

b06

Weak lensing effect on cosmic birefringence and its tomographic analysis

RESearch Center for Early Universe,
The University of Tokyo, J.Yokoyama lab. M2

Fumihiro Naokawa

T. Namikawa (IPMU), E. Komatsu (Max Planck Institute for Astrophysics)
K. Kamada (the University of Tokyo)



Cosmic Birefringence (宇宙複屈折)

◆ A new signal from CMB Minami & Komatsu (2020)

► Violating Parity Symmetry : beyond Standard Model ?

► Origin : Axion ?



Carroll et al. (1990), Harari & Sikivie (1992), Carroll (1998)

◆ Next generation CMB experiments → It requires precise theoretical predictions



icosahedron icon Cosmic Birefringence (宇宙複屈折)

- ◆ A new signal from CMB Minami & Komatsu (2020)

► Violating Parity Symmetry : beyond Standard Model ?

► Origin : Axion ?



Carroll et al. (1990), Harari & Sikivie (1992), Carroll (1998)

- ◆ Next generation CMB experiments → It requires precise theoretical predictions



icosahedron icon Weak Gravitational Lensing (重力レンズ)

- ◆ Crucial factor for precise CMB observations

- ◆ Especially important for ground-based observation



► ex) Simons Observatory (under development)

(As for C_l^{EE} in high l)

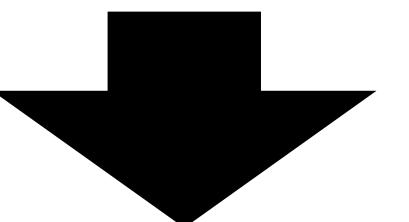
measurement error < Shift by Lensing ~ more than 10 %



CLASS

Lesgourges (2011)
(public code for CMB)

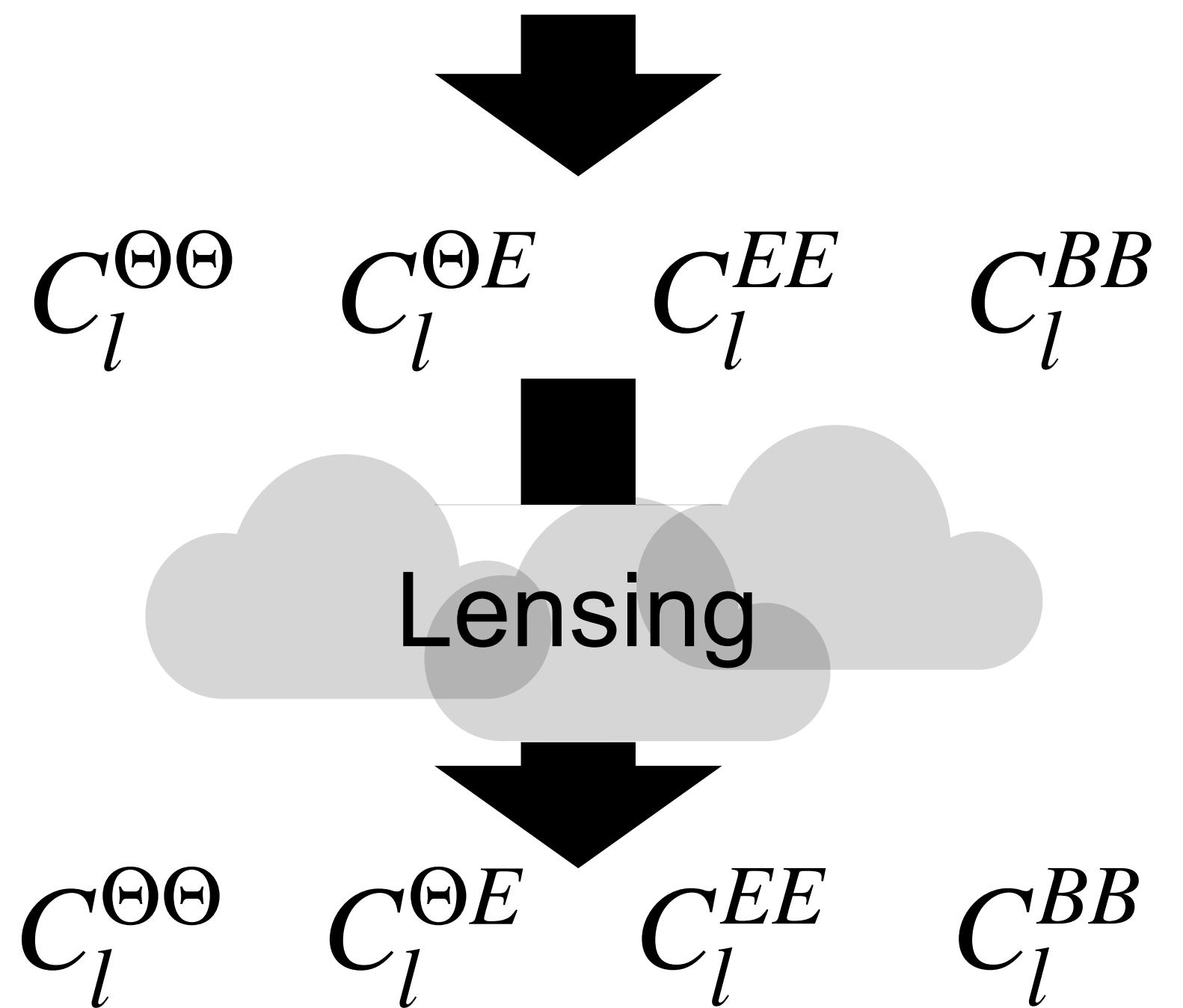
CLASS Lesgourges (2011)
(public code for CMB)



$$C_l^{\Theta\Theta} \quad C_l^{\Theta E} \quad C_l^{EE} \quad C_l^{BB}$$

CLASS

Lesgourgues (2011)
(public code for CMB)



CLASS Lesgourgues (2011)
(public code for CMB)

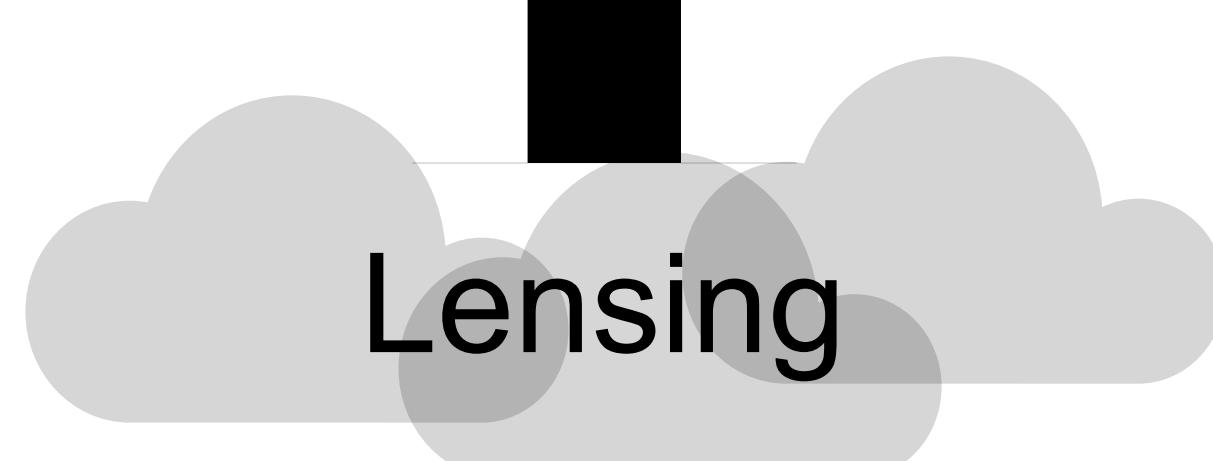
modify



CLASS +  Axion
(including cosmic birefringence)

Nakatsuka et al. (2022)

$$C_l^{\Theta\Theta} \quad C_l^{\Theta E} \quad C_l^{EE} \quad C_l^{BB}$$



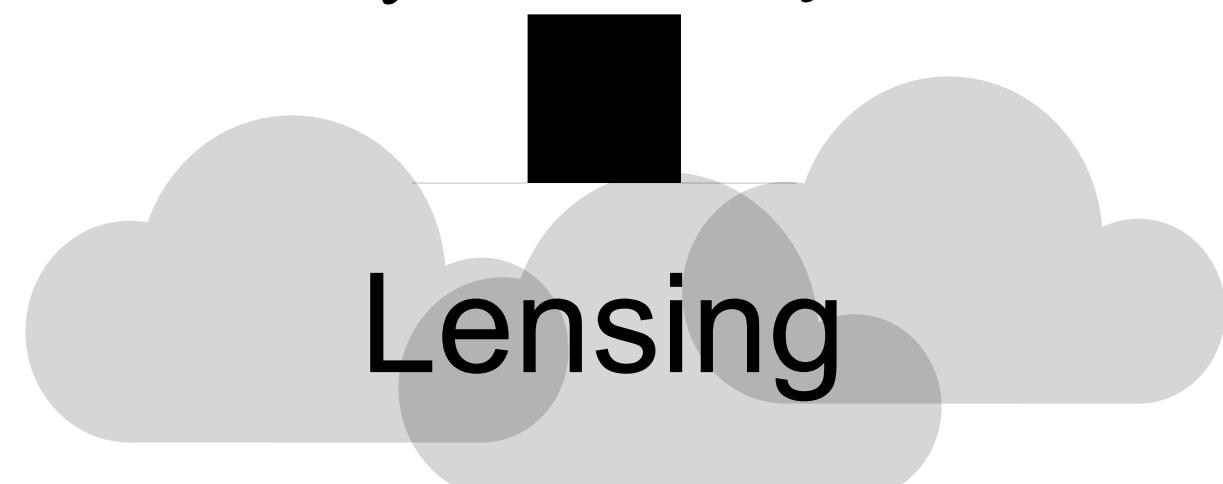
$$C_l^{\Theta\Theta} \quad C_l^{\Theta E} \quad C_l^{EE} \quad C_l^{BB}$$

$$C_l^{\Theta\Theta} \quad C_l^{\Theta E} \quad C_l^{EE} \quad C_l^{BB} \quad C_l^{\Theta B} \quad C_l^{EB}$$

$$C_l^{\Theta B} \quad C_l^{EB}$$

CLASS Lesgourgues (2011)
(public code for CMB)

$$C_l^{\Theta\Theta} \quad C_l^{\Theta E} \quad C_l^{EE} \quad C_l^{BB}$$



$$C_l^{\Theta\Theta} \quad C_l^{\Theta E} \quad C_l^{EE} \quad C_l^{BB}$$

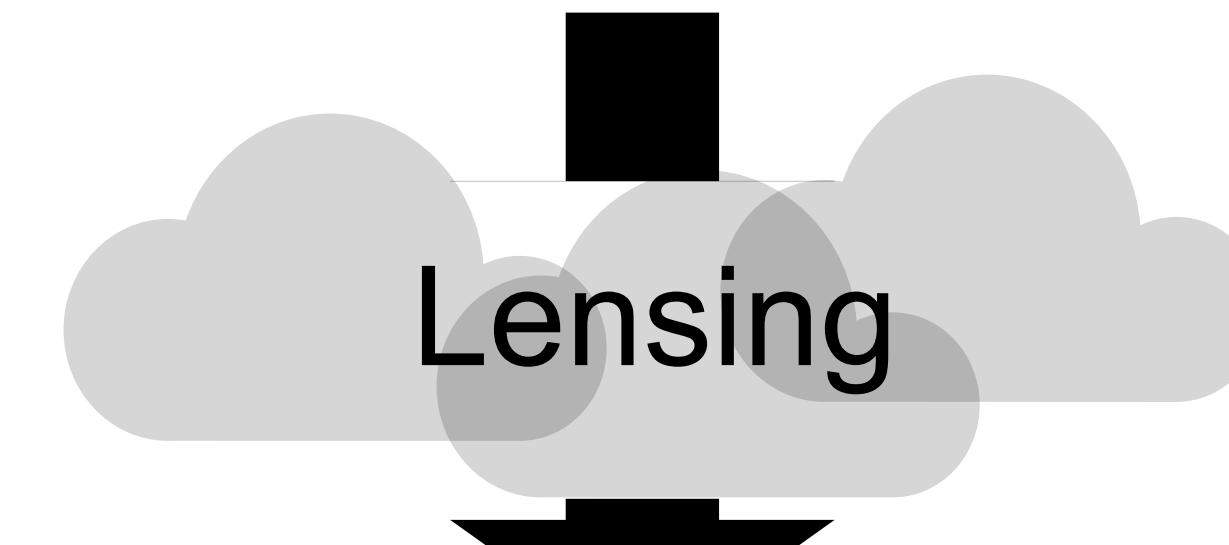
modify



CLASS +  Axion
(including cosmic birefringence)

Nakatsuka et al. (2022)

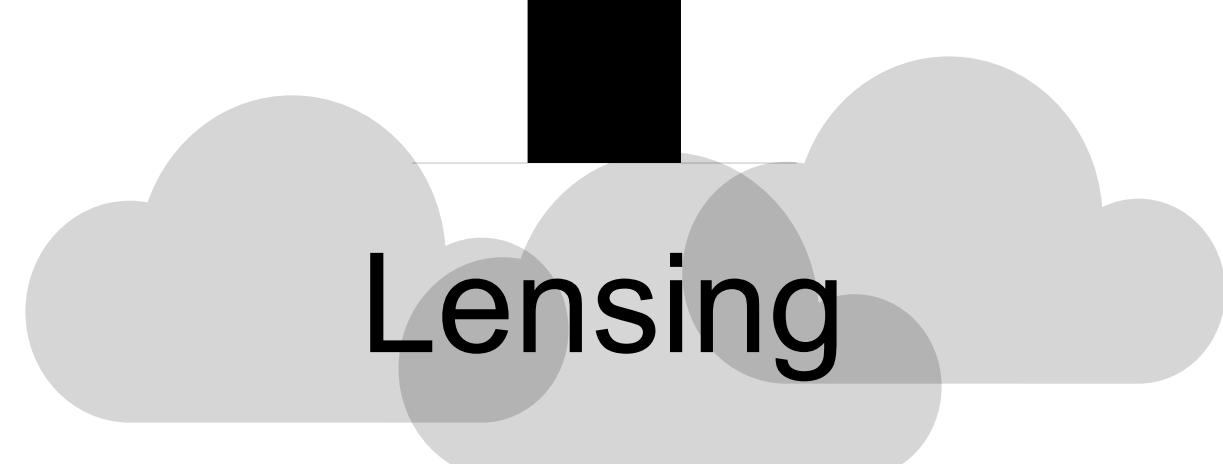
$$C_l^{\Theta\Theta} \quad C_l^{\Theta E} \quad C_l^{EE} \quad C_l^{BB} \quad C_l^{\Theta B} \quad C_l^{EB}$$



$$C_l^{\Theta\Theta} \quad C_l^{\Theta E} \quad C_l^{EE} \quad C_l^{BB}$$

CLASS Lesgourgues (2011)
(public code for CMB)

$$C_l^{\Theta\Theta} \quad C_l^{\Theta E} \quad C_l^{EE} \quad C_l^{BB}$$



$$C_l^{\Theta\Theta} \quad C_l^{\Theta E} \quad C_l^{EE} \quad C_l^{BB}$$

modify



CLASS +  Axion
(including cosmic birefringence)

Nakatsuka et al. (2022)

$$C_l^{\Theta\Theta} \quad C_l^{\Theta E} \quad C_l^{EE} \quad C_l^{BB}$$



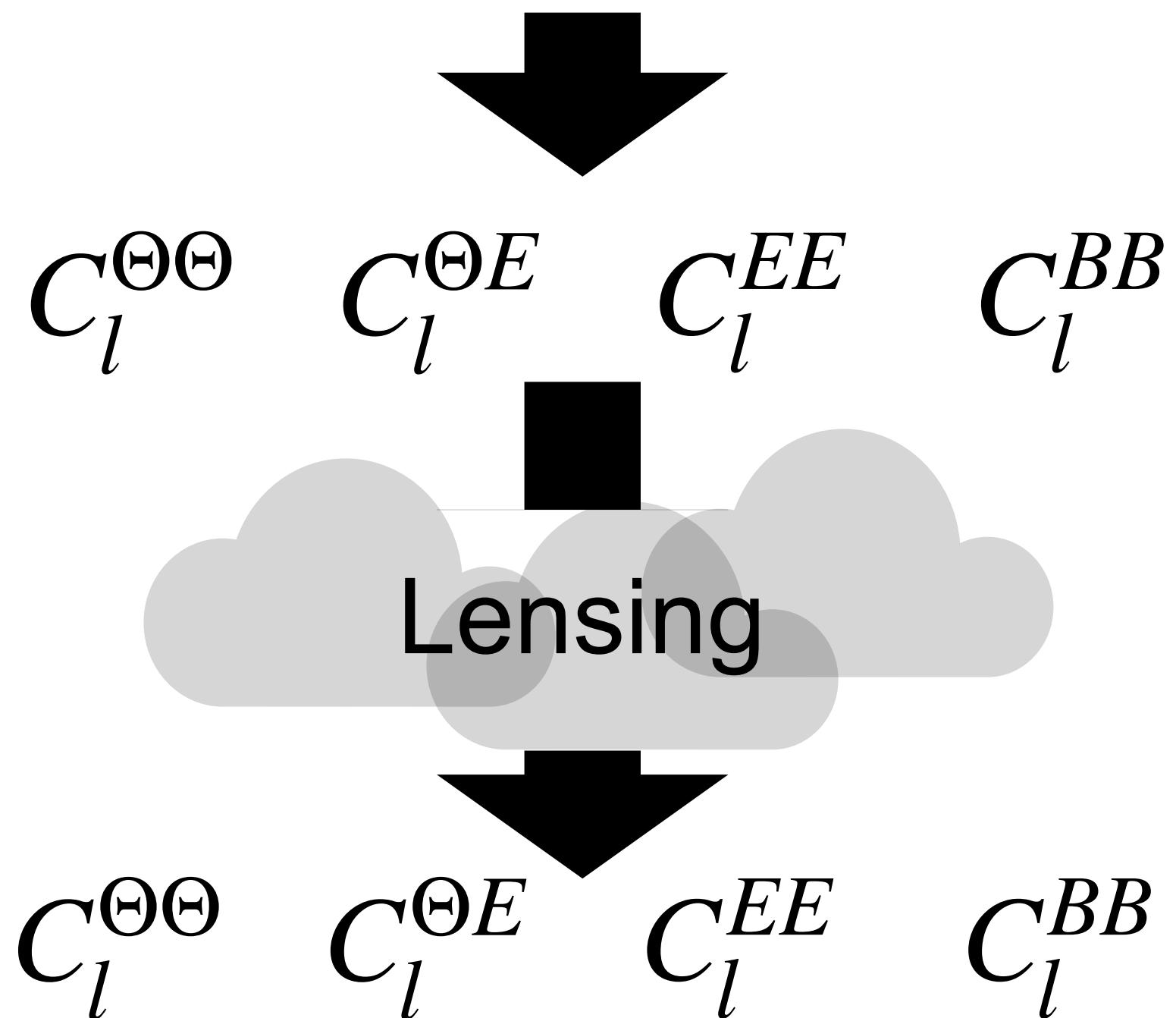
$$C_l^{\Theta\Theta} \quad C_l^{\Theta E} \quad C_l^{EE} \quad C_l^{BB}$$

$$C_l^{\Theta B} \quad C_l^{EB}$$



$$C_l^{\Theta B} \quad C_l^{EB}$$

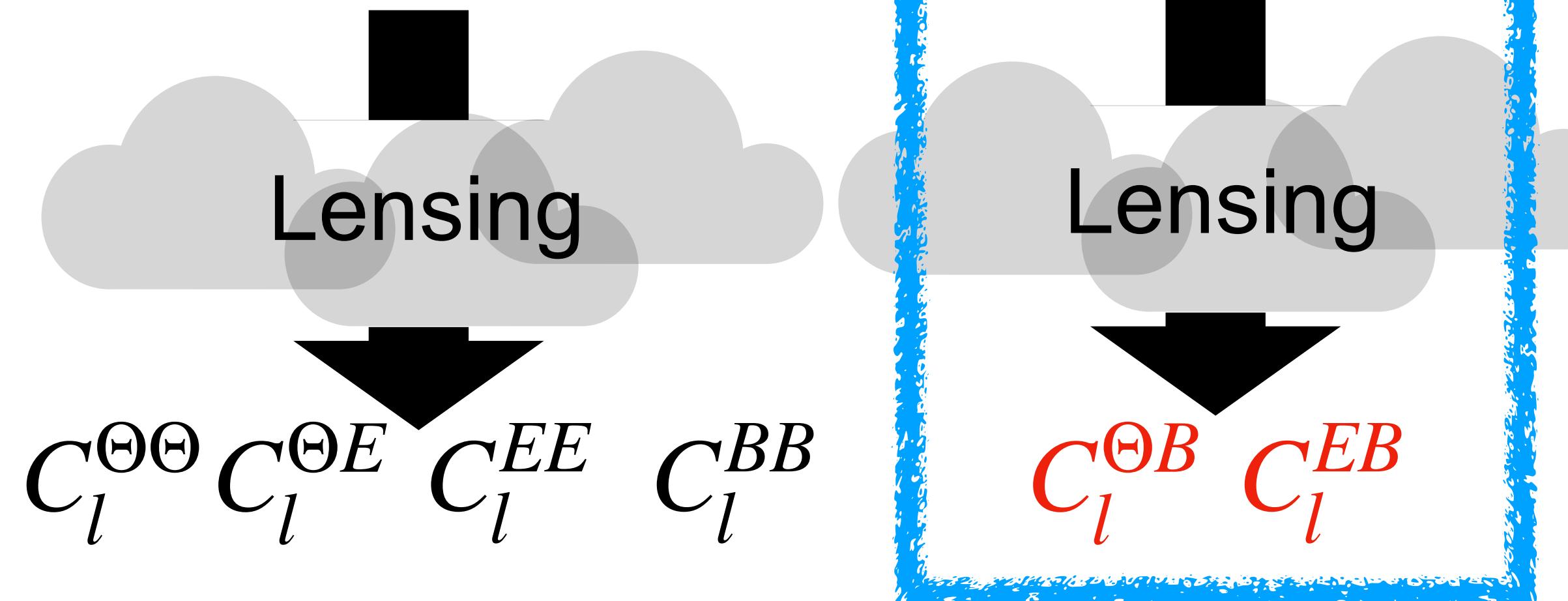
CLASS Lesgourgues (2011)
(public code for CMB)



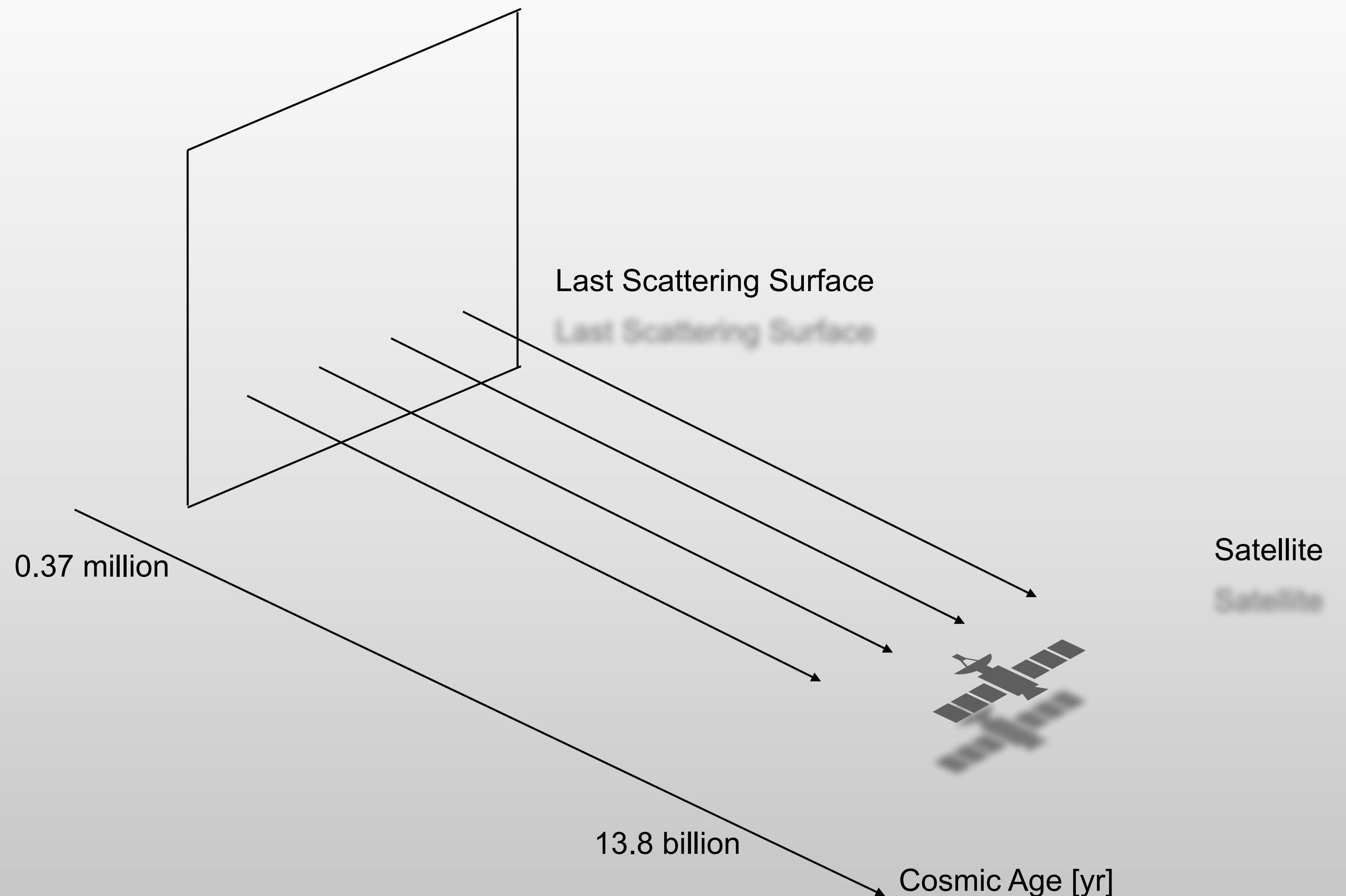
CLASS +  Axion
(including cosmic birefringence)

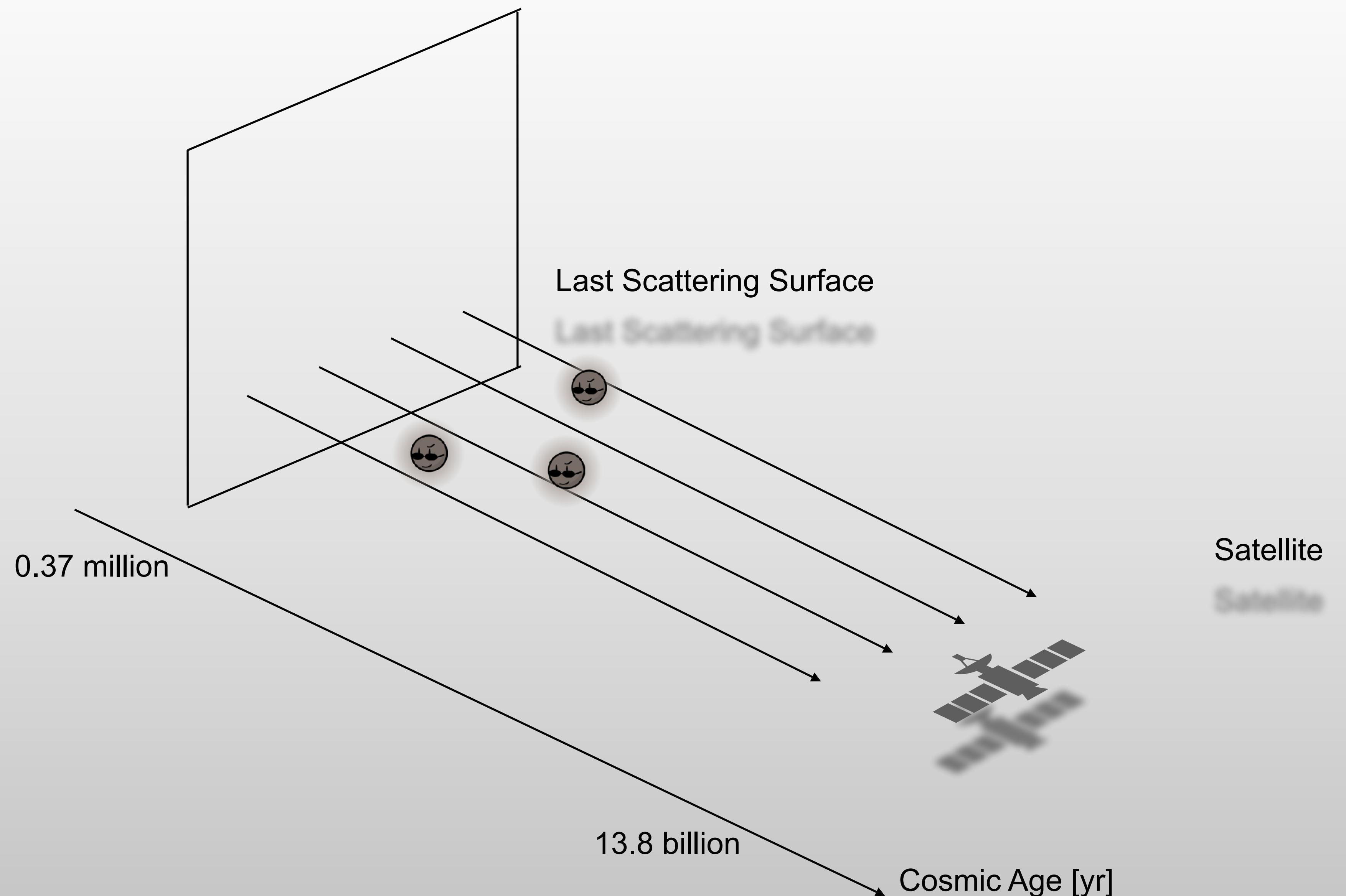
Nakatsuka et al. (2022)

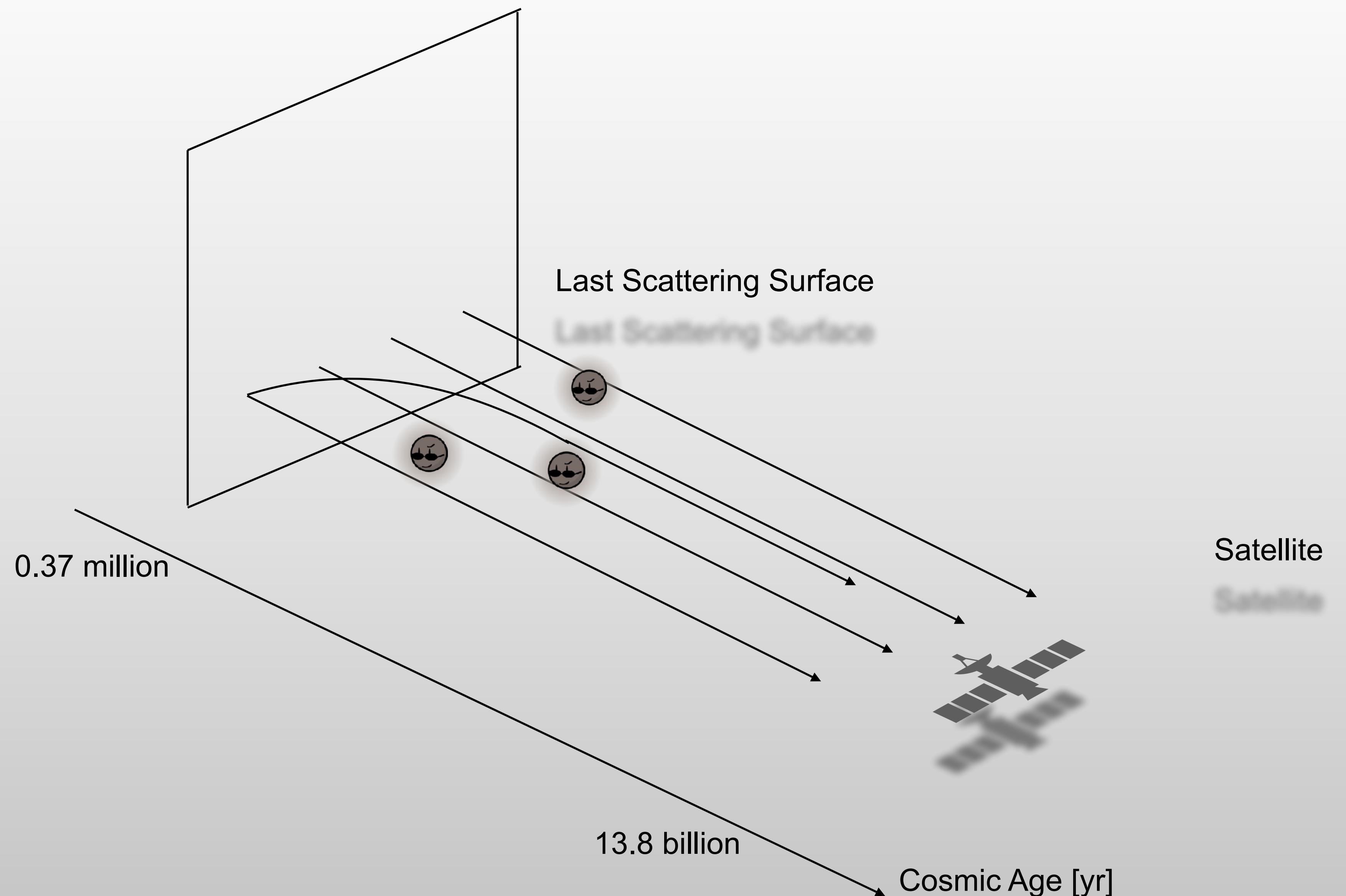
$$C_l^{\Theta\Theta} \quad C_l^{\Theta E} \quad C_l^{EE} \quad C_l^{BB}$$



