Aspects of Light Scalar Dark Matter

Mark Hertzberg, Tufts

“What is dark matter”, IPMU, March 7 2023

Part 1: Aspects of Superfluid Dark Matter

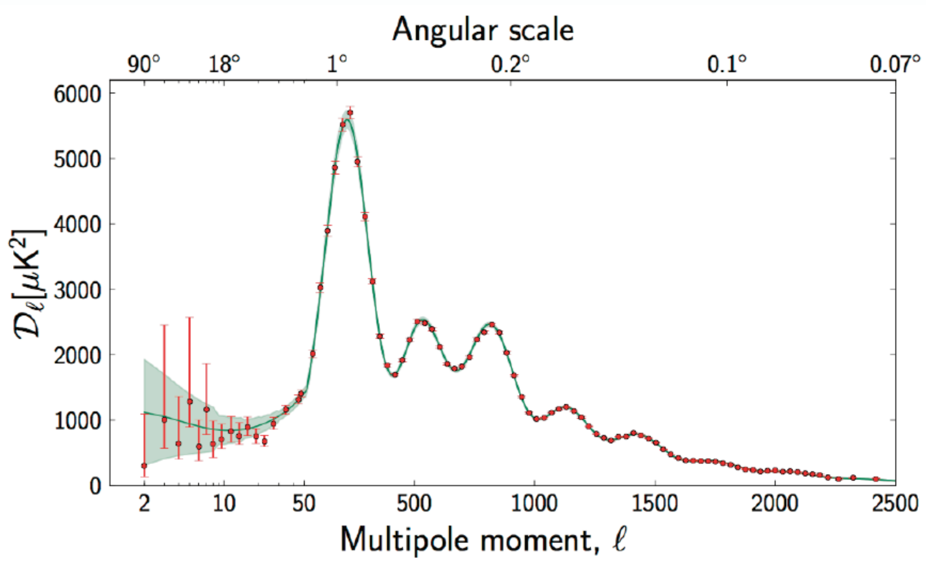
Based on work with:

Jacob Litterer

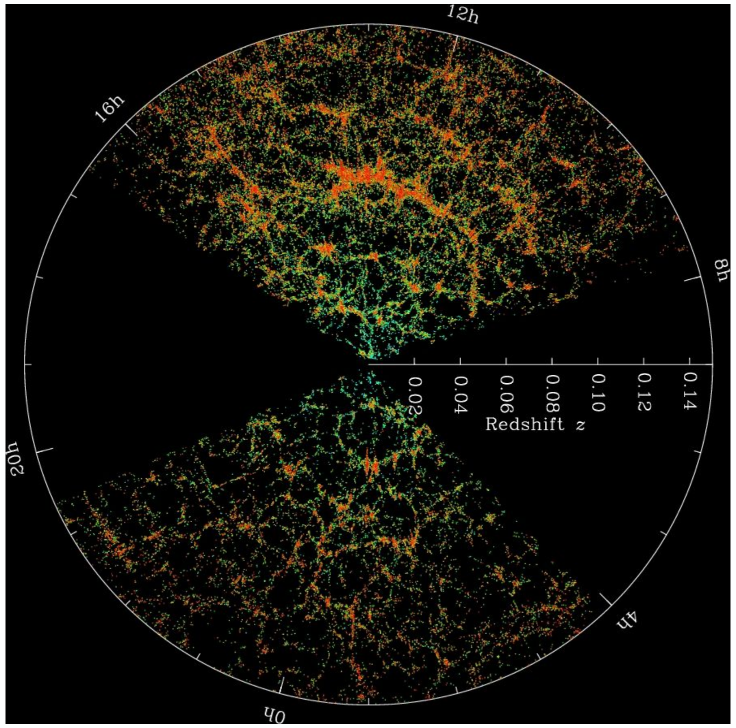
Neil Shah

M. Hertzberg, J. Litterer, N. Shah, JCAP 11 (2021) 2105.02241

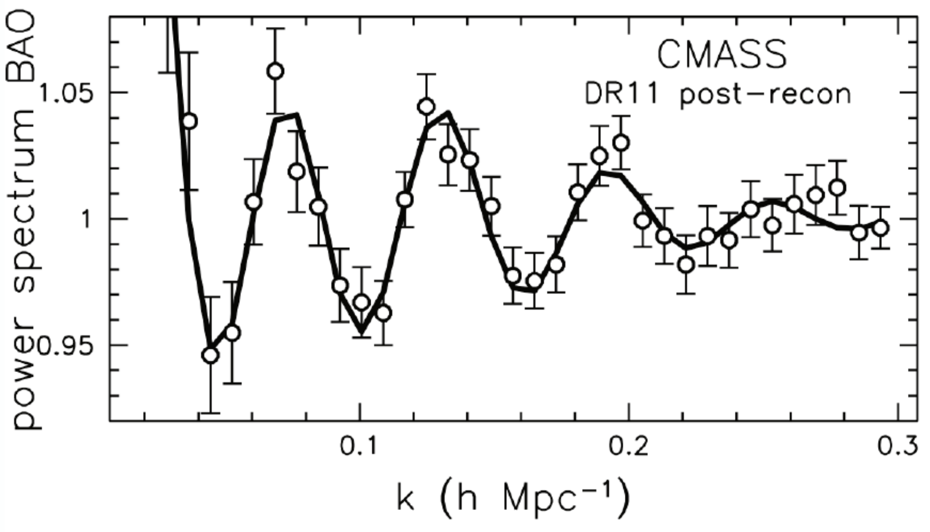
Tremendous Success of CDM on Large Scales



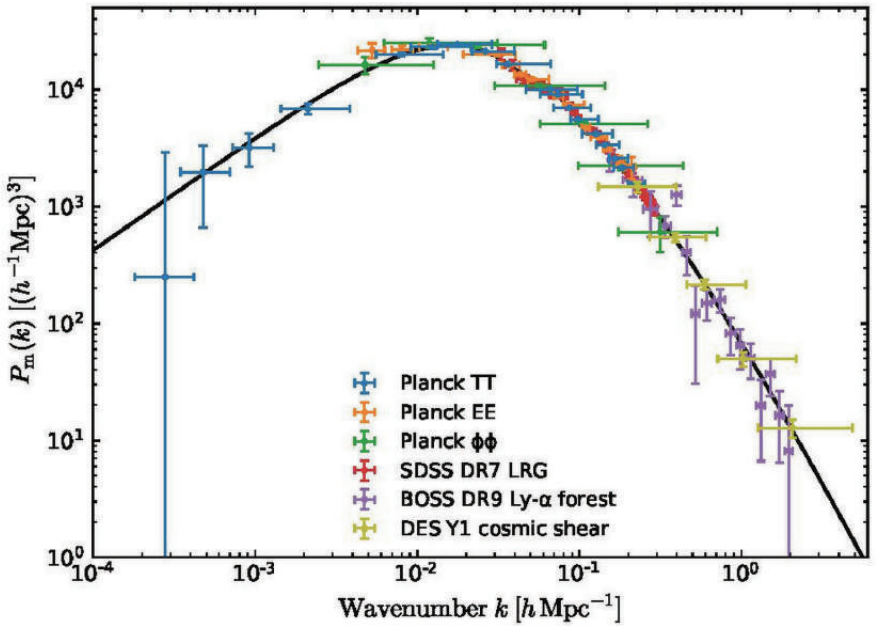
CMB (Planck)



Large Scale Structure (SDSS)

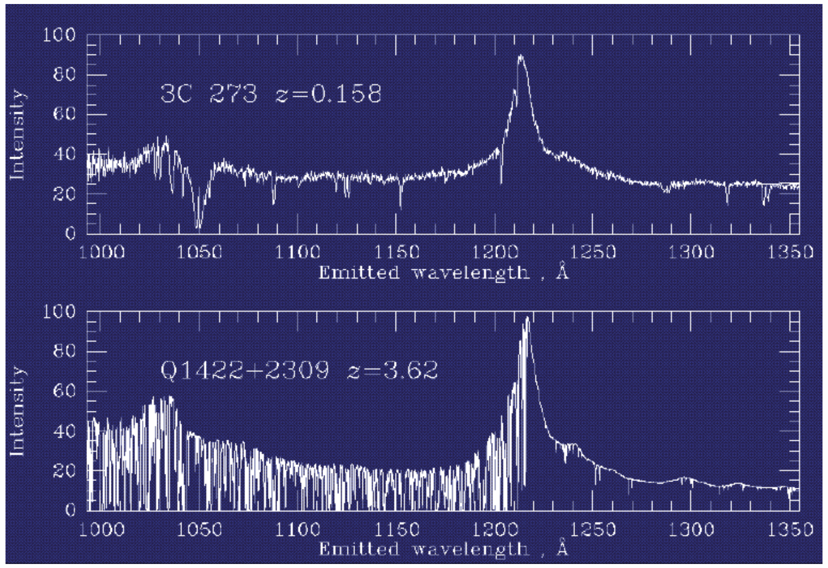


BAO (BOSS)



Concordance (Planck)

Galaxy Clustering (Hubble)

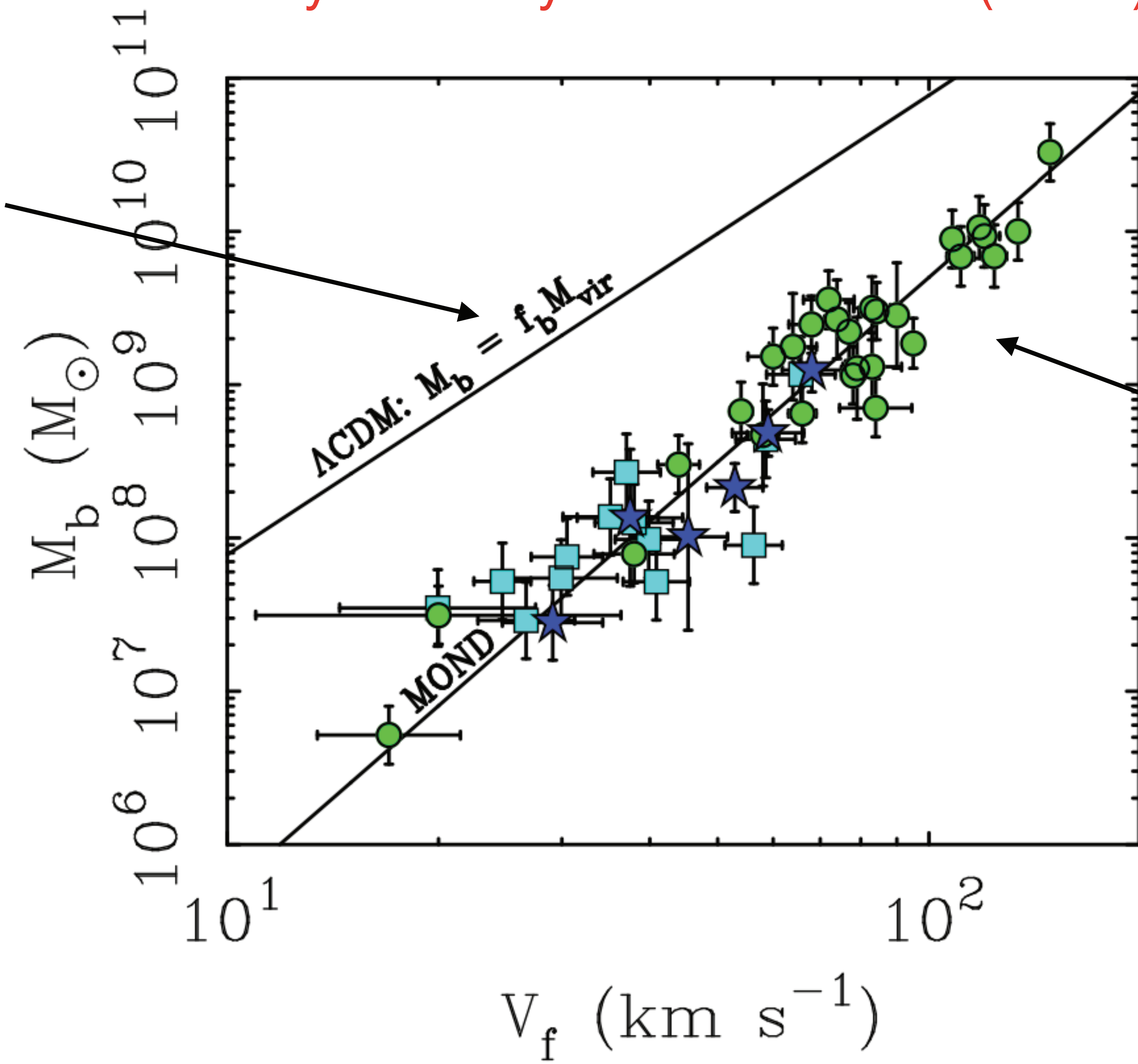


Lyman Alpha Forest (Keck)

Possible Difficulties with CDM on Galactic Scales?

