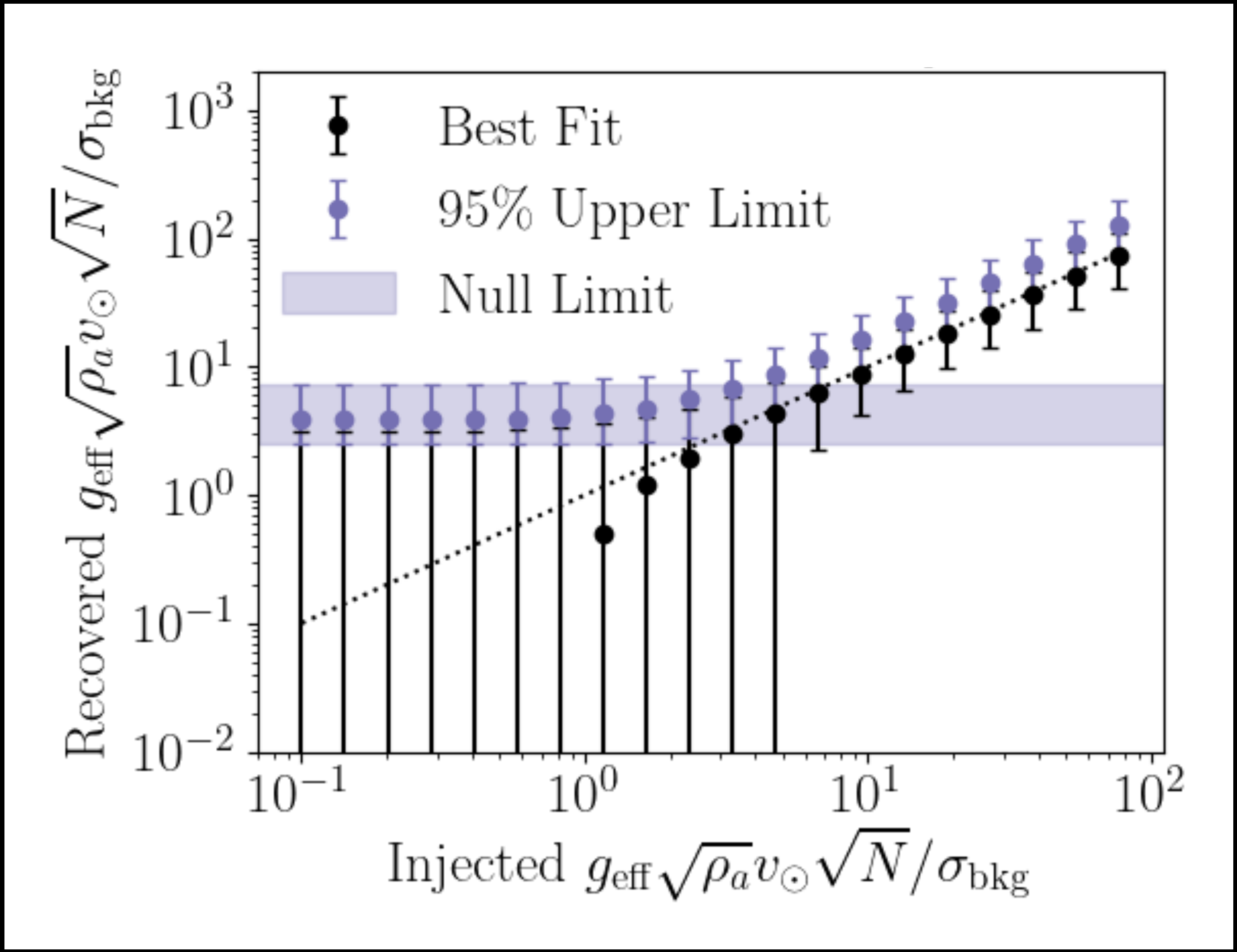
Signal Injection Tests



Uncertainty bands indicate spread over many Monte Carlo iterations

3D Stochastic Signal

Signal recovery and limit-setting procedures successful with stochastic model



Recovered Signal

Injected Signal

Incorrect Approaches

Across axion-fermion searches, stochastic behavior has been incorrectly modeled or ignored when it should not have been

Incorrect Model: 1D Deterministic

$$S_{\text{1D, det.}}(t) \propto g_{\text{eff}}\, v_{\odot} \cos\left(m_a t + \phi\right)\, \mathrm{m}_z(t)$$