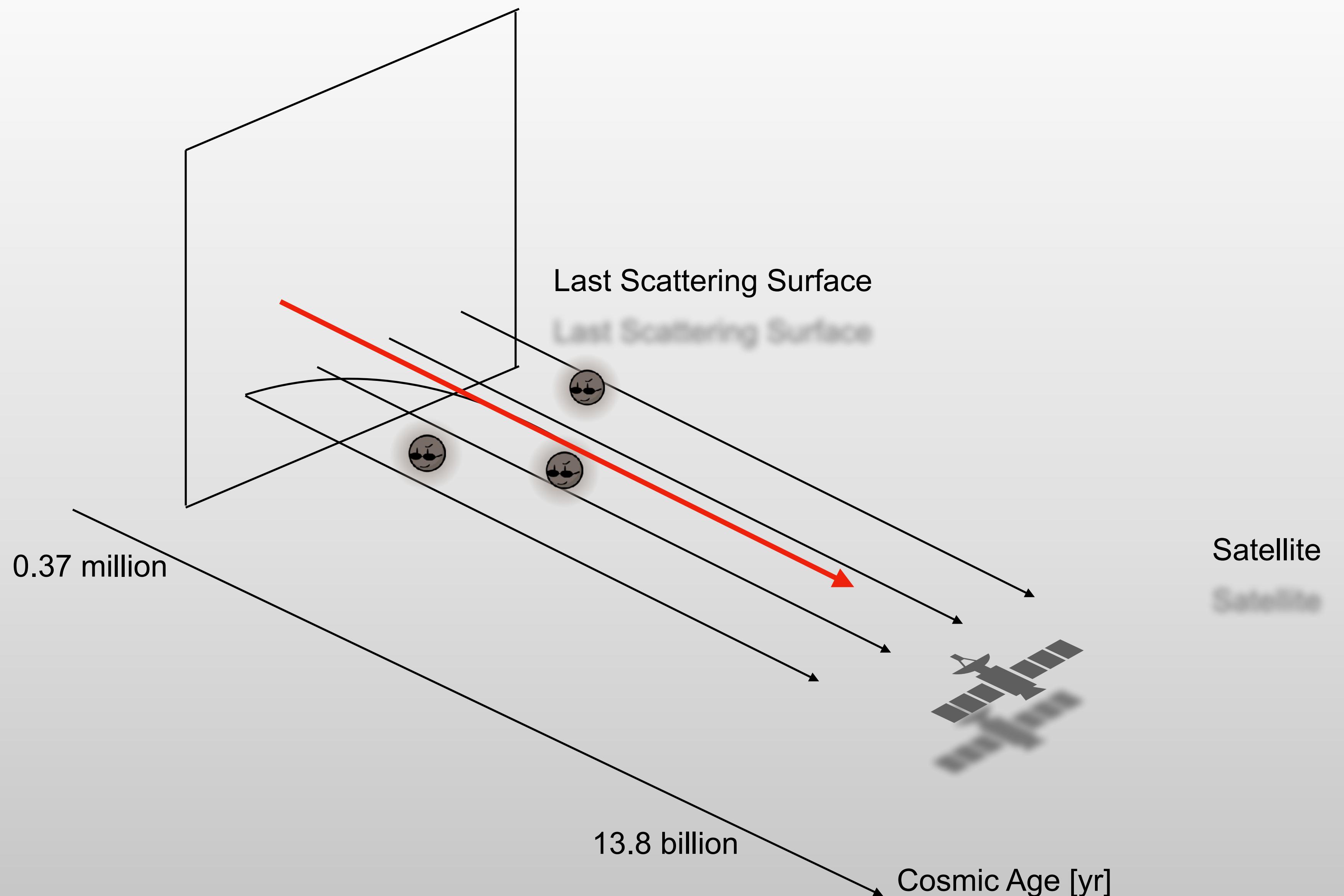
This work



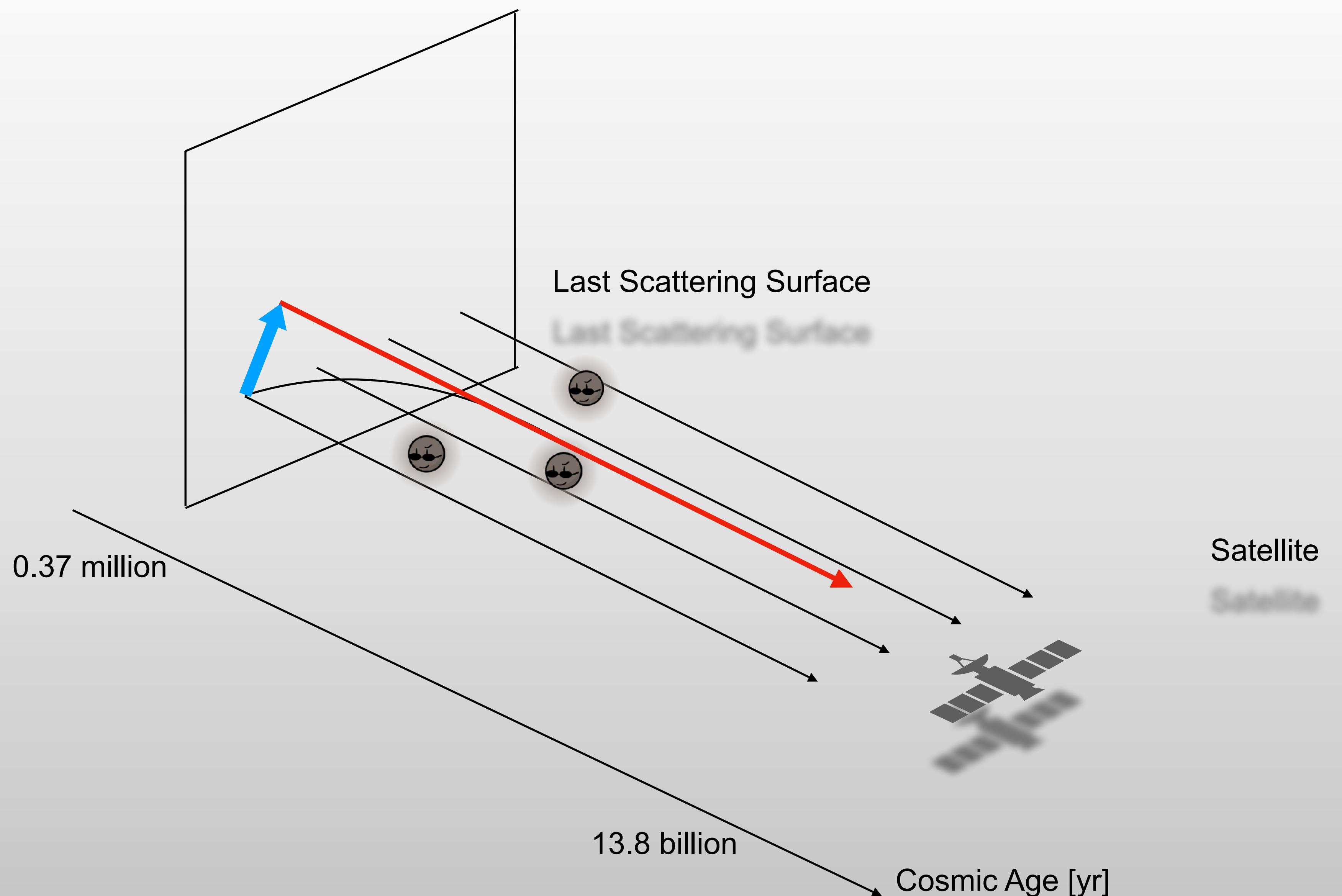
 : dark matter



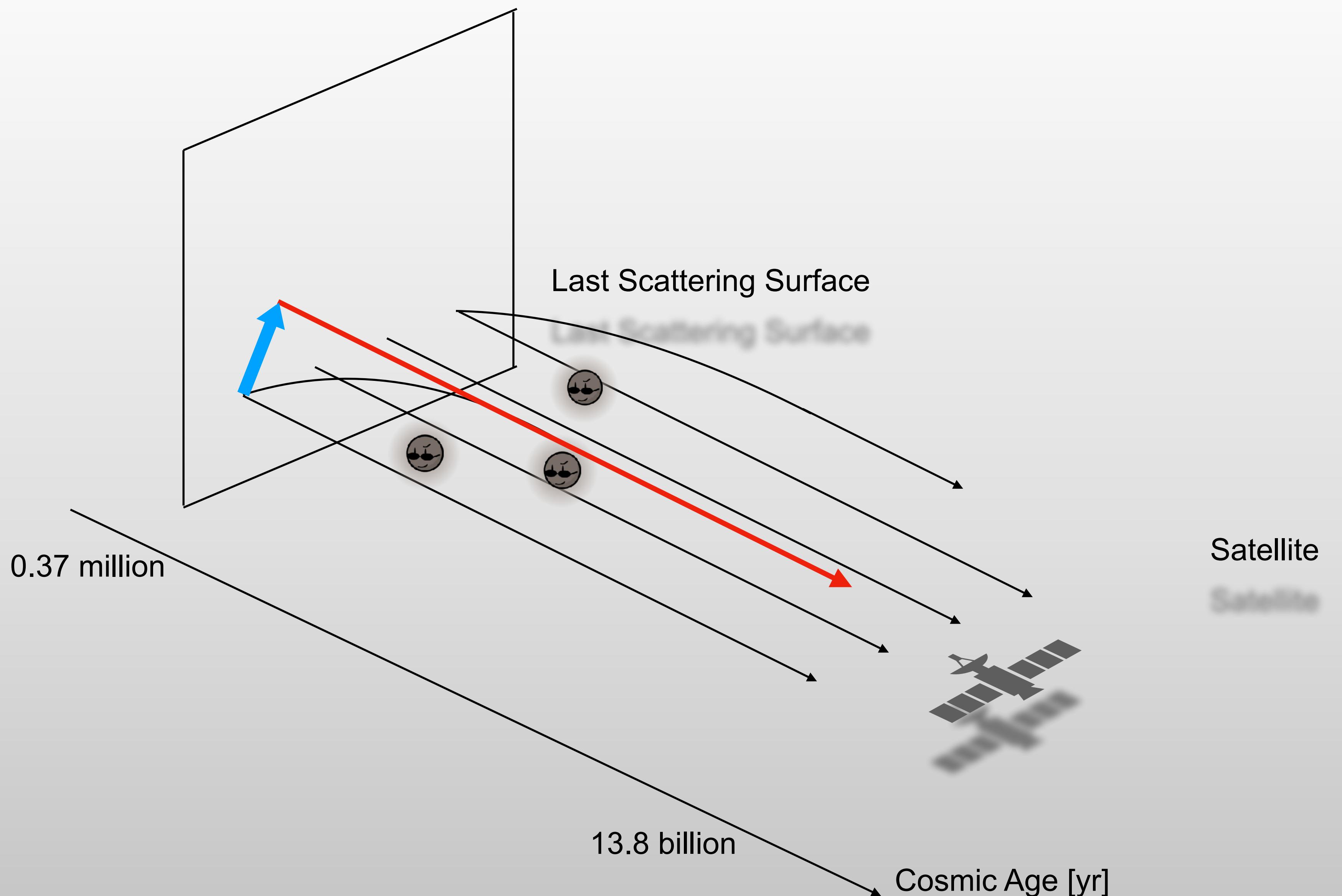
 : dark matter



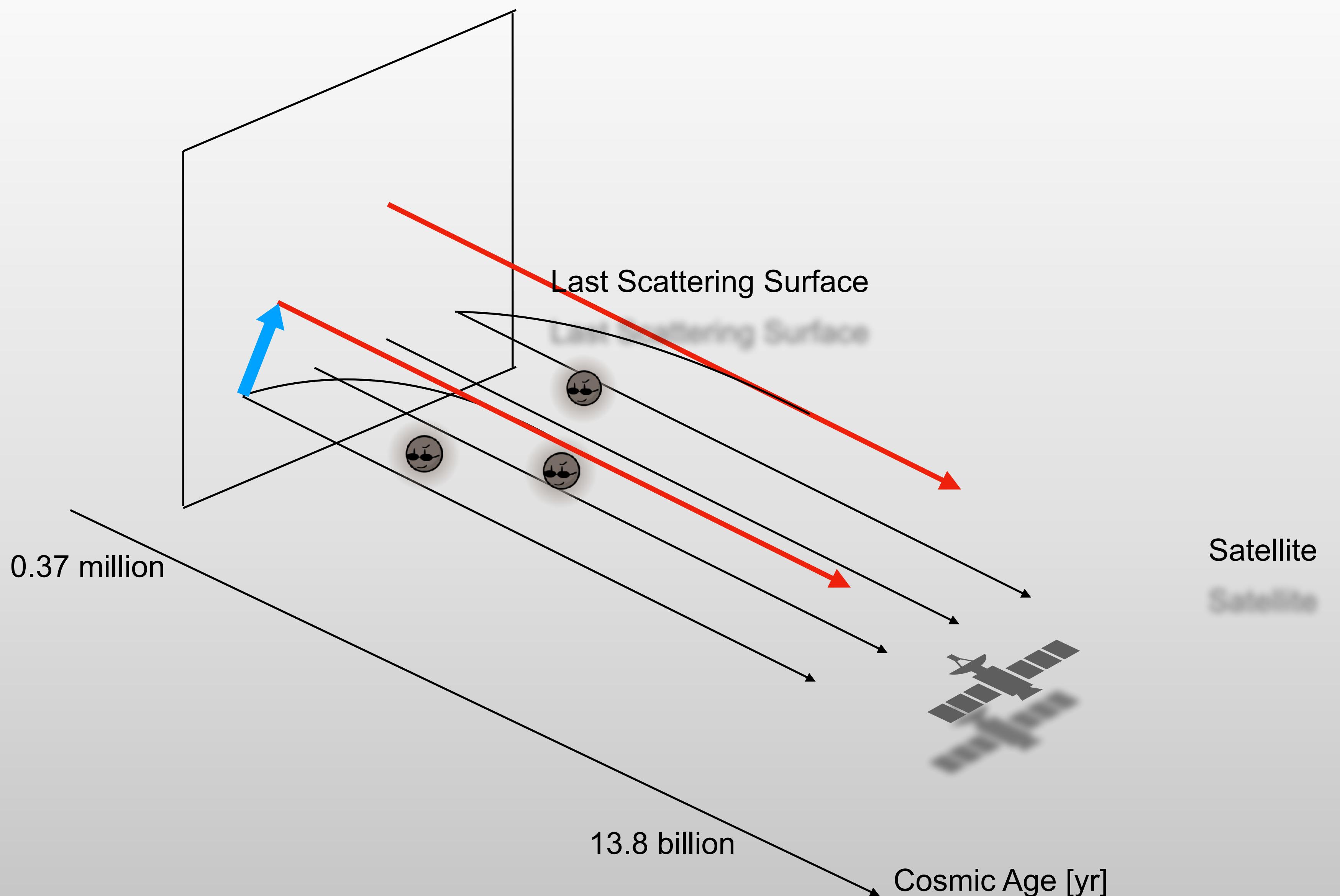
 : dark matter

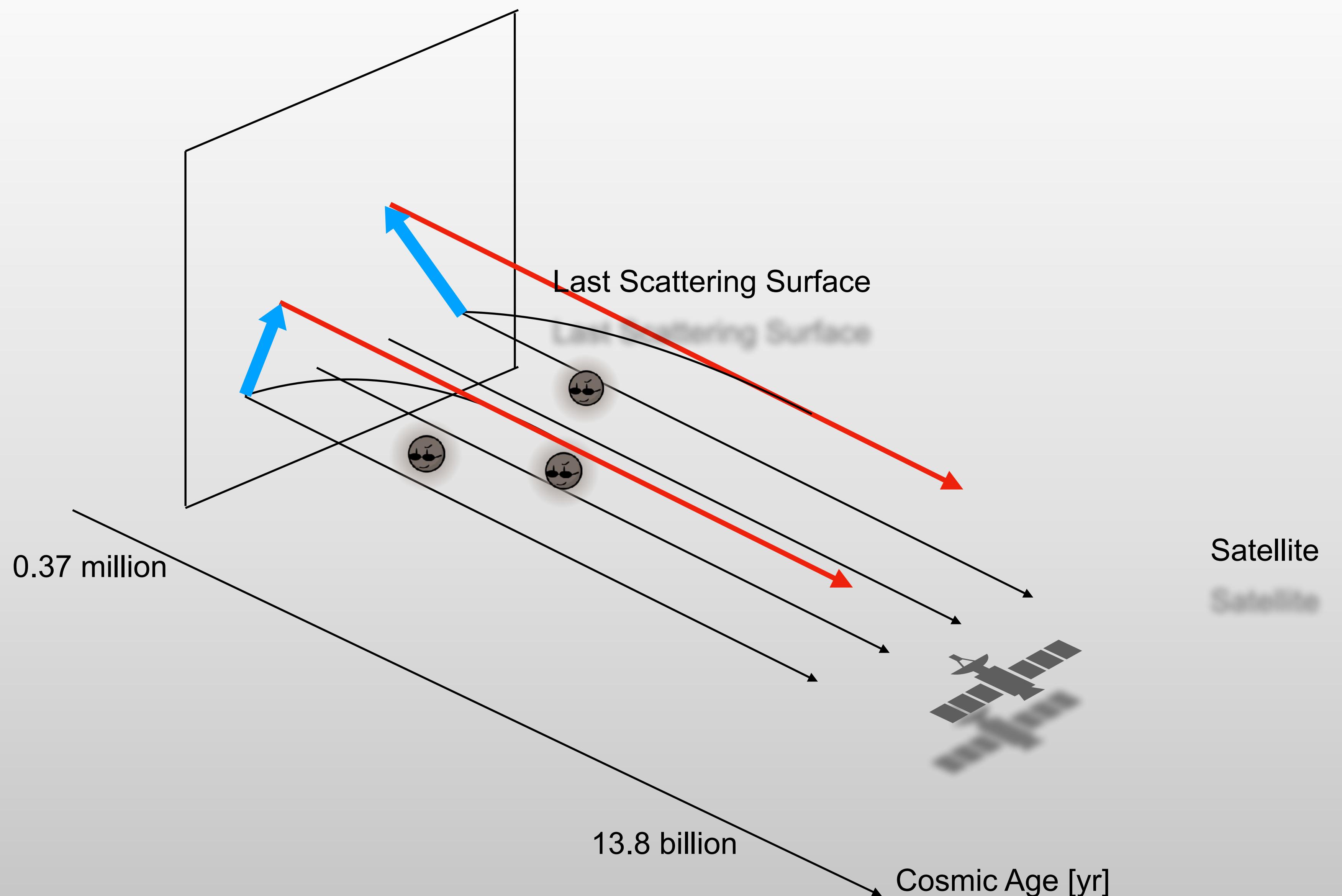


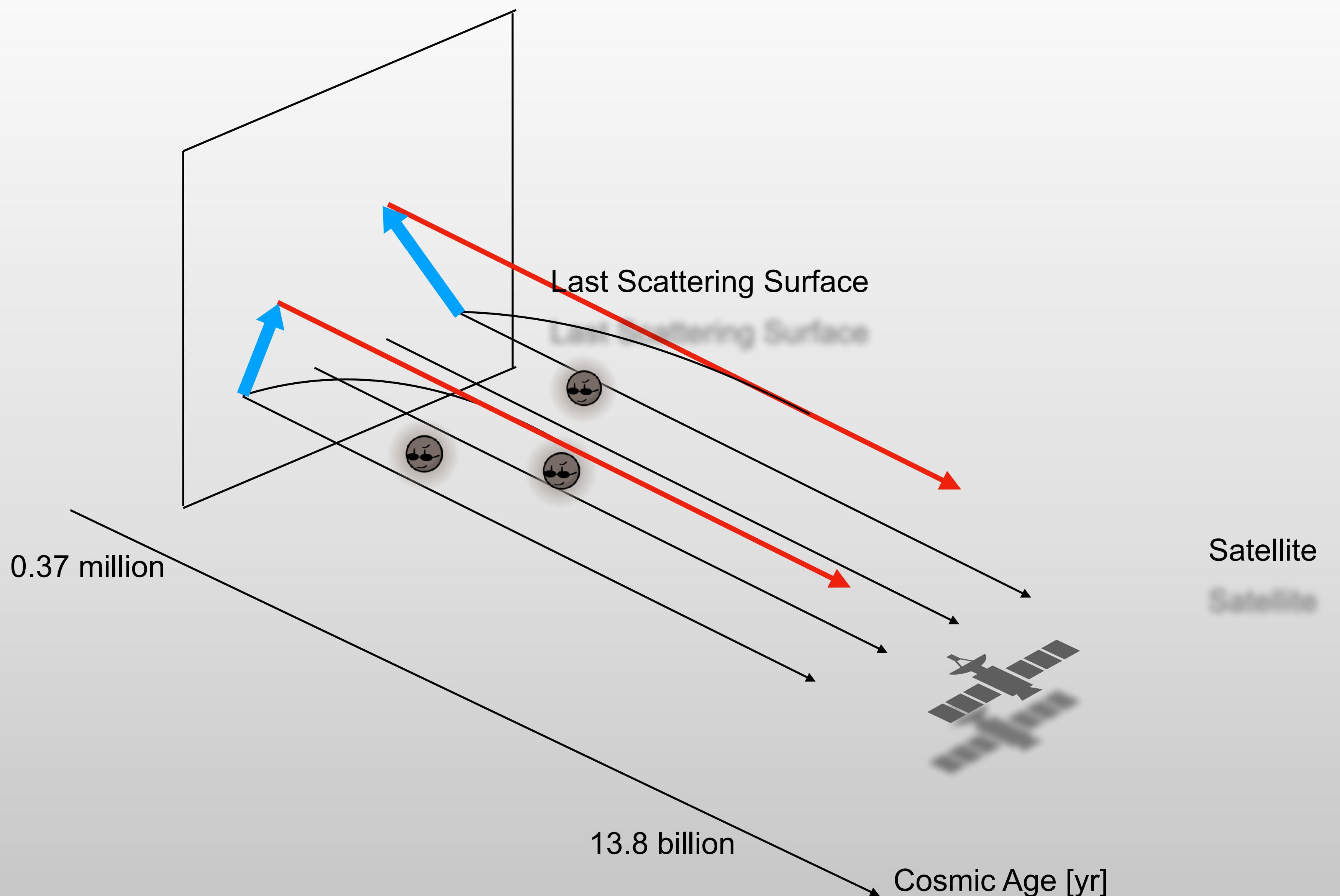
 : dark matter

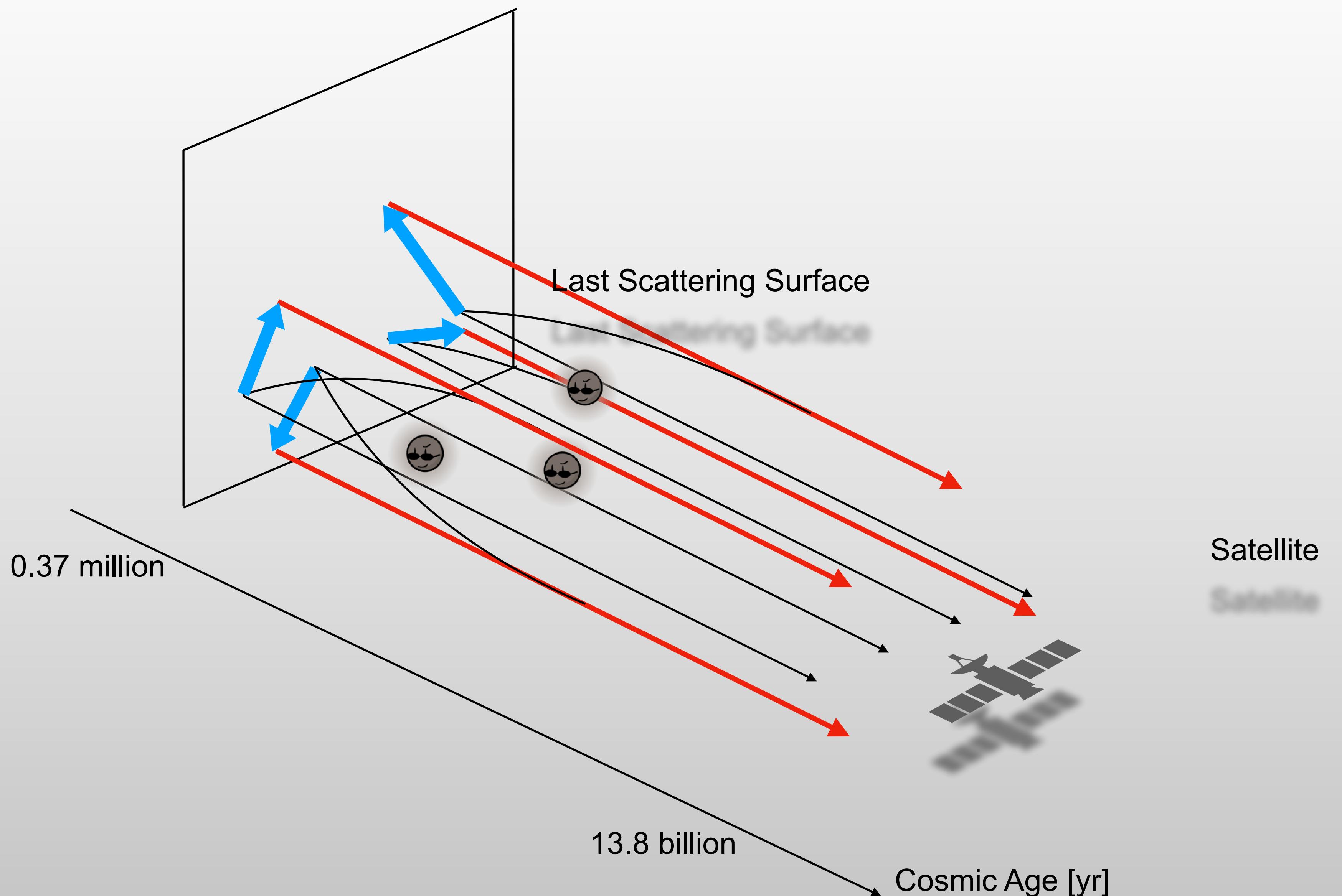


 : dark matter

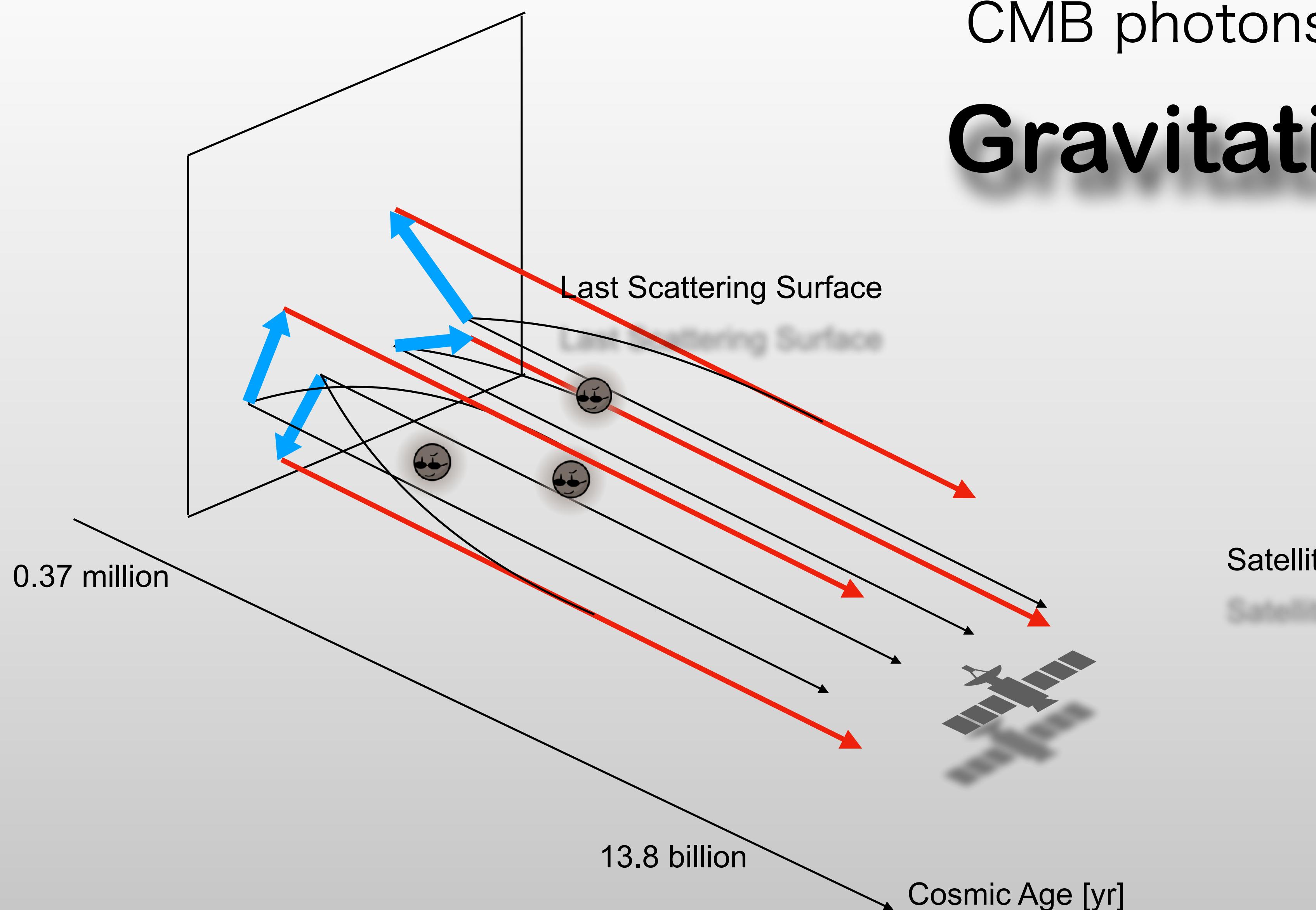


 : dark matter





 : dark matter



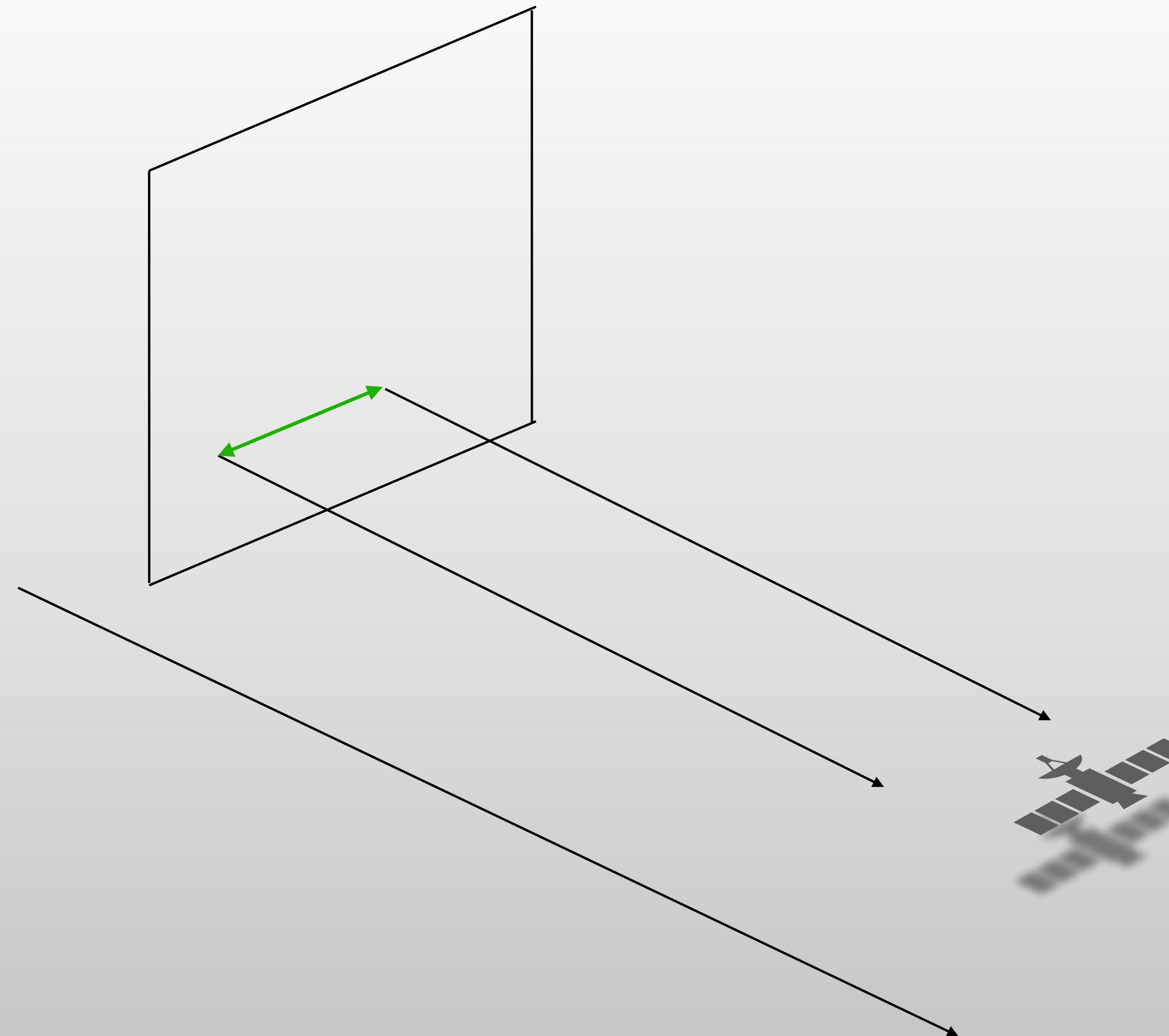
CMB photons are deflected

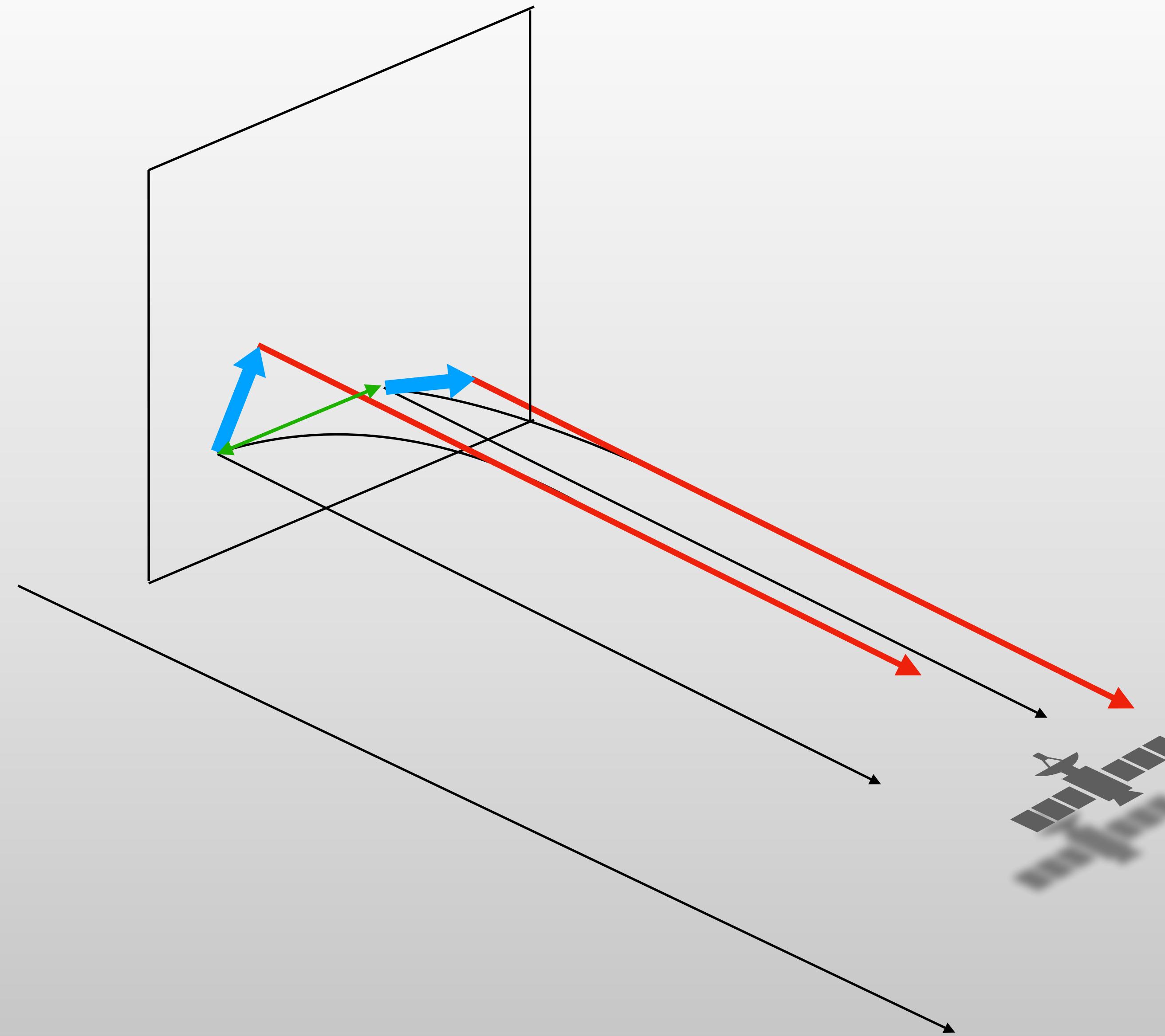
Gravitational Lensing

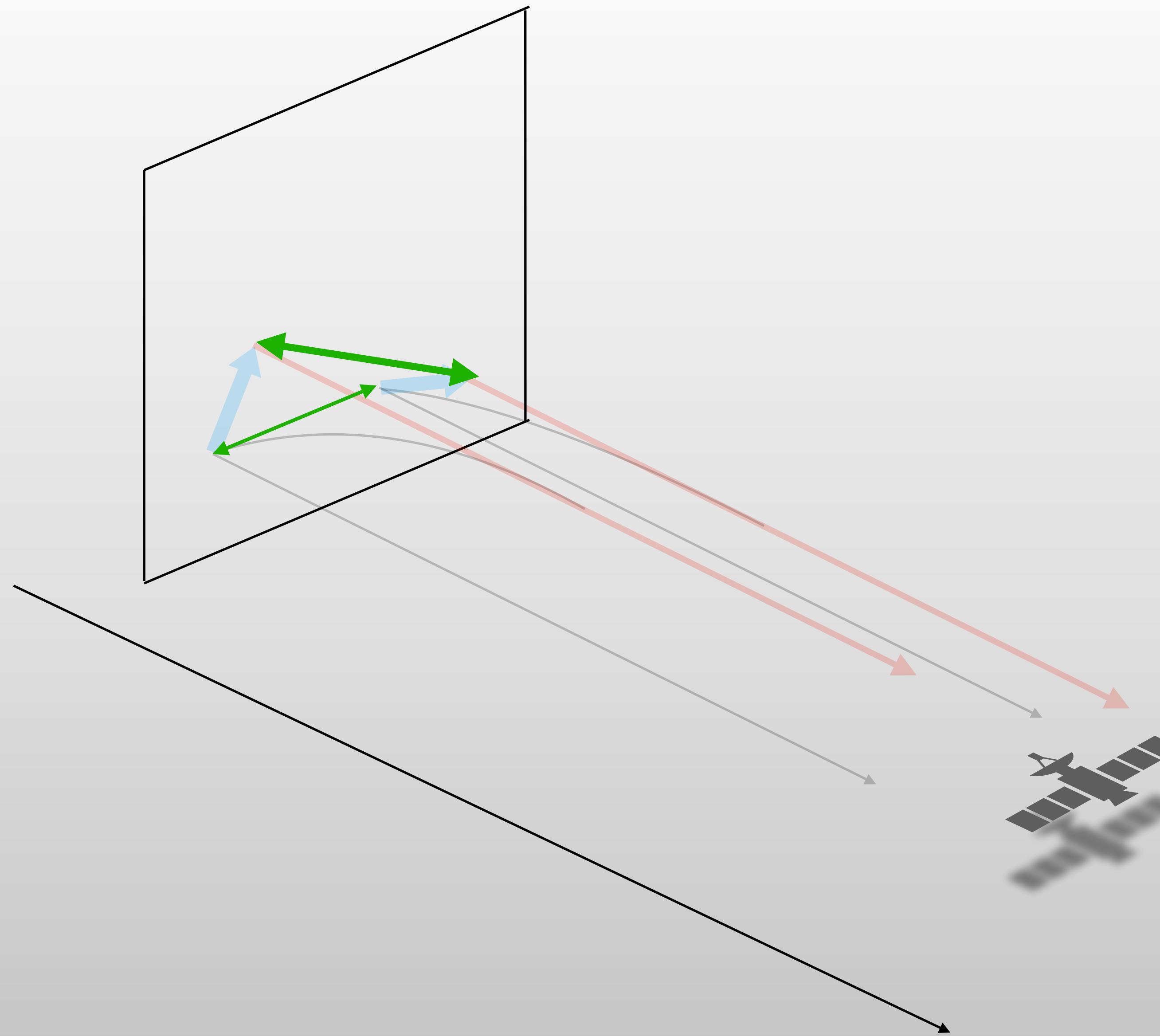
Lewis & Challinor(2006)

