Toward Quantum Gravity Constraints on Dark Matters

Toshifumi Noumi, supported by 公募研究 D03-08

(Kobe U → U Tokyo, Komaba from April 2023)

Goal: curve out the huge parameter space of dark matter models using consistency conditions of quantum gravity!

March 8 2023 @ DM symposium

Motivation

Experimental upper bounds on SM-DM interactions have been improving a lot!

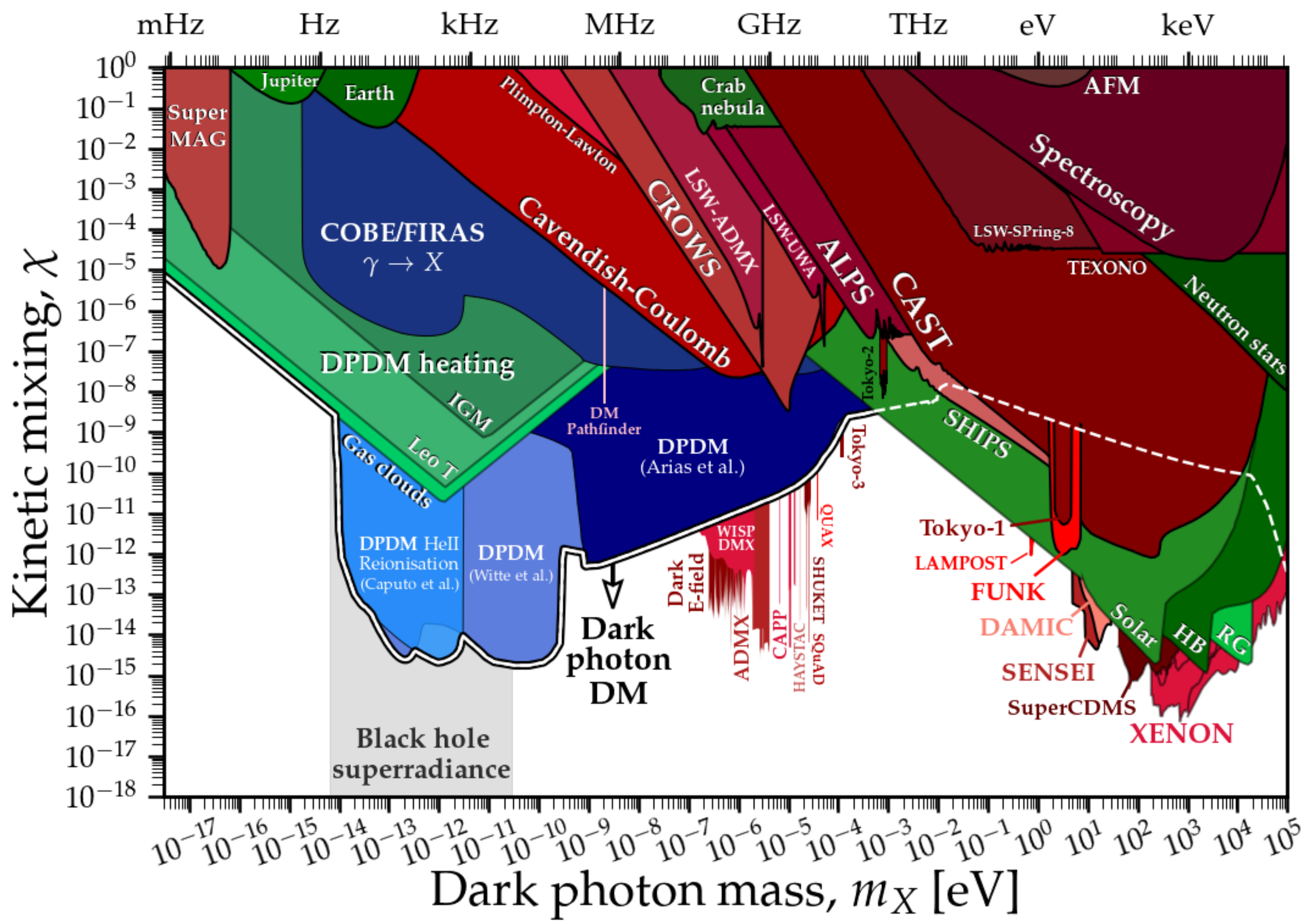
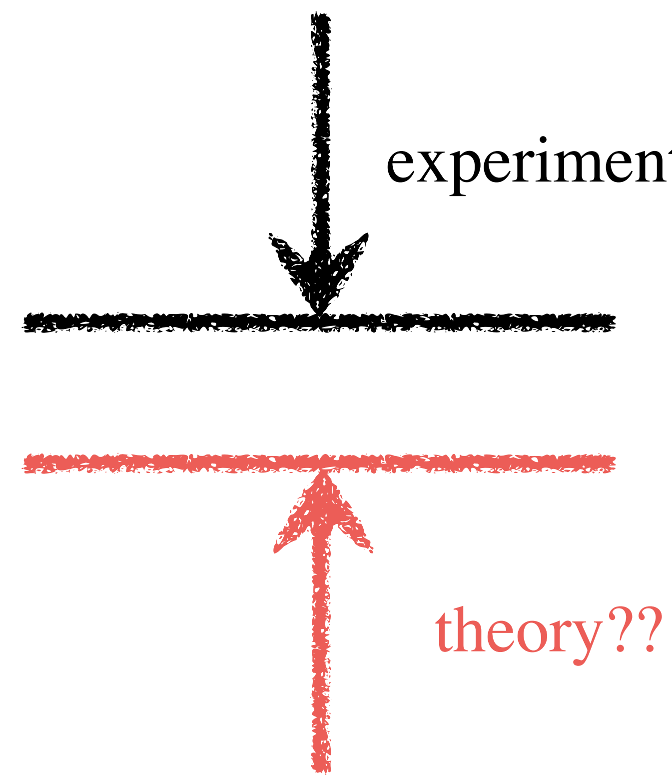