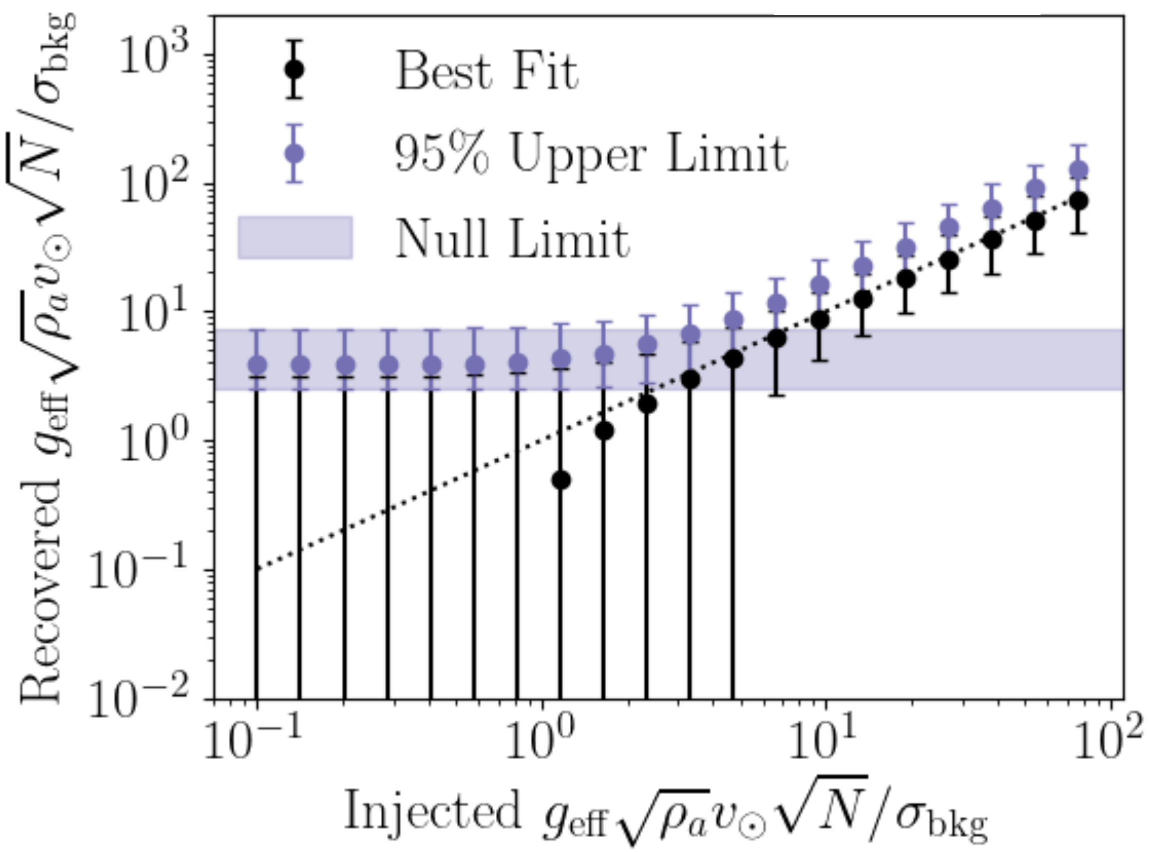
Correct Model

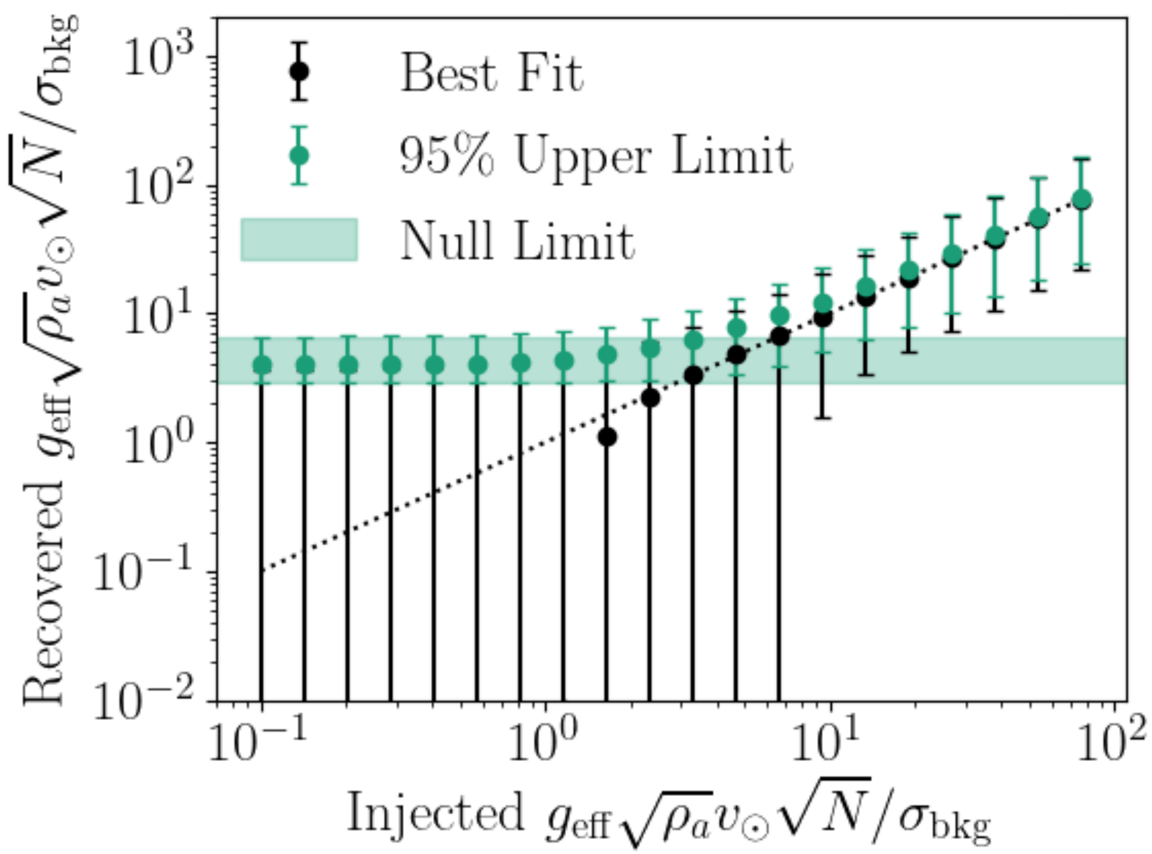
$$\begin{aligned} S_{\text{3D, stoch.}}(t) \propto\ & g_{\text{eff}} \sqrt{\sigma_v^2 + v_{\odot}^2}\, \alpha_z \cos\left(m_a t + \phi_z\right)\, \mathrm{m}_z(t) \\ & + g_{\text{eff}}\, \sigma_v\, \alpha_y \cos\left(m_a t + \phi_z\right)\, \mathrm{m}_y(t) \\ & + g_{\text{eff}}\, \sigma_v\, \alpha_x \cos\left(m_a t + \phi_x\right)\, \mathrm{m}_x(t) \end{aligned}$$

Signal Injection Tests

3D Stochastic Model

1D Deterministic Model





Assuming 1D Deterministic Model yields:

greater variability in recovered signal

95% upper limits are too strong at large coupling

Incorrect Approaches

Across axion-fermion searches, stochastic behavior has either been incorrectly modeled or ignored when it should not be

Incorrect Model: 1D Stochastic

$$S_{\text{1D, stoch.}}(t) \propto g_{\text{eff}}\, v_{\odot}\, \alpha \, \cos\left(m_a t + \phi\right)\, \mathrm{m}_z(t)$$