Baryonic Tully-Fisher Relation (BTFR)

$M_b \propto v_f^3$?



$\boxed{M_b \propto v_f^4}$

(from McGaugh 2011)

New Interactions on Galactic Scales?

$$a \propto \frac{M_{enc}}{R^2}$$ If instead: $$\frac{v^2}{R} = a \propto \frac{\sqrt{M_{enc}}}{R} \implies \boxed{M_b \propto v_f^4}$$

(Milgrom; “MOND”)

Implementing this is very difficult:

The unique theory of massless spin 2 particles at large distances is general relativity

(Feynman, Weinberg, Deser,…)

A possibility: we can add new degrees of freedom. In particular, new scalars could mediate a new long range (peculiar) interaction

Add a Real Scalar

(+ - - -)

$$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$S = \int d^4x \sqrt{-g} \left[\frac{\mathcal{R}}{16\pi G} + \mathcal{L}_M(\psi_i) - \beta\, \phi\, T_M + K(X) \right]$$

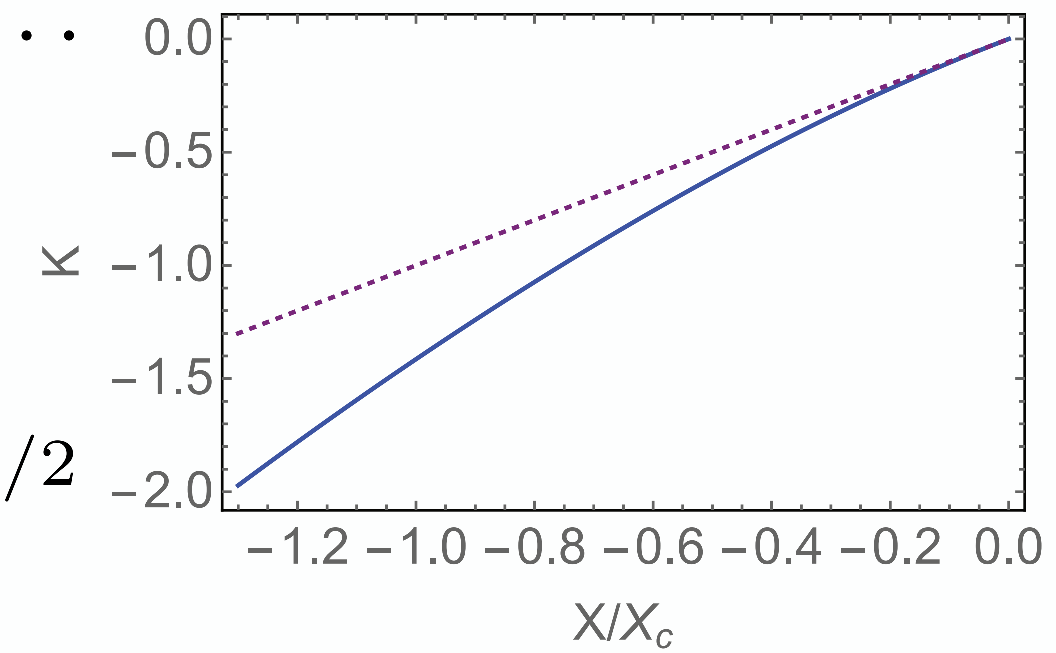
(Robust against quantum corrections, e.g., de Rham, Ribeiro, 2014)

EFT expansion $\quad K = X + c_2 X^2 + \ldots$

(3/2 scaling)

High densities/galactic scales $\quad \boxed{K = \tilde{\alpha} X \sqrt{|X|}}$

Example: $\quad K \propto -(1 - X/\mu)^{3/2}$



(Large ϕ, can stay within regime of validity of Effective Field Theory)

Add a Real Scalar

(+ - - -)

$$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$S = \int d^4x \sqrt{-g} \left[\frac{\mathcal{R}}{16\pi G} + \mathcal{L}_M(\psi_i) - \beta\, \phi\, T_M + K(X) \right]$$

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EFT expansion

$$K = X + c_2 X^2 + \ldots$$

(3/2 scaling)

High densities/galactic scales

$$\boxed{K = \tilde{\alpha} X \sqrt{|X|}}$$

Mediates a MOND-like force

$$-\frac{3\tilde{\alpha}}{2^{3/2}} \nabla \cdot (\nabla\phi |\nabla\phi|) = \beta\, T_M$$

$$\mathbf{a} \propto -\mathrm{sign}(\tilde{\alpha}) \frac{\sqrt{M_{enc}}}{R}\, \hat{r}$$

Add a Real Scalar

Two Problems

$$K = \tilde{\alpha} X \sqrt{|X|}$$

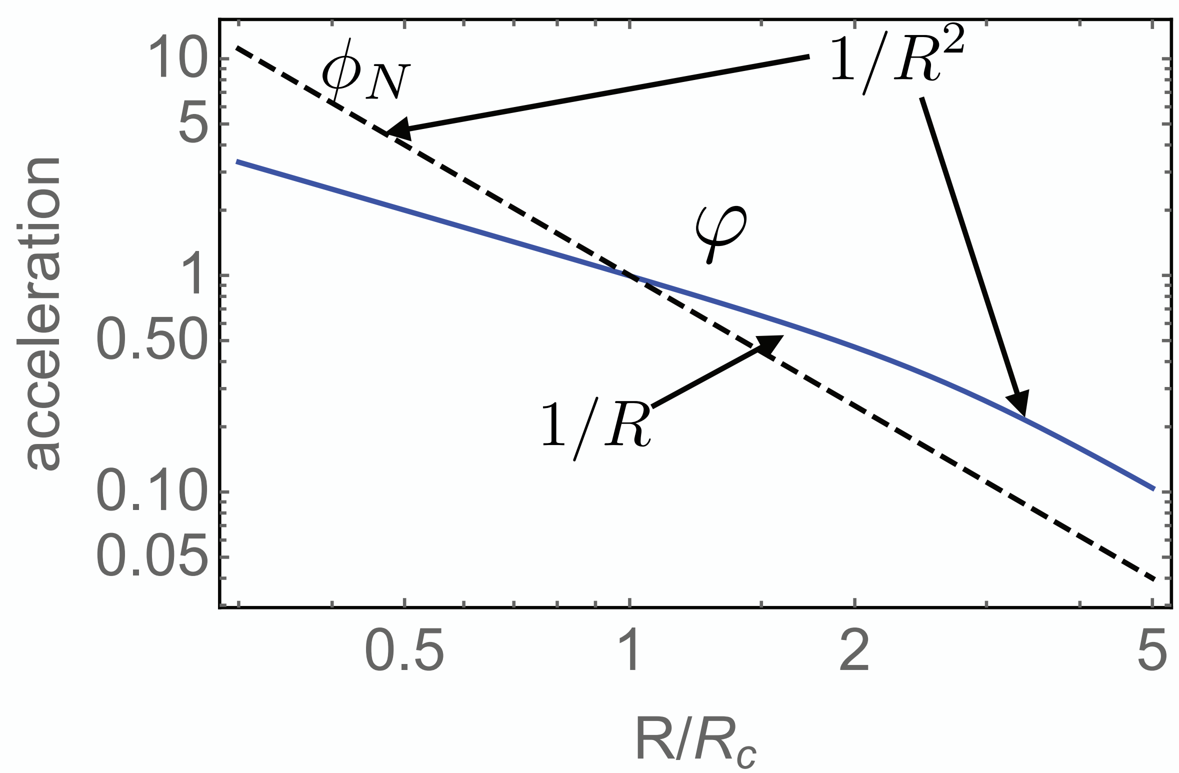
$$(c_s = \sqrt{2})$$

Theoretical: High energy perturbations on top of the MONDian solution are superluminal (related details later)

(e.g., Bruneton,…)

$$K = X$$

Phenomenological: Although the scalar becomes canonical at large scales, it introduces another $1/r^2$ force. So it is difficult to consistently obtain the desired galactic and large scale behaviors



SuperFluid Dark Matter (SFDM) - Complex Scalar

Clever idea: Use Spontaneous Symmetry Breaking

Complex Scalar Dark Matter

$U(1)$ Symmetry

Φ

$X = g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi^*$

Example:

$$\boxed{K = \left(X - m^2|\Phi|^2\right) + \frac{\Lambda^4}{6(\Lambda_c^2 + |\Phi|^2)^6}\left(X - m^2|\Phi|^2\right)^3}$$

(Quantum, from particle point of view
Classical, from field point of view)

Reproduces CDM on large scales

Allows for phase transition to superfluid
at galactic densities

$$\Phi = \rho\, e^{i(\theta + mt)}$$

Goldstone θ can act as long-ranged force mediator

Berezhiani, Khoury 2015

SuperFluid Dark Matter (SFDM) - Complex Scalar

$$\Phi = \rho\, e^{i(\theta + mt)}$$

Slowly varying phase θ and modulus ρ around superfluid condensate

$$-X + m^2|\Phi|^2 = (\nabla\rho)^2 - 2\,m\,\rho^2\,Y \qquad \text{with} \qquad Y \equiv \dot{\theta} - m\,\phi_N - \frac{(\nabla\theta)^2}{2m}$$

At tree-level, can integrate out heavy modulus (Higgs mode)

$$\rho^2 = \Lambda\sqrt{2m|Y|}$$

(3/2 scaling)

Find low energy effective action for massless Goldstone is

$$\boxed{K_{\text{eff}} = +\frac{2\Lambda(2m)^{3/2}}{3} Y\sqrt{|Y|}}$$

By coupling to baryons, can mediate MOND-like force — reproduce BTFR, and CDM on large scales

Berezhiani, Khoury 2015

Analysis of High Energy Perturbations ε_j

Decompose into components $\Phi = (\phi_1 + i\,\phi_2)/\sqrt{2}$

Expand around superfluid $\phi_j = \phi_j^b + \varepsilon_j \qquad (j = 1, 2) \qquad \left(K' \equiv \dfrac{\partial K}{\partial X}\right)$

Linear equation of motion for high energy perturbations

$$\sum_{j=1}^{2} \left[K' \eta^{\mu\nu} \delta^{ij} + K'' \partial^\mu \phi_i^b \partial^\nu \phi_j^b\right] \partial_\mu \partial_\nu \varepsilon_j = 0$$

Diagonalize to obtain Higgs normal mode perturbations and associated effective metric

$$\psi = \partial^\mu \phi_1^b \partial_\mu \varepsilon_1 + \partial^\mu \phi_2^b \partial_\mu \varepsilon_2$$

$$G_\phi^{\mu\nu} \partial_\mu \partial_\nu \psi = 0$$

$$\boxed{G_\phi^{\mu\nu} = K' g^{\mu\nu} + K''(\partial^\mu \phi_1^b \partial^\nu \phi_1^b + \partial^\mu \phi_2^b \partial^\nu \phi_2^b)}$$

M. Hertzberg, J. Litterer, N. Shah, JCAP 11 (2021) 2105.02241

Causal Propagation?

Obtain eigenvalues of effective metric $G_\phi^{\mu\nu} = K' g^{\mu\nu} + K''(\partial^\mu \phi_1^b \partial^\nu \phi_1^b + \partial^\mu \phi_2^b \partial^\nu \phi_2^b)$

Conditions for hyperbolicity

$$(A) \quad A \equiv K' > 0$$

$$(B) \quad B \equiv K' + 2XK'' > 0$$

(Aharanov, Komar, Susskind; Wald; Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi; Bruneton;...)