Fig: dark photon search as an example



Theoretical lower bounds, if exist, would be useful for comprehensive DM searches.

※ experiments + theories → close the window from both sides!

Satoshi Shirai’s talk:

Positivity bounds on gravitational scattering amplitudes may offer such a bound on SM-DM couplings!? Dark matters cannot be too dark!?

Contents

1. Gravitational positivity bounds

2. Implications for dark sector physics

Positivity bounds on gravitational scattering amplitudes provide

a criterion for a gravitational EFT to have a consistent UV completion.

The recipe of gravitational positivity bounds

1. Compute scattering amplitudes $\mathcal{M}(s,t)$ in your model taking into account gravity.

2. Perform IR expansion, e.g., as $\mathcal{M}(s,t) = (\text{graviton poles}) + \sum_{n=0}^{\infty} a_{2n} s^{2n} + \mathcal{O}(t)$.

3. Evaluate a cutoff-dependent quantity $B(\Lambda) := a_2 - \frac{2}{\pi}\int_{m_{\rm th}^2}^{\Lambda^2} ds \frac{\mathrm{Im}\mathcal{M}(s,t=0)}{s^3}$.

4. Then, $B(\Lambda) \gtrsim 0$ is required for the EFT to have a consistent UV completion.

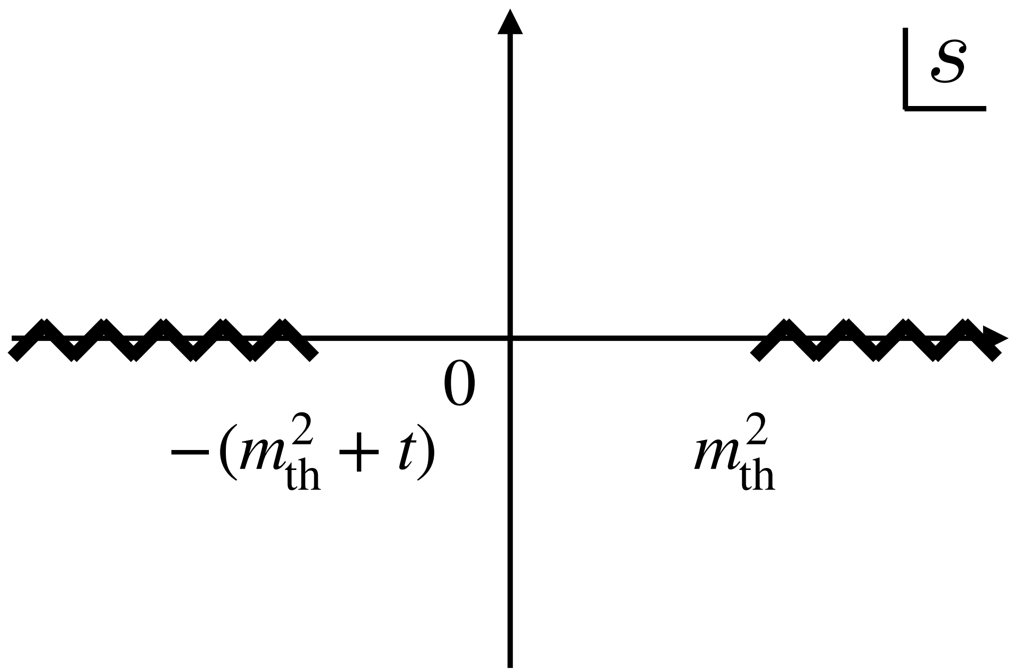
$\rightarrow$ Quantum gravity constraints on your gravitational model!