Correct Model

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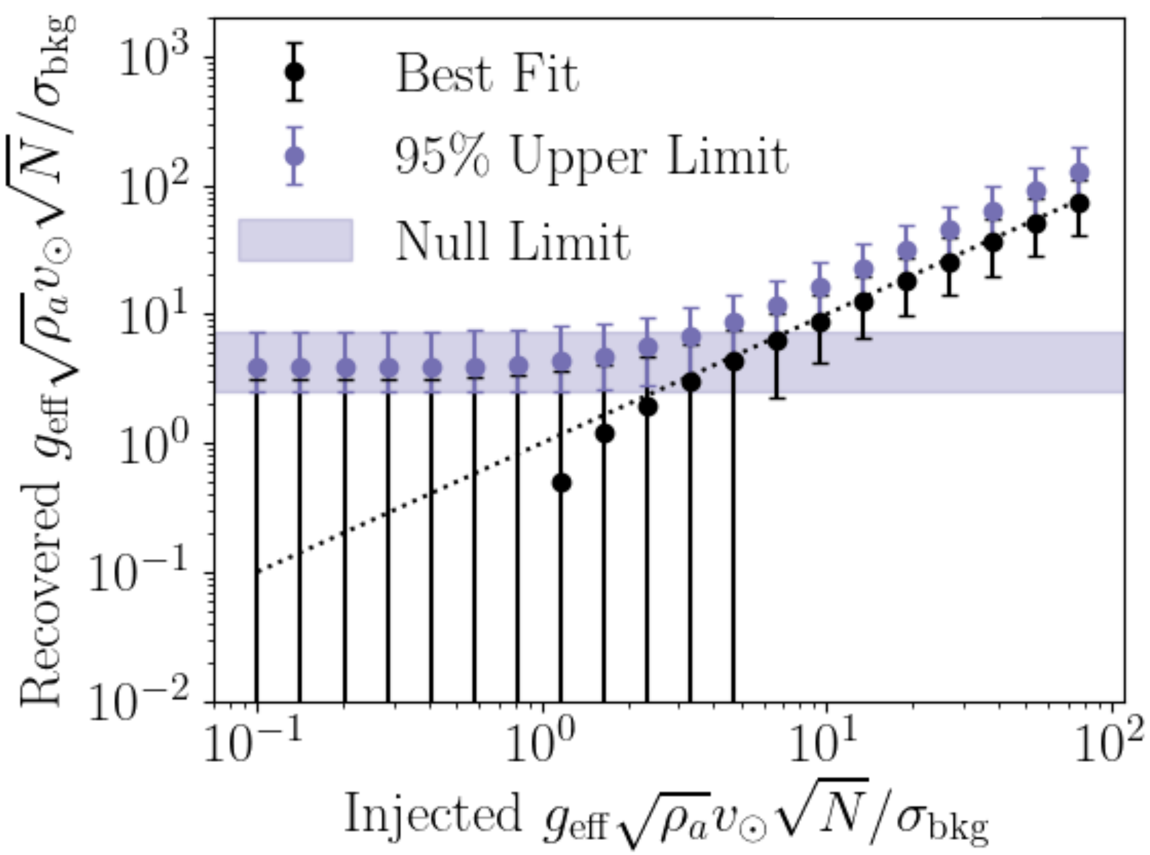
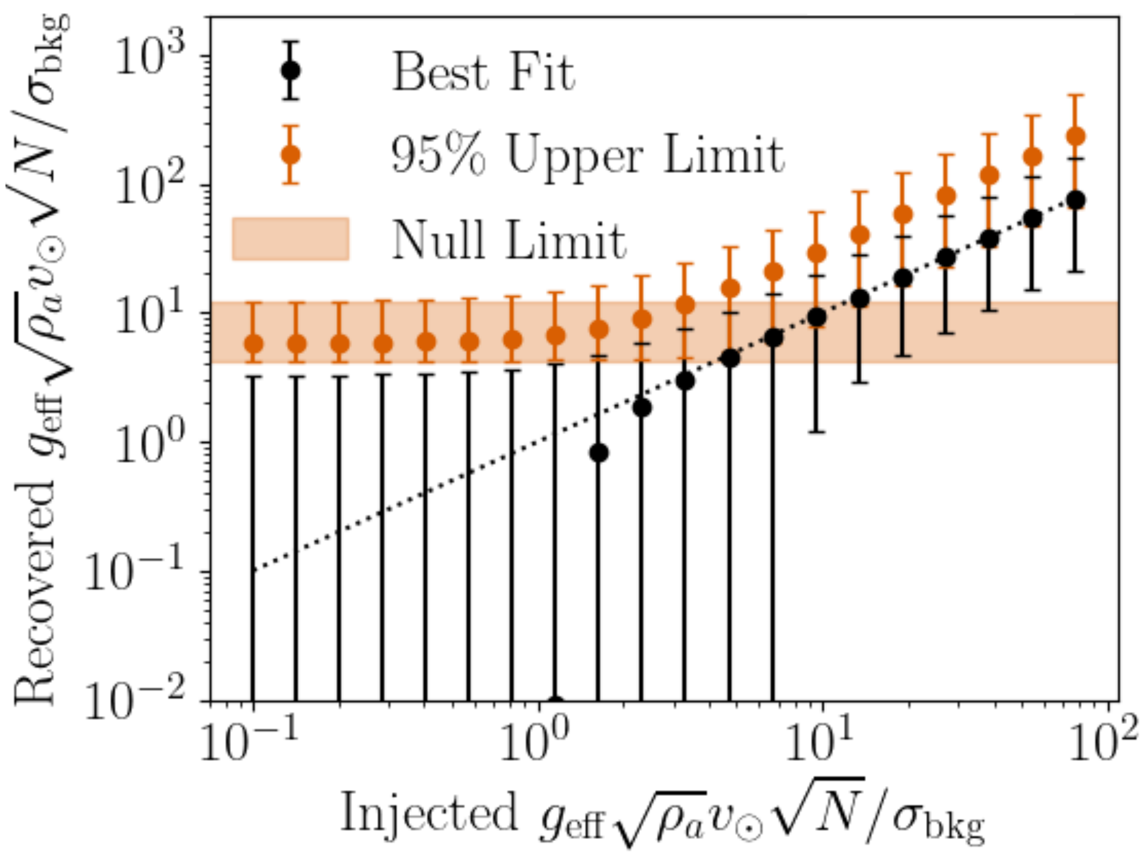
Signal Injection Tests