Condition for subluminality

$$(C) \quad C \equiv +K'' \geq 0$$

Evaluate in SFDM model $A > 0$ $\quad B = \dfrac{4\, m^3\, \Lambda^4\, Y}{\rho^8}$ $\quad C = \dfrac{2\, m\, \Lambda^4\, Y}{\rho^{10}}$

MOND regime $Y \approx -\dfrac{(\nabla\theta)^2}{2m}$ $\implies$ $\boxed{B < 0}$ (ghost-like) and $C < 0$

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Evaluate in SFDM model $A > 0$ $\quad B = \dfrac{4\, m^3\, \Lambda^4\, Y}{\rho^8}$ $\quad C = \dfrac{2\, m\, \Lambda^4\, Y}{\rho^{10}}$

Intermediate scales $Y \approx -\dfrac{(\nabla\theta)^2}{2m} \implies B < 0$ and $\boxed{C < 0}$ (superluminal)

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Causal Propagation? - General Analysis

Obtain eigenvalues of effective metric $G_\phi^{\mu\nu} = K' g^{\mu\nu} + K''(\partial^\mu \phi_1^b \partial^\nu \phi_1^b + \partial^\mu \phi_2^b \partial^\nu \phi_2^b)$

Conditions for hyperbolicity

$$(A) \quad A \equiv K' > 0$$

$$(B) \quad B \equiv K' + 2XK'' > 0$$

(Aharanov, Komar, Susskind; Wald; Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi; Bruneton;...)

Condition for subluminality

$$(C) \quad C \equiv +K'' \geq 0$$

Most general form

$$K = (X - m^2|\Phi|^2) \sum_{n=0} g_n \frac{\Lambda^{2n} \left(X - m^2|\Phi|^2\right)^n}{(\Lambda_c^2 + |\Phi|^2)^{3n}}$$

We proved that when g_n allow MOND regime $(\tilde{\alpha} > 0)$ $\Longrightarrow$ $\boxed{B < 0}$ (ghost-like) and $\boxed{C < 0}$ (superluminal)

M. Hertzberg, J. Litterer, N. Shah, JCAP 11 (2021) 2105.02241

Part 2: Aspects of Light Condensates in Galaxies

Based on work:

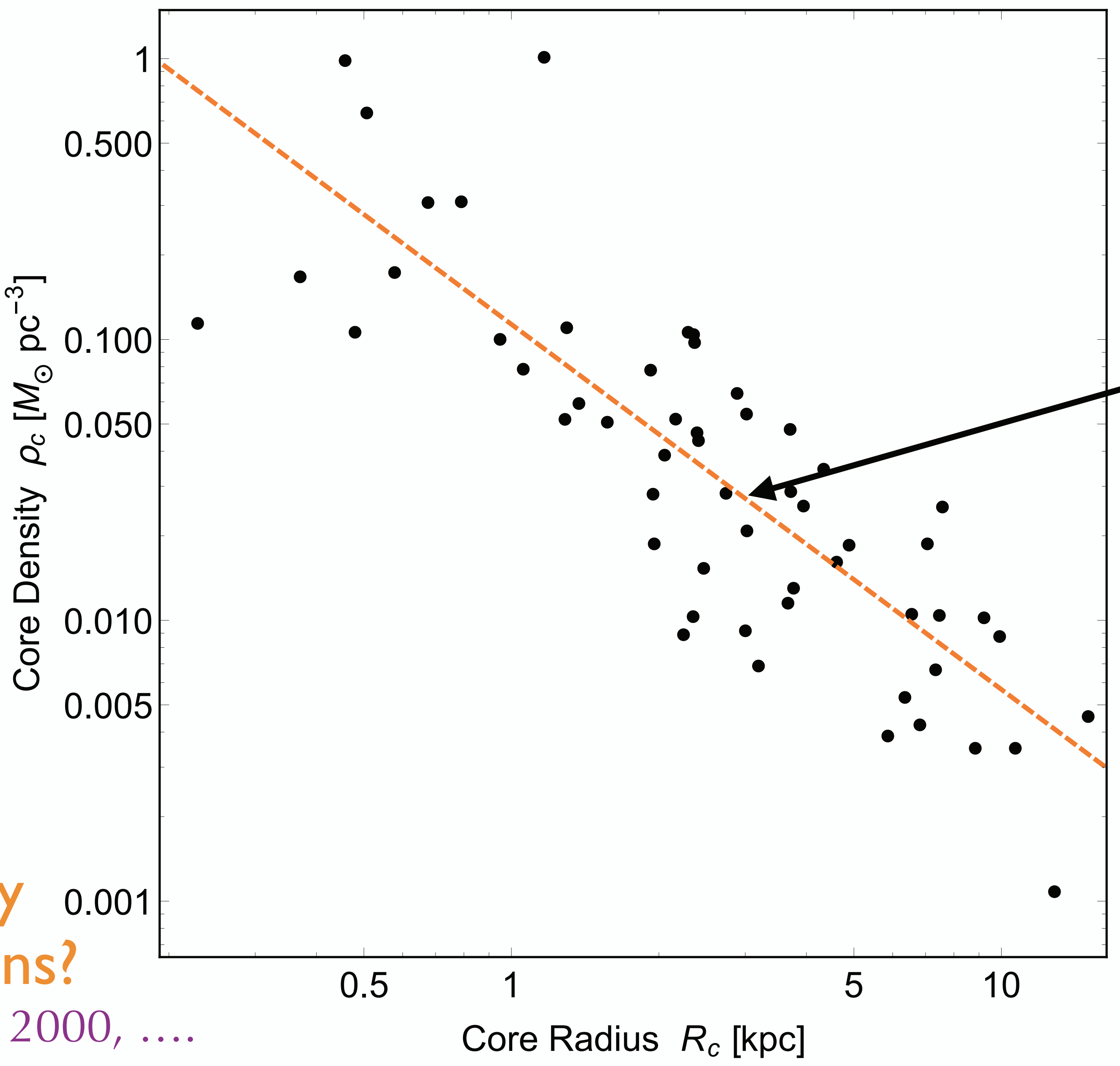
Deng, Hertzberg, Namjoo, Masoumi 1804.05921 (PRD)

New novel scalar interactions are difficult to solve BTFR.

What about scalars to solve the core-cusp problem?

$$\mathcal{L} = \frac{\mathcal{R}}{16\pi G} + \mathcal{L}_{SM} + \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4 + \ldots$$

Core-Cusp Problem (Data)



$$\rho_c \sim \frac{1}{R_c^\beta}$$

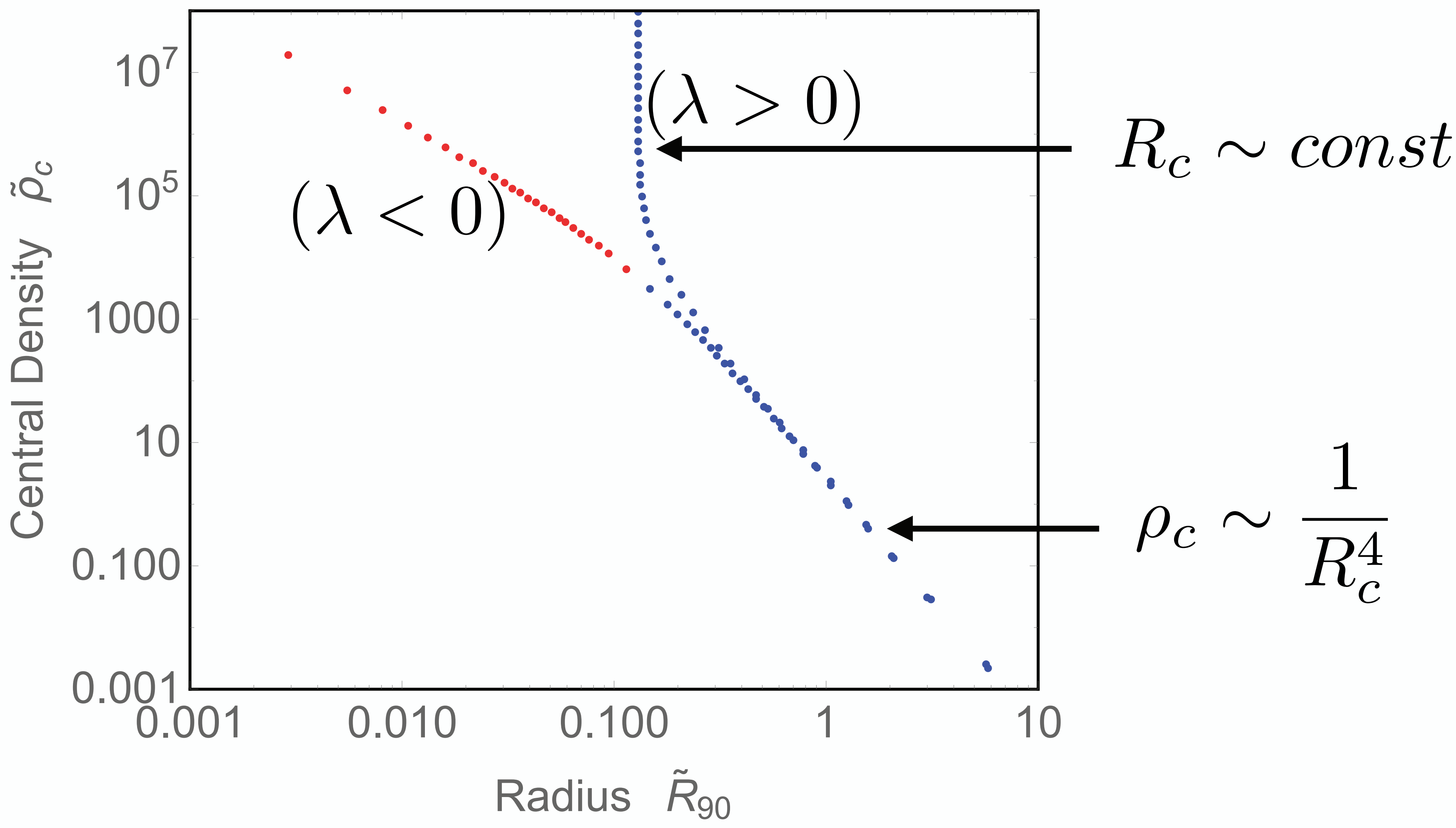
$$\beta \approx 1$$

Explained by ultralight axions?

Hu, Barkana, Gruzinov 2000, ….