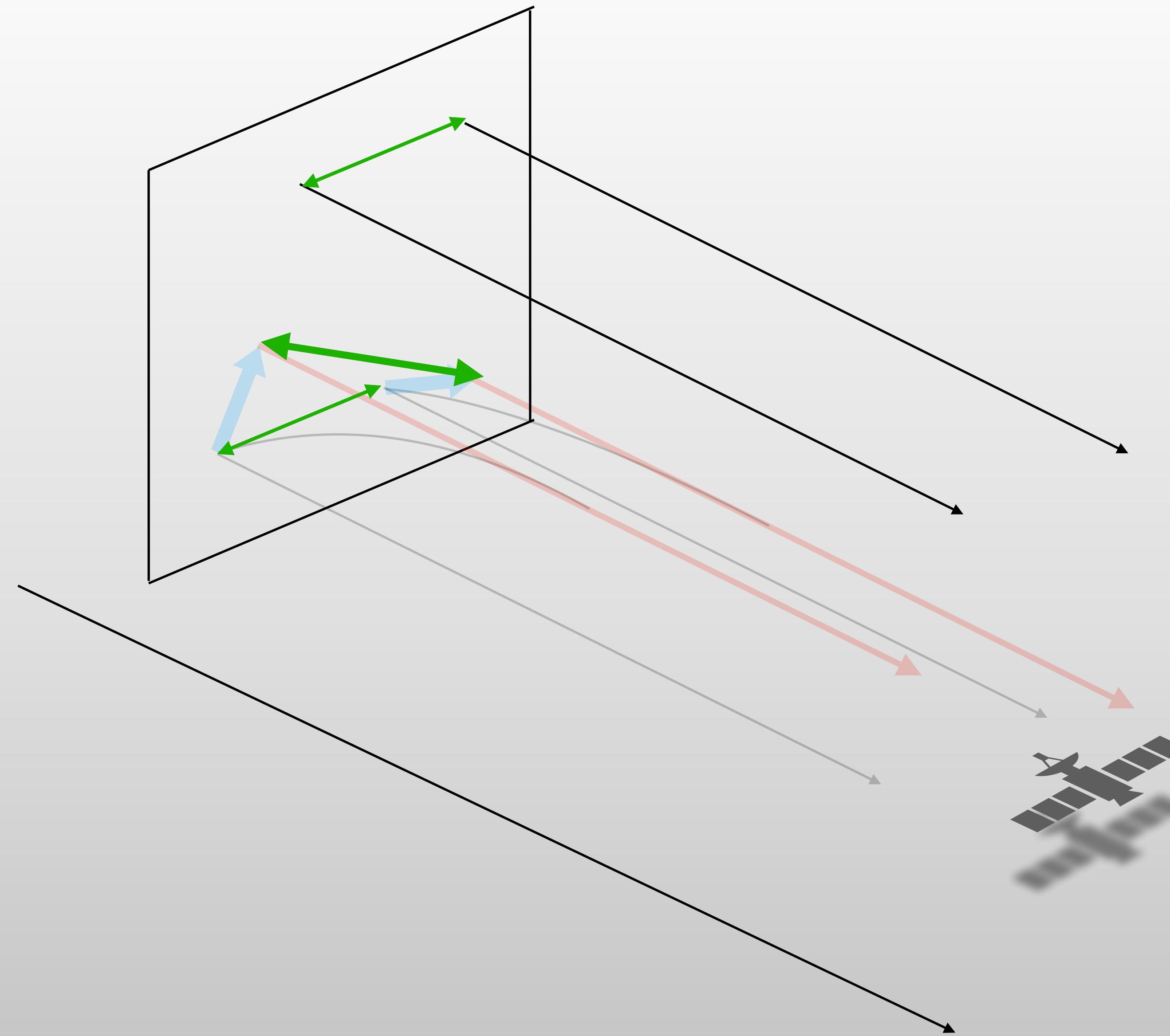
Satellite

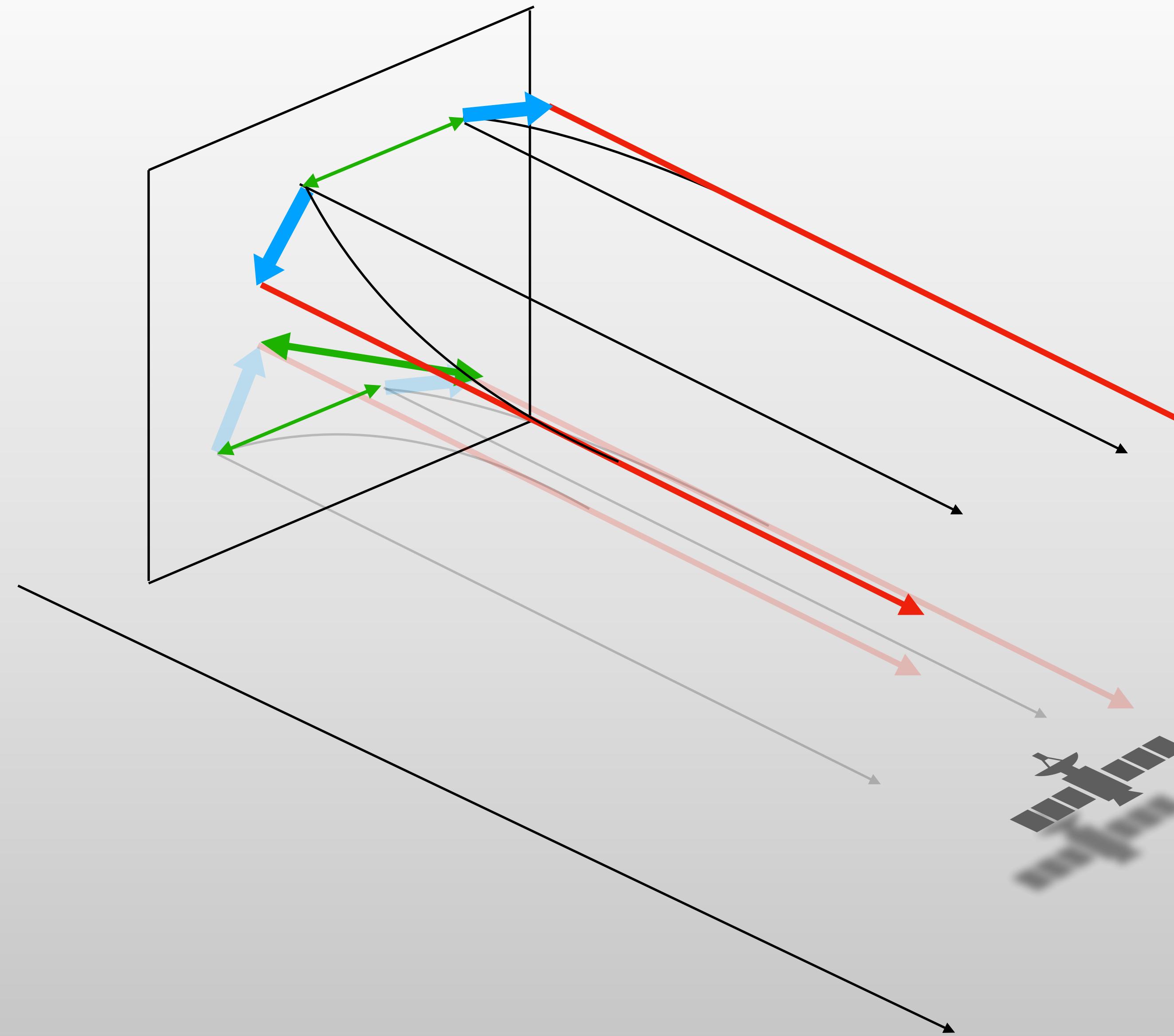
 : dark matter

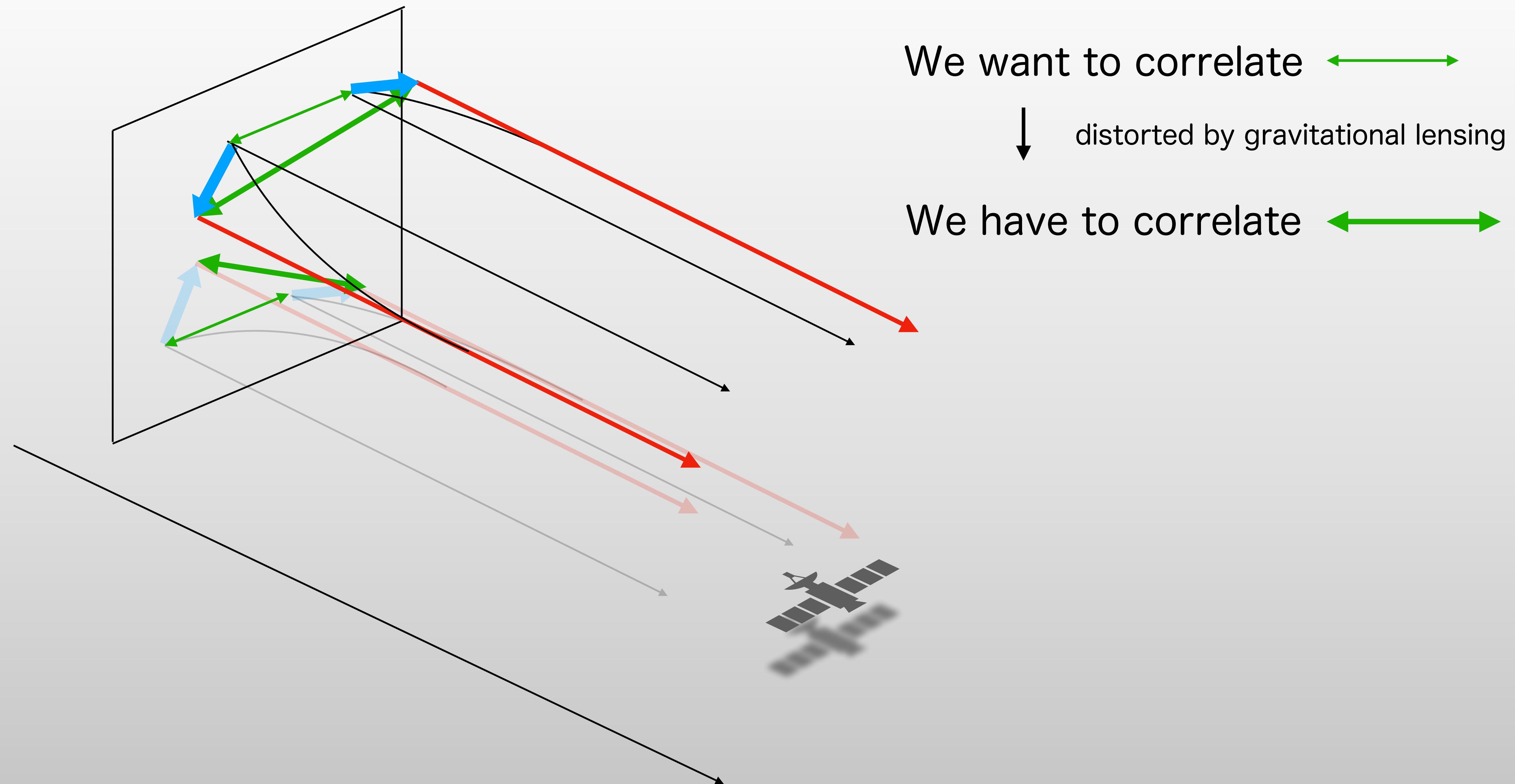


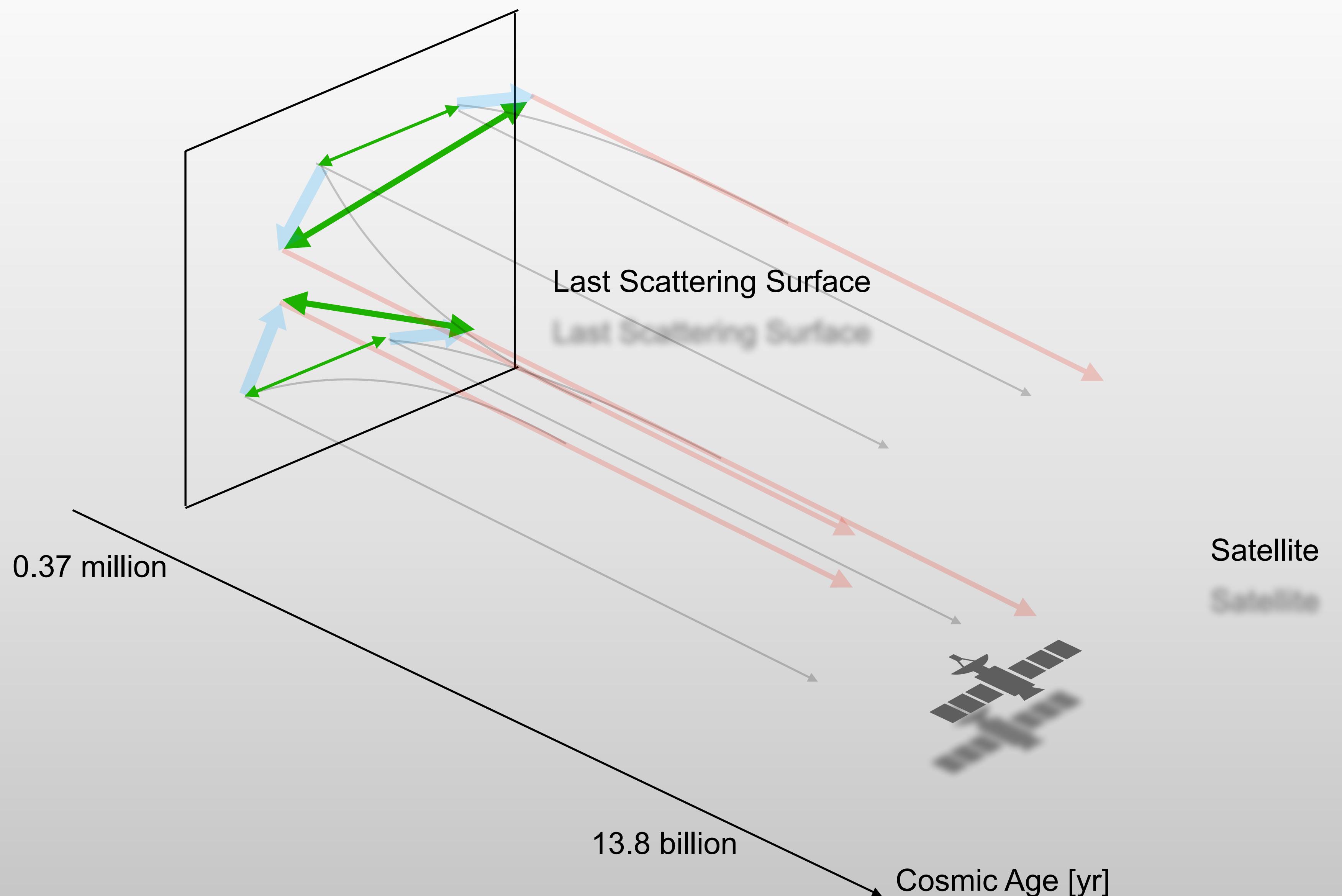


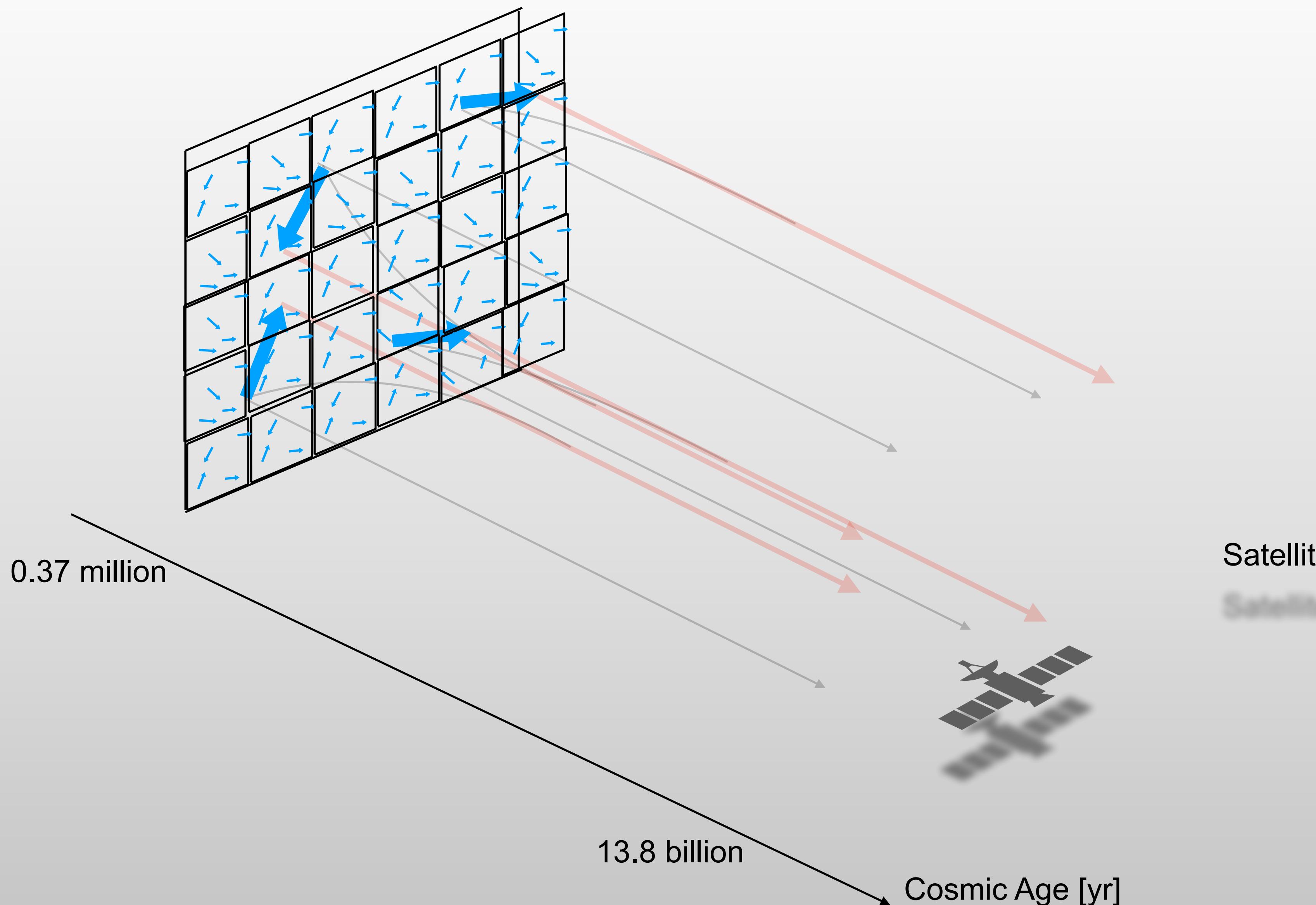


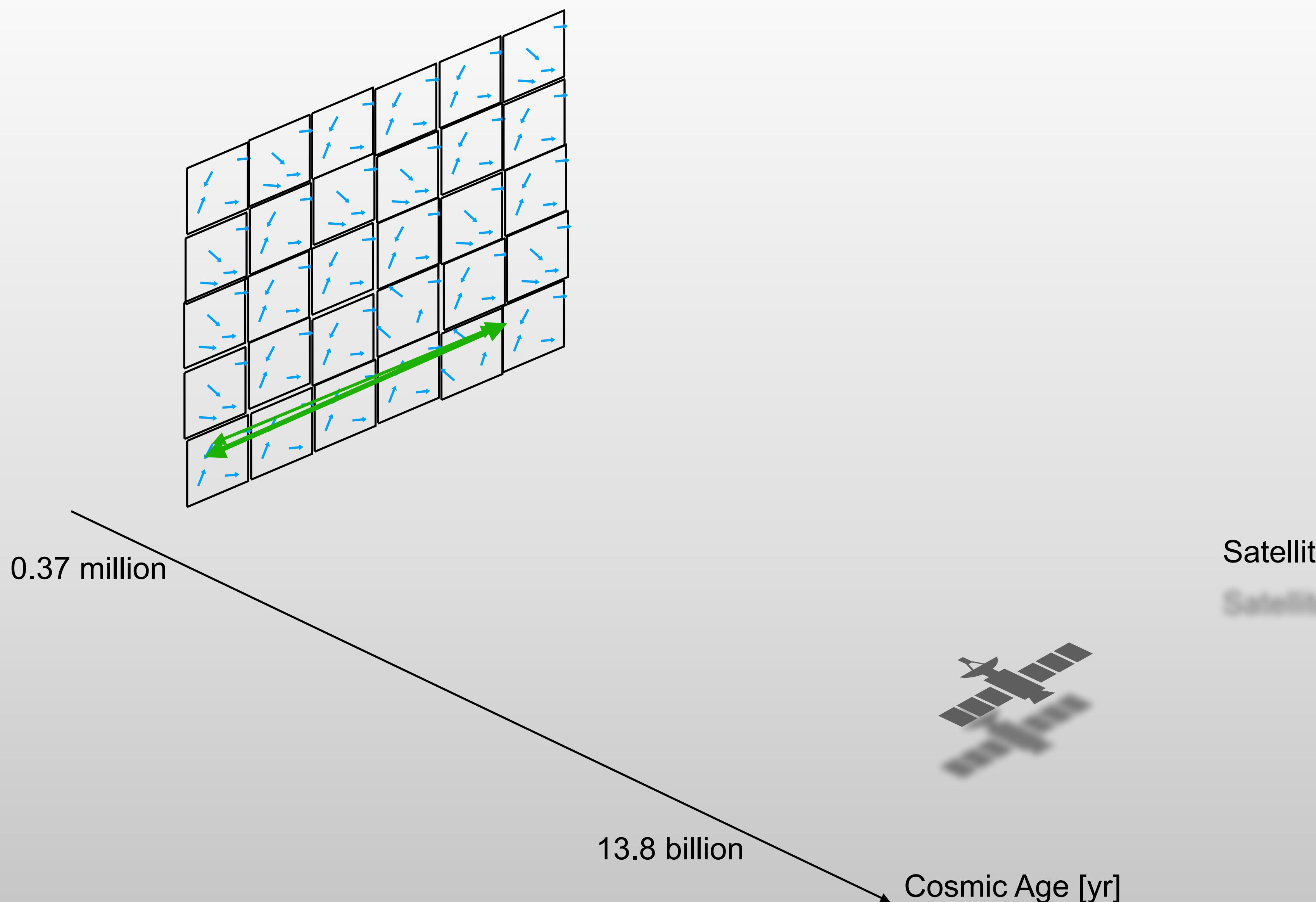


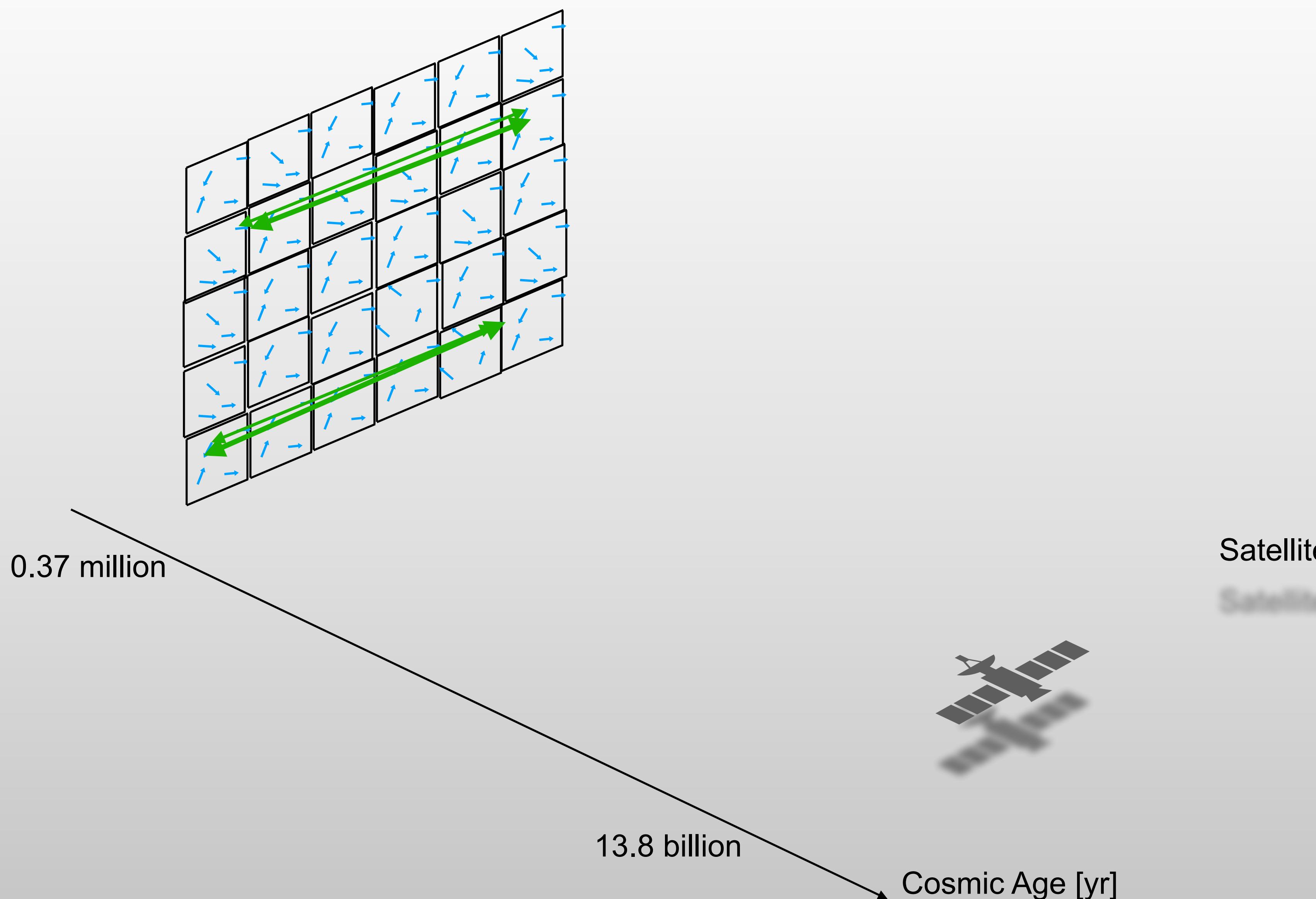


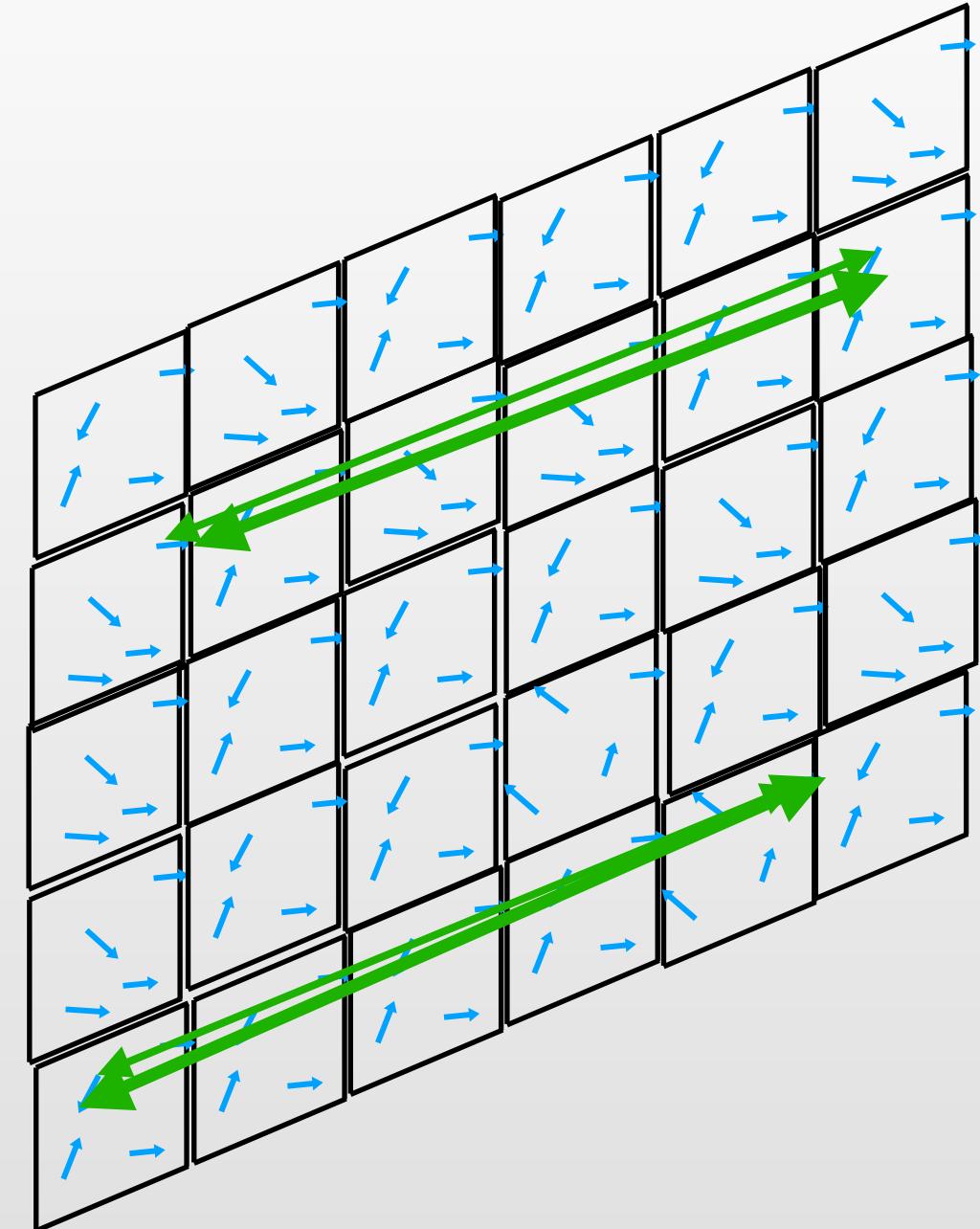






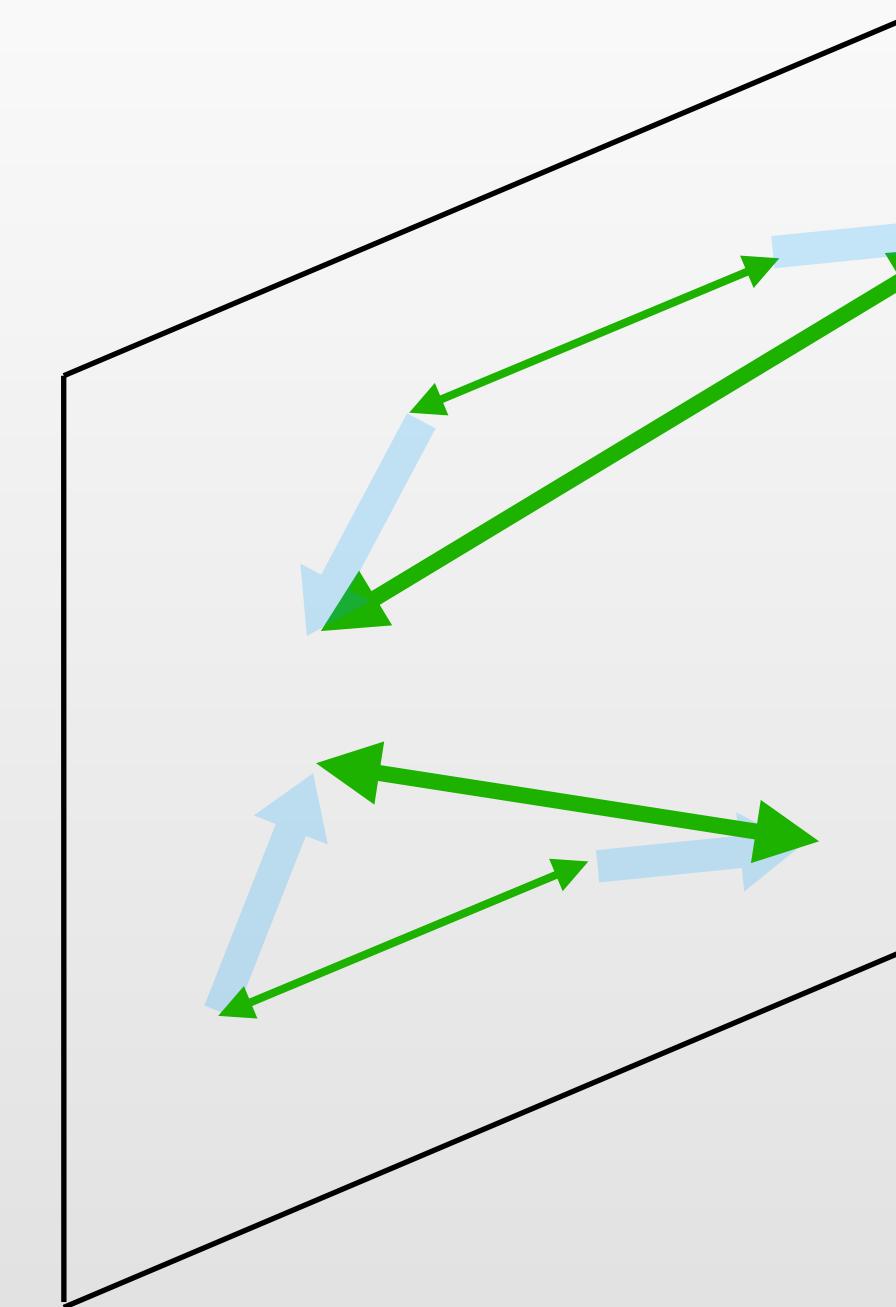






large scale

lensing effect is small



small scale

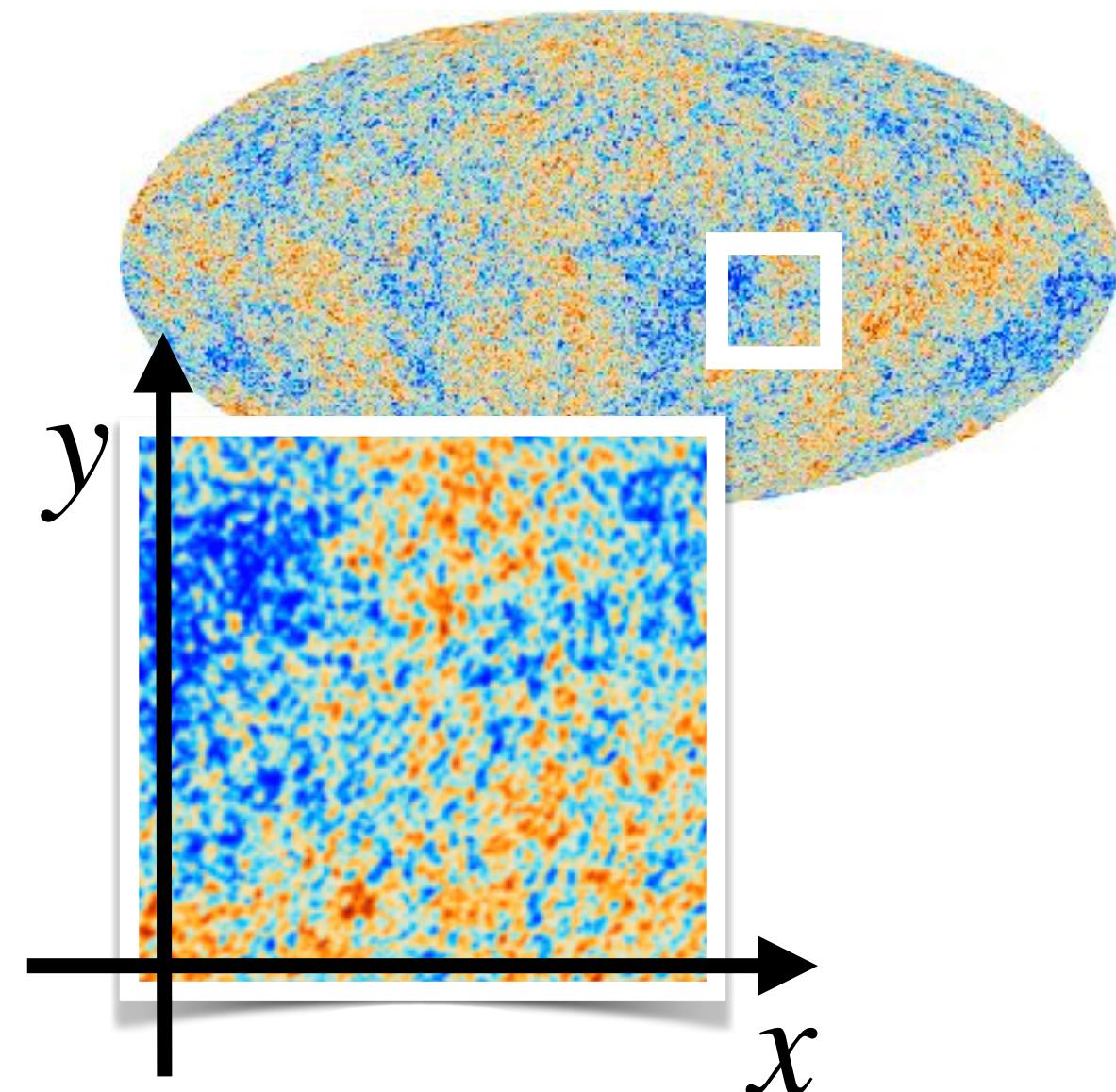
lensing effect is large

How to do lensing correction ?

How to do lensing correction ?

Temperature fluctuation

$$\Theta(x, y) \quad \left(= \frac{\delta T}{T}(x, y) \right)$$



© ESA and the Planck Collaboration

https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB#.Yyf0oyS8rWA.link

CC BY-SA 3.0 IGO

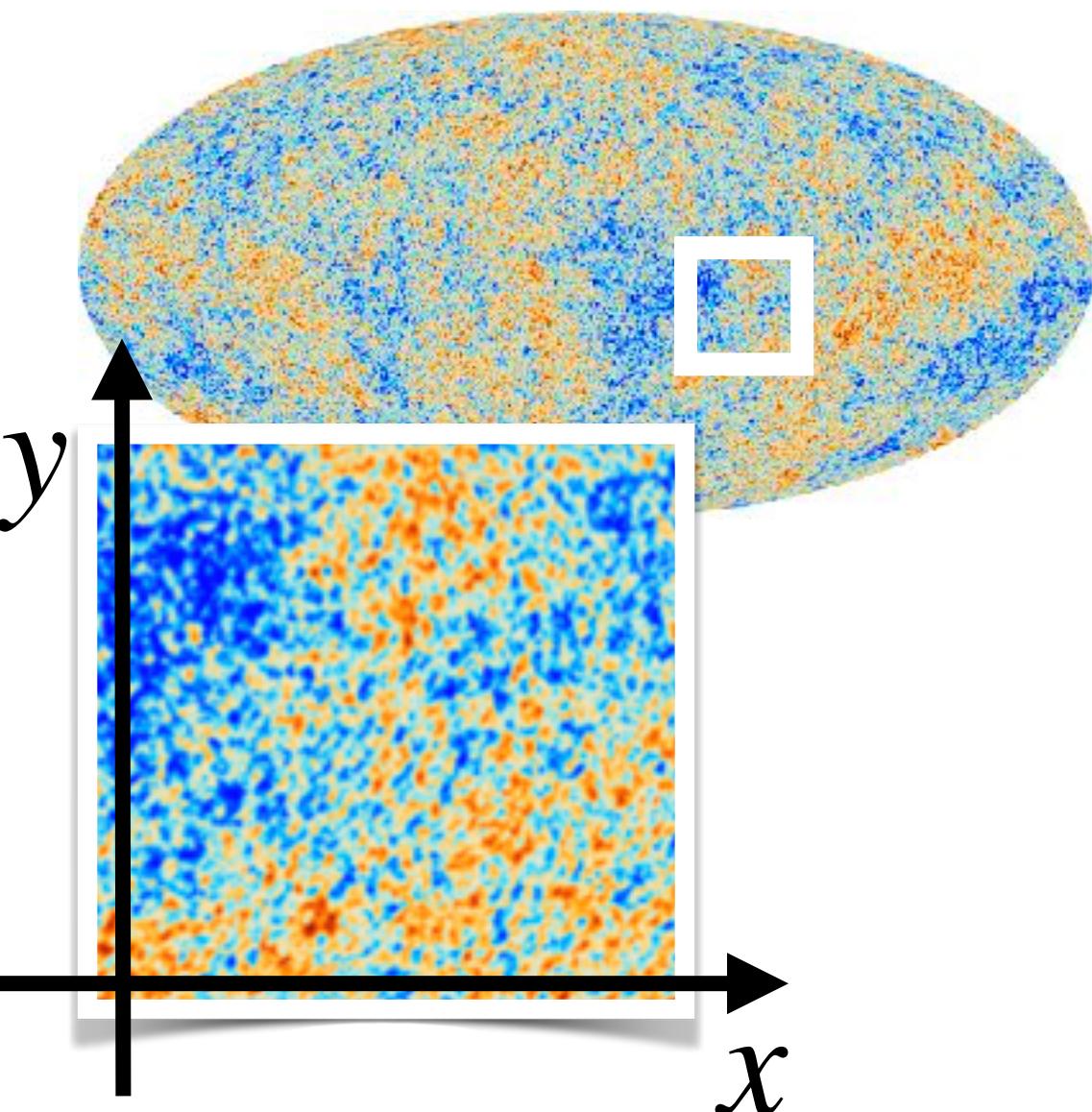
<https://creativecommons.org/licenses/by-sa/3.0/igo/>

Partly modified

How to do lensing correction ?

Temperature fluctuation

$$\Theta(x, y) \left(= \frac{\delta T}{T}(x, y) \right)$$



© ESA and the Planck Collaboration

https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB#.Yyf0oyS8rWA.link

CC BY-SA 3.0 IGO

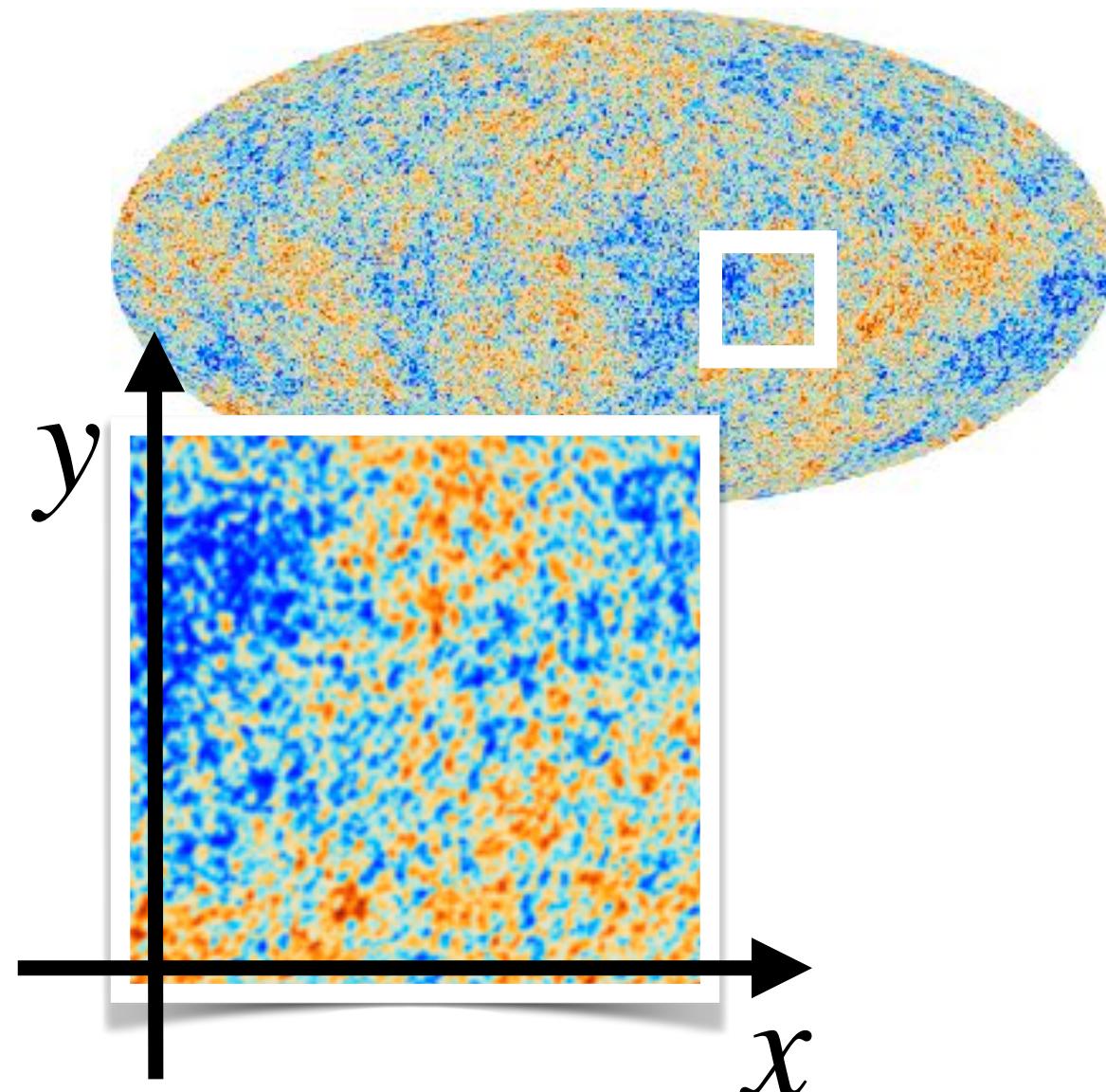
<https://creativecommons.org/licenses/by-sa/3.0/igo/>

Partly modified

How to do lensing correction ?

Temperature fluctuation

