Dark Matter and Quantum Gravity

Toshifumi Noumi

(Kobe U → U Tokyo, Komaba)

March 28 2023 @ VLDM 2023

1. Introduction

very light DMs = **very light** particles with **tiny couplings** to SM particles

→ How light DMs can be? How tiny the couplings can be?

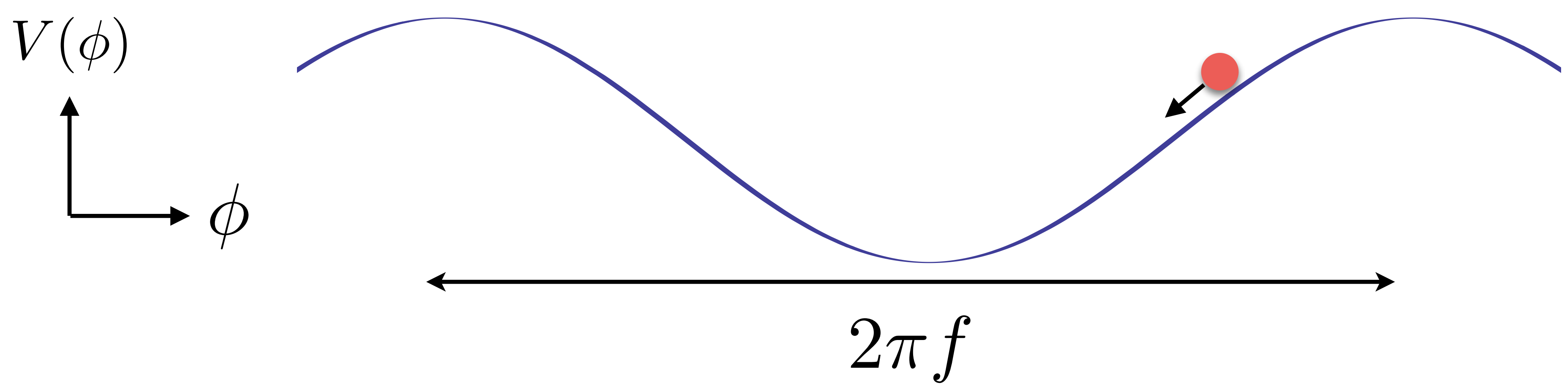
Recent studies in the Swampland Program are pointing an interesting possibility that such **lower bounds on masses & couplings** might exist in **quantum gravity**.

Quantum Gravity and Symmetries

It is widely believed that there exist **no global symmetries in quantum gravity**.

[…, Banks-Dixon '88, …, Banks-Seiberg '10, …, Ooguri-Harlow '18, …]

ex. axion potential $V(\phi) = \Lambda^4 \cos \frac{\phi}{f}$



- To make the axion very light, we need to consider **small Λ** or **large f**.

  Shift symmetry emerges in such limits → **QG obstruction to very light masses**.

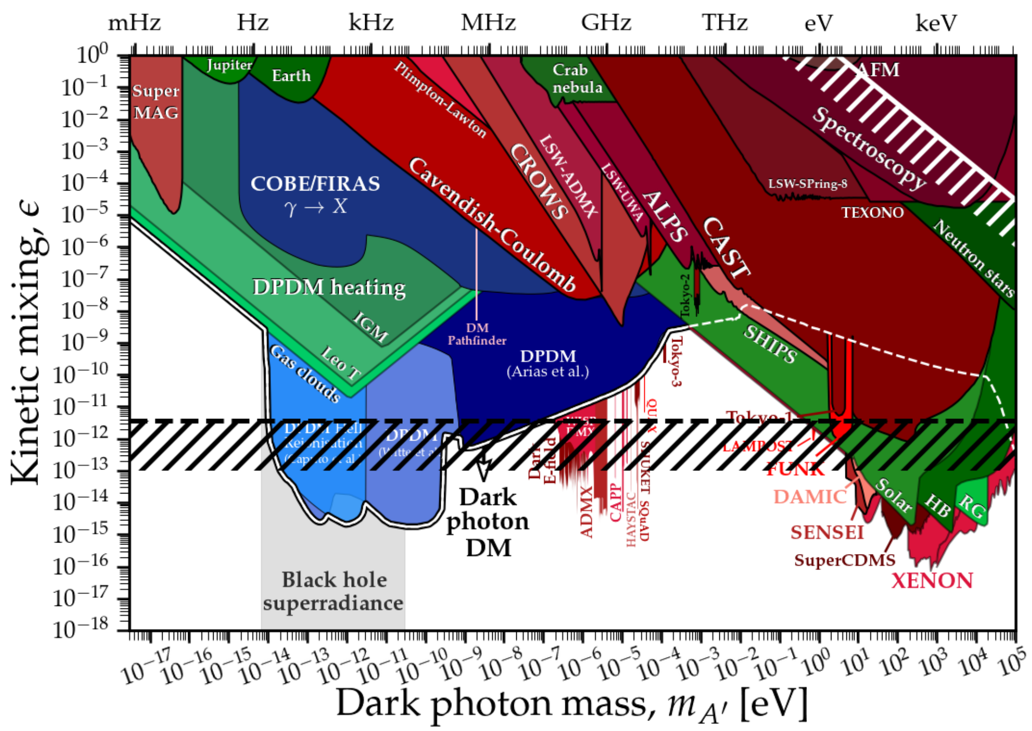
※ Fuzzy axion DM w/ $m \lesssim 10^{-21}$ eV has a conflict with Weak Gravity Conjecture.

- Large f means a tiny axion self-coupling → **QG obstruction to tiny couplings**.

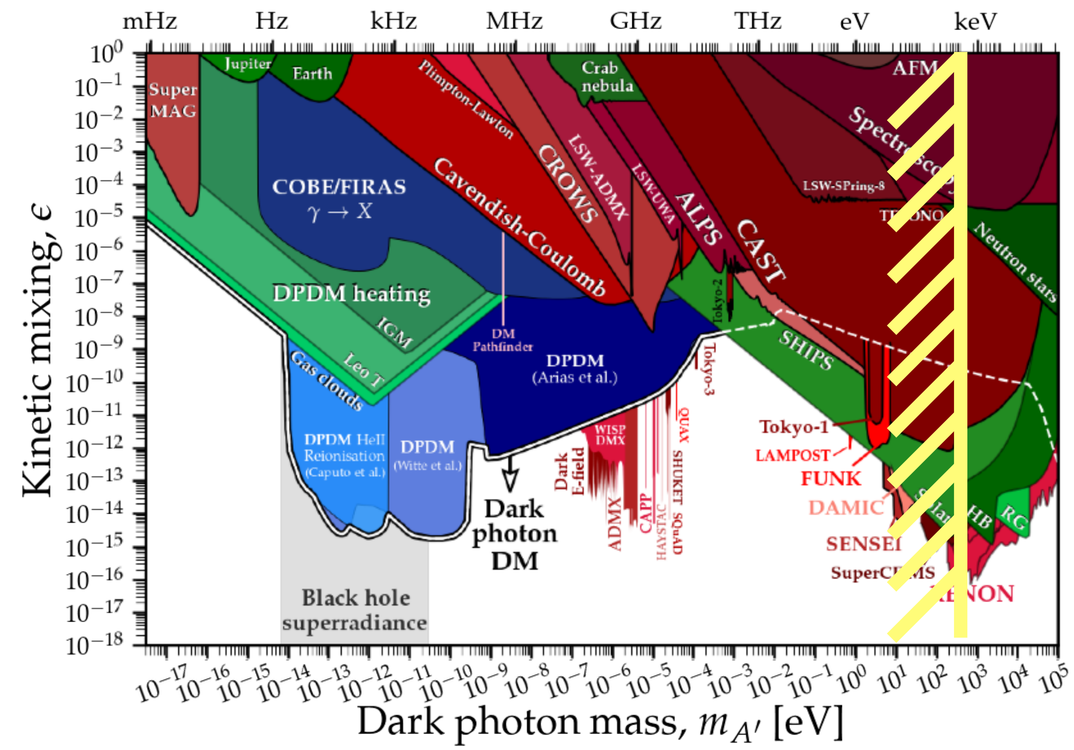
Various bounds on gauge couplings, Yukawa couplings, scalar potentials etc have been conjectured and studied in the context of the Swampland Program.

