

# Dark matter density profile estimation with cosmological models

Shunichi Horigome (Kavli IPMU)

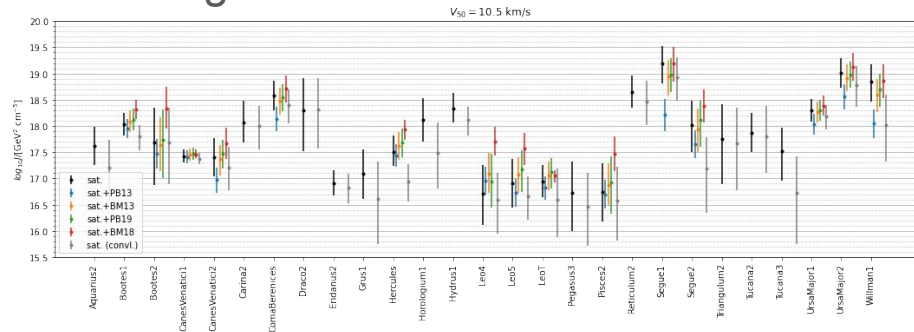
based on:

- [arXiv:2207.10378], collaboration with Kohei Hayashi and Shin'ichiro Ando
- [arXiv:in-prep], collaboration with Masato Shirasaki and Shin'ichiro Ando

# Dark matter density profile estimation with cosmological models

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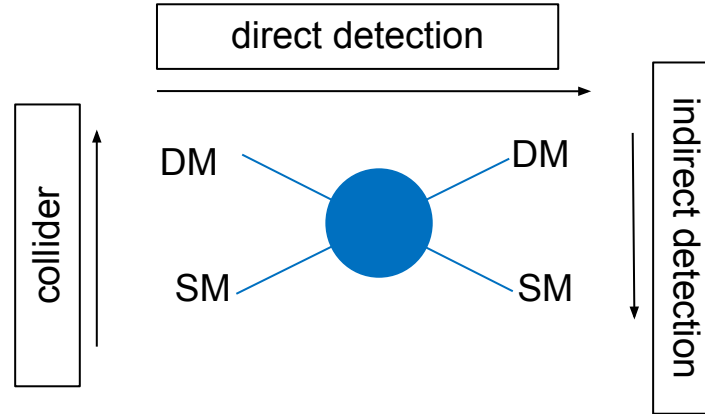
- Dark matter density profile of astronomical objects (dSph, MW) is important to study the nature of dark matter (mass, cross section, self-interaction)
- Cosmological models are useful to estimate these profiles by using stellar data
  - Satellite prior: prior on structural parameters of dSph dark matter profiles based on the extended Press-Schechter formalism
  - SHMR prior: empirical relation between stellar and dark matter mass
  - SIDM profile model: gravothermal fluid model calibrated by N-body simulation



# Introduction

# Introduction: Indirect detection

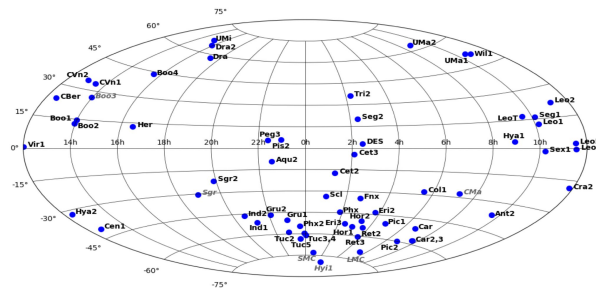
- Collider search
- Direct detection
- Indirect detection



# Introduction: Indirect detection

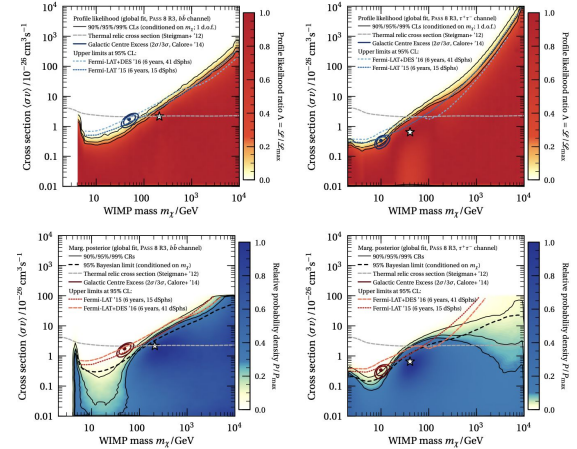
- e.g. Dwarf spheroidal galaxies (dSphs)
  - Containing large amount of DM
  - Good candidates for the indirect detection of the WIMP DM

e.g. Fornax dSph



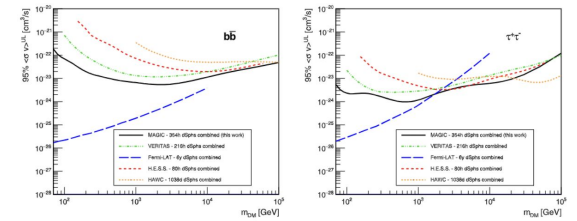
McConachie+(2020) [2007.05011]

Fermi-LAT



Hoof+(2020) [1812.06986]

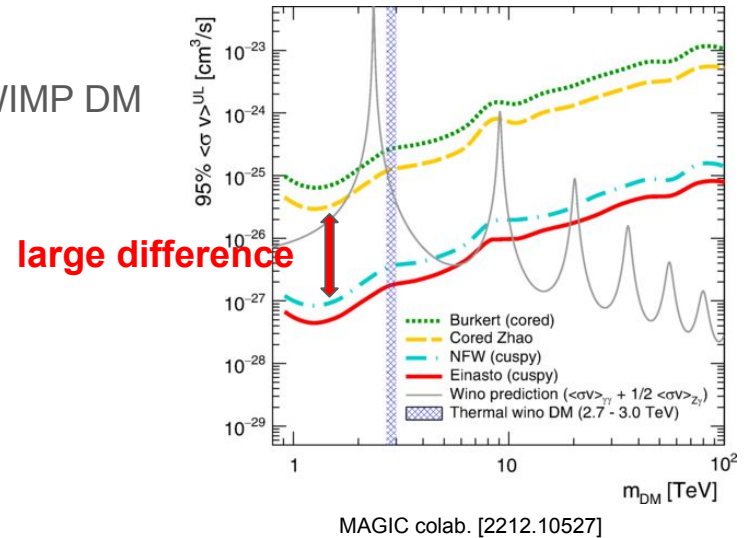
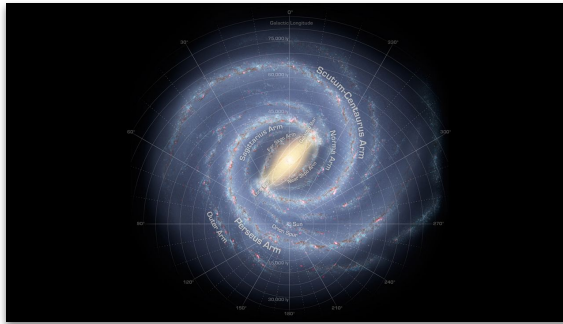
Combined (Fermi-LAT, HAWC, H.E.S.S., MAGIC, and VERITAS)



Armard+(2021) [2108.13646]

# Introduction: Indirect detection

- e.g. Galactic center
  - Containing large amount of DM
  - Good candidates for the indirect detection of the WIMP DM



# Problem: DM profile dependency

- The sensitivity of the indirect detection depends on the **DM profile** of targets
  - Indirect detection: DM annihilation into SM particles (gamma-ray etc.)
    - Signal flux:

$$\Phi(E, \Delta\Omega) = \left[ \frac{C\langle\sigma v\rangle}{4\pi m_{\text{DM}}^2} \sum_f b_f \left( \frac{dN_\gamma}{dE} \right)_f \right] \times \left[ \int_{\Delta\Omega} d\Omega \int_{\text{l.o.s.}} dl \rho_{\text{DM}}^2(l, \Omega) \right]$$

Particle physics factor


**Astronomical factor =  $J(\Delta\Omega)$**

# Questions

- How is the DM density profile...
  - from **observational** viewpoints?
    - How can we precisely determine the density profile?
  - from **theoretical** viewpoints?
    - How should the profile be in specific DM scenarios?
      - e.g. CDM v.s. SIDM
        - CDM ← “cuspy”
        - SIDM ← “cored”



# Questions

- How is the DM density profile...
    - from **observational** viewpoints? 1.
      - How can we precisely determine the density profile?
    - from **theoretical** viewpoints?
      - How should the profile be in specific DM scenarios?
        - e.g. CDM v.s. SIDM
          - CDM ← “cuspy”
          - SIDM ← “cored”
- 

# 1. Cosmological prior for the J-factor estimation of dwarf spheroidal galaxies

Shunichi Horigome, Kohei Hayashi, Shinichiro Ando

# Cosmological prior for the J-factor estimation of dwarf spheroidal galaxies

Shunichi Horigome, Kohei Hayashi, Shinichiro Ando

We estimate DM density profile of dSphs by

- Satellite prior:
  - cosmological model of CDM subhalo formation  
(semi-analytic model based on extended Press-Schechter formalizm)
- Stellar-to-halo mass relation (SHMR):
  - Empirical relation between stellar and DM mass
- velocity dependent likelihood:
  - probing velocity dispersion profile of dSph

# Jeans analysis

- Jeans equation: kinematical equation of dSph systems
  - Assumption: sphericity

$$\frac{1}{\nu_*(r)} \frac{\partial(\nu_*(r)\sigma_r^2(r))}{\partial r} + \frac{2\beta(r)\sigma_r^2(r)}{r} = -\frac{GM_{\text{DM}}(r)}{r^2}$$