$$\Theta(x, y) \left(= \frac{\delta T}{T}(x, y) \right) \xrightarrow{\text{Auto correlation}} \xi(\beta) \equiv \langle \Theta(\mathbf{x})\Theta(\mathbf{x} + \beta) \rangle$$



© ESA and the Planck Collaboration

https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB#.Yyf0oyS8rWA.link

CC BY-SA 3.0 IGO

<https://creativecommons.org/licenses/by-sa/3.0/igo/>

Partly modified

How to do lensing correction ?

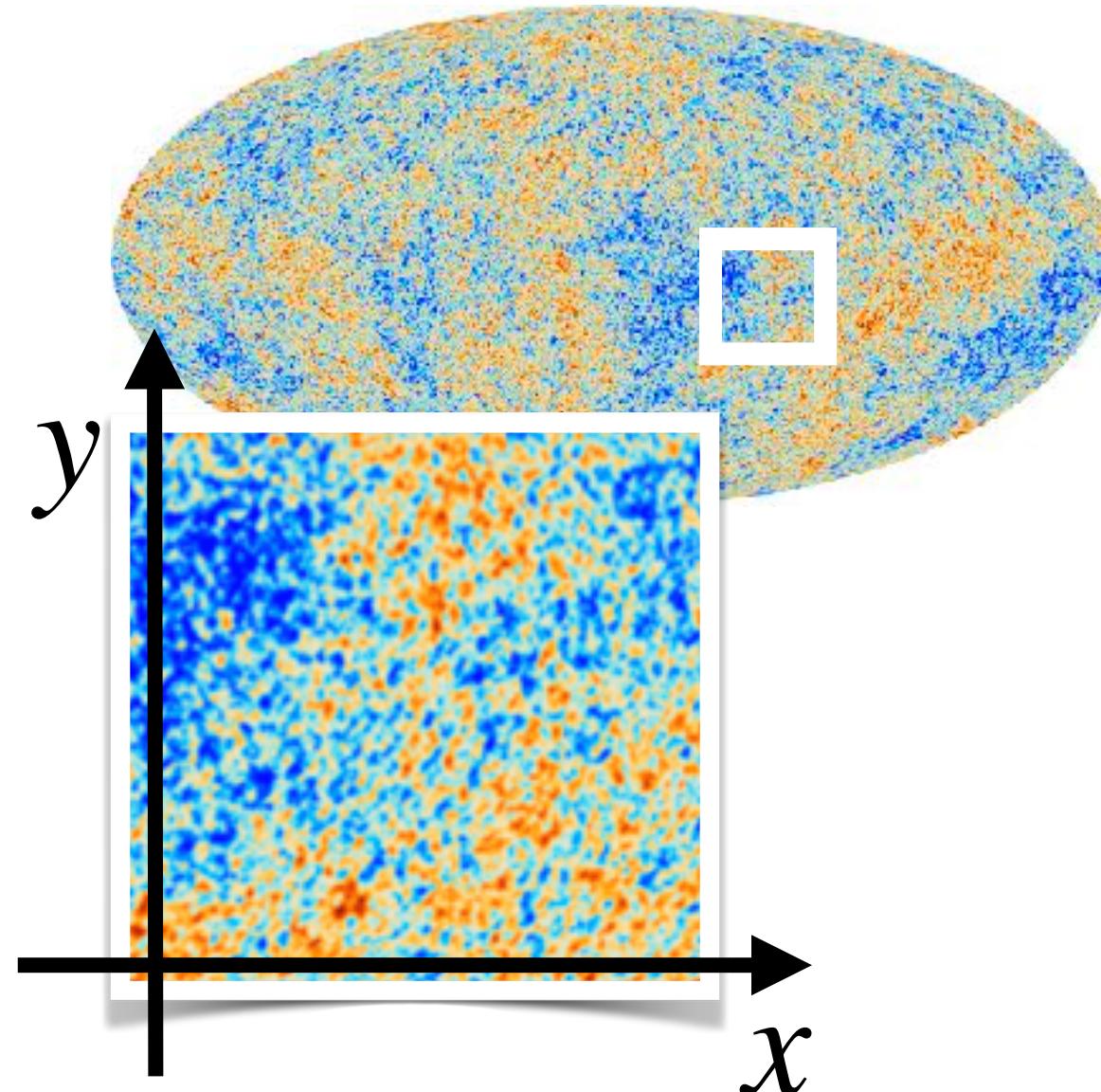
Temperature fluctuation

$$\Theta(x, y) \left(= \frac{\delta T}{T}(x, y) \right)$$

Auto correlation

$$\xi(\beta) \equiv <\Theta(\mathbf{x})\Theta(\mathbf{x} + \beta)>$$

Fourier



© ESA and the Planck Collaboration

https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB#.Yyf0oyS8rWA.link

CC BY-SA 3.0 IGO

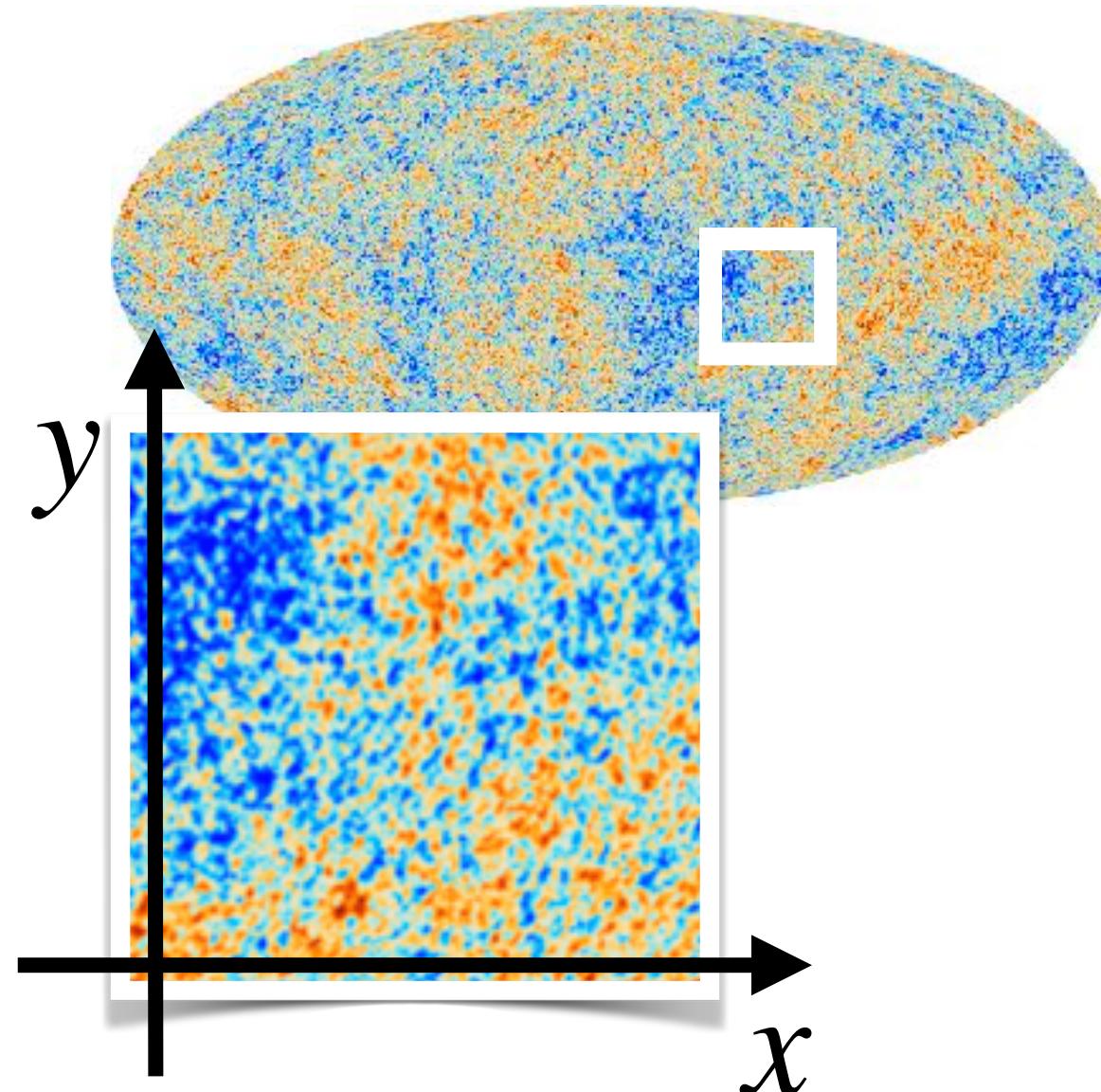
<https://creativecommons.org/licenses/by-sa/3.0/igo/>

Partly modified

How to do lensing correction ?

Temperature fluctuation

$$\Theta(x, y) \left(= \frac{\delta T}{T}(x, y) \right)$$



Auto correlation

$$\xi(\beta) \equiv <\Theta(\mathbf{x})\Theta(\mathbf{x} + \beta)>$$

Fourier

$$C_l^{\Theta\Theta} = \int d^2\beta \xi(\beta) \exp(i\mathbf{l} \cdot \beta)$$

Power spectrum

© ESA and the Planck Collaboration

https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB#.Yyf0oyS8rWA.link

CC BY-SA 3.0 IGO

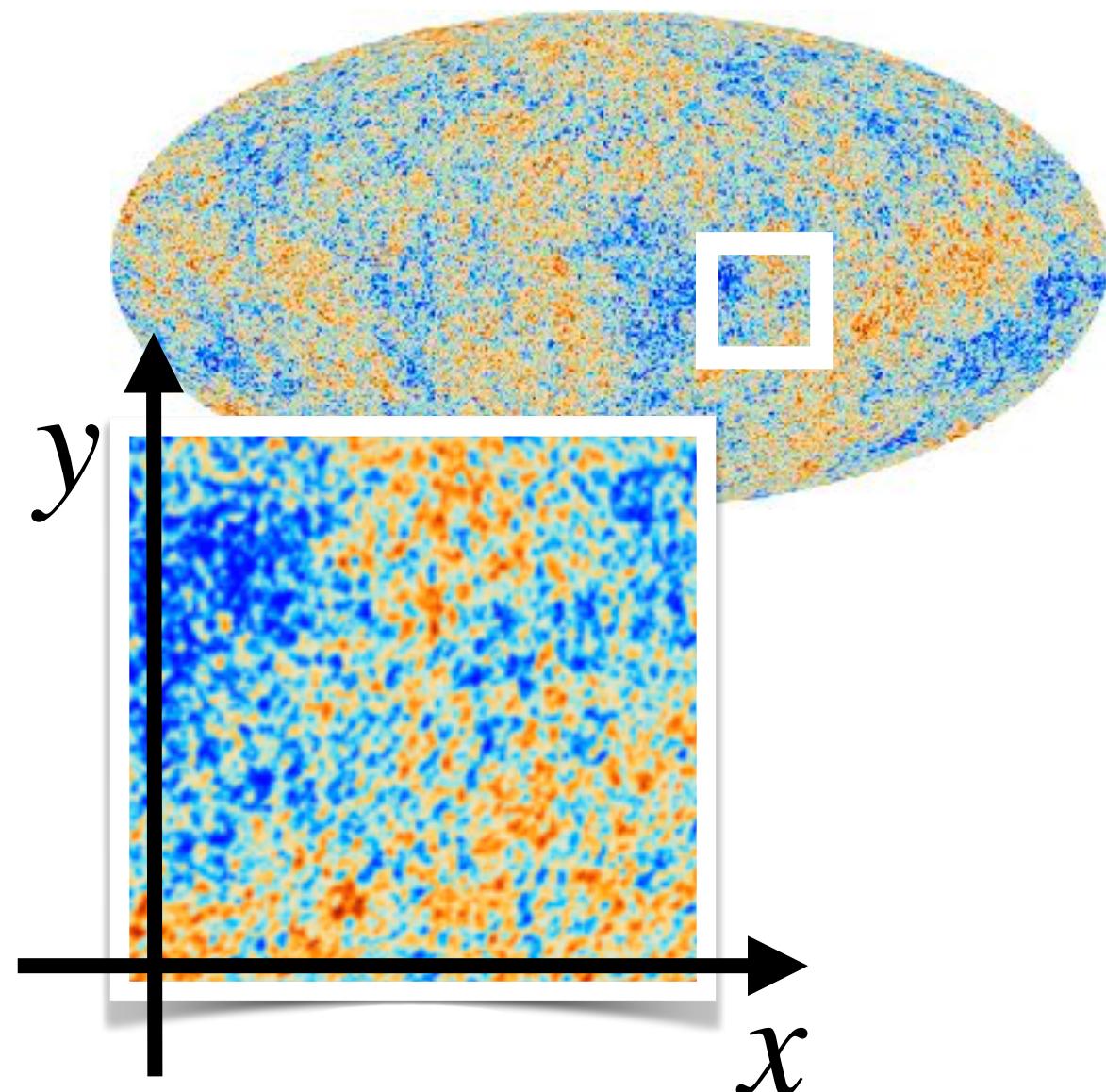
<https://creativecommons.org/licenses/by-sa/3.0/igo/>

Partly modified

How to do lensing correction ?