3D Stochastic Model



1D Stochastic Model



Assuming 1D Deterministic Model yields:

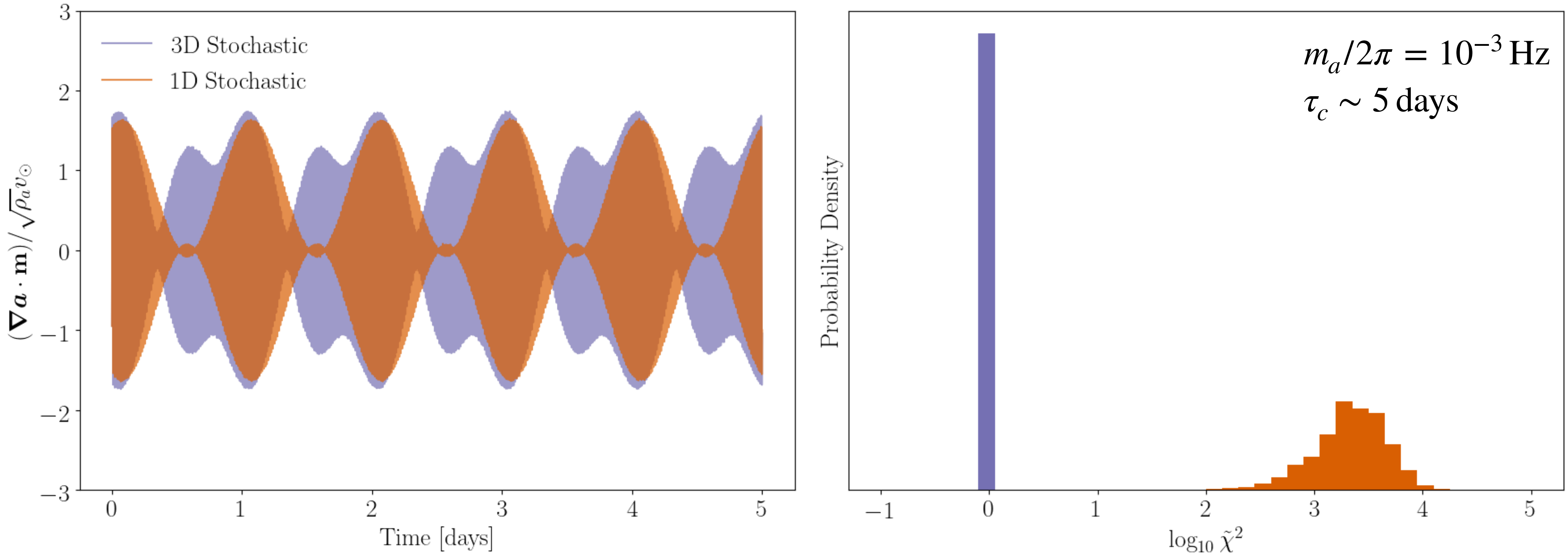
incorrect limits

greater variability in recovered signal

Incorrect Approaches

1D Stochastic Model misses signal information from x and y directions

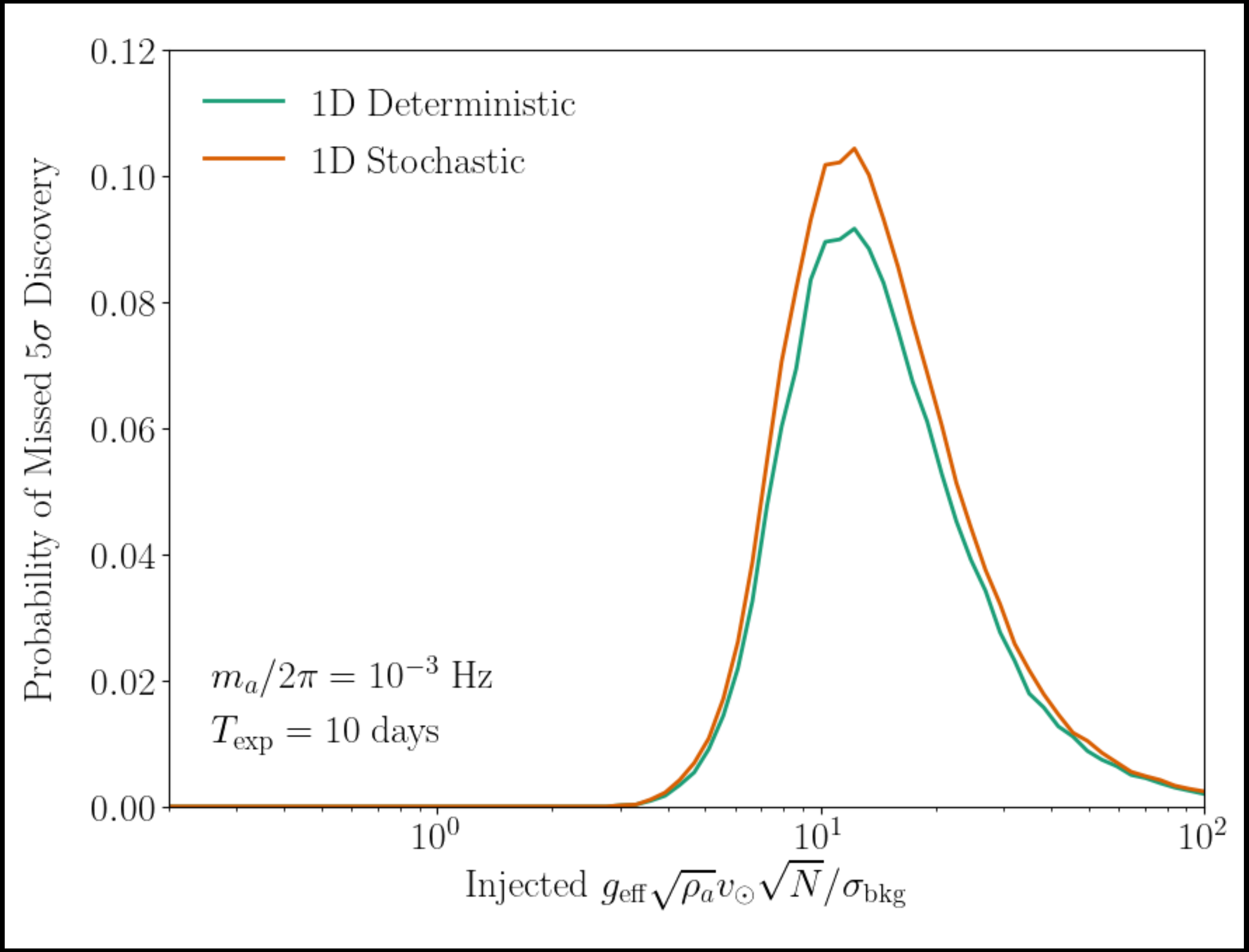
Provides poor fit to mock data generated with total axion gradient signal