(stellar distribution & velocity dispersion) ~ (inner dark matter mass)

- Observable: line-of-sight velocity dispersion (R-dependent)

$$\sigma_{\text{los}}^2(R) = \frac{2}{\Sigma(R)} \int_R^\infty dr \left(1 - \beta(r) \frac{R^2}{r^2}\right) \frac{\nu(r)\sigma_r^2(r)}{\sqrt{1 - R^2/r^2}}$$

- Models:
  - Stellar profile  $\Sigma(R)$ ,  $\nu(r)$ : Plummer model
  - DM profile  $\rho(r)$  : truncated NFW model
  - Anisotropy profile  $\beta(r)$  : constant model

# Likelihood

- Likelihood function

$$\mathcal{L}(\Theta) = \prod \mathcal{N}[v_i; v_{\text{dSph}}, \sigma_{\text{los}}^2(R_i) + \delta \sigma_i^2],$$

- Posterior probability

$$P(\Theta|D) = \frac{\mathcal{L}(\Theta)\pi(\Theta)}{\int d\Theta \mathcal{L}(\Theta)\pi(\Theta)},$$

- $\pi(\Theta)$  : prior

$$\pi = \begin{cases} \pi_{\text{photo.}} & \text{(without any cosmological priors)} \\ \pi_{\text{photo.}} \pi_{\text{sat.}} & \text{(satellite prior only)} \\ \pi_{\text{photo.}} \pi_{\text{sat.+SHMR}} & \text{(satellite \& SHMR prior)} \end{cases}$$

# Cosmological priors

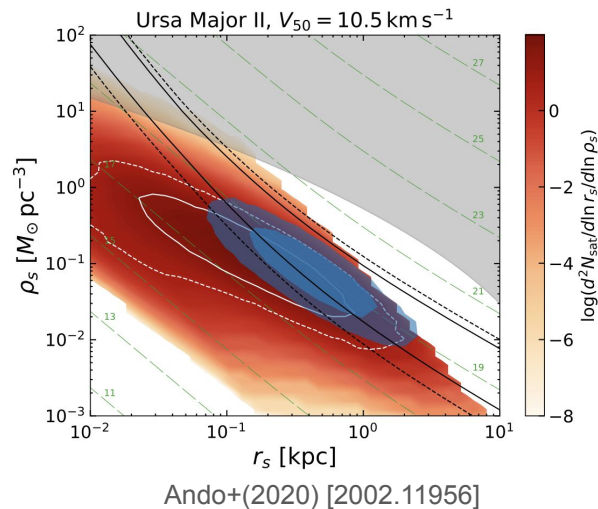
- Satellite prior [1803.07691, 2002.11956]
  - Accretion of subhalo: extended Press-Schechter (EPS) formalism
  - Tidal stripping effect: semi-analytical subhalo model calibrated by N-body simulation

$$\dot{m}(z) = -A \frac{m(z)}{\tau_{\text{dyn}}(z)} \left[ \frac{m(z)}{M(z)} \right]^\zeta$$

$$P_{\text{sat}}(r_s, \rho_s, r_t) \propto \frac{d^3 N_{\text{sat}}}{dr_s d\rho_s dr_t} = \frac{d^3 N_{\text{sh}}}{dr_s d\rho_s dr_t} P_{\text{form}}(V_{\text{peak}}).$$

$$P_{\text{form}}(V_{\text{peak}}) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{V_{\text{peak}} - V_{50}}{\sqrt{2}\sigma} \right) \right]$$

$V_{\text{peak}}$ : maximum circular velocity  
 $V_{50} = 10.5 \text{ km/s}$  or  $18 \text{ km/s}$

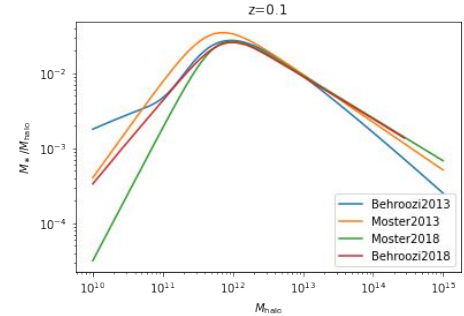


# Cosmological priors

- The stellar-to-halo mass relation (SHMR)
  - empirical relation between the stellar and DM halo mass of galaxies:  $M_{\text{star}} = f(M_{\text{halo}}, z)$
  - assumption:  $f(M_{\text{halo}}, z)$  is a monotonic function for  $M_{\text{halo}}$

## Models:

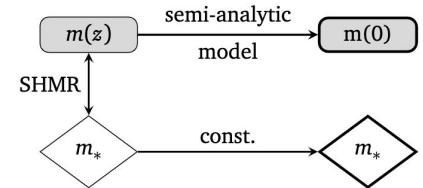
- Behroozi+(2013) [1207.6105]
  - calibrated by the Bolshoi simulation, complete model
- Moster+(2013) [1205.5807]
  - calibrated by the Millennium simulation, assuming simple double power law
- Behroozi+(2019) [1806.07893]
  - Updated dataset and models, model selection based on Bayes factor
- Moster+(2018) [1705.05373]
  - double-power law for efficiency evolution



## Prior:

$$\pi_{\text{SHMR}}(\rho_s, r_s, r_t) = \frac{\mathcal{N}(M_{*,\text{obs}} | M_*(M_{\text{halo}}), \sigma^2) \pi_{\text{satellite}}(\rho_s, r_s, r_t)}{\int d r_s d \rho_s d r_t \mathcal{N}(M_{*,\text{obs}} | M_*(M_{\text{halo}}), \sigma^2) \pi_{\text{satellite}}(\rho_s, r_s, r_t)}$$

$$M_{\text{halo}} \leftarrow (\rho_{s,0}, r_{s,0}, r_{t,0}) \longleftrightarrow (\rho_{s,a}, r_{s,a}, z) \xrightarrow{\text{SHMR}} M_{*,a} = M_{*,0}$$



# Target dSphs

- 8 classical + 26 ultrafaint dSph in [2002.11956]

## Classical:

Carina  
Draco  
Fornax  
Leo I  
Leo II  
Sculptor  
Sextans  
Ursa Minor

## UFD:

Aquarius 2  
Bootes I  
Bootes II  
Canes Venatici I  
Canes Venatici II  
Carina II  
Coma Berenices  
Draco II  
Eridanus II  
Grus I  
Hercules  
Horologium I  
Hydrus 1

Leo IV  
Leo T  
Leo V  
Pegasus III  
Pisces II  
Reticulum II  
Segue 1  
Segue 2  
Triangulum II  
Tucana II  
Tucana III  
Ursa Major I  
Ursa Major II