Temperature fluctuation

$$\Theta(x, y) \left(= \frac{\delta T}{T}(x, y) \right)$$



Auto correlation



$$\xi(\beta) \equiv \langle \Theta(\mathbf{x})\Theta(\mathbf{x} + \beta) \rangle$$

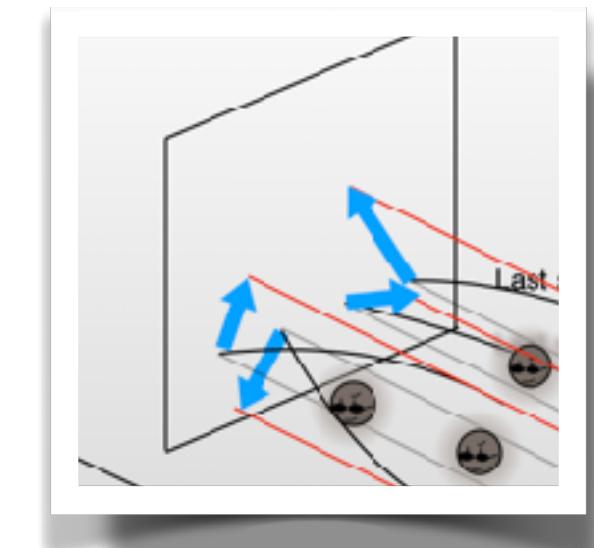
Fourier



Power spectrum

$$C_l^{\Theta\Theta} = \int d^2\beta \xi(\beta) \exp(i\mathbf{l} \cdot \beta)$$

Lensing distortion



$$\begin{aligned}\tilde{\mathbf{x}} &= (\tilde{x}, \tilde{y}) \\ &= (x, y) + \textcolor{blue}{d} \\ &= \mathbf{x} + \mathbf{d}\end{aligned}$$

© ESA and the Planck Collaboration

https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB#.Yyf0oyS8rWA.link

CC BY-SA 3.0 IGO

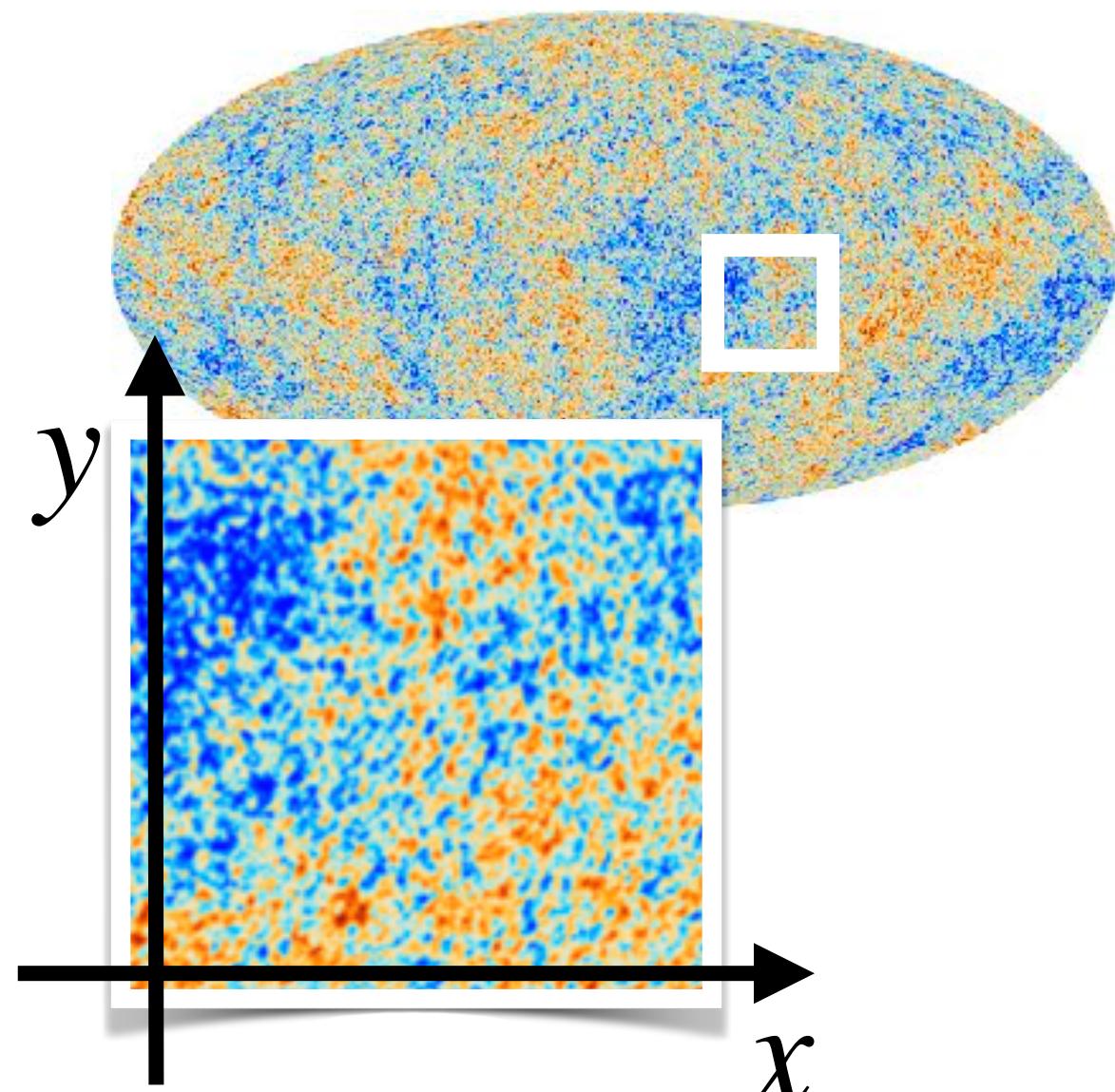
<https://creativecommons.org/licenses/by-sa/3.0/igo/>

Partly modified

How to do lensing correction ?

Temperature fluctuation

$$\Theta(x, y) \left(= \frac{\delta T}{T}(x, y) \right)$$



Auto correlation

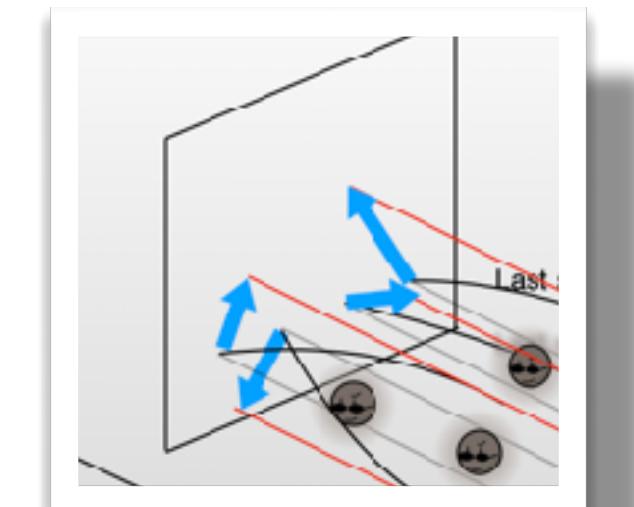
$$\xi(\beta) \equiv \langle \Theta(\mathbf{x})\Theta(\mathbf{x} + \beta) \rangle$$

Fourier

$$C_l^{\Theta\Theta} = \int d^2\beta \xi(\beta) \exp(i\mathbf{l} \cdot \beta)$$

Power spectrum

Lensing distortion



$$\begin{aligned}\tilde{\mathbf{x}} &= (\tilde{x}, \tilde{y}) \\ &= (x, y) + \mathbf{d} \\ &= \mathbf{x} + \mathbf{d}\end{aligned}$$

Auto correlation

© ESA and the Planck Collaboration

https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB#.Yyf0oyS8rWA.link

CC BY-SA 3.0 IGO

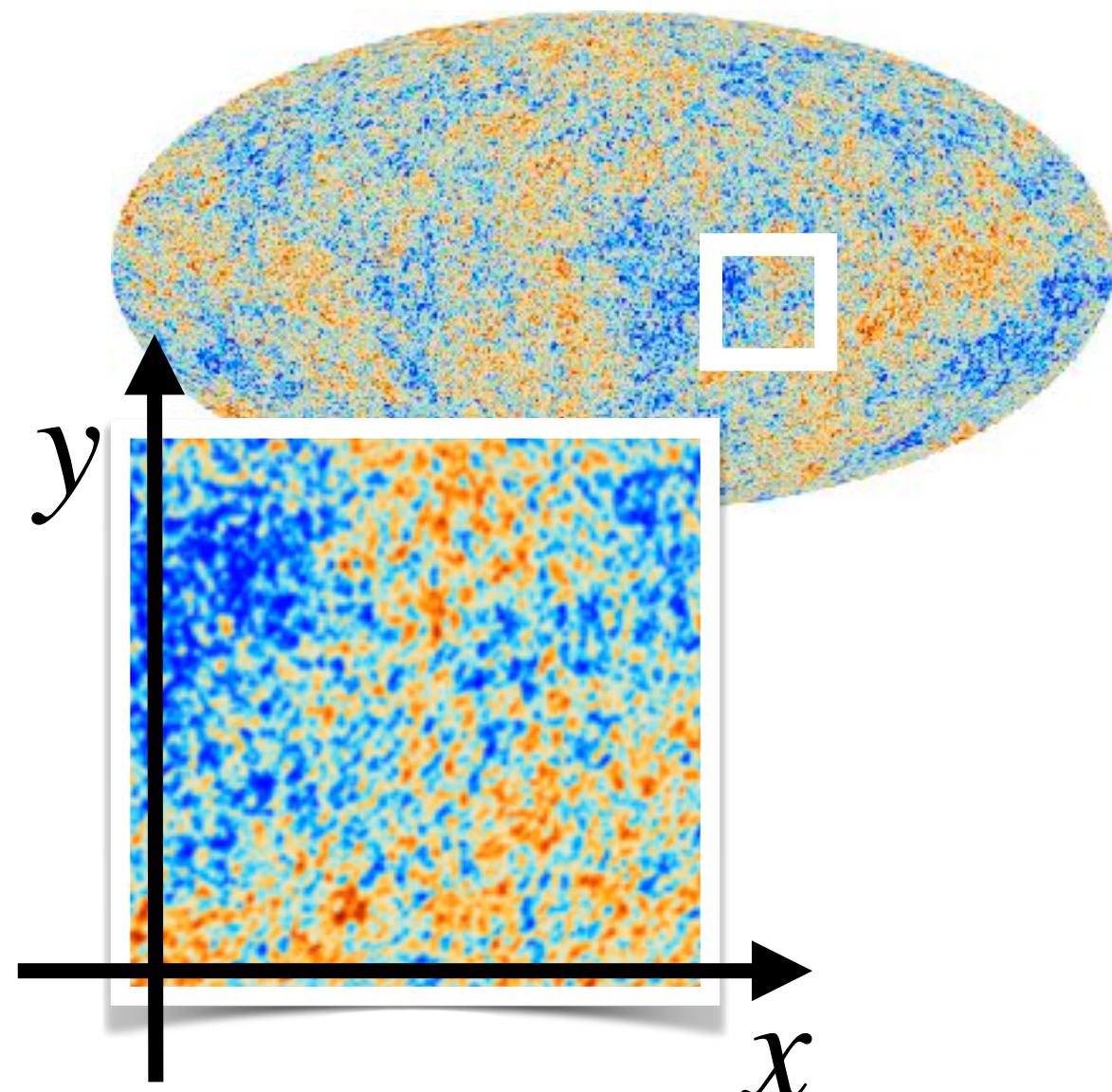
<https://creativecommons.org/licenses/by-sa/3.0/igo/>

Partly modified

How to do lensing correction ?

Temperature fluctuation

$$\Theta(x, y) \left(= \frac{\delta T}{T}(x, y) \right)$$



Auto correlation

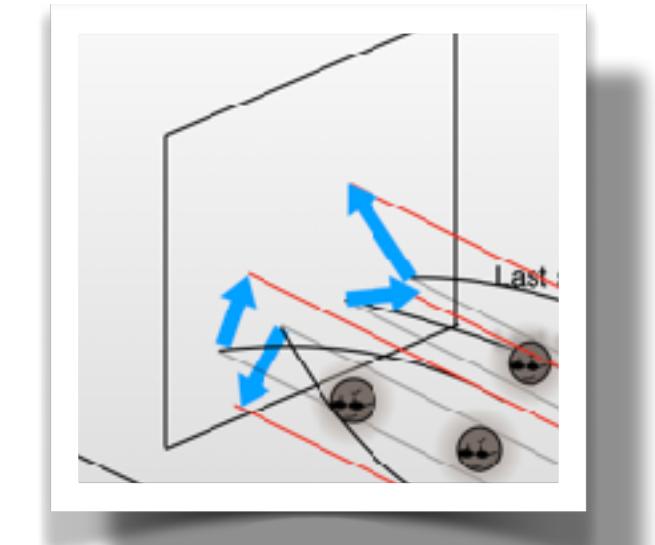
$$\longrightarrow \xi(\beta) \equiv <\Theta(\mathbf{x})\Theta(\mathbf{x} + \beta)>$$

Fourier

$$C_l^{\Theta\Theta} = \int d^2\beta \xi(\beta) \exp(i\mathbf{l} \cdot \beta)$$

Power spectrum

Lensing distortion



$$\begin{aligned}\tilde{\mathbf{x}} &= (\tilde{x}, \tilde{y}) \\ &= (x, y) + \text{d} \\ &= \mathbf{x} + \mathbf{d}\end{aligned}$$

$$\tilde{\xi}(\beta) \equiv <\tilde{\Theta}(\tilde{\mathbf{x}})\tilde{\Theta}(\tilde{\mathbf{x}} + \beta)>$$

Auto correlation

© ESA and the Planck Collaboration

https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB#.Yyf0oyS8rWA.link

CC BY-SA 3.0 IGO

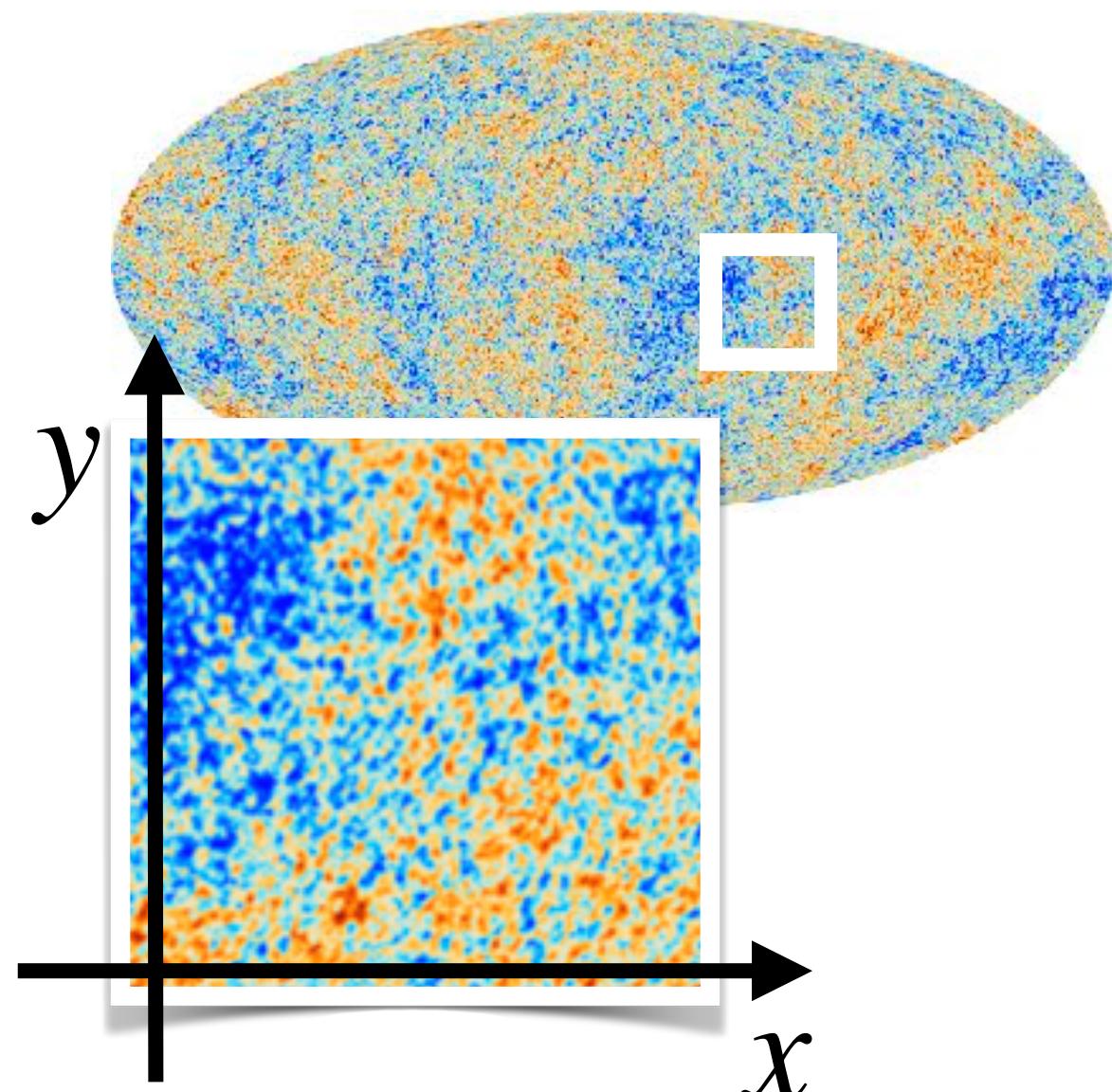
<https://creativecommons.org/licenses/by-sa/3.0/igo/>

Partly modified

How to do lensing correction ?

Temperature fluctuation

$$\Theta(x, y) \left(= \frac{\delta T}{T}(x, y) \right)$$

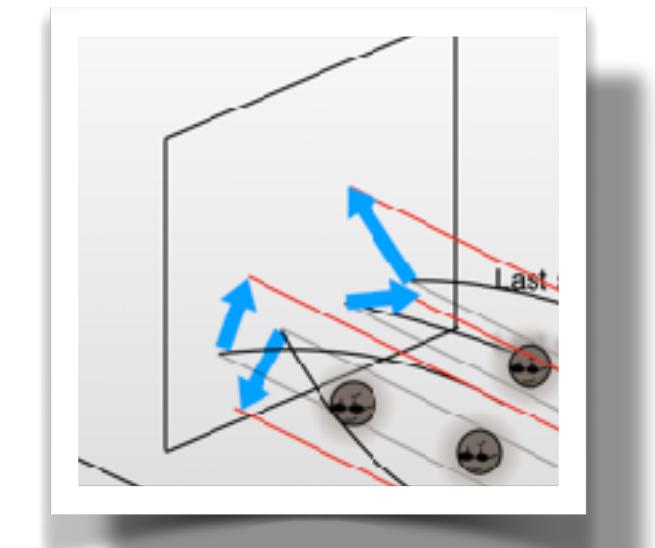


Auto correlation $\longrightarrow \xi(\beta) \equiv <\Theta(\mathbf{x})\Theta(\mathbf{x} + \beta)>$

Fourier $C_l^{\Theta\Theta} = \int d^2\beta \xi(\beta) \exp(i\mathbf{l} \cdot \beta)$

Power spectrum

Lensing distortion



$$\begin{aligned}\tilde{\mathbf{x}} &= (\tilde{x}, \tilde{y}) \\ &= (\mathbf{x}, \mathbf{y}) + \mathbf{d} \\ &= \mathbf{x} + \mathbf{d}\end{aligned}$$

Fourier

$$\tilde{\xi}(\beta) \equiv <\tilde{\Theta}(\tilde{\mathbf{x}})\tilde{\Theta}(\tilde{\mathbf{x}} + \beta)>$$

© ESA and the Planck Collaboration

https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB#.Yyf0oyS8rWA.link

CC BY-SA 3.0 IGO

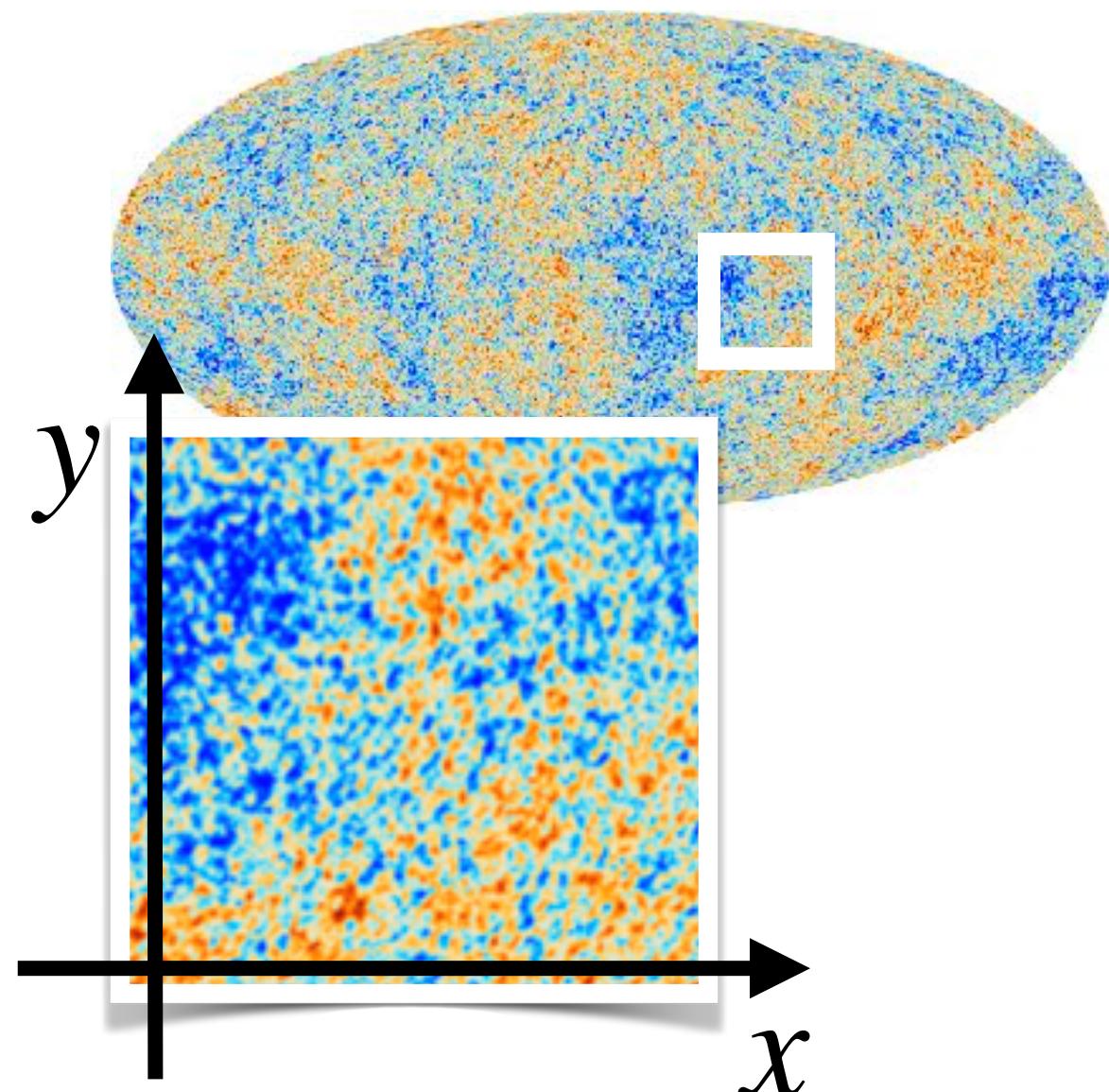
<https://creativecommons.org/licenses/by-sa/3.0/igo/>

Partly modified

How to do lensing correction ?

Temperature fluctuation

$$\Theta(x, y) \left(= \frac{\delta T}{T}(x, y) \right)$$



Auto correlation

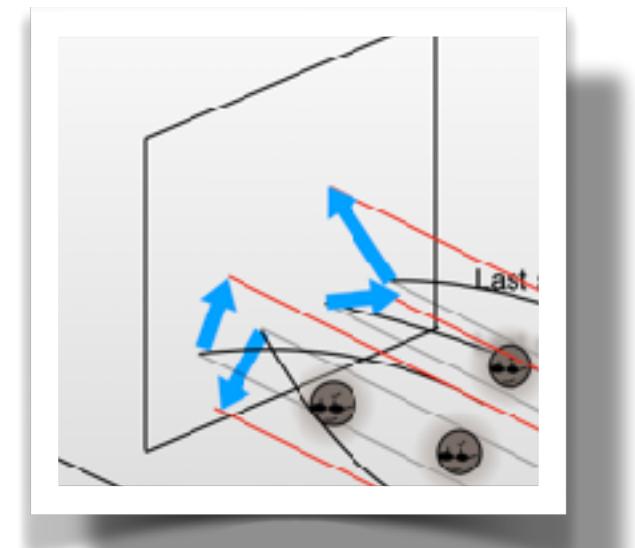
$$\xi(\beta) \equiv \langle \Theta(\mathbf{x})\Theta(\mathbf{x} + \beta) \rangle$$

Fourier

$$C_l^{\Theta\Theta} = \int d^2\beta \xi(\beta) \exp(i\mathbf{l} \cdot \beta)$$

Power spectrum

Lensing distortion



$$\begin{aligned}\tilde{\mathbf{x}} &= (\tilde{x}, \tilde{y}) \\ &= (\mathbf{x}, \mathbf{y}) + \mathbf{d} \\ &= \mathbf{x} + \mathbf{d}\end{aligned}$$

Fourier

Lensed Power spectrum

$$\tilde{C}_l^{\Theta\Theta} \simeq \int d^2\beta \tilde{\xi}(\beta) \exp(i\mathbf{l} \cdot \beta)$$

Auto correlation

$$\tilde{\xi}(\beta) \equiv \langle \tilde{\Theta}(\tilde{\mathbf{x}})\tilde{\Theta}(\tilde{\mathbf{x}} + \beta) \rangle$$

© ESA and the Planck Collaboration

https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB#.Yyf0oyS8rWA.link

CC BY-SA 3.0 IGO

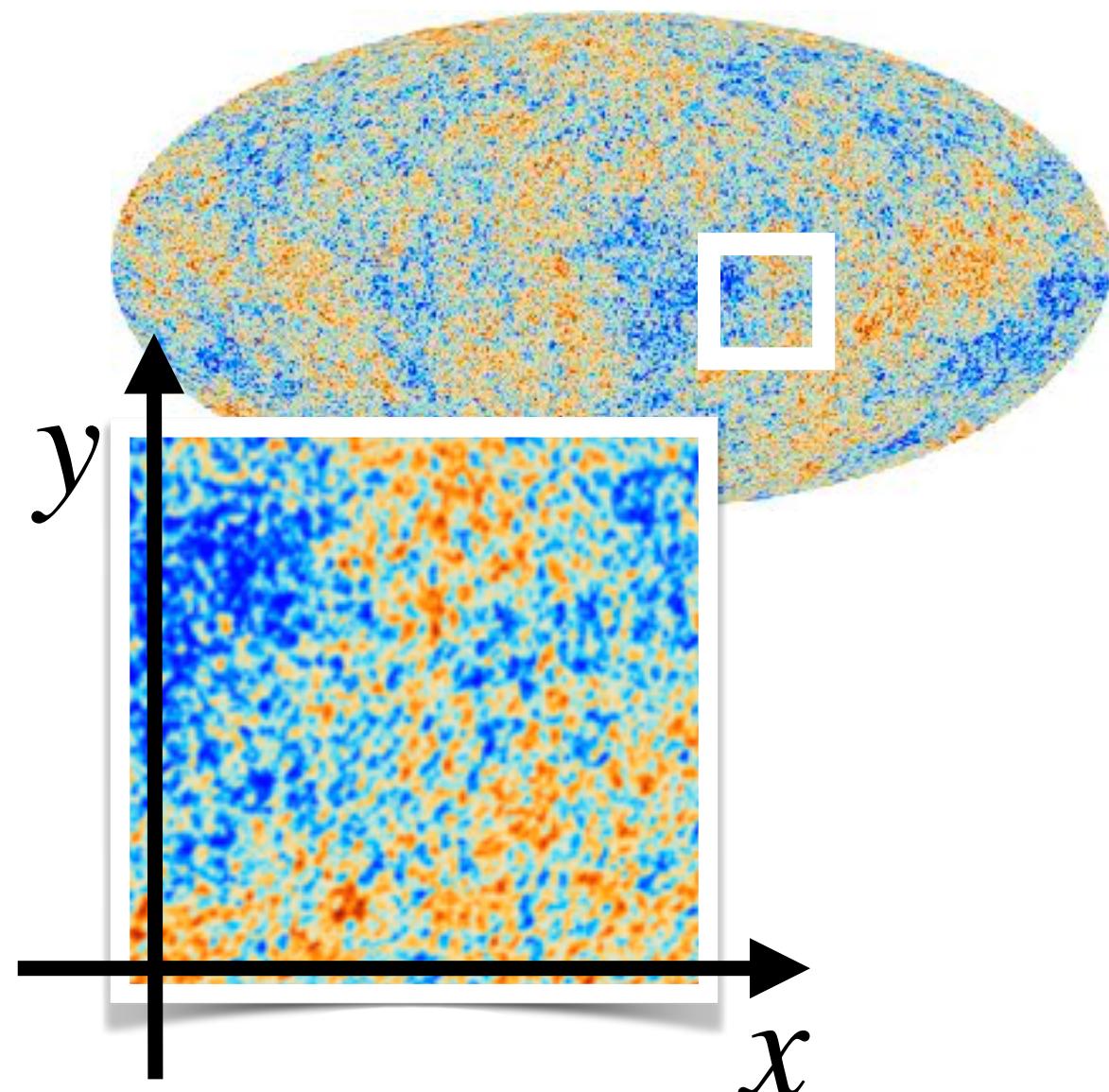
<https://creativecommons.org/licenses/by-sa/3.0/igo/>

Partly modified

How to do lensing correction ?

