1. Introduction to Primordial Black Holes (as dark matter)

2. Stellar microlensing constraints on PBH dark matter

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1. Introduction to Primordial Black Holes (as dark matter)

Motivation & history

Formation

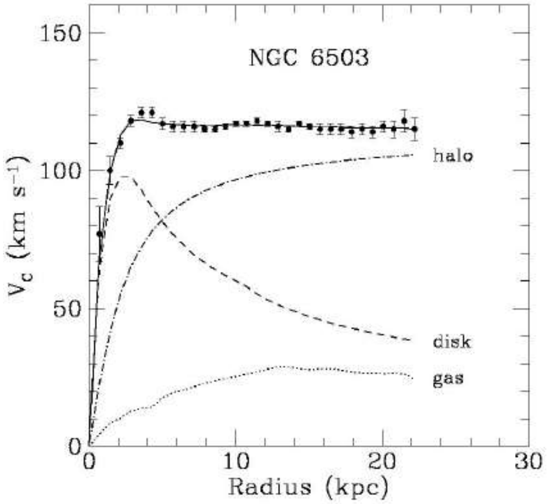
Observational constraints

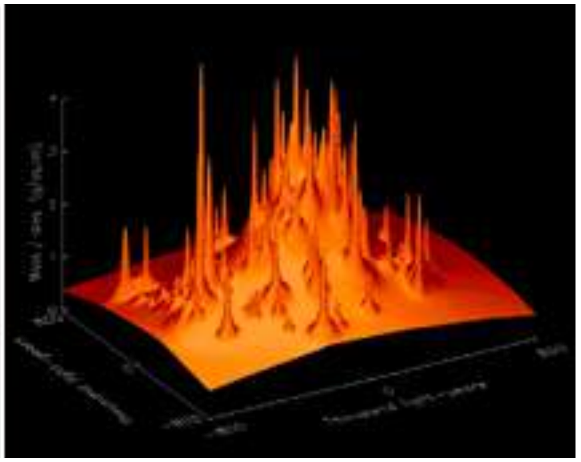
Reviews

Green & Kavanagh, J. Phys. G. arXiv:2007.10722, ‘PBHs as a dark matter candidate’
    Bradley Kavanagh’s PBH abundance constraint plotting code: https://github.com/bradkav/PBHbounds

Carr & Kühnel, Ann. Rev. Nuc. Part. Sci. arXiv:2006.02838, ‘PBHs as dark matter: recent developments’

Motivation & history

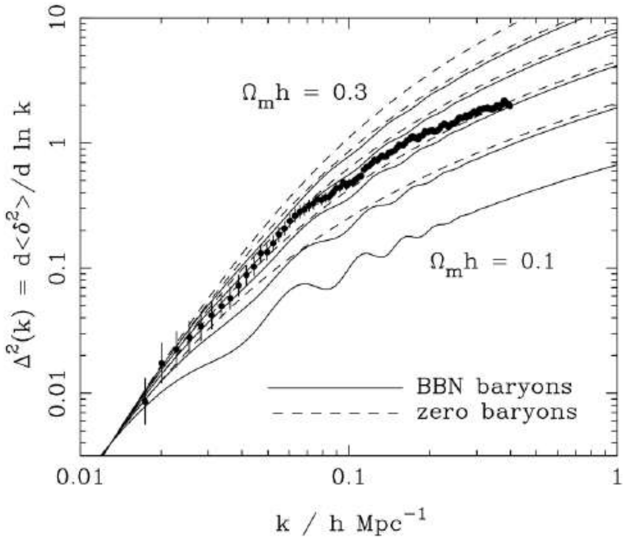


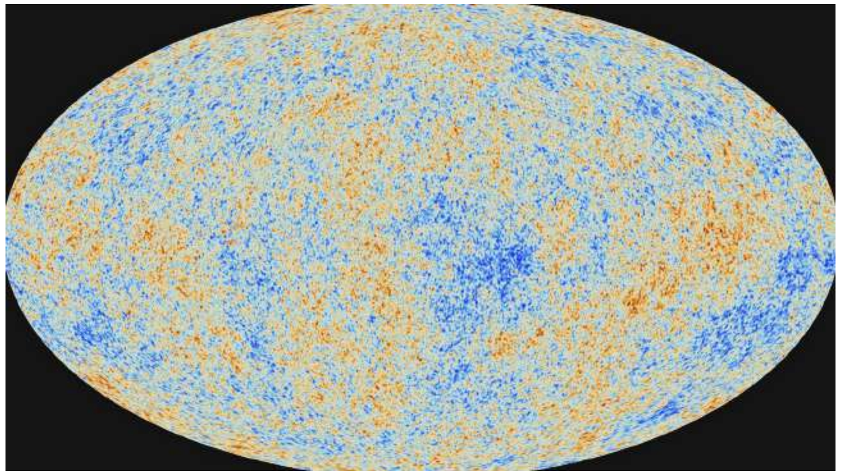


Lots of evidence for **non-baryonic cold dark matter** from diverse astronomical and cosmological observations

[galaxy rotation curves, galaxy clusters (galaxy velocities, X-ray gas, lensing), galaxy red-shift surveys, Cosmic Microwave Background]

assuming Newtonian gravity/GR is correct.

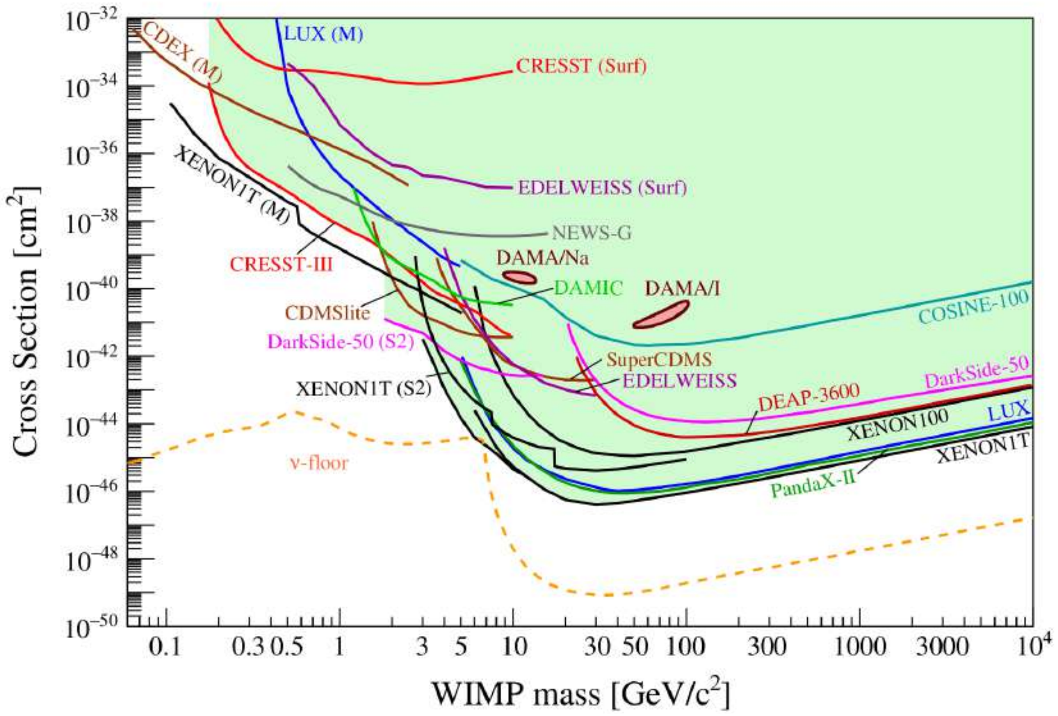




No sign (yet…) of well-motivated particle dark matter candidates in ‘direct detection’ experiments:

Weakly Interacting Massive Particles

elastic scattering
cross section

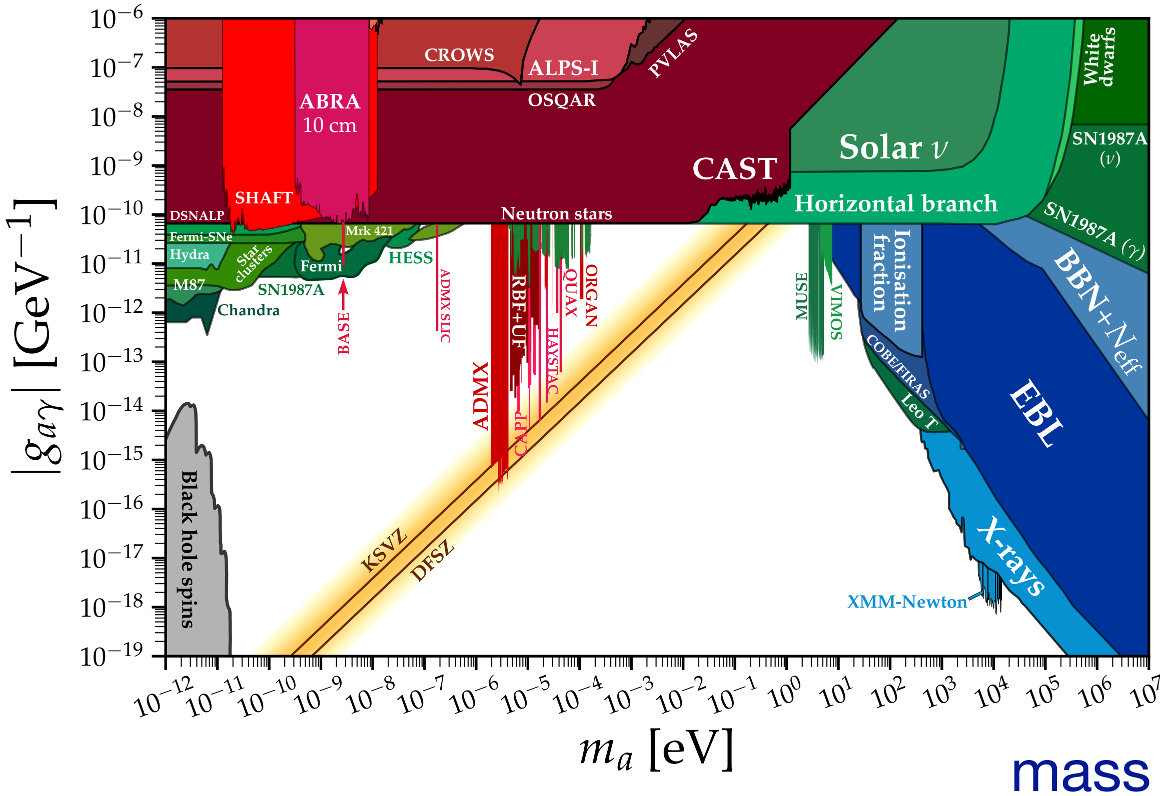


mass

APPEC committee report

axions/axion like particles

coupling with
photons



mass

O’Hare

Primordial Black Holes (PBHs) may form from over densities in the early Universe (before nucleosynthesis) and are therefore non-baryonic. Zel'dovich and Novikov; Hawking

PBHs evaporate (Hawking radiation), lifetime longer than the age of the Universe (and DM candidate) if $M \gtrsim 10^{15}\,\mathrm{g}$. MacGibbon

[Aside: evaporation of lighter PBHs can produce stable massive particle (i.e. DM) and have other cosmological consequences (talks: Perez-Gonzalez, Turner)]

A DM candidate which (unlike WIMPs, axions, sterile neutrinos,…) isn't a new particle, however their formation does usually require Beyond the Standard Model physics, e.g. inflation.

Was realised that PBHs are a cold dark matter (DM) candidate in the 1970s Hawking; Chapline

Wave of interest in ~Solar mass PBHs as DM in late 1990s, generated by excess of LMC microlensing events in MACHO collaboration's 2 year data set.

Nakamura et al. (1997): PBH binaries form in the early Universe and (**if** they survive to the present day) GWs from their coalescence detectable by LIGO.

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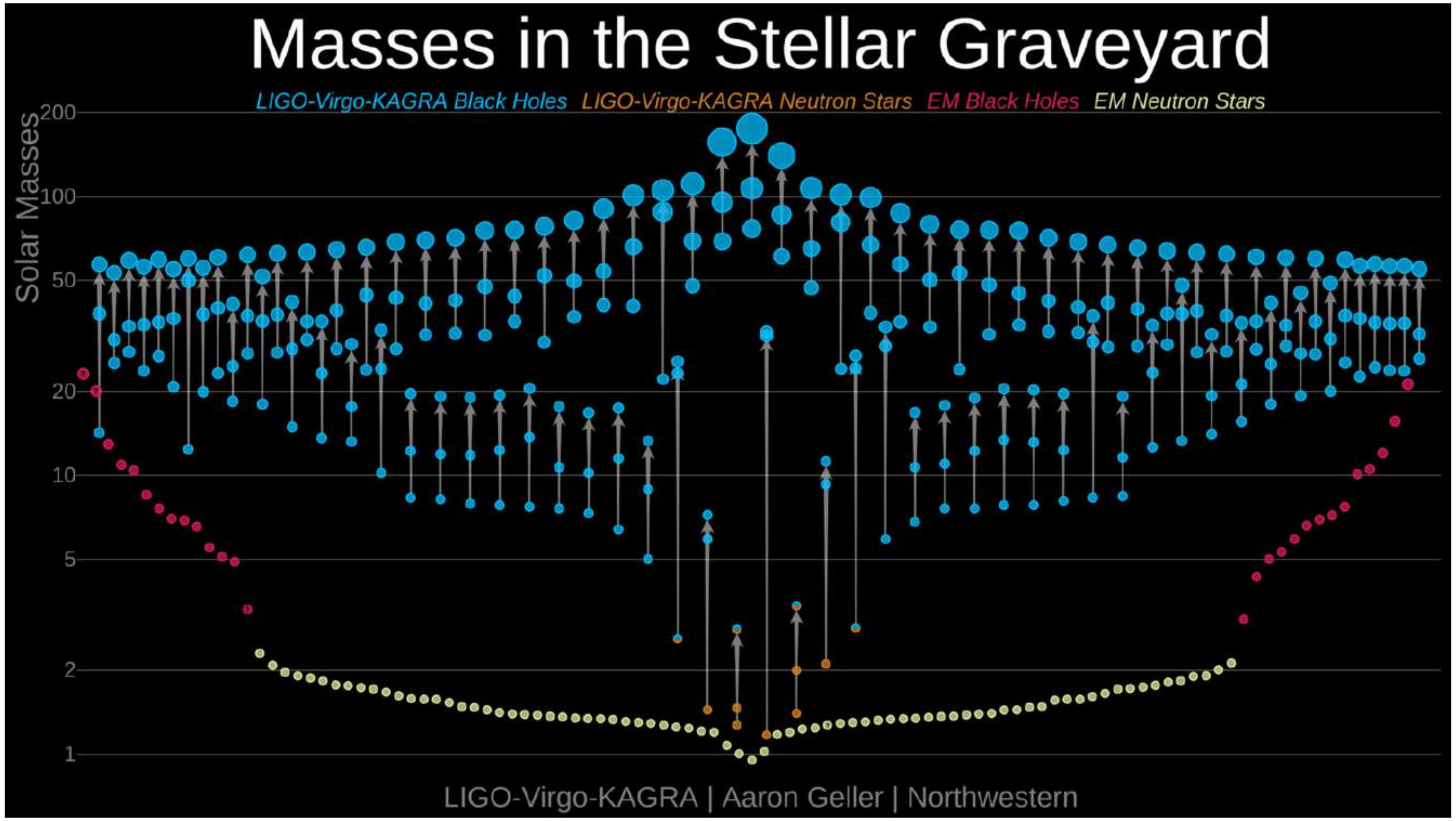
Could (some of) the BHs in the LIGO-Virgo BH binaries be primordial? (and also a significant component of the DM?) Bird et al.; Clesse & Garcia-Bellido; Sasaki et al.



$100M_\odot$

$10M_\odot$

LIGO-Virgo-KAGRA, Geller

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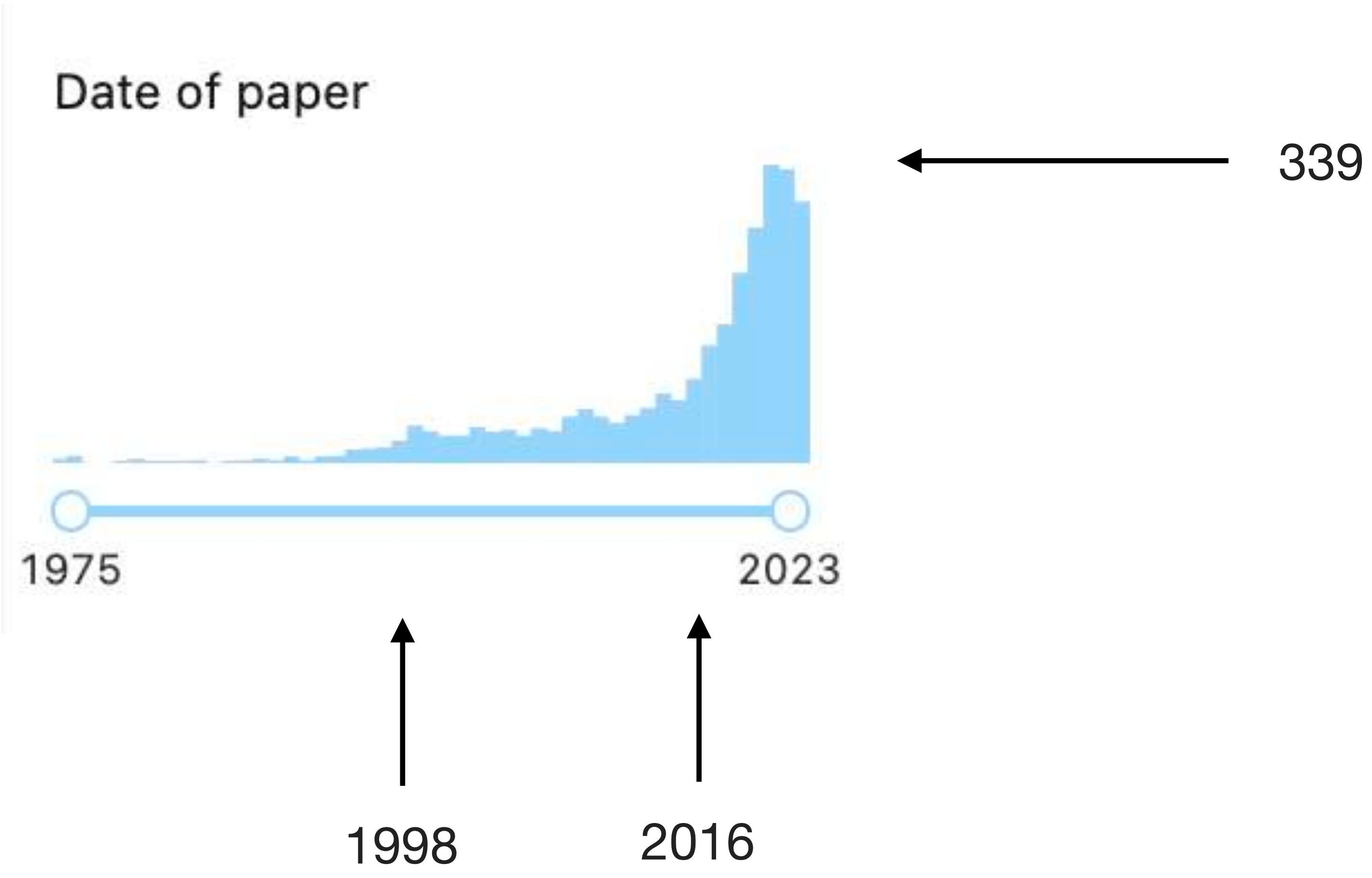

result of an inSPIRE search for 'primordial black hole'

Formation

Most ‘popular’ mechanism: collapse of large density perturbations during radiation domination. Zeldovich & Novikov; Hawking; Carr & Hawking

If a region is sufficiently over-dense, gravity overcomes pressure and it collapse to form a BH shortly after ‘horizon entry’.

other mechanisms:

- collapse of cosmic string loops Hawking; Polnarev & Zemboricz,
- bubble collisions Hawking, Moss & Stewart,
- fragmentation of inflaton scalar condensate Cotner & Kusenko,
- collapse of density perturbations during matter domination Khlopov & Polnarev,
- …
- trapped vacuum bubbles (talk: Escriva),
- confinement (talk: Zantedeschi),
- …

‘zero-th order’ analysis: Carr

i) criterion for PBH formation:

critical density:

$$\delta \geq \delta_{\rm c} \sim w = \frac{p}{\rho} = \frac{1}{3}$$

$$\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}}$$ density contrast (at horizon crossing)

PBH mass roughly equal to horizon mass:

$$M_{\rm PBH} \sim 10^{15}\,{\rm g} \left(\frac{t}{10^{-23}\,{\rm s}} \right) \sim M_{\odot} \left(\frac{t}{10^{-6}\,{\rm s}} \right)$$

Criterion is actually:

best specified in terms of compaction function,

depends on shape of perturbation (which depends on primordial power spectrum).

Harada,Yoo & Kohri; Germani & Musco; Musco; Escriva, Germani & Sheth. For overview see Escriva, Kuhnel & Tada
(talk: Harada)

ii) PBH abundance:

initial PBH mass fraction (fraction of universe in regions dense enough to form PBHs):

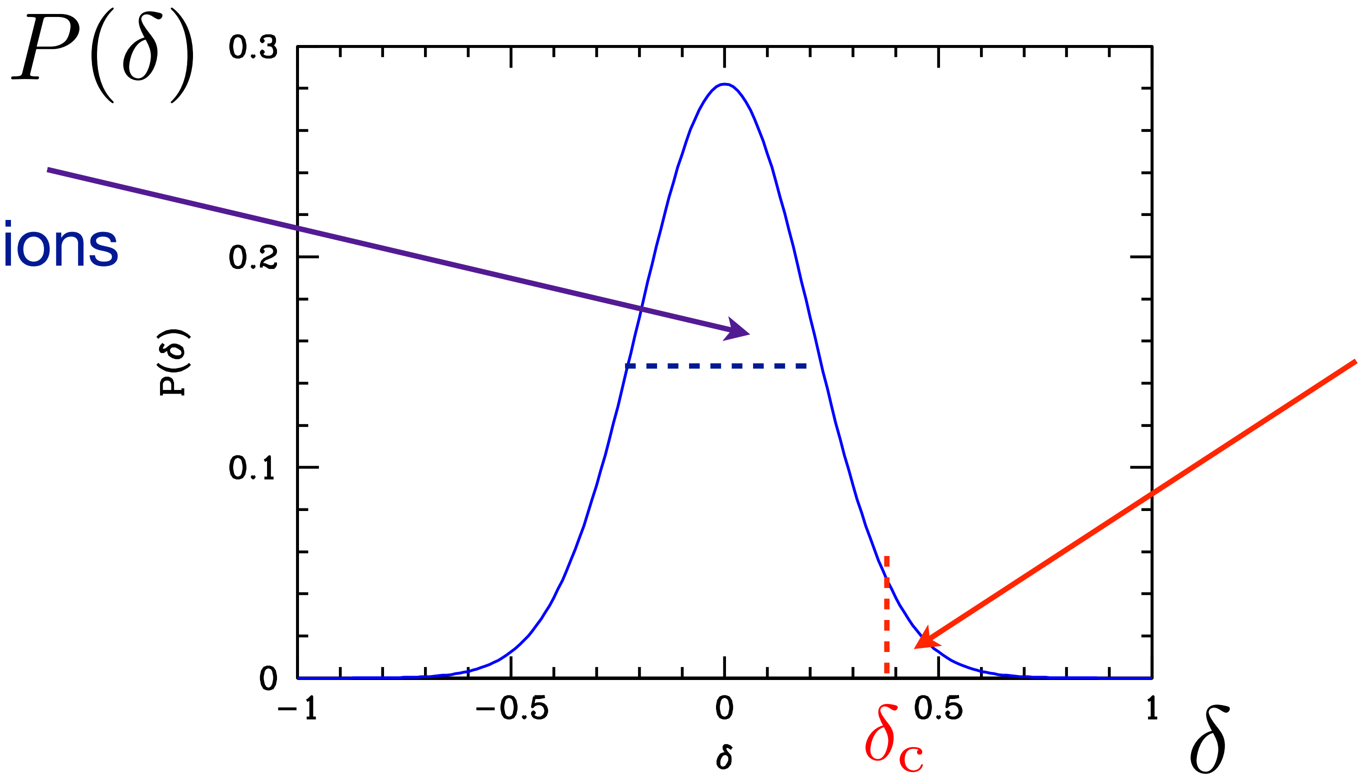
$$\beta(M) \sim \int_{\delta_{\rm c}}^{\infty} P(\delta(M_{\rm H}))\, {\rm d}\delta(M_{\rm H})$$

assuming a gaussian probability distribution:

$$\beta(M) = {\rm erfc}\left(\frac{\delta_{\rm c}}{\sqrt{2}\sigma(M_{\rm H})}\right)$$

β must be small, hence $\sigma \ll \delta_{\rm c}$ and

$$\beta(M) \sim \sigma(M_{\rm H}) \exp\left(-\frac{\delta_{\rm c}^2}{2\sigma^2(M_{\rm H})}\right)$$



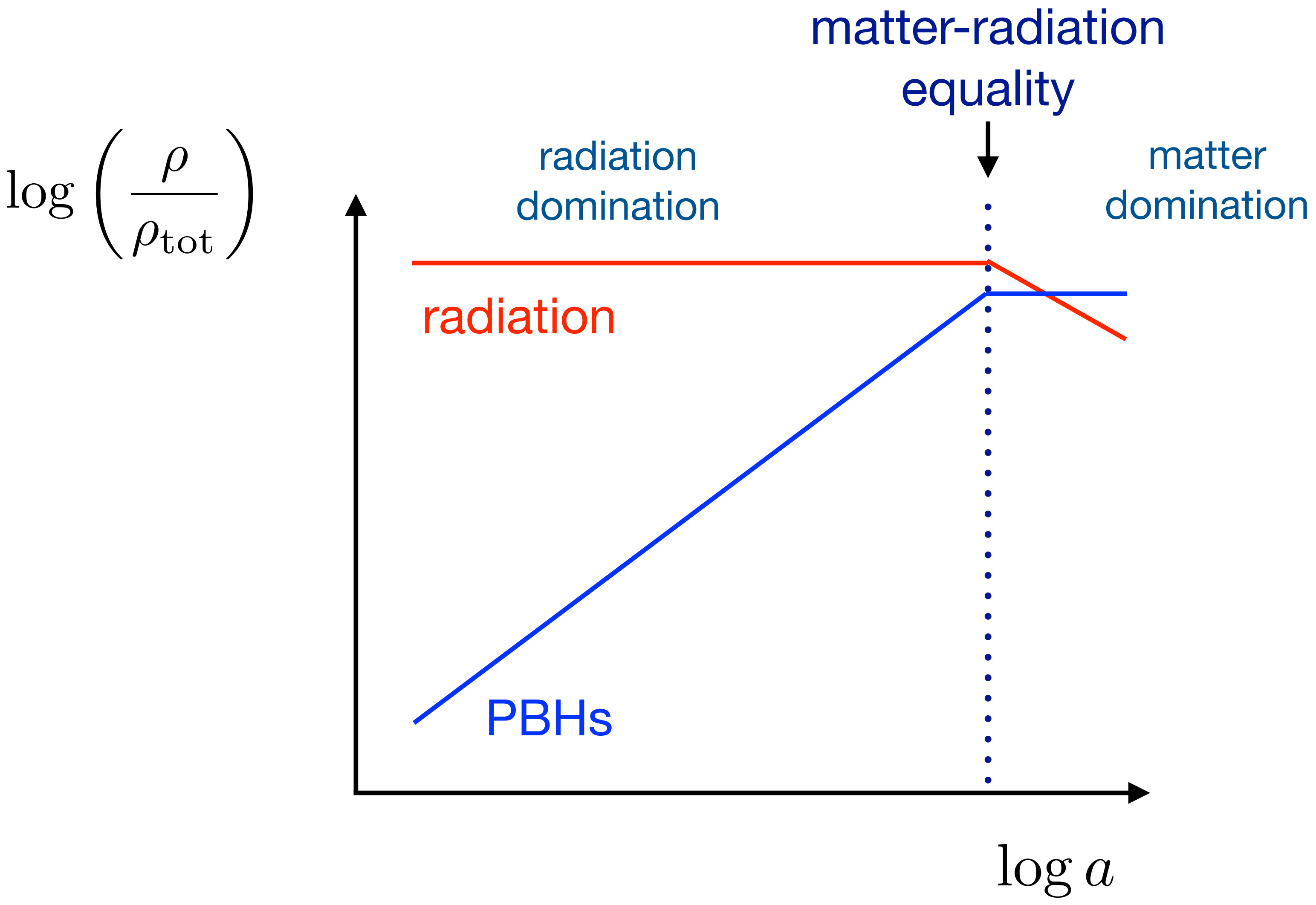
$\sigma(M_{\rm H})$ (mass variance)
typical size of fluctuations

PBH forming fluctuations

But this Press-Schechter, approach gives a biased estimate of the PBH mass fraction. e.g. Germani & Sheth (talk: Sheth)

Since PBHs are matter, during radiation domination the fraction of energy in PBHs grows with time:

$$\frac{\rho_{\rm PBH}}{\rho_{\rm rad}} \propto \frac{a^{-3}}{a^{-4}} \propto a$$



Relationship between **PBH initial mass fraction, β**, and **fraction of DM in form of PBHs, f**:

$$\beta(M) \sim 10^{-9} f \left(\frac{M}{M_\odot}\right)^{1/2}$$

i.e. initial mass fraction must be small.

On CMB scales the primordial perturbations have amplitude $\sigma(M_{\rm H}) \sim 10^{-5}$

If the primordial perturbations are very close to scale-invariant the number of PBHs formed will be completely negligible:

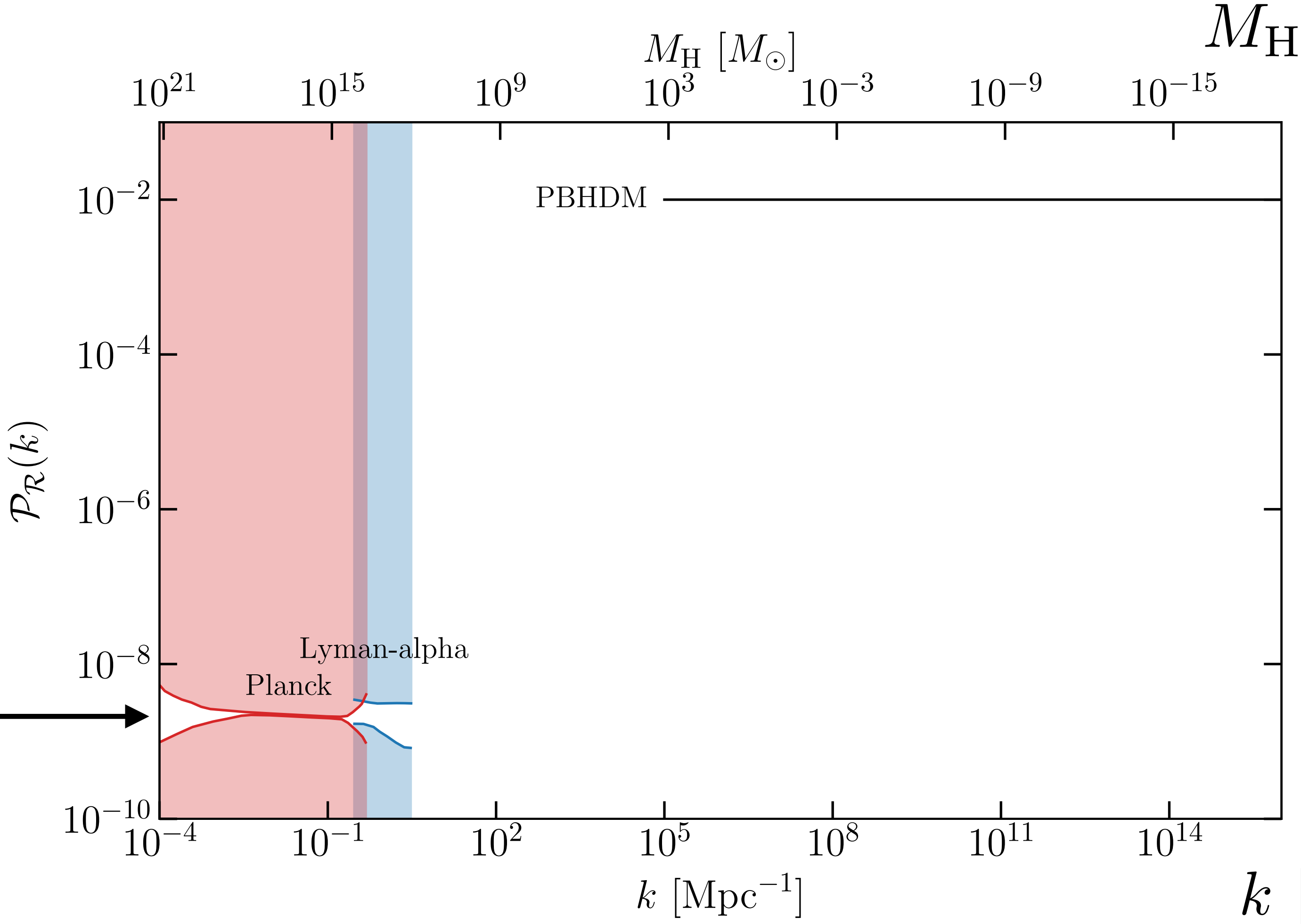
$$\beta(M) = {\rm erfc}\left(\frac{\delta_{\rm c}}{\sqrt{2}\sigma(M_{\rm H})}\right)$$

$$\beta(M) \sim {\rm erfc}(10^5) \sim \exp\left(-10^{10}\right)$$

To form an interesting number of PBHs the primordial perturbations must be significantly larger ($\sigma^2(M_{\rm H}) \sim 0.01$) on small scales than on cosmological scales.