$\rightarrow$ **Can we derive QG constraints on DM masses & DM-SM couplings?**

Curve out the DM theory space by QG constraints



lower bounds on couplings to SM



lower bounds on DM masses

Such QG lower bounds, if exist, would be useful for comprehensive DM searches.

※ experiments + theories → curve out the theory space from different directions!

In this talk,

I introduce our recent attempts toward derivation of such QG constraints

based on positivity bounds that follow from consistency of gravitational scattering.

Contents

Contents

Positivity bounds provide various UV-IR consistency relations that can be used as UV constraints on IR effective field theories.

The recipe of gravitational positivity bounds

1. Compute scattering amplitudes $\mathscr{M}(s,t)$ in your model taking into account gravity.

※ The model should be considered as an IR EFT since gravity is there.

2. Perform IR expansion, e.g., as $\mathscr{M}(s,t) = (\text{graviton poles}) + \sum_{n=0}^{\infty} a_{2n} s^{2n} + \mathcal{O}(t)$.

3. Evaluate a cutoff-dependent quantity $B(\Lambda) := a_2 - \frac{2}{\pi}\int_{m_{\rm th}^2}^{\Lambda^2} ds \frac{\mathrm{Im}\mathscr{M}(s,t=0)}{s^3}$.

4. Then, $B(\Lambda) \gtrsim 0$ is required for the EFT to have a consistent UV completion.

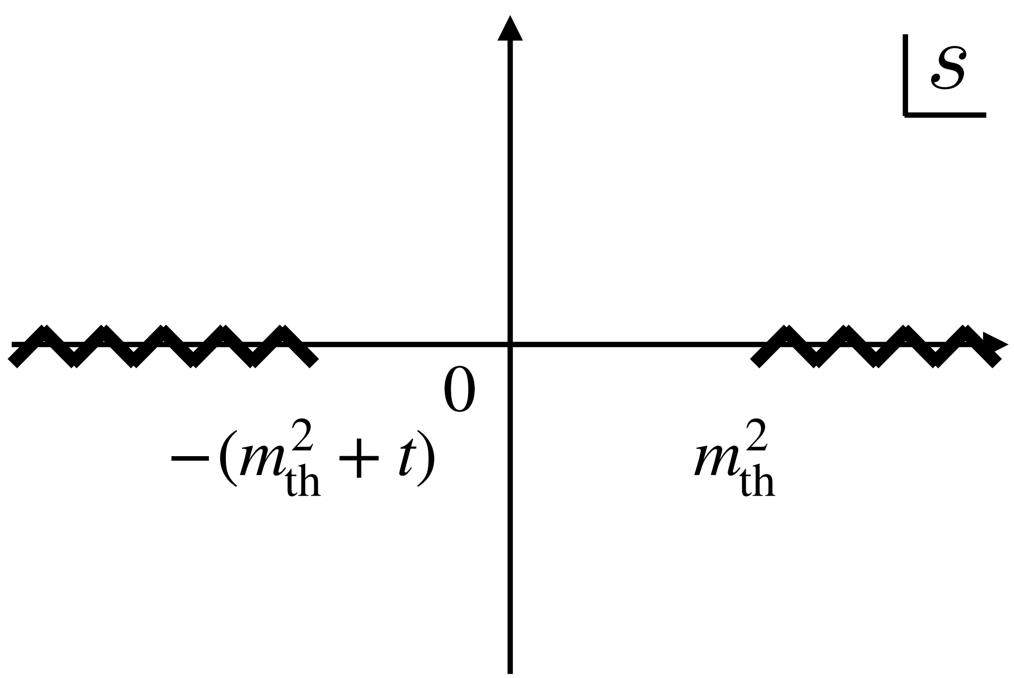
→ Quantum gravity constraints on your gravitational model!

The key idea of positivity bounds [ex. Adams et al '06]:

Analyticity of scattering amplitudes connects UV and IR.

Analyticity is the key

Consider a scattering amplitude $\mathscr{M}(s,t)$ in the forward limit $t \to -0$.

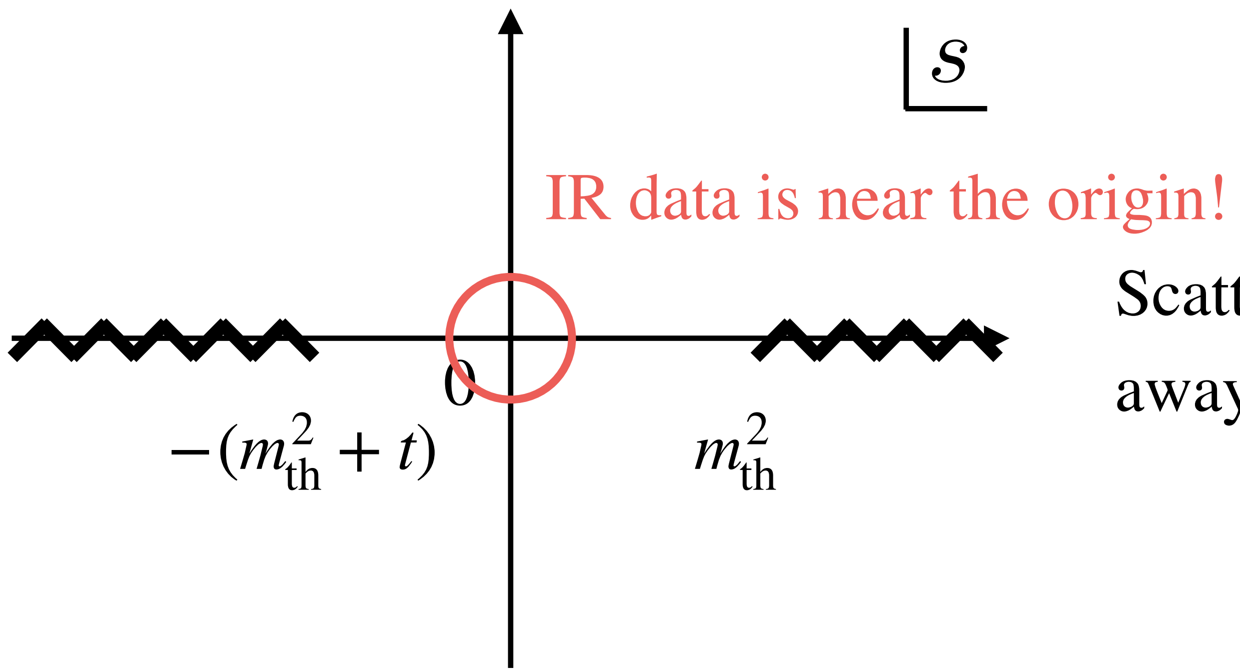


analytic structure of $\mathscr{M}(s,t)$

Scattering amplitudes are analytic away from the real axis! (cf. causality)

Analyticity is the key

Consider a scattering amplitude $\mathcal{M}(s,t)$ in the forward limit $t \to -0$.