Temperature fluctuation

$$\Theta(x, y) \left(= \frac{\delta T}{T}(x, y) \right)$$



Auto correlation

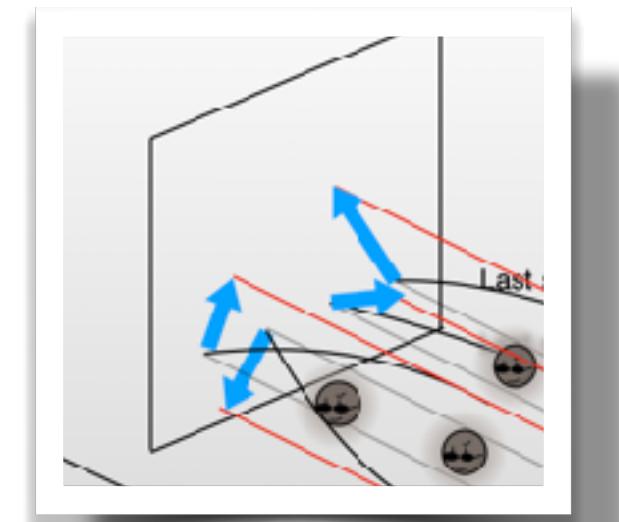
$$\xi(\beta) \equiv \langle \Theta(\mathbf{x})\Theta(\mathbf{x} + \beta) \rangle$$

Fourier

$$C_l^{\Theta\Theta} = \int d^2\beta \xi(\beta) \exp(i\mathbf{l} \cdot \beta)$$

Power spectrum

Lensing distortion



$$\begin{aligned}\tilde{\mathbf{x}} &= (\tilde{x}, \tilde{y}) \\ &= (x, y) + \mathbf{d} \\ &= \mathbf{x} + \mathbf{d}\end{aligned}$$

Fourier

$$\tilde{\xi}(\beta) \equiv \langle \tilde{\Theta}(\tilde{\mathbf{x}})\tilde{\Theta}(\tilde{\mathbf{x}} + \beta) \rangle$$

Lensed Power spectrum

$$\tilde{C}_l^{\Theta\Theta} \simeq \int d^2\beta \tilde{\xi}(\beta) \exp(i\mathbf{l} \cdot \beta)$$

$$\simeq \frac{1}{\sqrt{2}} \int_0^\infty \frac{dl'}{\sqrt{2\pi\epsilon l'}} \sqrt{l'} C_{l'}^{\Theta\Theta} \exp\left[-\frac{(l - l')^2}{2(\epsilon l')^2}\right]$$

ϵ is a constant $\ll 1$

© ESA and the Planck Collaboration

https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB#.Yyf0oyS8rWA.link

CC BY-SA 3.0 IGO

<https://creativecommons.org/licenses/by-sa/3.0/igo/>

Partly modified

$$C_l^{EE}, \ C_l^{BB}, \ C_l^{EB}, \ C_l^{\Theta E}, \ C_l^{\Theta B}$$

$$\tilde{C}_l^{EE}, \quad \tilde{C}_l^{BB}, \quad \tilde{C}_l^{EB}, \quad \tilde{C}_l^{\Theta E}, \quad \tilde{C}_l^{\Theta B}$$



$$C_l^{EE}, \quad C_l^{BB}, \quad C_l^{EB}, \quad C_l^{\Theta E}, \quad C_l^{\Theta B}$$

$$\tilde{C}_l^{EE}, \quad \tilde{C}_l^{BB}, \quad \tilde{C}_l^{EB}, \quad \tilde{C}_l^{\Theta E}, \quad \tilde{C}_l^{\Theta B}$$



$$C_l^{EE}, \quad C_l^{BB}, \quad C_l^{EB}, \quad C_l^{\Theta E}, \quad C_l^{\Theta B}$$

$$\tilde{C}_l^{EE} + \tilde{C}_l^{BB} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} (C_{l'}^{EE} + C_{l'}^{BB}) A_+$$

$$\tilde{C}_l^{EE} - \tilde{C}_l^{BB} + i\tilde{C}_l^{\Theta B} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} (C_{l'}^{EE} - C_{l'}^{BB} + iC_{l'}^{\Theta B}) A_-$$

$$\tilde{C}_l^{\Theta E} + i\tilde{C}_l^{\Theta B} = - 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} (C_{l'}^{\Theta E} + iC_{l'}^{\Theta B}) A_X$$

$A_+, \quad A_-, \quad A_X$: lensing factor

$$\tilde{C}_l^{EE}, \quad \tilde{C}_l^{BB}, \quad \tilde{C}_l^{EB}, \quad \tilde{C}_l^{\Theta E}, \quad \tilde{C}_l^{\Theta B}$$



$$C_l^{EE}, \quad C_l^{BB}, \quad C_l^{EB}, \quad C_l^{\Theta E}, \quad C_l^{\Theta B}$$

$$\underline{\tilde{C}_l^{EE} + \tilde{C}_l^{BB}} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} \underline{(C_{l'}^{EE} + C_{l'}^{BB})} A_+$$

lensed

unlensed

$$\underline{\tilde{C}_l^{EE} - \tilde{C}_l^{BB} + i\tilde{C}_l^{\Theta B}} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} \underline{(C_{l'}^{EE} - C_{l'}^{BB} + iC_{l'}^{\Theta B})} A_-$$

lensed

unlensed

$$\underline{\tilde{C}_l^{\Theta E} + i\tilde{C}_l^{\Theta B}} = -2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} \underline{(C_{l'}^{\Theta E} + iC_{l'}^{\Theta B})} A_X$$

lensed

unlensed

 $A_+, \quad A_-, \quad A_X$: lensing factor



$$\underline{\tilde{C}_l^{EE} + \tilde{C}_l^{BB}} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} (\underline{C_{l'}^{EE} + C_{l'}^{BB}}) A_+$$

lensed

unlensed

$$\underline{\tilde{C}_l^{EE} - \tilde{C}_l^{BB} + i\tilde{C}_l^{\Theta E}} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} (\underline{C_{l'}^{EE} - C_{l'}^{BB} + iC_{l'}^{\Theta E}}) A_-$$

lensed

unlensed

$$\underline{\tilde{C}_l^{\Theta E} + i\tilde{C}_l^{\Theta B}} = -2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} (\underline{C_{l'}^{\Theta E} + C_{l'}^{\Theta B}}) A_X$$

lensed

unlensed

A_+, A_-, A_X : lensing factor

$$\tilde{C}_l^{EE}, \quad \tilde{C}_l^{BB}, \quad \tilde{C}_l^{EB}, \quad \tilde{C}_l^{\Theta E}, \quad \tilde{C}_l^{\Theta B}$$



$$C_l^{EE}, \quad C_l^{BB}, \quad C_l^{EB}, \quad C_l^{\Theta E}, \quad C_l^{\Theta B}$$

$$\underline{\tilde{C}_l^{EE} + \tilde{C}_l^{BB}} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} \underline{(C_{l'}^{EE} + C_{l'}^{BB})} A_+$$

lensed

unlensed

$$\underline{\tilde{C}_l^{EE} - \tilde{C}_l^{BB} + i\tilde{C}_l^{EB}} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} \underline{(C_{l'}^{EE} - C_{l'}^{BB} + iC_{l'}^{EB})} A_-$$

lensed

unlensed

$$\underline{\tilde{C}_l^{\Theta E} + i\tilde{C}_l^{\Theta B}} = -2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} \underline{(C_{l'}^{\Theta E} + iC_{l'}^{\Theta B})} A_X$$

lensed

unlensed

 $A_+, \quad A_-, \quad A_X$: lensing factor

$$\tilde{C}_l^{EE}, \tilde{C}_l^{BB}, \tilde{C}_l^{EB}, \tilde{C}_l^{\Theta E}, \tilde{C}_l^{\Theta B}$$



$$C_l^{EE}, C_l^{BB}, C_l^{EB}, C_l^{\Theta E}, C_l^{\Theta B}$$

$$\underline{\tilde{C}_l^{EE} + \tilde{C}_l^{BB}} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} \underline{(C_{l'}^{EE} + C_{l'}^{BB})} A_+$$

lensed

unlensed

$$\underline{\tilde{C}_l^{EE} - \tilde{C}_l^{BB} + i\tilde{C}_l^{EB}} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} \underline{(C_{l'}^{EE} - C_{l'}^{BB} + iC_{l'}^{EB})} A_-$$

lensed

unlensed

$$\underline{\tilde{C}_l^{\Theta E} + i\tilde{C}_l^{\Theta B}} = -2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} \underline{(C_{l'}^{\Theta E} + iC_{l'}^{\Theta B})} A_X$$

lensed

unlensed

Parity even

$$C_l^{EE}, C_l^{BB}, C_l^{\Theta E} \\ \rightarrow \text{real part}$$

Parity odd

$$C_l^{EB}, C_l^{\Theta B}$$

\rightarrow imaginary part

A_+, A_-, A_X : lensing factor

$$\tilde{C}_l^{EE}, \tilde{C}_l^{BB}, \tilde{C}_l^{EB}, \tilde{C}_l^{\Theta E}, \tilde{C}_l^{\Theta B}$$



$$C_l^{EE}, C_l^{BB}, C_l^{EB}, C_l^{\Theta E}, C_l^{\Theta B}$$

$$\underline{\tilde{C}_l^{EE} + \tilde{C}_l^{BB}} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} (\underline{C_{l'}^{EE} + C_{l'}^{BB}}) A_+$$

lensed

unlensed

$$\underline{\tilde{C}_l^{EE} - \tilde{C}_l^{BB}} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} (\underline{C_{l'}^{EE} - C_{l'}^{BB}}) A_-$$

lensed

unlensed

$$\underline{\tilde{C}_l^{\Theta E}} = -2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} (\underline{C_{l'}^{\Theta E}}) A_X$$

lensed

unlensed

$$\tilde{C}_l^{EB} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} C_{l'}^{EB} A_-$$

$$\tilde{C}_l^{\Theta B} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} C_{l'}^{\Theta B} A_X$$

Parity even

$$C_l^{EE}, C_l^{BB}, C_l^{\Theta E}$$

→ real part

Parity odd

$$C_l^{EB}, C_l^{\Theta B}$$

→ imaginary part

 A_+, A_-, A_X : lensing factor

$$\tilde{C}_l^{EE}, \tilde{C}_l^{BB}, \tilde{C}_l^{EB}, \tilde{C}_l^{\Theta E}, \tilde{C}_l^{\Theta B}$$



$$C_l^{EE}, C_l^{BB}, C_l^{EB}, C_l^{\Theta E}, C_l^{\Theta B}$$

$$\underline{\tilde{C}_l^{EE} + \tilde{C}_l^{BB}} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} (\underline{C_{l'}^{EE} + C_{l'}^{BB}}) A_+$$

lensed

unlensed

$$\underline{\tilde{C}_l^{EE} - \tilde{C}_l^{BB}} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} (\underline{C_{l'}^{EE} - C_{l'}^{BB}}) A_-$$

lensed

unlensed

$$\underline{\tilde{C}_l^{\Theta E}} = -2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} (\underline{C_{l'}^{\Theta E}}) A_X$$

lensed

unlensed

$$\underline{\tilde{C}_l^{EB}} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} \underline{C_{l'}^{EB}} A_-$$

lensed

unlensed

$$\underline{\tilde{C}_l^{\Theta B}} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} \underline{C_{l'}^{\Theta B}} A_X$$

lensed

unlensed

Parity even

$$C_l^{EE}, C_l^{BB}, C_l^{\Theta E}$$

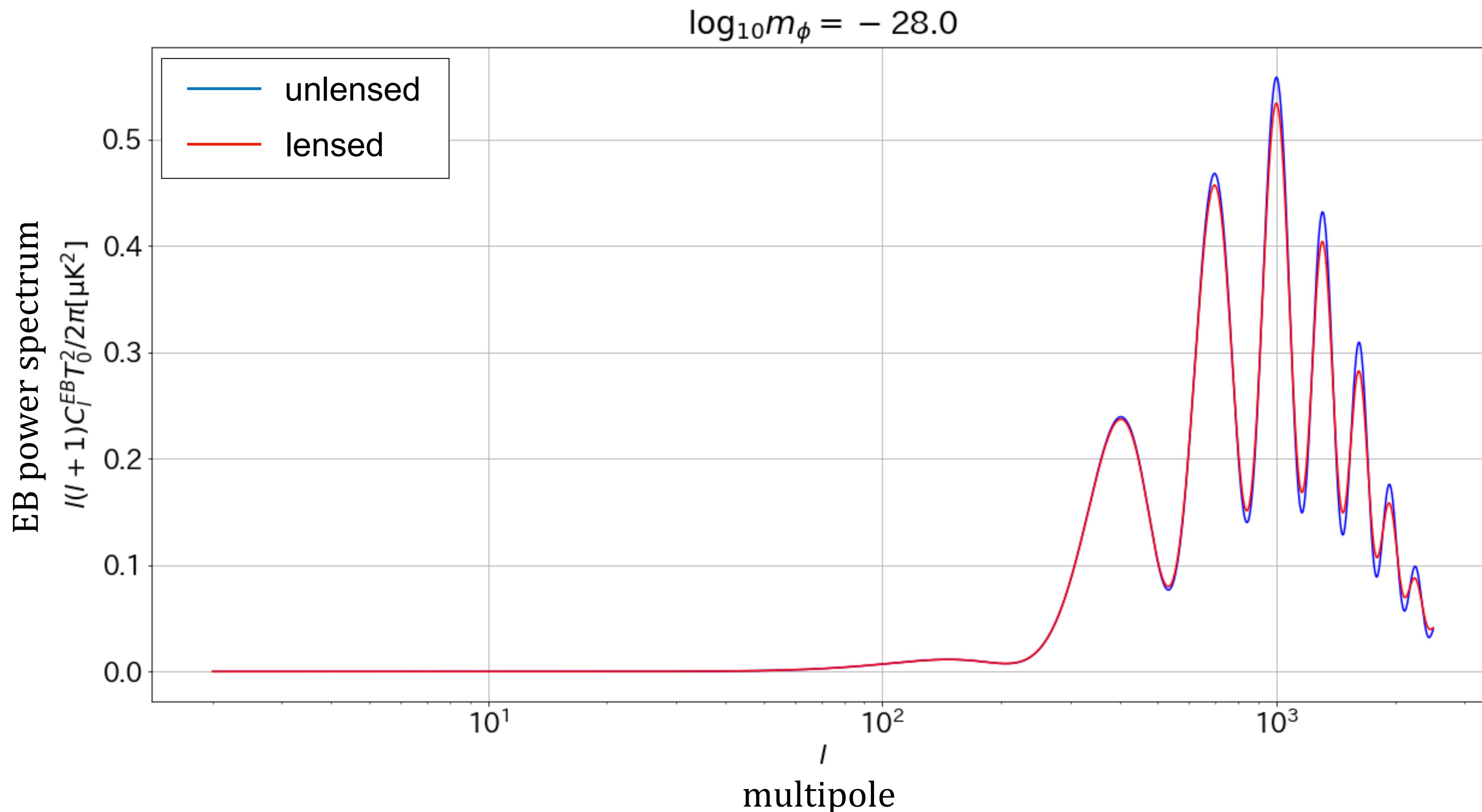
→ real part

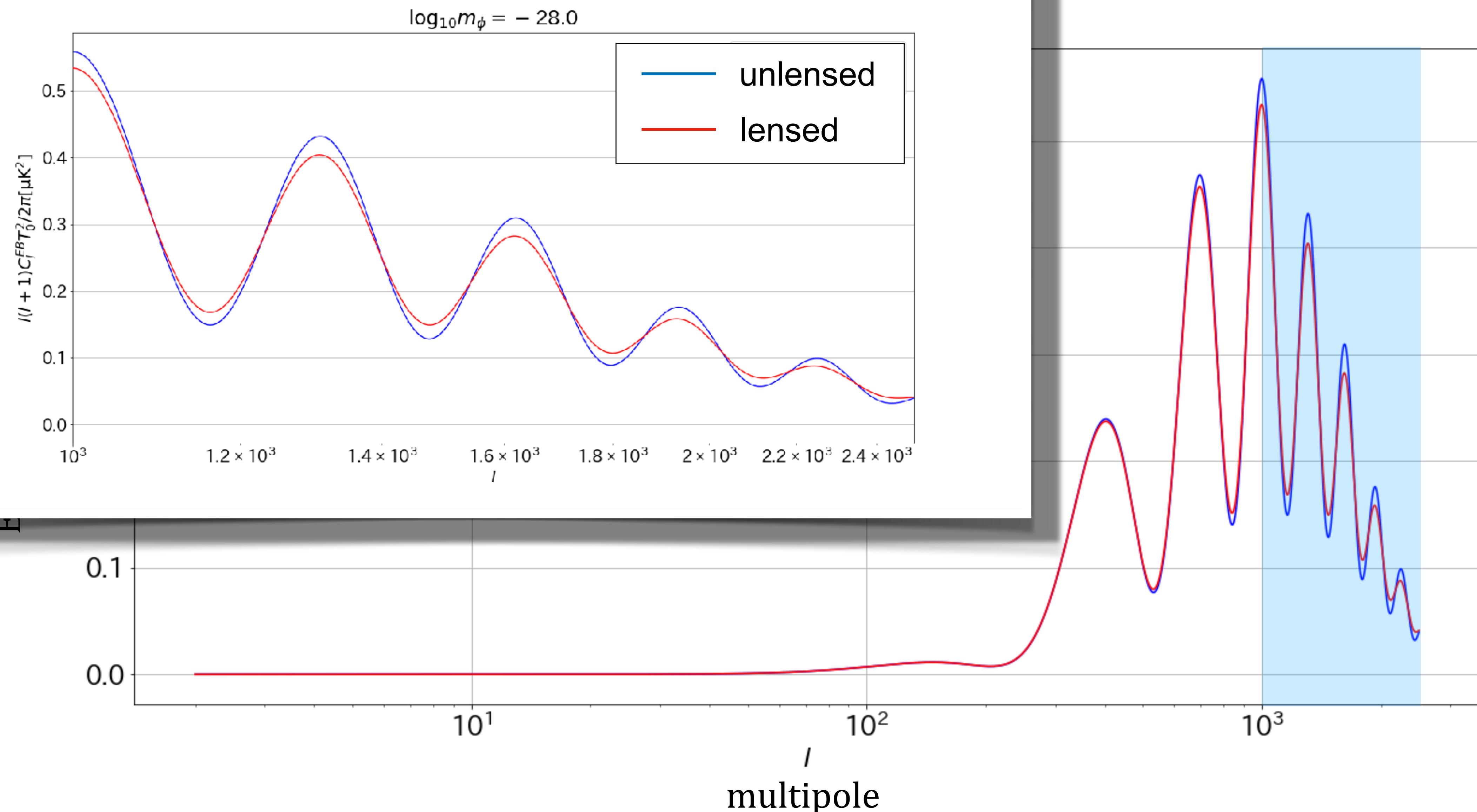
Parity odd

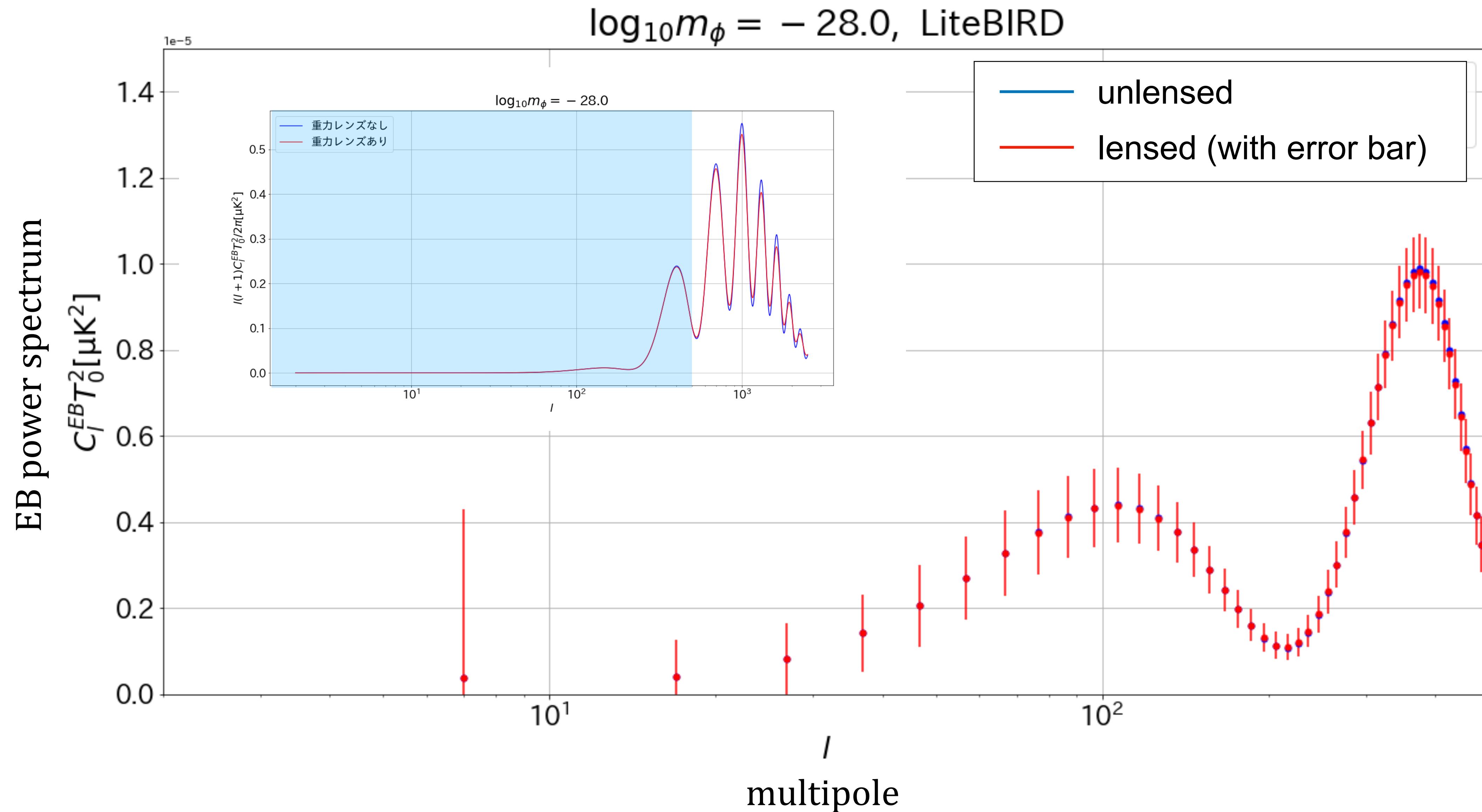
$$C_l^{EB}, C_l^{\Theta B}$$

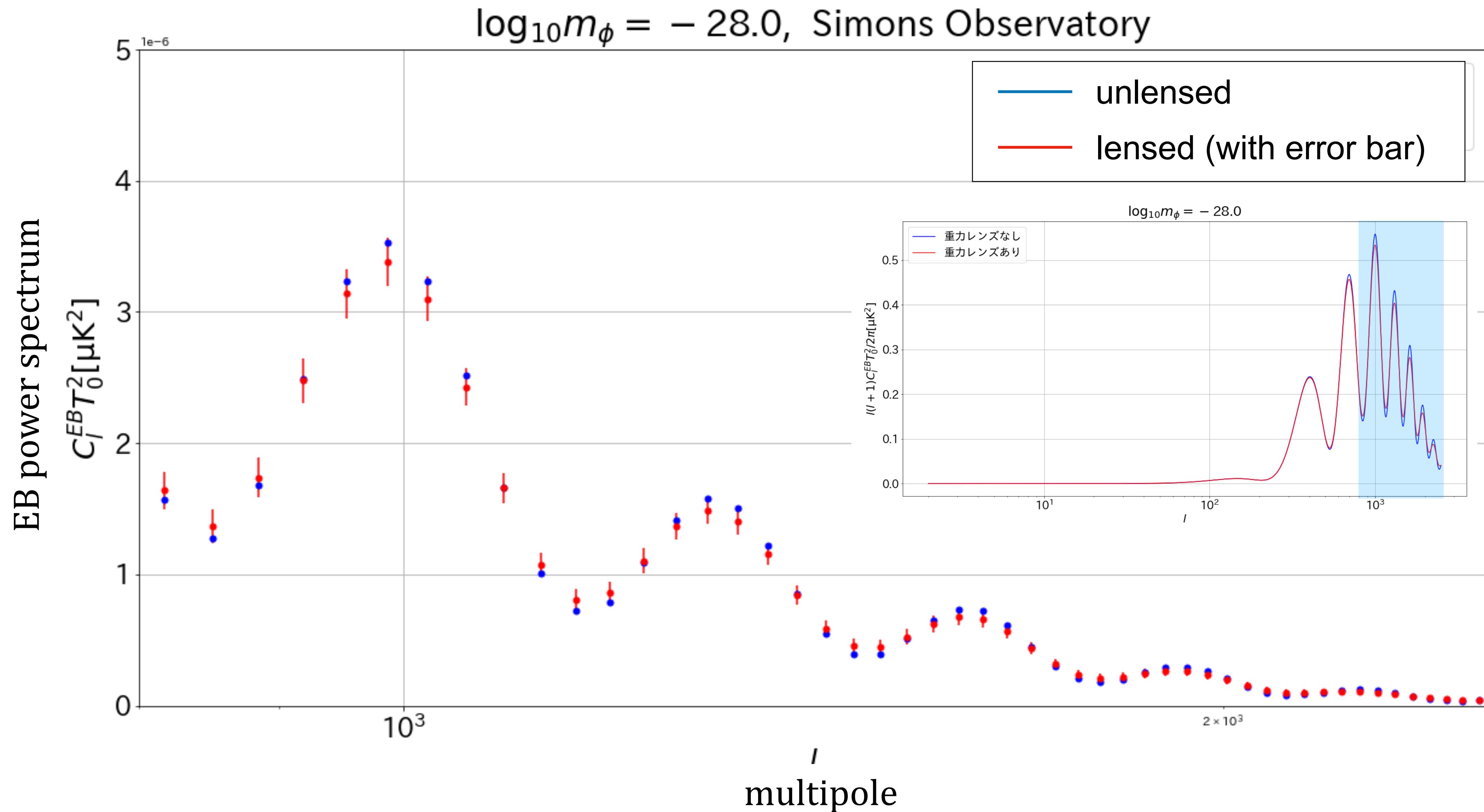
→ imaginary part

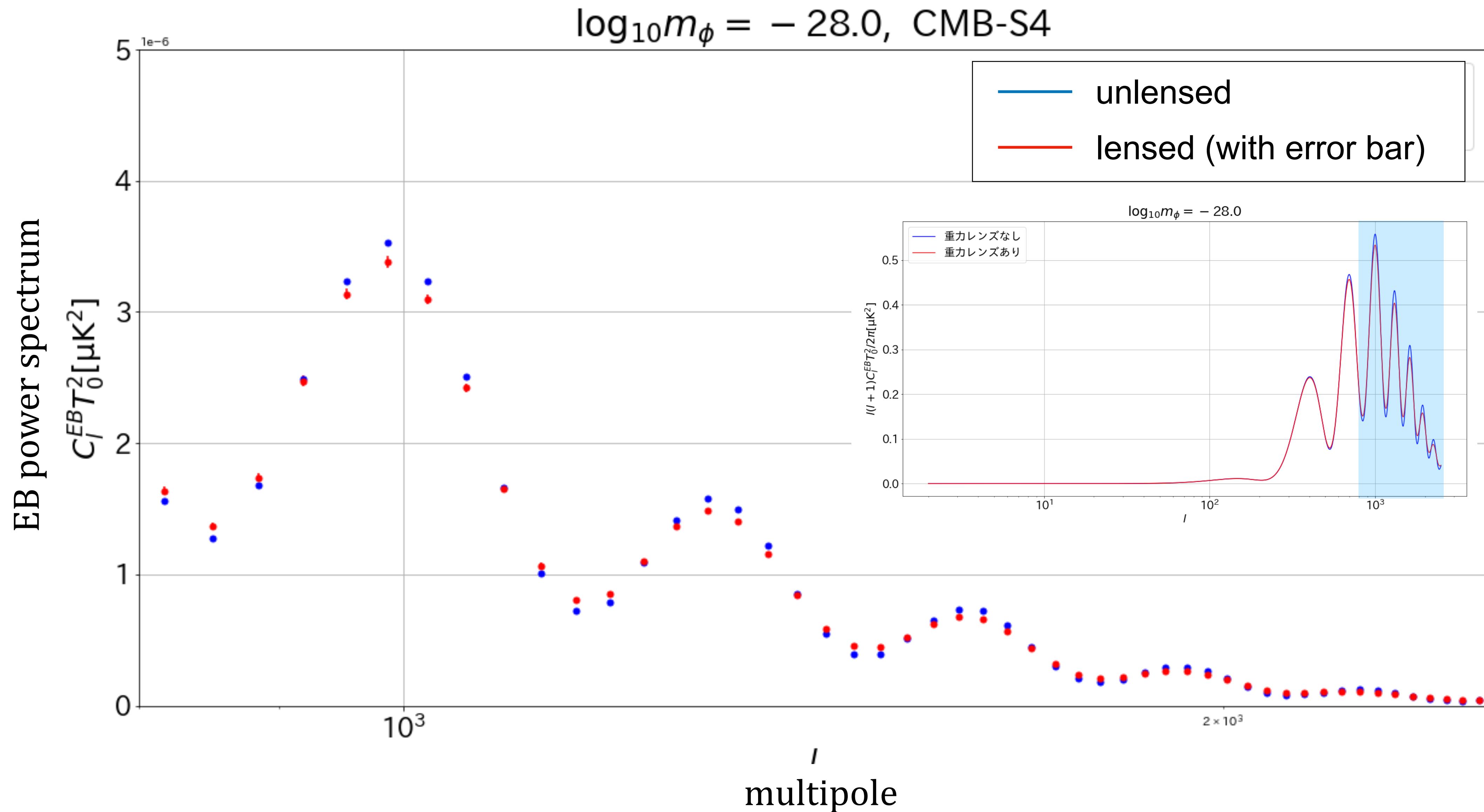
 A_+, A_-, A_X : lensing factor







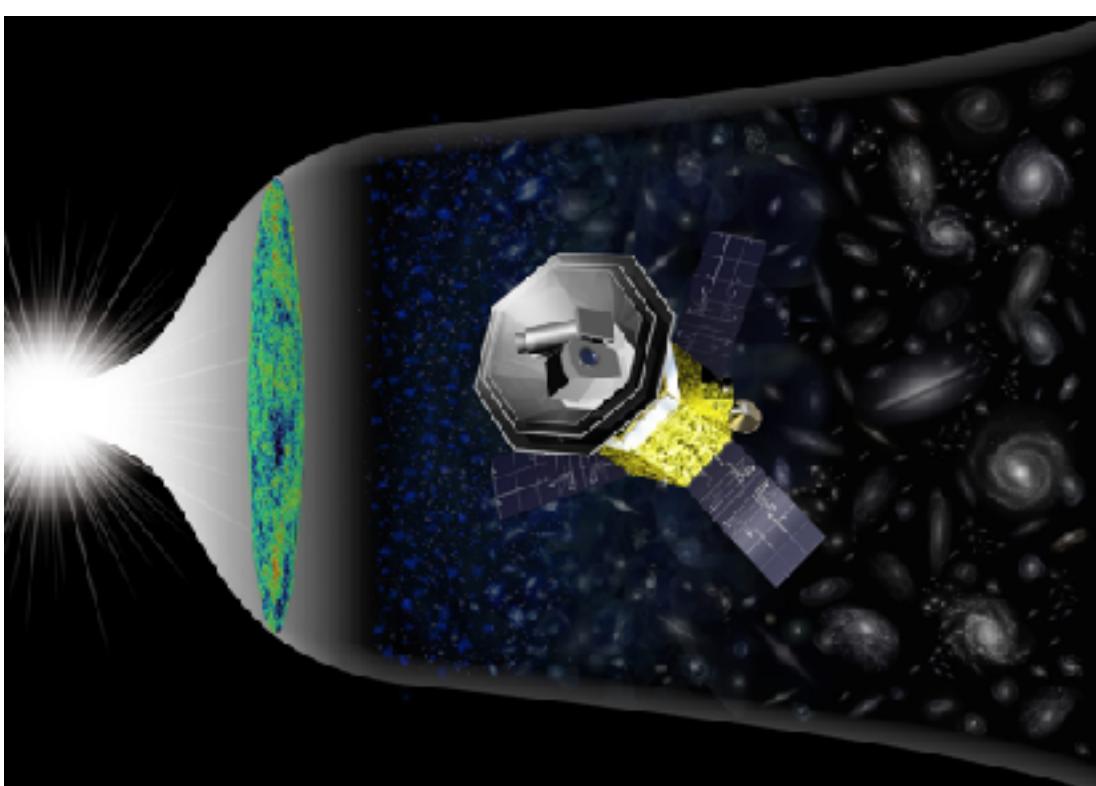




We conducted a forecast of parameters search of ALPs by Cosmic Birefringence to estimate lensing effect on it.

Mock observational C_l^{EB} **with lensing**

LiteBIRD



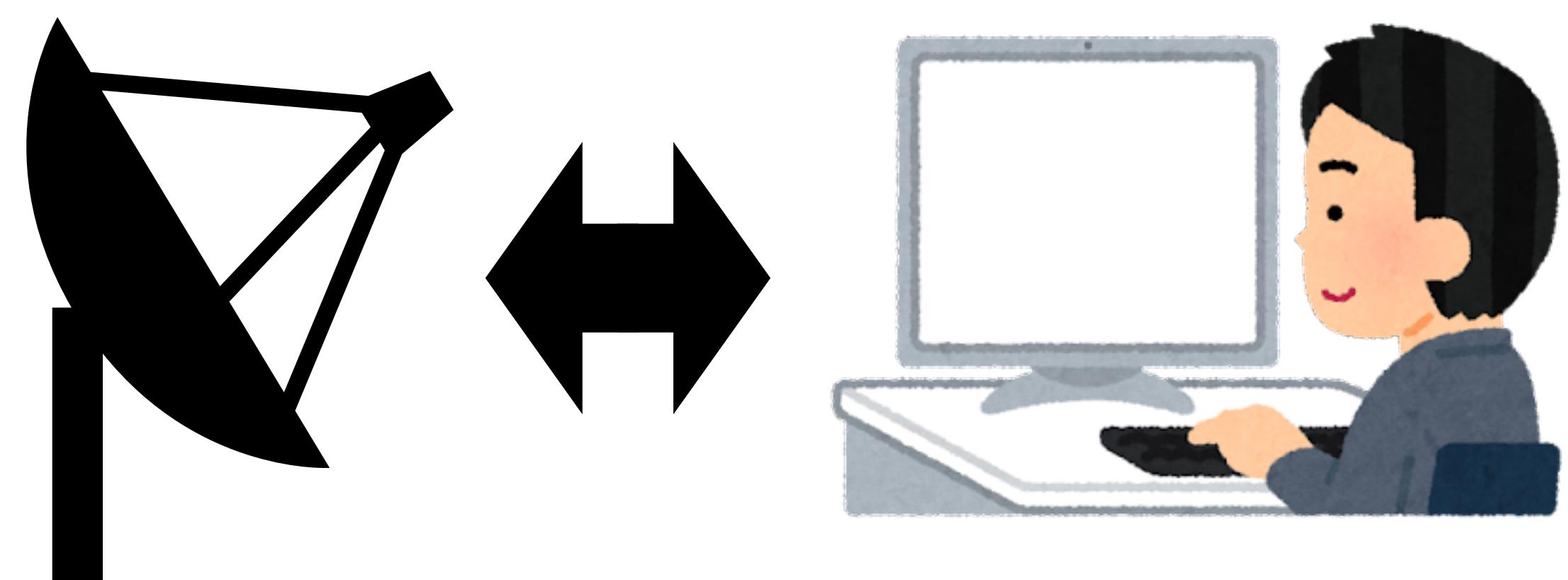
Credit : ISAS/JAXA

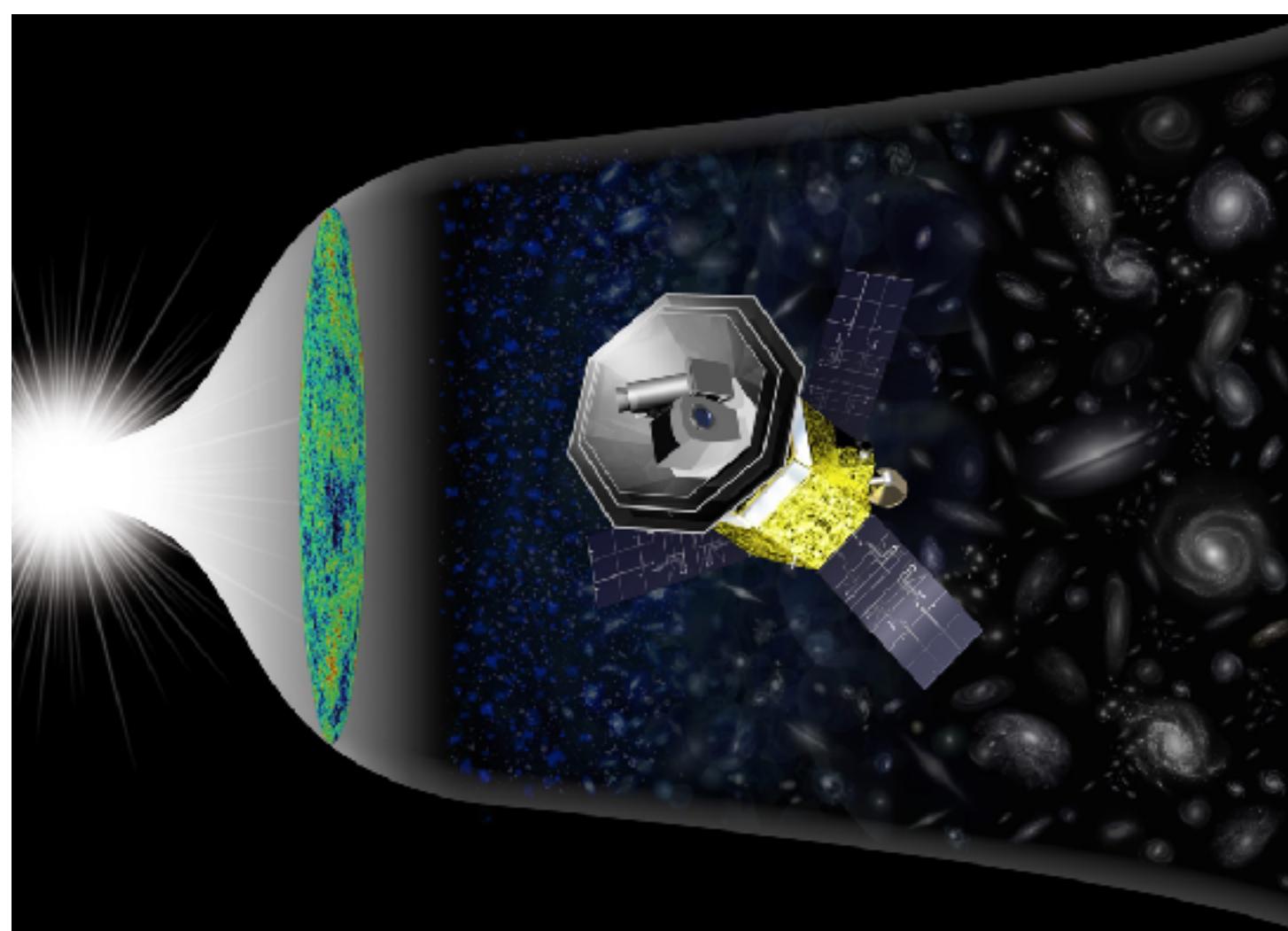
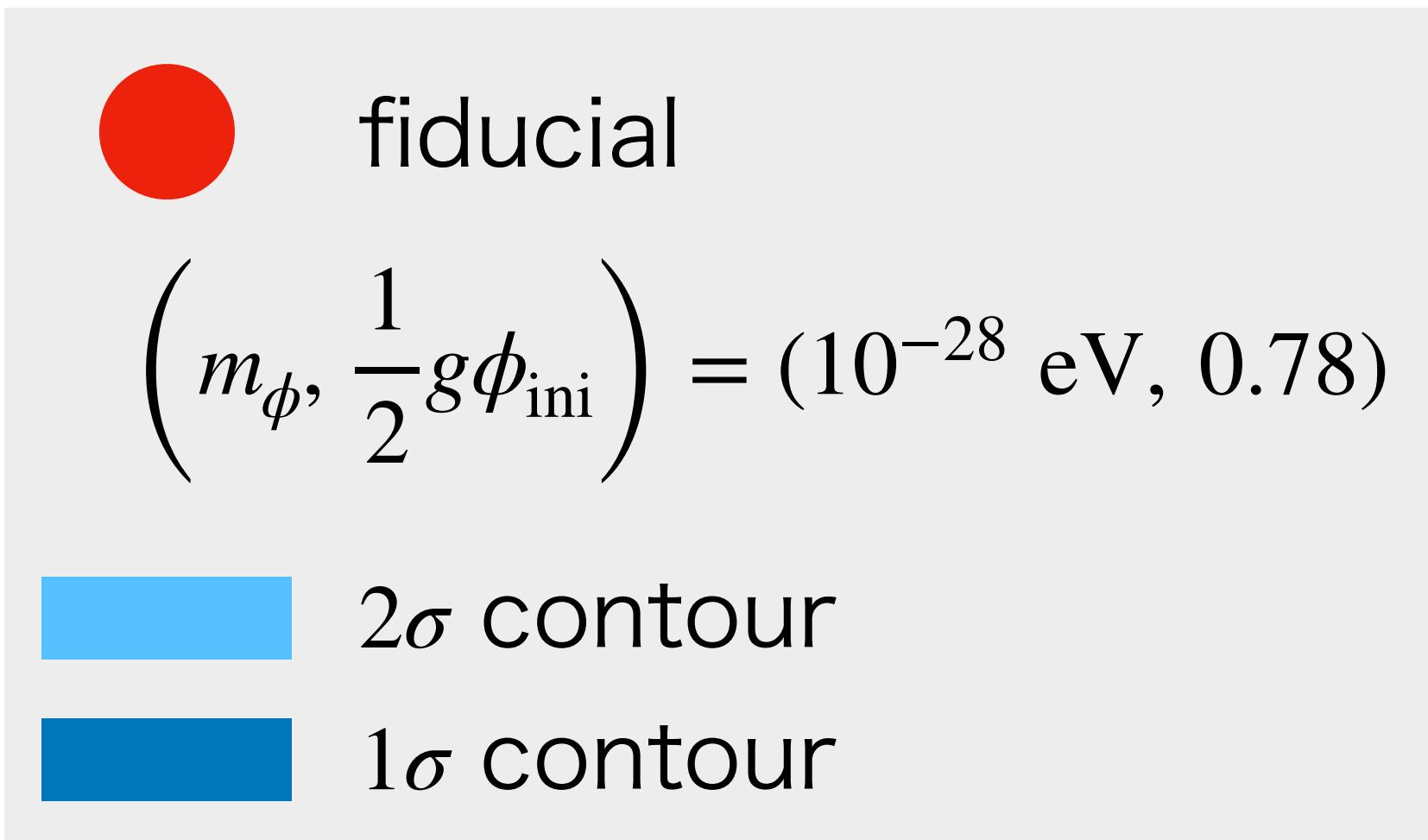
Simons Observatory