Data from: Rodriguez, del Popolo, Marra, de Oliveira 2017

Core Density Vs Core Radius (Ultralight Scalar in BEC)



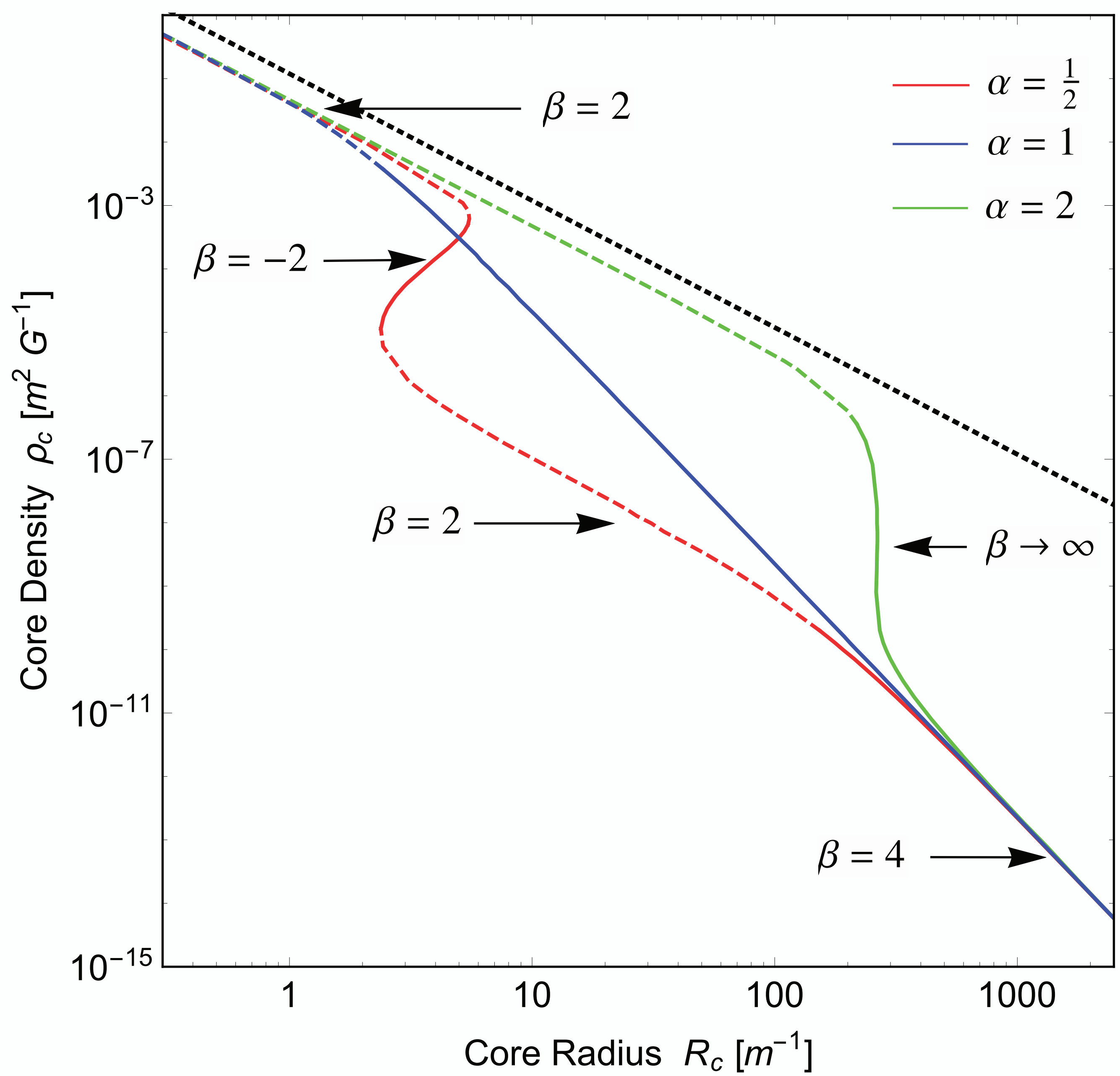
$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4$$

Core Density Vs Core Radius (Ultralight Scalar in BEC)

Extension to general potentials, polytropes, full relativistic

$$V(\phi) \propto ((1 + \phi^2/F^2)^\alpha - 1)$$

Solid = Stable
Dashed = Unstable

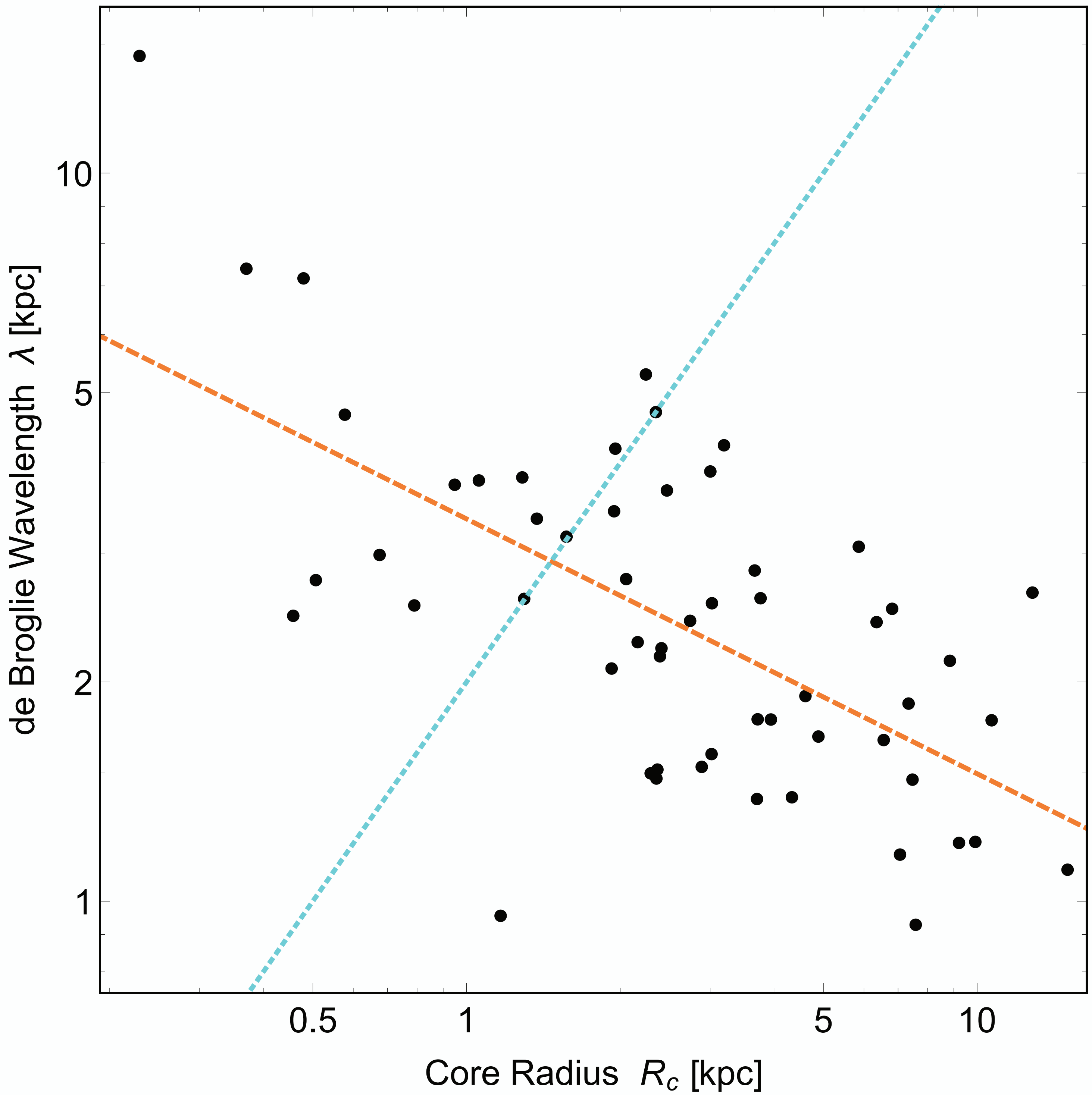


$$\rho_c \sim \frac{1}{R_c^\beta}$$

Never obtain $\beta \sim 1$ and stable

Deng, Hertzberg, Namjoo, Masoumi 1804.05921 (PRD)

Core Density Vs Core Radius (Ultralight Scalar in BEC)



Deng, Hertzberg, Namjoo, Masoumi 1804.05921 (PRD)

Part 3: Aspects of Quantum Dark Matter

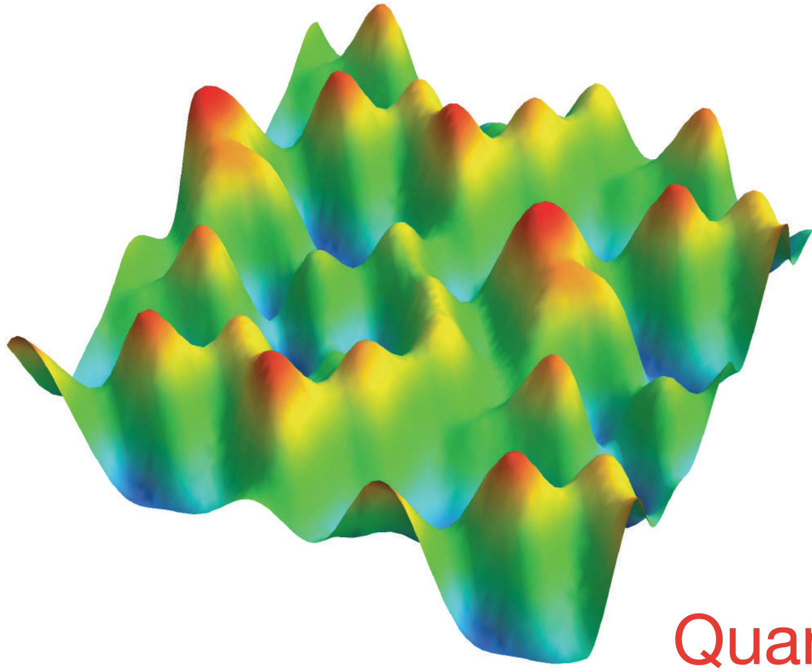
Based on work with:

Itamar Allali 

Allali, Hertzberg 2005.12287 (JCAP), 2012.12903 (PRD), 2103.15892 (PRL)

Quantum: Any aspects of light scalar DM not captured by classical field theory?

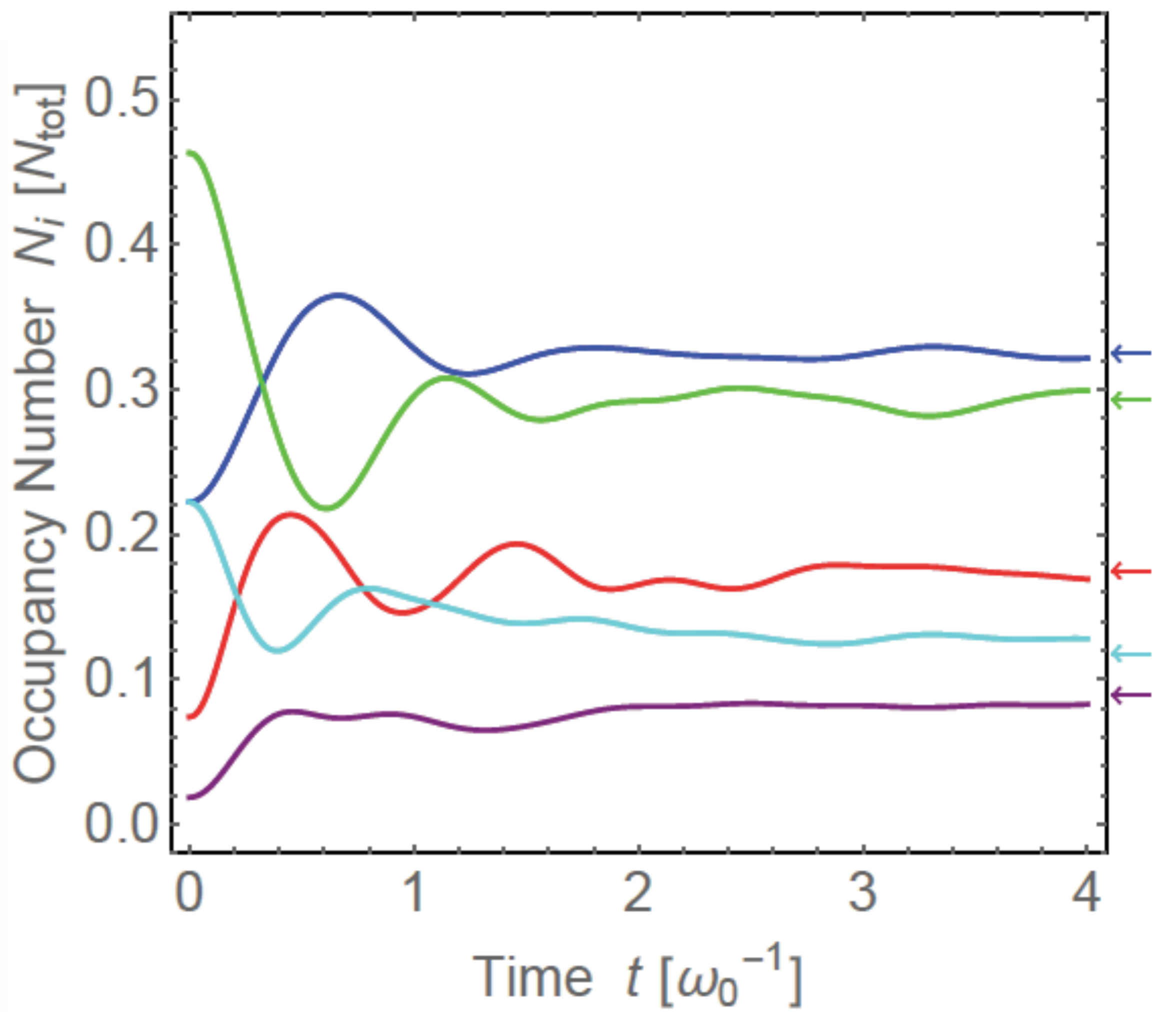
Example: Quantum radiation (even at high occupancy) for appreciable self-couplings
Hertzberg 1003.3459 (PRD); Hertzberg, Rompineve, Yang 2010.07927 (PRD)