$\mathcal{P}_{\mathcal{R}}(k) \sim \sigma^2$

$M_{\rm H}\ [M_\odot]$ ~ PBH mass



amplitude required for $f_{\rm PBH} \sim 1$

measured amplitude on cosmological scales

$k\ [{\rm Mpc}^{-1}]$

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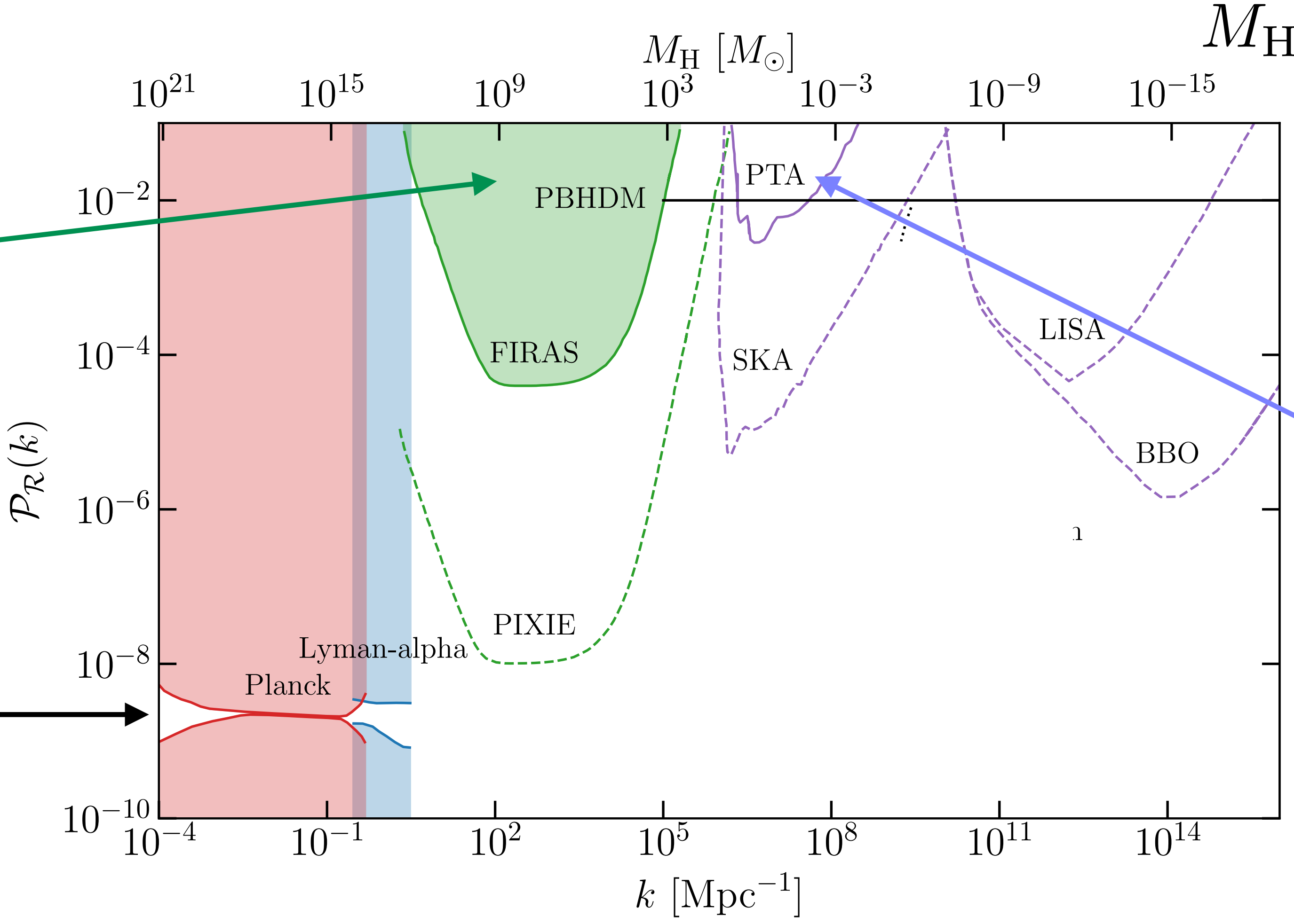


$\mathrm{pc}^{-1}]$

CMB spectral distortions
Carr & Lidsey

measured amplitude on cosmological scales

amplitude required for $f_{\rm PBH} \sim 1$

scalar induced gravitational waves
Ananda, Clarkson & Wands
Saito & Yokoyama

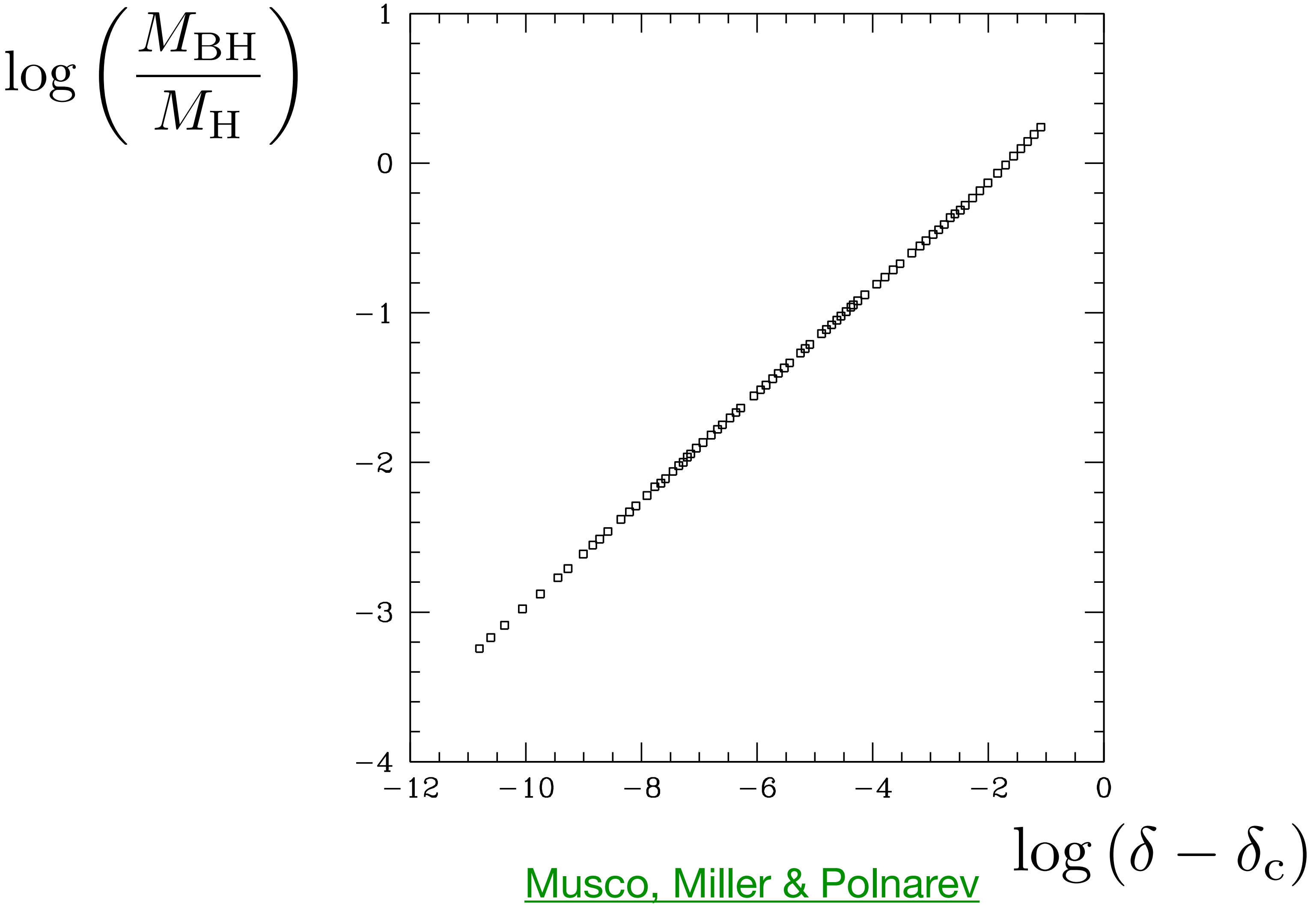
deviations from simple scenario:

i) critical collapse

Niemeyer & Jedamzik

BH mass depends on size of fluctuation it forms from:

$$M = k M_{\rm H} (\delta - \delta_{\rm c})^{\gamma}$$



Musco, Miller & Polnarev

Get PBHs with range of masses produced even if they all form at the same time i.e. we don't expect the PBH MF to be a delta-function

ii) non-gaussianity of probability distribution

Since PBHs are formed from rare large density fluctuations, changes in the shape of the tail of the probability distribution (i.e. non-gaussianity) can significantly affect the PBH abundance. Bullock & Primack; Ivanov;… Francolini et al.

Relationship between density perturbations and curvature perturbations is non-linear, so even if curvature perturbations are gaussian (large) density perturbations won't be. Kawasaki & Nakatsuka; De Luca et al.; Young, Musco & Byrnes

(talks: Kristiano, Kawaguchi)

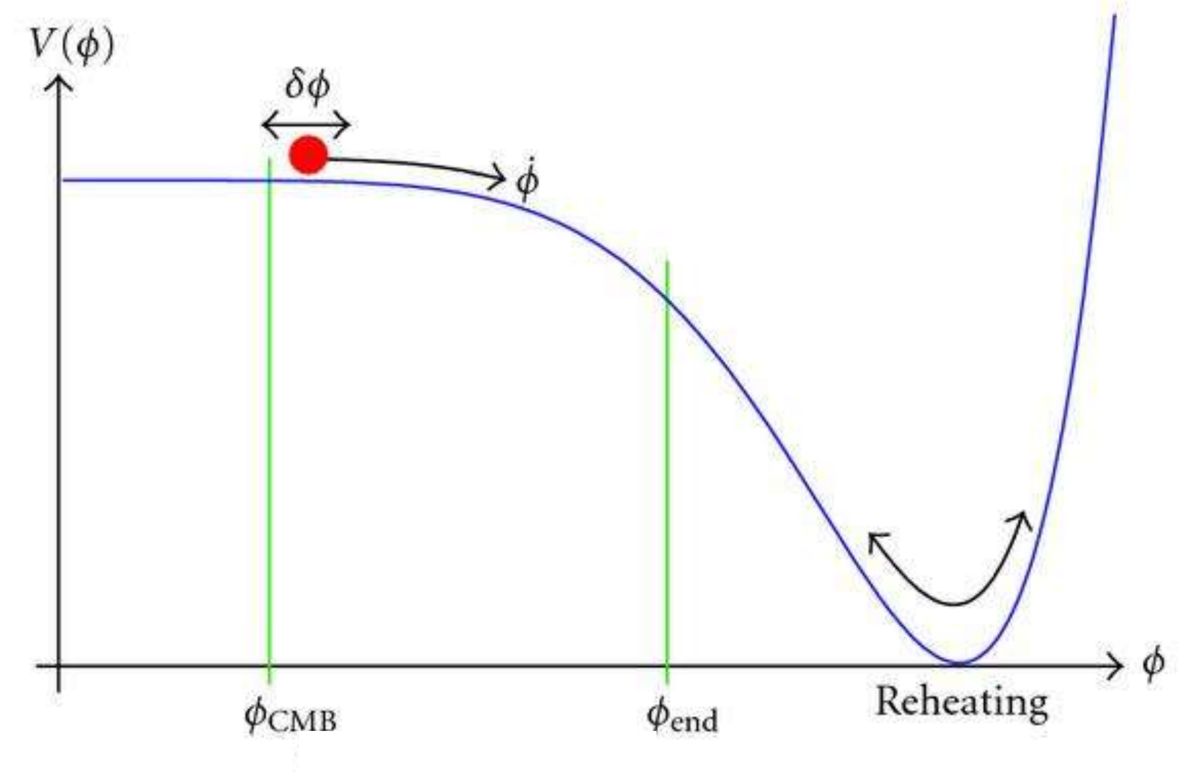
Inflation: a brief crash course

A postulated period of accelerated expansion in the early Universe, proposed to solve various problems with the Big Bang (flatness, horizon & monopole).

Driven by a 'slowly rolling' scalar field.

Quantum fluctuations in scalar field generate density perturbations.

Scale dependence of primordial perturbations depends on shape of potential:



Yadav & Wandelt

in slow-roll approx

$$\sigma^2(M_{\rm H}) \propto \frac{V^3}{(V')^2}$$

Scales probed by:

Large scale structure
& the CMB

Primordial Black Holes

inflation models that produce large perturbations

In slow-roll approx*:

$$\sigma \propto \frac{V^{3/2}}{V'}$$

A plateau in the potential can generate large perturbations which form an interesting abundance of PBHs. Ivanov, Naselsky, Novikov

* in 'ultra-slow-roll' limit, $V' \rightarrow 0$, this expression isn't accurate (and USR also affects probability distribution of fluctuations).

Requirements for a PBH producing inflation model:

i) produce measured power spectrum (amplitude and scale dependence) on cosmological scales,

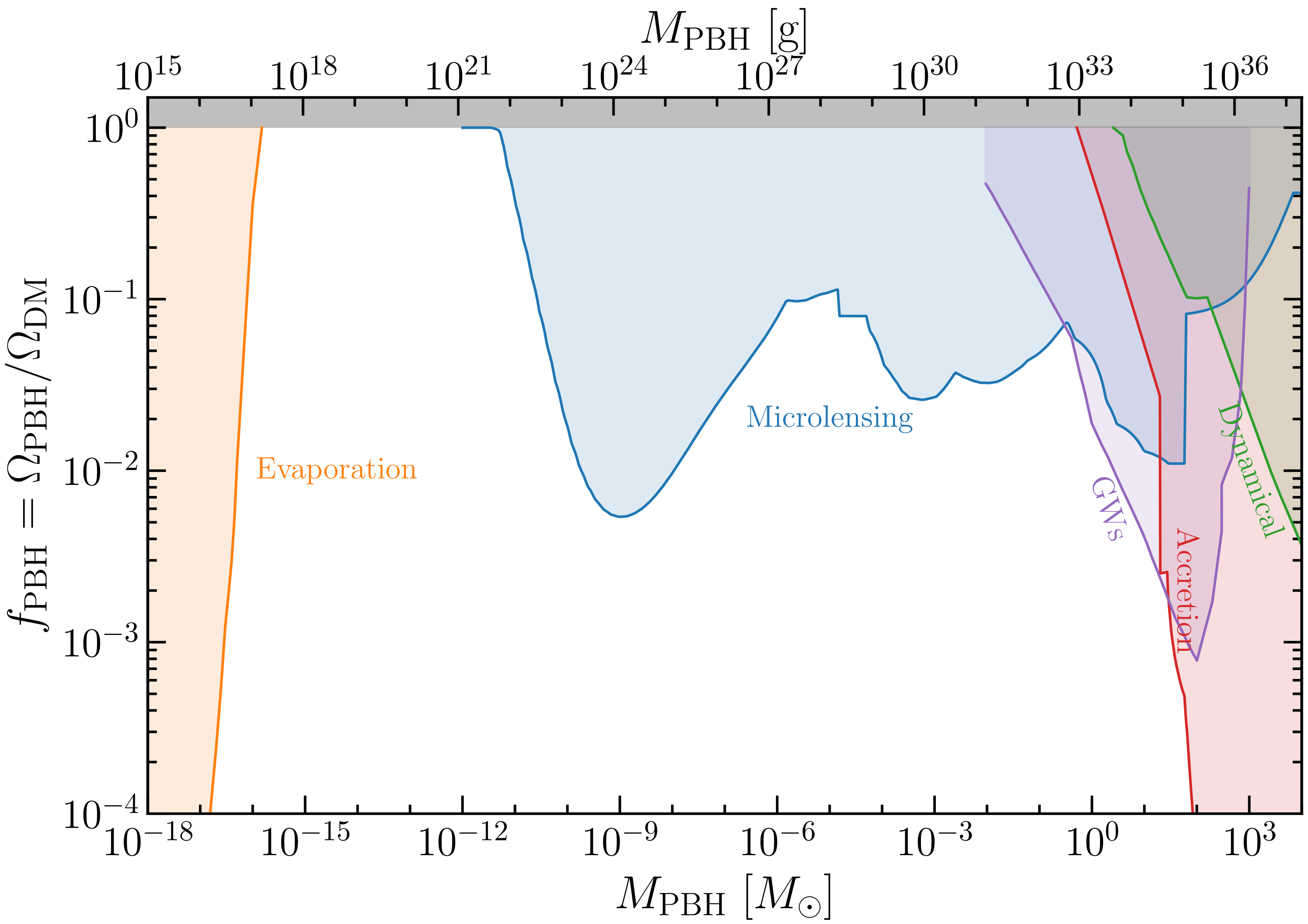
ii) amplitude of perturbations ~3.5 orders of magnitude larger on some smaller scale,

iii) inflation ends.

(talks: Ketov, Wang)

Observational constraints

(assuming a delta-function PBH mass function)



evaporation

lensing

gravitational waves

accretion

dynamical

https://github.com/bradkav/PBHbounds

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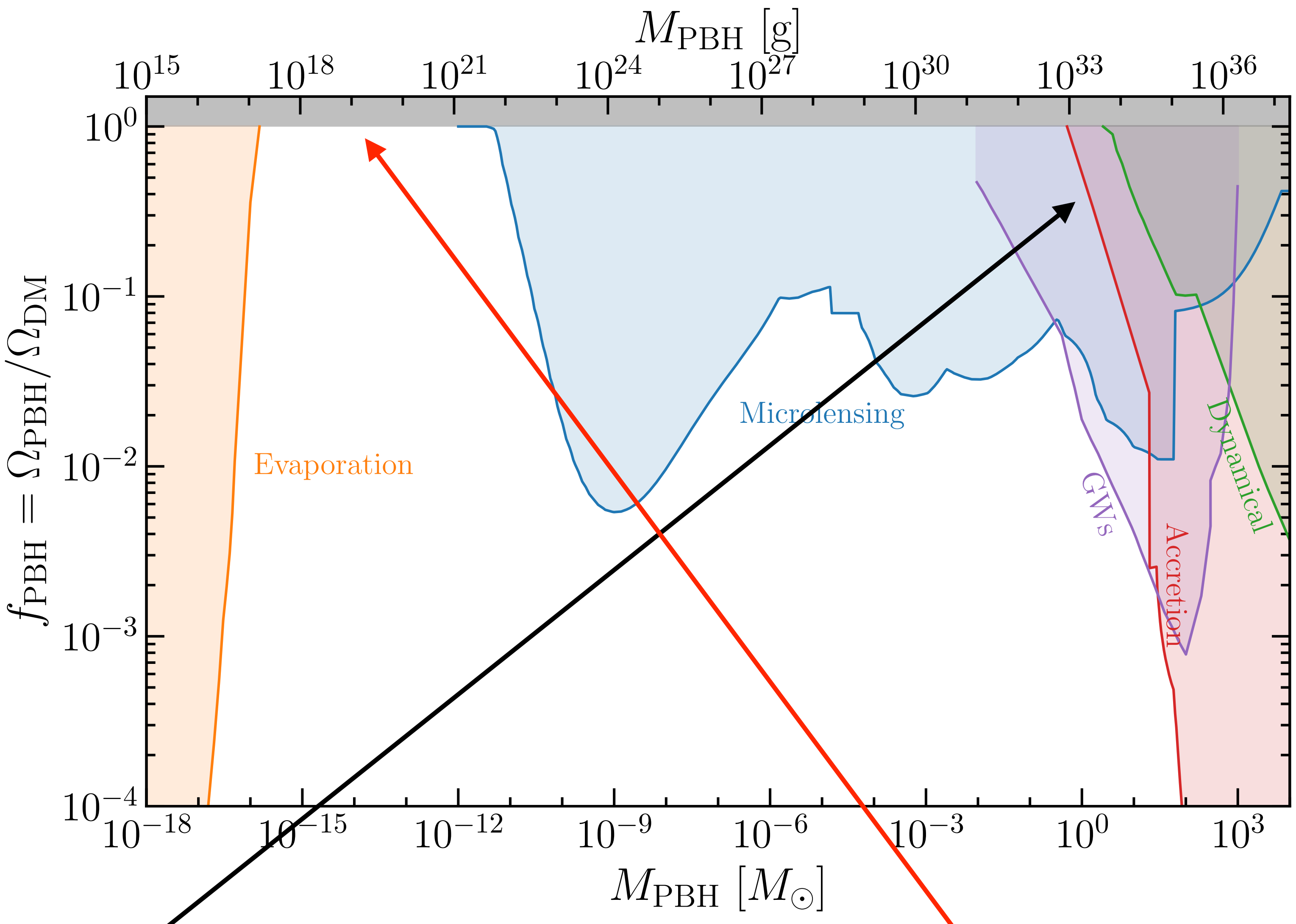


evaporation
lensing
gravitational waves
accretion
dynamical

https://github.com/bradkav/PBHbounds

multi-Solar mass Primordial Black Holes making up all of the DM appears to be excluded.

However there is a hard to probe, open window for very light (asteroid mass) PBHs.

(talks: Kühnel, Kohri, Kuroyanagi,Takahashi)

2. Stellar microlensing constraints on PBH dark matter

How robust are the stellar microlensing constraints?

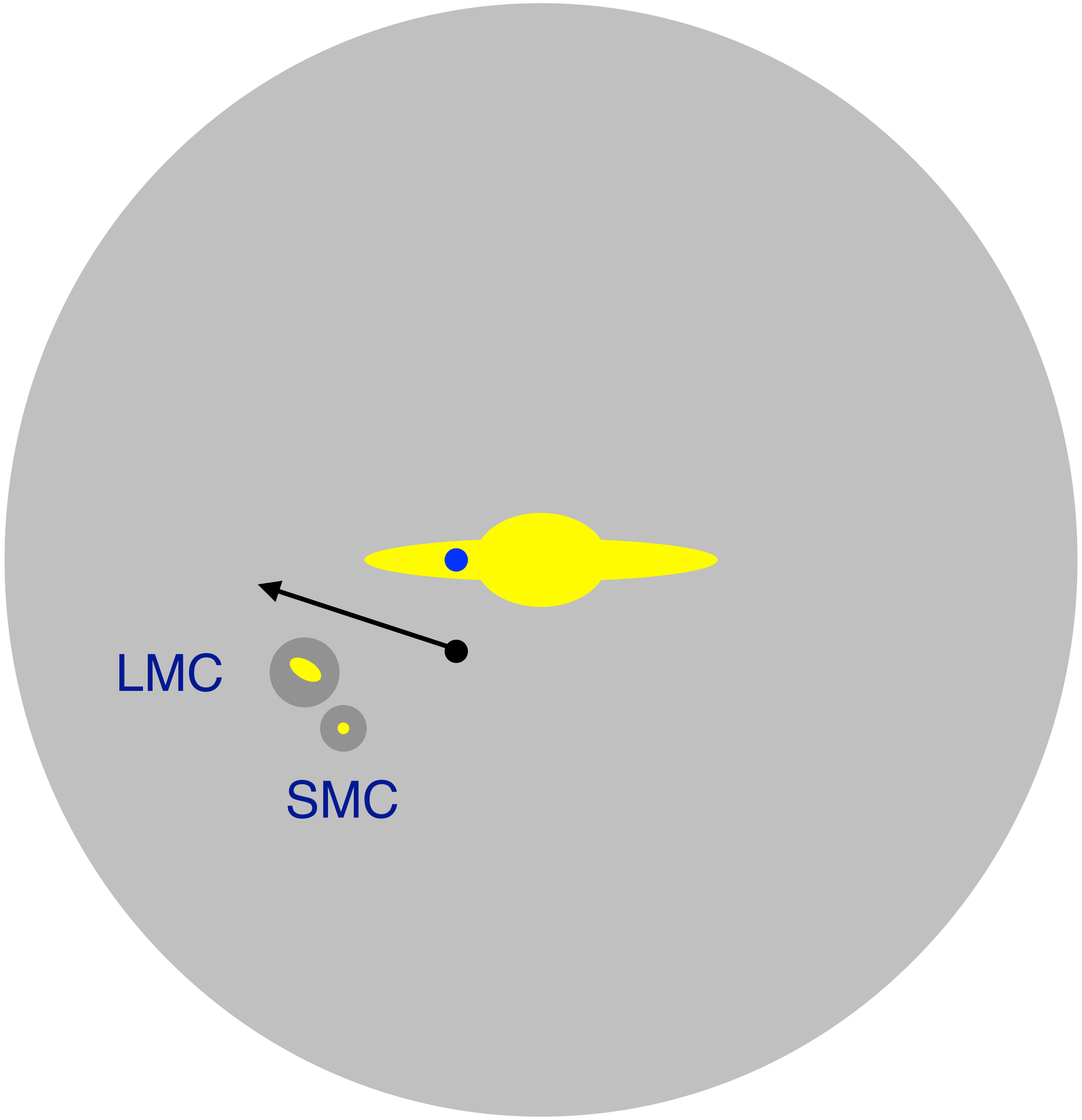
How does realistic modelling of

- DM distribution within the Milky Way
- PBH mass function
- PBH clustering

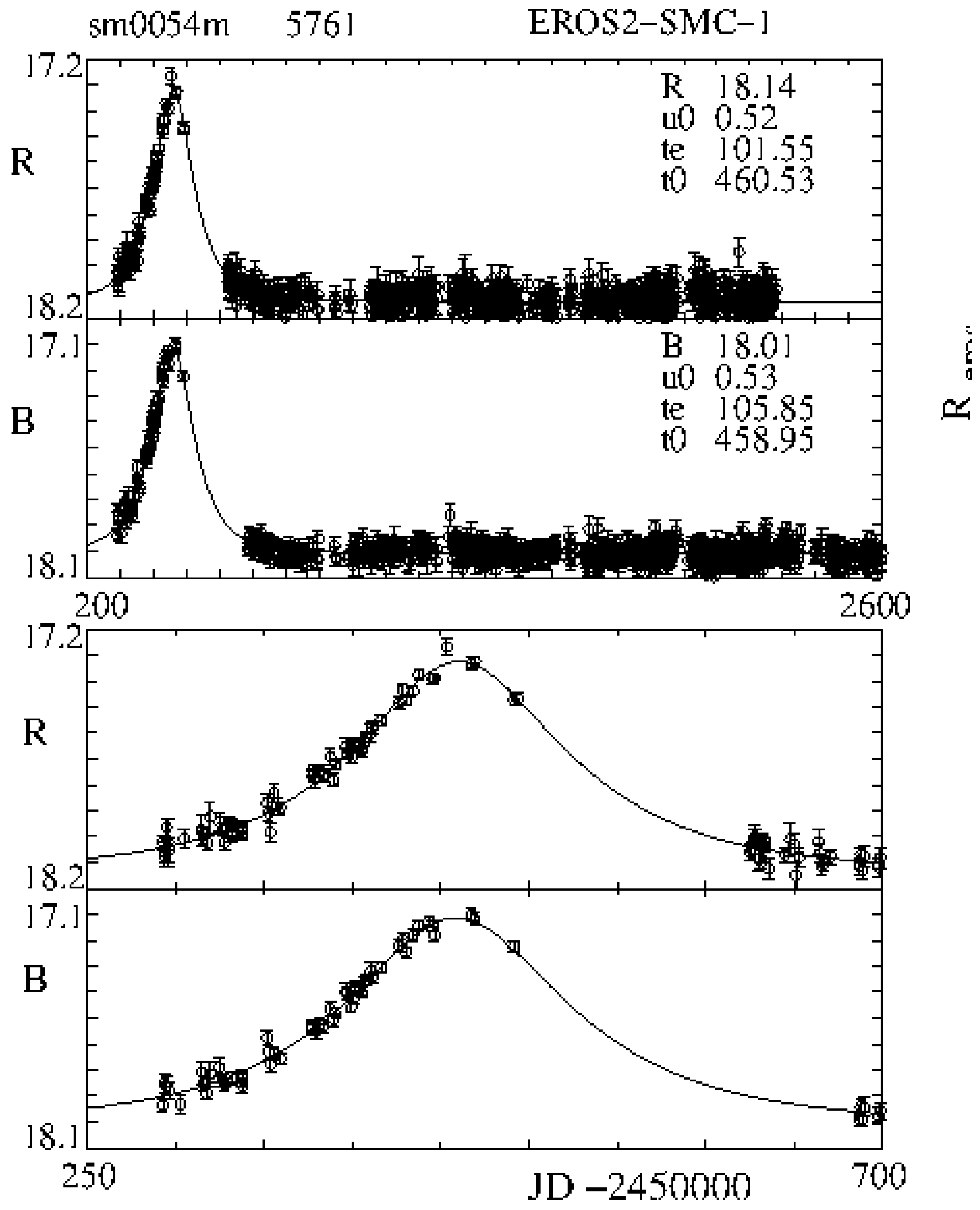
effect the constraints?

Stellar microlensing

Observe temporary (achromatic) brightening of background star when compact object passes close to the line of sight. Paczynski



Not to scale!

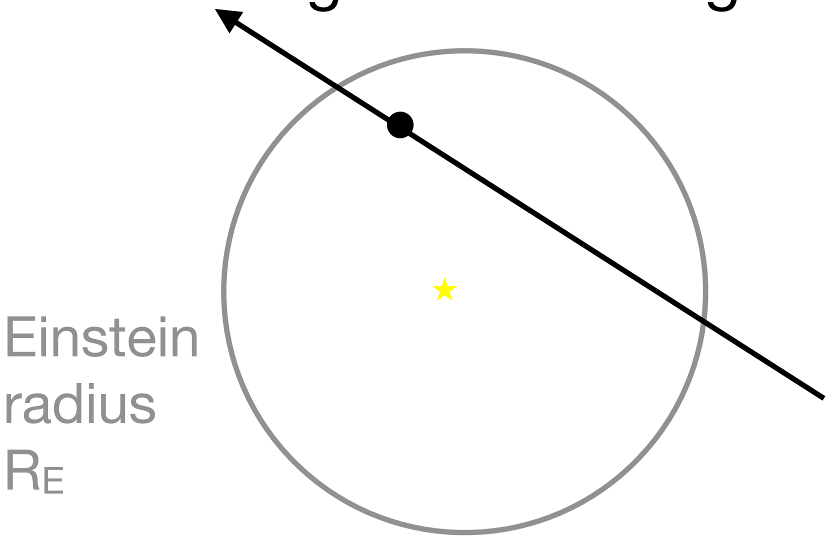


EROS

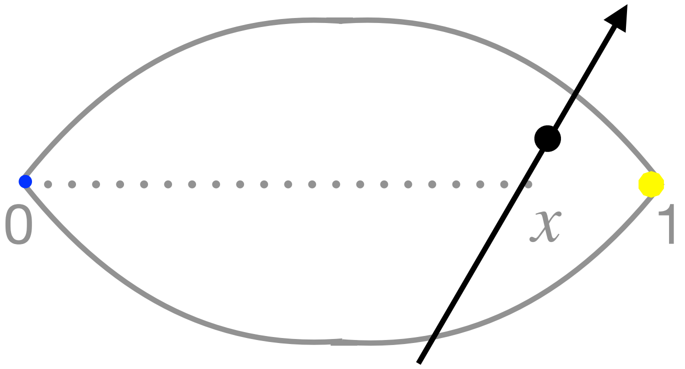
Einstein radius:

$$R_E \approx 3\times10^{-4}\,\mathrm{pc}\sqrt{x(1-x)}\left(\frac{M}{10M_\odot}\right)^{1/2}\left(\frac{D_s}{50\,\mathrm{kpc}}\right)^{1/2}$$

along the line of sight



perpendicular to the line of sight



‘Duration’ of event (Einstein diameter crossing time):

$$\hat{t} = \frac{2R_E}{v} \approx 3\,\mathrm{yr}\sqrt{x(1-x)}\left(\frac{M}{10M_\odot}\right)^{1/2}\left(\frac{D_s}{50\,\mathrm{kpc}}\right)^{1/2}\left(\frac{v}{200\,\mathrm{km\,s}^{-1}}\right)^{-1}$$

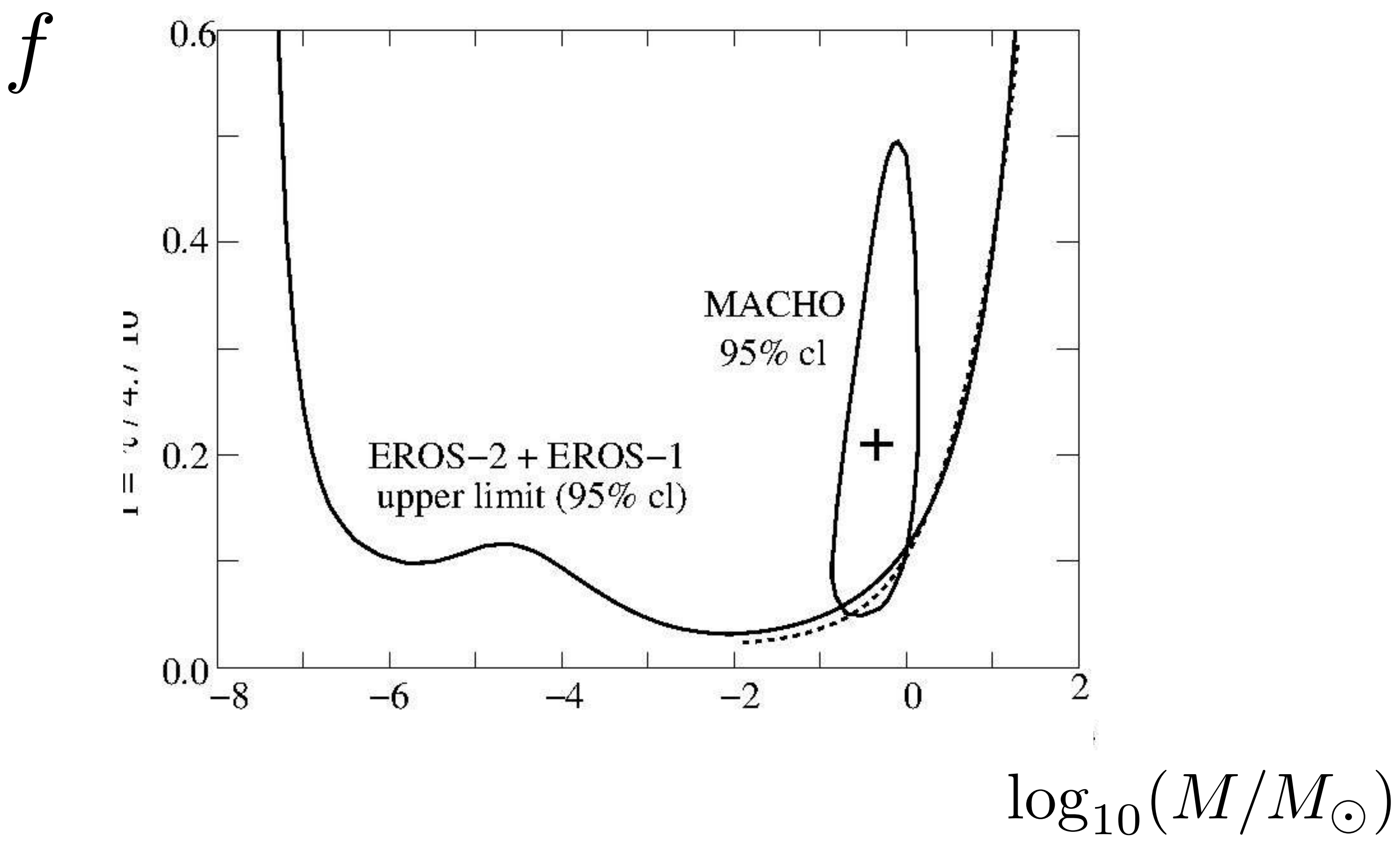
EROS

Monitored 67 million stars in LMC and SMC for 6.7 years. EROS
Use bright stars in sparse fields (to avoid complications due to 'blending'-contribution to baseline flux from unresolved neighbouring star).

1 SMC event (also seen by MACHO collab.) consistent with expectations for self-lensing (SMC is aligned along line of sight).

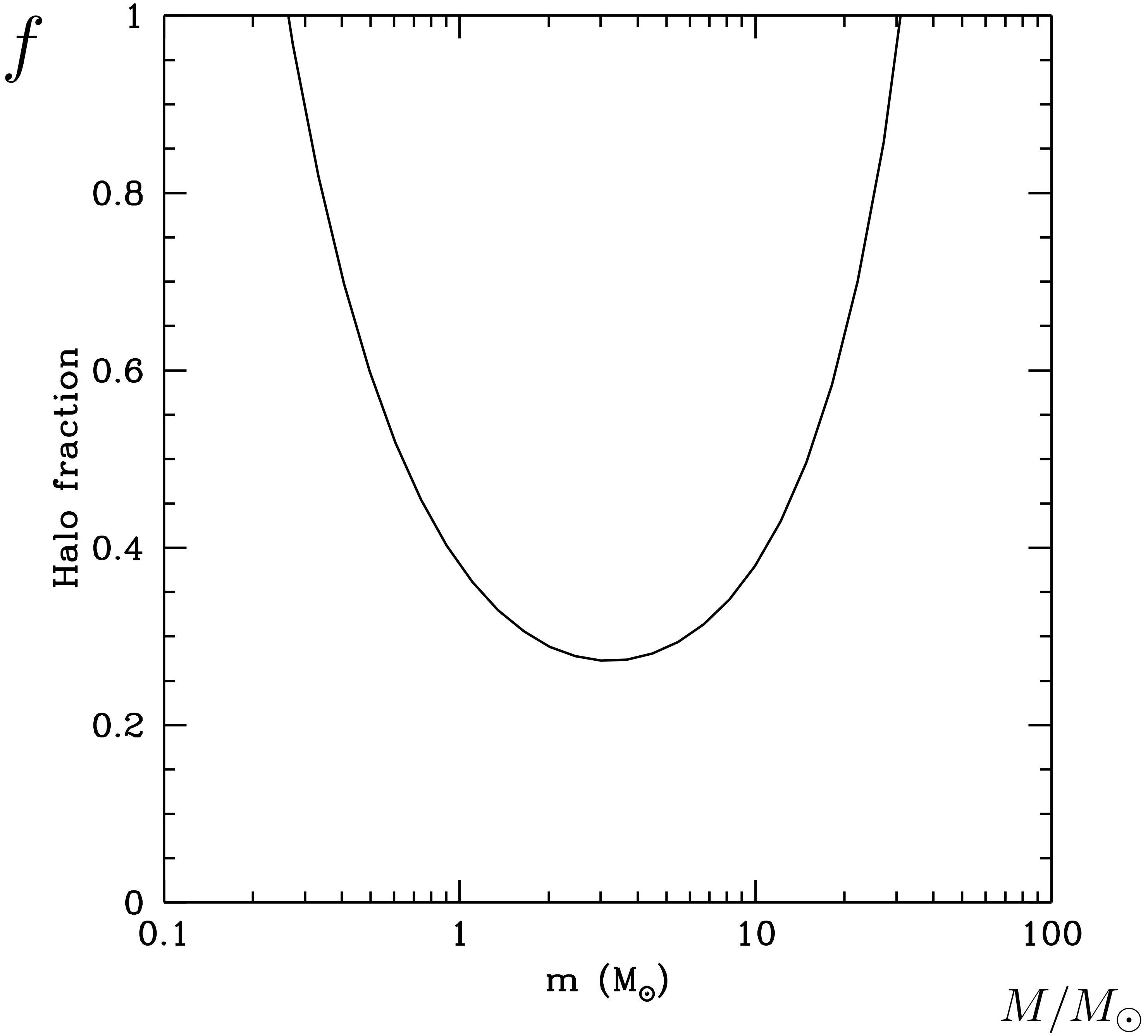
Earlier candidate events eliminated: 7 varied again and 3 identified as supernovae.