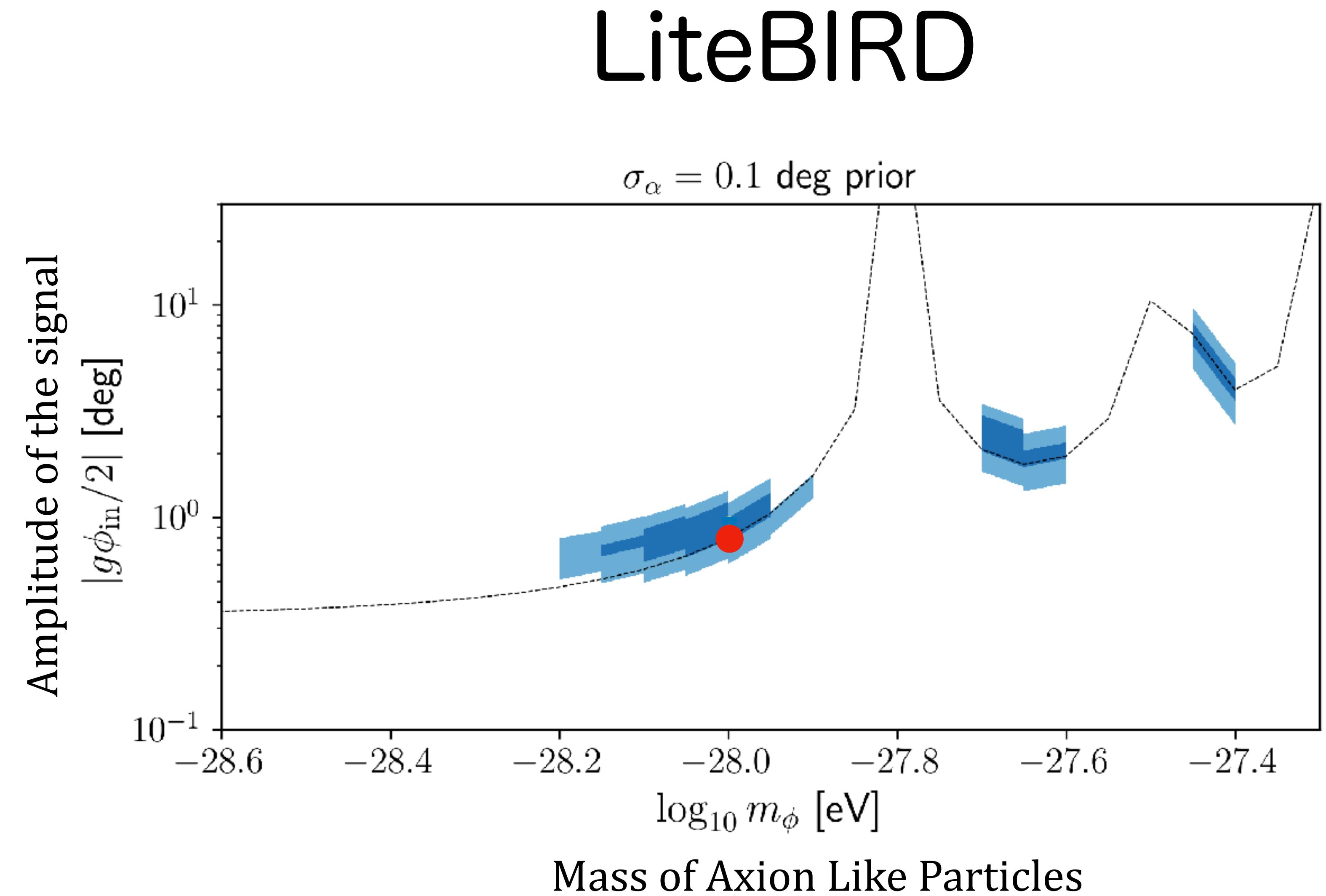
C_l^{EB} **without lensing**

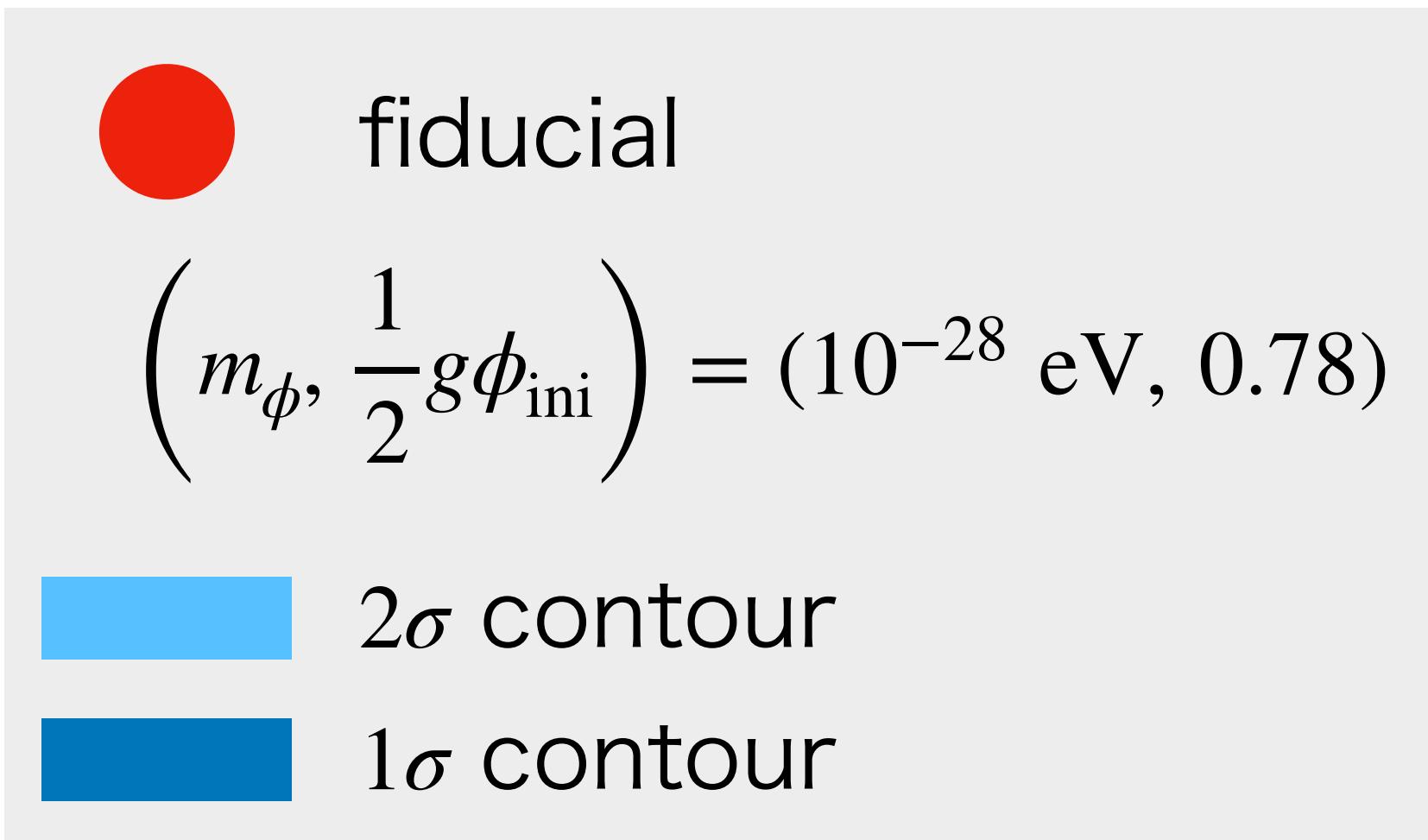
CMB-S4



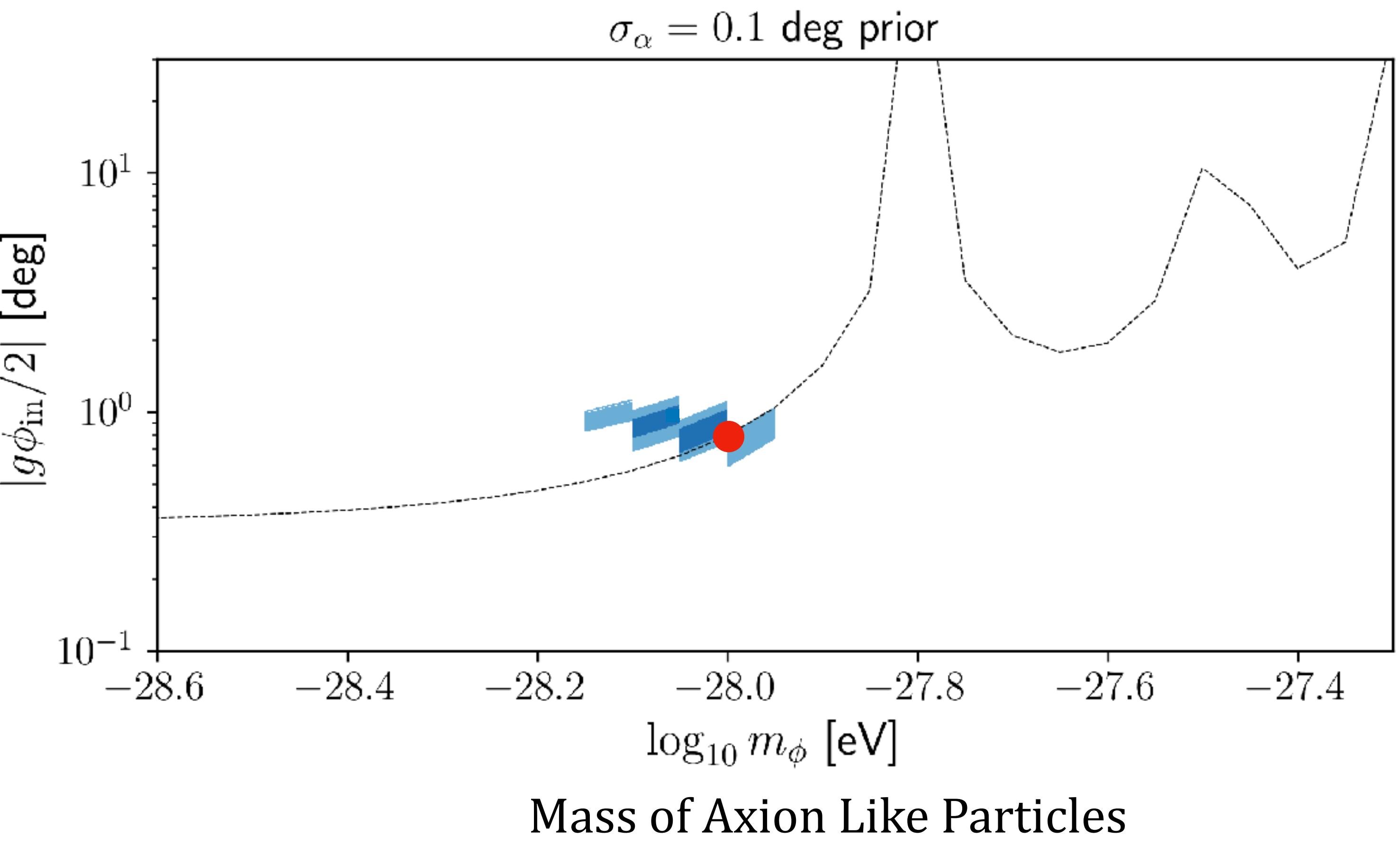


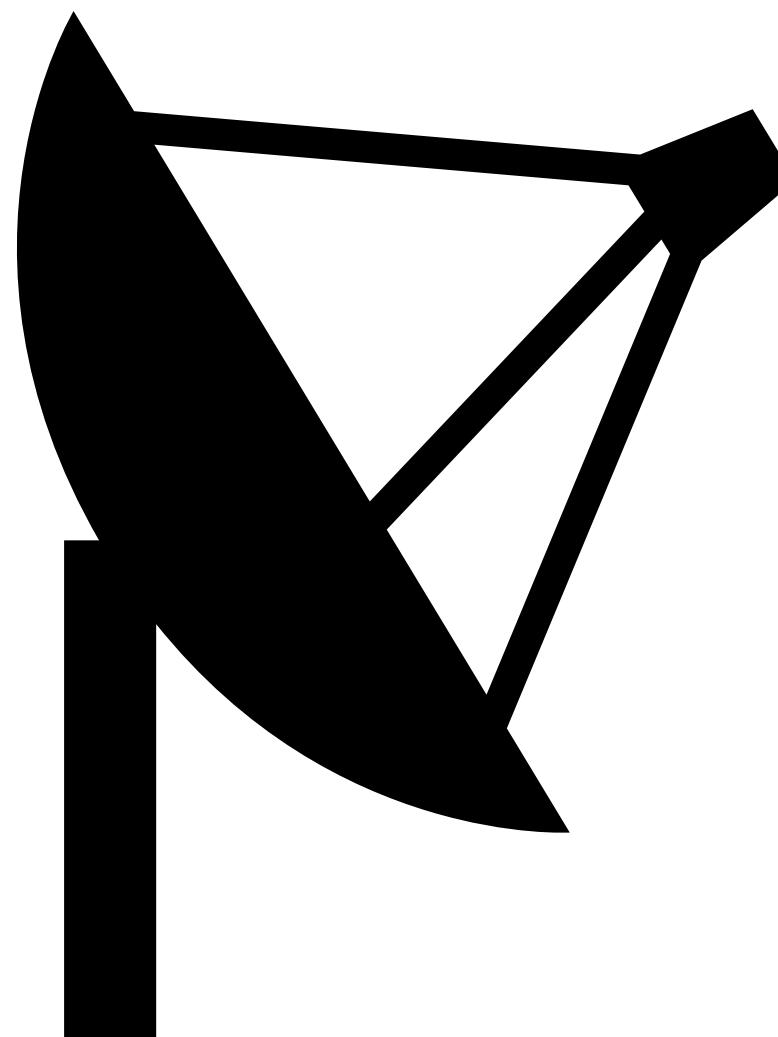
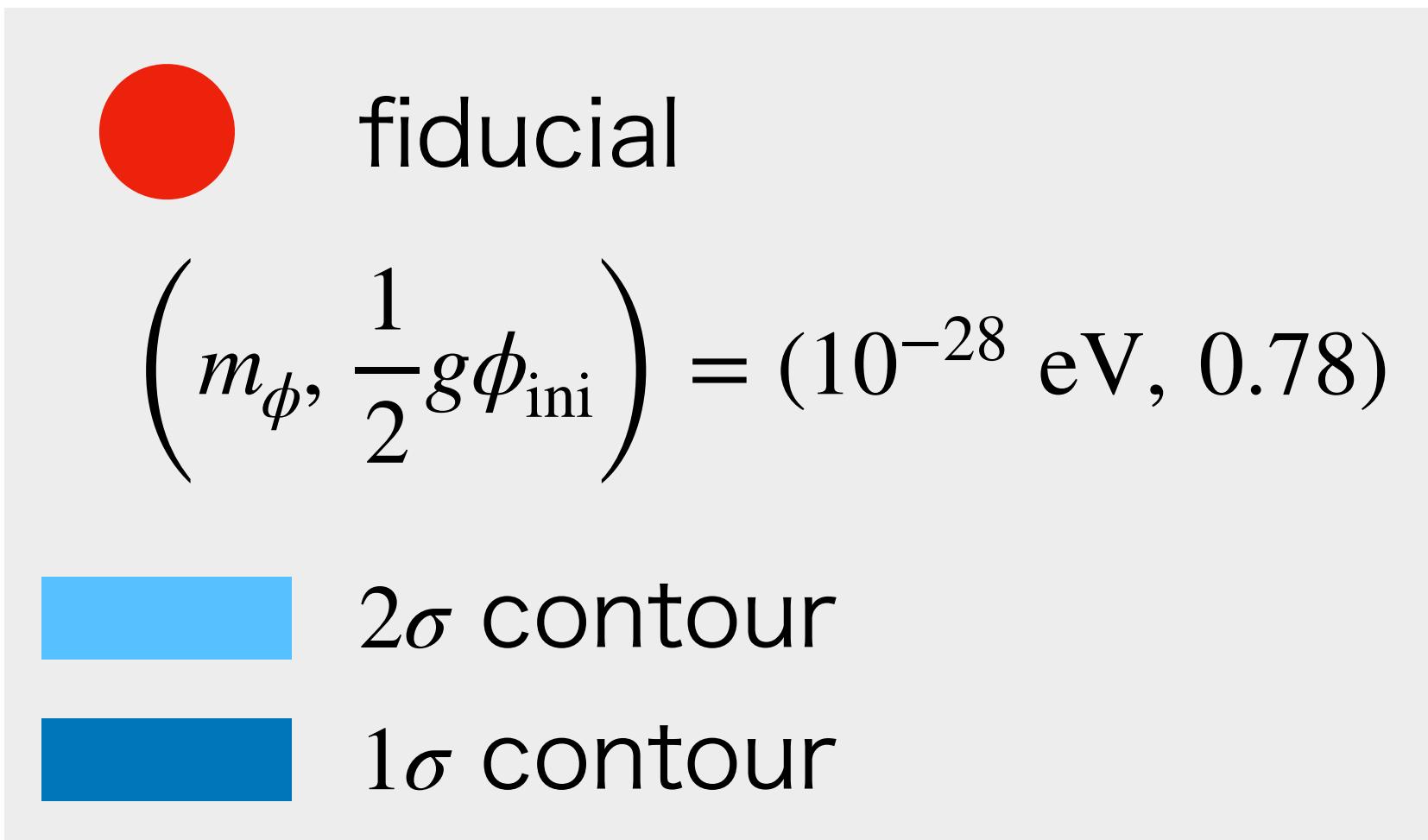
Credit : ISAS/JAXA





Amplitude of the signal

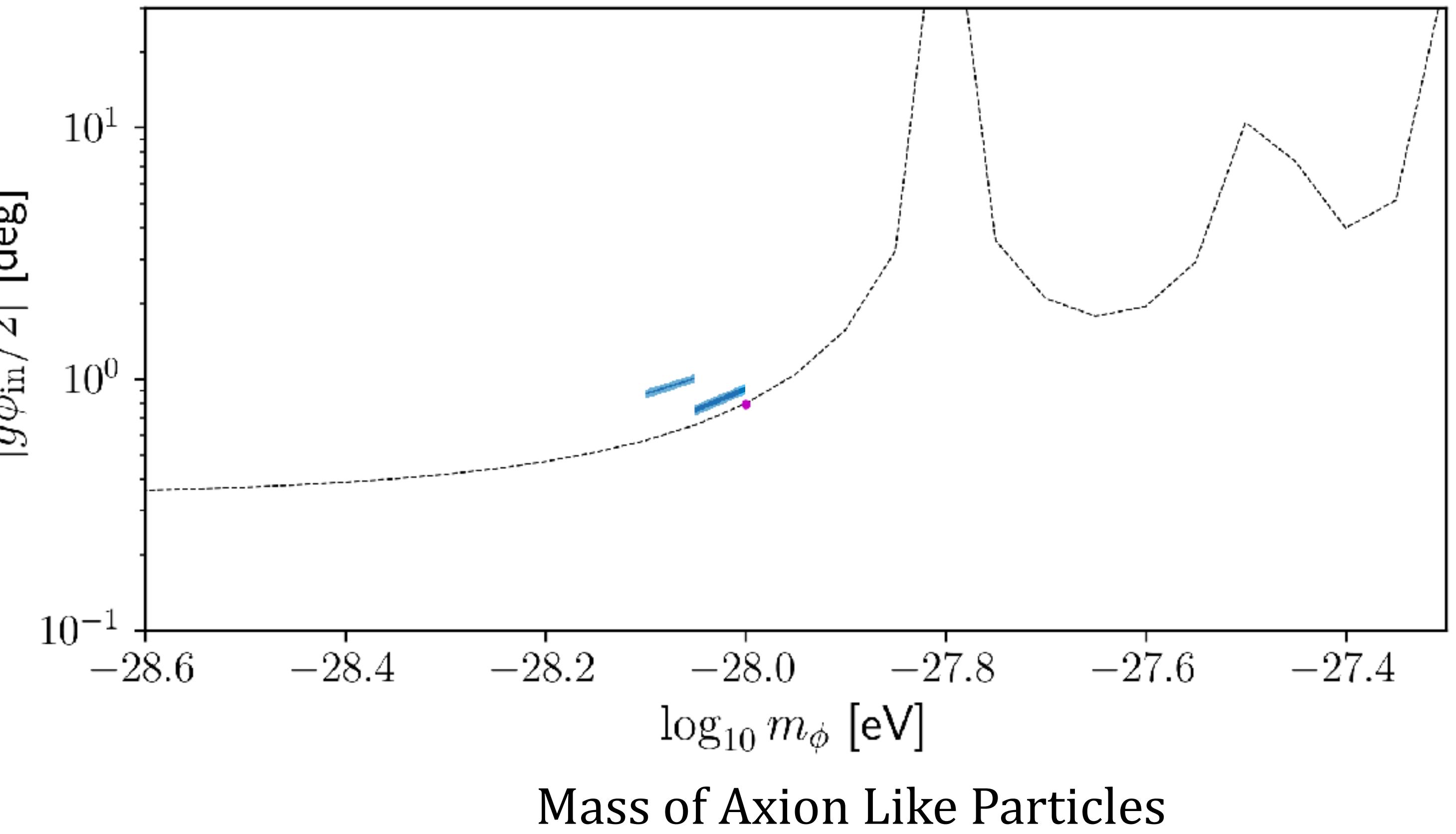


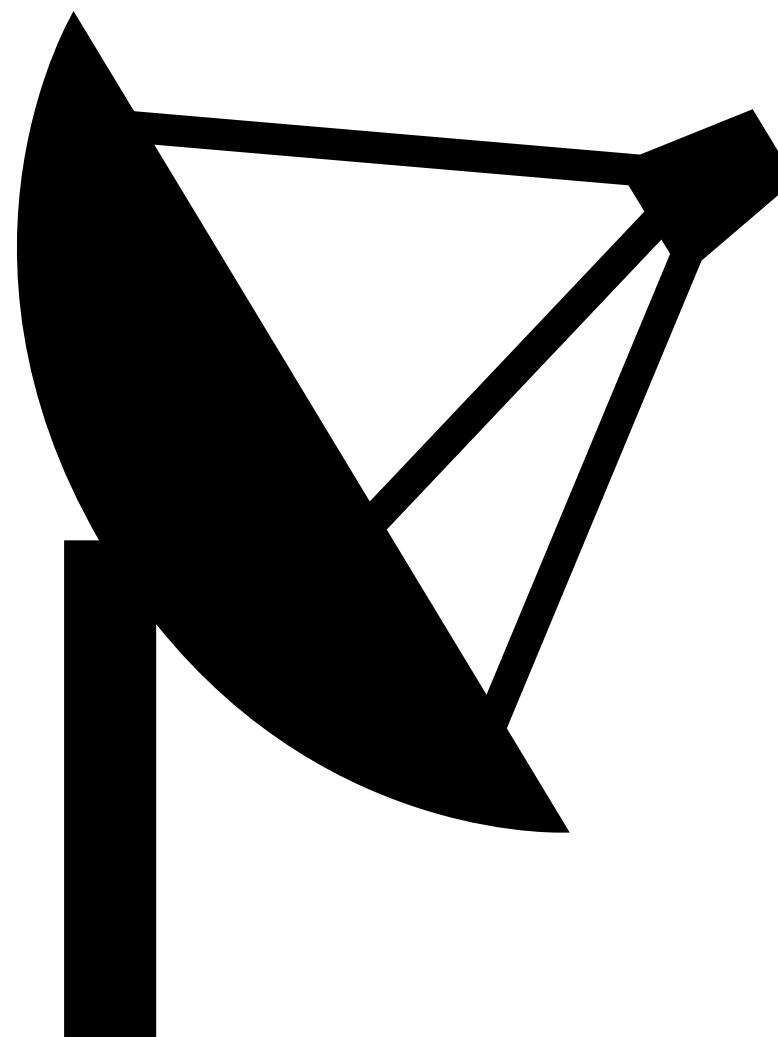
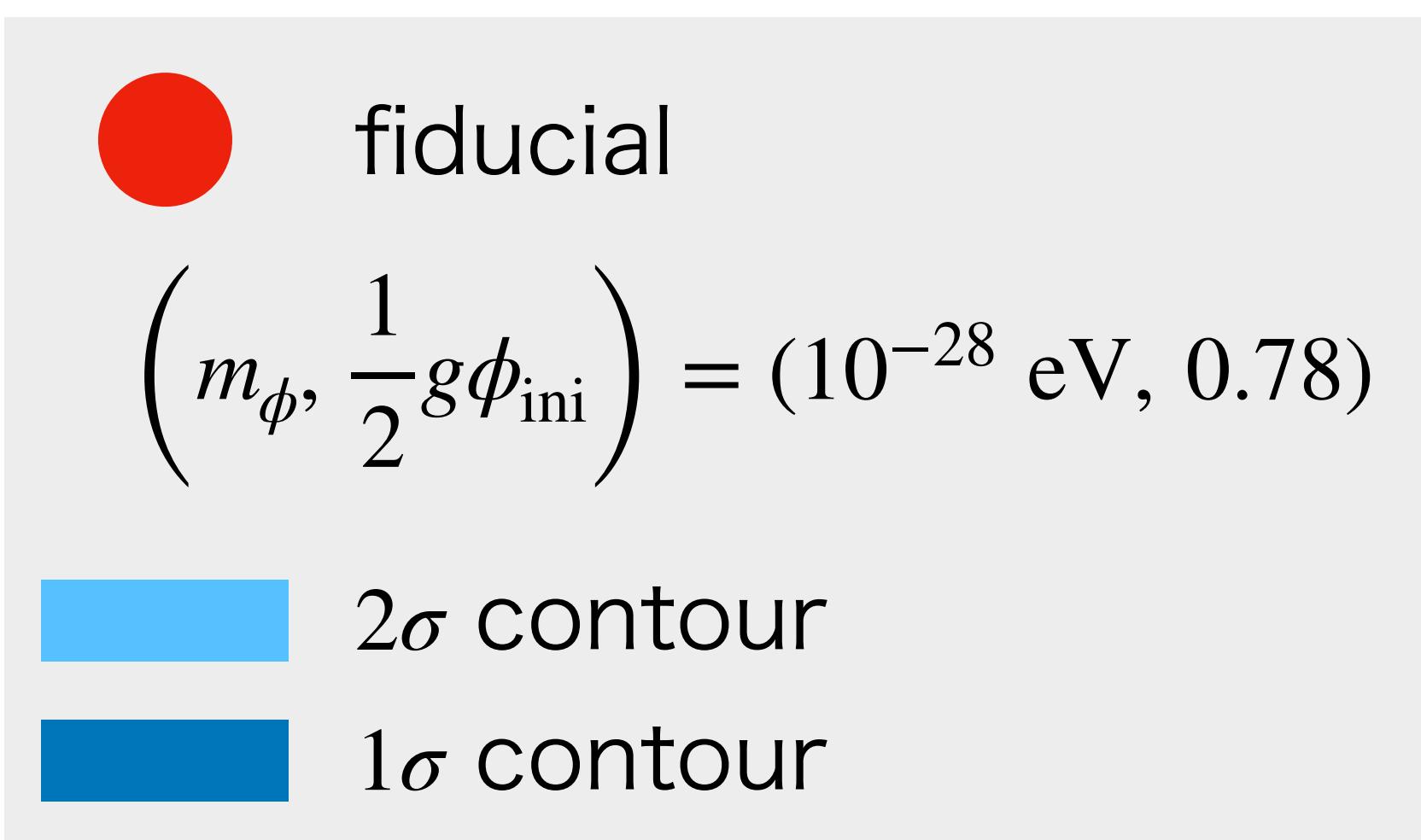