The key idea of positivity bounds [ex. Adams et al '06]:

Analyticity of scattering amplitudes connects UV and IR.

Analyticity is the key

Consider a scattering amplitude $\mathscr{M}(s,t)$ in the forward limit $t \to -0$.



analytic structure of $\mathscr{M}(s,t)$

Scattering amplitudes are analytic away from the real axis! (cf. causality)

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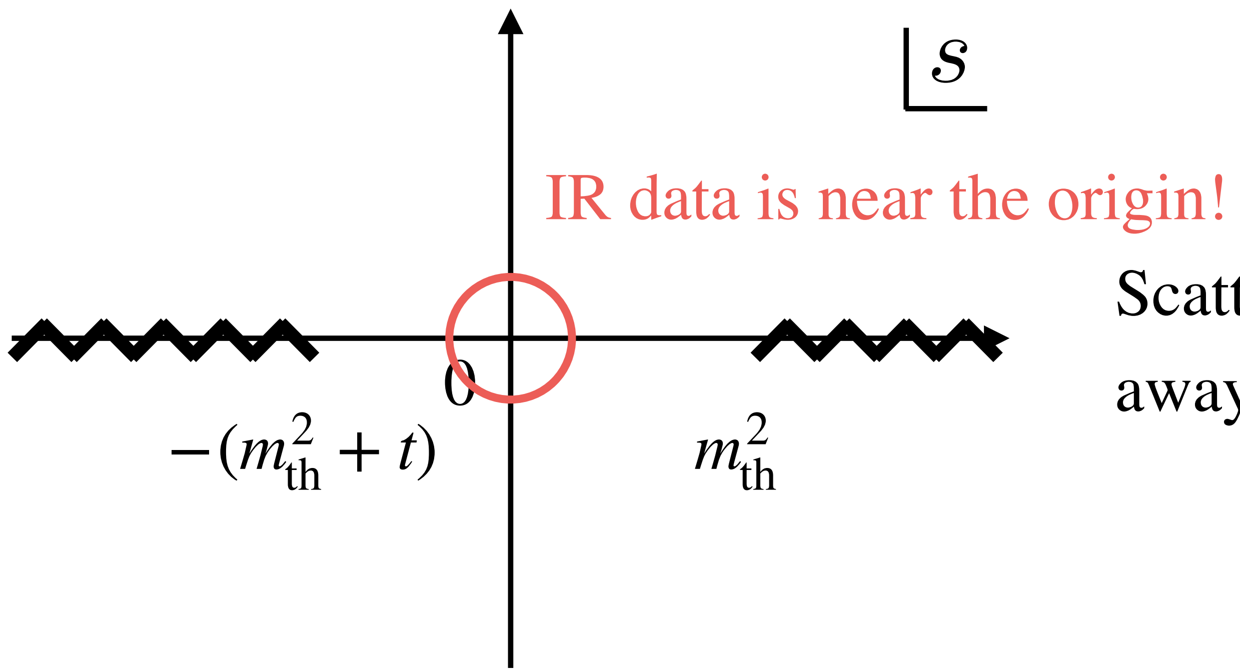