Constraints on fraction of halo in compact objects, f, (DF MF):



MACHO long duration

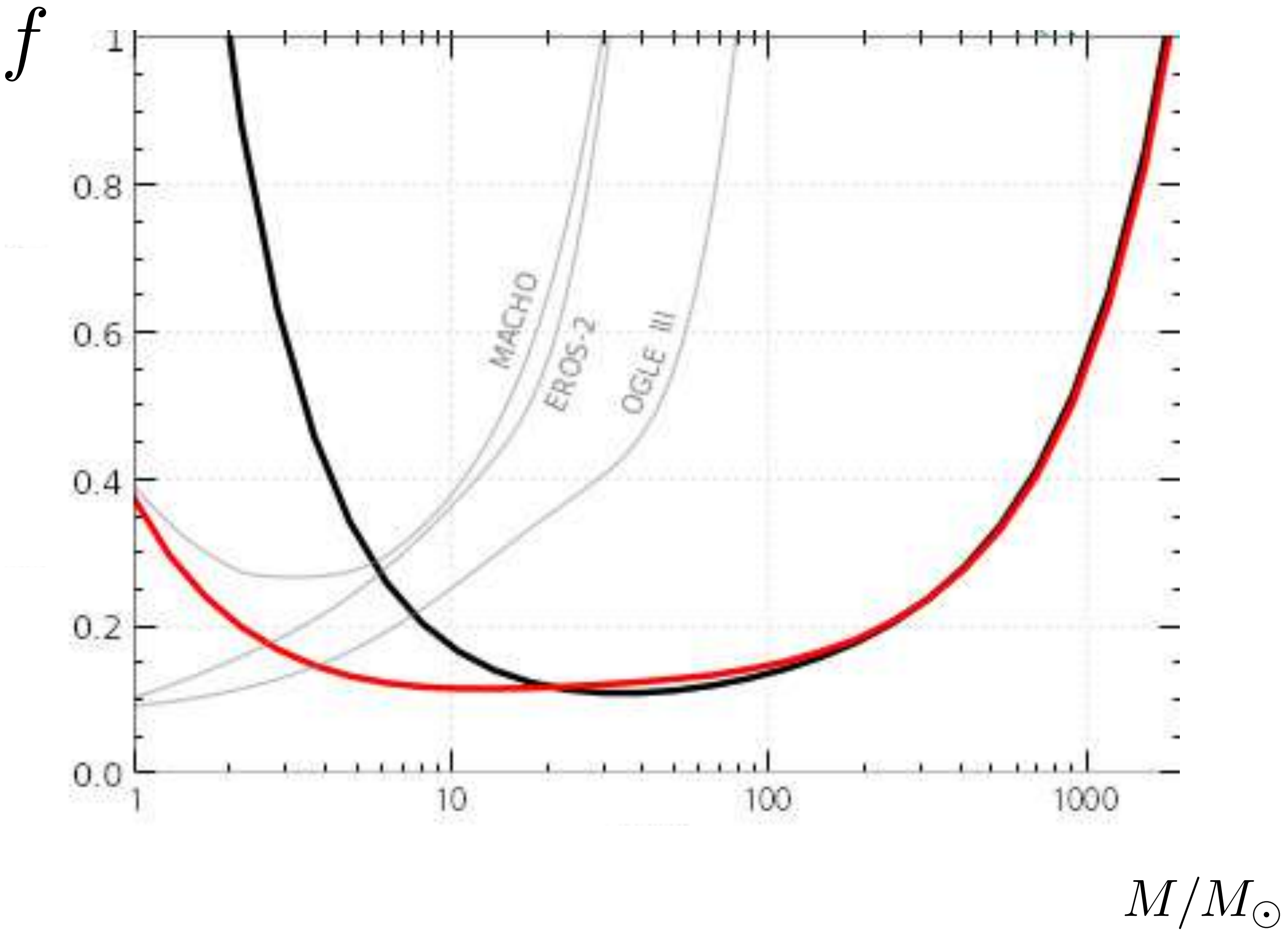
MACHO null search for long (> 150 day) duration events MACHO:



combined long duration

Combined data from EROS-2 and MACHO (14.1 million objects, over 10.6 years):

Blaineau et. al.



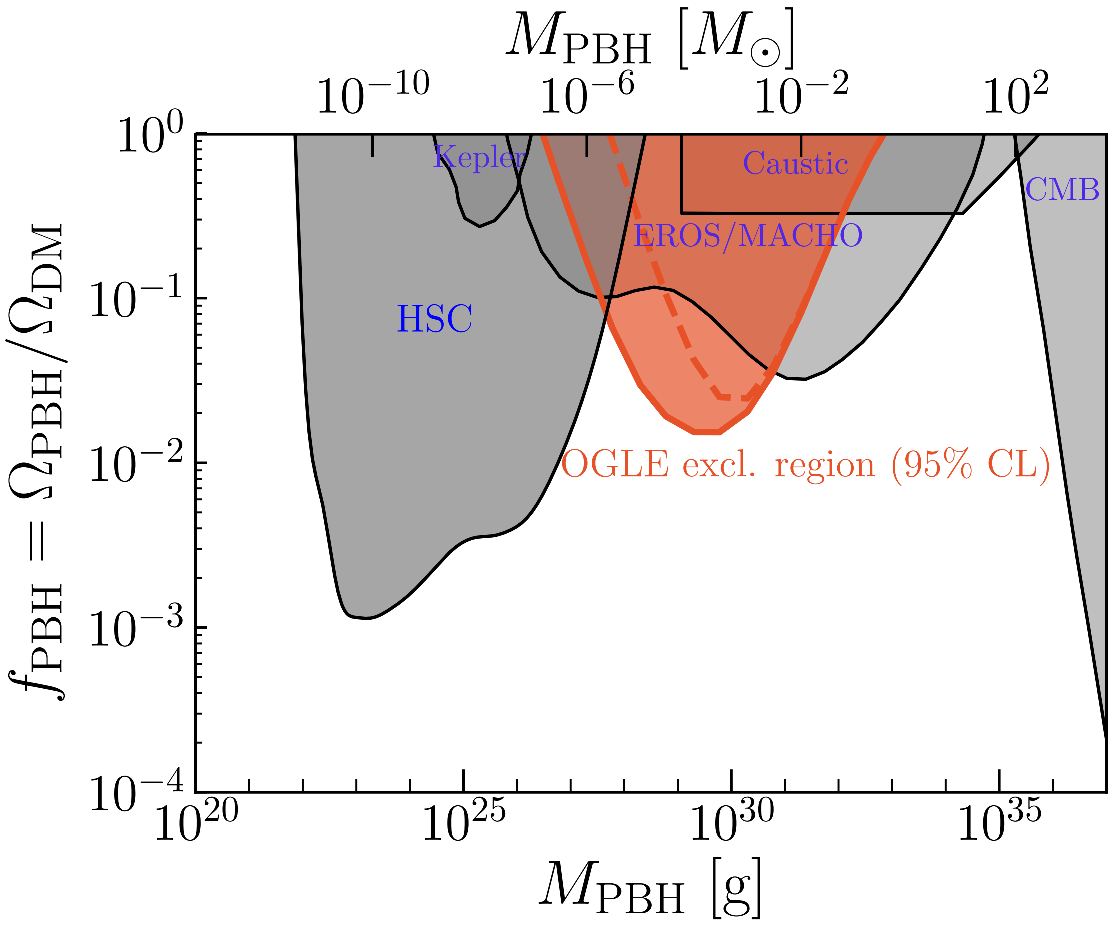
Einstein radius
crossing time
> 100 days
> 200 days

Including OGLE-III and IV data would extend timescale to 30 years, and constraints to $\sim 3000 M_\odot$.

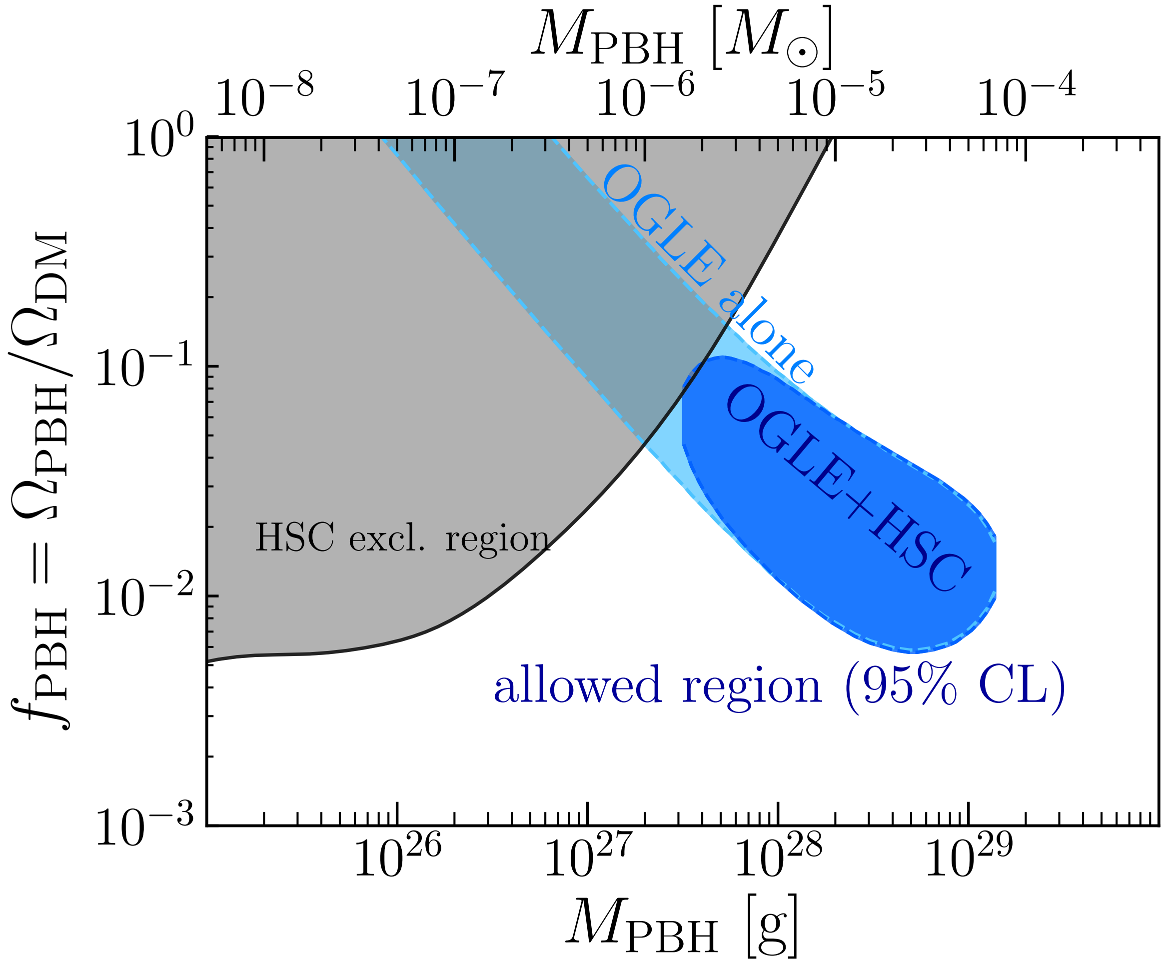
OGLE: Galactic bulge

Observed events consistent with expectations from stars (except for 6 ultra-short (0.1-0.3) day events)

Exclusion limit
assuming no PBH lensing observed



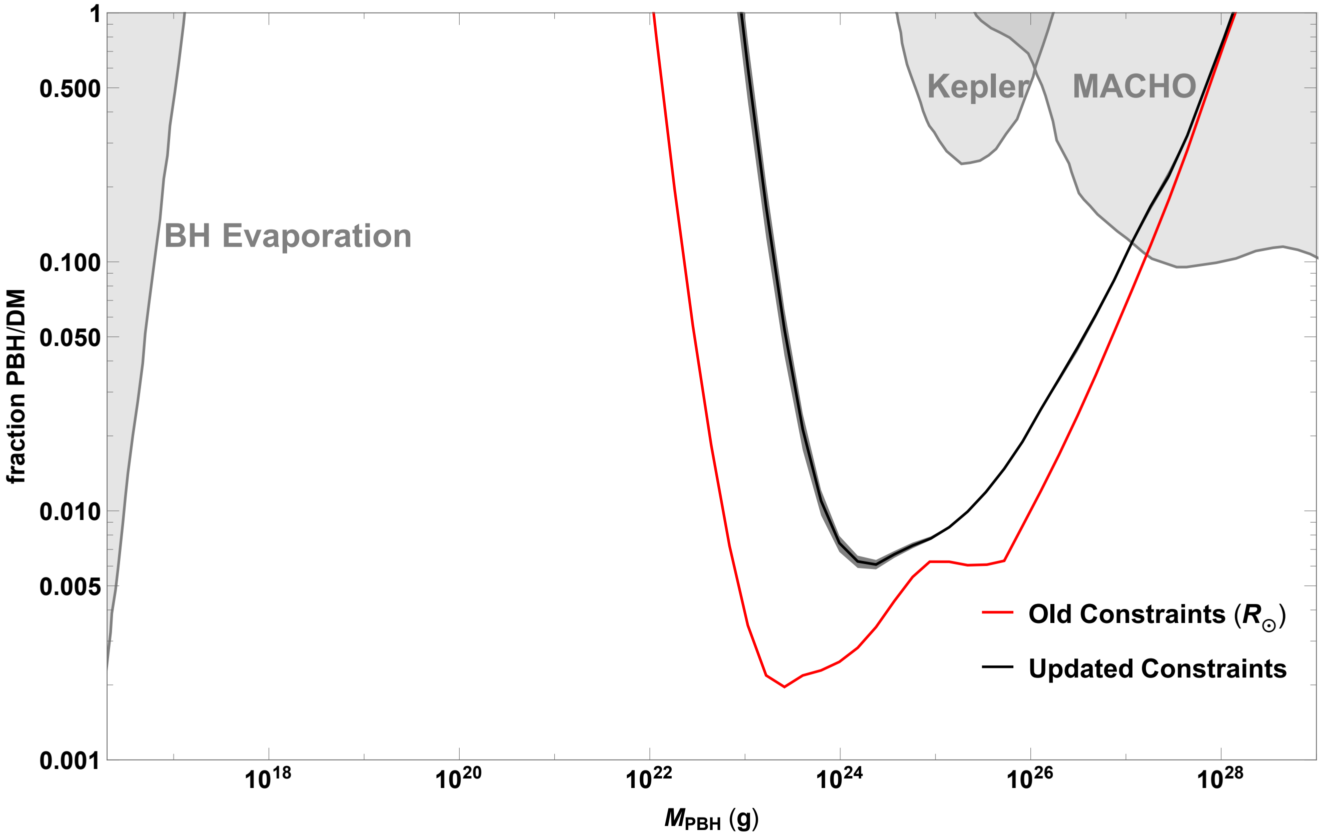
Allowed region
assuming 6 ultra-short events are
due to PBHs



Niikura et al.

HSC: M31

Subaru HSC observations have higher cadence than EROS/MACHO, so sensitive to shorter duration events and hence lighter compact objects. Niikura et al.



Smyth et al.

Finite size of source stars and effects of wave optics (Schwarzschild radius of BH comparable to wavelength of light) leads to reduction in maximum magnification. Sugiyama, Kurita & Takada and references therein.

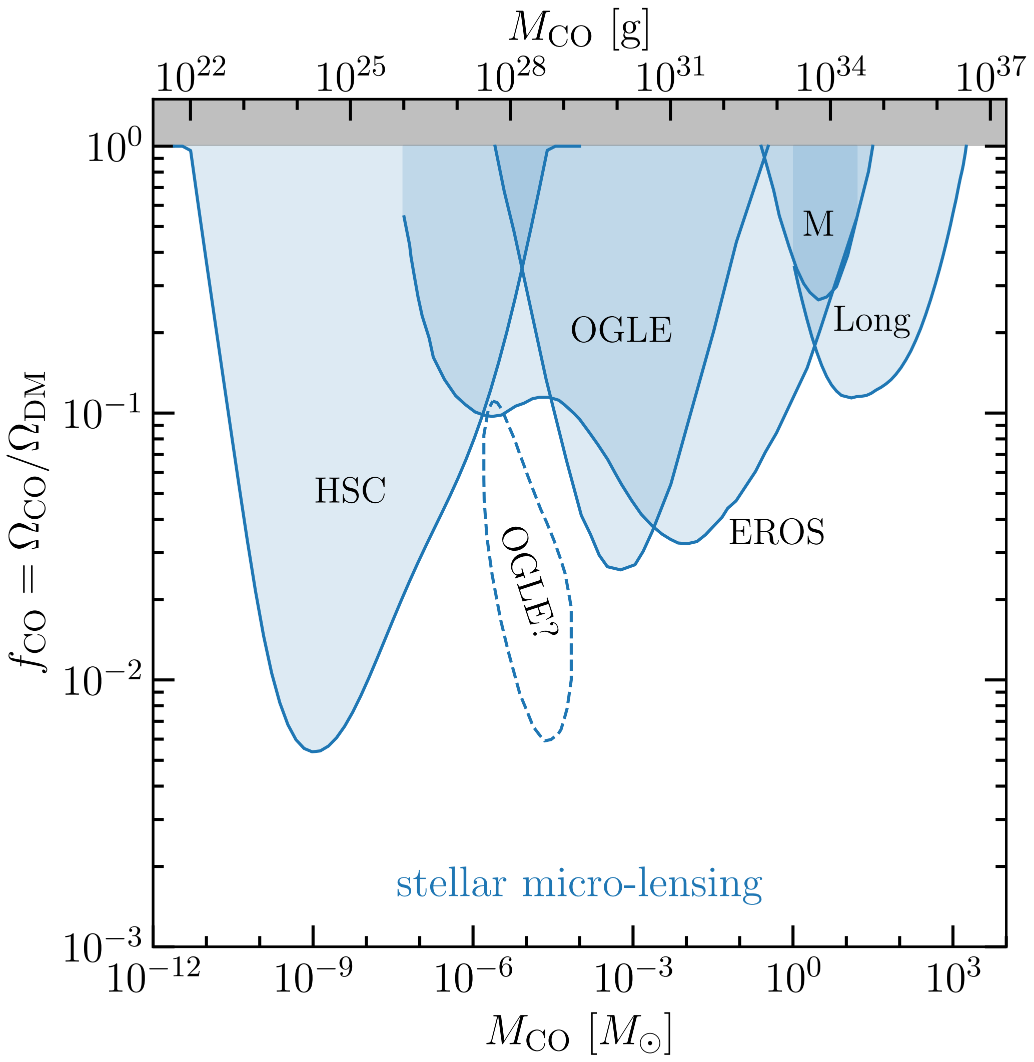
And only large stars are bright enough for microlensing to be observed. Montero-Camacho et al.; Smyth et al.

compilation of stellar microlensing constraints

M31 (HSC, Croon et al.), Galactic bulge (OGLE),LMC/SMC (MACHO, EROS, OGLE, combined long duration).

fraction of dark matter
in form of compact objects

$$f_{\mathrm{CO}} = \frac{\Omega_{\mathrm{CO}}}{\Omega_{\mathrm{DM}}}$$



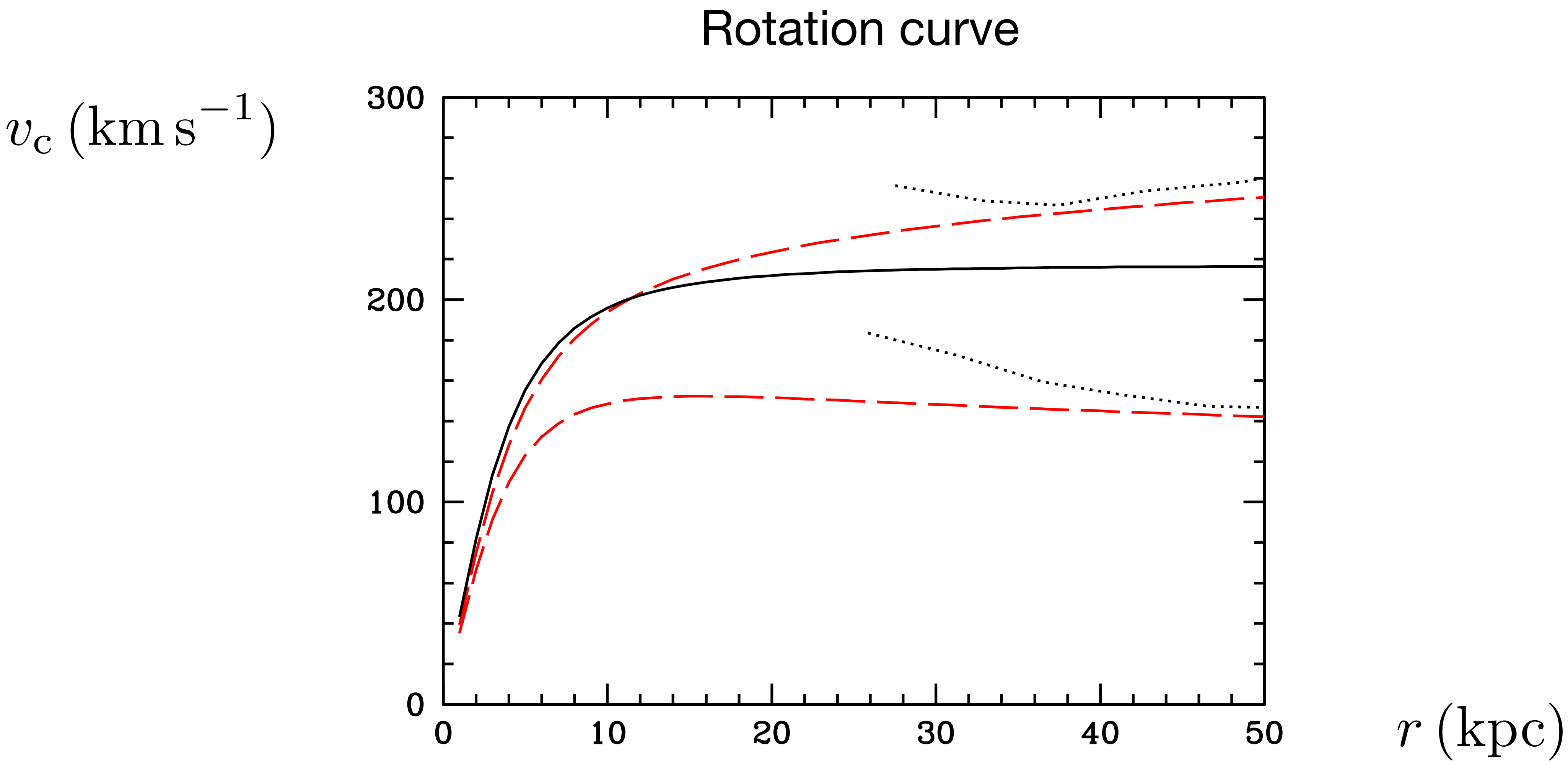
mass in grams

mass in Solar masses

Effect of MW DM distribution on constraints

Evans power law halo models: self-consistent halo models, which allow for non-flat rotation curves.

Traditionally used in microlensing studies [Alcock et al. MACHO collab.; Hawkins] since there are analytic expressions for velocity distribution.



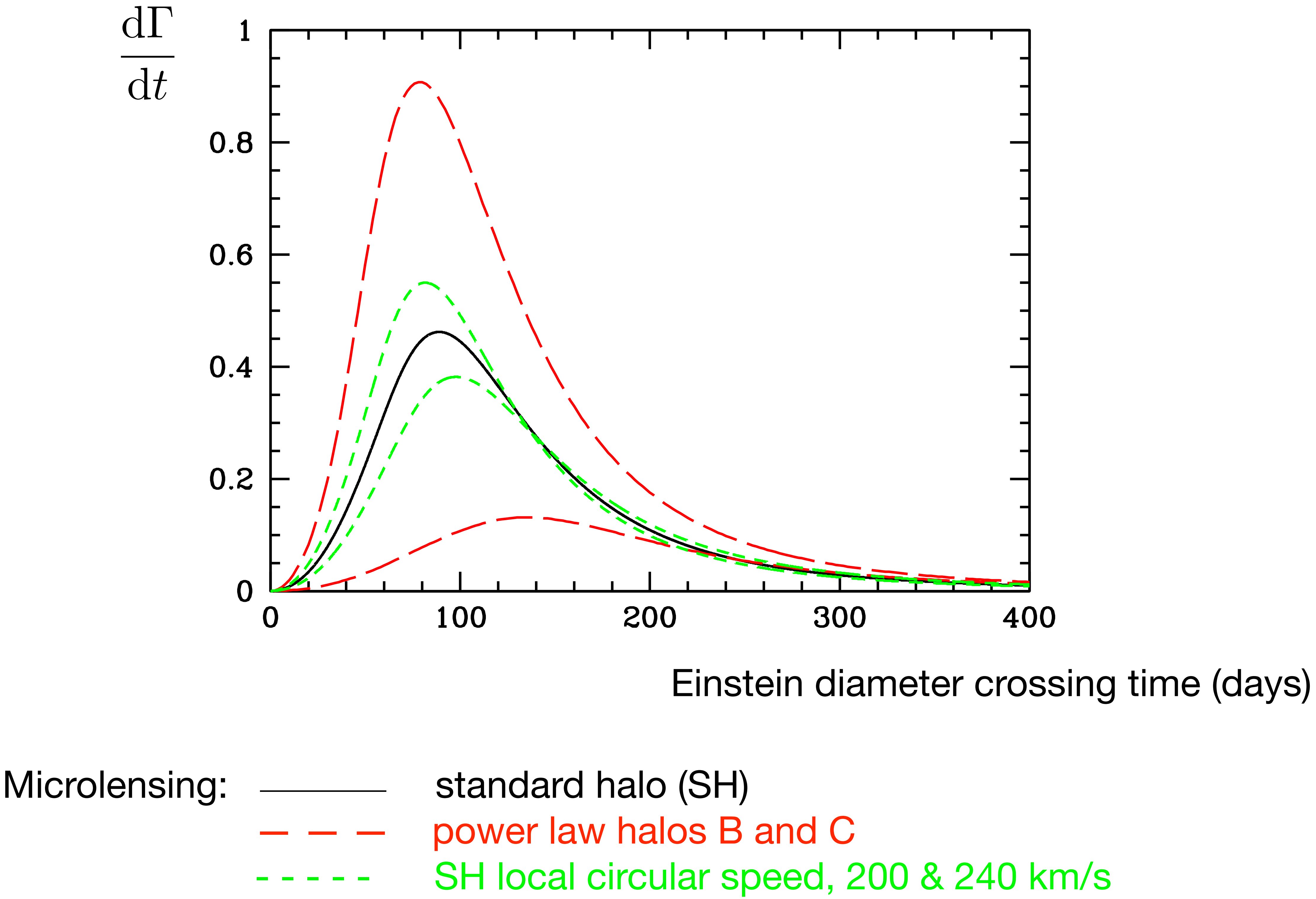
——— standard halo (SH)

— — — top: power law halo B (massive halo, rising rotation curve)
bottom: power law halo C (light halo falling rotation curve)

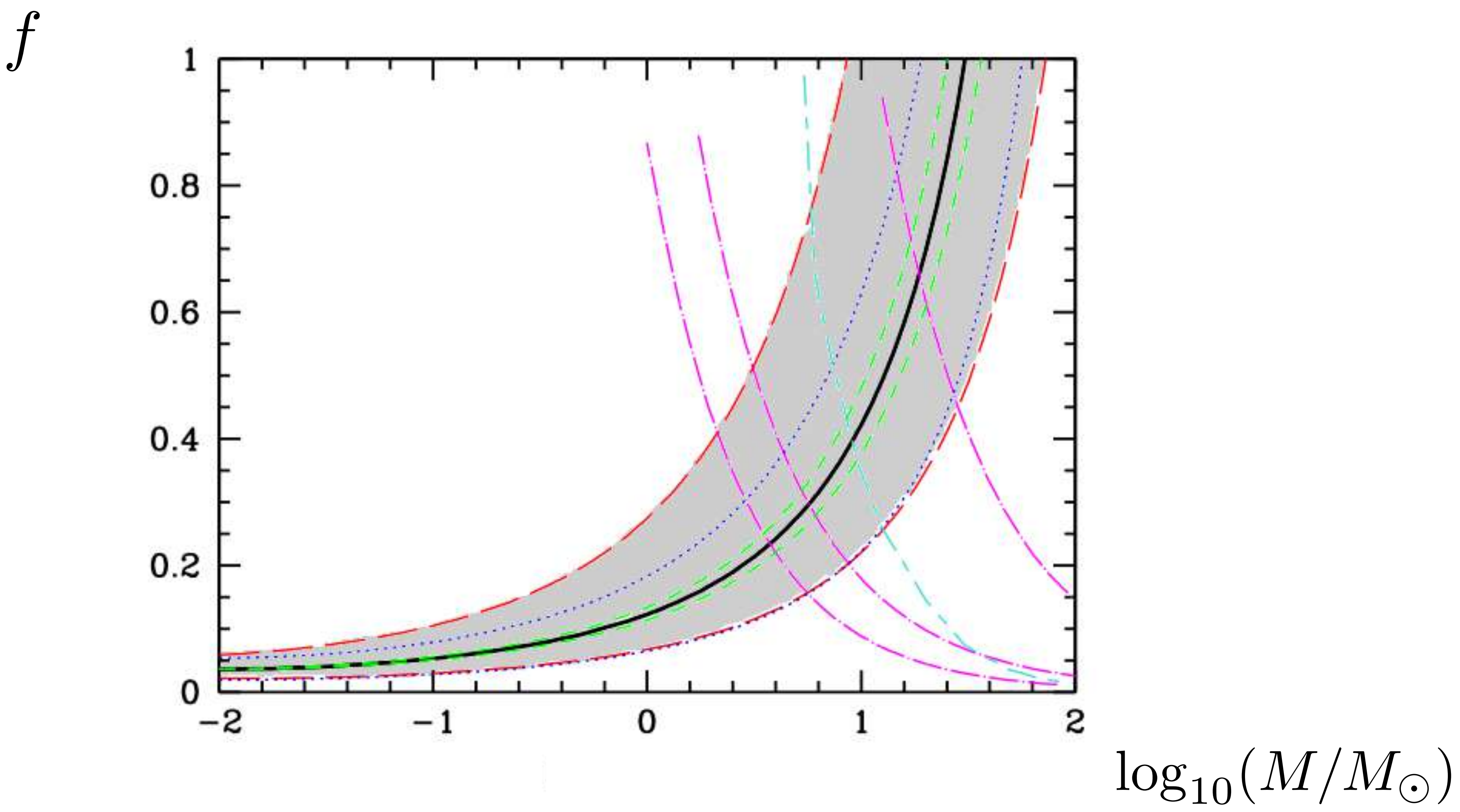
……….. envelope of MW rotation curve data [Bhattacharjee et al.]

Microlensing differential event rate

(f=1 M= 1 $M_\odot$, and perfect detection efficiency)



EROS constraints on halo fraction for delta-function MF



——— standard halo (SH)

- - - - - SH local circular speed, 200 & 240 km/s

………. SH local density, 0.005 and 0.015 $M_\odot\,\mathrm{pc}^{-3}$

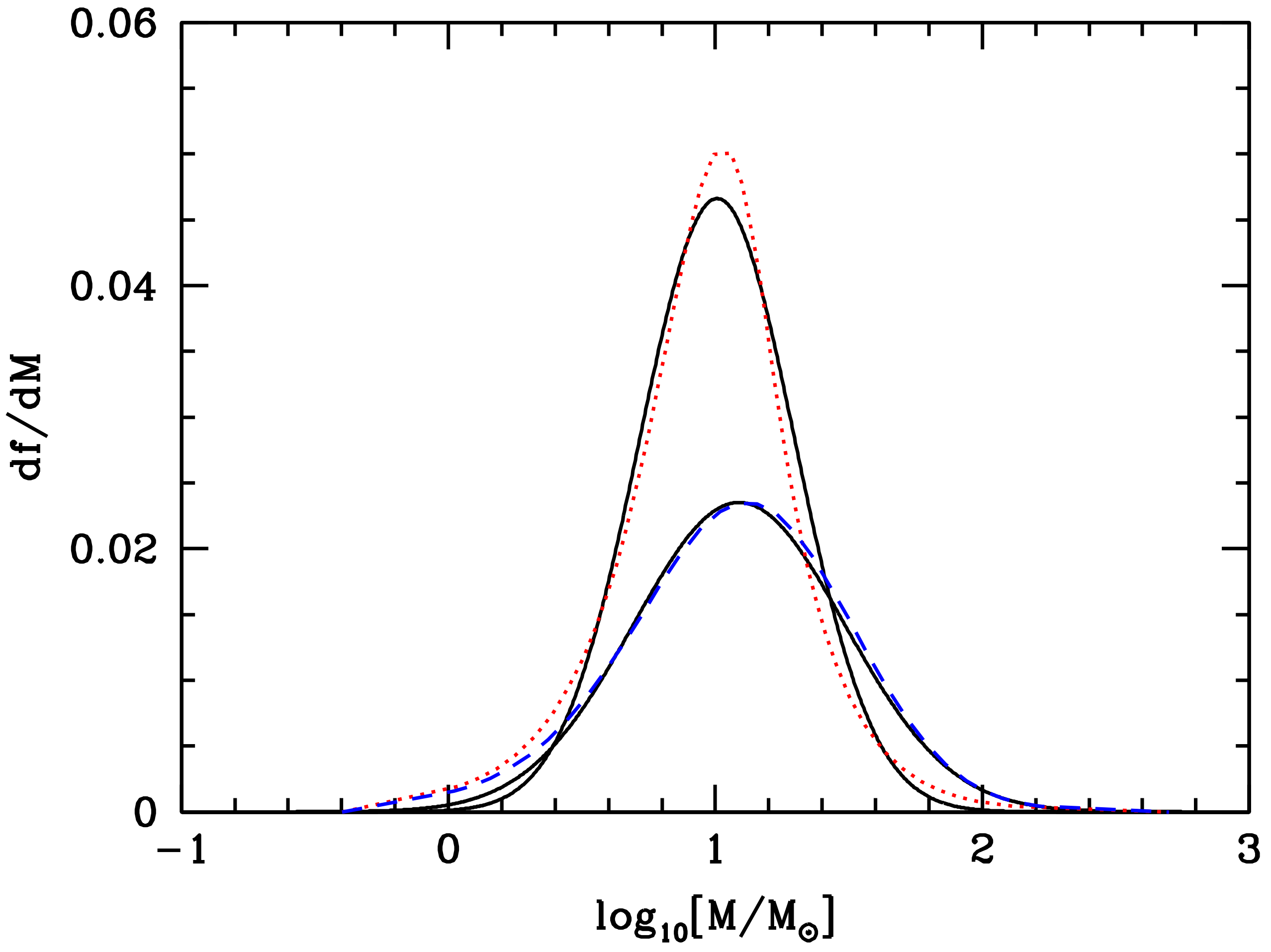
— — — power law halos C and B

——— Brandt dwarf galaxy constraints

constraints on (realistic) extended mass functions

Extended MFs produced by broad peak in power spectrum, moderately well approximated by a **log-normal distribution**: Green; Kannike et al.