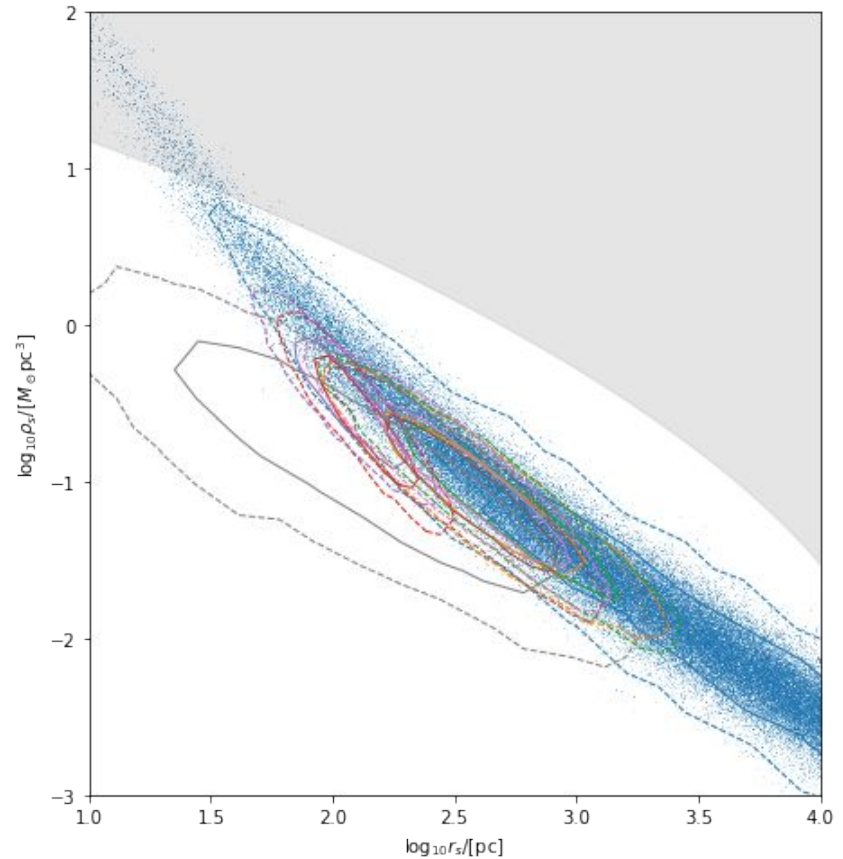


# Results

- Posterior density function

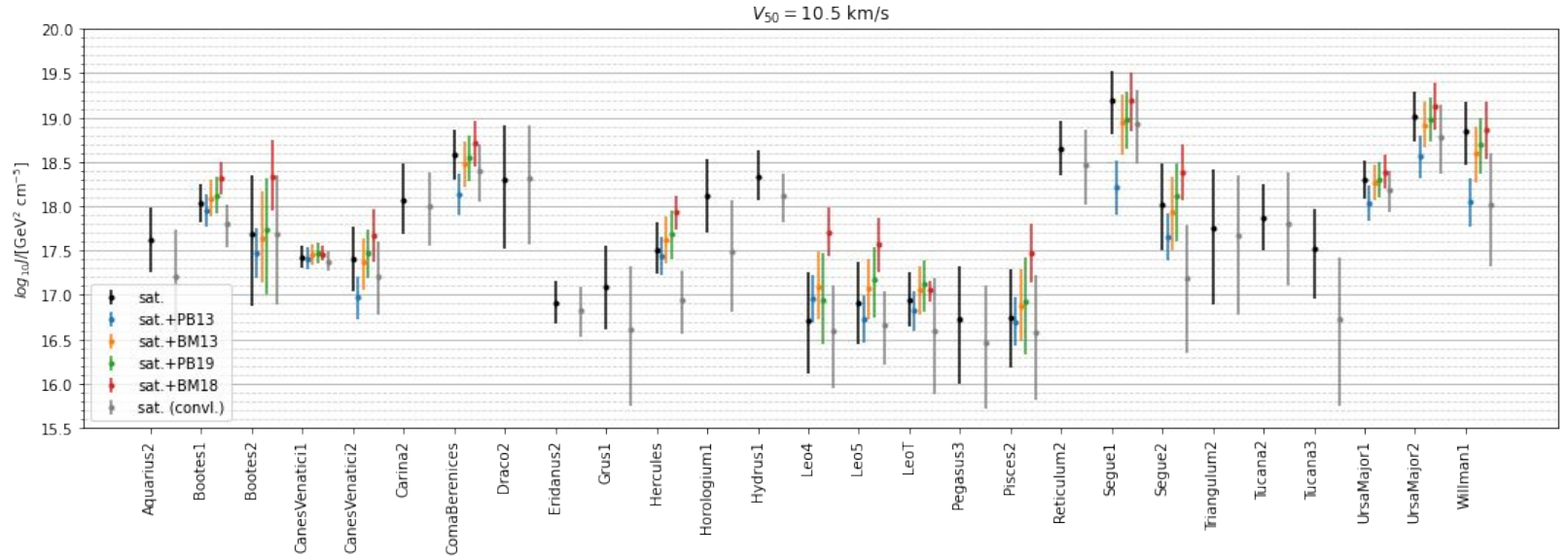
e.g. Coma Berenices

- satellite prior only
- likelihood only
- likelihood + satellite ( $V_{50} = 10.5$  km/s)
- likelihood + satellite ( $V_{50} = 18$  km/s)
- likelihood + satellite ( $V_{50} = 10.5$  km/s) + SHMR(Behroozi)
- likelihood + satellite ( $V_{50} = 18$  km/s) + SHMR(Behroozi)
- likelihood + satellite ( $V_{50} = 10.5$  km/s) + SHMR(Moster)
- likelihood + satellite ( $V_{50} = 18$  km/s) + SHMR(Moster)



# Results

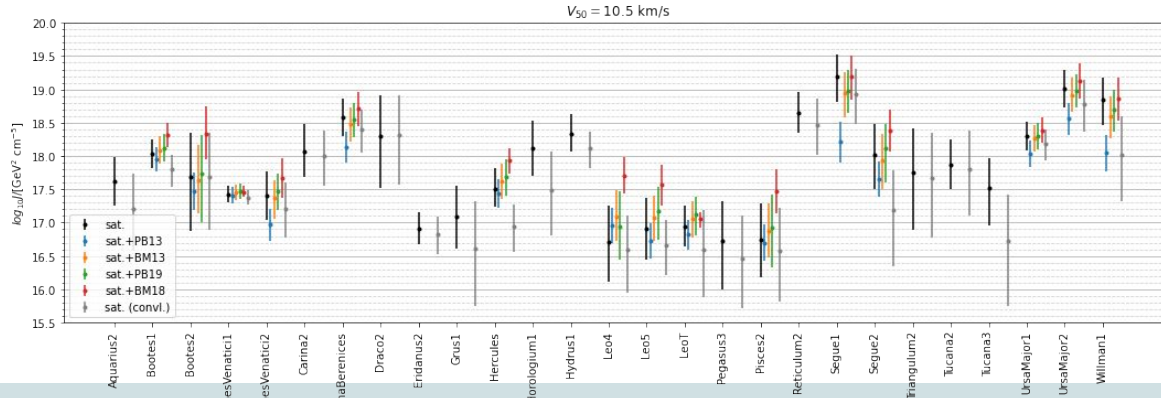
- J-factor



# Results

- J-factor

- slightly larger than those of the velocity independent analysis
  - radial dependence of the likelihood excludes too compact or faint DM halo having small J-factor
    - Note: anisotropy profile dependence in the velocity dispersion
- SHMR priors can decrease J-factor uncertainty (upto ~50%) but results are model dependent
  - Test of SHMR models by using dSphs?



# Results

- J-factors vs. Bayes factors
  - Deviated results have less Bayes factor values
    - Results of the cosmological prior analysis is stable in terms of their Bayes factors

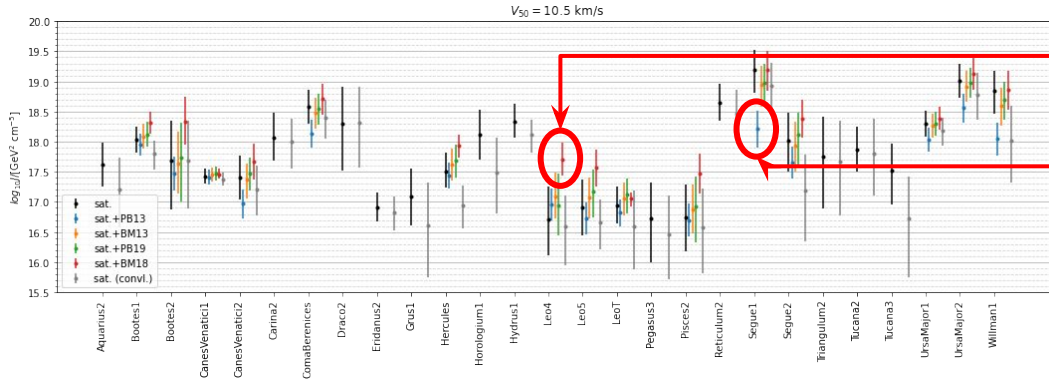



Table 6: The logarithm of Bayes factors of each model calculated according to Eq. (20). Column 1 shows the Bayes factor of  $\text{sat}_{18}$  to a reference model  $\text{sat}_{10.5}$  for each dSphs. Columns 2-5 shows the Bayes factors of the satellite prior and SHMR analyses to the satellite prior only analysis  $\text{sat}_{10.5}$  as a reference, so as Columns 6-9 not for  $\text{sat}_{10.5}$  but  $\text{sat}_{18}$  cases. By definition, positive (negative) values mean that the corresponding model is more (less) credible than the reference model.

	$\text{sat}_{18}/\text{sat}_{10.5}$ w/o SHMR	$\text{sat}_{10.5}$				$\text{sat}_{18}$			
		PB13	BM13	PB19	BM18	PB13	BM13	PB19	BM18
Aquarius2	0.77	-	-	-	-	-	-	-	-
Bootes1	-0.01	0.34	0.20	0.17	0.09	0.12	0.10	0.21	0.11
Bootes2	-0.09	0.05	0.06	-0.05	-0.16	0.07	0.04	0.01	-0.01
CanesVenatic1	0.49	0.36	0.49	0.07	0.34	0.33	-0.01	-0.31	-0.03
CanesVenatic2	1.29	-0.70	0.64	0.92	2.08	-2.61	-0.70	0.15	0.71
Carina2	-0.14	-	-	-	-	-	-	-	-
ComaBerenices	1.06	-1.71	-0.09	0.35	1.75	-3.07	-0.52	0.07	0.64
Draco2	0.16	-	-	-	-	-	-	-	-
Eridanus2	0.79	-	-	-	-	-	-	-	-
Grus1	-0.30	-	-	-	-	-	-	-	-
Hercules	0.88	0.58	0.96	0.59	1.06	-0.04	-0.06	-0.07	0.15
Horologium1	1.12	-	-	-	-	-	-	-	-
Hydrus1	-0.17	-	-	-	-	-	-	-	-
Leo4	0.16	0.34	0.02	0.13	-0.93	0.44	0.15	0.01	-0.72
Leo5	-0.03	0.34	0.51	0.04	0.24	-0.47	-0.15	0.31	0.45
LeoT	1.39	0.65	1.85	1.43	0.84	-0.01	0.47	0.08	-0.61
Pegasus3	1.28	-	-	-	-	-	-	-	-
Pisces2	0.27	0.49	0.26	-0.04	-0.07	0.29	-0.01	-0.11	-0.28
Reticulum2	0.96	-	-	-	-	-	-	-	-
Segue1	0.08	-2.63	1.00	-0.27	1.36	-4.29	-1.01	-0.05	-0.41
Segue2	0.08	0.11	0.12	0.21	0.26	-0.17	-0.20	0.04	-0.10
Triangulum2	-0.65	-	-	-	-	-	-	-	-
Tucana2	-0.13	-	-	-	-	-	-	-	-
Tucana3	-2.75	-	-	-	-	-	-	-	-
UrsaMajor1	1.06	-5.26	-0.16	0.61	1.84	-5.30	-0.45	-0.08	0.80
UrsaMajor2	1.25	-4.98	-1.11	0.05	1.67	-5.93	-0.63	-0.21	0.37
Willman1	2.07	-3.05	-1.05	-0.62	1.71	-4.90	-1.26	-0.37	-0.29

# Questions

- How is the DM density profile...
    - from **observational** viewpoints? 1.
      - How can we precisely determine the density profile?
    - from **theoretical** viewpoints?
      - How should the profile be in specific DM scenarios?
        - e.g. CDM v.s. SIDM
          - CDM ← “cuspy”
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- 

## 2. Comparison of MW-sized halo profiles in cosmological SIDM simulations and gravothermal fluid models

Masahiro Shirasaki, Shunichi Horigome, Shinichiro Ando

## 2. Comparison of MW-sized halo profiles in cosmological SIDM simulations and gravothermal fluid models

Masahiro Shirasaki, Shunichi Horigome, Shinichiro Ando

What is the physics underlying the formation of core-like profiles?