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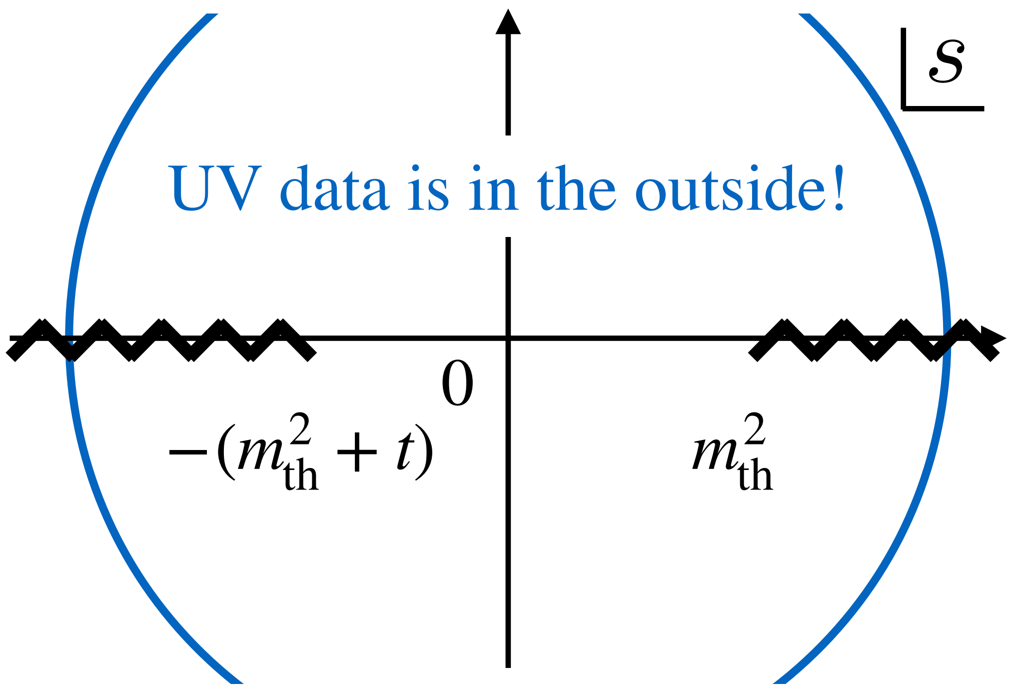

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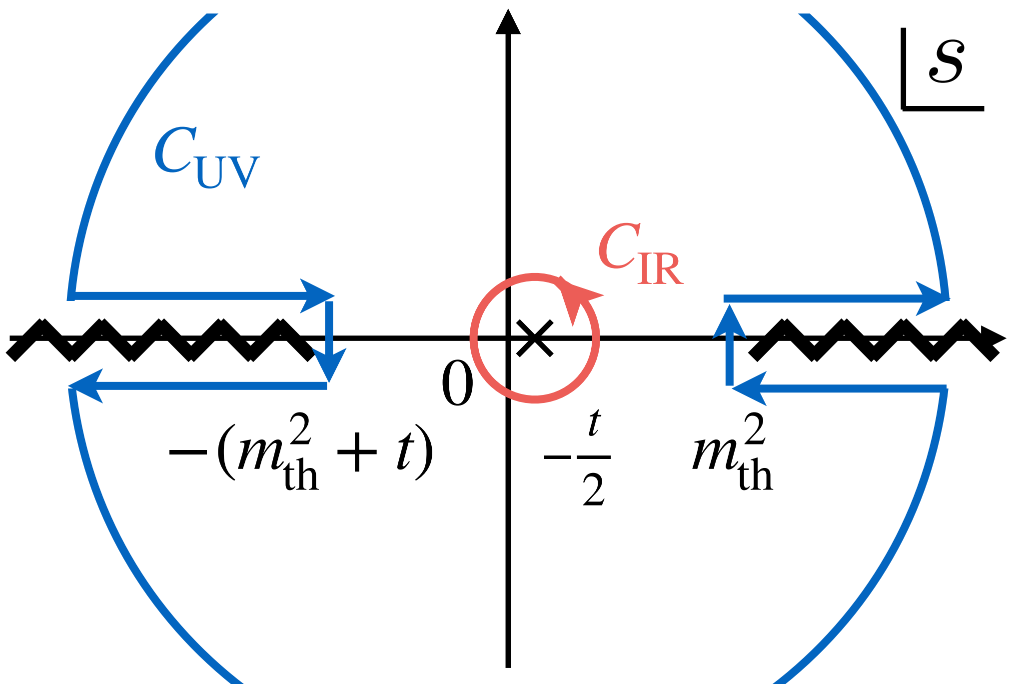


analytic structure of $\mathcal{M}(s,t)$

Scattering amplitudes are analytic away from the real axis! (cf. causality)

By deform the integration contour, we can connect UV and IR:

$$\text{IR data} \to \oint_{C_{\rm IR}} \frac{ds}{2\pi i} \frac{\mathcal{M}(s,t)}{(s+\frac{t}{2})^3} = \oint_{C_{\rm UV}} \frac{ds}{2\pi i} \frac{\mathcal{M}(s,t)}{(s+\frac{t}{2})^3} \leftarrow \text{UV data}$$

Careful analysis gives various **UV-IR relations** (dispersion relations).