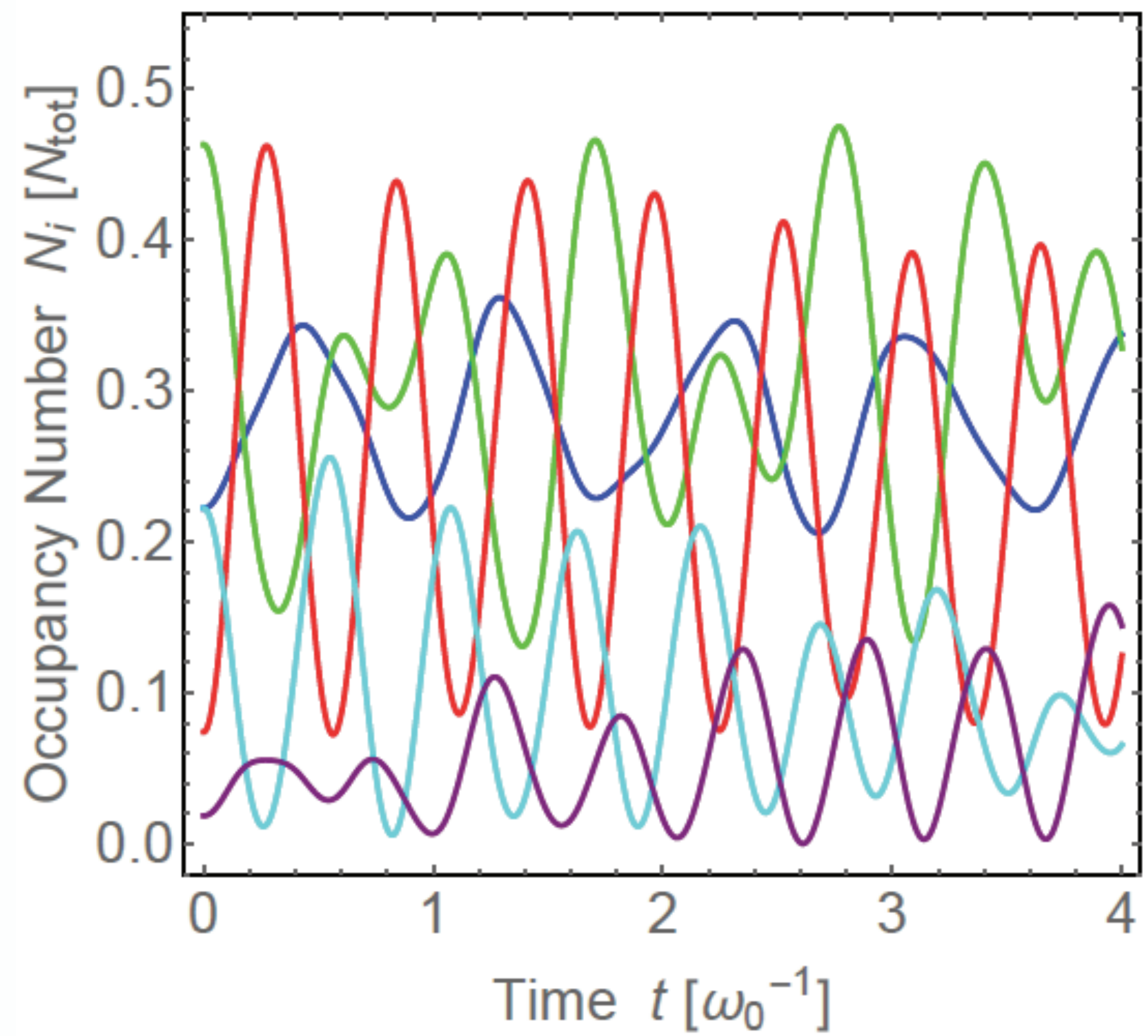
Quantum Average vs Classical State

(Toy example)

Quantum

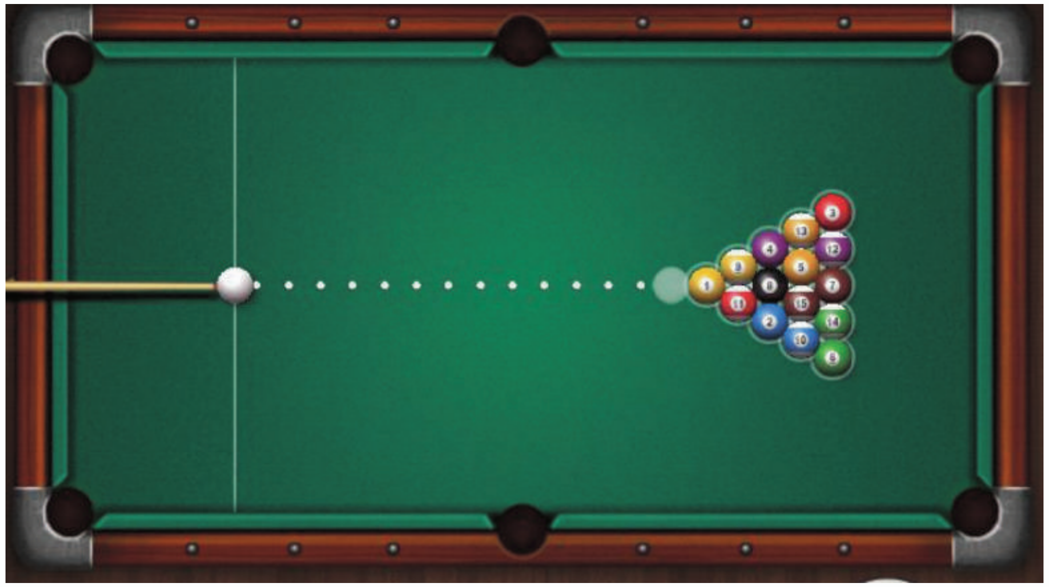


Classical



Sikivie, Todarello, 1607.00949

Dynamics can launch states into Schrodinger cat-like states



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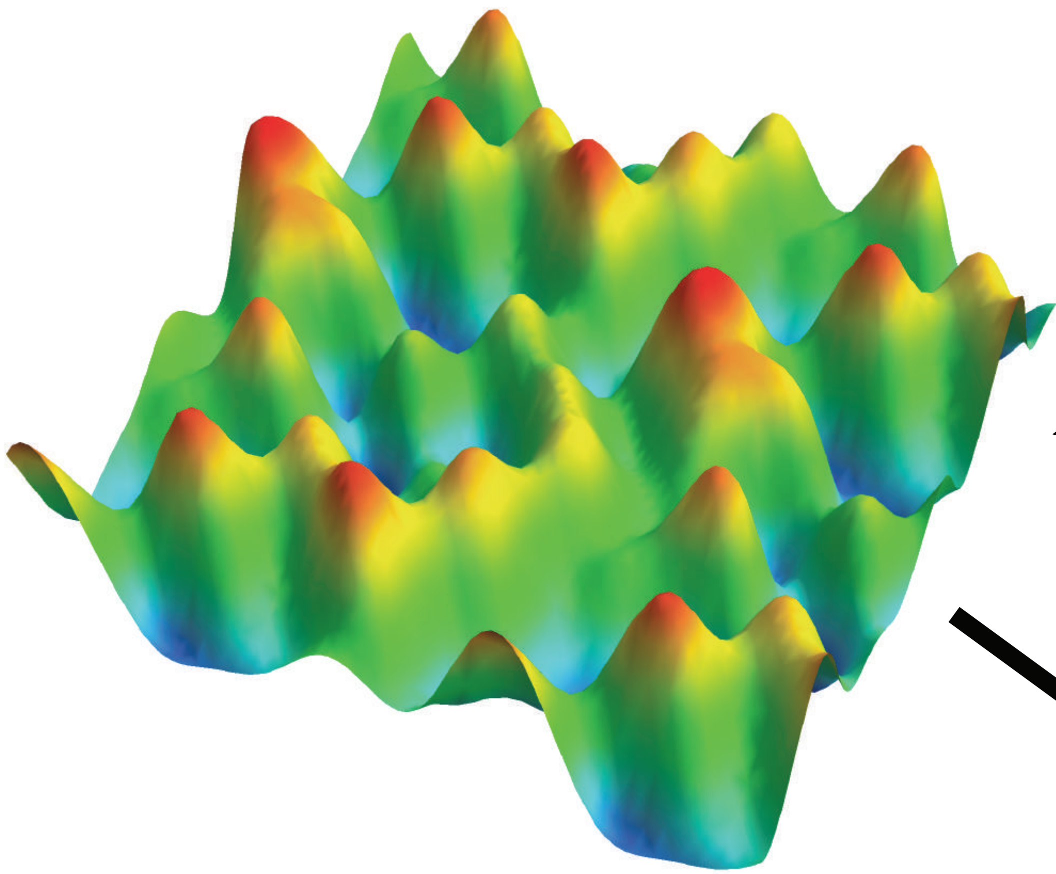
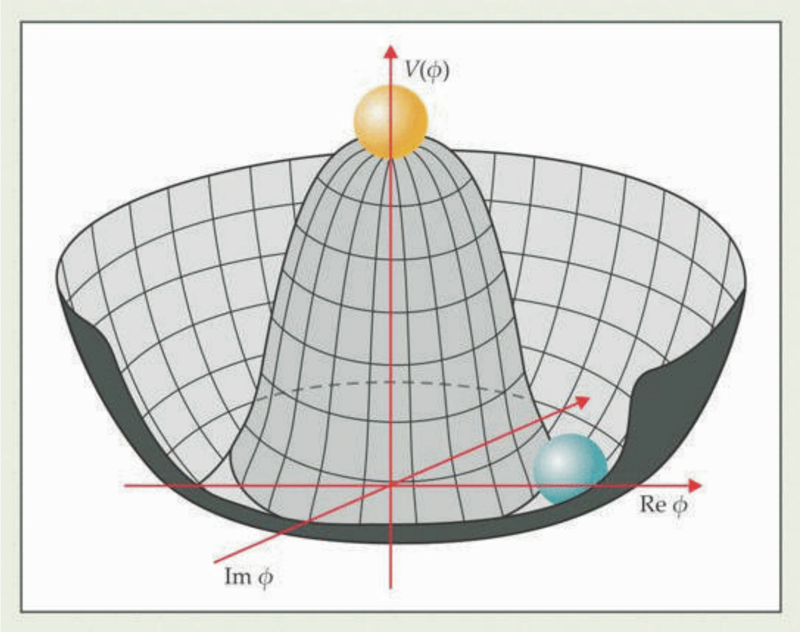
Schrodinger Cat Billiards