...we assume the SIDM halo can be described by the

“gravothermal fluid model”

and test it by a cosmological N-body simulation

# Gravothermal fluid model

“What is dark matter?”

← ... a “gravitationally bound and thermally conducting fluid” [Balberg+(2002)]

$$\frac{\partial M}{\partial r} = 4\pi r^2 \rho$$

mass conservation

$$\frac{\partial(\rho\sigma_v^2)}{\partial r} = -G \frac{M\rho}{r}$$

Hydrostatic equilibrium

$$\frac{L}{4\pi r^2} = -\kappa \frac{\partial T}{\partial r} = -\kappa \rho \frac{\partial}{\partial r} \left( \frac{3\sigma_v^2}{2} \right)$$

Heat conduction

$$\frac{\partial L}{\partial r} = -4\pi r^2 \rho \left[ \left( \frac{\partial}{\partial t} \right)_M \frac{3\sigma_v^2}{2} + p \left( \frac{\partial}{\partial t} \right)_M \frac{1}{\rho} \right] = -4\pi r^2 \rho v^2 \left( \frac{\partial}{\partial t} \right)_M \log \left( \frac{\sigma_v^3}{\rho} \right)$$

1st law of thermodynamics

Model parameter: thermal conductivity  $\kappa$  ← calibrated by **isolated** N-body simulation [Koda+(2011)]

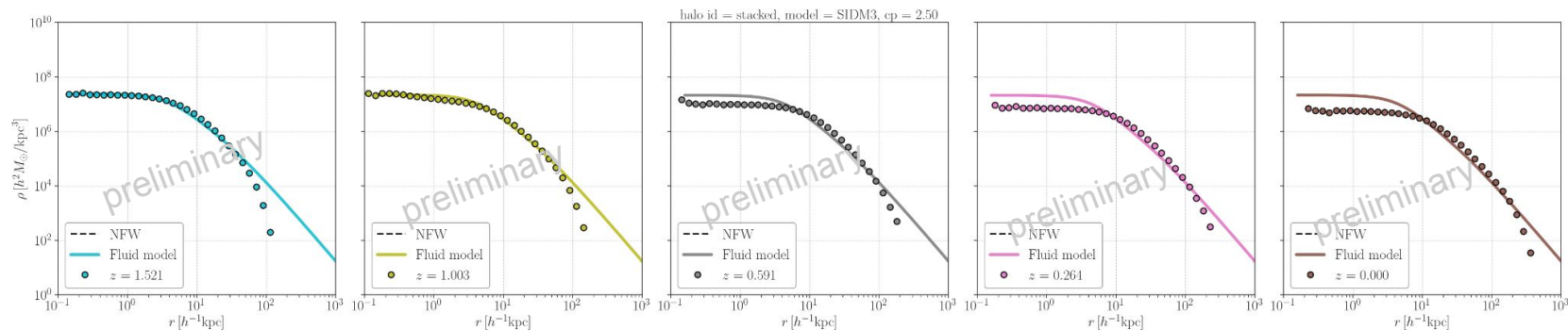


# Application to cosmological N-body simulation

- How about **cosmological** N-body simulation?
  - Simulation data: [Ebisu, Ishiyama, Hayashi(2022)]
    - $1024^3$  particles in the comoving volume of  $(8 \text{ Mpc}/h)^3$
    - particle mass:  $4.1 \times 10^4 h^{-1} M_\odot$
    - CDM and two different SIDM runs ( $\sigma/m = 1, 3 \text{ cm}^2 \text{ g}^{-1}$ ) are available
    - Select 9 Milky-Way-sized halos and measure the density and velocity dispersion profiles
    - Mass accretion history (MAH) of each halo is also available

# Result

- Roughly fit higher z halo
- discrepancy in lower z halo
  - mass accretion?



# Summary

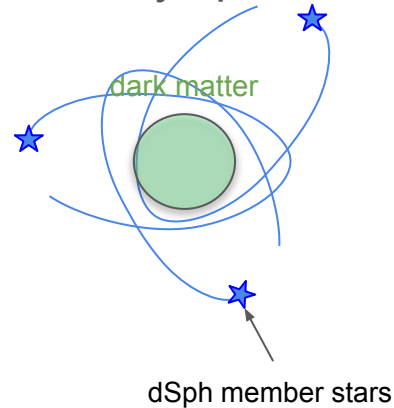
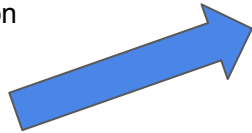
- Dark matter density profile of (sub-)halos plays an important the indirect detection method, but they are not well understood yet
- From observational viewpoint, we decreased the uncertainty of dark matter density profile of dSphs by introducing
  - Satellite prior: DM subhalo formation model based on extended Press-Schechter formalizm
  - Stellar-to-halo mass relation (SHMR): Empirical relation between stellar and DM mass
  - velocity dependent likelihood: probing velocity dispersion profile of dSph
- From theoretical viewpoint, we study density profile of SIDM halos by gravothermal fluid model
  - potentially useful to cosmological N-body simulation (in progress)

# Backup slides

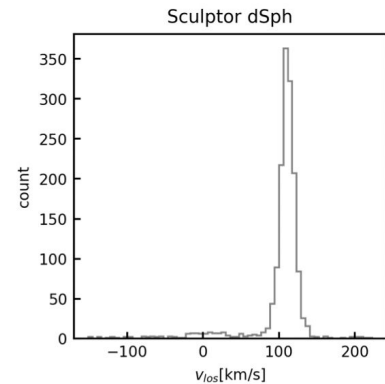
# How to estimate DM density profile

- dSph member stars move in the gravitational potential yielded by DM mass density
- Velocity of member stars is observed by spectroscopic telescope

spectroscopic observation

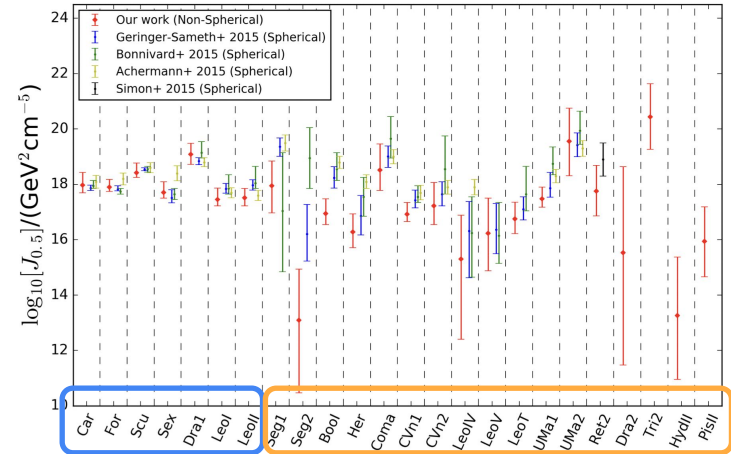


e.g. Stellar velocity distribution of the Sculptor dSph



# J-factor uncertainty

- J-factor has large uncertainty
  - Limited number of dataset
    - **Classical**: O(100)
    - **Ultrafaint**: O(10)



Hayashi et al. [1603.08046]

- We use "cosmological prior" to improve the accuracy

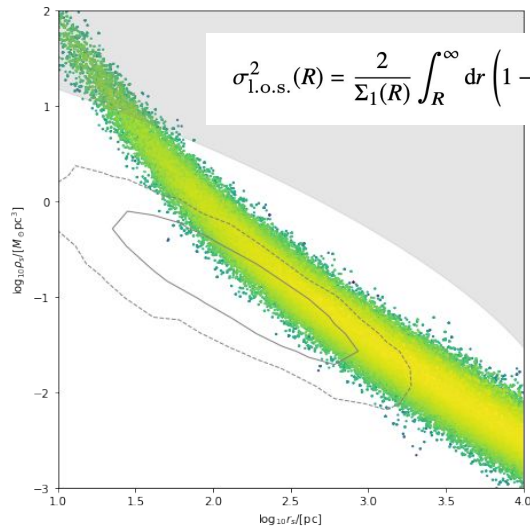
# Priors

- Photometry prior: for stellar distribution
  - half-light radius determined by photometric observation