In non-gravitational theories, this gives the dispersion relation:

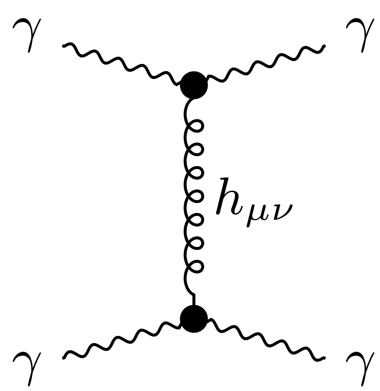
$$a_2 = \int_{m_{\rm th}^2}^{\infty} \frac{{\rm Im}\mathscr{M}(s, t = 0)}{s^3}, \quad \mathscr{M}(s, t = 0) = \sum_{n=0}^{\infty} a_{2n} s^{2n}.$$

This implies $B(\Lambda) := a_2 - \int_{m_{\rm th}^2}^{\Lambda^2} \frac{{\rm Im}\mathscr{M}(s, t = 0)}{s^3} = \int_{\Lambda^2}^{\infty} \frac{{\rm Im}\mathscr{M}(s, t = 0)}{s^3} \geq 0,$

which is called the positivity bounds.

In the presence of gravity, the IR expansion is modified as

$$\mathscr{M}(s,t) = -\frac{s^2}{M_{\rm Pl}^2 t} + \sum_{n=0}^{\infty} a_{2n} s^{2n} + \mathcal{O}(t).$$



The t-channel graviton exchange dominates in the forward limit, so that a careful analysis is required to derive positivity bounds.

[Tokuda-Aoki-Hirano '20] performed such a careful study in gravitational EFTs.

See also Hamada-TN-Shiu '18, Herrero-Valea et al '20, Bellazzini et al'19, Alberte et al '20, Arkani-Hamed et al '20, Caron-Huot et al '21, TN-Tokuda '22.

Gravitational positivity bounds [Tokuda-Aoki-Hirano '20]

Finding 1

Consistent dispersion relations require Reggization of gravitational amplitudes:

$$\mathrm{Im}\mathscr{M}(s,t) \simeq f(t)\left(\frac{s}{M_s^2}\right)^{2+\alpha' t+\alpha'' t^2+\cdots} \quad (s > M_{\mathrm{Regge}} : \text{Reggeization scale}).$$

cf. In string theory, an infinite higher spin tower is responsible for this.

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Finding 2

If we perform IR expansion $\mathscr{M}(s,t) = -\dfrac{s^2}{M_{\mathrm{Pl}}^2 t} + \sum_{n=0}^{\infty} a_{2n} s^{2n} + \mathcal{O}(t)$,

and define a cutoff-dependent quantity $B(\Lambda) := a_2 - \dfrac{2}{\pi}\displaystyle\int_{m_{\mathrm{th}}^2}^{\Lambda^2} ds \frac{\mathrm{Im}\mathscr{M}(s,t=0)}{s^3}$,

dispersion relations imply $B(\Lambda) \geq -\dfrac{1}{M_{\mathrm{Pl}}^2}\left(\dfrac{f'(0)}{f(0)} - \dfrac{\alpha''}{\alpha'}\right) := \pm \dfrac{1}{M_{\mathrm{Pl}}^2 M^2}$.

※ M carries information of Regge amplitudes (ex. $M \sim M_s$ for tree-level string).

※ Positivity bounds w/o gravity $B(\Lambda) \geq 0$ is reproduced in the limit $M_{\mathrm{Pl}} \to \infty$.

In the following I discuss phenomenological implications of $B(\Lambda) \geq \pm \frac{1}{M_{\rm Pl}^2 M^2}$.

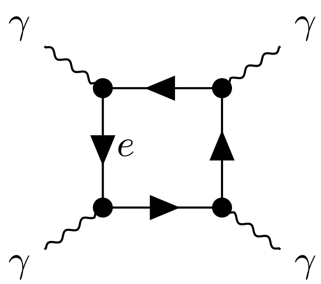
※ $B(\Lambda)$ is calculable in the standard Feynman rule for a given gravitational EFT.