Reduced Chi-Squared Test Statistic

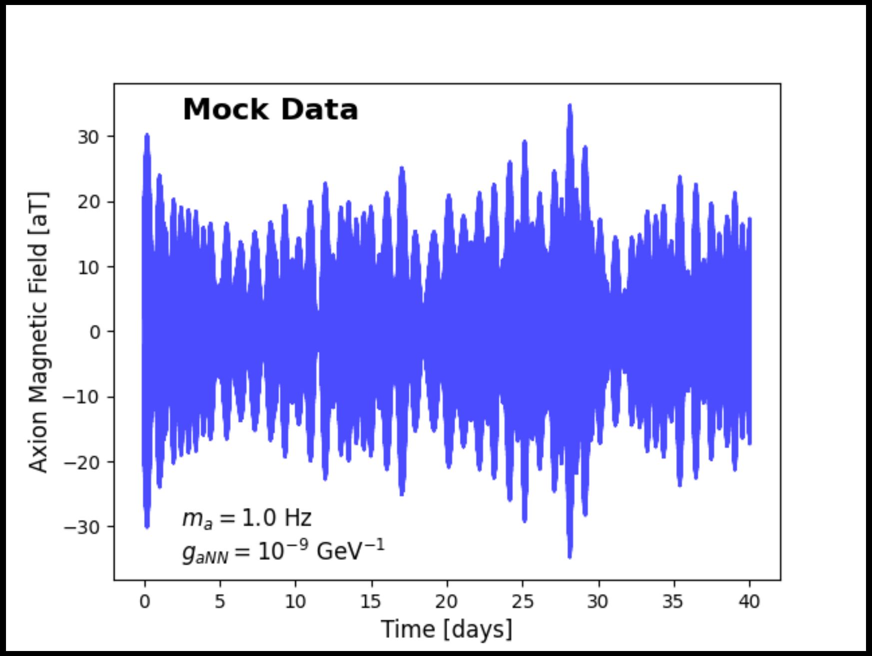
Potential Failure to Discover Signal

Improper modeling of stochastic signal can potentially result in failure to discover a true signal



All Coherence Times

For a time series of N data points $\{D(t_n)\,|\,n \in 1,\ldots,N\}$, the total likelihood is



$$\mathscr{L}(\mathbf{D}) = \frac{1}{\sqrt{(2\pi)^{6N} \det \mathbf{\Sigma}}} e^{-\frac{1}{2}\mathbf{D}^T \mathbf{\Sigma}^{-1} \mathbf{D}}$$

The combined covariance matrix is

$$\mathbf{\Sigma} = \mathbf{\Sigma}_{\rm sig} + \mathbf{\Sigma}_{\rm bkg} \qquad \text{where} \qquad \mathbf{\Sigma}_{{\rm sig},nm} = \langle S(t_n)\, S(t_m) \rangle$$