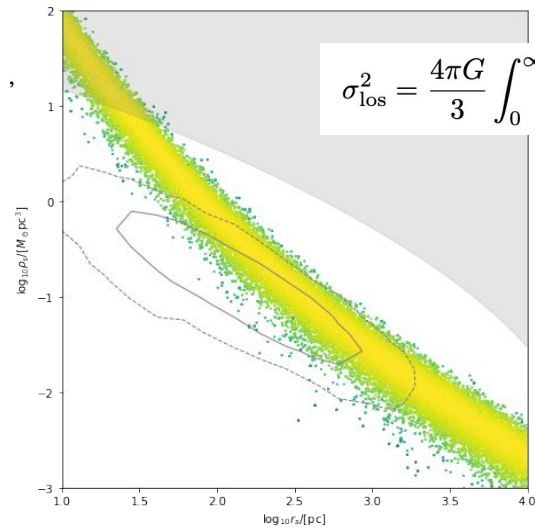
Name	$\log_{10} r_e$ /[pc]		
Aquarius 2	$2.094 \pm 0.078$	Leo 4	$2.013 \pm 0.053$
Boötes 1	$2.204 \pm 0.015$	Leo T	$2.125 \pm 0.051$
Boötes 2	$1.523 \pm 0.068$	Leo 5	$1.571 \pm 0.181$
CanesVenatici 1	$2.529 \pm 0.017$	Pegasus 3	$1.616 \pm 0.158$
CanesVenatici 2	$1.732 \pm 0.086$	Pisces 2	$1.678 \pm 0.072$
Carina	$2.392 \pm 0.005$	Reticulum 2	$1.495 \pm 0.018$
Carina 2	$1.870 \pm 0.045$	Sagittarius	$3.191 \pm 0.020$
ComaBerenices	$1.757 \pm 0.029$	Sculptor	$2.359 \pm 0.004$
Draco	$2.256 \pm 0.005$	Segue 1	$1.295 \pm 0.062$
Draco 2	$1.121 \pm 0.182$	Segue 2	$1.528 \pm 0.038$
Eridanus 2	$2.196 \pm 0.046$	Sextans 1	$2.538 \pm 0.004$
Fornax	$2.849 \pm 0.003$	Triangulum 2	$1.096 \pm 0.134$
Grus 1	$1.267 \pm 0.459$	Tucana 2	$2.212 \pm 0.073$
Hercules	$2.080 \pm 0.042$	Tucana 3	$1.640 \pm 0.058$
Horologium 1	$1.488 \pm 0.097$	UrsaMajor 1	$2.176 \pm 0.024$
Hydrus 1	$1.727 \pm 0.030$	UrsaMajor 2	$1.930 \pm 0.022$
Leo 1	$2.353 \pm 0.004$	UrsaMinor	$2.434 \pm 0.006$
Leo 2	$2.217 \pm 0.005$	Willman 1	$1.304 \pm 0.045$

# Results

- vs. radial independent analysis [2002.11956]
  - radial dependence of the likelihood break the degeneracy of the parameter



$$\sigma_{1.o.s.}^2(R) = \frac{2}{\Sigma_1(R)} \int_R^{\infty} dr \left( 1 - \beta_{\text{ani}} \frac{R^2}{r^2} \right) \frac{v_1(r) \sigma_r^2(r)}{\sqrt{1 - R^2/r^2}},$$

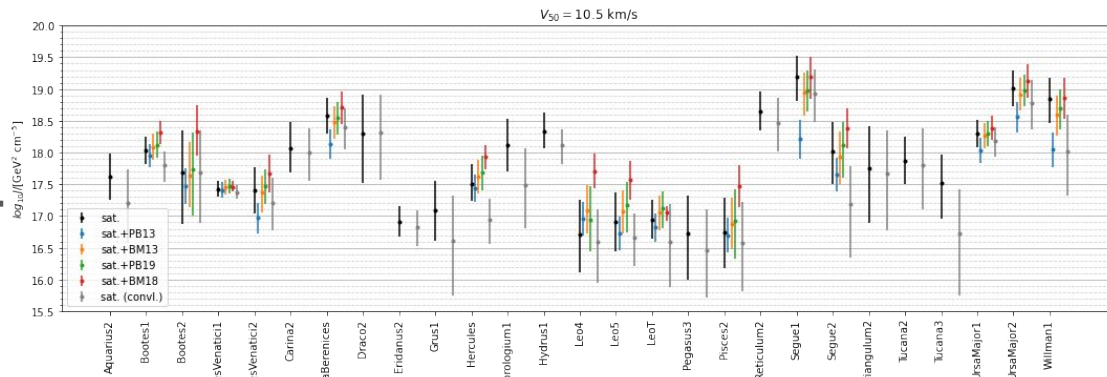
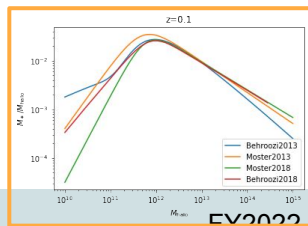
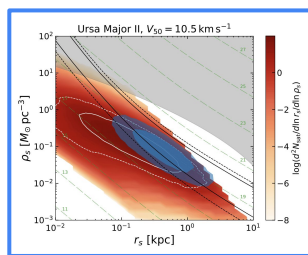


$$\sigma_{\text{los}}^2 = \frac{4\pi G}{3} \int_0^{\infty} dr r v_{\star}(r) M(r),$$



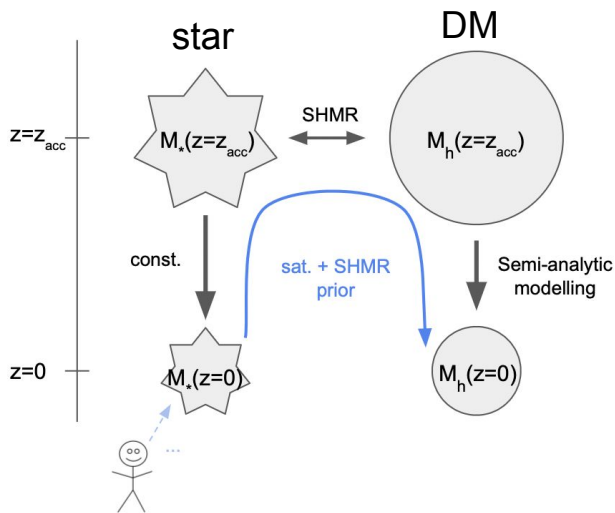
# Cosmological prior for the J-factor estimation of dwarf spheroidal galaxies

- Dwarf spheroidal galaxies (dSph) play important roles for dark matter detection but their dark matter halo profiles have large uncertainties
- For the halo profile estimation of dSphs, we apply two **cosmological priors**:
  - **Satellite prior**: constraint distribution of halo parameter based on a structure formation model
  - **Stellar-to-halo mass relation prior**: empirical relation between stellar mass and halo mass
- The cosmological priors are useful to decrease the uncertainty in the estimation and give a better understanding of dSphs



# SHMR

- The stellar-to-halo mass relation (SHMR)
  - empirical relation between the stellar and DM halo mass of galaxies:  $M_{\text{star}} = f(M_{\text{halo}}, z)$
  - assumption:  $f(M_{\text{halo}}, z)$  is a monotonic function for  $M_{\text{halo}}$



# MCMC Analysis

- Jeans analysis
  - 6 Parameters
  - Prior choices
    - photometry only
    - photometry + satellite
    - photometry + satellite + SHMR
  - Bayesian analysis to calculate posterior probability
    - MCMC tool: emcee 3.0.2

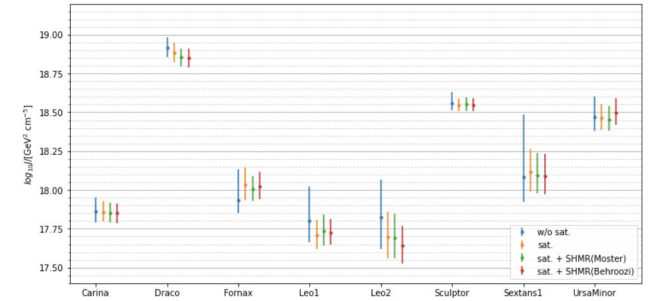
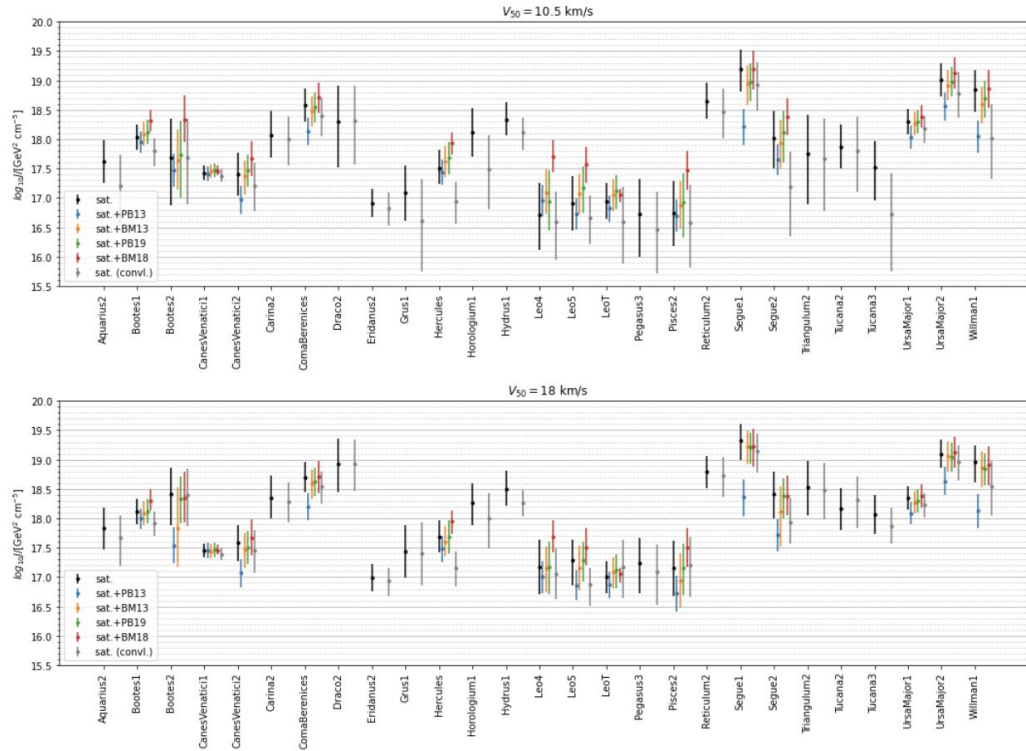
parameter	min.	max.
$\log_{10} R_e / [\text{pc}]$	1.0	3.5
$\log_{10} r_s / [\text{pc}]$	0.0	5.0
$\log_{10} \rho_s / [M_\odot \text{pc}^{-3}]$	-4.0	4.0
$\log_{10} r_t / [\text{pc}]$	0.0	5.0
$-\log_{10}(1 - \beta_{\text{ani}})$	-1.0	1.0
$v_{\text{dSph}} / [\text{km s}^{-1}]$	-1000	1000