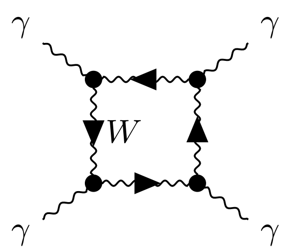
Contents

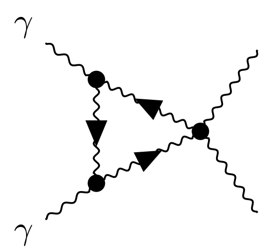
Gravitational Electroweak Theory and SM [Aoki-Loc-TN-Tokuda ’21]

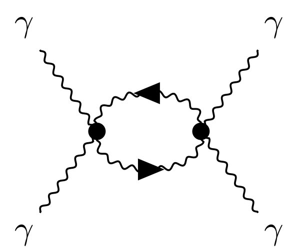
Light-by-light scattering in gravitational EW theory

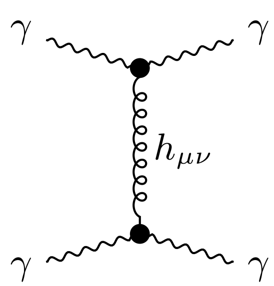
\# $\gamma\gamma \to \gamma\gamma$ scattering at one-loop: $\mathscr{M} = \mathscr{M}_{\text{QED}} + \mathscr{M}_{\text{weak}} + \mathscr{M}_{\text{grav}}$.

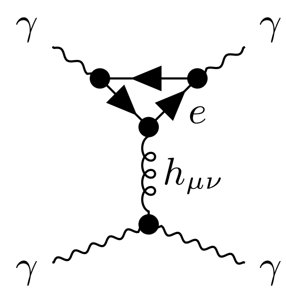
$\mathscr{M}_{\text{QED}} =$ 

, $\mathscr{M}_{\text{weak}} =$ 

$+$ 

$+$ 

$\mathscr{M}_{\text{grav}} \simeq$ 

$+$ 

\# $B(\Lambda)$ from each sector: $B(\Lambda) = B_{\text{QED}}(\Lambda) + B_{\text{weak}}(\Lambda) + B_{\text{grav}}(\Lambda)$

$$B_{\text{QED}}(\Lambda) = \frac{2e^4}{\pi^2\Lambda^4}\left(\ln\frac{\Lambda}{m} - \frac{1}{4}\right), \quad B_{\text{weak}}(\Lambda) = \frac{4e^4}{\pi^2 m_W^2 \Lambda^2}, \quad B_{\text{grav}}(\Lambda) \simeq -\frac{11e^2}{180\pi^2 m_e^2 M_{\text{Pl}}^2}$$

- Non-gravitational contributions $B_{\text{non-grav}} = B_{\text{QED}} + B_{\text{weak}}$ vanish for $\Lambda \to \infty$.

- Gravitational contribution is negative!

Gravitational Positivity

Gravitational positivity $B(\Lambda) > \pm \dfrac{1}{M_{\rm Pl}^2 M^2}$ implies

$$B_{\text{non-grav}}(\Lambda) + B_{\text{grav}}(\Lambda) = \frac{4e^4}{\pi^2 m_W^2 \Lambda^2} - \frac{11e^2}{180\pi^2 m_e^2 M_{\rm Pl}^2} > \pm \frac{1}{M_{\rm Pl}^2 M^2}.$$

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Consider the following two cases:

1) $M \gg m_e$

RHS is negligible, so that a nontrivial bound appears:

$$B_{\rm weak}(\Lambda) > -B_{\rm grav}(\Lambda) \quad \rightleftarrows \quad \frac{m_W}{M_{\rm Pl}} < \sqrt{\frac{720}{11}}\, e\, \frac{m_e}{\Lambda} \quad \rightleftarrows \quad \Lambda < \sqrt{\frac{1440}{11}} y_e \sin\theta_W M_{\rm Pl}$$

- Explains the hierarchy between the EW scale and the Planck scale??

- A WGC type bound on the Yukawa coupling and the Weinberg angle.

- A similar analysis for SM implies $\Lambda \sim 10^{16}$ GeV (grand unification??)

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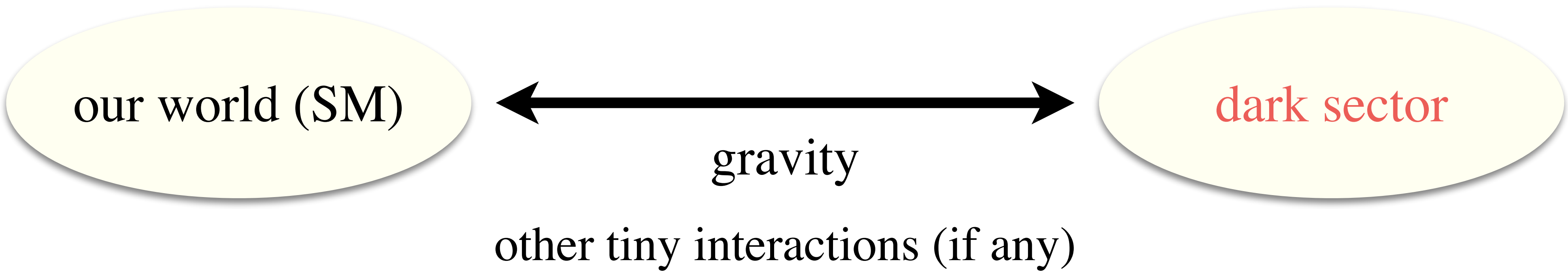
2) If it is violated, negative sign and $M \lesssim m_e$ are required on RHS