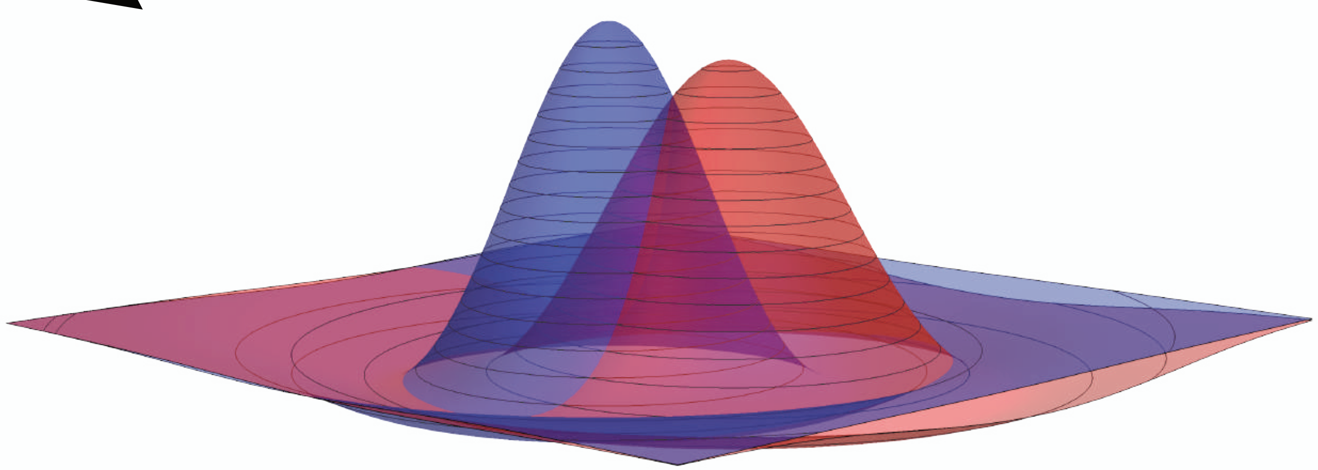
Albrecht, Phillips

Dynamics can launch states into Schrodinger cat-like states

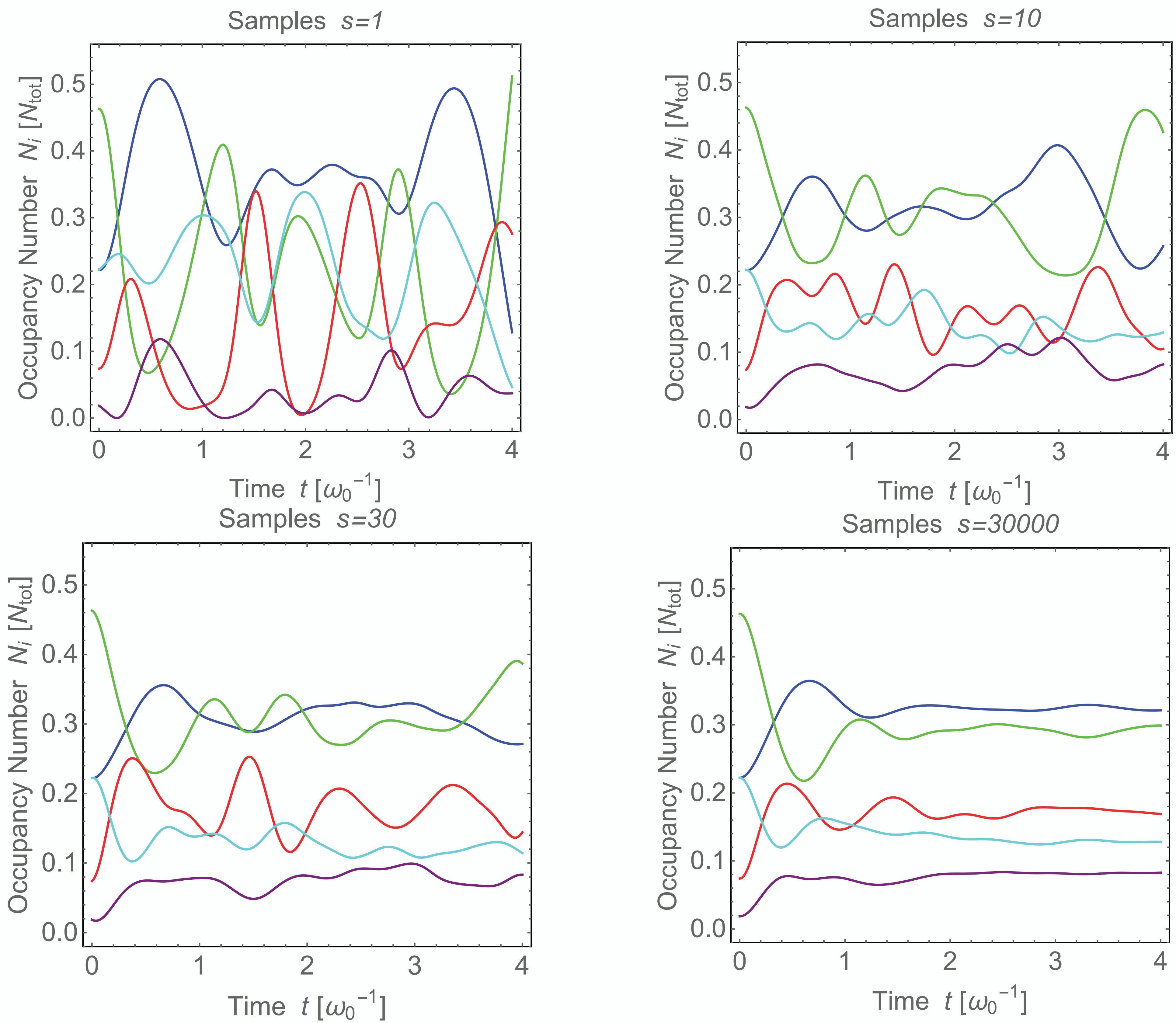




Dark Matter Schrodinger Cat (Axions)



Stochastic Classical Treatment

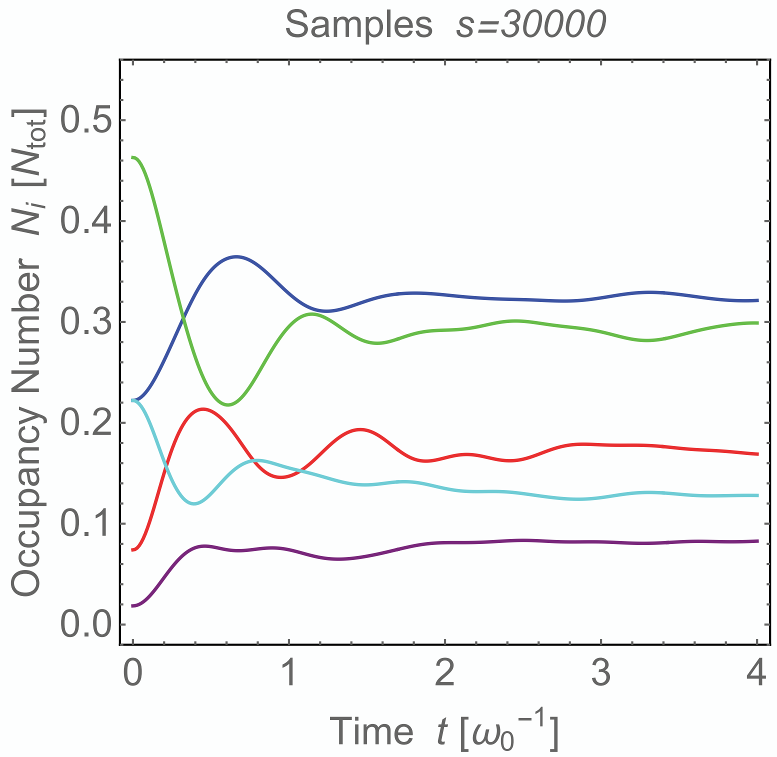


Matches quantum average

Hertzberg 1609.01342 (JCAP)

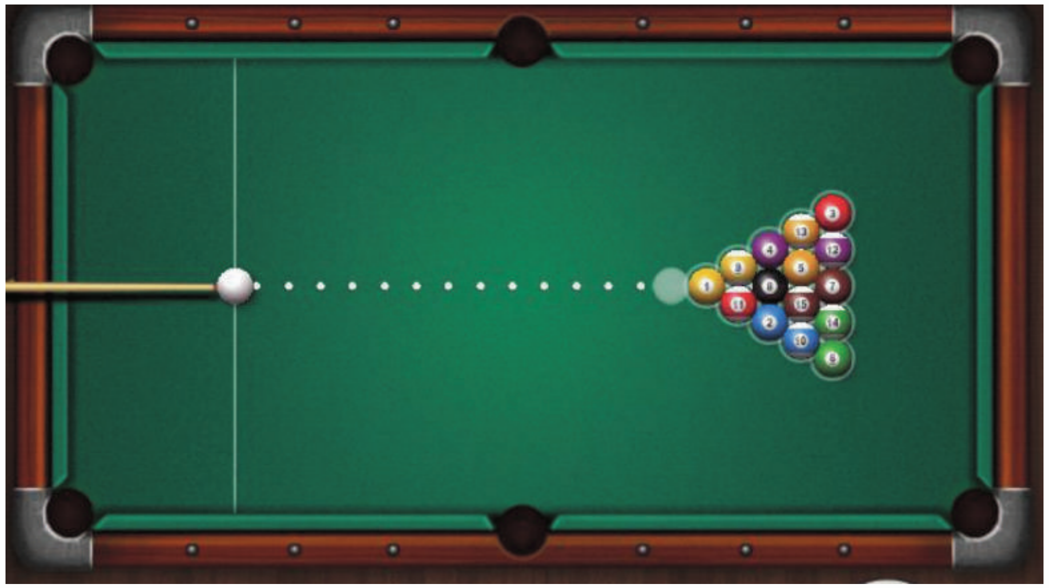
Quantum correlation functions are captured by stochastic classical averaging.

Caveat: Precision requires sampling Wigner distribution



(Related to Stochastic Inflation)