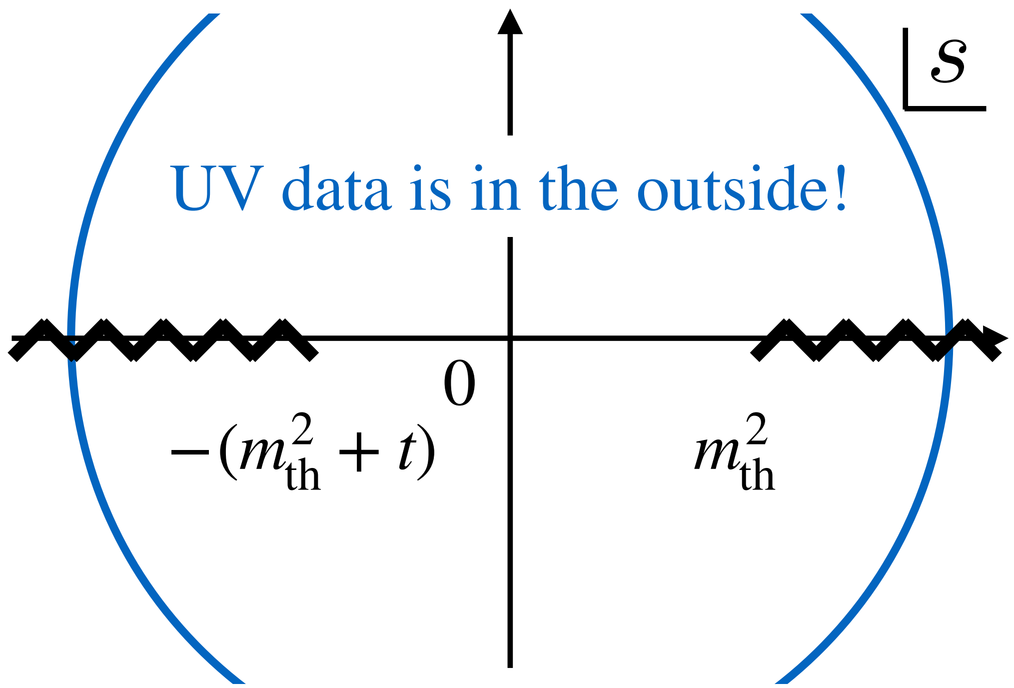


analytic structure of $\mathcal{M}(s,t)$

Scattering amplitudes are analytic away from the real axis! (cf. causality)

Analyticity is the key

Consider a scattering amplitude $\mathscr{M}(s,t)$ in the forward limit $t \to -0$.

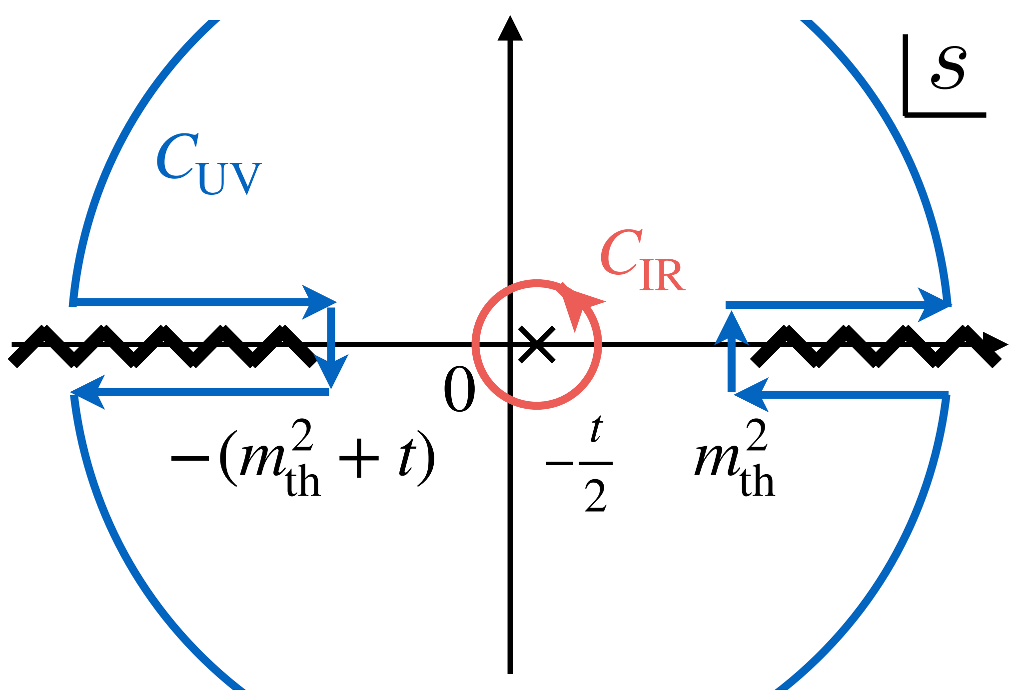


analytic structure of $\mathscr{M}(s,t)$

Scattering amplitudes are analytic away from the real axis! (cf. causality)

Analyticity is the key

Consider a scattering amplitude $\mathcal{M}(s,t)$ in the forward limit $t \to -0$.



analytic structure of $\mathcal{M}(s,t)$

Scattering amplitudes are analytic away from the real axis! (cf. causality)

By deforming the integration contour, we can connect UV and IR:

$$\text{IR data} \to \quad \oint_{C_{\rm IR}} \frac{ds}{2\pi i} \frac{\mathcal{M}(s,t)}{(s+\frac{t}{2})^3} = \oint_{C_{\rm UV}} \frac{ds}{2\pi i} \frac{\mathcal{M}(s,t)}{(s+\frac{t}{2})^3} \quad \leftarrow \text{UV data}$$

Careful analysis gives various **UV-IR relations** (dispersion relations).

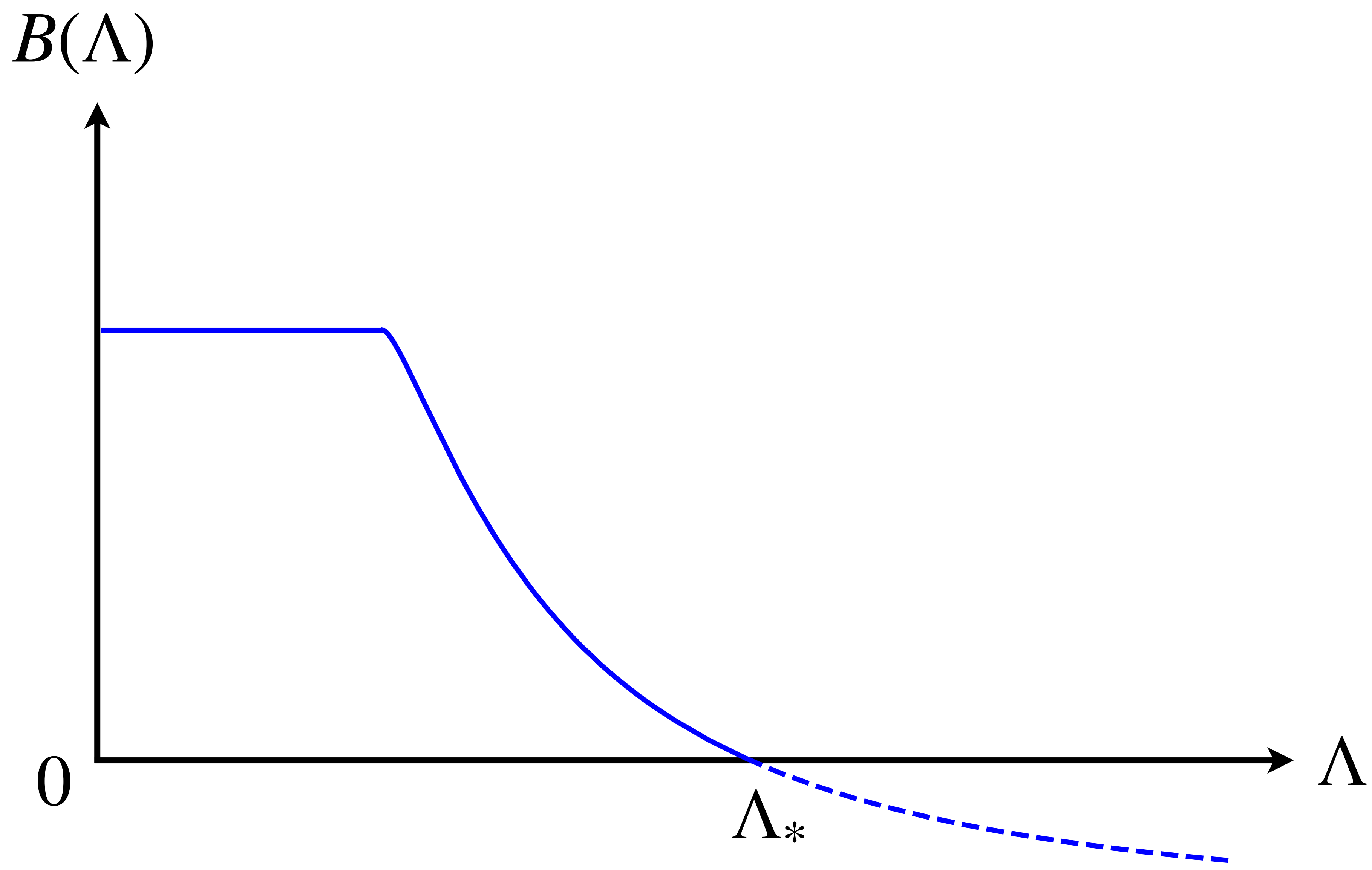
In non-gravitational theories, this gives the dispersion relation:

$$a_2 = \int_{m_{\rm th}^2}^{\infty} \frac{{\rm Im}\mathcal{M}(s,t=0)}{s^3}, \quad \mathcal{M}(s,t=0) = \sum_{n=0}^{\infty} a_{2n} s^{2n}.$$

This implies $B(\Lambda) := a_2 - \int_{m_{\rm th}^2}^{\Lambda^2} \frac{{\rm Im}\mathcal{M}(s,t=0)}{s^3} = \int_{\Lambda^2}^{\infty} \frac{{\rm Im}\mathcal{M}(s,t=0)}{s^3} \geq 0,$

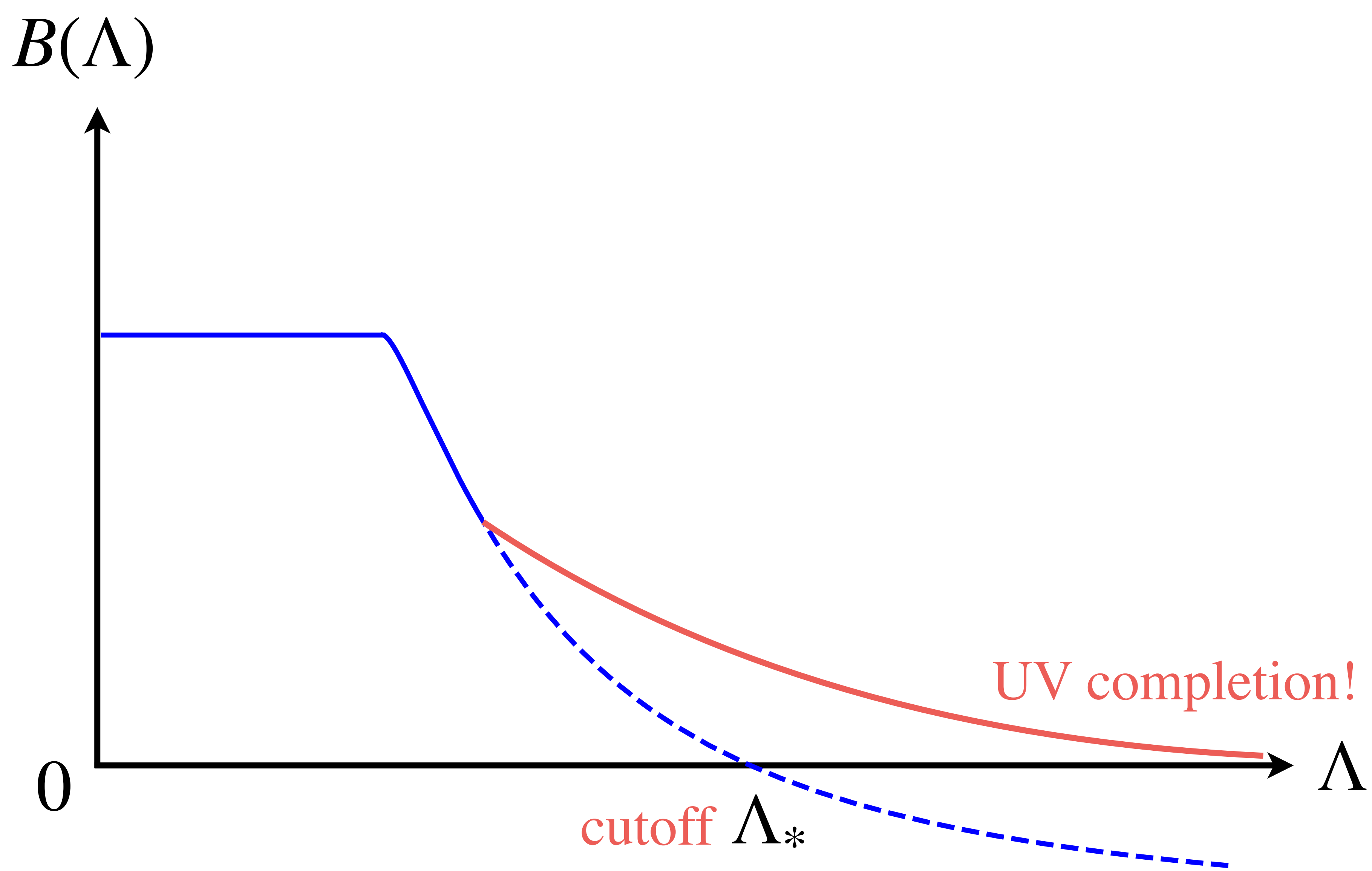
which is called the positivity bounds.

$B(\Lambda)$ in IR effective field theories



- $B(\Lambda) < 0$ for $\Lambda > \Lambda_*$, then the EFT is not trustable above Λ_*.

$B(\Lambda)$ in IR effective field theories



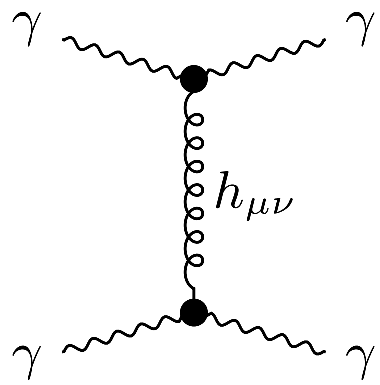
- $B(\Lambda) < 0$ for $\Lambda > \Lambda_*$, then the EFT is not trustable above Λ_*.

- The amplitude has to be modified such that $B(\Lambda) \geq 0$ for all Λ.

How the story changes in the presence of gravity?

In the presence of gravity, the IR expansion is modified as

$$\mathcal{M}(s,t) = -\frac{s^2}{M_{\rm Pl}^2 t} + \sum_{n=0}^{\infty} a_{2n} s^{2n} + \mathcal{O}(t).$$



The t-channel graviton exchange dominates in the forward limit,

so that a careful analysis is required to derive positivity bounds.

[Tokuda-Aoki-Hirano '20] performed such a careful study in gravitational EFTs.

See also Hamada-TN-Shiu '18, Herrero-Valea et al '20, Bellazzini et al'19, Alberte et al '20, Arkani-Hamed et al '20, Caron-Huot et al '21, TN-Tokuda '22.