※ This means that Regge amplitudes highly depend on IR physics,

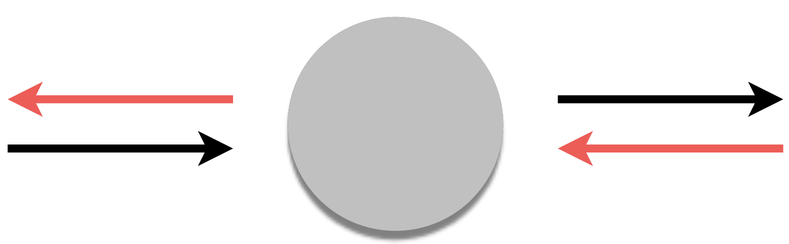
which seems nontrivial ($M \sim M_{\text{string}} \gg m_e$ in tree-level string).

Implications for dark sector physics [Sato-TN-Tokuda '22]

Dark sector cannot be too dark?



- Consider scattering of SM particles and dark sector particles:



$= \mathscr{M}_{\text{non-grav}} + \mathscr{M}_{\text{grav}}$

- Positivity implies $B_{\text{non-grav}}(\Lambda) > -B_{\text{grav}}(\Lambda) \pm \dfrac{1}{M_{\text{Pl}}^2 M^2}$

※ To our knowledge, $B_{\text{grav}}(\Lambda) < 0$ is quite generic.

- Under the assumption "$M \gg m_e$," we have $B_{\text{non-grav}}(\Lambda) > -B_{\text{grav}}(\Lambda)$.

$\rightarrow B_{\text{non-grav}}(\Lambda)$ cannot be too small: dark sector cannot be too dark?

Dark photon models

In [TN-Sato-Tokuda '22],

we performed a concrete analysis in dark photon models as an illustrative example.

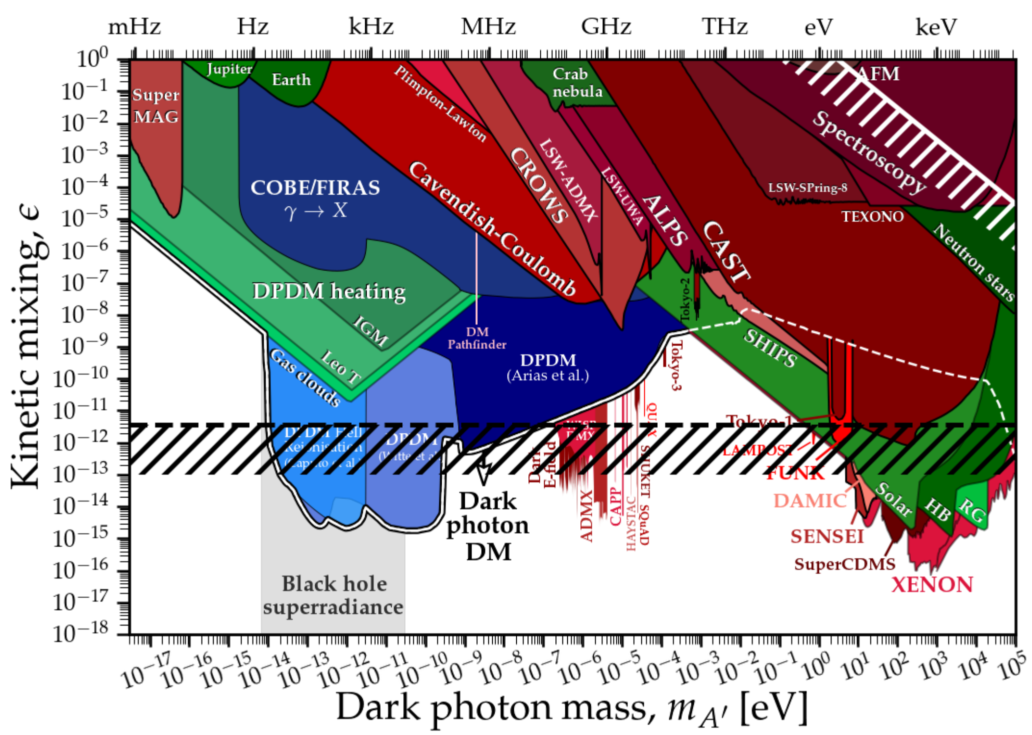
The value of $B(\Lambda)$ and therefore implications of gravitational positivity bounds depend on details of dark photon scenarios.