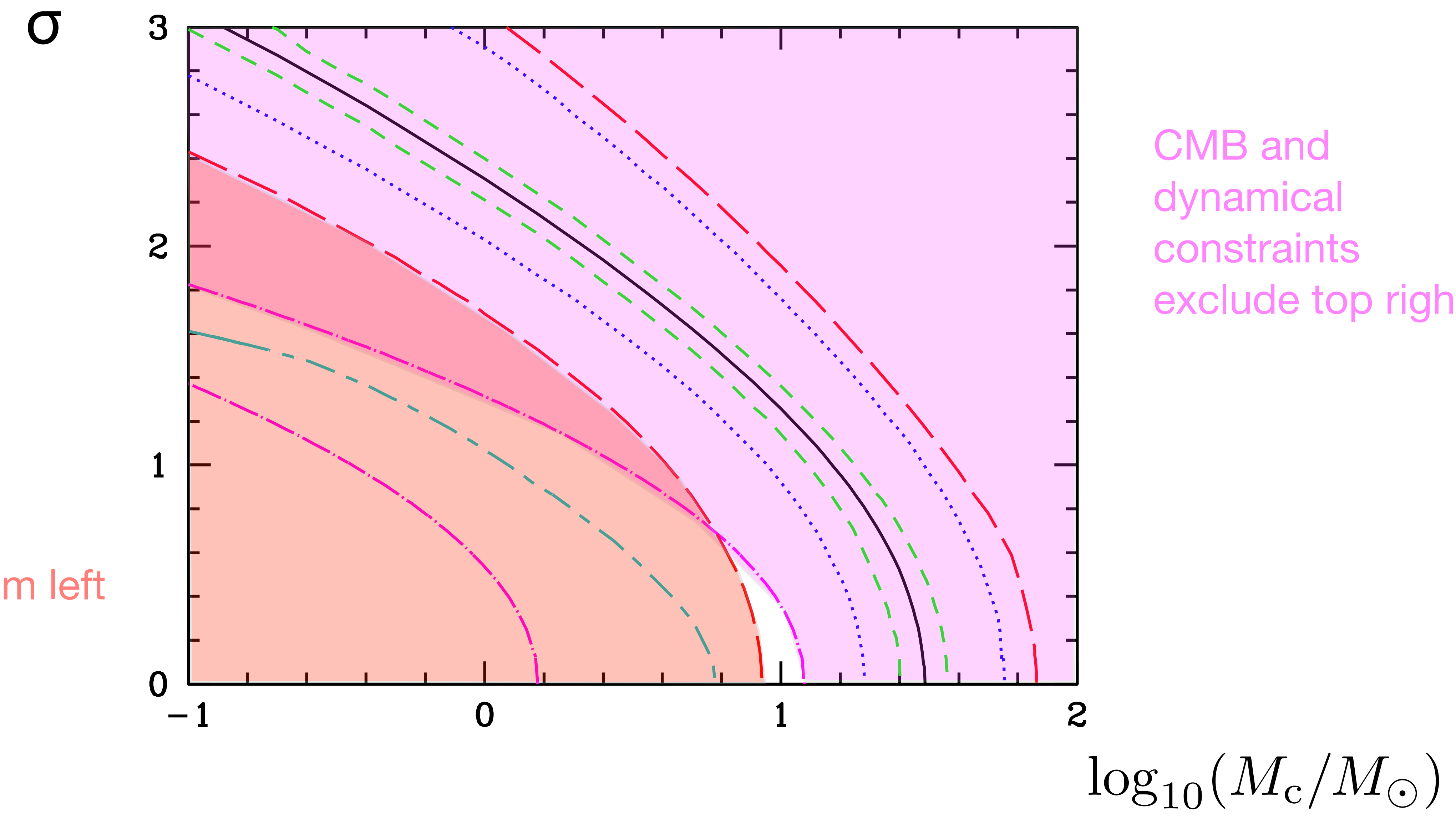
$$M\frac{\mathrm{d}n}{\mathrm{d}M} \propto \exp\left\{-\frac{\left[\log\left(M/M_{\mathrm{c}}\right)\right]^2}{2\sigma^2}\right\}$$



axion-like curvaton

running mass inflation

EROS microlensing constraints on width of log-normal MF with f=1



CMB and
dynamical
constraints
exclude top right

microlensing
constraints
exclude bottom left

—— standard halo (SH)

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………. SH local density, 0.005 and 0.015

- - - - - SH local circular speed, 200 & 240 km/s

—— Brandt dwarf galaxy constraints

Clustering of PBHs formed from collapse of large density perturbations

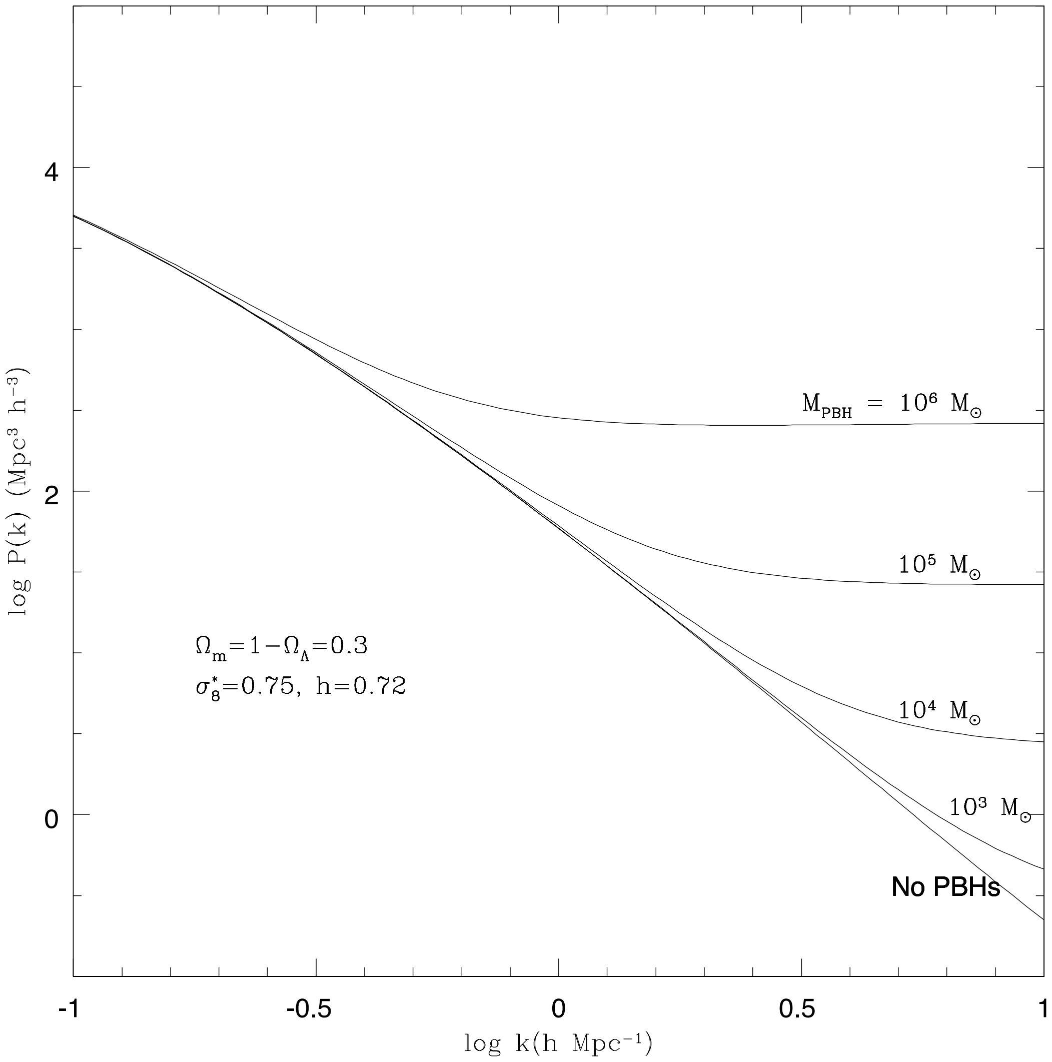
PBHs don’t form in clusters Ali-Haïmoud (previous work Chisholm extrapolated an expression for the correlation function beyond its range of validity).

However there are additional isocurvature perturbations (due to Poisson fluctuations in PBH distribution) and PBH clusters form shortly after matter-radiation equality.
Afshordi, Macdonald & Spergel; Inman & Ali-Haïmoud; Jedamzik

power spectrum

$P(k)$

$(\propto k^{n_s})$



Afshordi, Macdonald & Spergel

increasing PBH mass

no PBHs

k = comoving wavenumber

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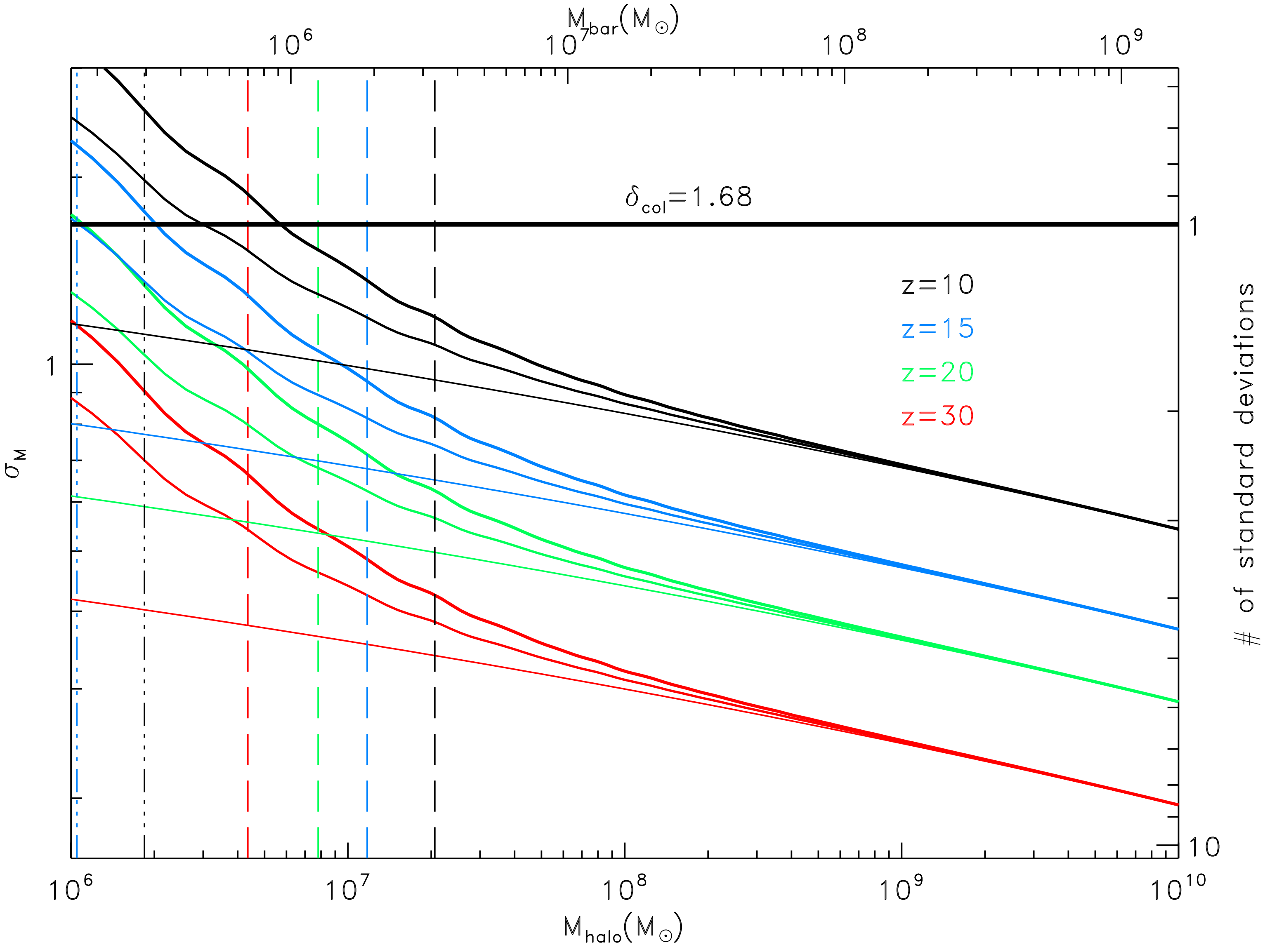


mass variance

threshold for collapse

z= 10, 15, 20, 30

$M_{PBH}=30\ M_\odot$

$M_{PBH}=15\ M_\odot$

no PBHs

halo mass

Kashlinksy

Approximate analytic calculation

c.f. Afshordi, Macdonald & Spergel; Jedamzik

PBH DM has additional isocurvature perturbations due to Poisson fluctuations in their distribution:

$$\delta(N) = \frac{\Delta N}{N} = \frac{1}{\sqrt{N}}$$

growth factor for isocurvature perturbations:

$$D(a) \approx \left(1 + \frac{3}{2}\frac{a}{a_{\rm eq}}\right)$$

spherical top hat collapse:

collapse occurs when:

$$D(a_{\rm col})\delta(N) = \delta_{\rm critical} \approx 1.69$$

final halo/cluster density:

$$\rho_{\rm cl} \approx 178 \rho_{\rm DM}(a_{\rm coll})$$

radius of cluster:

$$r_{\rm cl} \approx 0.01 \left(\frac{M_{\rm PBH}}{M_{\odot}}\right)^{1/3} N^{5/6}\,{\rm pc}$$

For $M_{\rm PBH} = M_{\odot}$, N=10 (100) clusters form at $z_{\rm coll}$ ≈1200 (320) and have $r_{\rm cl}$≈ 0.06 (0.5) pc.

N-body simulations

Inman & Ali-Haïmoud

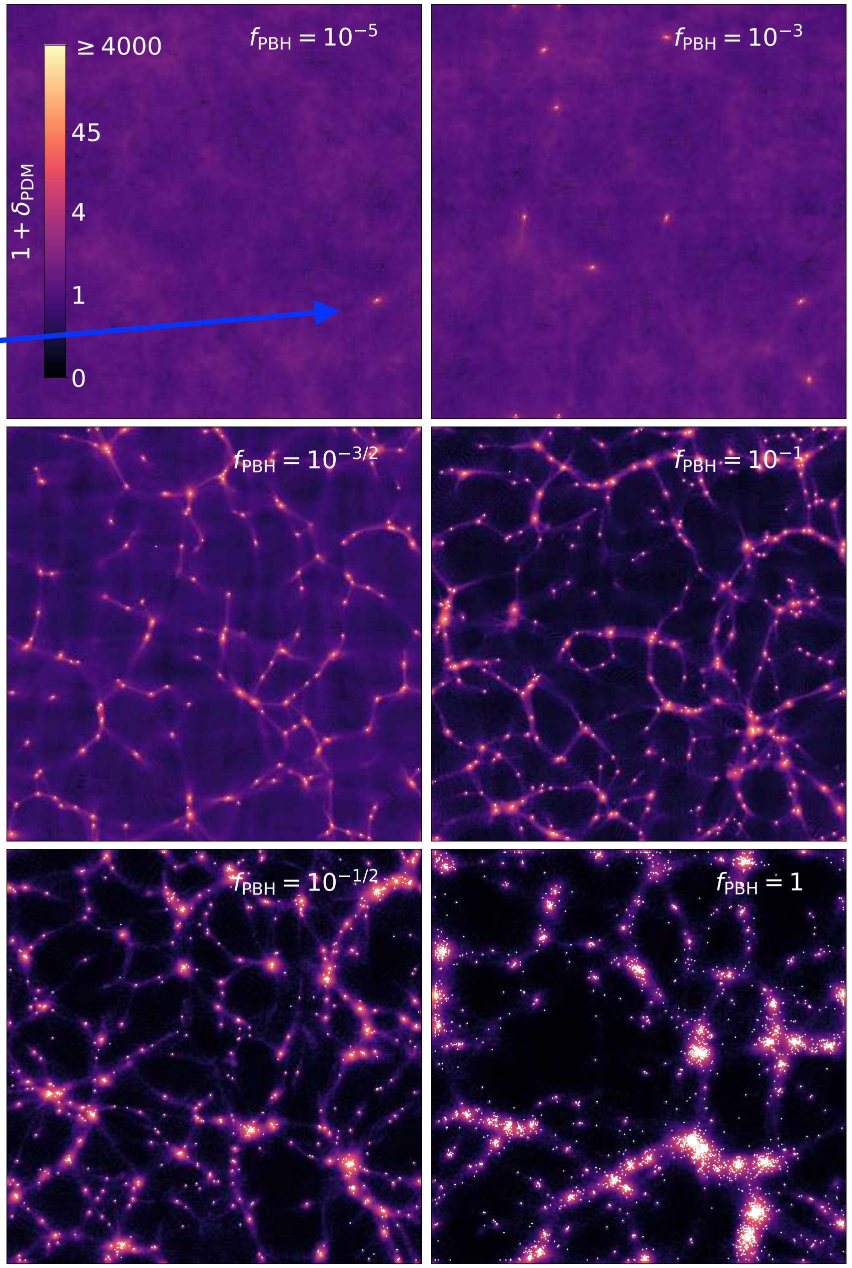
Simulate a L = 30 h^{-1} kpc box, with $M_{\rm PBH} = 20 h^{-1} M_\odot$ from radiation domination to z = 99, for $f_{\rm PBH}$ = 1 and also $f_{\rm PBH}$ < 1 + particle dark matter.

matter field at z=100

$f_{\rm PBH} = 10^{-5}$

$f_{\rm PBH} = 10^{-3}$

a PBH!



$f_{\rm PBH} = 10^{-3/2}$

$f_{\rm PBH} = 10^{-1}$

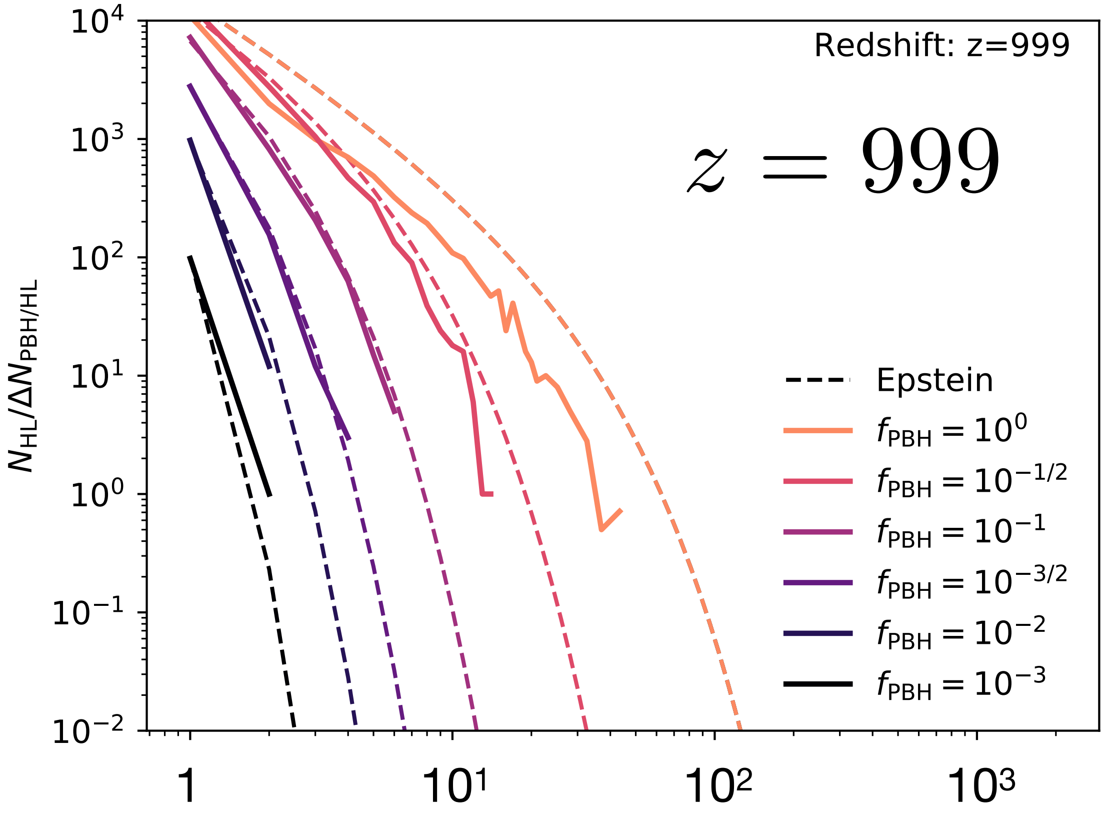
$f_{\rm PBH} = 10^{-1/2}$

$f_{\rm PBH} = 1$

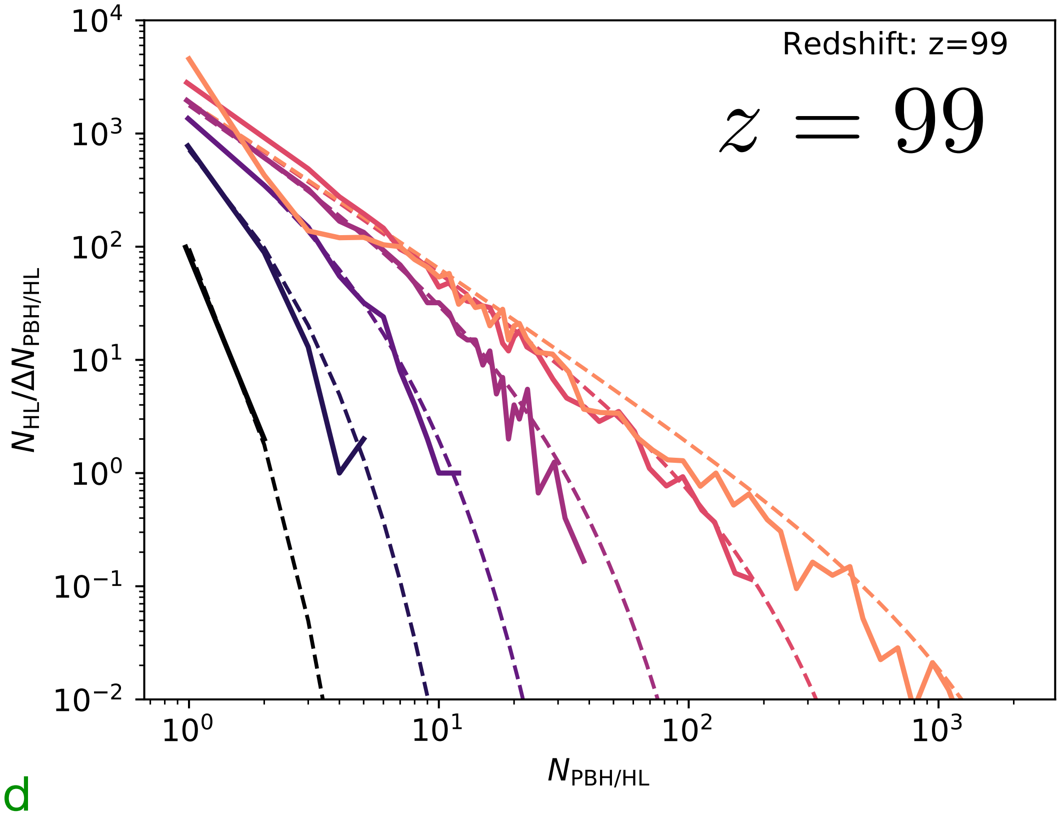
Inman & Ali-Haïmoud

Clusters containing small numbers of PBHs always most abundant, but abundance of clusters containing large numbers of PBHs increases with time.

halo mass function (number of halos containing a given number of PBHs)



decreasing f_{PBH}



Inman & Ali-Haïmoud

Evolution of PBH clusters (and in particular PBH binaries) through to the present day is a challenging open problem. e.g. Jedamzik; Trashorras et al….

Clusters containing $\lesssim 10^3$ PBHs will evaporate by present day. Afshordi, Macdonald & Spergel; Jedamzik

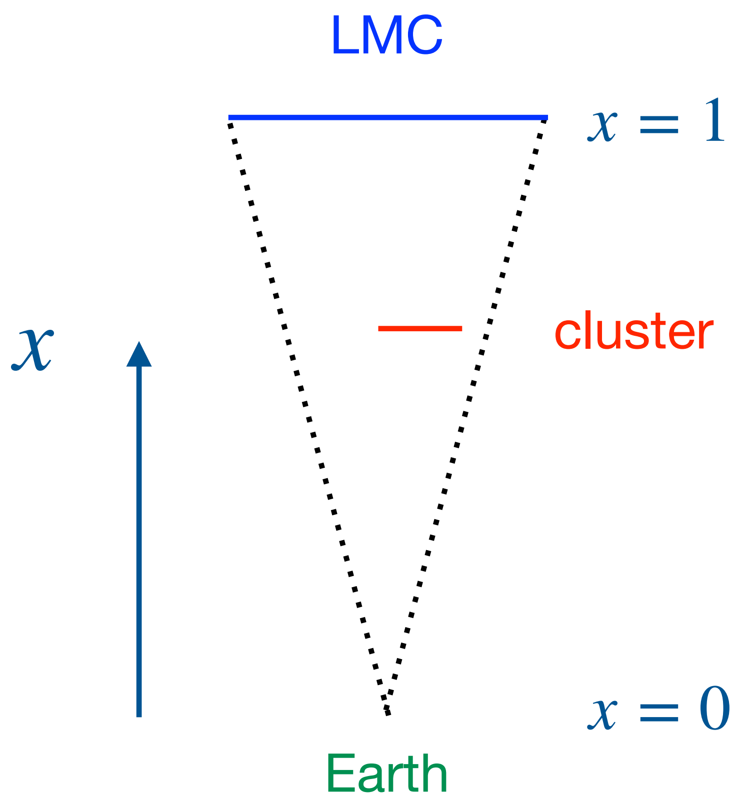
Effect of clustering on LMC microlensing constraints

Gorton & Green (see also Petač, Lavalle & Jedamzik)

For PBHs formed from collapse of density perturbations during radiation, clusters are sufficiently extended that PBHs lens individually (separation of PBHs $\gg R_E$).

Microlensing from a single cluster:

looking down on line of sight



LMC

$x = 1$

x

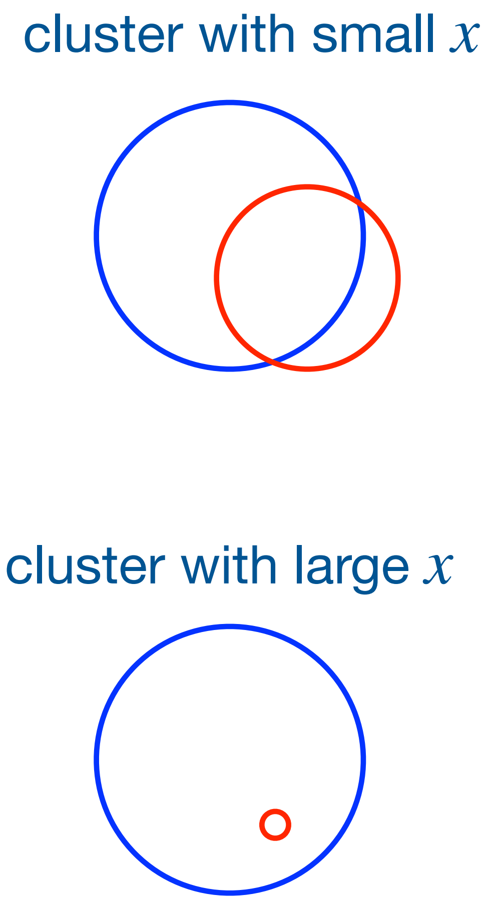
cluster

$x = 0$

Earth

x = fractional
line of sight dist

looking along line of sight



cluster with small x

cluster with large x

probability of finding a cluster at line of sight distance x is proportional to cross sectional area of ‘cone’ to LMC $\propto x^2$

all the PBHs in a given cluster cause events with the same duration:

$$\hat{t} = \frac{2R_{\rm E}(x)}{v} \propto [x(1-x)]^{1/2}$$

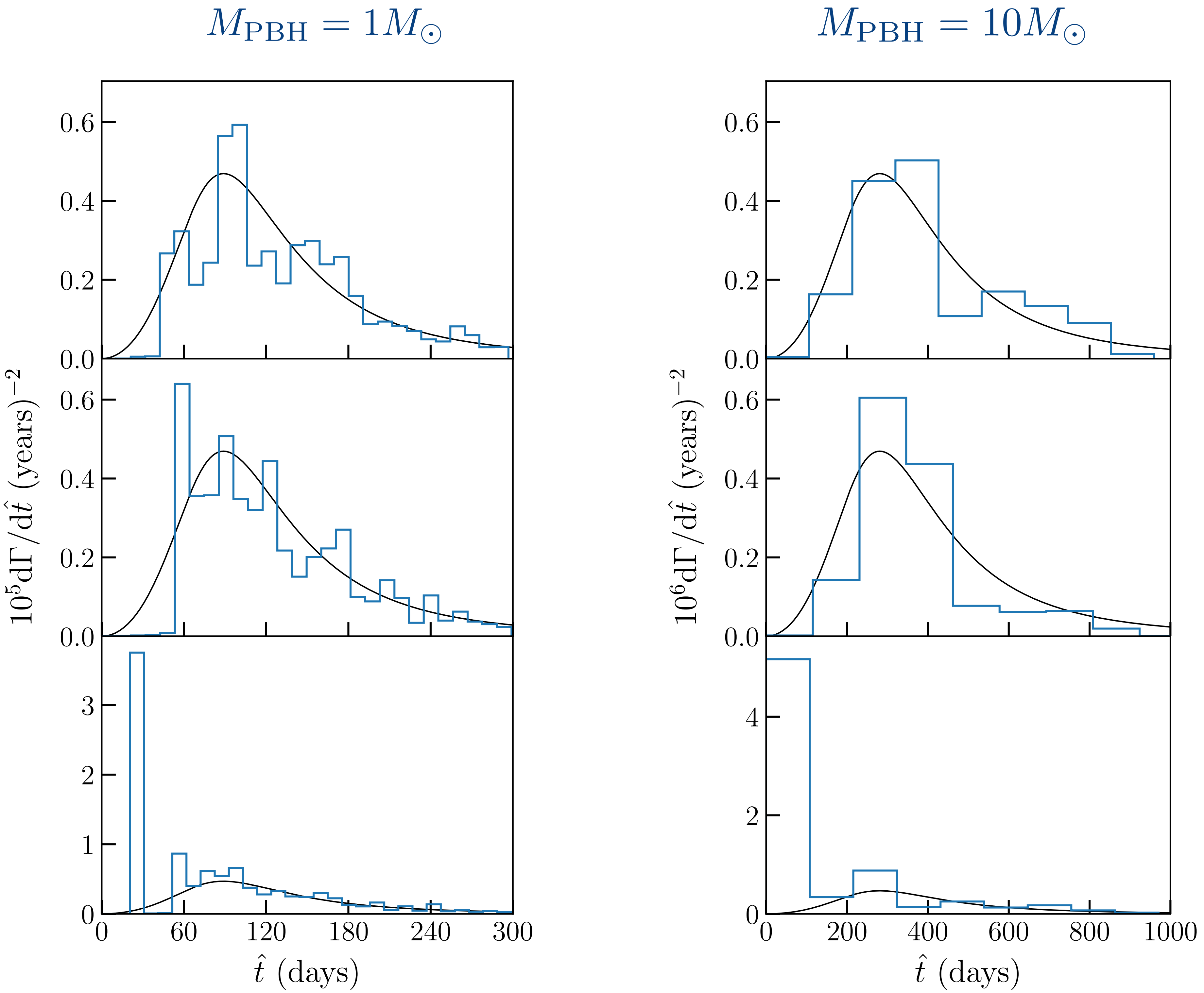
rate at which cluster causes microlensing events is proportional to solid angle subtended by cluster times Einstein radius:

$$\propto \frac{[x(1-x)]^{1/2}}{x^2}$$

Close clusters (small x) are rare, but if one intersects the line of sight if produces short duration events at a high rate.

LMC microlensing differential event rate for clustered DM and standard smooth DM

all of the DM in clusters containing $N_{cl}=10^6$ PBHs with mass



Typical realisations

No close cluster.
Deficit of short
duration events.

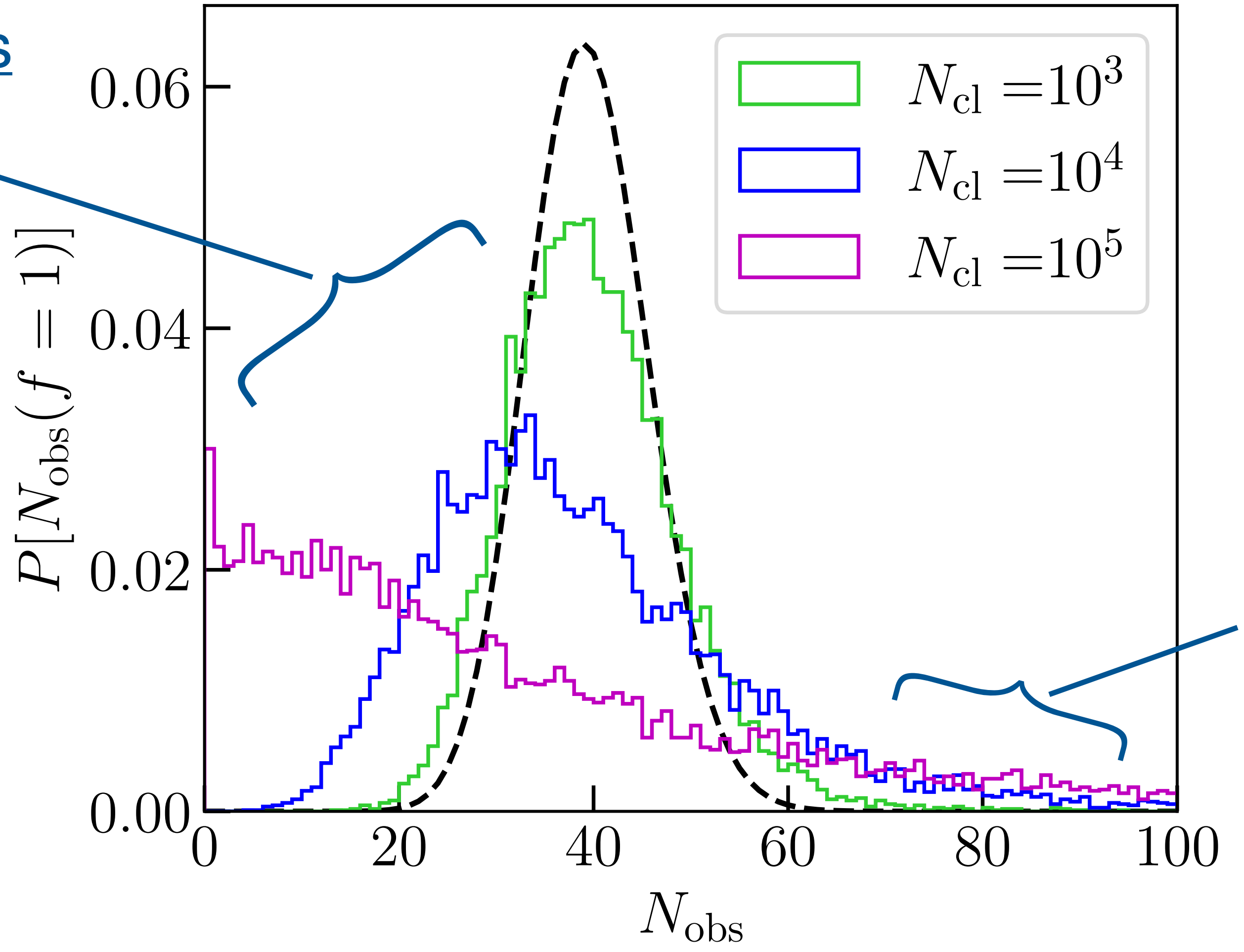
Rare realisation

Close cluster.
Excess of short
duration events.

Probability distribution of number of events in a long duration microlensing survey if all of the DM is in PBHs clusters containing $N_{\rm cl}$ PBHs with mass $M_{\rm PBH} = 10^3 M_\odot$

Typical realisations

No close cluster.
Deficit of events.



Rare realisations

Close cluster.
Excess of events.

Change in constraints is negligible apart (possibly) from at largest $M_{\rm PBH}$ probed by stellar microlensing.

(**if** all of the DM is in PBH clusters containing $N_{\rm cl} = 10^3$ PBHs with mass $M_{\rm PBH} = 10^3 M_\odot$ constraint on $f_{\rm PBH}$ from long duration microlensing survey weakens from 0.076 to 0.096).

Petaĉ, Lavalle & Jedamzik; Gorton & Green.

Summary

Primordial Black Holes can form in the early Universe, for instance from the collapse of large density perturbations during radiation domination.

To produce an interesting number of PBHs, amplitude of perturbations must be ~3 orders of magnitude larger on small scales than on cosmological scales.

There are numerous constraints on the abundance of PBHs from gravitational lensing, their evaporation, dynamical effects, accretion and other astrophysical processes.

Solar mass PBHs probably can't make up all of the dark matter, but lighter, (10^{17}-10^{22})g, PBHs could.

Uncertainties in the dark matter distribution (density profile and clustering) have a non-negligible effect on stellar microlensing constraints on multi-Solar mass compact objects (but not sufficient for multi-Solar mass PBHs formed from the collapse of large density perturbations to make up all of the DM).