Gravitational positivity bounds [Tokuda-Aoki-Hirano '20]

Finding 1

Consistent dispersion relations require Reggization of gravitational amplitudes:

$$\mathrm{Im}\mathscr{M}(s,t) \simeq f(t)\left(\frac{s}{M_s^2}\right)^{2+\alpha' t+\alpha'' t^2+\cdots} \quad (s > M_{\mathrm{Regge}} : \text{Reggeization scale}).$$

cf. In string theory, an infinite higher spin tower is responsible for this.

Gravitational positivity bounds [Tokuda-Aoki-Hirano '20]

Finding 1

Consistent dispersion relations require Reggization of gravitational amplitudes:

$$\mathrm{Im}\mathscr{M}(s,t) \simeq f(t)\left(\frac{s}{M_s^2}\right)^{2+\alpha' t+\alpha'' t^2+\cdots} \quad (s > M_{\mathrm{Regge}} : \text{Reggeization scale}).$$

cf. In string theory, an infinite higher spin tower is responsible for this.

Finding 2

If we perform IR expansion $\mathscr{M}(s,t) = -\dfrac{s^2}{M_{\mathrm{Pl}}^2 t} + \displaystyle\sum_{n=0}^{\infty} a_{2n} s^{2n} + \mathscr{O}(t)$,

and define a cutoff-dependent quantity $B(\Lambda) := a_2 - \dfrac{2}{\pi}\displaystyle\int_{m_{\mathrm{th}}^2}^{\Lambda^2} ds \frac{\mathrm{Im}\mathscr{M}(s,t=0)}{s^3}$,

dispersion relations imply $B(\Lambda) \geq -\dfrac{1}{M_{\mathrm{Pl}}^2}\left(\dfrac{f'(0)}{f(0)} - \dfrac{\alpha''}{\alpha'}\right) := \pm \dfrac{1}{M_{\mathrm{Pl}}^2 M^2}$.

※ M carries information of Regge amplitudes (ex. $M \sim M_s$ for tree-level string).

※ Positivity bounds w/o gravity $B(\Lambda) \geq 0$ is reproduced in the limit $M_{\mathrm{Pl}} \to \infty$.

In the following I discuss phenomenological implications of $B(\Lambda) \geq \pm \frac{1}{M_{\rm Pl}^2 M^2}$.

※ $B(\Lambda)$ is calculable in the standard Feynman rule for a given gravitational EFT.

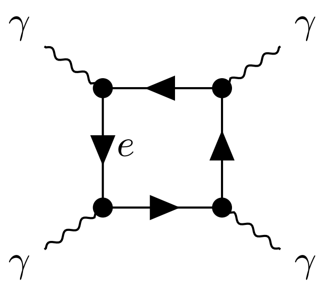
Contents

Gravitational Electroweak Theory and SM [Aoki-Loc-TN-Tokuda '21]

Light-by-light scattering in gravitational EW theory

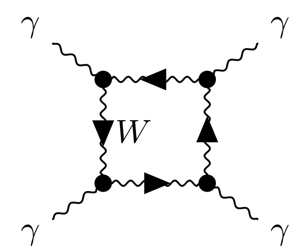
$\gamma\gamma \to \gamma\gamma$ scattering at one-loop: $\mathscr{M} = \mathscr{M}_{\text{QED}} + \mathscr{M}_{\text{weak}} + \mathscr{M}_{\text{grav}}$.

$\mathscr{M}_{\text{QED}} =$

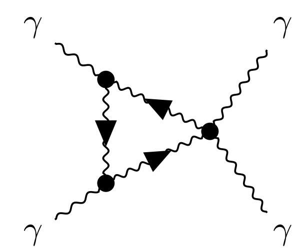


,

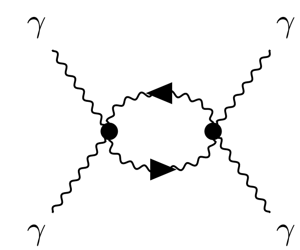
$\mathscr{M}_{\text{weak}} =$



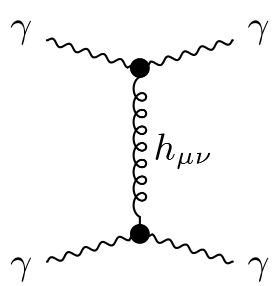
$+$



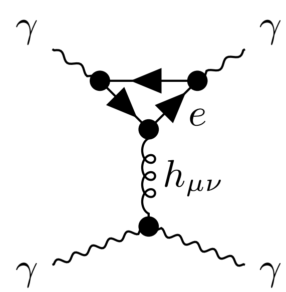
$+$



$\mathscr{M}_{\text{grav}} \simeq$



$+$



$B(\Lambda)$ from each sector: $B(\Lambda) = B_{\text{QED}}(\Lambda) + B_{\text{weak}}(\Lambda) + B_{\text{grav}}(\Lambda)$

$$B_{\text{QED}}(\Lambda) = \frac{2e^4}{\pi^2\Lambda^4}\left(\ln\frac{\Lambda}{m} - \frac{1}{4}\right), \quad B_{\text{weak}}(\Lambda) = \frac{4e^4}{\pi^2 m_W^2 \Lambda^2}, \quad B_{\text{grav}}(\Lambda) \simeq -\frac{11e^2}{180\pi^2 m_e^2 M_{\text{Pl}}^2}$$

- Non-gravitational contributions $B_{\text{non-grav}} = B_{\text{QED}} + B_{\text{weak}}$ vanish for $\Lambda \to \infty$.

- Gravitational contribution is negative!

Gravitational Positivity

Gravitational positivity $B(\Lambda) > \pm \dfrac{1}{M_{\rm Pl}^2 M^2}$ implies

$$B_{\text{non-grav}}(\Lambda) + B_{\text{grav}}(\Lambda) = \frac{4e^4}{\pi^2 m_W^2 \Lambda^2} - \frac{11e^2}{180\pi^2 m_e^2 M_{\rm Pl}^2} > \pm \frac{1}{M_{\rm Pl}^2 M^2}.$$

Gravitational Positivity

Gravitational positivity $B(\Lambda) > \pm \frac{1}{M_{\rm Pl}^2 M^2}$ implies

$$B_{\text{non-grav}}(\Lambda) + B_{\text{grav}}(\Lambda) = \frac{4e^4}{\pi^2 m_W^2 \Lambda^2} - \frac{11e^2}{180\pi^2 m_e^2 M_{\rm Pl}^2} > \pm \frac{1}{M_{\rm Pl}^2 M^2}.$$

Consider the following two cases:

1) $M \gg m_e$

RHS is negligible, so that a nontrivial bound appears:

$$B_{\text{weak}}(\Lambda) > -B_{\text{grav}}(\Lambda) \quad \rightleftarrows \quad \frac{m_W}{M_{\rm Pl}} < \sqrt{\frac{720}{11}}\, e\, \frac{m_e}{\Lambda} \quad \rightleftarrows \quad \Lambda < \sqrt{\frac{1440}{11}} y_e \sin\theta_W M_{\rm Pl}$$

- Explains the hierarchy between the EW scale and the Planck scale??

- A WGC type bound on the Yukawa coupling and the Weinberg angle.

- A similar analysis for SM implies $\Lambda \lesssim 10^{16}$ GeV (grand unification??)