In our previous paper, we focused on the Stuckelberg case and considered

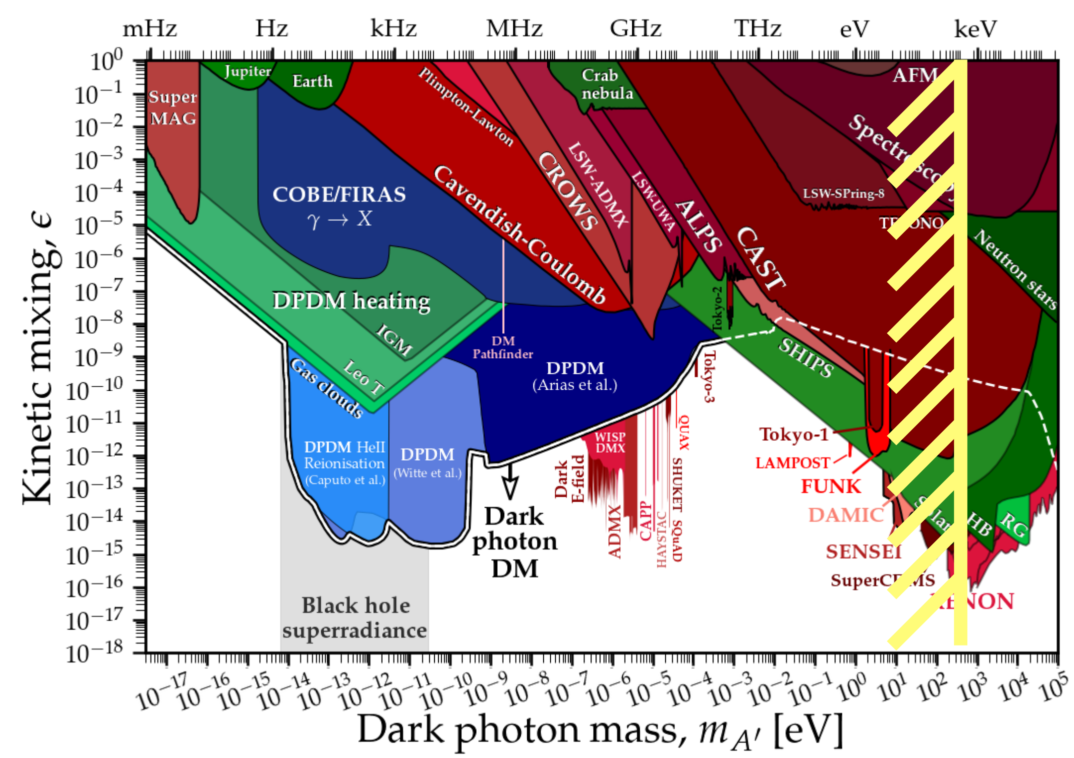
A) SM-DM interactions are only through dark photon-photon kinetic mixing

B) There exists a spin 1 particle charged under both U(1)'s.

Achievements in FY2022



dark photon model A



dark photon model B

Under the assumptions "$M \gg m_e$" made in the SM analysis,
we showed that unitarity of gravitational scattering can be useful
to curved out the DM theory space from a complementary direction.
Interesting interplay between theory, pheno, and experiments!

Prospects for FY2023-2024

After presenting our work at the DM symposium of the last year etc,
we started collaboration with the C01 group!

With my collaborators (Aoki, Sato, Tokuda) and the C01 members (Saito, Shirai, Yamazaki),
we are working on (See also the next talk by Katsuki Aoki.)

1) comprehensive unitarity analysis of DM models coupled to gravity

2) theoretical studies on gravitational S-matrix bootstrap

We used the grant to cover travel expenses for discussion, workshop etc,
which was very useful especially for my student Sota Sato.

My proposal for the FY2023-2024 公募研究 along this line has been approved!

I hope to make progress with the C01 group and interact more also with other groups!


Thank you very much!
山口県
Sota Sato 佐藤爽太
(D1 student @ Kobe U)
photo taken after the workshop @ Shimonoseki