Back-up slides

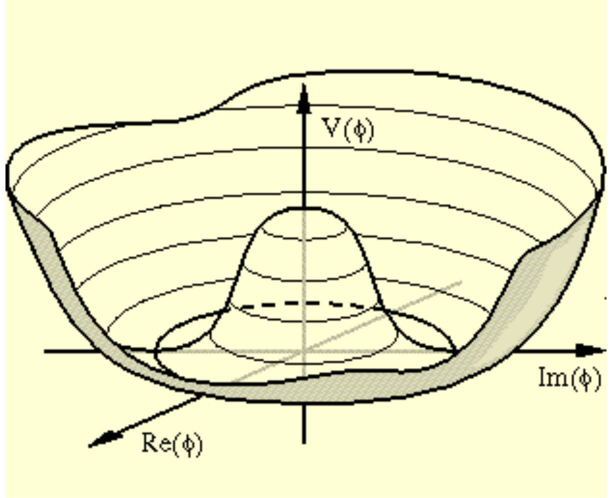
PBH formation: (some) other mechanisms

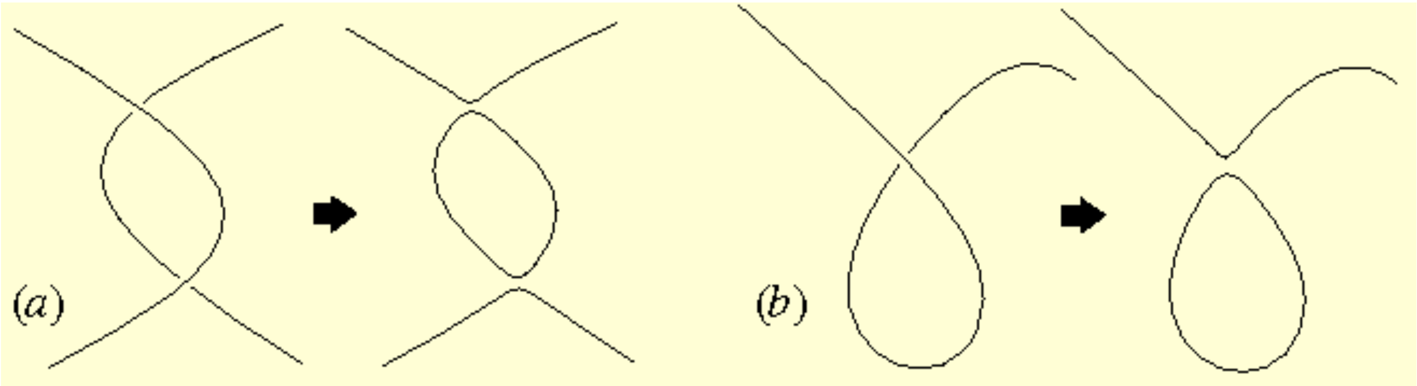
PBH formation: (some) other mechanisms

Collapse of cosmic string loops Hawking; Polnarev & Zemboricz;

Cosmic strings are 1d topological defects formed during symmetry breaking phase transition.

String intercommute producing loops.





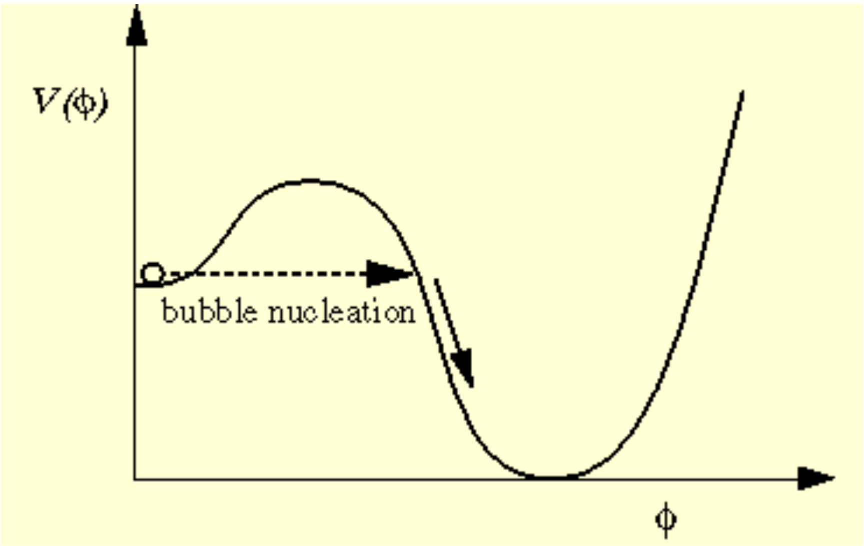
Small probability that loop will get into configuration where all dimensions lie within Schwarzschild radius (and hence collapse to from a PBH with mass of order the horizon mass at that time).

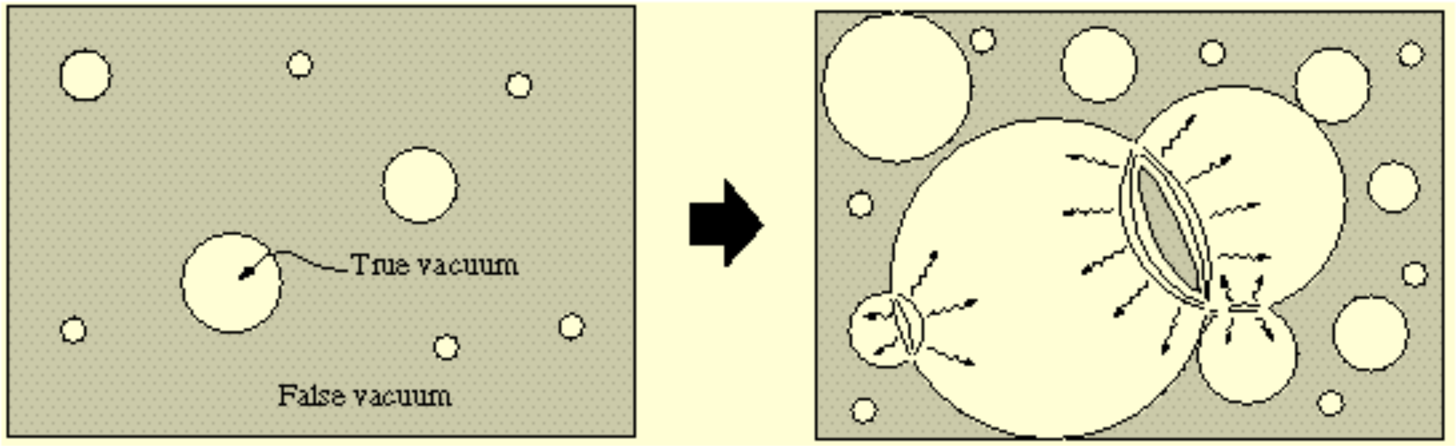
Probability is time independent, therefore PBHs have extended mass spectrum.

Bubble collisions

Hawking

1st order phase transitions occur via the nucleation of bubbles.





PBHs can form when bubbles collide (but bubble formation rate must be fine tuned).

PBH mass is of order horizon mass at phase transition.

Fragmentation of inflaton scalar condensate into oscillons/Q-balls

Cotner & Kusenko; Cotner, Kusenko & Takhistov

Scalar field with flat potential forms condensate at end of inflation, fragments into lumps (oscillons/Q-balls) which can come to dominate universe and have large density fluctuations that can produce PBHs.

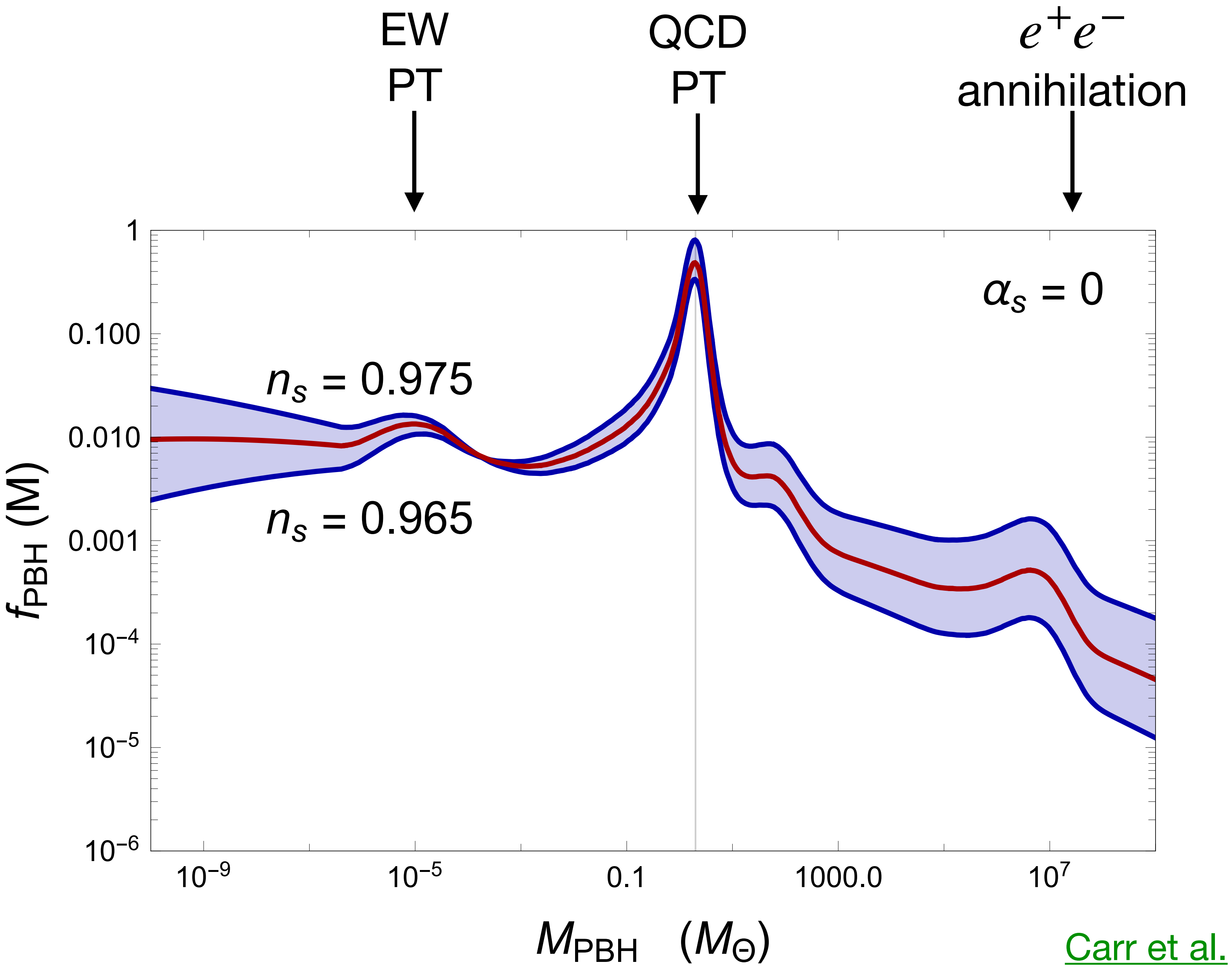
Mass smaller than horizon mass and spin can be of order 1.

ii) effect of phase transitions

Decrease in pressure leads to reduction in threshold for collapse and hence increase in PBH abundance

e.g. the QCD phase transition when the horizon mass is ~Solar mass. Jedamzik

fraction of DM
in PBHs



Carr et al.

n.b. amplitude of power spectrum A ≈ 0.02 assumed.

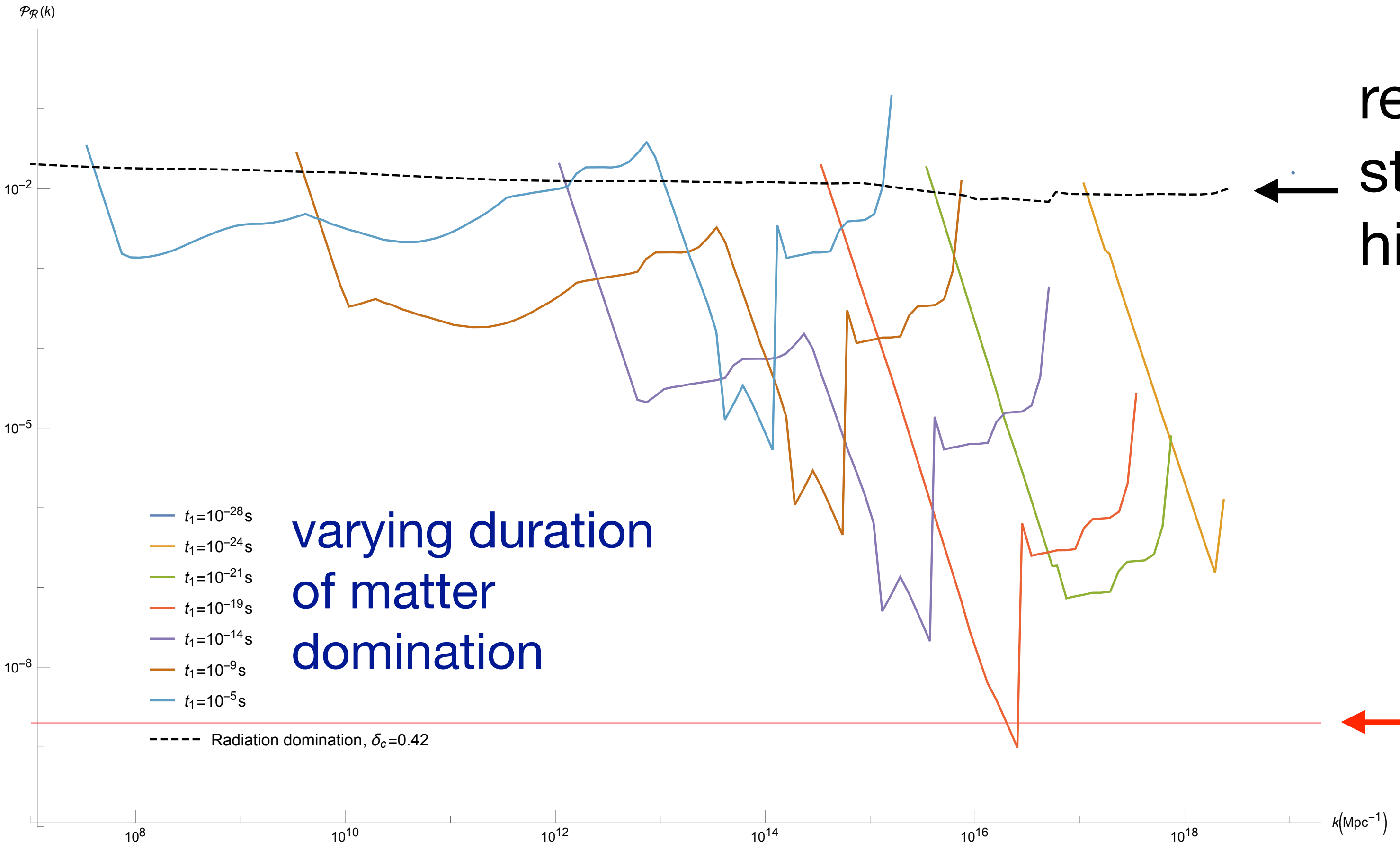
PBH formation during an early (pre nucleosynthesis) period of matter domination

During matter domination PBHs can form from smaller fluctuations (no pressure to resist collapse) in this case fluctuations must be sufficiently spherically symmetric Yu, Khlopov & Polnarev; Harada et al. and

$$\beta(M) \approx 0.056\sigma^{5(+1.5?)}$$

The required increase in the amplitude of the perturbations is reduced Georg, Sengör & Watson; Georg & Watson; Carr, Tenkanen & Vaskonen; Cole & Byrnes:

Primordial curvature perturbation power spectrum



varying duration of matter domination

required amplitude for standard cosmological history

amplitude on CMB scales

Cole & Byrnes

k

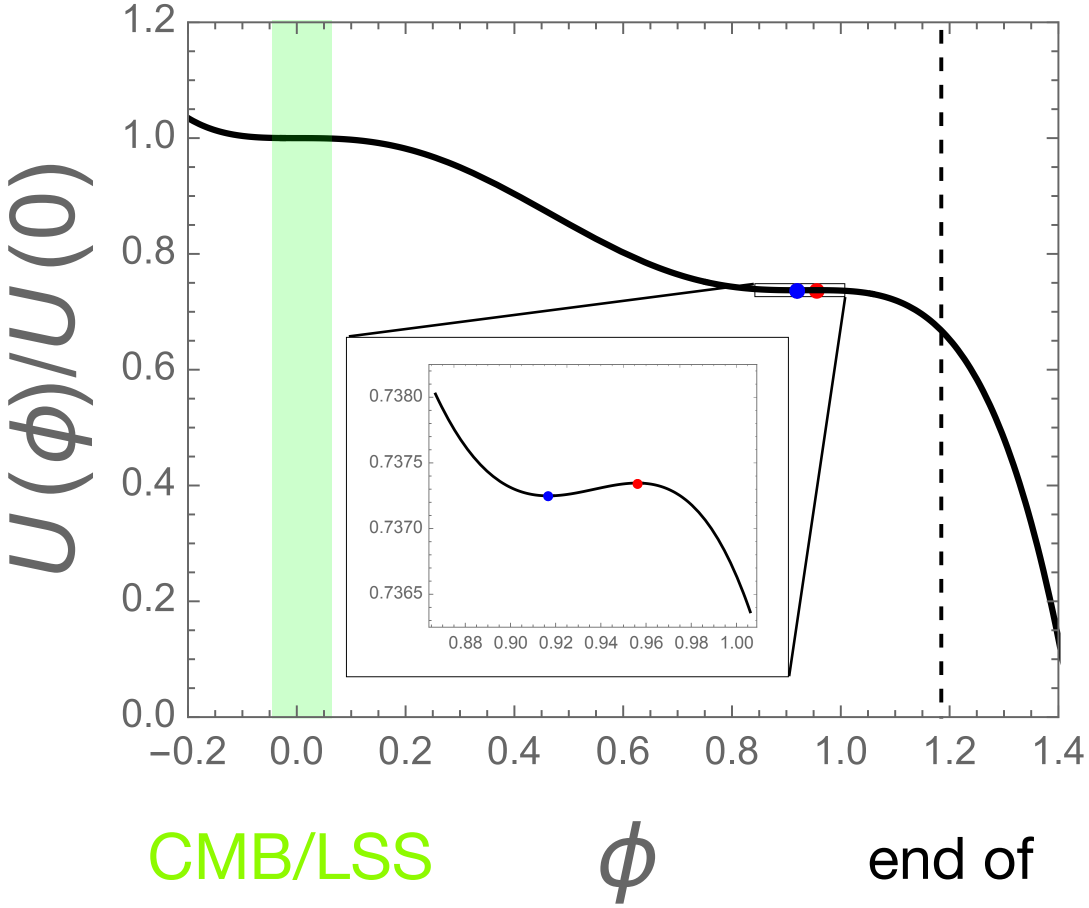
Inflation models that produce large perturbations

single field

Potential needs to be fine-tuned so that field goes past local min, but with reduced speed.

Ballesteros & Taoso; Herzberg & Yamada

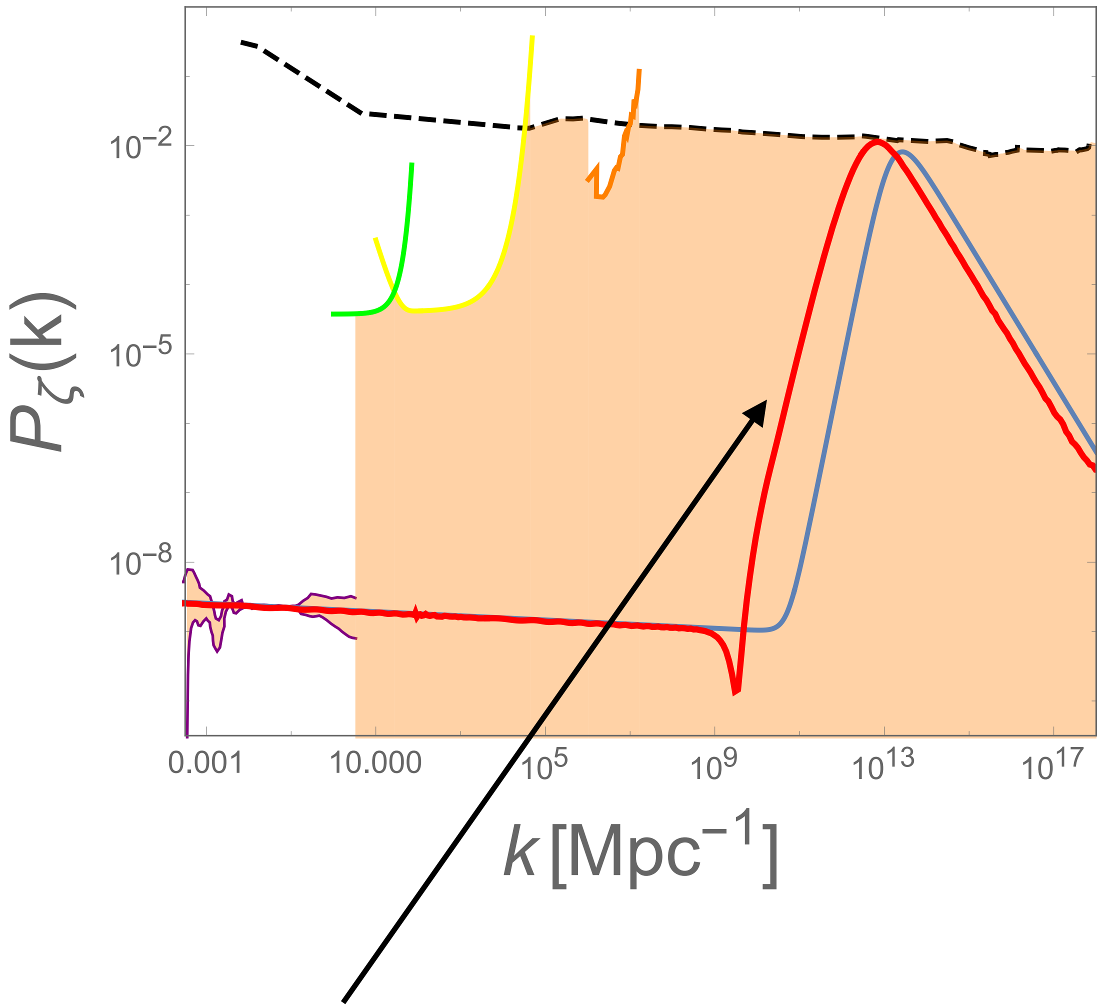
potential



CMB/LSS

end of inflation

primordial power spectrum

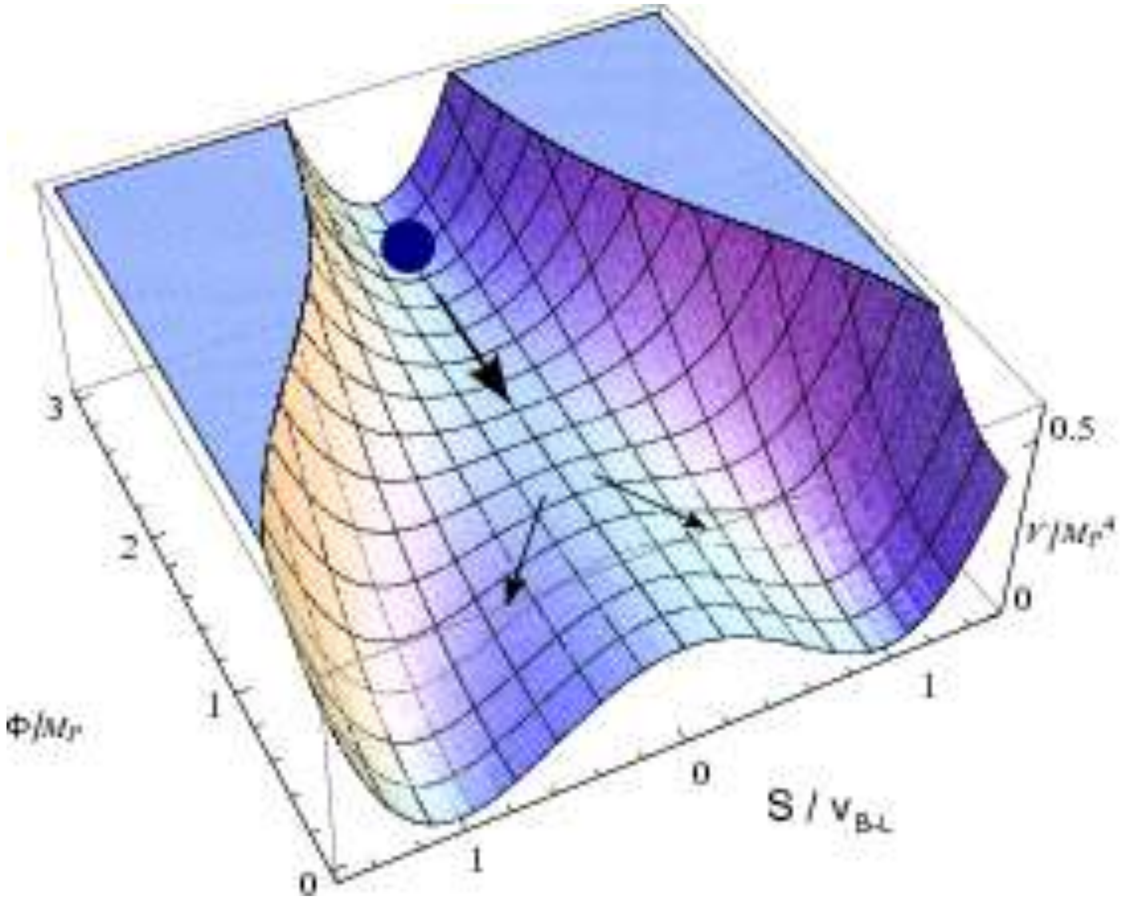


$\sim k^4$ Byrnes, Cole & Patil

multi-field models

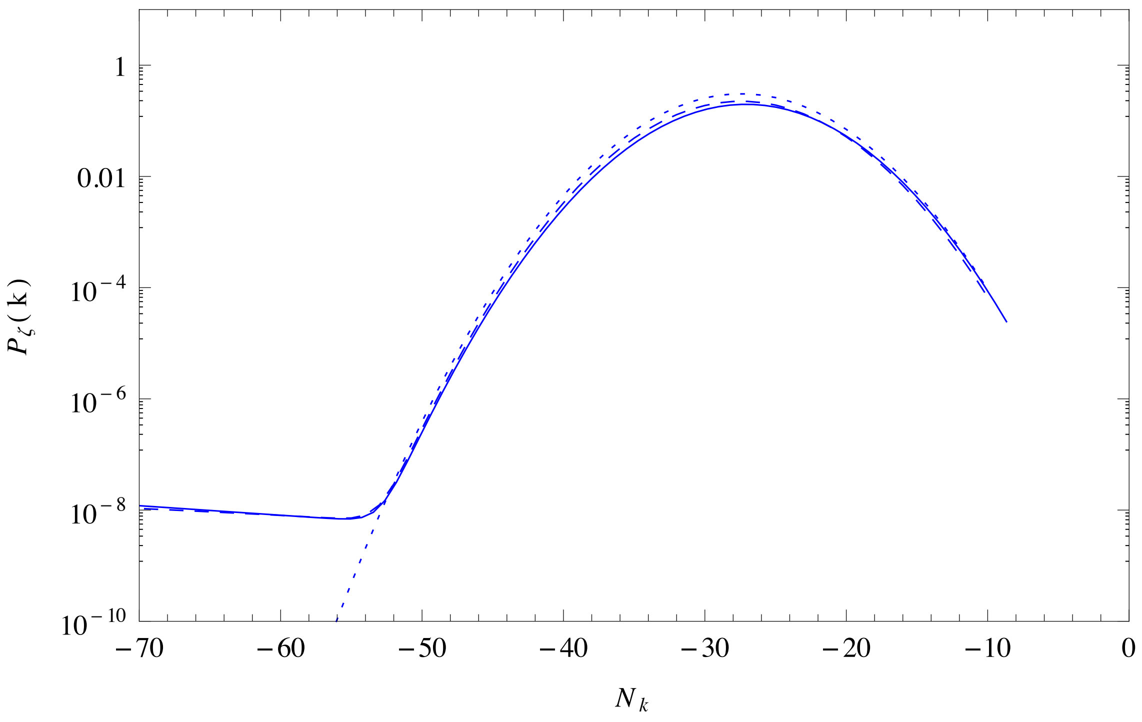
e.g. hybrid inflation with a mild waterfall transition Garcia-Bellido, Linde & Wands

potential



Buchmuller

primordial power spectrum



Clesse & Garcia-Bellido

various others for reviews see Öszoy & Tasinato; Escriva, Kuhnel & Tada

running mass, double inflation, axion-like curvaton, reduced sound speed, multi-field models with rapid turns in field space,…

axion-like curvaton

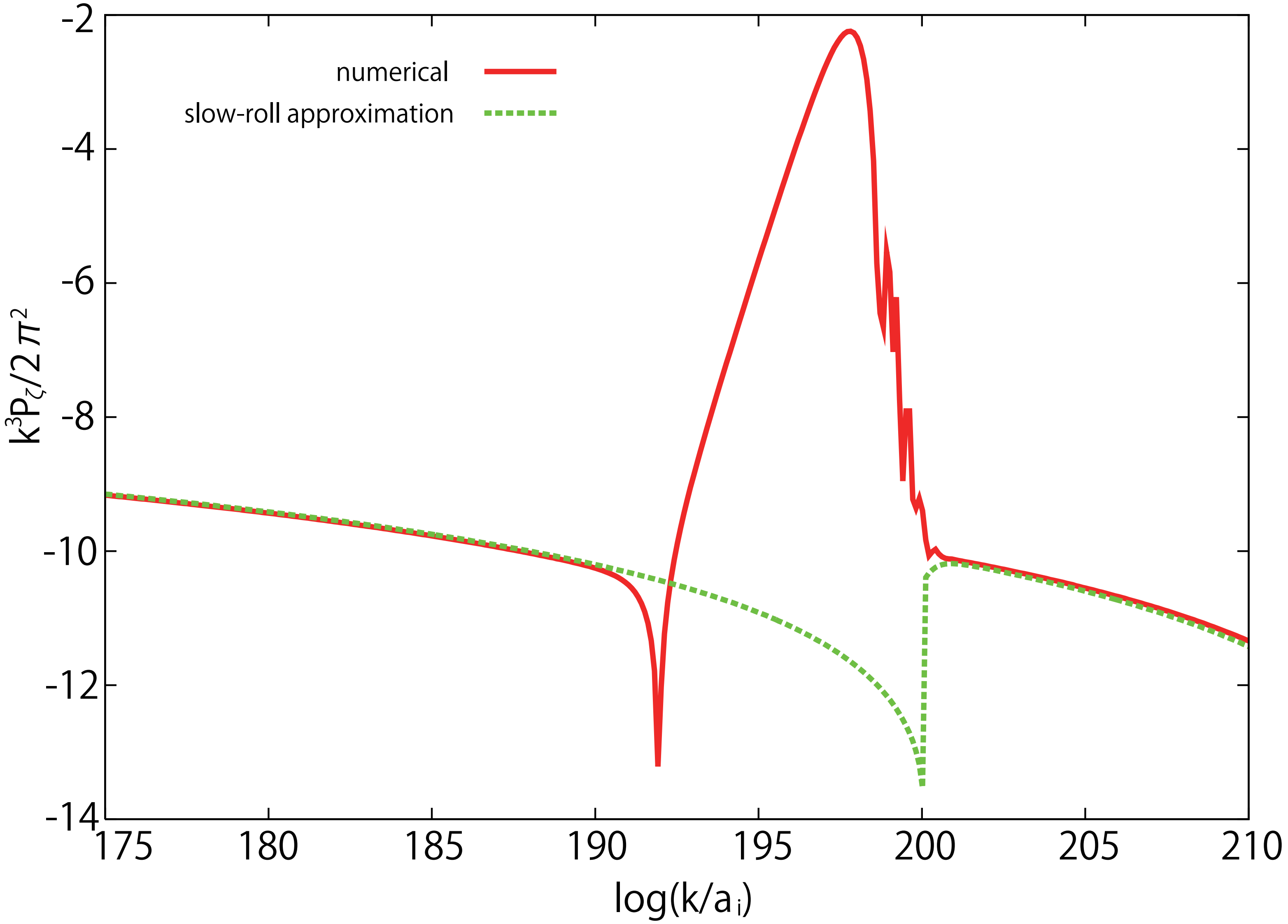
Kawasaki, Kitajima & Yanagida

Large scale perturbations generated by inflaton, small scale (PBH forming) perturbations by curvaton (a spectator field during inflation gets fluctuations and decays afterwards producing perturbations Lyth & Wands)

b) double inflation

Saito, Yokoyama & Nagata; Kannike et al.

Perturbations on scales which leave the horizon close to the end of the 1st period, of inflation get amplified during the 2nd period.