Dynamics can launch states into Schrodinger cat-like states



Quantumness destroyed due to DECOHERENCE

Schrodinger Cat Billiards

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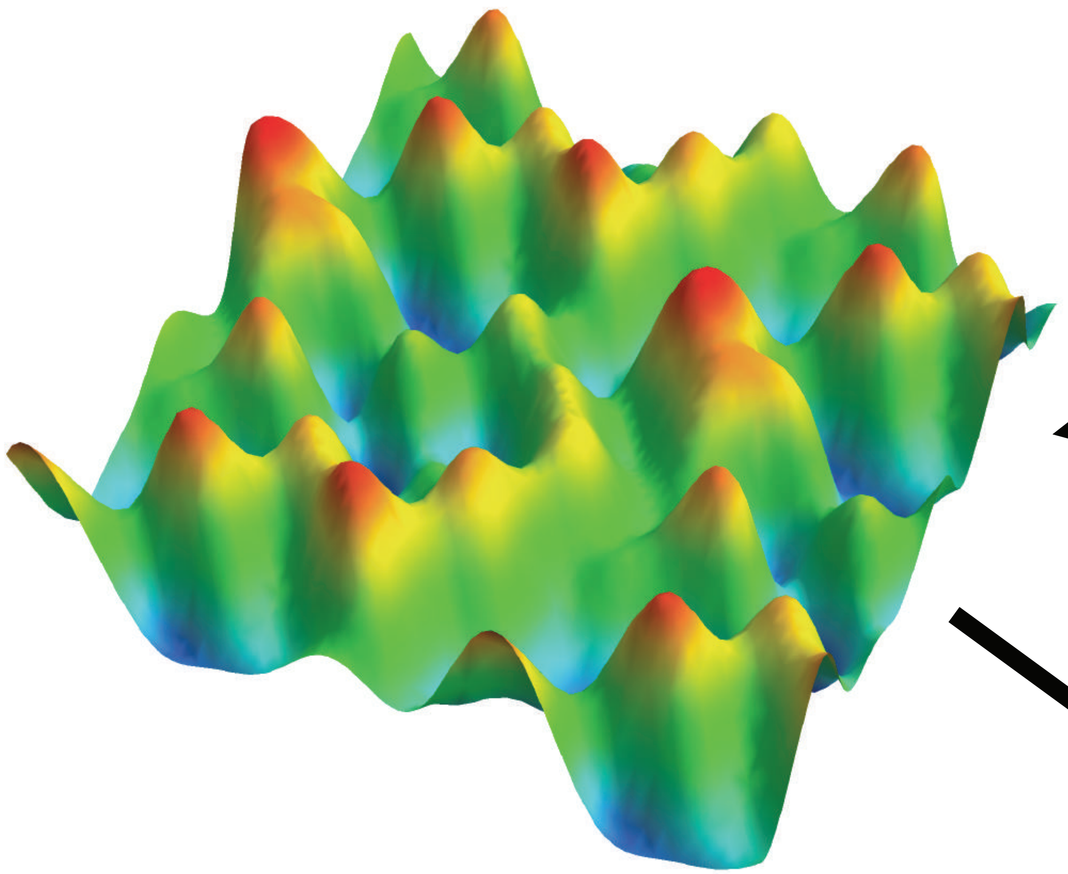
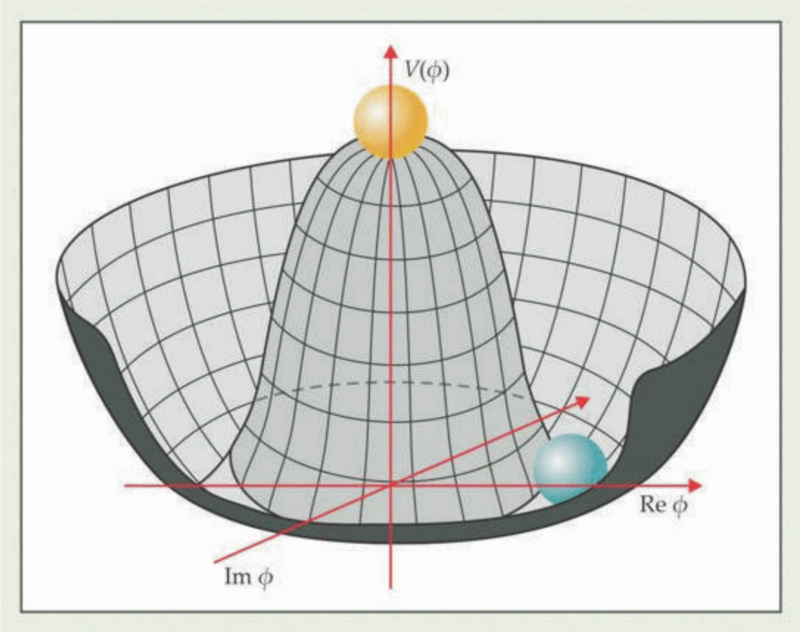
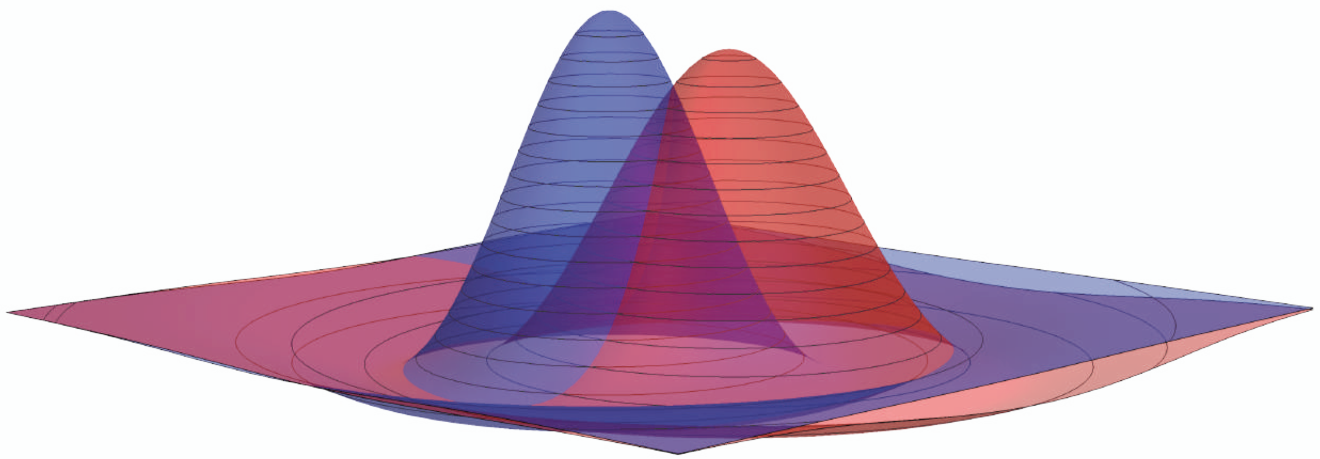
Albrecht, Phillips

Dynamics can launch states into Schrodinger cat-like states





Dark Matter Schrodinger Cat (Axions)



Quantumness destroyed due to DECOHERENCE???

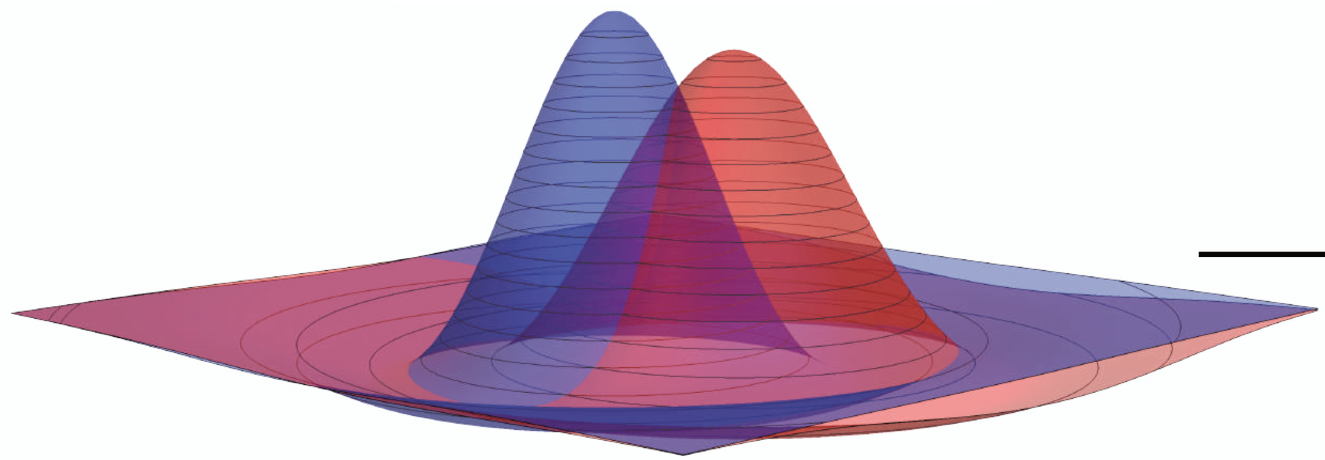
Less clear because dark matter has tiny (non-gravitational) interactions

Entanglement from Gravitational Scattering

$$|\mathrm{DM}_1\rangle + |\mathrm{DM}_2\rangle$$

Probe particle $|\psi\rangle$

(e.g., baryon,…)



$|\psi_1\rangle$

$|\psi_2\rangle$

$$|\Psi_{\mathrm{ini}}\rangle = (|\mathrm{DM}_1\rangle + |\mathrm{DM}_2\rangle)\,|\psi\rangle$$

Product State

$$|\Psi_{\mathrm{fin}}\rangle = |\mathrm{DM}_1\rangle\,|\psi_1\rangle + |\mathrm{DM}_2\rangle\,|\psi_2\rangle$$

Entangled State

Trace Out Probe Particle

$$\hat{\rho} \equiv |\Psi\rangle \langle\Psi|$$

Full Density Matrix

$$\hat{\rho}_{\mathrm{red}} = \mathrm{Tr}_{|\psi\rangle}[\hat{\rho}]$$

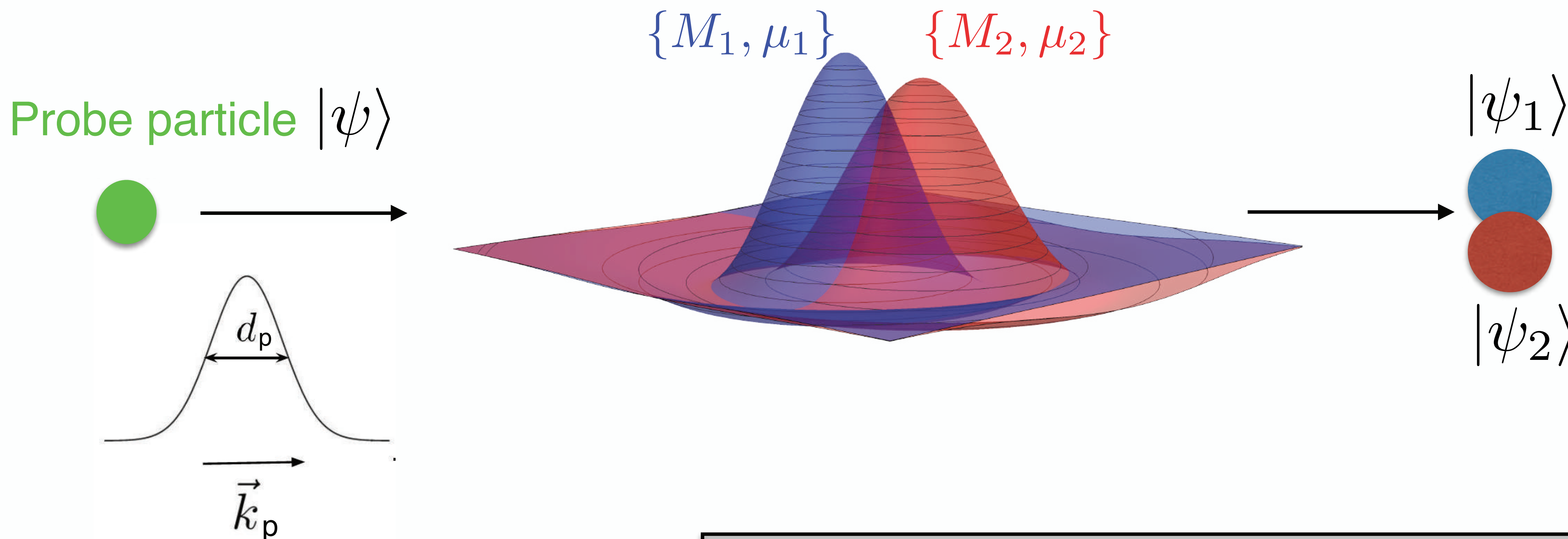
Reduced Density Matrix

$$= |\mathrm{DM}_1\rangle \langle\mathrm{DM}_1| + \boxed{\langle\psi_2|\psi_1\rangle} |\mathrm{DM}_1\rangle \langle\mathrm{DM}_2| + \boxed{\langle\psi_1|\psi_2\rangle} |\mathrm{DM}_2\rangle \langle\mathrm{DM}_1| + |\mathrm{DM}_2\rangle \langle\mathrm{DM}_2|$$

Off diagonal elements;
controlling true quantum effects

Result for Overlap of Probe Particle States

We evolve a Gaussian wave packet using perturbation theory



$$|\langle\psi_1|\psi_2\rangle|^2 = 1 - 2\Delta$$

$$\boxed{\Delta_0 = \frac{2G^2m^4}{\hbar^4k^2d^2}\left[\frac{M_1^2}{\mu_1^2}\chi_{11} + \frac{M_2^2}{\mu_2^2}\chi_{22} - 2\frac{M_1M_2}{\mu_1\mu_2}\chi_{12}\right]}$$

Allali, Hertzberg 2005.12287 (JCAP)

Decoherence Rate from N-Probe Particles

Off diagonal element of density matrix

$$\prod_{n=1}^{N} |\langle\psi_1|\psi_2\rangle|_n = \prod_{n=1}^{N}(1-\Delta_b) \sim e^{-\sum_{n=1}^{N}\Delta_b}$$