# J-factors (table)

	w/o SHMR			PB13		BM13		PB19		BM18	
	flat	sat <sub>10.5</sub>	sat <sub>18</sub>	sat <sub>10.5</sub>	sat <sub>18</sub>	sat <sub>10.5</sub>	sat <sub>18</sub>	sat <sub>10.5</sub>	sat <sub>18</sub>	sat <sub>10.5</sub>	sat <sub>18</sub>
Aquarius2	18.2 <sup>+0.6</sup> <sub>-0.6</sub>	17.6 <sup>+0.4</sup> <sub>-0.4</sub>	17.8 <sup>+0.3</sup> <sub>-0.4</sub>	-	-	-	-	-	-	-	-
Bootes1	18.2 <sup>+0.3</sup> <sub>-0.3</sub>	18.0 <sup>+0.2</sup> <sub>-0.2</sub>	18.1 <sup>+0.2</sup> <sub>-0.2</sub>	17.9 <sup>+0.2</sup> <sub>-0.2</sub>	18.0 <sup>+0.2</sup> <sub>-0.2</sub>	18.1 <sup>+0.2</sup> <sub>-0.2</sub>	18.1 <sup>+0.2</sup> <sub>-0.2</sub>	18.1 <sup>+0.2</sup> <sub>-0.2</sub>	18.1 <sup>+0.2</sup> <sub>-0.2</sub>	18.3 <sup>+0.2</sup> <sub>-0.2</sub>	18.3 <sup>+0.2</sup> <sub>-0.2</sub>
Bootes2	16.6 <sup>+2.8</sup> <sub>-4.9</sub>	17.7 <sup>+0.7</sup> <sub>-0.8</sub>	18.4 <sup>+0.5</sup> <sub>-0.5</sub>	17.5 <sup>+0.3</sup> <sub>-0.3</sub>	17.5 <sup>+0.3</sup> <sub>-0.3</sub>	17.6 <sup>+0.3</sup> <sub>-0.3</sub>	17.8 <sup>+0.7</sup> <sub>-0.7</sub>	17.7 <sup>+0.2</sup> <sub>-0.2</sub>	18.3 <sup>+0.4</sup> <sub>-0.4</sub>	18.3 <sup>+0.4</sup> <sub>-0.4</sub>	18.4 <sup>+0.4</sup> <sub>-0.4</sub>
CanesVenatic1	17.6 <sup>+0.3</sup> <sub>-0.2</sub>	17.4 <sup>+0.1</sup> <sub>-0.1</sub>	17.5 <sup>+0.1</sup> <sub>-0.1</sub>	17.4 <sup>+0.1</sup> <sub>-0.1</sub>	17.4 <sup>+0.1</sup> <sub>-0.1</sub>	17.5 <sup>+0.1</sup> <sub>-0.1</sub>	17.4 <sup>+0.1</sup> <sub>-0.1</sub>	17.5 <sup>+0.1</sup> <sub>-0.1</sub>	17.5 <sup>+0.1</sup> <sub>-0.1</sub>	17.5 <sup>+0.1</sup> <sub>-0.1</sub>	17.5 <sup>+0.1</sup> <sub>-0.1</sub>
CanesVenatic2	17.9 <sup>+0.5</sup> <sub>-0.5</sub>	17.4 <sup>+0.4</sup> <sub>-0.4</sub>	17.6 <sup>+0.3</sup> <sub>-0.3</sub>	17.0 <sup>+0.2</sup> <sub>-0.2</sub>	17.1 <sup>+0.2</sup> <sub>-0.2</sub>	17.4 <sup>+0.3</sup> <sub>-0.3</sub>	17.5 <sup>+0.3</sup> <sub>-0.3</sub>	17.5 <sup>+0.3</sup> <sub>-0.3</sub>	17.5 <sup>+0.3</sup> <sub>-0.3</sub>	17.7 <sup>+0.3</sup> <sub>-0.3</sub>	17.7 <sup>+0.3</sup> <sub>-0.3</sub>
Carina2	18.4 <sup>+0.6</sup> <sub>-0.5</sub>	18.1 <sup>+0.4</sup> <sub>-0.4</sub>	18.4 <sup>+0.4</sup> <sub>-0.4</sub>	-	-	-	-	-	-	-	-
ComaBerenices	19.0 <sup>+0.4</sup> <sub>-0.4</sub>	18.6 <sup>+0.3</sup> <sub>-0.3</sub>	18.7 <sup>+0.3</sup> <sub>-0.3</sub>	18.1 <sup>+0.2</sup> <sub>-0.2</sub>	18.2 <sup>+0.2</sup> <sub>-0.2</sub>	18.5 <sup>+0.2</sup> <sub>-0.3</sub>	18.6 <sup>+0.2</sup> <sub>-0.3</sub>	18.5 <sup>+0.3</sup> <sub>-0.3</sub>	18.6 <sup>+0.2</sup> <sub>-0.2</sub>	18.7 <sup>+0.3</sup> <sub>-0.3</sub>	18.7 <sup>+0.3</sup> <sub>-0.3</sub>
Draco2	16.8 <sup>+2.5</sup> <sub>-4.8</sub>	18.3 <sup>+0.6</sup> <sub>-0.8</sub>	18.9 <sup>+0.4</sup> <sub>-0.5</sub>	-	-	-	-	-	-	-	-
Eridanus2	17.3 <sup>+0.4</sup> <sub>-0.4</sub>	16.9 <sup>+0.2</sup> <sub>-0.2</sub>	17.0 <sup>+0.2</sup> <sub>-0.2</sub>	-	-	-	-	-	-	-	-
Grus1	17.4 <sup>+0.9</sup> <sub>-0.9</sub>	17.1 <sup>+0.5</sup> <sub>-0.5</sub>	17.4 <sup>+0.4</sup> <sub>-0.4</sub>	-	-	-	-	-	-	-	-
Hercules	17.9 <sup>+0.4</sup> <sub>-0.4</sub>	17.5 <sup>+0.3</sup> <sub>-0.3</sub>	17.7 <sup>+0.3</sup> <sub>-0.3</sub>	17.4 <sup>+0.2</sup> <sub>-0.2</sub>	17.5 <sup>+0.2</sup> <sub>-0.2</sub>	17.6 <sup>+0.3</sup> <sub>-0.3</sub>	17.6 <sup>+0.3</sup> <sub>-0.3</sub>	17.7 <sup>+0.3</sup> <sub>-0.3</sub>	17.7 <sup>+0.3</sup> <sub>-0.3</sub>	17.9 <sup>+0.2</sup> <sub>-0.2</sub>	18.0 <sup>+0.2</sup> <sub>-0.2</sub>
Horologium1	19.1 <sup>+0.7</sup> <sub>-0.6</sub>	18.1 <sup>+0.4</sup> <sub>-0.4</sub>	18.3 <sup>+0.3</sup> <sub>-0.3</sub>	-	-	-	-	-	-	-	-
Hydrus1	18.5 <sup>+0.6</sup> <sub>-0.3</sub>	18.3 <sup>+0.3</sup> <sub>-0.3</sub>	18.5 <sup>+0.3</sup> <sub>-0.3</sub>	-	-	-	-	-	-	-	-
Leo4	15.6 <sup>+1.9</sup> <sub>-0.3</sub>	16.7 <sup>+0.5</sup> <sub>-0.5</sub>	17.2 <sup>+0.5</sup> <sub>-0.5</sub>	17.0 <sup>+0.3</sup> <sub>-0.3</sub>	17.0 <sup>+0.3</sup> <sub>-0.3</sub>	17.1 <sup>+0.4</sup> <sub>-0.4</sub>	17.1 <sup>+0.4</sup> <sub>-0.4</sub>	16.9 <sup>+0.5</sup> <sub>-0.5</sub>	17.2 <sup>+0.4</sup> <sub>-0.4</sub>	17.7 <sup>+0.3</sup> <sub>-0.3</sub>	17.7 <sup>+0.3</sup> <sub>-0.3</sub>
Leo5	17.2 <sup>+0.8</sup> <sub>-0.8</sub>	16.9 <sup>+0.6</sup> <sub>-0.6</sub>	17.3 <sup>+0.4</sup> <sub>-0.4</sub>	16.7 <sup>+0.3</sup> <sub>-0.3</sub>	16.9 <sup>+0.3</sup> <sub>-0.3</sub>	17.1 <sup>+0.4</sup> <sub>-0.4</sub>	17.2 <sup>+0.4</sup> <sub>-0.4</sub>	17.2 <sup>+0.4</sup> <sub>-0.4</sub>	17.3 <sup>+0.3</sup> <sub>-0.3</sub>	17.6 <sup>+0.3</sup> <sub>-0.3</sub>	17.5 <sup>+0.3</sup> <sub>-0.3</sub>
LeoT	17.6 <sup>+0.4</sup> <sub>-0.4</sub>	16.9 <sup>+0.3</sup> <sub>-0.3</sub>	17.0 <sup>+0.3</sup> <sub>-0.3</sub>	16.8 <sup>+0.2</sup> <sub>-0.2</sub>	16.9 <sup>+0.2</sup> <sub>-0.2</sub>	17.1 <sup>+0.3</sup> <sub>-0.3</sub>	17.1 <sup>+0.2</sup> <sub>-0.2</sub>	17.1 <sup>+0.3</sup> <sub>-0.3</sub>	17.1 <sup>+0.3</sup> <sub>-0.3</sub>	17.1 <sup>+0.1</sup> <sub>-0.1</sub>	17.1 <sup>+0.1</sup> <sub>-0.1</sub>
Pegasus3	17.8 <sup>+1.0</sup> <sub>-2.1</sub>	16.7 <sup>+0.2</sup> <sub>-0.7</sub>	17.2 <sup>+0.4</sup> <sub>-0.5</sub>	-	-	-	-	-	-	-	-
Pisces2	17.2 <sup>+0.9</sup> <sub>-0.9</sub>	16.7 <sup>+0.6</sup> <sub>-0.6</sub>	17.2 <sup>+0.5</sup> <sub>-0.5</sub>	16.7 <sup>+0.3</sup> <sub>-0.3</sub>	16.7 <sup>+0.3</sup> <sub>-0.3</sub>	16.9 <sup>+0.4</sup> <sub>-0.4</sub>	16.9 <sup>+0.5</sup> <sub>-0.5</sub>	16.9 <sup>+0.5</sup> <sub>-0.6</sub>	17.2 <sup>+0.4</sup> <sub>-0.5</sub>	17.5 <sup>+0.3</sup> <sub>-0.3</sub>	17.5 <sup>+0.3</sup> <sub>-0.3</sub>
Reticulum2	19.0 <sup>+0.4</sup> <sub>-0.4</sub>	18.7 <sup>+0.3</sup> <sub>-0.3</sub>	18.8 <sup>+0.3</sup> <sub>-0.3</sub>	-	-	-	-	-	-	-	-
Segue1	19.7 <sup>+0.4</sup> <sub>-0.4</sub>	19.2 <sup>+0.3</sup> <sub>-0.3</sub>	19.3 <sup>+0.3</sup> <sub>-0.3</sub>	18.2 <sup>+0.3</sup> <sub>-0.3</sub>	18.4 <sup>+0.3</sup> <sub>-0.3</sub>	18.9 <sup>+0.3</sup> <sub>-0.3</sub>	19.2 <sup>+0.3</sup> <sub>-0.3</sub>	19.0 <sup>+0.3</sup> <sub>-0.3</sub>	19.2 <sup>+0.3</sup> <sub>-0.3</sub>	19.2 <sup>+0.3</sup> <sub>-0.3</sub>	19.2 <sup>+0.3</sup> <sub>-0.3</sub>
Segue2	18.0 <sup>+0.7</sup> <sub>-0.7</sub>	18.0 <sup>+0.5</sup> <sub>-0.5</sub>	18.4 <sup>+0.4</sup> <sub>-0.4</sub>	17.7 <sup>+0.3</sup> <sub>-0.3</sub>	17.7 <sup>+0.3</sup> <sub>-0.3</sub>	17.9 <sup>+0.4</sup> <sub>-0.4</sub>	18.1 <sup>+0.4</sup> <sub>-0.6</sub>	18.1 <sup>+0.5</sup> <sub>-0.5</sub>	18.4 <sup>+0.3</sup> <sub>-0.4</sub>	18.4 <sup>+0.3</sup> <sub>-0.3</sub>	18.4 <sup>+0.3</sup> <sub>-0.3</sub>
Triangulum2	14.4 <sup>+2.9</sup> <sub>-3.9</sub>	17.7 <sup>+0.7</sup> <sub>-0.9</sub>	18.5 <sup>+0.4</sup> <sub>-0.4</sub>	-	-	-	-	-	-	-	-
Tucana2	18.1 <sup>+0.6</sup> <sub>-0.5</sub>	17.9 <sup>+0.4</sup> <sub>-0.4</sub>	18.2 <sup>+0.4</sup> <sub>-0.4</sub>	-	-	-	-	-	-	-	-
Tucana3	15.7 <sup>+1.8</sup> <sub>-0.3</sub>	17.5 <sup>+0.5</sup> <sub>-0.5</sub>	18.1 <sup>+0.3</sup> <sub>-0.3</sub>	-	-	-	-	-	-	-	-
UrsaMajor1	18.7 <sup>+0.3</sup> <sub>-0.3</sub>	18.3 <sup>+0.2</sup> <sub>-0.2</sub>	18.3 <sup>+0.2</sup> <sub>-0.2</sub>	18.0 <sup>+0.2</sup> <sub>-0.2</sub>	18.1 <sup>+0.2</sup> <sub>-0.2</sub>	18.3 <sup>+0.2</sup> <sub>-0.2</sub>	18.3 <sup>+0.2</sup> <sub>-0.2</sub>	18.3 <sup>+0.2</sup> <sub>-0.2</sub>	18.3 <sup>+0.2</sup> <sub>-0.2</sub>	18.4 <sup>+0.2</sup> <sub>-0.2</sub>	18.4 <sup>+0.2</sup> <sub>-0.2</sub>
UrsaMajor2	19.5 <sup>+0.4</sup> <sub>-0.4</sub>	19.0 <sup>+0.3</sup> <sub>-0.3</sub>	19.1 <sup>+0.3</sup> <sub>-0.3</sub>	18.6 <sup>+0.2</sup> <sub>-0.2</sub>	18.6 <sup>+0.2</sup> <sub>-0.2</sub>	18.9 <sup>+0.3</sup> <sub>-0.3</sub>	19.1 <sup>+0.3</sup> <sub>-0.3</sub>	19.0 <sup>+0.2</sup> <sub>-0.2</sub>	19.0 <sup>+0.3</sup> <sub>-0.3</sub>	19.1 <sup>+0.3</sup> <sub>-0.3</sub>	19.1 <sup>+0.3</sup> <sub>-0.3</sub>
Willman1	19.5 <sup>+0.4</sup> <sub>-0.4</sub>	18.8 <sup>+0.3</sup> <sub>-0.4</sub>	19.0 <sup>+0.3</sup> <sub>-0.3</sub>	18.0 <sup>+0.3</sup> <sub>-0.3</sub>	18.1 <sup>+0.3</sup> <sub>-0.3</sub>	18.6 <sup>+0.3</sup> <sub>-0.3</sub>	18.9 <sup>+0.3</sup> <sub>-0.3</sub>	18.7 <sup>+0.3</sup> <sub>-0.3</sub>	18.8 <sup>+0.3</sup> <sub>-0.3</sub>	18.9 <sup>+0.3</sup> <sub>-0.3</sub>	18.9 <sup>+0.3</sup> <sub>-0.3</sub>