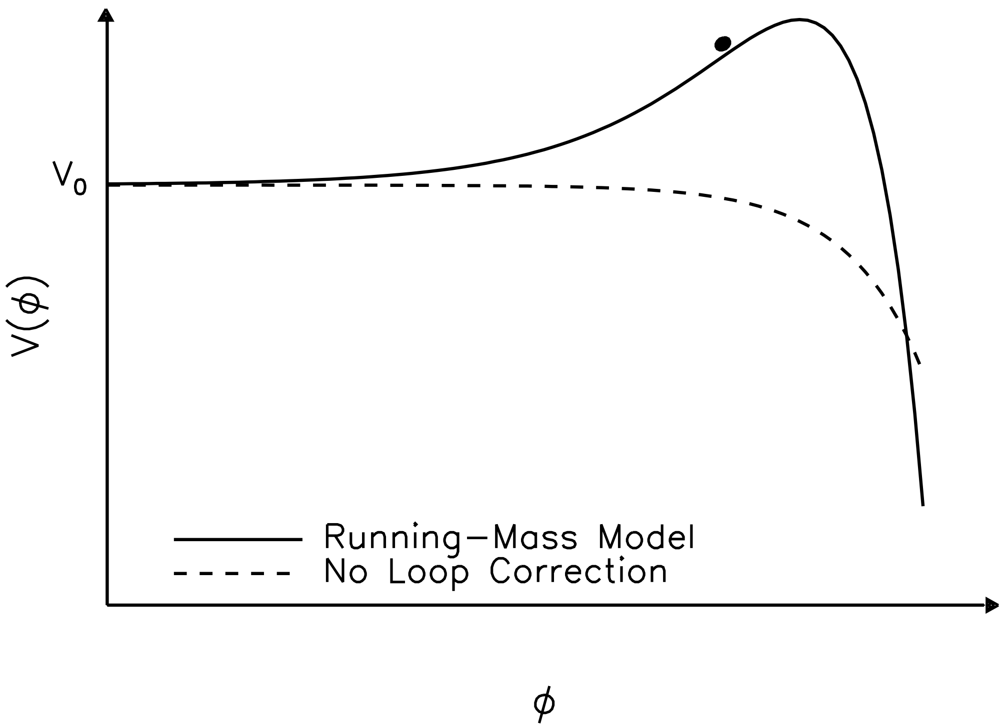
Also double inflation models where large scale perturbations are produced during 1st period, and small scale (PBH forming) perturbations during 2nd (Kawasaki et al.; Kannike et al.; Inomata et al.)

ii) monotonically increasing power spectrum

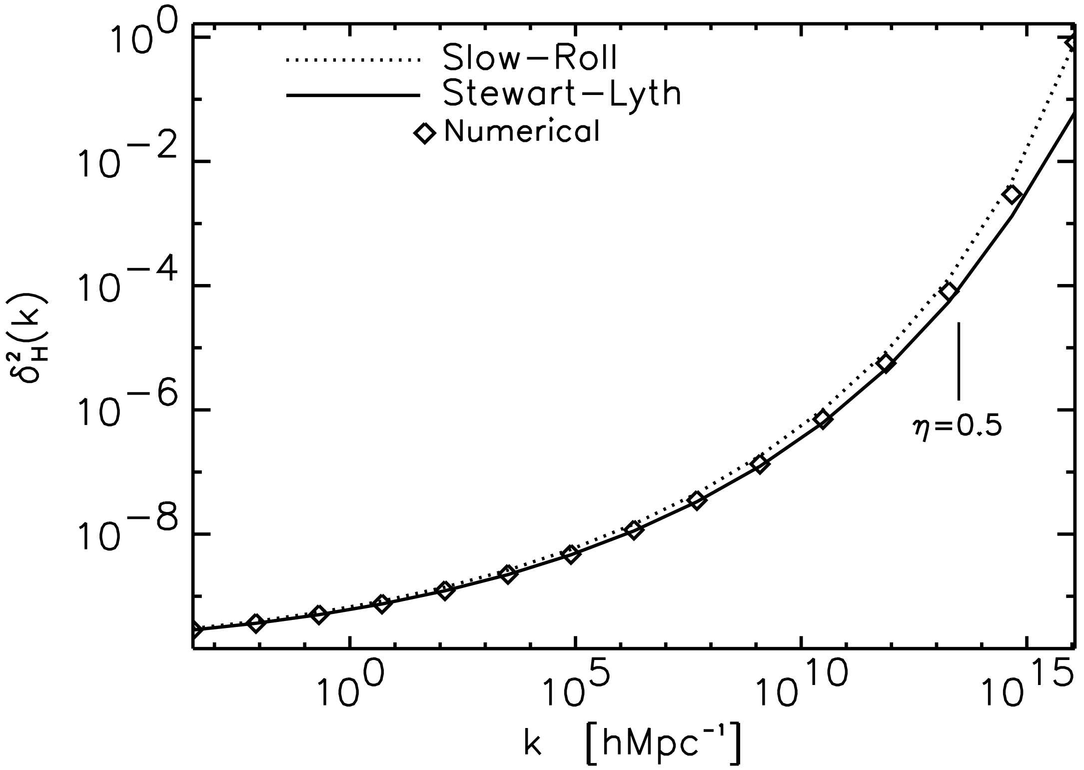
running-mass inflation Stewart

$$V(\phi) = V_0 + \frac{1}{2}m_\phi^2(\phi)\phi^2$$

potential



primordial power spectrum



Leach, Grivell, Liddle

An aside: ‘Pitfalls of a power-law parameterisation of the primordial power spectrum for primordial black hole formation’ 1805.05178

It is common to parameterise the primordial power spectrum as:

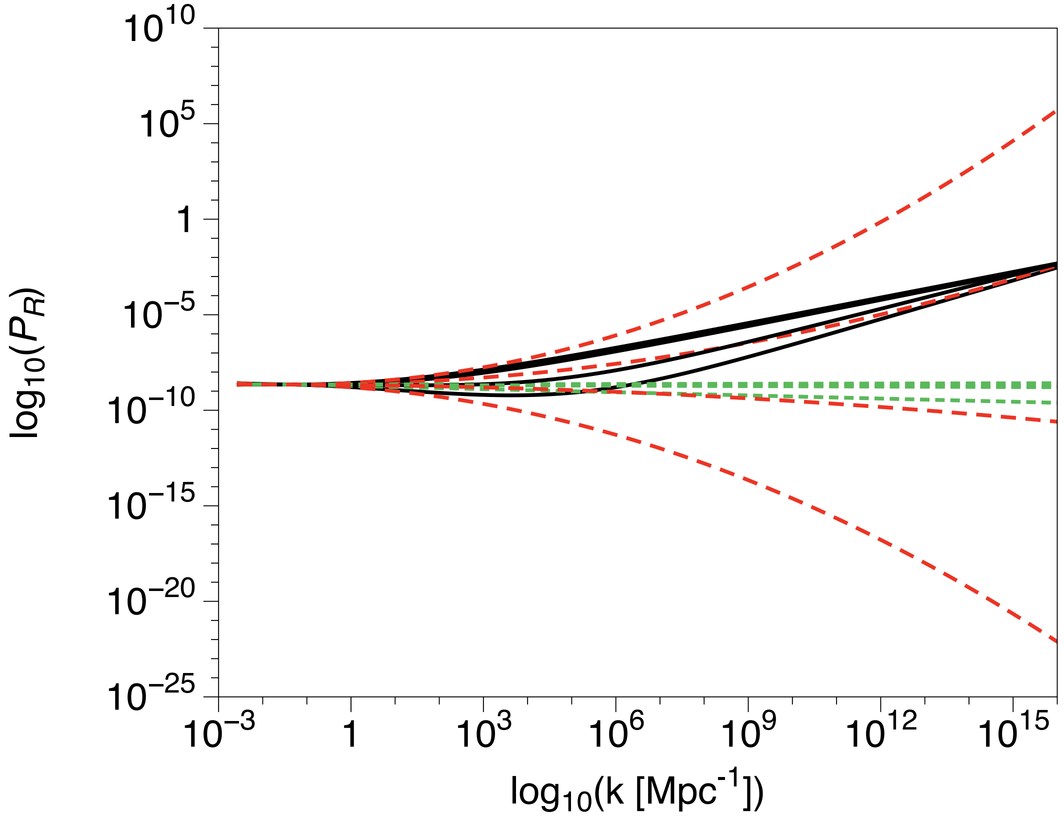
$$\mathcal{P}_{\mathcal{R}}(k) = A_{\rm s}\left(\frac{k}{k_0}\right)^{n_{\rm s}(k)-1} \quad \text{with } n_{\rm s}(k) = n_{\rm s}|_{k_0} + \alpha_{\rm s}\ln\left(\frac{k}{k_0}\right) + \beta_{\rm s}\ln^2\left(\frac{k}{k_0}\right) + \ldots\,,$$

For slow-roll inflation $(n_{\rm s}-1) \sim \mathcal{O}(\epsilon)\,, \quad \alpha_{\rm s} \sim \mathcal{O}(\epsilon^2)\,, \quad \beta_{\rm s} \sim \mathcal{O}(\epsilon^3)$ where $\epsilon < 1$

The expansion of $n_{\rm s}$ is therefore valid only if $\epsilon \ln\left(\frac{k}{k_0}\right) \ll 1$

This holds over cosmological scales, but not down to PBH forming scales:

Power spectra of some PBH producing inflation models:



full calculation

1st order in expansion

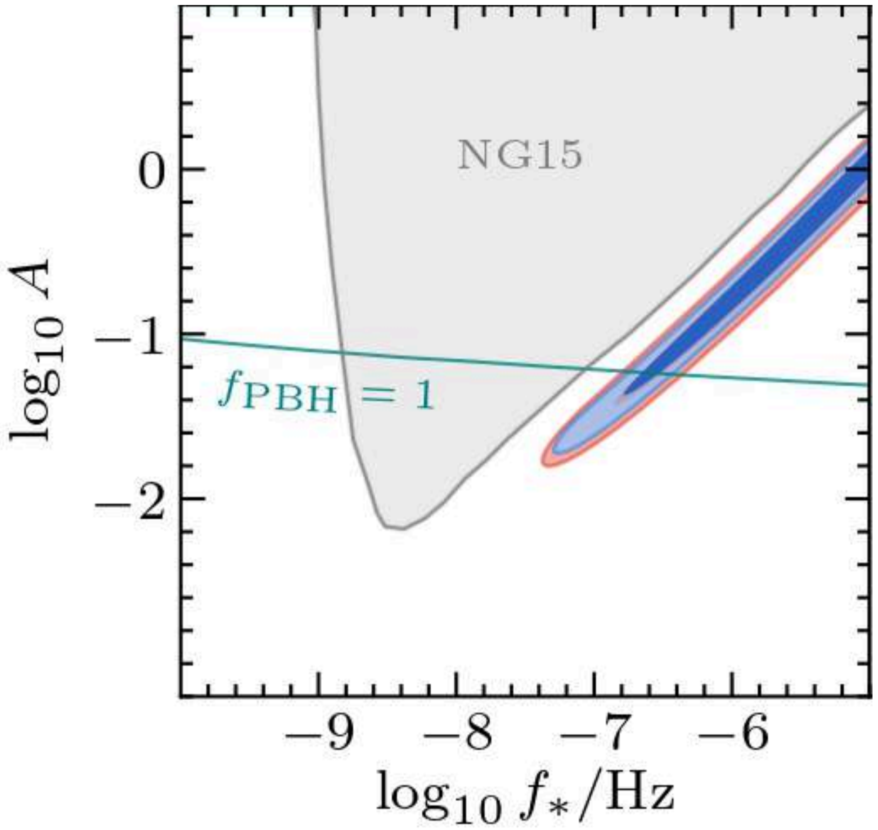
2nd order in expansion

NANOGrav (pulsar timing array) 15 year data set

Interpretation in terms of scalar-induced gravitational waves Afzal et al.

for delta function primordial power spectrum

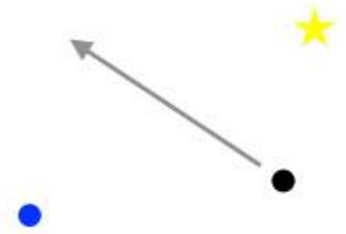
$$\mathcal{P}_{\mathcal{R}}(k) = A\,\delta(\ln k - \ln k_\star)$$



$$\underbrace{\qquad\qquad}_{(10^{-3} - 10^{-4})M_\odot}$$

Observational constraints

other microlensing



Gravitational lensing where separation of images is micro-arcsecond, too small to resolve, but can detect variations in magnification.

supernovae: magnification distribution Zumalacarregui & Seljak

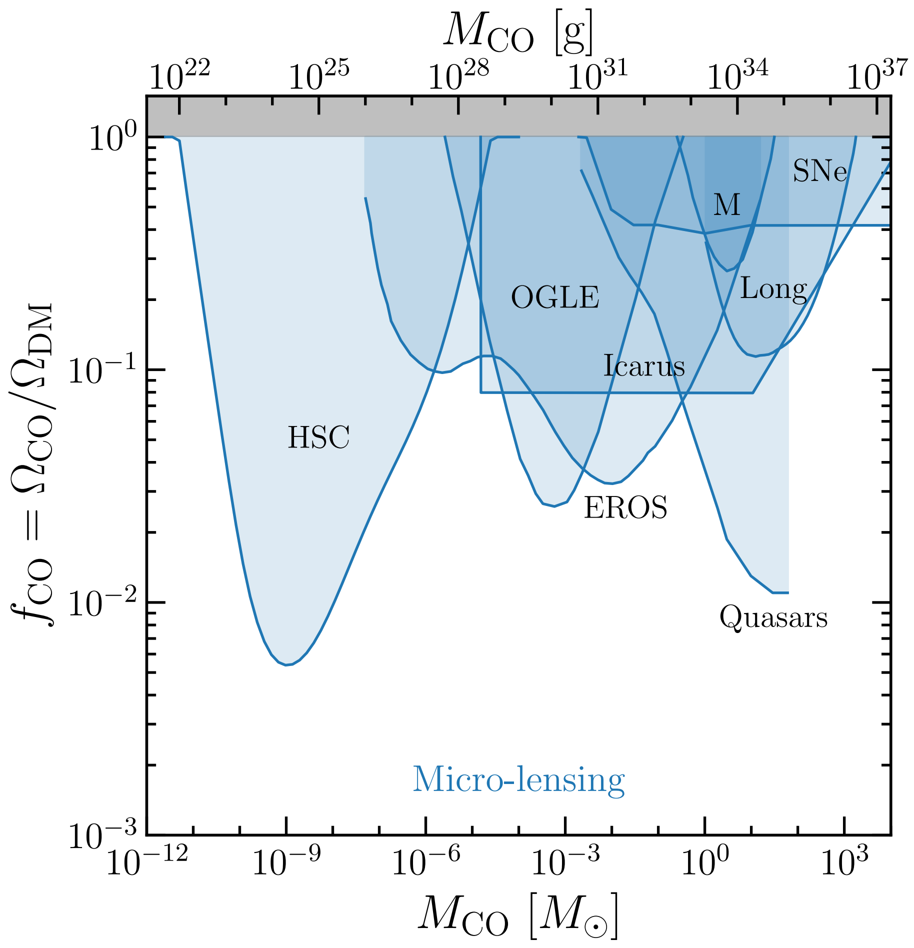
luminosity-redshift relation Dhawan & Mörtsell

Icarus: caustic crossing event Oguri et al.

quasars: flux ratios of multiply-lensed systems Esteban-Gutierrez et al.

fraction of dark matter in form of compact objects

$$f_{\mathrm{CO}} = \frac{\Omega_{\mathrm{CO}}}{\Omega_{\mathrm{DM}}}$$



mass in grams

mass in Solar masses

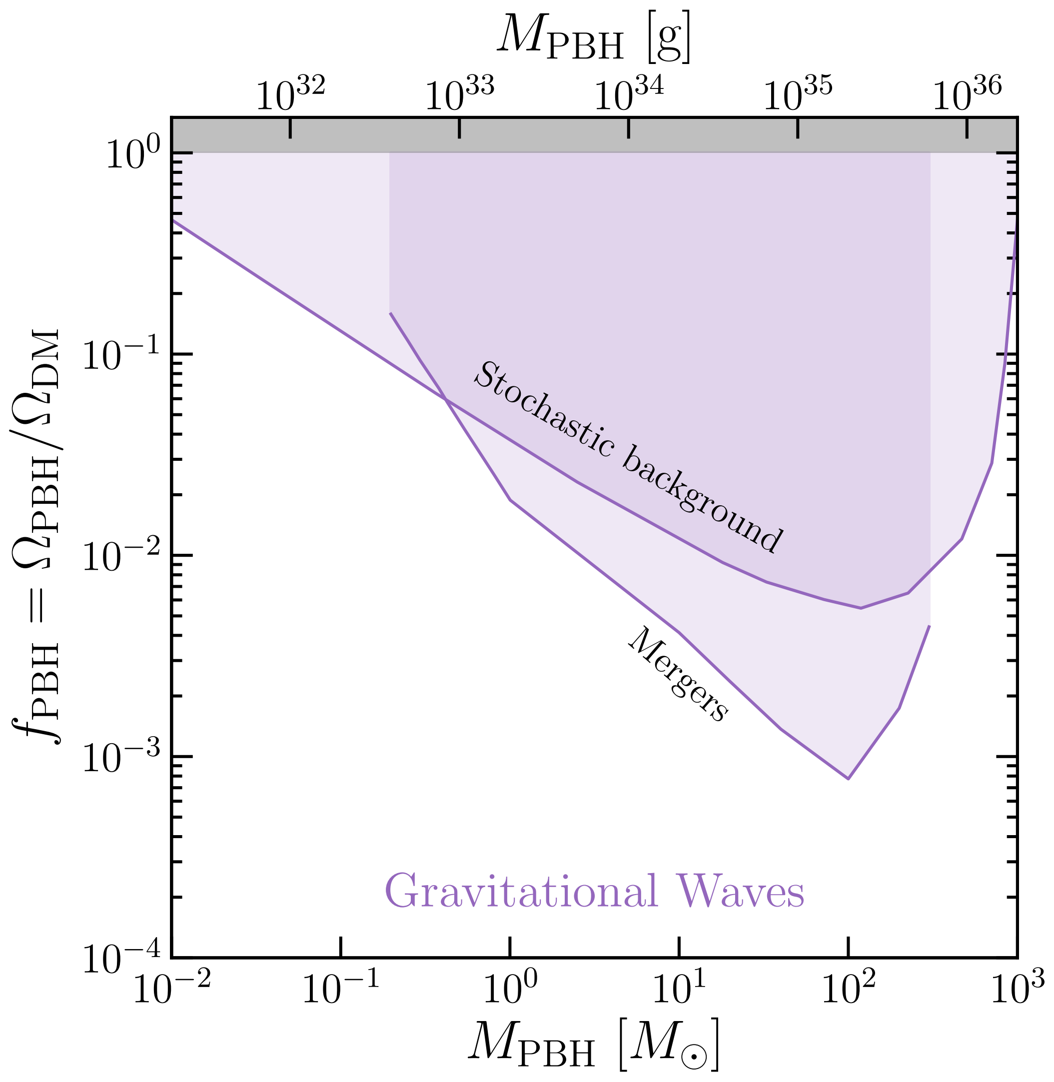
gravitational waves from PBH-PBH binary mergers

PBH binaries can form at early times (from chance proximity). Nakamura et al.

If orbits aren't significantly perturbed subsequently, then their mergers are orders of magnitude larger than the merger rate measured by LIGO. Ali-Haïmoud, Kovetz & Kamionkowski

Also comparable constraints from stochastic GW from mergers. Wang et al.

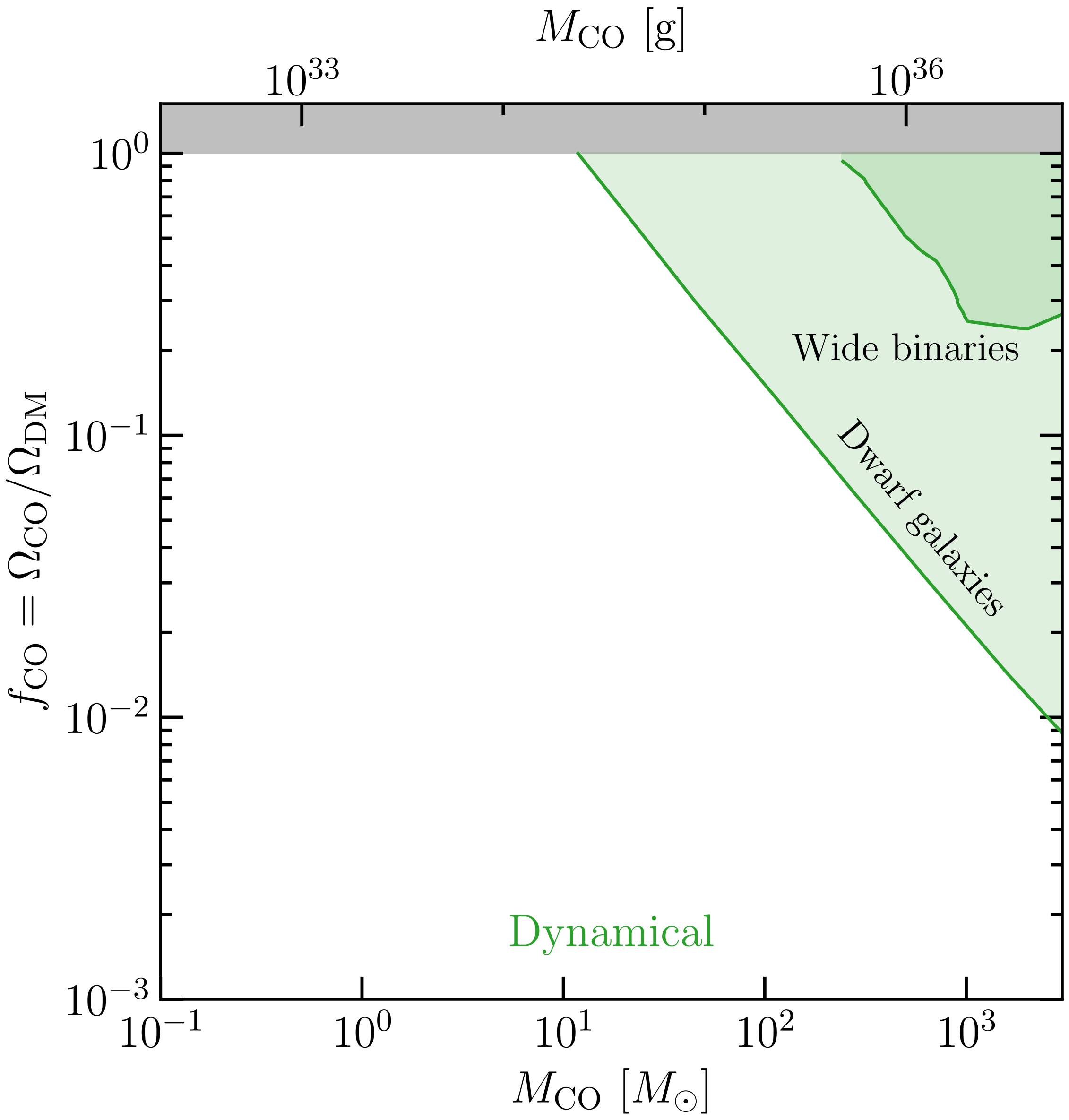
$$f_{\rm PBH} = \frac{\Omega_{\rm PBH}}{\Omega_{\rm DM}}$$



dynamical effects

dwarf galaxies: stars are dynamically heated and size of stellar component increased
Brandt; Koushiappas & Loeb; Zhu et al.; Stegmann et al.

wide binaries: dynamically heated, separations increased, and widest binaries disrupted. Yoo, Chaname & Gould; … Monroy-Rodriguez & Allen; Tyler, Green & Goodwin



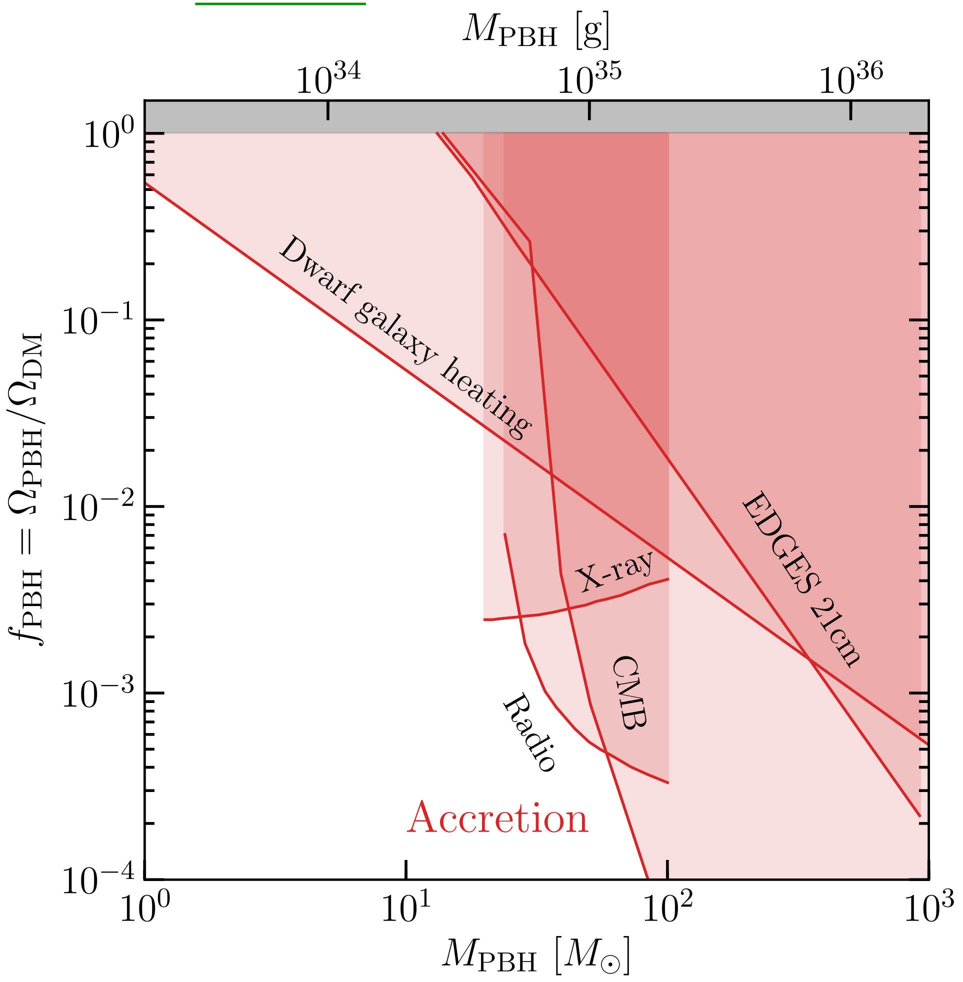
accretion

Radiation emitted due to gas accretion onto PBHs can modify the recombination history of the universe, constrained by

- distortion of CMB anisotropies Ricotti et al; Ali-Haïmoud & Kamionkowski; … Poulin et al.…
- EDGES 21cm measurements Hektor et al.;

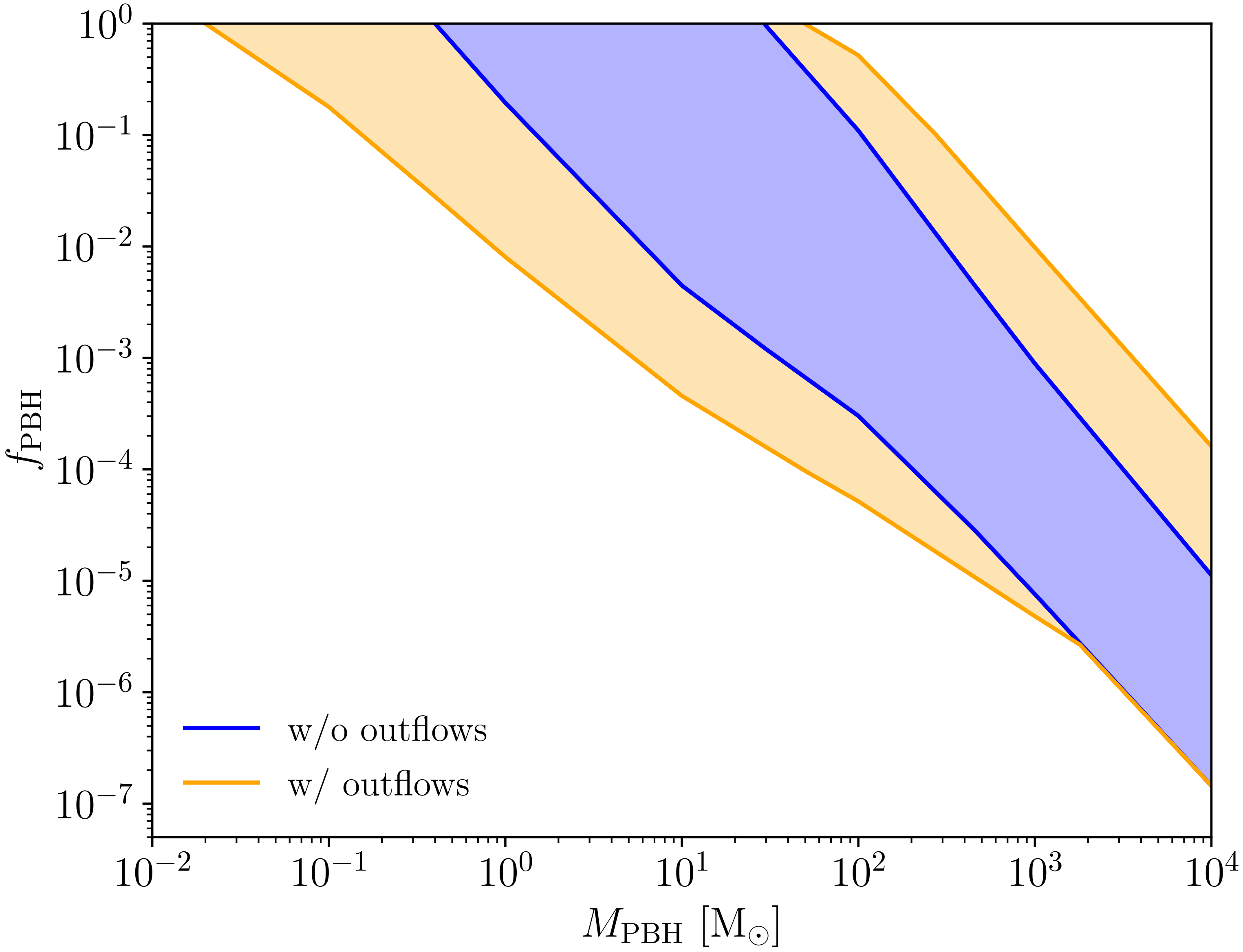
Accretion onto PBHs today constrained by

- X-ray and radio emission in MW Gaggero et al; Inoue & Kusenko; Manshanden et al.
- gas-heating in dwarf galaxies Lu et al.



uncertainty in constraint from distortion of CMB anisotropies

from geometry of accretion (spherical or disc) Poulin et al. and outflows Piga et al.



Piga et al.

constraints on asteroid mass PBHs from interactions with stars

Stars can capture asteroid mass PBHs through dynamical friction, accretion onto PBH can then destroy the star. Capela, Pshirkov & Tinyakov; Pani & Loeb; Montero-Camacho et al.

Transit of asteroid mass PBH through white dwarf heats it, due to dynamical friction, causing it to explode. Graham, Rajendran & Varela

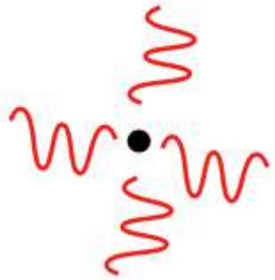
Montero-Camacho et al. **No current constraints**, but potential future constraints from

i) survival of neutron stars in globular cluster **if** it has DM halo (need high DM density, low velocity-dispersion environment),

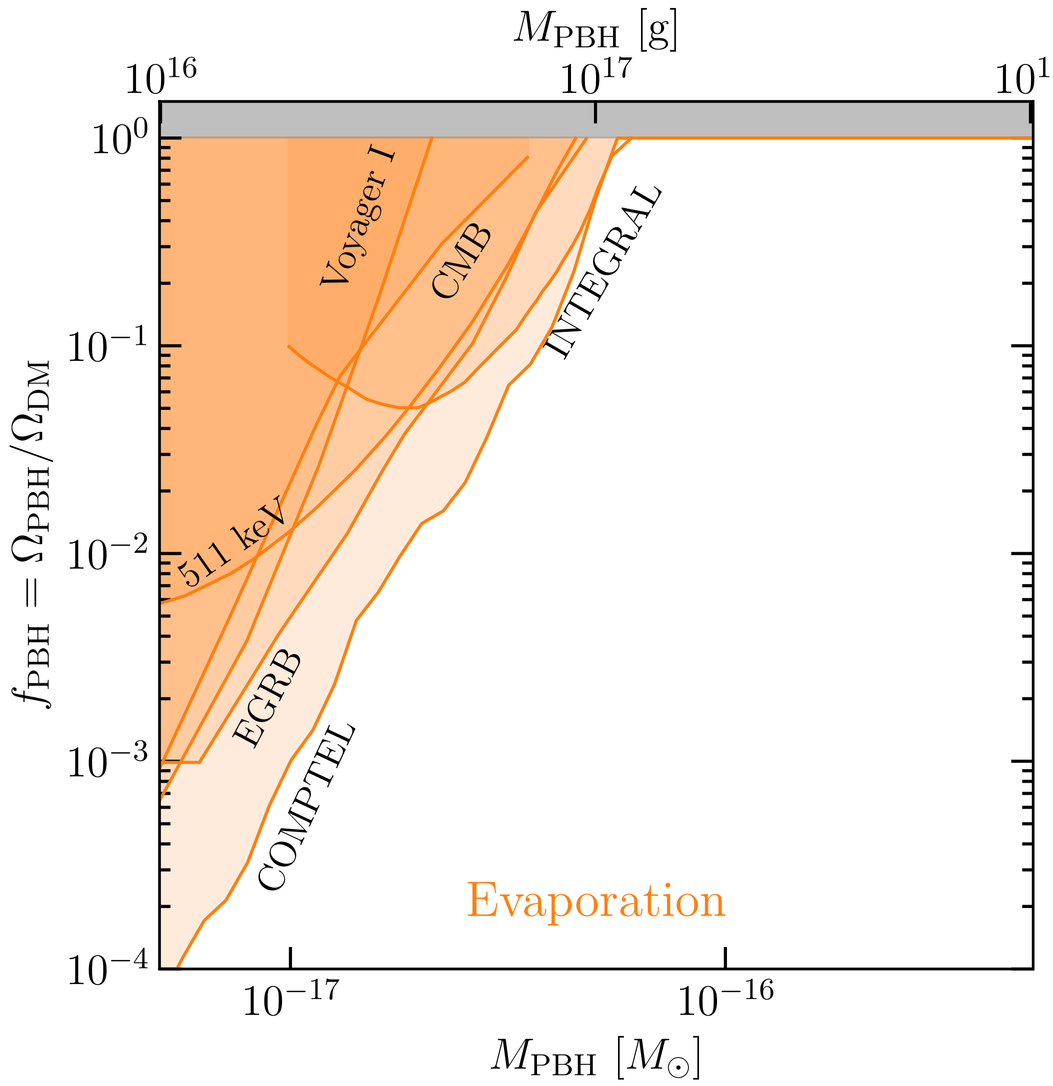
ii) signatures of star being destroyed.

Esser & Tinyakov potential constraints from disruption of main sequence stars in dwarf galaxies, due to PBH capture during star formation.

constraints on light PBHs from evaporation products



Evaporation products (gamma rays, $e^{\pm}$,…) from PBHs reaching the end of their lifetime would be detectable/have observable consequences.

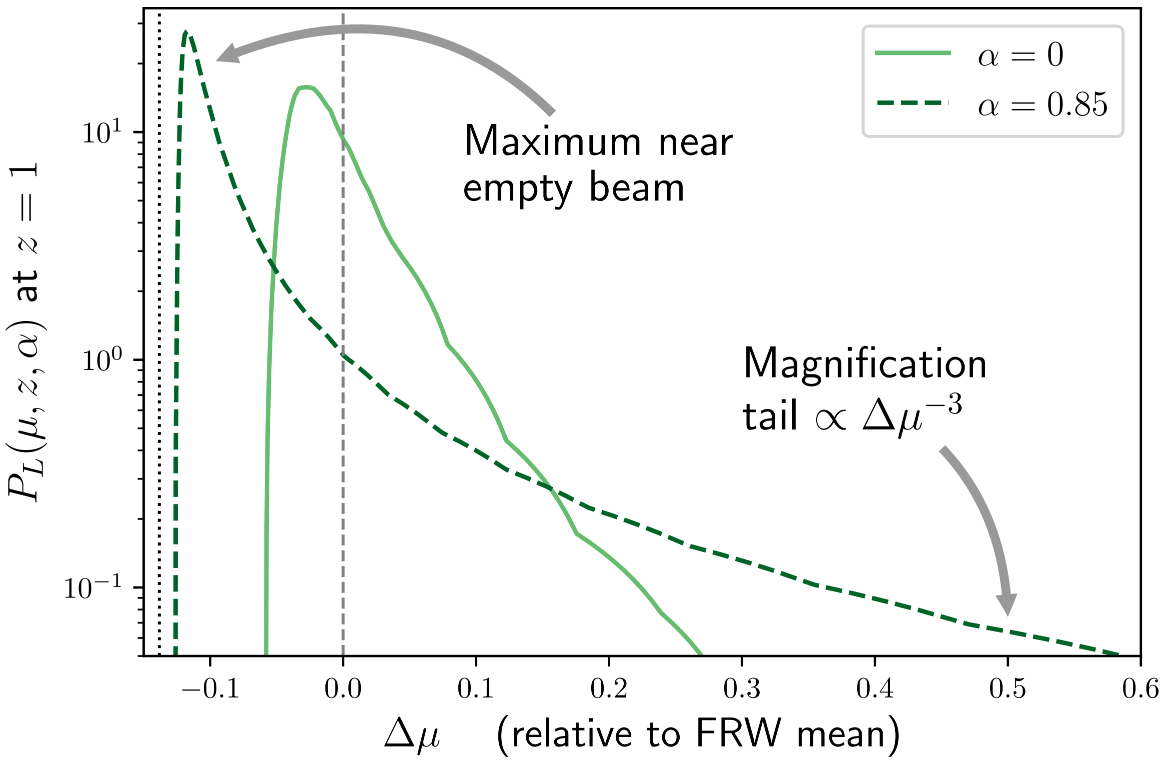


See also Auffinger review.