Decoherence rate

$$\Gamma_{\rm dec} = n\,v \int d^2b\,\Delta_b$$

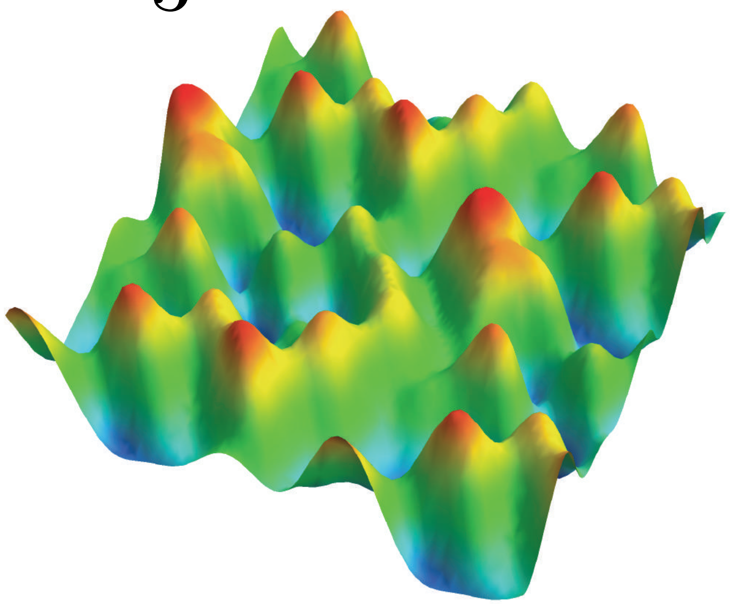
$$\boxed{\Gamma_{\rm dec} = \frac{4\pi G^2 m^4 n\,v}{\hbar^4 k^2}\left[\frac{M_1^2}{\mu_1^2}\chi_{11} + \frac{M_2^2}{\mu_2^2}\chi_{22} - 2\frac{M_1 M_2}{\mu_1\mu_2}\chi_{12}\right]}$$

Allali, Hertzberg 2005.12287 (JCAP)

Application to Light Diffuse scalar DM (axions)

Diffuse scalars (axions)

$$\mu_i \sim \frac{1}{\lambda_{dB,a}} \sim \frac{2\pi}{m_a v_{vir}} \qquad M_i \sim \frac{4\pi}{3}\rho_{DM}\lambda_{dB,a}^3$$



Probe: Diffuse baryons

$$k_p \sim m_p\, v_{vir}$$

Decoherence Rate

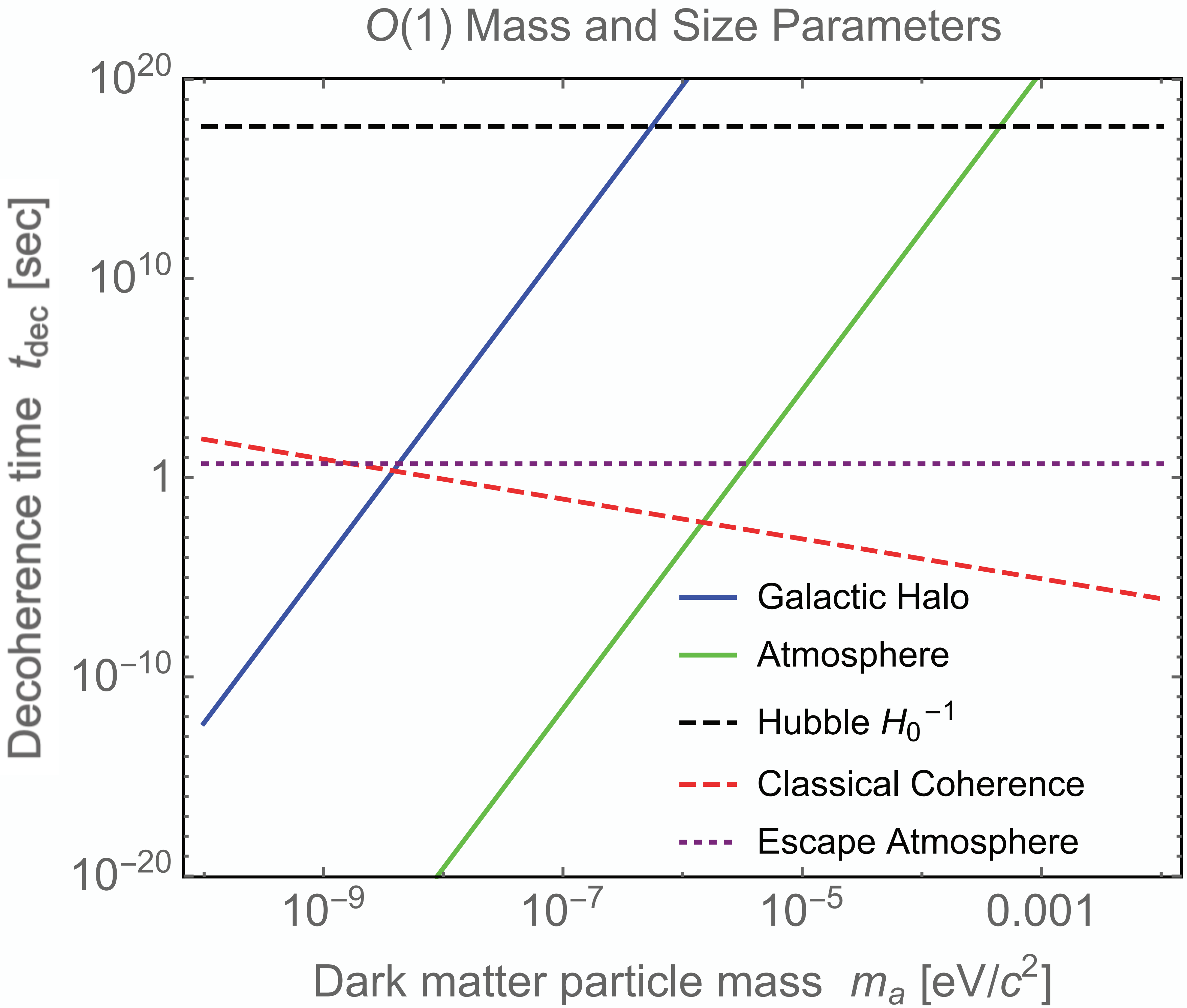
$$\boxed{\Gamma_{dec} \sim \frac{G^2\, m_p\, \rho_b\, \rho_{DM}^2}{m_a^8\, v_{vir}^9}}$$

Decoherence Time

$$t_{dec} = 1/\Gamma_{dec} \propto m_a^8$$

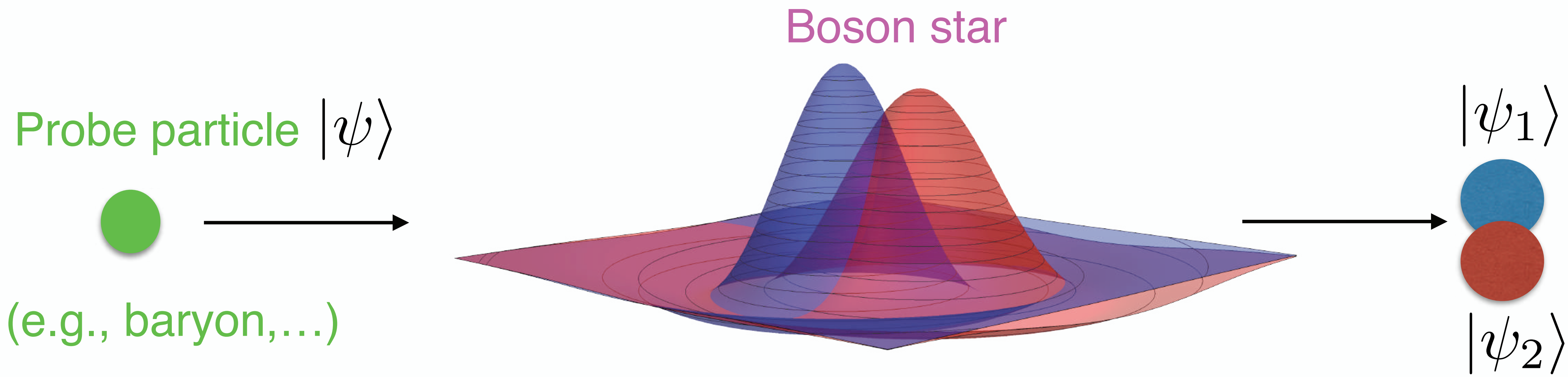
Allali, Hertzberg 2005.12287 (JCAP)

Application to Light Diffuse scalar DM (axions)



Allali, Hertzberg 2005.12287 (JCAP)

Application to Boson Stars - Gravitational Condensates



Boson stars are much denser

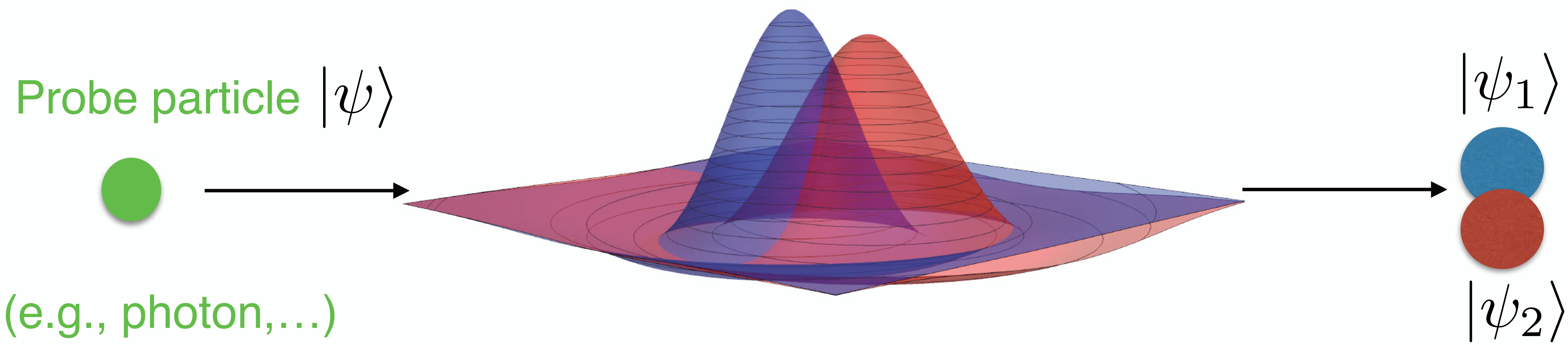
Decoherence Rate

$$\boxed{\Gamma_{\rm dec} \gtrsim \frac{\hbar^2 m_p \rho_p}{v_p m_a^4} \sim 10^{21}\,\mathrm{sec}^{-1}\left(\frac{1\,\mathrm{eV}}{m_a\,c^2}\right)^4}$$

Extremely rapid decoherence —> Very classical

Allali, Hertzberg 2005.12287 (JCAP)

General Relativistic Extension



Rigorous quantum gravity calculation — General Relativity is a well behaved effective theory

Allali, Hertzberg 2012.12903 (PRD), 2103.15892 (PRL)

General Relativistic Extension

Starting from QFT, can derive RSE for 1-particle sub-space: (ignoring spin)

$$(i\partial_t - \sqrt{-\nabla^2 + m^2})\psi(\mathbf{x},t) = \left(\Phi(\mathbf{x},t)\sqrt{-\nabla^2 + m^2} - \frac{\Psi(\mathbf{x},t)\nabla^2}{\sqrt{-\nabla^2 + m^2}}\right)\psi(\mathbf{x},t)$$

Decoherence Rate
for superposition of
different phases

$$\boxed{\Gamma_{dec} \propto \exp\left[-\frac{m_a^2 E_p^2}{\mu^2 k^2}\right] \sim \exp\left[-\frac{1}{v_a^2 v_p^2}\right]}$$

So the phase is rather robust against decoherence

Open question: are there observables in direct detection related to this?

Allali, Hertzberg 2012.12903 (PRD), 2103.15892 (PRL)

Conclusions

1. Superfluid dark matter (phonon with 1/r force) is a novel way (only known?) to obtain success of CDM on large scales and success of MOND (BTFR) on galactic scales.

We studied a general class of models, and proved that high energy perturbations always violate hyperbolicity — ghost like behavior — in MOND regime, while intermediate regimes can exhibit superluminality. (Problems in related models too.)

2. An ultralight scalar has been suggested to explain the cores at centers of galaxies.

But we showed that it is difficult to fit the pattern across galaxies.

3. Macroscopic quantum states of light scalar dark matter might exist and has been suggested by some to make the classical field theory approximation inaccurate.

We showed that classical stochastic averaging reproduces the quantum correlation functions; assuming a positive definite Wigner distribution exists initially.

Also, we found that very light scalar DM undergoes rapid decoherence of spatial profile, while phase superpositions may be long lived.