	w/o SHMR		SHMR <sub>Moster</sub>	SHMR <sub>Behroozi</sub>
	flat	sat.	sat.	sat.
Carina	17.9 <sup>+0.1</sup> <sub>-0.1</sub>	17.9 <sup>+0.1</sup> <sub>-0.1</sub>	17.9 <sup>+0.1</sup> <sub>-0.1</sub>	17.9 <sup>+0.1</sup> <sub>-0.1</sub>
Draco	18.9 <sup>+0.1</sup> <sub>-0.1</sub>	18.9 <sup>+0.1</sup> <sub>-0.1</sub>	18.9 <sup>+0.1</sup> <sub>-0.1</sub>	18.8 <sup>+0.1</sup> <sub>-0.1</sub>
Fornax	17.9 <sup>+0.2</sup> <sub>-0.2</sub>	18.0 <sup>+0.1</sup> <sub>-0.1</sub>	18.0 <sup>+0.1</sup> <sub>-0.1</sub>	18.0 <sup>+0.1</sup> <sub>-0.1</sub>
Leo1	17.8 <sup>+0.2</sup> <sub>-0.1</sub>	17.7 <sup>+0.1</sup> <sub>-0.1</sub>	17.7 <sup>+0.1</sup> <sub>-0.1</sub>	17.7 <sup>+0.1</sup> <sub>-0.1</sub>
Leo2	17.8 <sup>+0.2</sup> <sub>-0.2</sub>	17.7 <sup>+0.2</sup> <sub>-0.2</sub>	17.7 <sup>+0.2</sup> <sub>-0.2</sub>	17.6 <sup>+0.1</sup> <sub>-0.1</sub>
Sculptor	18.6 <sup>+0.2</sup> <sub>-0.1</sub>	18.5 <sup>+0.0</sup> <sub>-0.0</sub>	18.6 <sup>+0.0</sup> <sub>-0.1</sub>	18.5 <sup>+0.0</sup> <sub>-0.0</sub>
Sextans1	18.1 <sup>+0.4</sup> <sub>-0.4</sub>	18.1 <sup>+0.1</sup> <sub>-0.1</sub>	18.1 <sup>+0.1</sup> <sub>-0.1</sub>	18.1 <sup>+0.1</sup> <sub>-0.1</sub>
UrsaMinor	18.5 <sup>+0.1</sup> <sub>-0.1</sub>	18.5 <sup>+0.1</sup> <sub>-0.1</sub>	18.5 <sup>+0.1</sup> <sub>-0.1</sub>	18.5 <sup>+0.1</sup> <sub>-0.1</sub>

# J-factors



# Difference of Jeans analyses

- [\[2002.11956\]](#): velocity dispersion averaged over total system

$$\sigma_{\text{los}}^2 = \frac{4\pi G}{3} \int_0^\infty dr r \nu_\star(r) M(r),$$