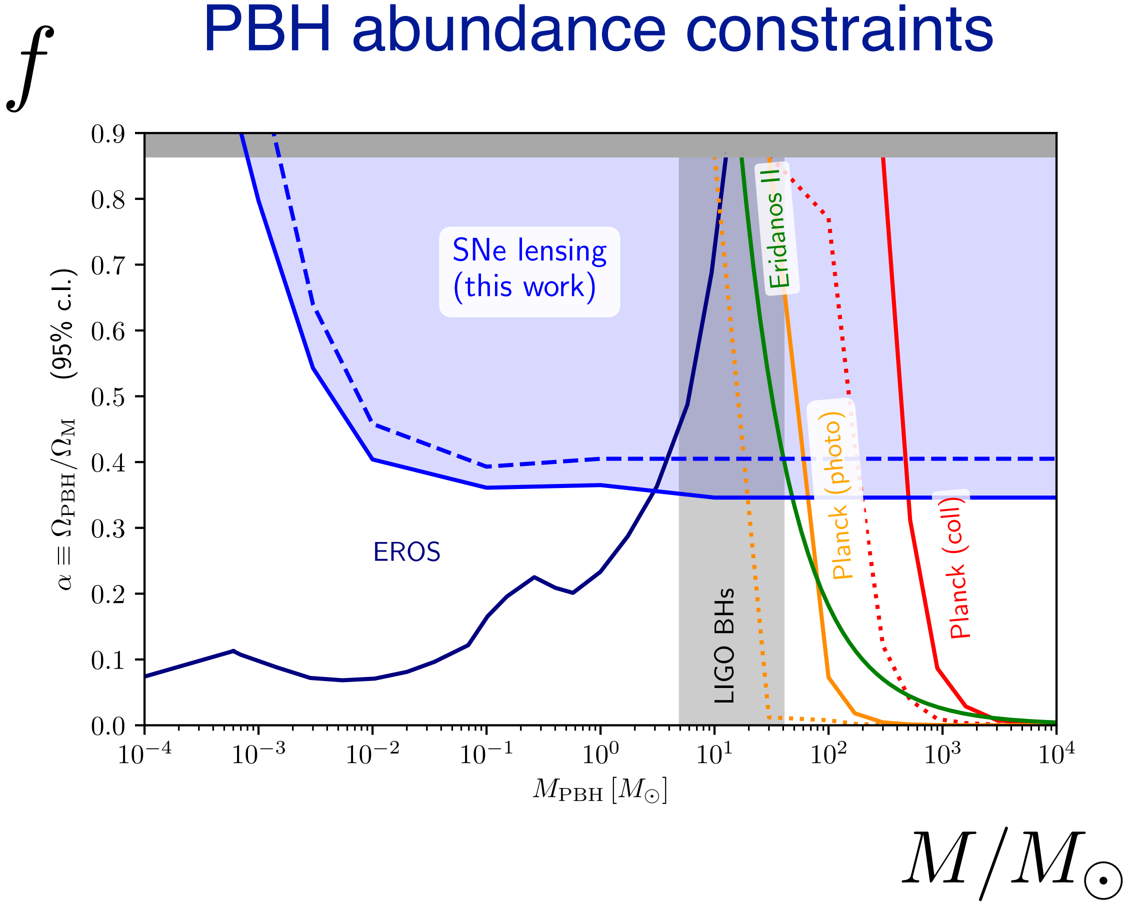
supernova microlensing

Lensing magnification distribution of type 1a SNe affected (most lines of sight are demagnified relative to mean, plus long-tail of high magnifications): Zumalacarregui & Seljak

magnification distribution



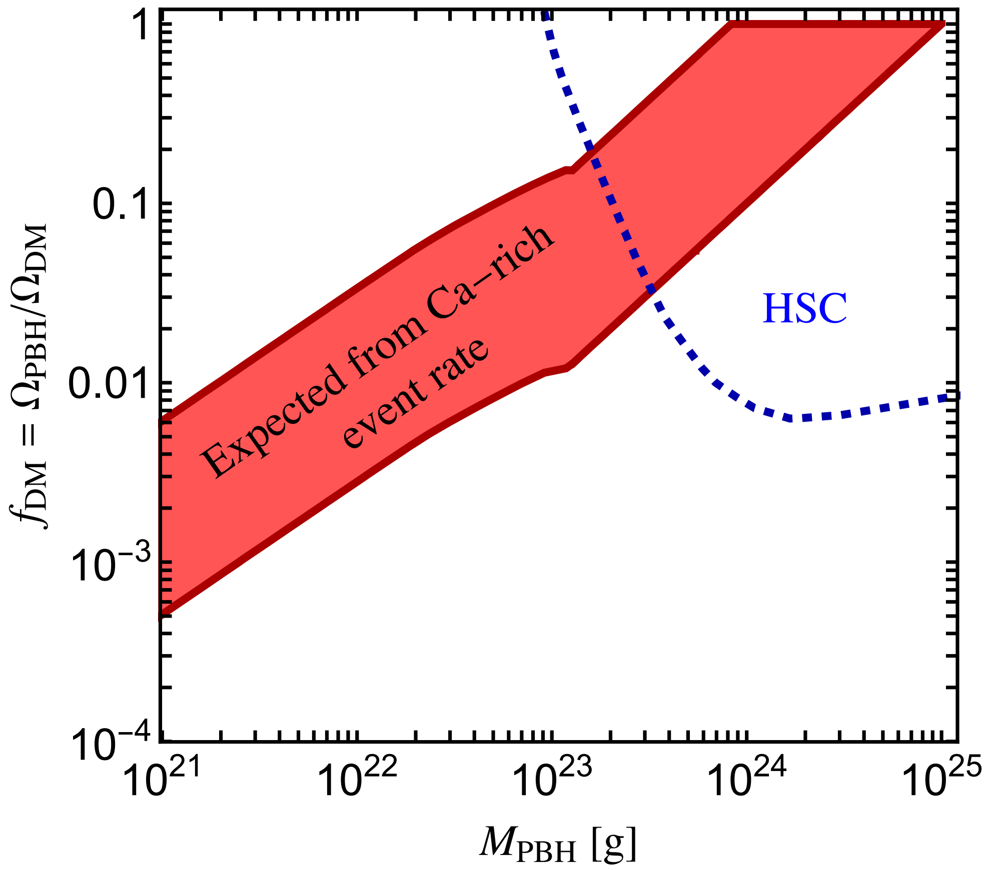
PBH abundance constraints



Garcia-Bellido, Clesse & Fleury argue priors on cosmological parameters are overly restrictive and physical size of supernovae have been underestimated.

Transit of asteroid mass PBH through white dwarf heats it, due to dynamical friction, causing it to explode. Graham, Rajendran & Varela.

Population of faint, Calcium-rich supernovae mostly located at large distances from centre of host galaxy, could be due to PBHs interacting with low mass white dwarfs in dwarf galaxies??
Smirnov et al.



Smirnov et al.

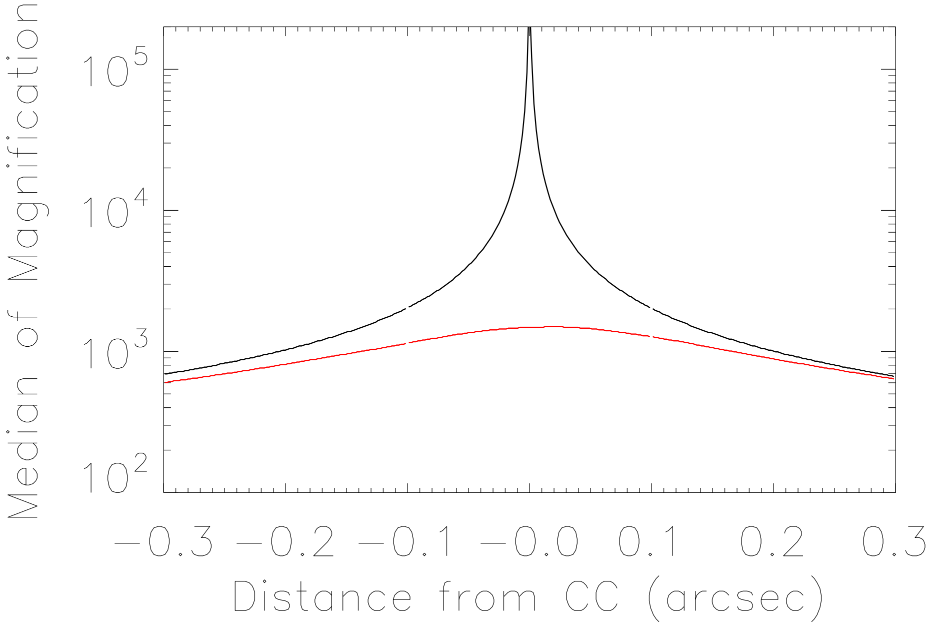
But observational signature of PBH-induced white dwarf explosion not yet reliably calculated. Montero-Camacho et al.

Icarus

When a distant star crosses a galaxy cluster caustic get huge magnification which can be increased by microlensing by compact objects (stars, black holes,..) in cluster. Miralda-Escude.

However if large fraction of DM is in compact objects magnification is reduced.

magnification Kelly et al.

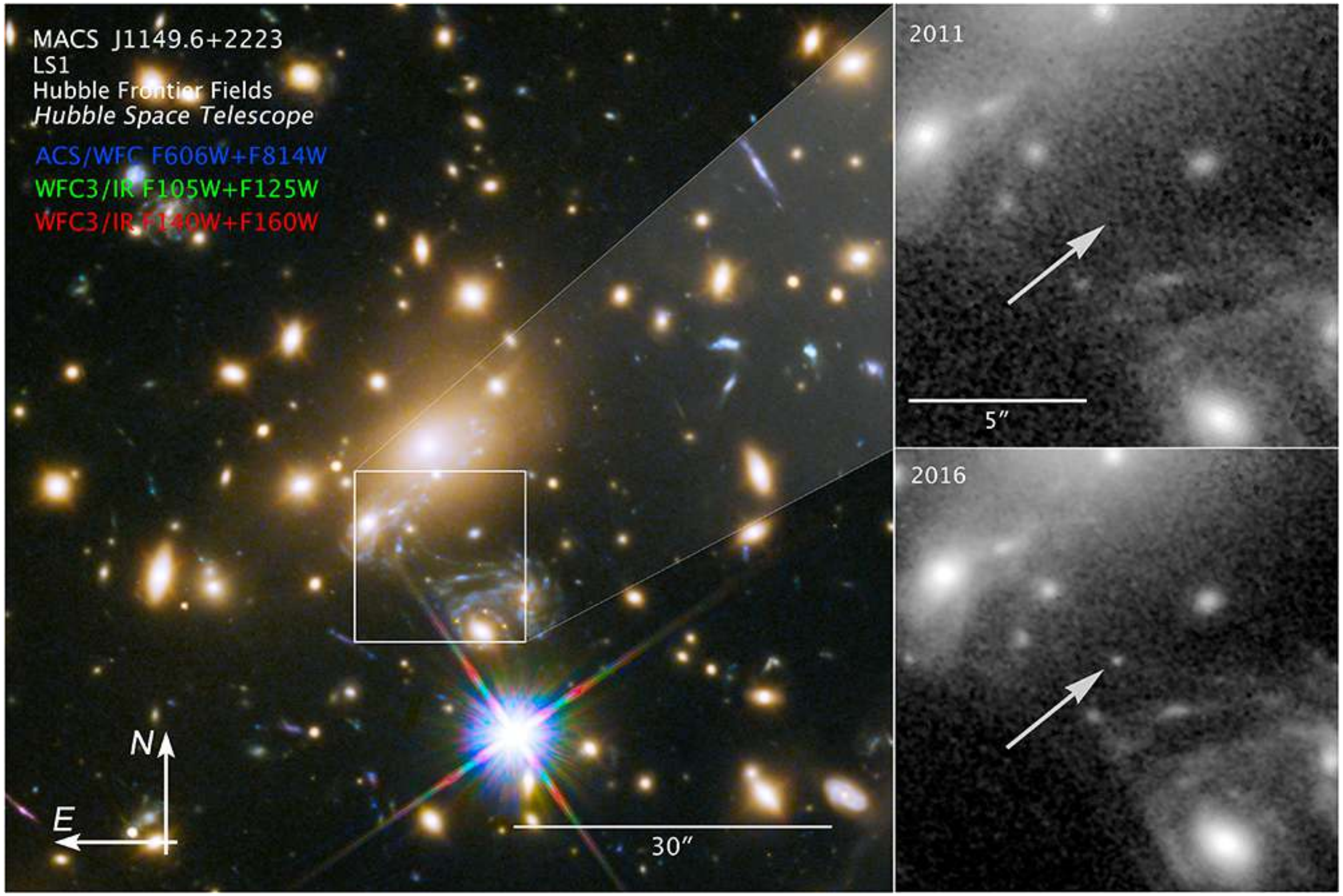


smooth DM, compact DM

Icarus is first (serendipitously) observed event involving a star at red-shift 1.5. Kelly et al.

Constraint from Icarus: $f < 0.08$ (but factor of 2 uncertainty in transverse velocity leads to similar uncertainty on f). Oguri et al.

Icarus is first (serendipitously) observed event involving a star at red-shift 1.5. Kelly et al.



Kelly et al.

Constraint from Icarus: $f < 0.08$ (but factor of 2 uncertainty in transverse velocity leads to similar uncertainty on f). Oguri et al.

constraints on light PBHs from evaporation products

Extragalactic gamma-rays background (EGRET/Fermi) Carr, Kohri, Sendouda & Yokoyama

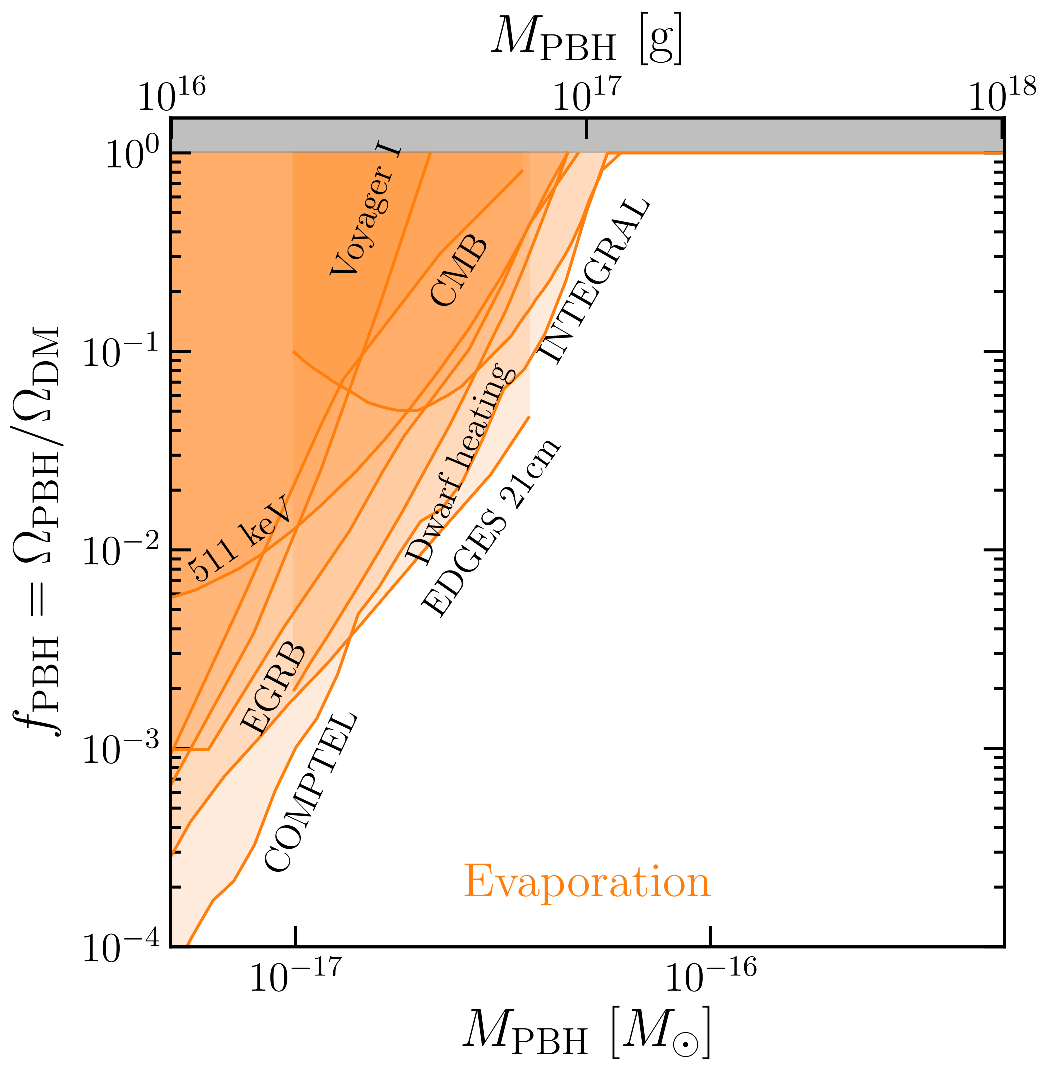
MeV galactic diffuse flux (INTEGRAL) Laha, Munoz & Slatyer (COMPTEL) Coogan, Morrison & Profumo

damping of CMB anisotropies during recombination (Planck) Poulin et al.; Clark et al.

$e^{\pm}$ flux (Voyager 1) Boudaud & Cirelli

511 keV line from $e^{\pm}$ annihilation (INTEGRAL) DeRocco & Graham; Laha

heating of ISM in dwarf galaxy Kim



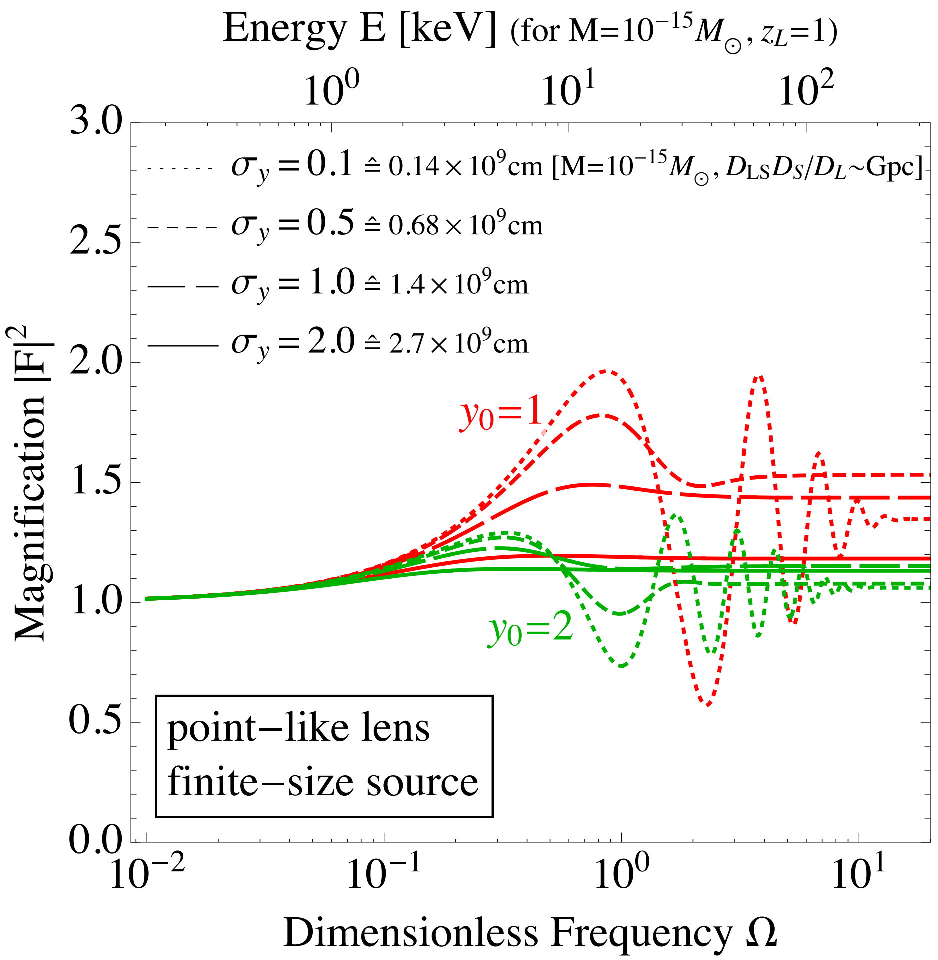
how to constrain asteroid mass PBHs??

Femtolensing of GRBs

Different path lengths lead to phase differences, and hence interference fringes in energy spectrum of lensed GRBs. Gould

Barnacka, Glickenstein & Moderski constraints from Fermi Gamma Ray Burst monitor.

BUT Katz, Kopp, Sibiryakov, Xue most GRBs not point-like, and (less significantly) geometric optics approximation also breaks down:



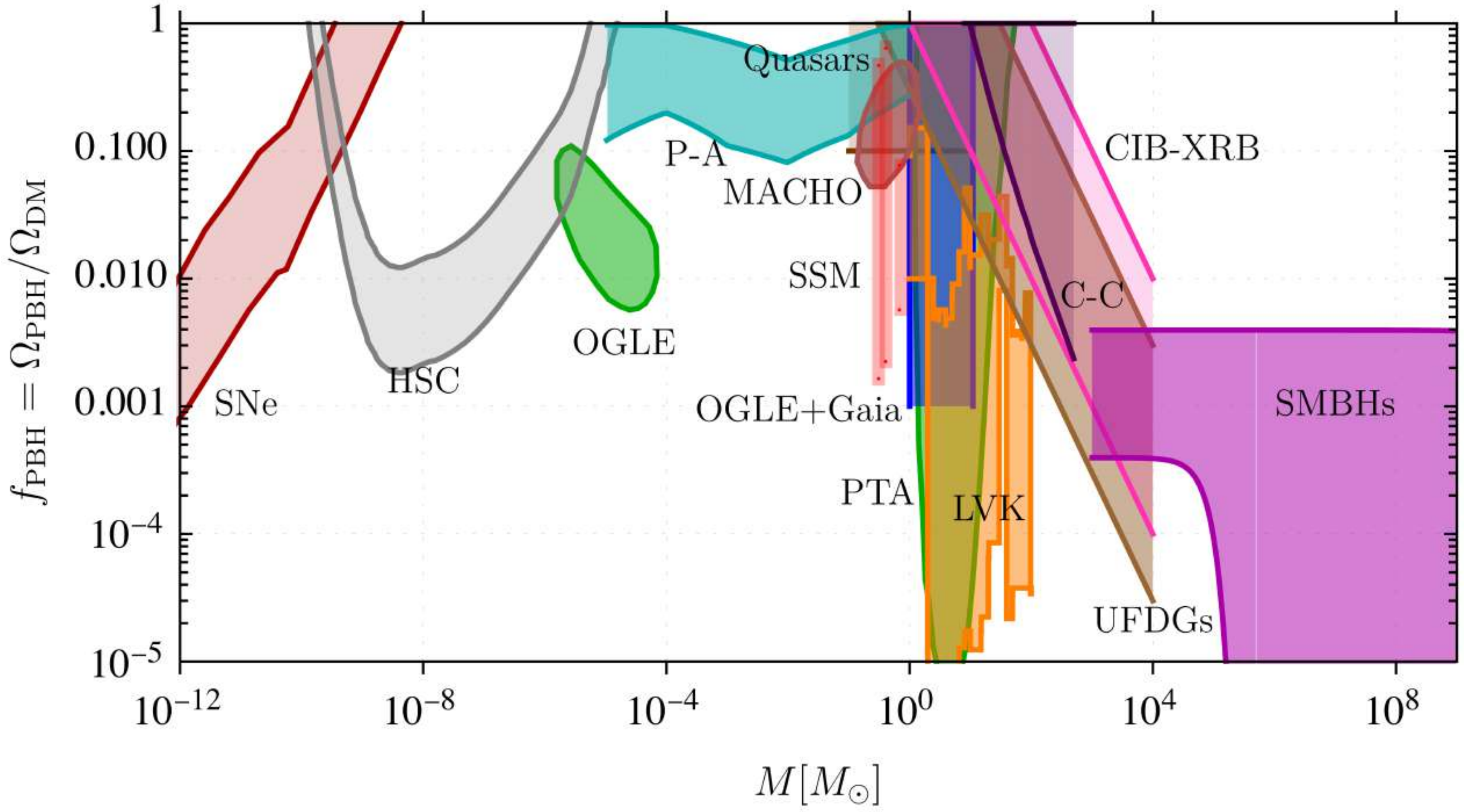
↓ increasing angular size of source

y0 = position of source centre in terms of β/θ_E

Constraints could be achieved in a future with a sample of GRBs with well-measured red-shift and spectra, and small size (which is expected to correspond to sub-milli-second variability).

Observational signatures ???

Carr, Clesse, Garcia-Bellido, Hawkins & Kuhnel, arXiv:2306.03903, ‘Observational evidence for PBHs: a positivist perspective’ and references therein.



SNe: trigger explosions of white dwarfs → calcium-rich supernovae

HSC, OGLE, PA, MACHO, OGLE-Gaia: microlensing

PTA: scalar induced gravitational waves detected by pulsar timing arrays

LVK: LIGO-Virgo-Kagra gravitational wave events

C-C: producing cores in density profiles of dwarf galaxies

CIB-XRB: accretion + clustering explains correlations in infra-red and X-ray backgrounds

UFDGs: clustering explains minimum mass & size of ultra faint dwarf galaxies

SMBHs: provide seeds for super massive black holes

Method for applying delta-function constraints to extended mass functions:

Carr, Raidal, Tenkanen, Vaskonen& Veermae, see also Bellomo, Bernal, Raccanelli & Verde:

If $f_{max}(M)$ is the maximum allowed PBH fraction for a delta-function MF, an extended mass function ψ(M) has to satisfy:

$$\int \mathrm{d}M \, \frac{\psi(M)}{f_{\mathrm{max}}(M)} \leq 1$$

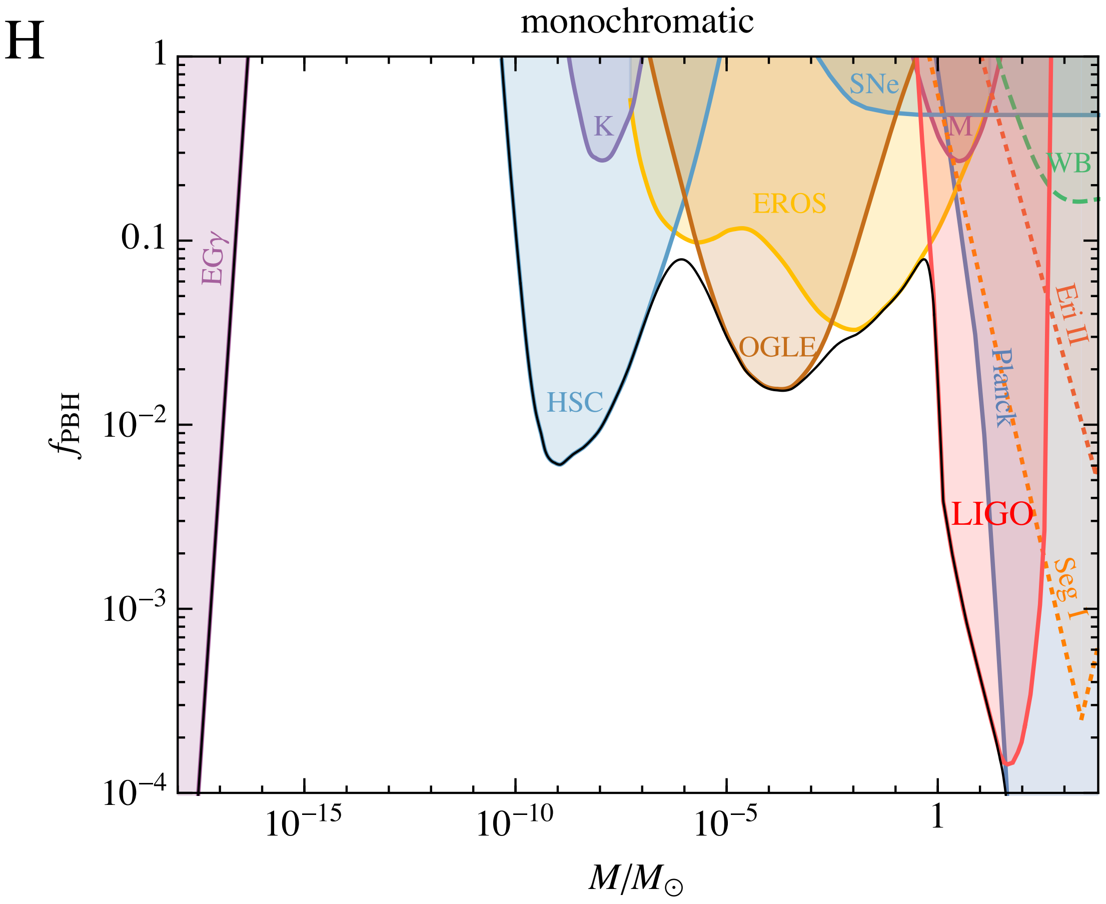
For more realistic extended mass functions, constraints on f are smeared out, and gaps between constraints are ‘filled in’:

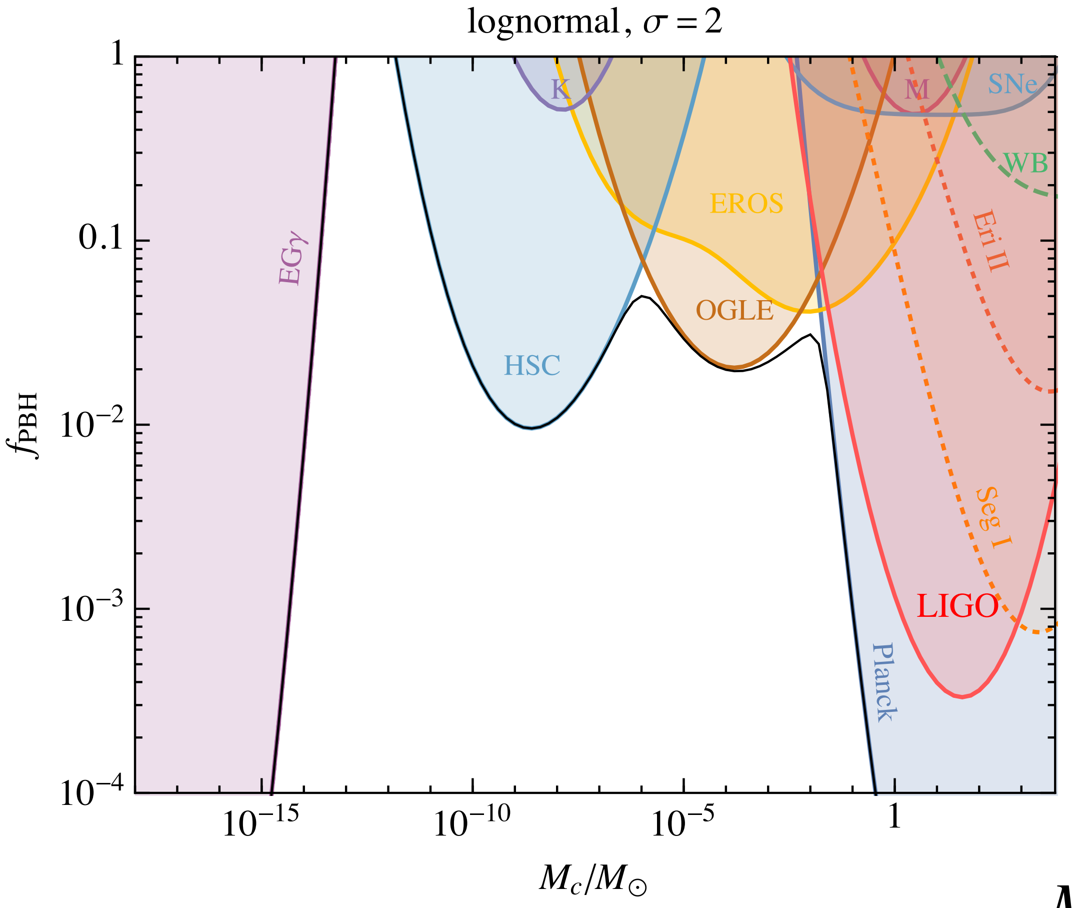
Green; Carr et al.

monochromatic

log-normal
(fixed width)

f_{PBH}

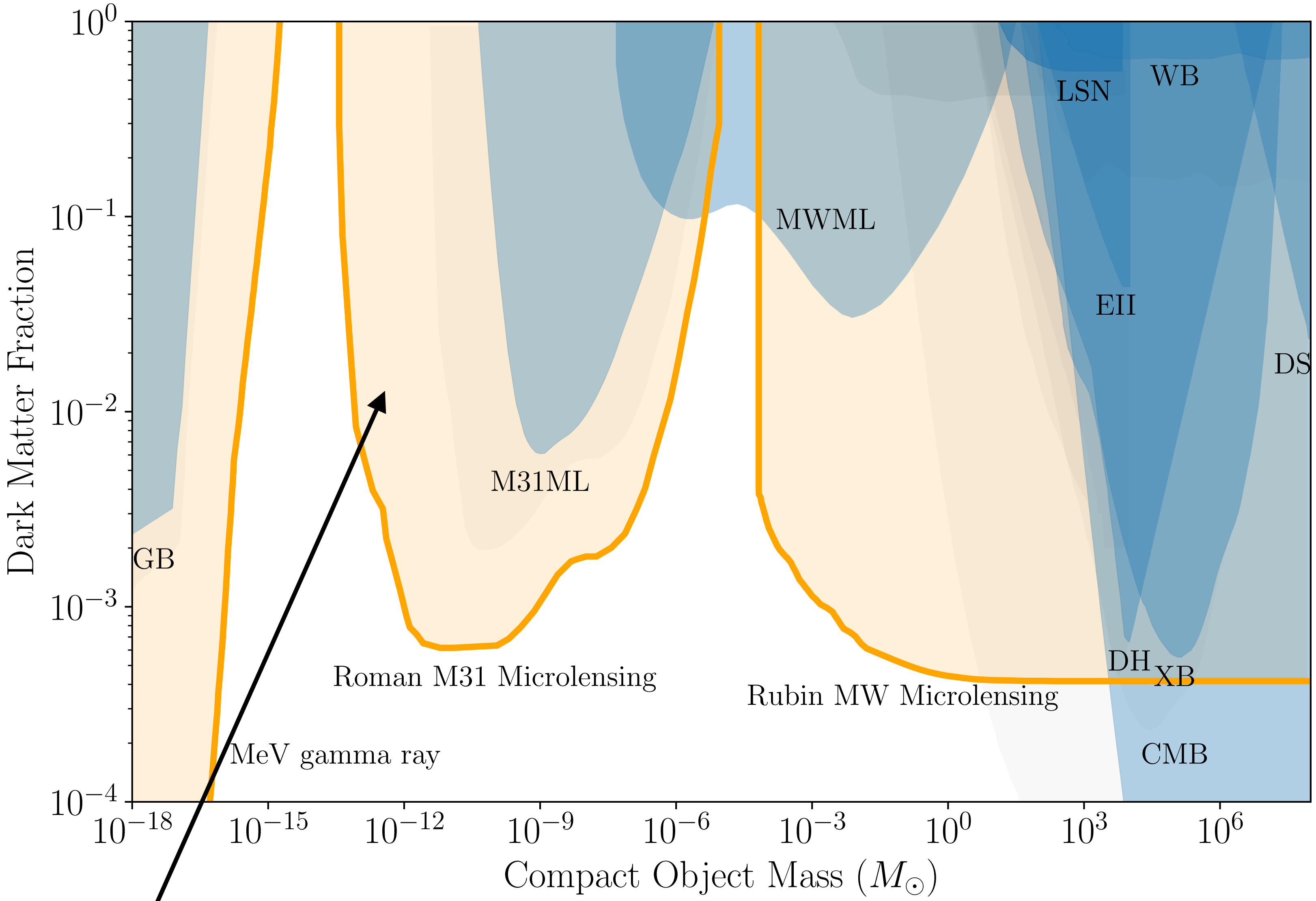




$\frac{M_c}{M_\odot}$

Carr et al.

Future constraints



Bird et al. (Snowmass PBH white paper)

But for $M \lesssim 10^{-12} M_\odot$ microlensing amplification reduced due to:

i) finite source size

ii) wave optics (wavelength of light similar to Schwarzschild radius of PBH).

Sugiyama et al. and references therein.

Open questions

i) how to probe asteroid mass PBHs?

femtolensing of GRBs Gould need small GRBs Katz et al.

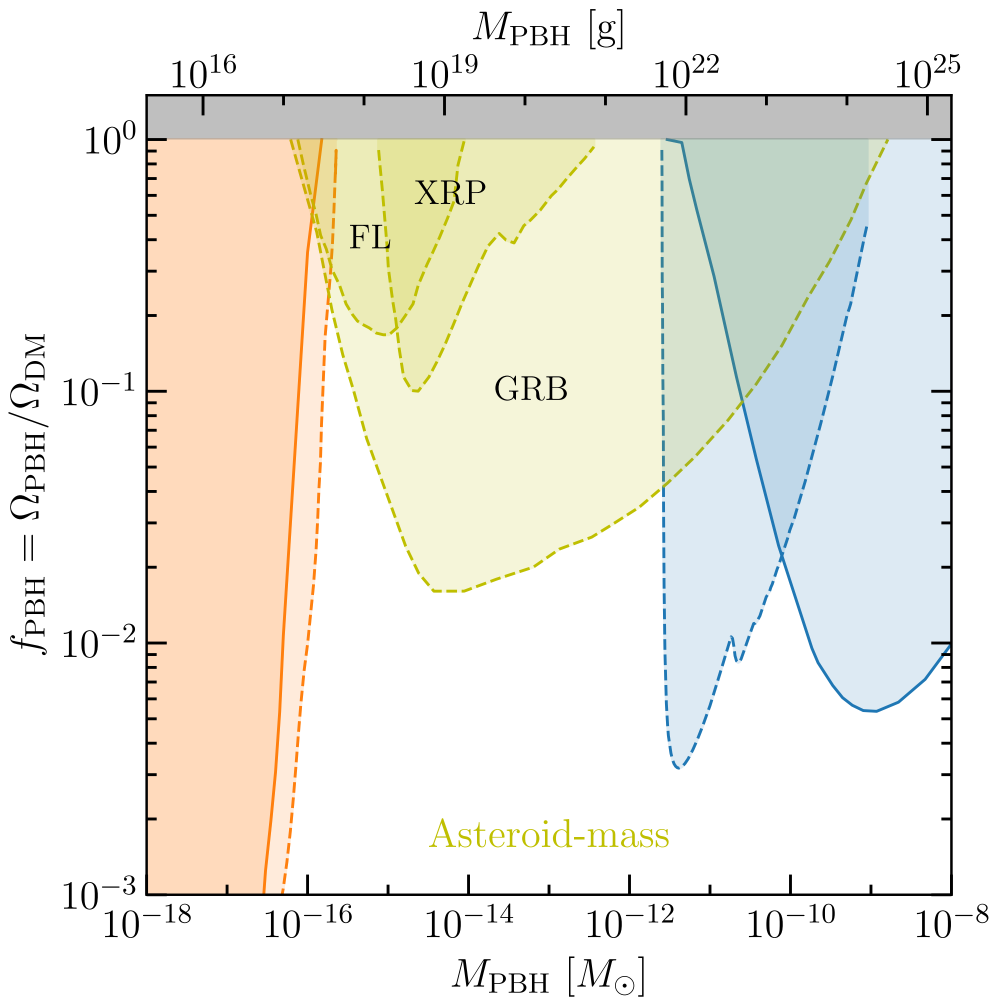
GRB lensing parallax Nemiroff & Gould; Jung & Kim

microlensing of X-ray pulsars Bai & Orlofsky

interactions with stars? see e.g. Montero-Camacho et al.

evaporation

future:
MeV gamma-rays
Coogan et al.