Princeton Comagnetometer Experiment

Princeton Comagnetometer Experiment

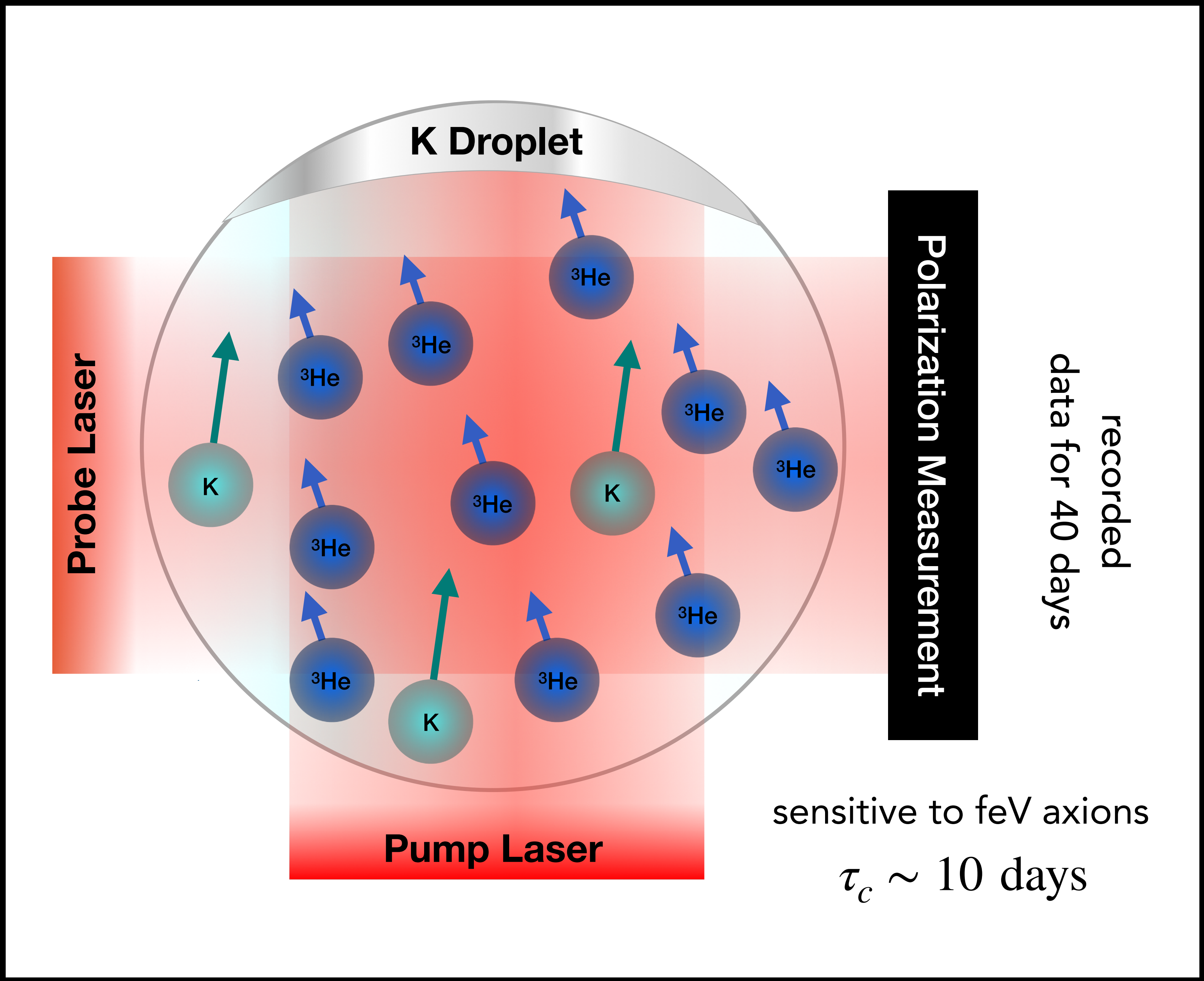
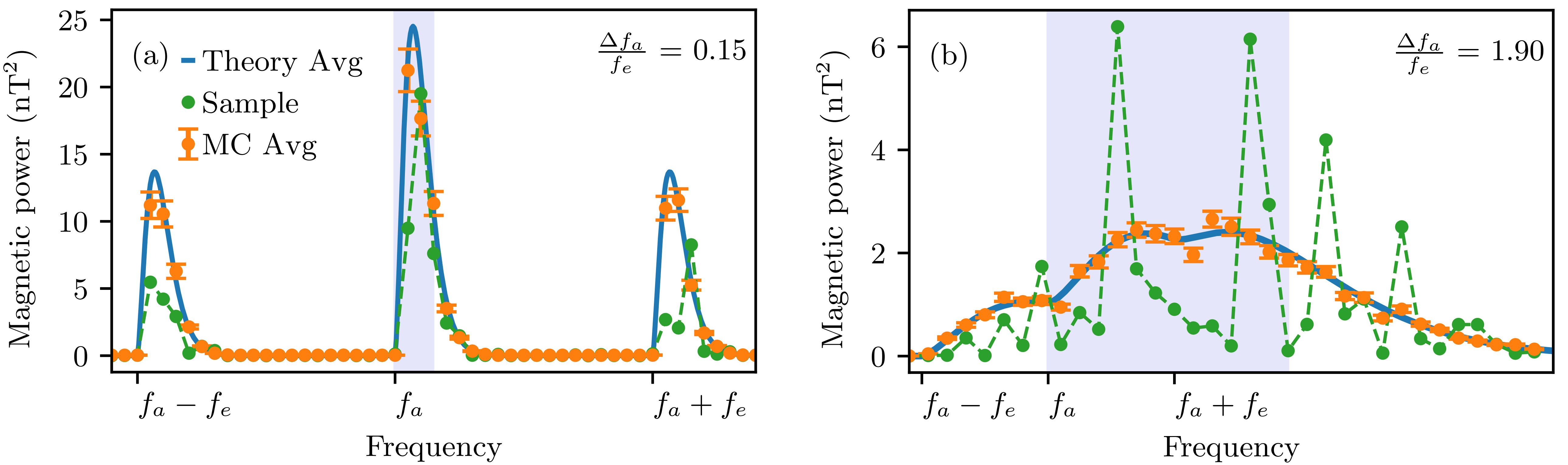


Image Credit: Bloch et al. [1907.03767]

Lee, Lisanti, Terrano, and Romalis [2209.03289]