Amplitude of the signal

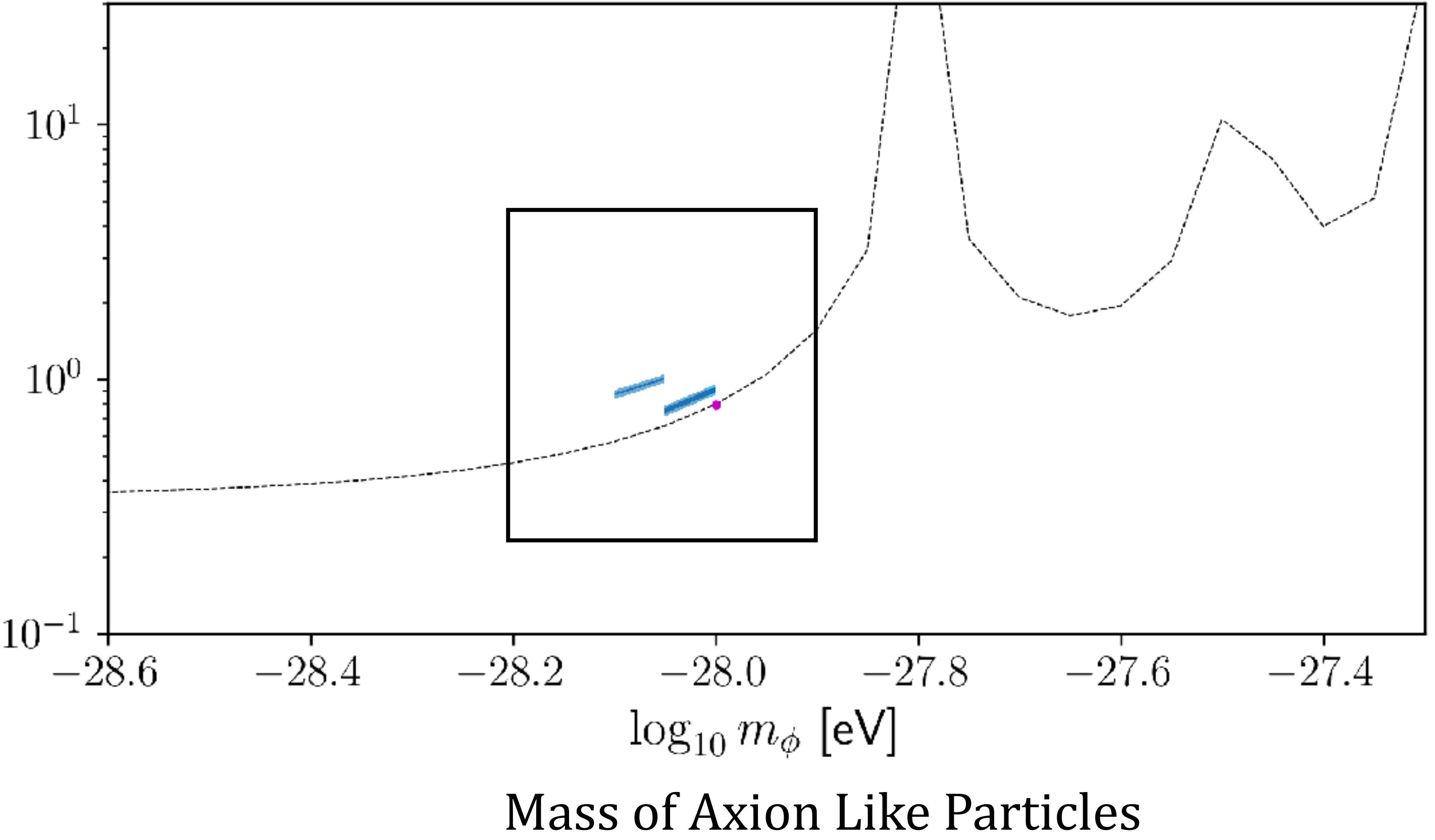
 $|g\phi_{\text{ini}}/2| [\text{deg}]$

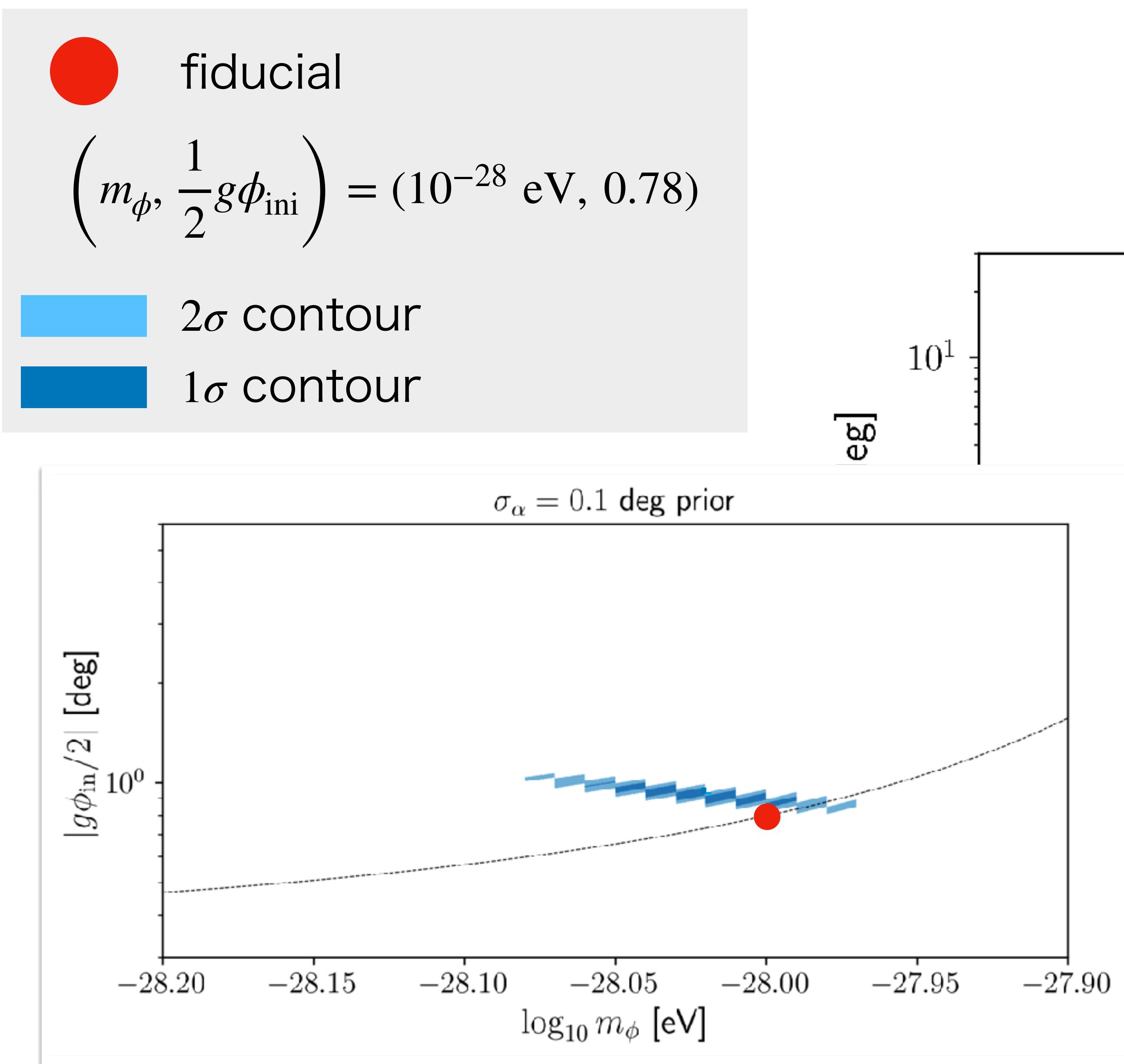
CMB - S4

 $\sigma_\alpha = 0.1 \text{ deg prior}$ 



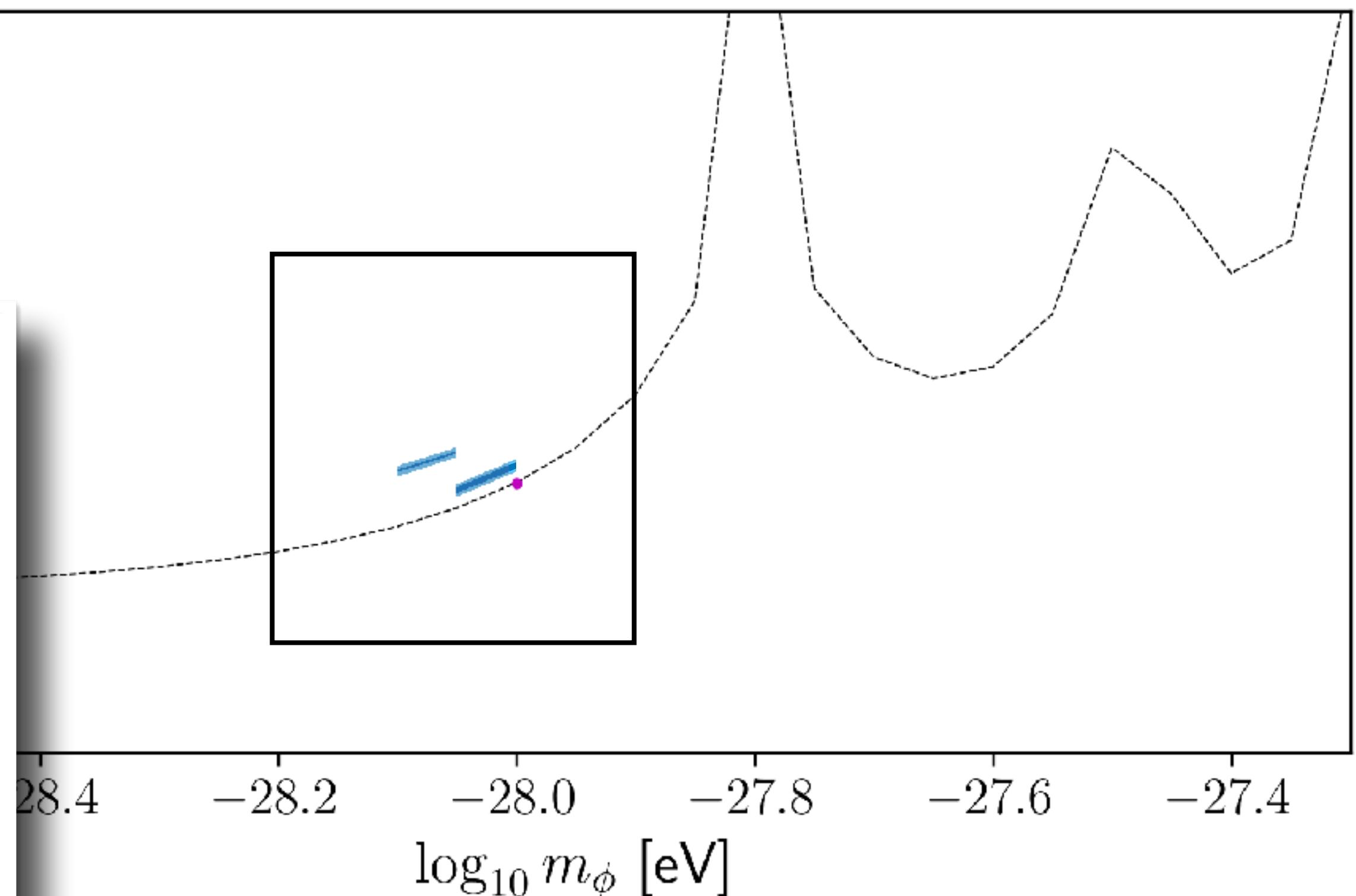
Amplitude of the signal

 $|g\phi_{\text{ini}}/2| [\text{deg}]$ 



CMB - S4

$\sigma_\alpha = 0.1 \text{ deg prior}$



Motivation

- icosahedron icon We need to improve the **precision of theoretical prediction** of Cosmic Birefringence.
- icosahedron icon **Gravitational lensing** correction should be crucial.

Method

- icosahedron icon We have developed lensing correction tool for EB & TB

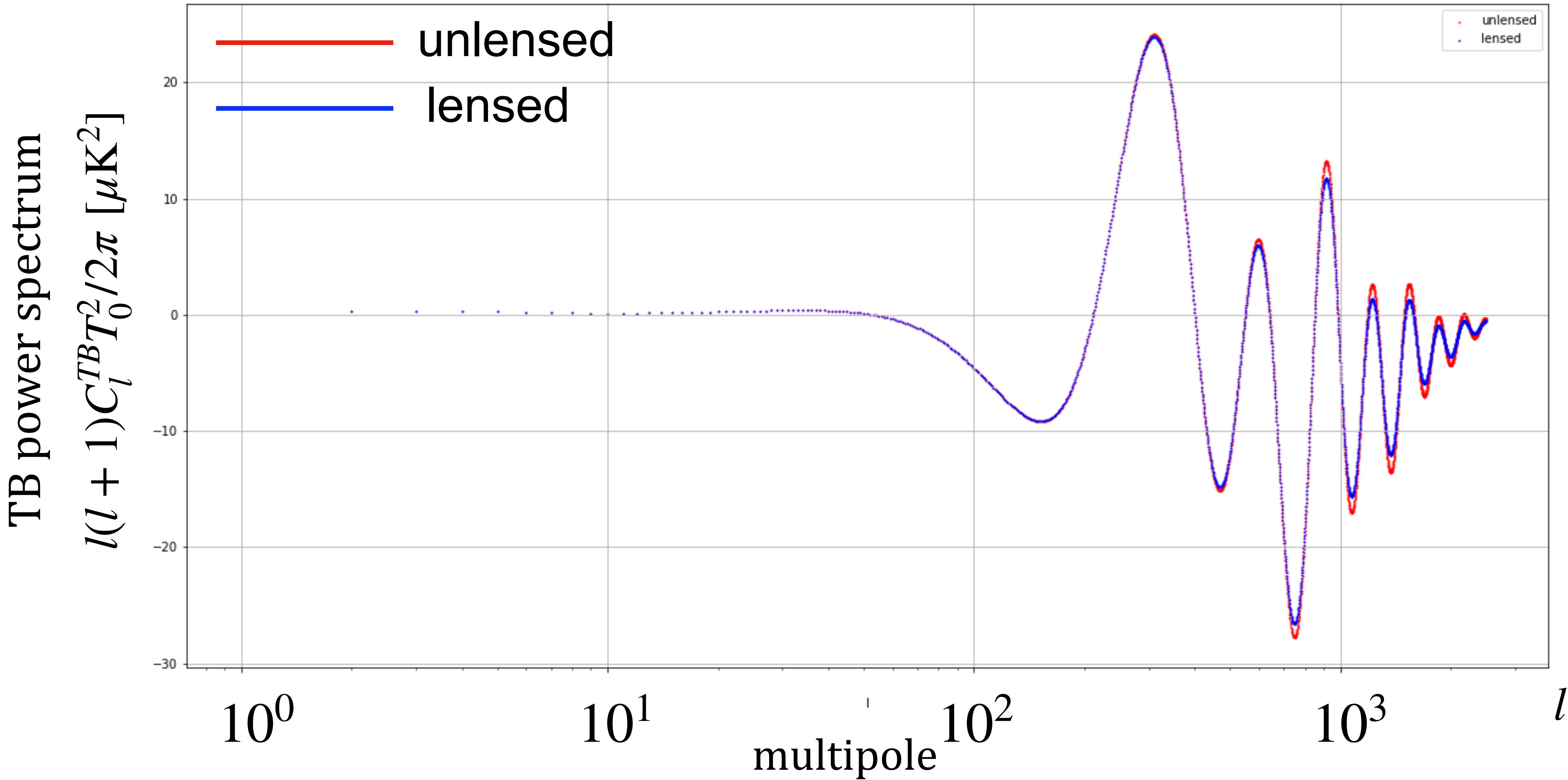


Results

- icosahedron icon Lensing effect on EB is **detectable** in the future ground-based observations.
- icosahedron icon Estimated parameters of ALPs by Cosmic Birefringence can be **biased without lensing correction**.



Backup



例：温度ゆらぎ・Eモードパワースペクトル

例：温度ゆらぎ・Eモードパワースペクトル

$$\tilde{C}_l^{\Theta E} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} C_{l'}^{\Theta E} \exp \left[-\frac{l^2}{2} \sigma_0^2(\beta) \right] J_2(l'\beta) J_2(l\beta)$$

例：温度ゆらぎ・Eモードパワースペクトル

$$\tilde{C}_l^{\Theta E} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} C_{l'}^{\Theta E} \exp \left[-\frac{l^2}{2} \sigma_0^2(\beta) \right] J_2(l'\beta) J_2(l\beta)$$

レンジング後

例：温度ゆらぎ・Eモードパワースペクトル

$$\tilde{C}_l^{\Theta E} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} C_{l'}^{\Theta E} \exp \left[-\frac{l^2}{2} \sigma_0^2(\beta) \right] J_2(l'\beta) J_2(l\beta)$$

レンジング後 レンジング前

例：温度ゆらぎ・Eモードパワースペクトル

$$\tilde{C}_l^{\Theta E} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} C_{l'}^{\Theta E} \exp \left[-\frac{l^2}{2} \sigma_0^2(\beta) \right] J_2(l'\beta) J_2(l\beta)$$

レンジング後

レンジング前

↓

$$\begin{aligned} & \beta \ll 1 \text{ として、 } d(\cos \beta) = -\sin \beta d\beta \simeq -\beta d\beta \\ & \sigma_0^2(\beta) \simeq \epsilon \beta, \epsilon \simeq 0.7 \end{aligned}$$

例：温度ゆらぎ・Eモードパワースペクトル

$$\tilde{C}_l^{\Theta E} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} C_{l'}^{\Theta E} \exp \left[-\frac{l^2}{2} \sigma_0^2(\beta) \right] J_2(l'\beta) J_2(l\beta)$$

レンジング後

レンジング前

\downarrow
 $\beta \ll 1$ として、 $d(\cos \beta) = -\sin \beta d\beta \simeq -\beta d\beta$
 $\sigma_0^2(\beta) \simeq \epsilon \beta$, $\epsilon \simeq 0.7$

$$\tilde{C}_l^{\Theta E} = \int l' dl' C_{l'}^{\Theta E} \int_0^\infty \beta d\beta \exp \left[-\frac{l^2}{2} \epsilon^2 \beta^2 \right] J_2(l'\beta) J_2(l\beta)$$

例：温度ゆらぎ・Eモードパワースペクトル

$$\tilde{C}_l^{\Theta E} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} C_{l'}^{\Theta E} \exp \left[-\frac{l^2}{2} \sigma_0^2(\beta) \right] J_2(l'\beta) J_2(l\beta)$$

レンジング後

レンジング前

$$\downarrow \quad \begin{aligned} & \beta \ll 1 \text{ として、 } d(\cos \beta) = -\sin \beta d\beta \simeq -\beta d\beta \\ & \sigma_0^2(\beta) \simeq \epsilon \beta, \epsilon \simeq 0.7 \end{aligned}$$

$$\tilde{C}_l^{\Theta E} = \int l' dl' C_{l'}^{\Theta E} \int_0^\infty \beta d\beta \exp \left[-\frac{l^2}{2} \epsilon^2 \beta^2 \right] J_2(l'\beta) J_2(l\beta)$$

$$\downarrow \quad \begin{aligned} & \text{ベッセル関数の積分公式} \\ & \text{修正ベッセル関数の漸近表現} \end{aligned}$$

例：温度ゆらぎ・Eモードパワースペクトル

$$\tilde{C}_l^{\Theta E} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} C_{l'}^{\Theta E} \exp \left[-\frac{l^2}{2} \sigma_0^2(\beta) \right] J_2(l'\beta) J_2(l\beta)$$

レンジング後

レンジング前

\downarrow

$\beta \ll 1$ として、 $d(\cos \beta) = -\sin \beta d\beta \simeq -\beta d\beta$

$\sigma_0^2(\beta) \simeq \epsilon \beta$, $\epsilon \simeq 0.7$

$$\tilde{C}_l^{\Theta E} = \int l' dl' C_{l'}^{\Theta E} \int_0^\infty \beta d\beta \exp \left[-\frac{l^2}{2} \epsilon^2 \beta^2 \right] J_2(l'\beta) J_2(l\beta)$$

\downarrow

ベッセル関数の積分公式

修正ベッセル関数の漸近表現

$$\sqrt{l} \tilde{C}_l^{\Theta E} = \int_0^\infty \frac{dl'}{\sqrt{2\pi \epsilon l'}} \sqrt{l'} C_{l'}^{\Theta E} \exp \left[-\frac{(l-l')^2}{2(\epsilon l')^2} \right]$$

例：温度ゆらぎ・Eモードパワースペクトル

$$\tilde{C}_l^{\Theta E} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} C_{l'}^{\Theta E} \exp \left[-\frac{l^2}{2} \sigma_0^2(\beta) \right] J_2(l'\beta) J_2(l\beta)$$

レンジング後 レンジング前

$$\sqrt{l} \tilde{C}_l^{\Theta E} = \int_0^\infty \frac{dl'}{\sqrt{2\pi\epsilon l'}} \sqrt{l'} C_{l'}^{\Theta E} \exp \left[-\frac{(l-l')^2}{2(\epsilon l')^2} \right]$$

レンジング後 レンジング前

$\beta \ll 1$ として、 $d(\cos \beta) = -\sin \beta d\beta \simeq -\beta d\beta$

$\sigma_0^2(\beta) \simeq \epsilon \beta$, $\epsilon \simeq 0.7$

$$\tilde{C}_l^{\Theta E} = \int l' dl' C_{l'}^{\Theta E} \int_0^\infty \beta d\beta \exp \left[-\frac{l^2}{2} \epsilon^2 \beta^2 \right] J_2(l'\beta) J_2(l\beta)$$

↓

ベッセル関数の積分公式
修正ベッセル関数の漸近表現

$$\sqrt{l} \tilde{C}_l^{\Theta E} = \int_0^\infty \frac{dl'}{\sqrt{2\pi\epsilon l'}} \sqrt{l'} C_{l'}^{\Theta E} \exp \left[-\frac{(l-l')^2}{2(\epsilon l')^2} \right]$$

例：温度ゆらぎ・Eモードパワースペクトル

$$\tilde{C}_l^{\Theta E} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} C_{l'}^{\Theta E} \exp \left[-\frac{l^2}{2} \sigma_0^2(\beta) \right] J_2(l'\beta) J_2(l\beta)$$

レンジング後 レンジング前

$$\sqrt{l} \tilde{C}_l^{\Theta E} = \int_0^\infty \frac{dl'}{\sqrt{2\pi\epsilon l'}} \sqrt{l'} C_{l'}^{\Theta E} \exp \left[-\frac{(l-l')^2}{2(\epsilon l')^2} \right]$$

レンジング後 レンジング前

$\beta \ll 1$ として、 $d(\cos \beta) = -\sin \beta d\beta \simeq -\beta d\beta$

$\sigma_0^2(\beta) \simeq \epsilon \beta$, $\epsilon \simeq 0.7$

$$\tilde{C}_l^{\Theta E} = \int l' dl' C_{l'}^{\Theta E} \int_0^\infty \beta d\beta \exp \left[-\frac{l^2}{2} \epsilon^2 \beta^2 \right] J_2(l'\beta) J_2(l\beta)$$

↓

ベッセル関数の積分公式
修正ベッセル関数の漸近表現

$$\sqrt{l} \tilde{C}_l^{\Theta E} = \int_0^\infty \frac{dl'}{\sqrt{2\pi\epsilon l'}} \sqrt{l'} C_{l'}^{\Theta E} \exp \left[-\frac{(l-l')^2}{2(\epsilon l')^2} \right]$$

例：温度ゆらぎ・Eモードパワースペクトル

$$\tilde{C}_l^{\Theta E} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} C_{l'}^{\Theta E} \exp \left[-\frac{l^2}{2} \sigma_0^2(\beta) \right] J_2(l'\beta) J_2(l\beta)$$

レンジング後 レンジング前

$$\sqrt{l} \tilde{C}_l^{\Theta E} = \int_0^\infty \frac{dl'}{\sqrt{2\pi\epsilon l'}} \sqrt{l'} C_{l'}^{\Theta E} \exp \left[-\frac{(l-l')^2}{2(\epsilon l')^2} \right]$$

レンジング後 レンジング前

$\beta \ll 1$ として、 $d(\cos \beta) = -\sin \beta d\beta \simeq -\beta d\beta$

$\sigma_0^2(\beta) \simeq \epsilon\beta$, $\epsilon \simeq 0.7$

$$\tilde{C}_l^{\Theta E} = \int l' dl' C_{l'}^{\Theta E} \int_0^\infty \beta d\beta \exp \left[-\frac{l^2}{2} \epsilon^2 \beta^2 \right] J_2(l'\beta) J_2(l\beta)$$

ガウシアン

$$\exp \left[-\frac{(l-l')^2}{2(\epsilon l')^2} \right]$$

$l' = l$ が中心

l が大きい \rightarrow 分散が大きい

l が小さい \rightarrow 分散が小さい

による畳み込み積分

ベッセル関数の積分公式

修正ベッセル関数の漸近表現

$$\sqrt{l} \tilde{C}_l^{\Theta E} = \int_0^\infty \frac{dl'}{\sqrt{2\pi\epsilon l'}} \sqrt{l'} C_{l'}^{\Theta E} \exp \left[-\frac{(l-l')^2}{2(\epsilon l')^2} \right]$$

例：温度ゆらぎ・Eモードパワースペクトル

$$\tilde{C}_l^{\Theta E} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} C_{l'}^{\Theta E} \exp \left[-\frac{l^2}{2} \sigma_0^2(\beta) \right] J_2(l'\beta) J_2(l\beta)$$

レンジング後 レンジング前

\downarrow
 $\beta \ll 1$ として、 $d(\cos \beta) = -\sin \beta d\beta \simeq -\beta d\beta$
 $\sigma_0^2(\beta) \simeq \epsilon \beta$, $\epsilon \simeq 0.7$

$$\tilde{C}_l^{\Theta E} = \int l' dl' C_{l'}^{\Theta E} \int_0^\infty \beta d\beta \exp \left[-\frac{l^2}{2} \epsilon^2 \beta^2 \right] J_2(l'\beta) J_2(l\beta)$$

\downarrow
 ベッセル関数の積分公式
 修正ベッセル関数の漸近表現

$$\sqrt{l} \tilde{C}_l^{\Theta E} = \int_0^\infty \frac{dl'}{\sqrt{2\pi\epsilon l'}} \sqrt{l'} C_{l'}^{\Theta E} \exp \left[-\frac{(l-l')^2}{2(\epsilon l')^2} \right]$$

$$\sqrt{l} \tilde{C}_l^{\Theta E} = \int_0^\infty \frac{dl'}{\sqrt{2\pi\epsilon l'}} \sqrt{l'} C_{l'}^{\Theta E} \exp \left[-\frac{(l-l')^2}{2(\epsilon l')^2} \right]$$

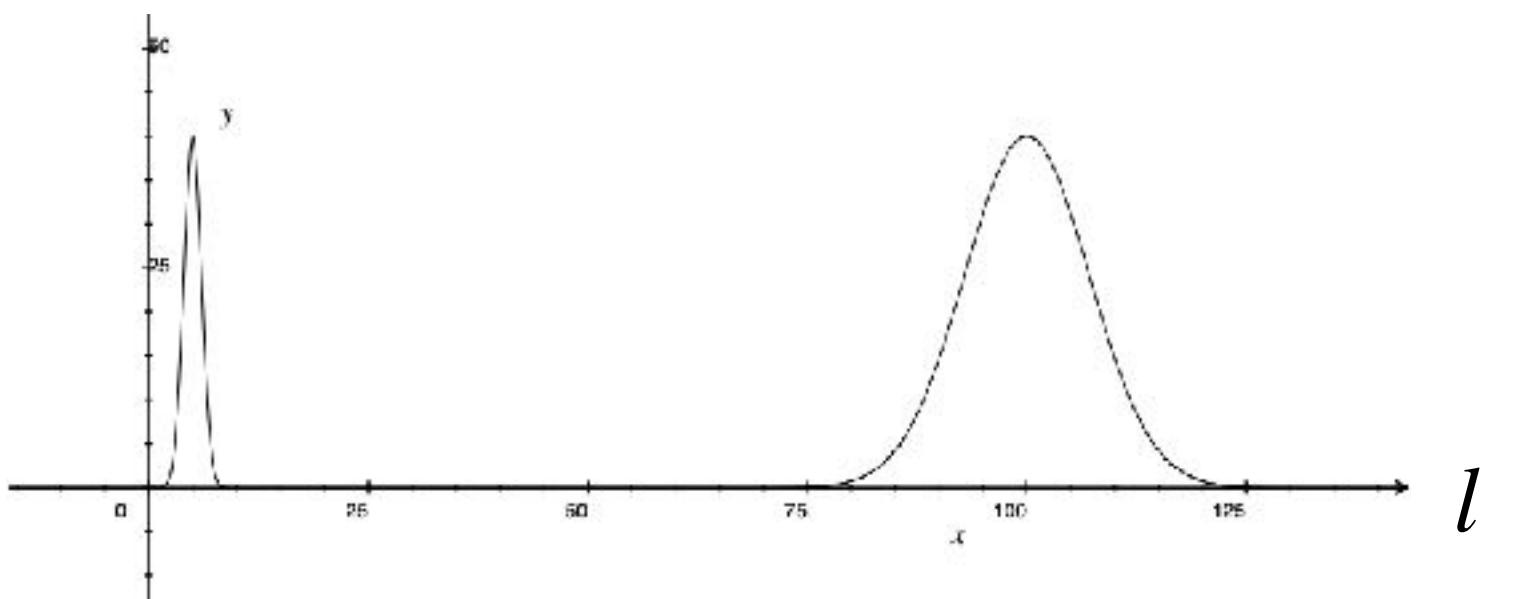
レンジング後 レンジング前

ガウシアン

$$\exp \left[-\frac{(l-l')^2}{2(\epsilon l')^2} \right]$$

$l' = l$ が中心
 l が大きい \rightarrow 分散が大きい
 l が小さい \rightarrow 分散が小さい

による畳み込み積分



例：温度ゆらぎ・Eモードパワースペクトル

$$\tilde{C}_l^{\Theta E} = 2\pi \int_{-1}^1 d(\cos \beta) \int \frac{l' dl'}{2\pi} C_{l'}^{\Theta E} \exp \left[-\frac{l^2}{2} \sigma_0^2(\beta) \right] J_2(l'\beta) J_2(l\beta)$$

レンジング後 レンジング前

$$\sqrt{l} \tilde{C}_l^{\Theta E} = \int_0^\infty \frac{dl'}{\sqrt{2\pi\epsilon l'}} \sqrt{l'} C_{l'}^{\Theta E} \exp \left[-\frac{(l-l')^2}{2(\epsilon l')^2} \right]$$

レンジング後 レンジング前

$\beta \ll 1$ として、 $d(\cos \beta) = -\sin \beta d\beta \simeq -\beta d\beta$

$\sigma_0^2(\beta) \simeq \epsilon\beta$, $\epsilon \simeq 0.7$

$$\tilde{C}_l^{\Theta E} = \int l' dl' C_{l'}^{\Theta E} \int_0^\infty \beta d\beta \exp \left[-\frac{l^2}{2} \epsilon^2 \beta^2 \right] J_2(l'\beta) J_2(l\beta)$$

ベッセル関数の積分公式

修正ベッセル関数の漸近表現

$$\sqrt{l} \tilde{C}_l^{\Theta E} = \int_0^\infty \frac{dl'}{\sqrt{2\pi\epsilon l'}} \sqrt{l'} C_{l'}^{\Theta E} \exp \left[-\frac{(l-l')^2}{2(\epsilon l')^2} \right]$$

ガウシアン

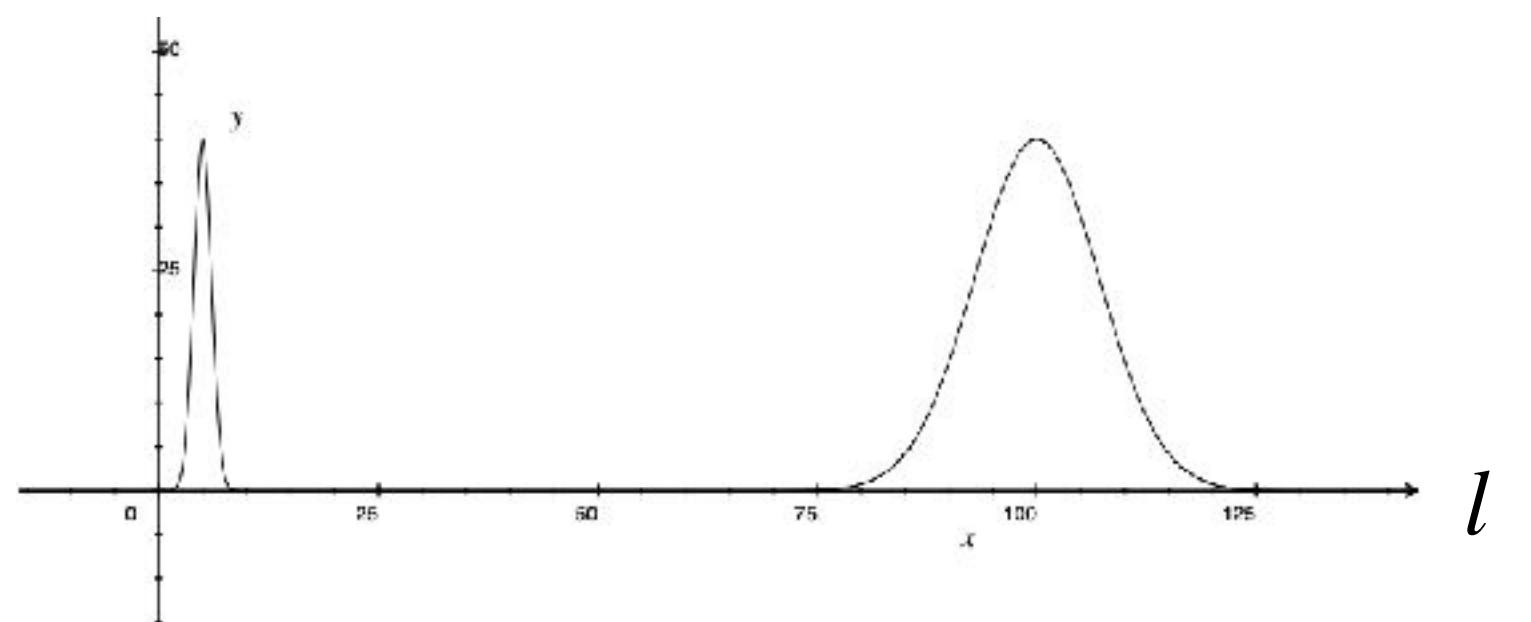
$$\exp \left[-\frac{(l-l')^2}{2(\epsilon l')^2} \right]$$

$l' = l$ が中心

l が大きい \rightarrow 分散が大きい

l が小さい \rightarrow 分散が小さい

による畳み込み積分



大きな l でパワースペクトルがなまされるような効果