Gravitational Positivity

Gravitational positivity $B(\Lambda) > \pm \dfrac{1}{M_{\rm Pl}^2 M^2}$ implies

$$B_{\text{non-grav}}(\Lambda) + B_{\text{grav}}(\Lambda) = \frac{4e^4}{\pi^2 m_W^2 \Lambda^2} - \frac{11e^2}{180\pi^2 m_e^2 M_{\rm Pl}^2} > \pm \frac{1}{M_{\rm Pl}^2 M^2}.$$

Consider the following two cases:

1) $M \gg m_e$

RHS is negligible, so that a nontrivial bound appears:

$$B_{\text{weak}}(\Lambda) > -B_{\text{grav}}(\Lambda) \;\rightleftarrows\; \frac{m_W}{M_{\rm Pl}} < \sqrt{\frac{720}{11}}\, e\, \frac{m_e}{\Lambda} \;\rightleftarrows\; \Lambda < \sqrt{\frac{1440}{11}} y_e \sin\theta_W M_{\rm Pl}$$

- Explains the hierarchy between the EW scale and the Planck scale??

- A WGC type bound on the Yukawa coupling and the Weinberg angle.

- A similar analysis for SM implies $\Lambda \lesssim 10^{16}$ GeV (grand unification??)

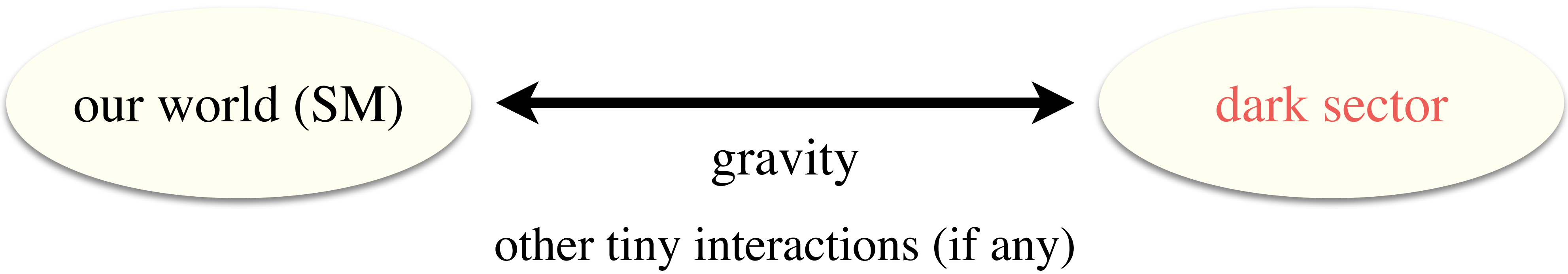
2) If it is violated, negative sign and $M \lesssim m_e$ are required on RHS

※ This means that Regge amplitudes highly depend on IR physics,

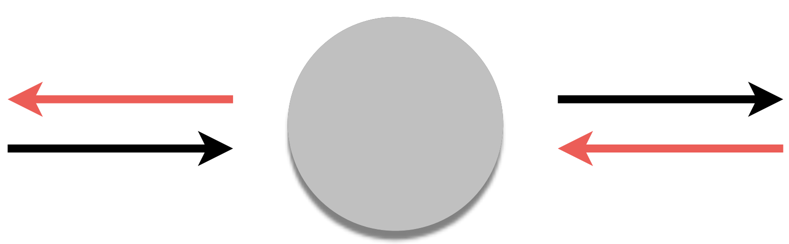
which seems nontrivial ($M \sim M_{\text{string}} \gg m_e$ in tree-level string).

Implications for dark sector physics [Sato-TN-Tokuda ’22]

Dark sector cannot be completely dark?



- Consider scattering of SM particles and dark sector particles:

 $= \mathscr{M}_{\text{non-grav}} + \mathscr{M}_{\text{grav}}$

- Positivity implies $B_{\text{non-grav}}(\Lambda) > -B_{\text{grav}}(\Lambda) \pm \dfrac{1}{M_{\text{Pl}}^2 M^2}$

※ To our knowledge, $B_{\text{grav}}(\Lambda) < 0$ is quite generic.

- Under the assumption “$M \gg m_e$,” we have $B_{\text{non-grav}}(\Lambda) > -B_{\text{grav}}(\Lambda)$.

$\rightarrow B_{\text{non-grav}}(\Lambda)$ cannot be too small: dark sector cannot be completely dark?

Dark photon models

In [TN-Sato-Tokuda '22],

we performed a concrete analysis in dark photon models as an illustrative example.

The value of $B(\Lambda)$ and therefore implications of gravitational positivity bounds depend on details of dark photon scenarios.

In our previous paper, we focused on the Stuckelberg case and considered

A) SM-DM interactions are only through dark photon-photon kinetic mixing

B) There exists a spin 1 particle charged under both U(1)'s.

Scenario A: kinetic mixing only

Lagrangian after diagonalization of kinetic terms:

$$\mathscr{L} \simeq \frac{M_{\rm Pl}^2}{2} R + \mathscr{L}_{\rm SM} - \frac{1}{4} F_X^2 - \frac{1}{2} m_X^2 X^2 + \epsilon\, e\, X^\mu J_\mu^{\rm EM}.$$

1. $\gamma X_T \to \gamma X_T$ (transverse modes)

$$B_{\rm others}(\Lambda) > -B_{\rm GR}(\Lambda) \quad \rightleftarrows \quad \frac{2e^4\epsilon^2}{\pi^2 m_W^2 \Lambda^2} > \frac{11e^2}{720\pi^2 m_e^2 M_{\rm Pl}^2}$$

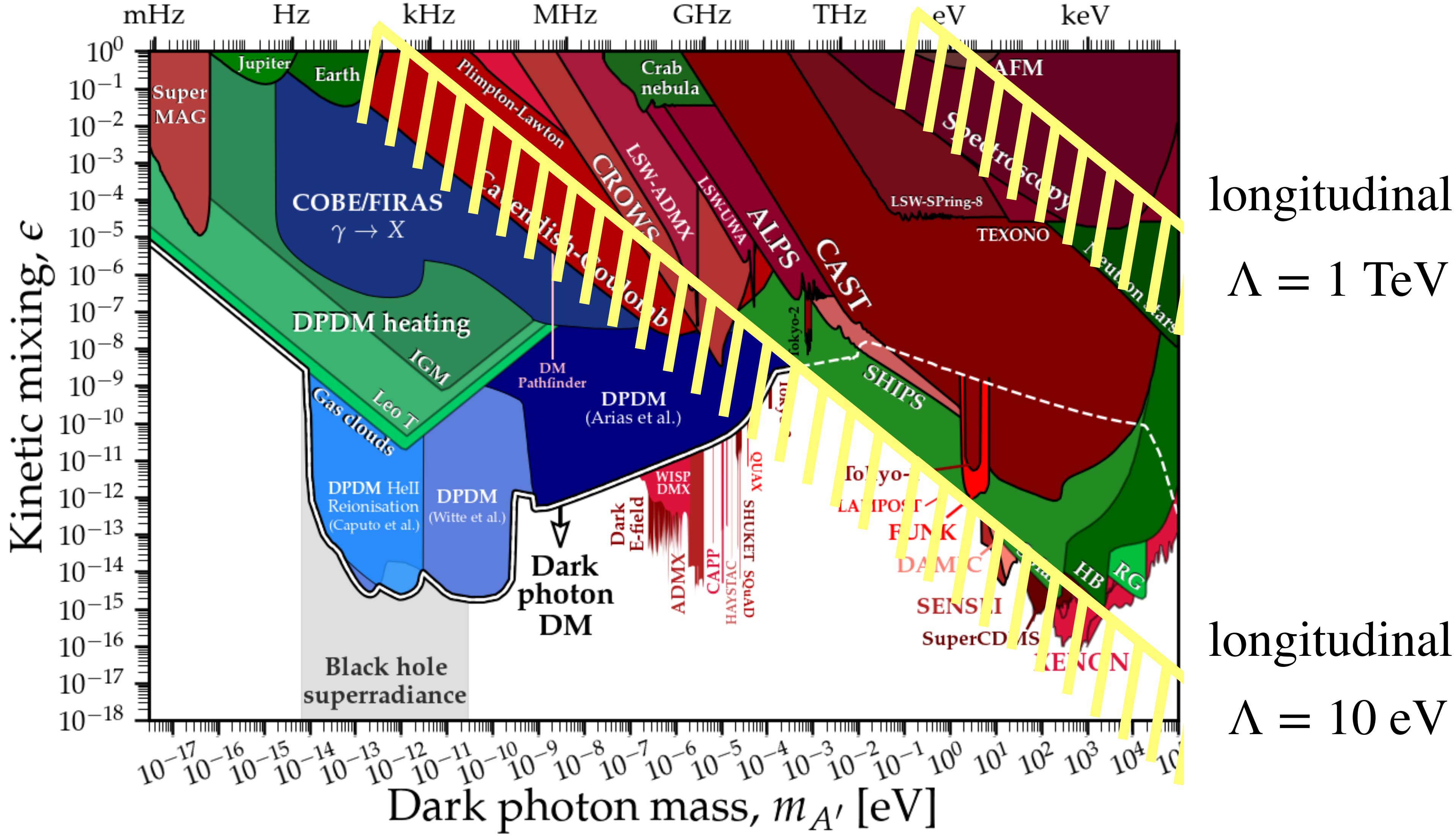
$$\rightleftarrows \quad \epsilon > \sqrt{\frac{11}{1440e^2}} \frac{m_W \Lambda}{m_e M_{\rm Pl}} = 1.9 \times 10^{-11} \frac{\Lambda}{1\,{\rm TeV}}.$$