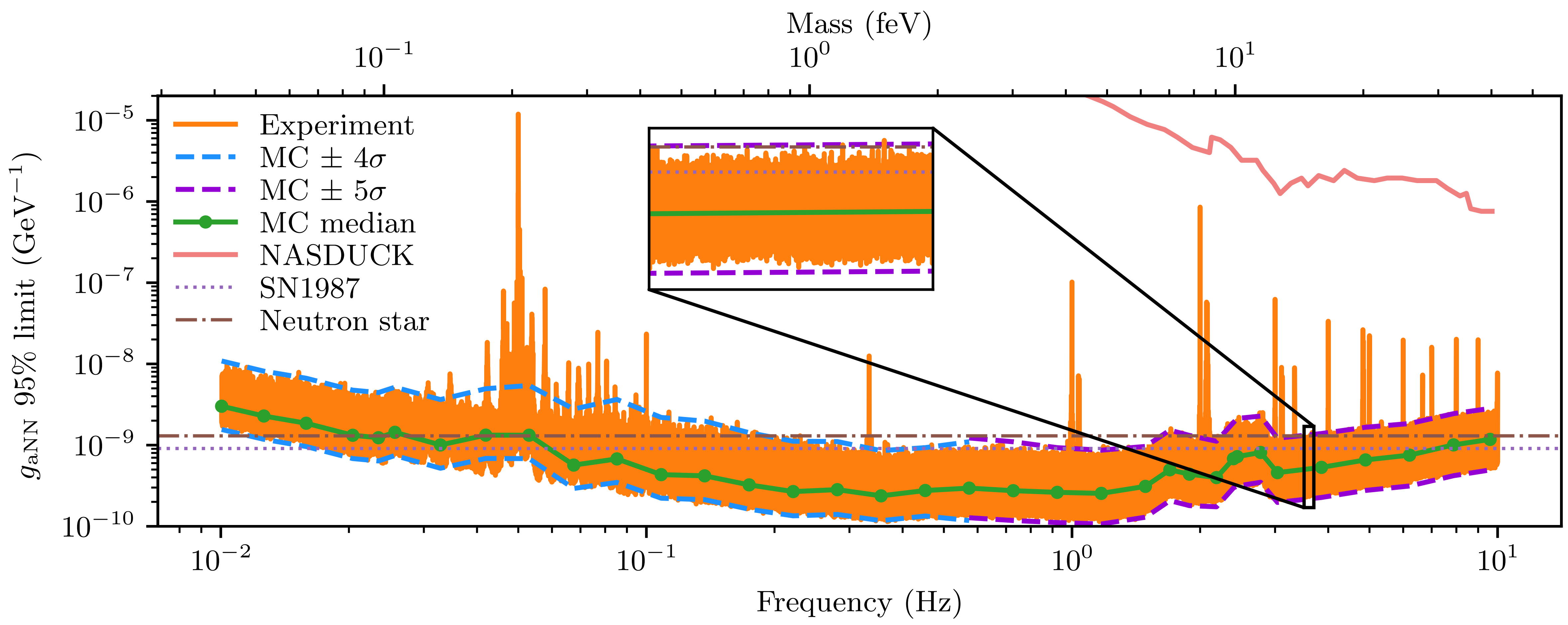
Some Mock Data Examples

Experiment is sensitive to 0.4-4 feV axions,
covering a wide range of signal shapes

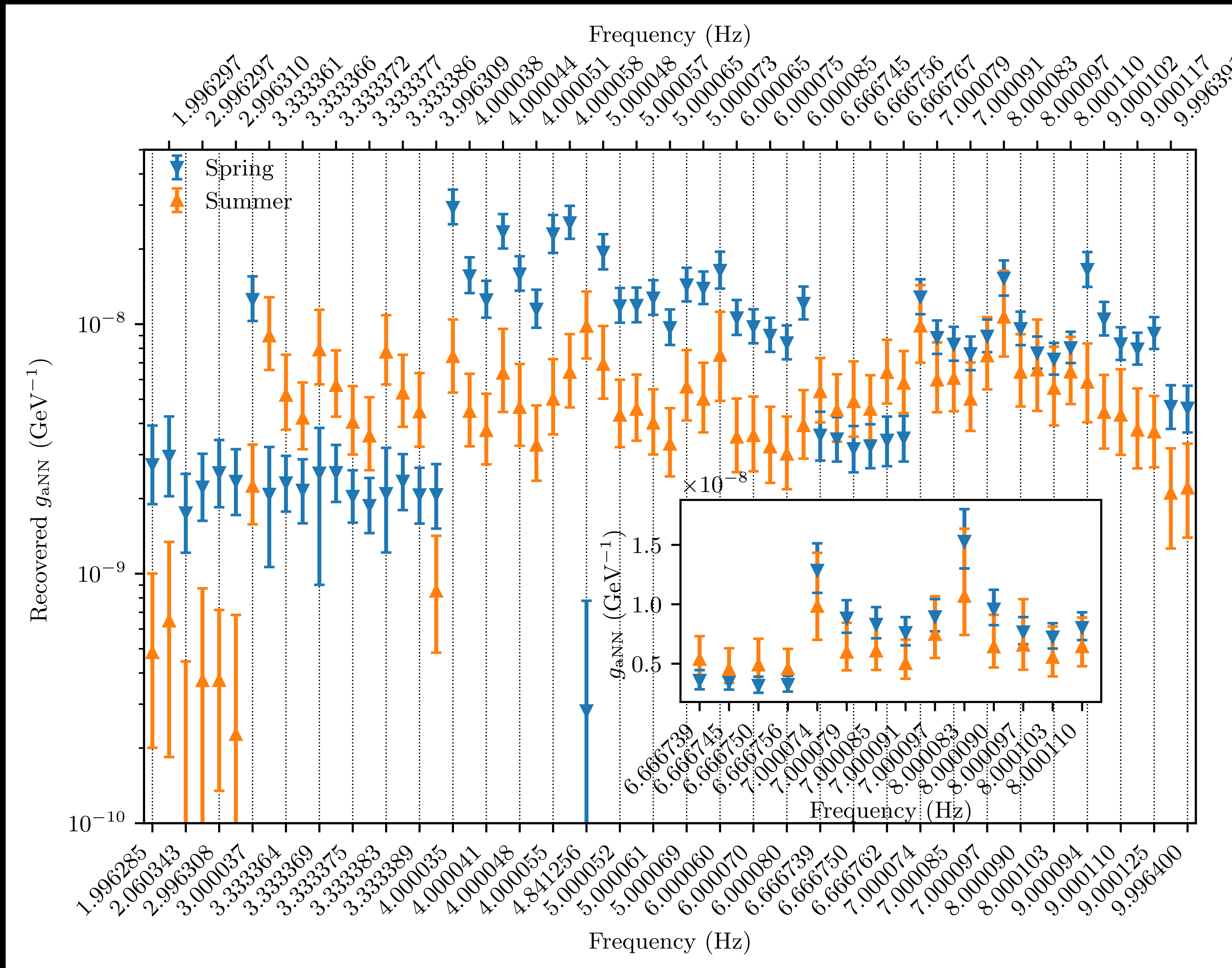


Search Limits

New limits are about 5 orders of magnitude more stringent than previous laboratory constraints in this mass range