- This work: radial dependent velocity dispersion calculated by the spherical Jeans equation

$$\sigma_{\text{l.o.s.}}^2(R) = \frac{2}{\Sigma_1(R)} \int_R^\infty dr \left( 1 - \beta_{\text{ani}} \frac{R^2}{r^2} \right) \frac{v_1(r) \sigma_r^2(r)}{\sqrt{1 - R^2/r^2}},$$

# Models

- Plummer model

$$\nu(r) = \frac{3}{4\pi R_e^3} \left(1 + \left(\frac{r}{R_e}\right)^2\right)^{-5/2},$$
$$\Sigma(R) = \frac{1}{\pi} \left(1 + \frac{R^2}{R_e^2}\right)^{-2},$$

- Truncated NFW model

- Outermost halo is striped by tidal force

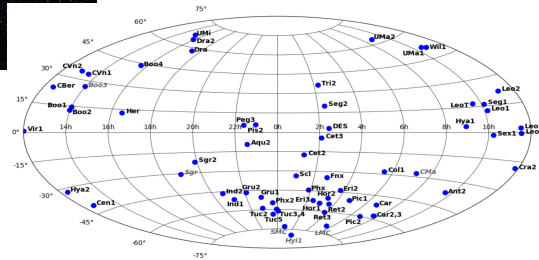
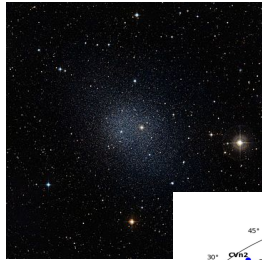
$$\rho(r) = \begin{cases} \rho_s \left(\frac{r}{r_s}\right)^{-1} \left(1 + \frac{r}{r_s}\right)^{-2} & (0 \leq r \leq r_t) \\ 0 & (r_t < r) \end{cases},$$

$$M(r) = \begin{cases} 4\pi\rho_s r_s^3 \left(\log\left(1 + \frac{r}{r_s}\right) - \frac{r}{r+r_s}\right) & (0 \leq r \leq r_t) \\ 4\pi\rho_s r_s^3 \left(\log\left(1 + \frac{r_t}{r_s}\right) - \frac{r_t}{r_t+r_s}\right) & (r_t < r), \end{cases}$$

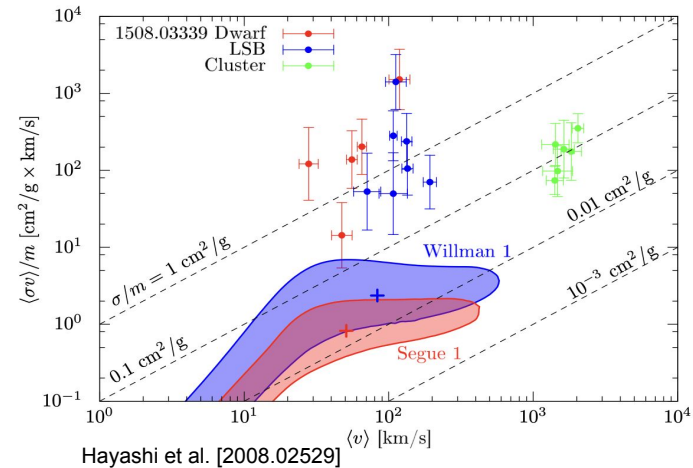
# dSphs and DM detection

- Dwarf spheroidal galaxies (dSphs)
  - inner DM halo profile gives constraints on DM self-interaction

e.g. Fornax dSph



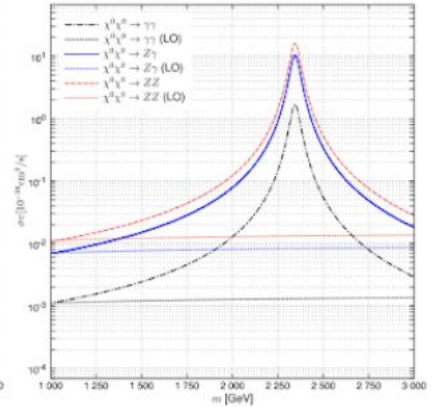
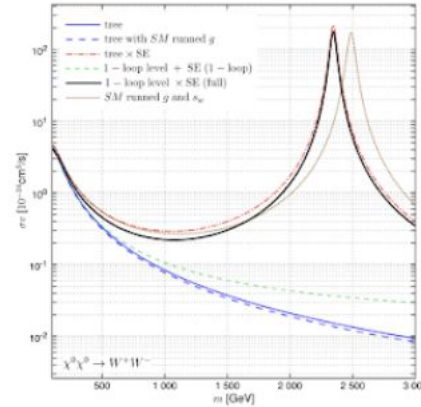
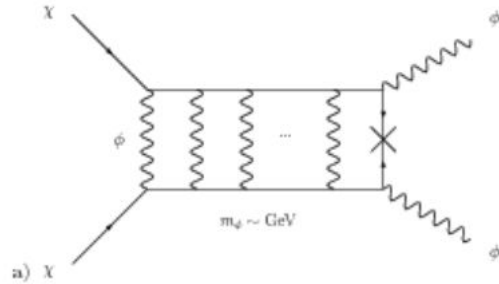
McConachie et al. [2007.05011]





# Sommerfeld effect

- Sommerfeld effect:
  - nonrelativistic effect of scattering



# Difference of SHMRs



For low-mass haloes ( $M_h < 10^{11} M_\odot$ ), the best-fitting model has a weaker upturn in the SMHM ratio than found by Behroozi et al. (2013e); this is because Behroozi et al. (2013e) assumed a strong surface-brightness incompleteness correction for faint galaxies that is no longer observationally supported (Williams et al. 2016). This will make it easier to reconcile observed galaxy counts with the HI mass function and observed HI gas fractions in faint galaxies (Popping et al. 2015).

The SMHM relation for star-forming vs. quiescent galaxies depends on the correlation between galaxy and halo assembly (§4.2) and the evolution of the SMHM relation (see also Moster et al. 2018). As shown in Fig. 38, there remain significant differences across studies (see also Wechsler & Tinker 2018). Our model is flexible in terms of both the SMHM relation evolution and the galaxy–halo assembly correlation, and suggests that the stellar mass–halo mass relation at fixed halo mass is similar for star-forming and quiescent central galaxies, matching the conclusion in Zu & Mandelbaum (2016). The results of Moster et al. (2018) and Rodríguez-Puebla et al. (2015) give opposite conclusions of higher and lower (respectively) median stellar masses for quiescent compared to star-forming galaxies, despite using the same underlying data (correlation functions in the SDSS) to constrain their models. In part, these divergent conclusions arise because correlation functions and weak lensing measurements are both very sensitive to satellite clustering; hence, small changes to the satellite halo occupation can lead to large changes in the inferred occupation for central galaxies. Applying a cut to first remove satellites before measuring clustering, environment, or lensing (as in both this study and Zu & Mandelbaum 2016) is hence necessary to robustly determine SMHM differences for star-forming and quiescent central galaxies. As noted in Zu & Mandelbaum (2016) and Moster et al. (2018), having an equivalent median stellar mass at fixed halo mass does *not* imply that the median halo mass at fixed stellar mass will be equal for star-forming and quiescent galaxies. Because the ratio of star-forming to quiescent galaxies drops rapidly with increasing halo mass, it is much more likely in this case that a given massive star-forming galaxy will be hosted by a lower-mass halo than a massive quiescent galaxy.

Behroozi+(2019) [1806.07893]

# Gravothermal fluid model

“What

$$\kappa = b_* \sigma_v \left[ \left( \frac{1}{\lambda} \right) + \left( \frac{b_* \sigma_v t_r}{C_* H_g} \right) \right]^{-1}$$

thermally conducting fluid” [Balberg+(2002)]

$$\frac{\partial M}{\partial t} = 4\pi r^2 \rho$$

where  $H_g \equiv \sqrt{\sigma_v^2 / (4\pi G \rho)}$  is the gravitational scale height of the system,  $\lambda = (\rho \sigma / m)^{-1}$  is the collisional scale for the mean free path,  $t_r \equiv \lambda / (a \sigma_v)$  is the relaxation time with a coefficient of order of unity being  $a$ , and we adopt  $a = \sqrt{16/\pi}$  for hard-sphere scattering of particles with a Maxwell-Boltzmann velocity distribution (Reif 1965).

In the limit of  $\lambda \ll H_g$ , the thermal conductivity is given by  $\kappa \approx (3/2)(k_B/m)b_*\rho\lambda^2/(at_r)$  and  $b_*$  can be regarded as an effective impact parameter among particle collisions. In the limit of  $\lambda \gg H_g$ , one finds  $\kappa \approx (3/2)(k_B/m)C_*\rho H_g^2/t_r$ , reproducing an empirical formula of gravothermal collapse of globular clusters (Lynden-Bell & Eggleton 1980).

$$\frac{L}{4\pi r^2} = -\kappa \frac{\partial T}{\partial r} = -\kappa \rho \frac{\partial}{\partial r} \left( \frac{3\sigma_v^2}{2} \right)$$

Heat conduction

$$4\pi r^2 \rho \left[ \left( \frac{\partial}{\partial t} \right)_M \frac{3\sigma_v^2}{2} + p \left( \frac{\partial}{\partial t} \right)_M \frac{1}{\rho} \right] = -4\pi r^2 \rho v^2 \left( \frac{\partial}{\partial t} \right)_M \log \left( \frac{\sigma_v^3}{\rho} \right)$$

1st law of thermodynamics

$\kappa$  ← calibrated by isolated N-body simulation