2. $\gamma X_T \to \gamma X_T$ (longitudinal modes)

$$B_{\rm others}(\Lambda) > -B_{\rm GR}(\Lambda) \quad \rightleftarrows \quad \frac{e^4\epsilon^2 m_X^2}{2\pi^2 m_W^4 \Lambda^2} > \frac{11e^2}{720\pi^2 m_e^2 M_{\rm Pl}^2}$$

$$\rightleftarrows \quad \epsilon > \sqrt{\frac{11}{360e^2}} \frac{m_W^2 \Lambda}{m_e m_X M_{\rm Pl}} = 3.0 \frac{\Lambda}{1\,{\rm TeV}} \frac{1\,{\rm eV}}{M_X}.$$

Scenario A: kinetic mixing only



If we make the assumptions "$M \gg m_e$" discussed in the SM analysis, other SM-DM interactions are needed to save the above plotted region.

※ In the heavier regime, there is a target of collider experiments.

Scenario B: bi-charged spin 1 particle

Suppose that there exists a bi-charged massive vector boson V.

Consider the longitudinal scattering $\gamma X_L \to \gamma X_L$ ($\tilde{e}$: dark photon gauge coupling)

$$B_{\text{others}}(\Lambda) > -B_{\text{GR}}(\Lambda) \rightleftarrows \frac{e^2\tilde{e}^2 m_X^2}{2\pi^2 m_V^4 \Lambda^2} > \frac{11e^2}{720\pi^2 m_e^2 M_{\text{Pl}}^2}$$

$$\rightleftarrows m_V < (m_V^2\Lambda)^{1/3} < 1.3\ \text{TeV}\left(\frac{\tilde{e}}{e}\right)^{1/3}\left(\frac{m_X}{10^3\ \text{eV}}\right)^{1/3}.$$

※ dark photon mass cannot be too small, since the vector boson V is coupled to photon.

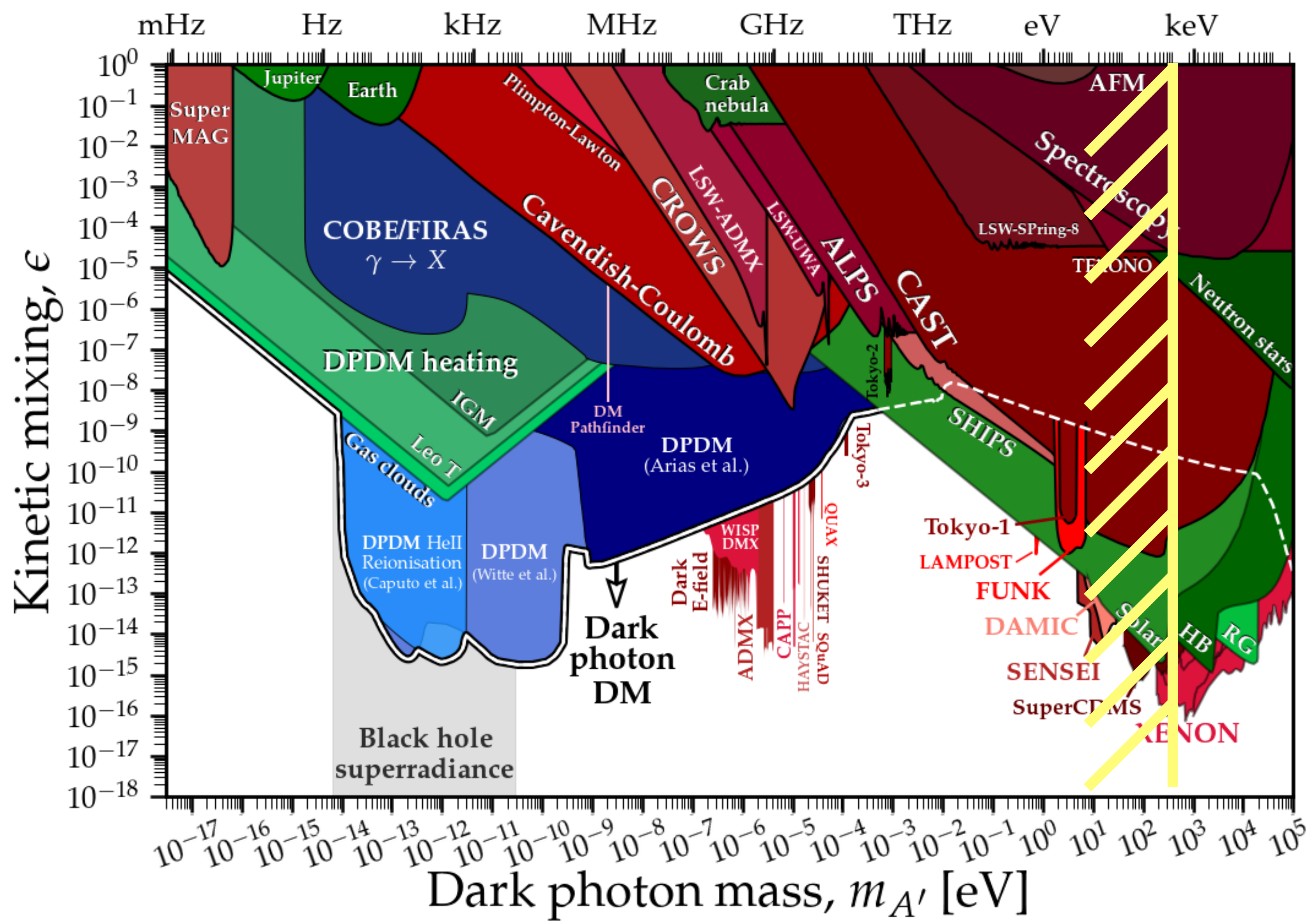
※ if V were spin 0 or spin 1/2, the situation becomes worse.

※ fine-tuning is generically needed to keep the kinetic mixing tiny.