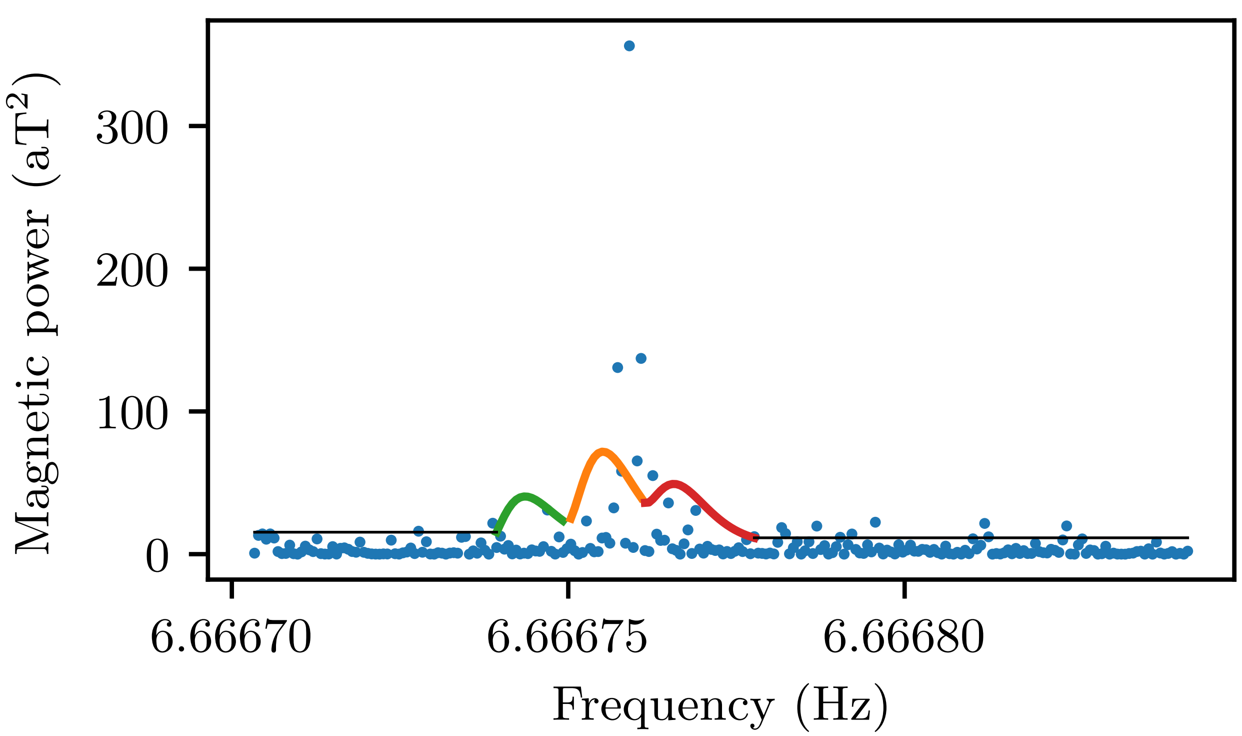


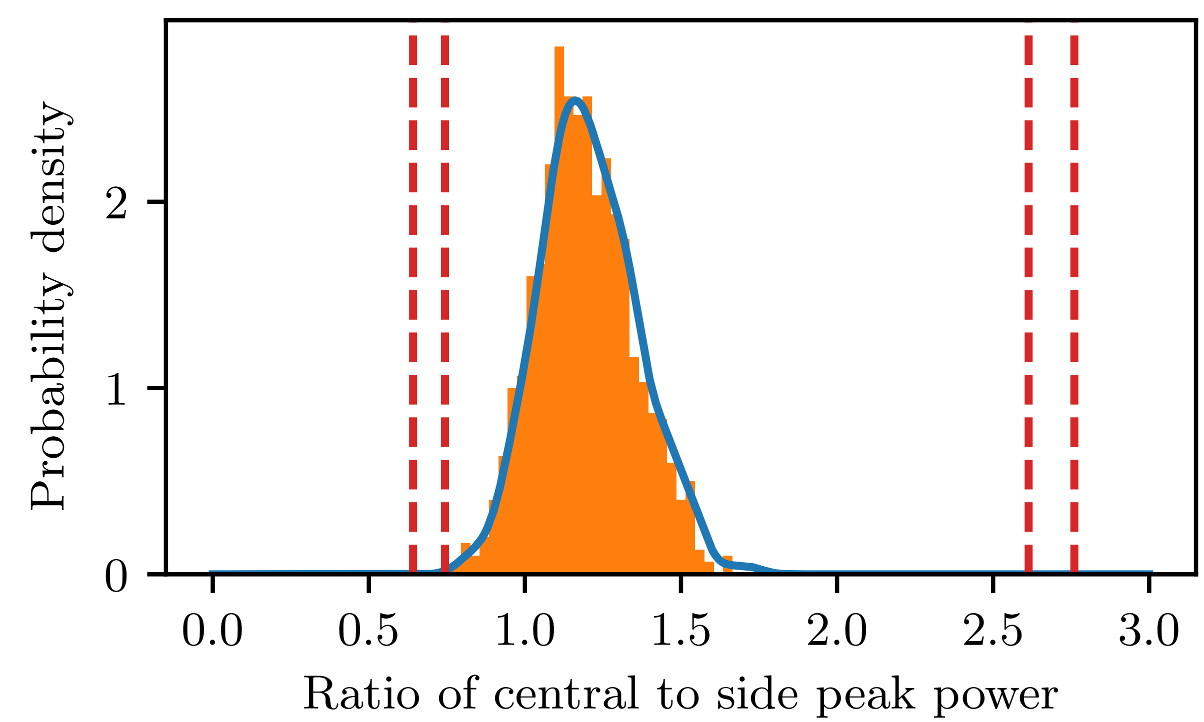
High Significance Peaks



Signal Lineshape Test

Lineshapes of high-significance peaks are not consistent with axion interpretation





Conclusions

Comagnetometers are sensitive probes for ultralight dark matter

Stochastic behavior of axions has important experimental ramifications

Re-analysis of data from Princeton Comagnetometer Experiment sets world-leading constraints on 0.4-4 feV axions