We can also think of it as a lower bound on the dark photon mass:

$$B_{\text{others}}(\Lambda) > -B_{\text{GR}}(\Lambda) \rightleftarrows m_X > 4.7 \times 10^2\ \text{eV} \times \frac{e}{\tilde{e}}\left(\frac{M_V}{1\ \text{TeV}}\right)^2 \frac{\Lambda}{1\ \text{TeV}}.$$

Scenario B ($M_V = \Lambda = 1\text{TeV},\ \tilde{e} = e$)



Lesson 1: The bounds depend on details of the dark photon model.

Lesson 2: Very light & very tiny is generically in conflict w/the bounds.

Summary and prospects

Summary

The recipe of gravitational positivity bounds

1. Compute scattering amplitudes $\mathscr{M}(s,t)$ in your model taking into account gravity.

※ The model should be considered as an IR EFT since gravity is there.

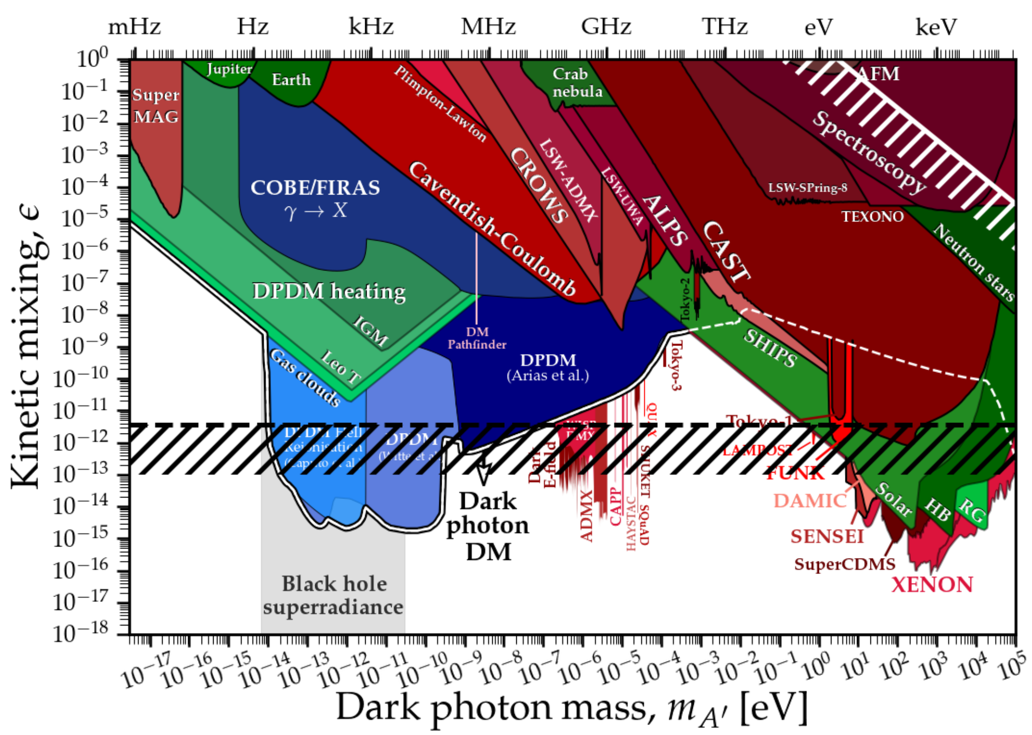
2. Perform IR expansion, e.g., as $\mathscr{M}(s,t) = (\text{graviton poles}) + \sum_{n=0}^{\infty} a_{2n} s^{2n} + \mathcal{O}(t)$.

3. Evaluate a cutoff-dependent quantity $B(\Lambda) := a_2 - \frac{2}{\pi}\int_{m_{\text{th}}^2}^{\Lambda^2} ds \frac{\text{Im}\mathscr{M}(s,t=0)}{s^3}$.

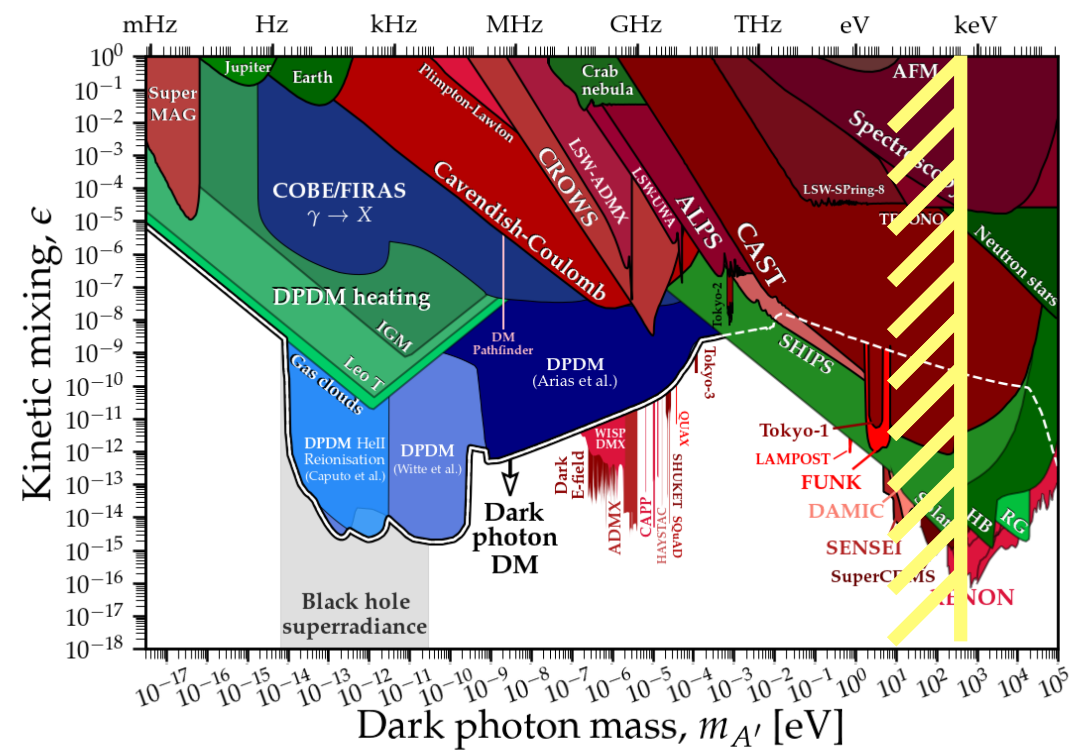
4. Then, $B(\Lambda) = \pm \frac{1}{M_{\text{Pl}}^2 M^2}$ is required for the EFT to have a consistent UV completion.

→ Quantum gravity constraints on your gravitational model!

Summary



dark photon model A



dark photon model B

Under the assumptions “$M \gg m_e$” made in the SM analysis,
we showed that unitarity of gravitational scattering can be useful
to curved out the DM theory space from a complementary direction.
Interesting interplay between theory, pheno, and experiments!

Prospects

1) comprehensive unitarity analysis of DM models coupled to gravity

- dark photon w/Higgs [Aoki-TN-Tokuda-Saito-Sato-Shirai-Yamazaki to appear]
- B-L gauge boson (implications for neutrino masses?)
- axion-photon coupling, …

※ There are several theoretical works necessary for such generalizations

2) theoretical studies on gravitational S-matrix bootstrap

- How generic is the assumption “$M \gg m_e$” is?
- Implications to/from string compactification.
- Positivity bounds w/unstable external particles, …

Prospects

1) comprehensive unitarity analysis of DM models coupled to gravity

- dark photon w/Higgs [Aoki-TN-Tokuda-Saito-Sato-Shirai-Yamazaki to appear]

- B-L gauge boson (implications for neutrino masses?)

- axion-photon coupling, …

※ There are several theoretical works necessary for such generalizations

2) theoretical studies on gravitational S-matrix bootstrap

- How generic is the assumption "$M \gg m_e$" is?

- Implications to/from explicit string compactification.

- Positivity bounds w/unstable external particles, …

Thank you!