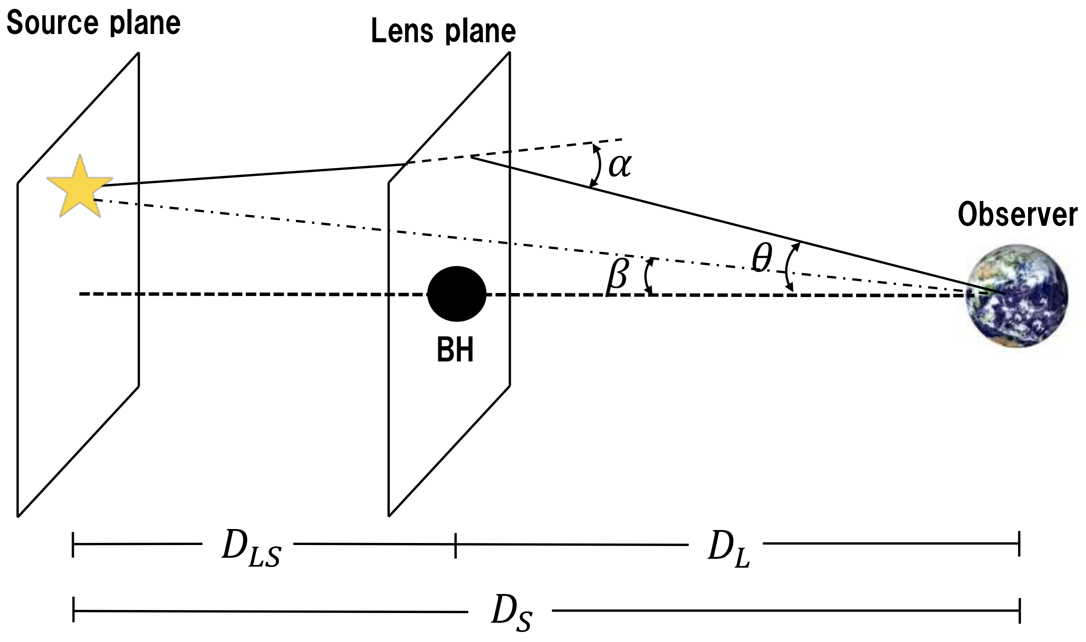
stellar microlensing

future:
white dwarfs in LMC
Sugiyama et al.

Stellar microlensing

gravitational lensing

for an intro see e.g. Sasaki et al.



$$x = \frac{D_L}{D_S}$$

Sasaki et al.

Lens equation: $\theta D_{\rm S} = D_{\rm S}\beta + D_{\rm LS}\alpha$

deflection $\alpha = \dfrac{4GM_{\rm BH}}{D_{\rm L}\theta}$

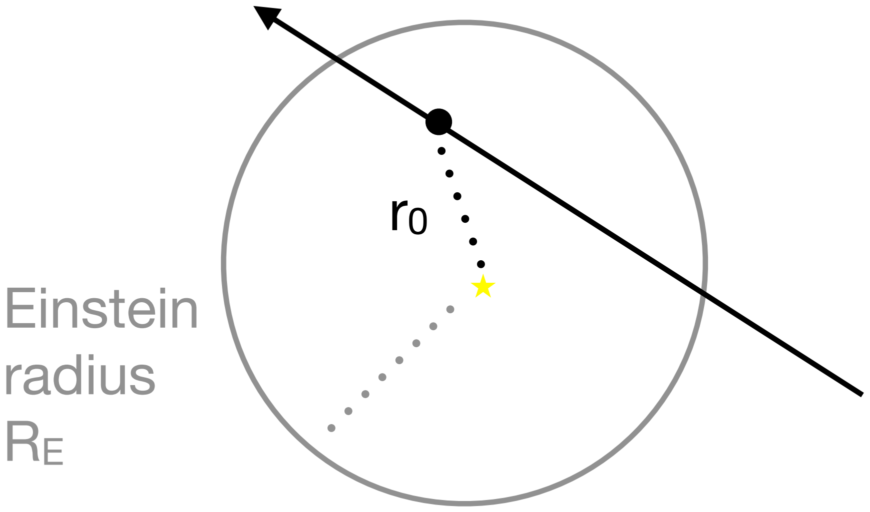
Lens equation on lens plane: $r^2 - r_0 r - R_{\rm E}^2 = 0$

$r = D_L\theta$ $\qquad$ $r_0 = D_L\beta$ $\qquad$ Einstein radius: $R_E = \sqrt{\dfrac{4GMD_LD_{LS}}{D_S}}$

Image positions: $r_{1,2} = \dfrac{1}{2}\left(r_0 \pm \sqrt{r_0^2 + 4R_E^2}\right)$

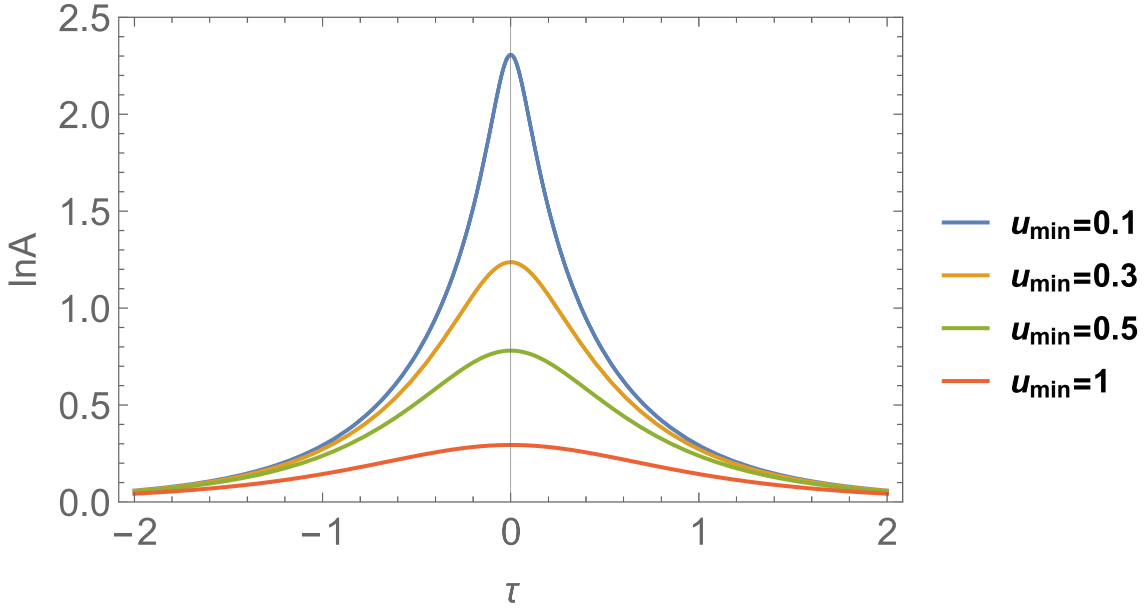
Angular separation: $\Delta \sim \dfrac{R_E}{D_L} = 0.3\,\text{mas}\left(\dfrac{M}{10\,M_\odot}\right)^{1/2}\left(\dfrac{D_S}{100\,\text{kpc}}\right)^{-1/2}\sqrt{\dfrac{1-x}{x}}$

along the line of sight



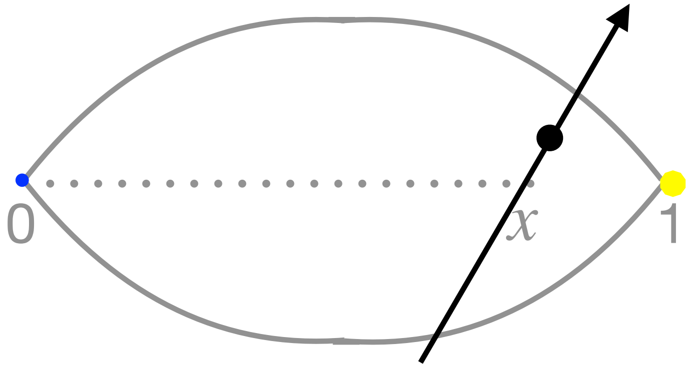
$$u = \frac{r_0}{R_E}$$

amplification: $$A = \frac{u^2+2}{u\sqrt{u^2+4}}$$



Sasaki et al.

perpendicular to the line of sight



Einstein radius:

$$R_E \approx 3\times 10^{-4}\,\mathrm{pc}\sqrt{x(1-x)}\left(\frac{M}{10M_\odot}\right)^{1/2}\left(\frac{D_s}{50\,\mathrm{kpc}}\right)^{1/2}$$

‘Duration’ of event (Einstein diameter crossing time):

$$\hat{t} = \frac{2R_E}{v} \approx 3\,\mathrm{yr}\sqrt{x(1-x)}\left(\frac{M}{10M_\odot}\right)^{1/2}\left(\frac{D_s}{50\,\mathrm{kpc}}\right)^{1/2}\left(\frac{v}{200\,\mathrm{km\,s^{-1}}}\right)^{-1}$$

Differential event rate,
assuming a delta-function lens mass function and a spherical halo with a Maxwellian velocity distribution (and neglecting the transverse velocity of the microlensing tube): [Griest]

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{t}} = \frac{32 L u_{\mathrm{T}}^4}{\hat{t}^4 M v_{\mathrm{c}}^2} \int_0^1 \rho(x)\, R_{\mathrm{E}}^4(x)\, \exp\left[-\frac{4R_{\mathrm{E}}^2(x)}{\hat{t}^2 v_{\mathrm{c}}^2}\right] \mathrm{d}x$$

$\rho(x)$ = compact object density distribution

$\hat{t}$ = Einstein ***diameter*** crossing time (as used by the MACHO collab., EROS & OGLE use Einstein radius crossing time)

v_c = local circular speed (usually taken to be 220 km/s, ~10s% uncertainty)

Expected number of events:

$$N_{\mathrm{exp}} = E \int_0^\infty \frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{t}}\, \epsilon(\hat{t})\, \mathrm{d}\hat{t}$$

E = exposure (number of stars times duration of obs.)

$\epsilon(\hat{t})$ = efficiency (prob. that an event of duration $\hat{t}$ is observed)

Standard halo model

cored isothermal sphere:

$$\rho(r) = \rho_0 \frac{R_{\rm c}^2 + R_0^2}{R_{\rm c}^2 + r^2}$$

$\rho_0 = 0.008\, M_\odot\, {\rm pc}^{-3}$, local dark matter density

$R_{\rm c} = 5\, {\rm kpc}$, core radius

$R_0 = 8.5\, {\rm kpc}$, Solar radius

‘Backgrounds’

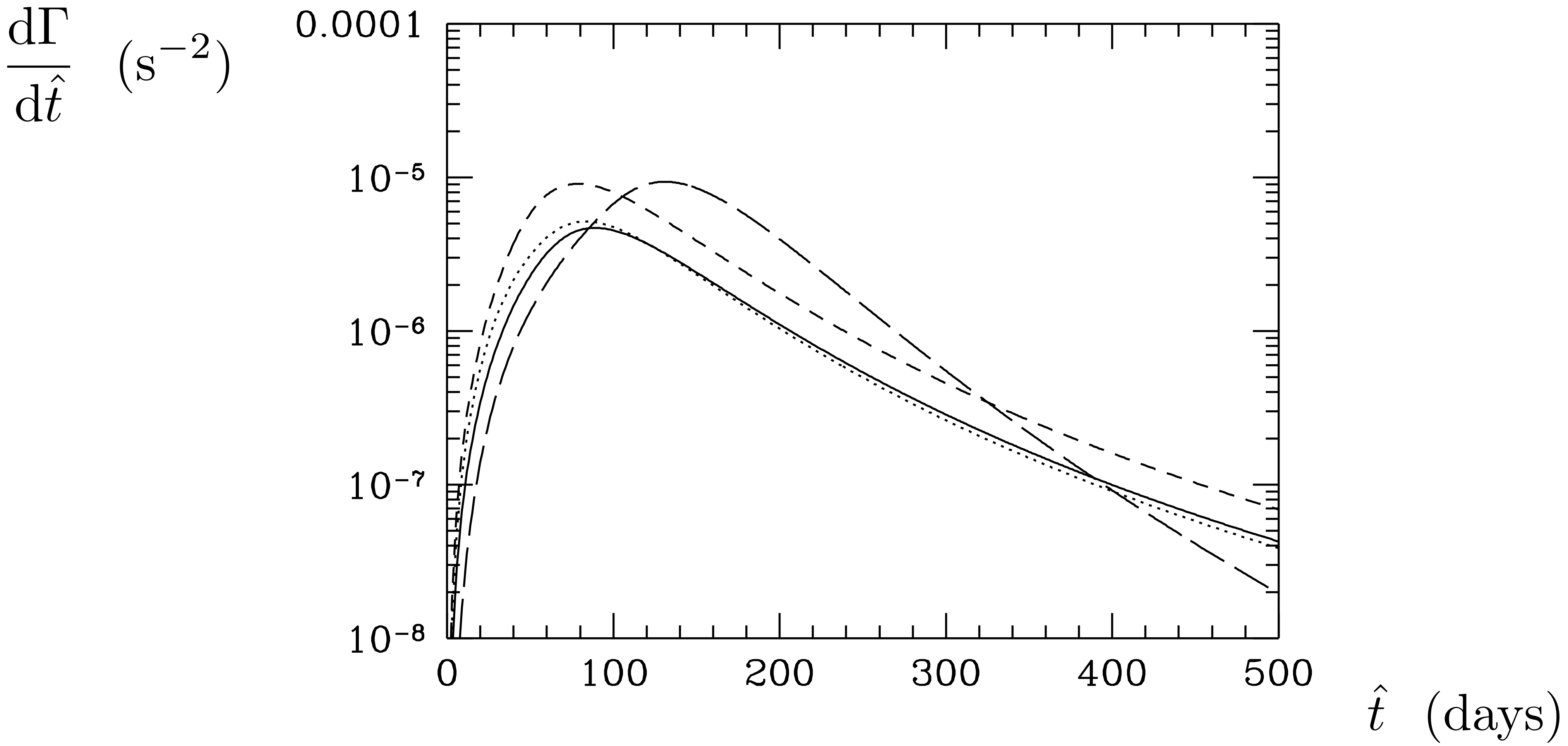
i) variable stars, supernovae in background galaxies

cuts/fits developed to eliminate them (but some events only rejected years later, after star’s brightness varied a 2nd time!)

ii) lensing by stars in MW or Magellanic Clouds themselves (‘self-lensing’)

model and include in event rate calculation

Differential event rate for $M = 1\,M_\odot$ and halo fraction f=1:
($\hat{t} \propto M^{1/2}$, $\mathrm{d}\Gamma/\mathrm{d}\hat{t} \propto M^{-1}$)



______ = standard halo model

. . . . = standard halo model including transverse velocity

- - - - = Evans power law model: massive halo with rising rotation curve, $v_\mathrm{c} \propto R^{0.2}$

_ _ _ _ = Evans power law model: flattened halo with falling rotation curve, $v_\mathrm{c} \propto R^{-0.2}$

velocity anisotropy can affect rate at ~10% level [De Paolis, Ingrosso & Jetzer]

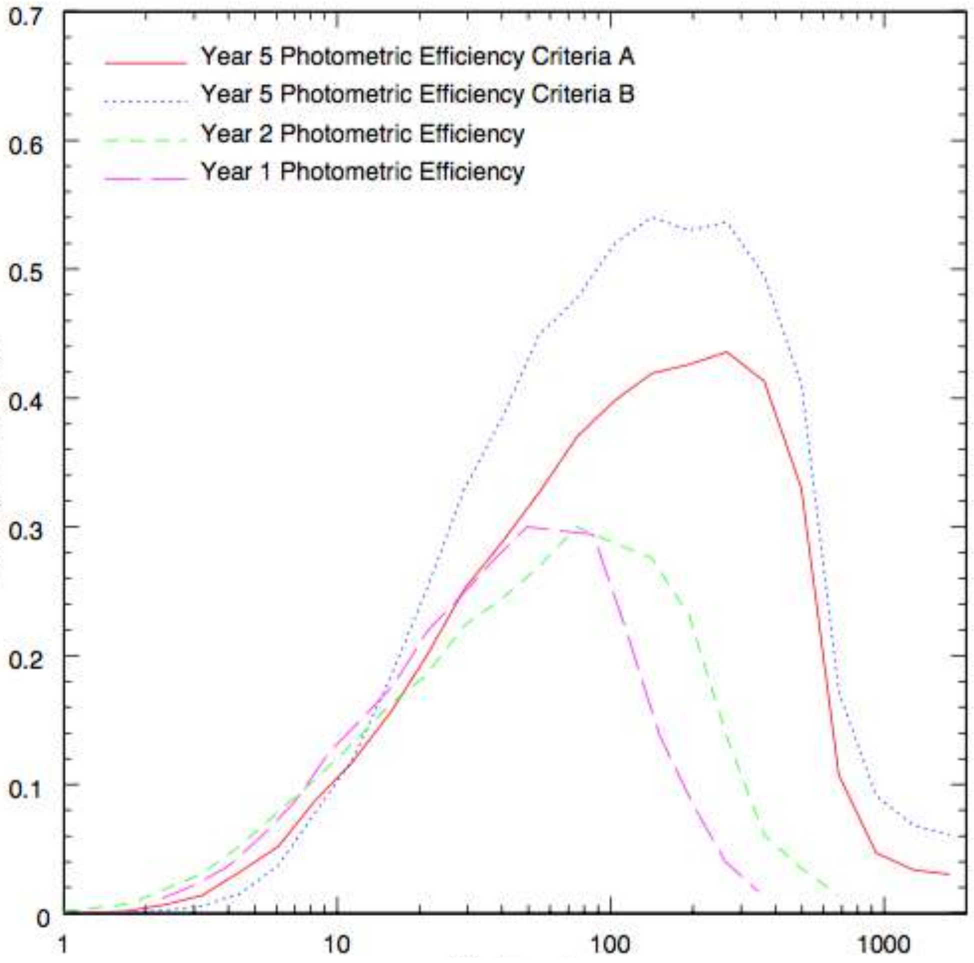
Observations

MACHO

Monitored 12 million stars in LMC for 5.7 years.

Found 13/17 events (for selection criteria A/B, B less restrictive-picks-up exotic events).

Detection efficiency



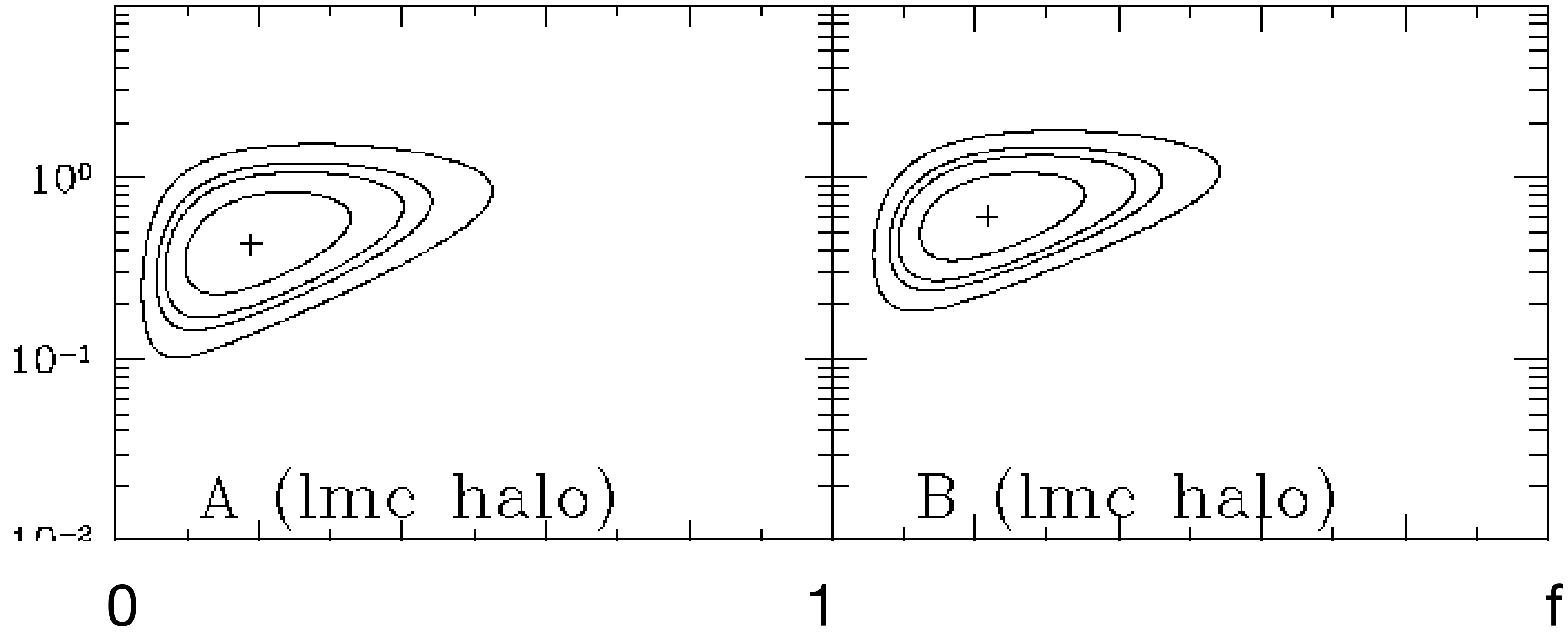
$\hat{t}$

5 years A
5 years B
2 years
1 year

Measurement of fraction of halo in compact objects, f,
(assuming a delta-function mass function):

selection criteria A B

$M/M_\odot$



68%, 90%, 95% and 99% confidence contours

BUT

LMC-5: lens identified (using HST obs & parallax fit) as a low mass MW disc star. [MACHO]

LMC-9: (only satisfied criteria B) lens is a binary, allowing measurement of low projected velocity, which suggests lens is in LMC (or source is also binary). [MACHO]

LMC-14: source is binary, and lens most likely to lie in LMC. [MACHO]

LMC-20: (only satisfied criteria B) lens identified (using Spitzer obs) as a MW thick disc star. [Kallivayalil et al.]

LMC-22: (only satisfied criteria B) supernova or an AGN in background galaxy. [MACHO]

LMC-23: varied again, so not microlensing [EROS/OGLE]

AND

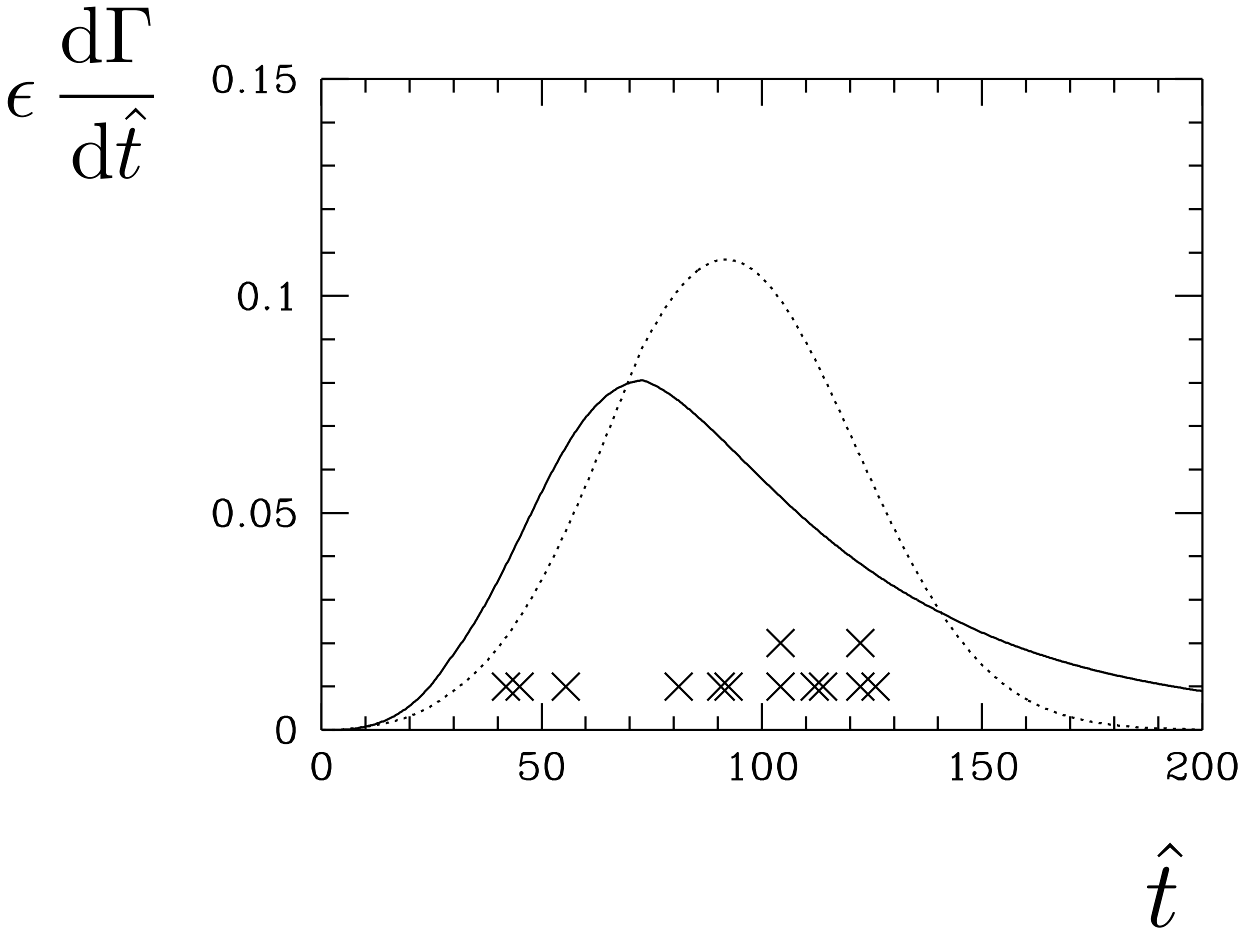
Distribution of timescales is narrower than expected for lenses in MW halo (assuming standard halo model) [Green & Jedamzik]

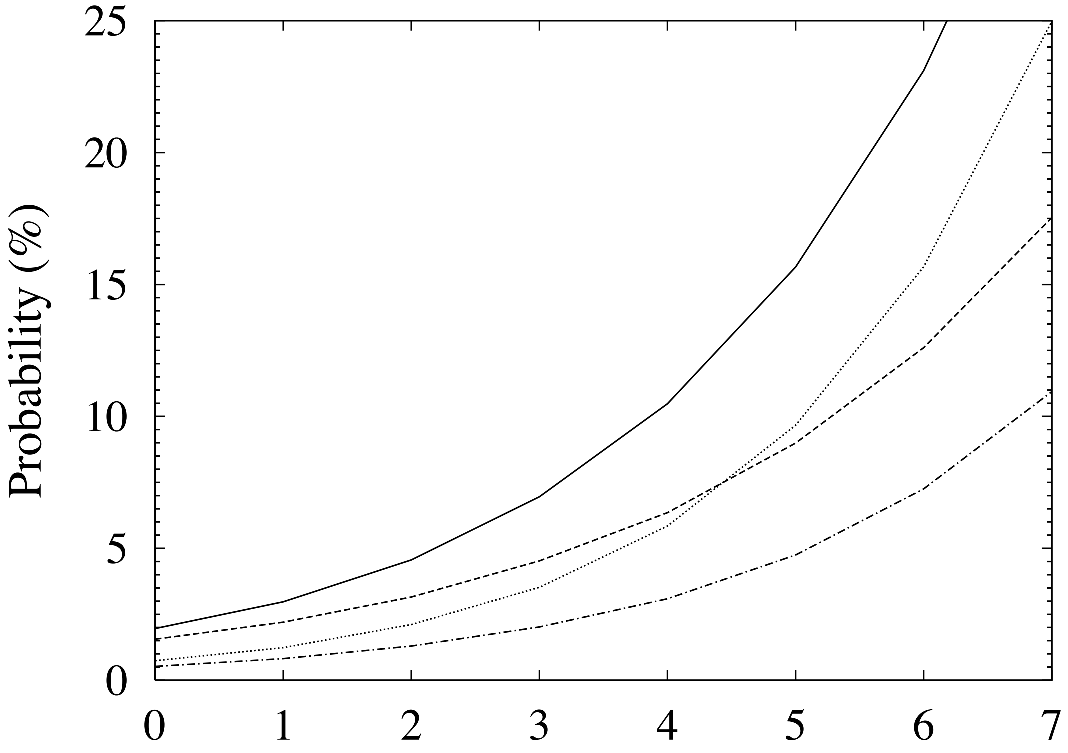
X durations of observed events

_____ best fit distribution assuming standard halo model + delta-function mass function

- - - - - best fit gaussian differential event rate

Probability of width of distribution being as small as observed, as a function of number of background events





OGLE: LMC & SMC

OGLE-II and III monitored 41 million stars in LMC and SMC for 12 years.

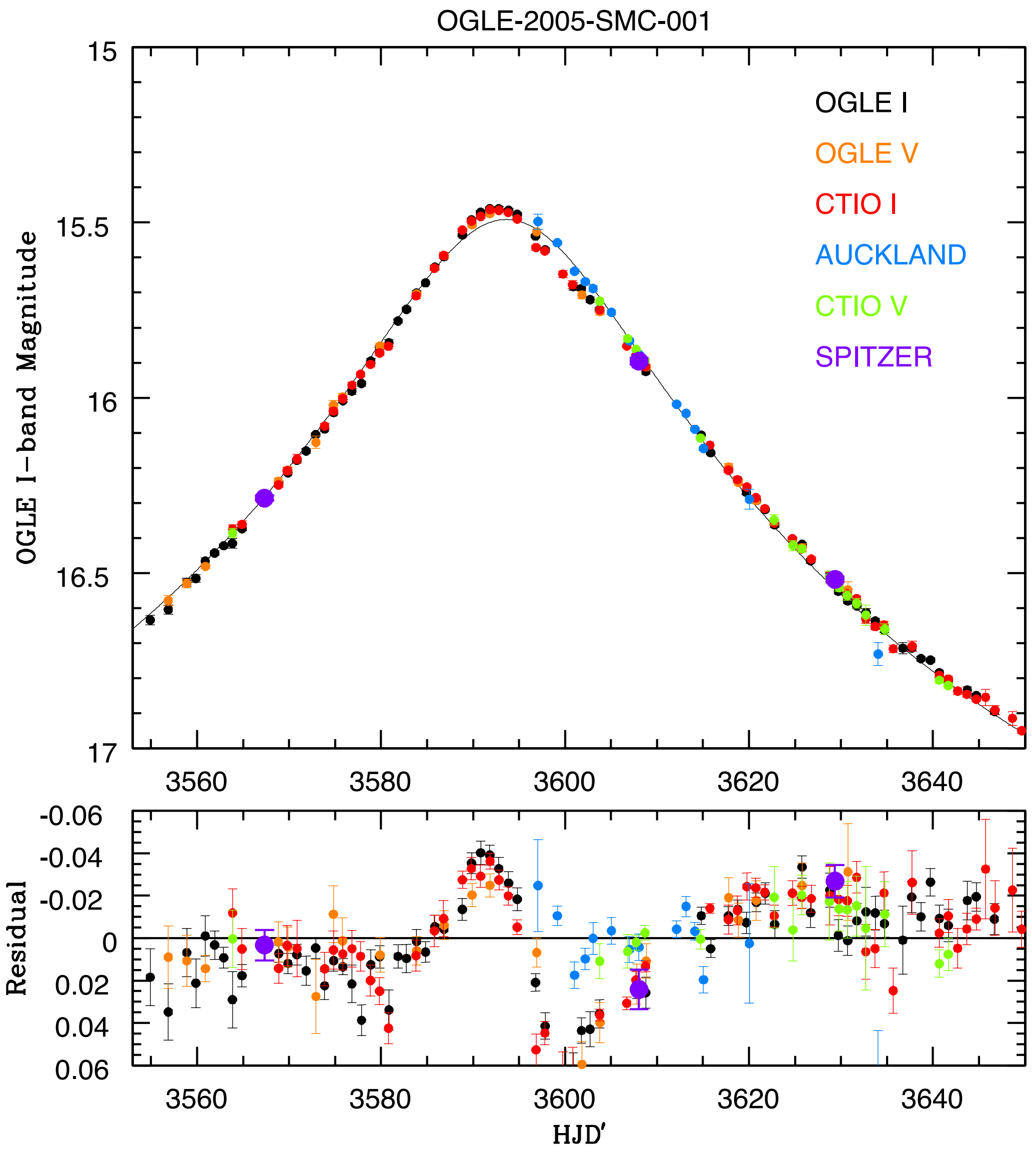
Total of 8 events. All but 1 (SMC-02) consistent (number/duration/lensed star location/detailed modelling of light curve including parallax) with lens being a star in the MW or MCs.

SMC-02

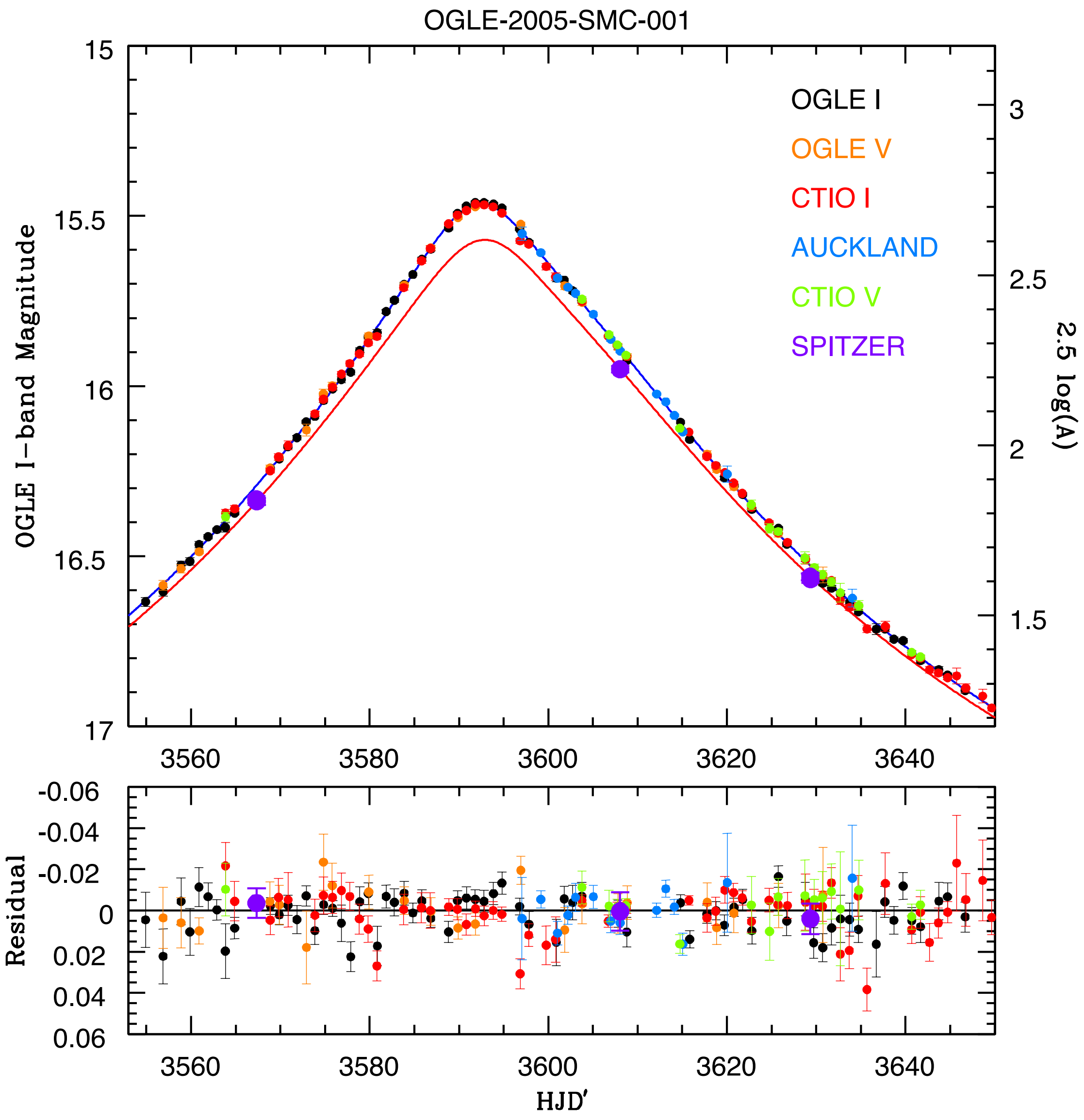
Light curve shows parallax effect and additional Spitzer observations find deviation from single lens model [Dong et al.].

Consistent with lens being a ~10 Solar mass BH binary in MW halo.

standard microlensing fit

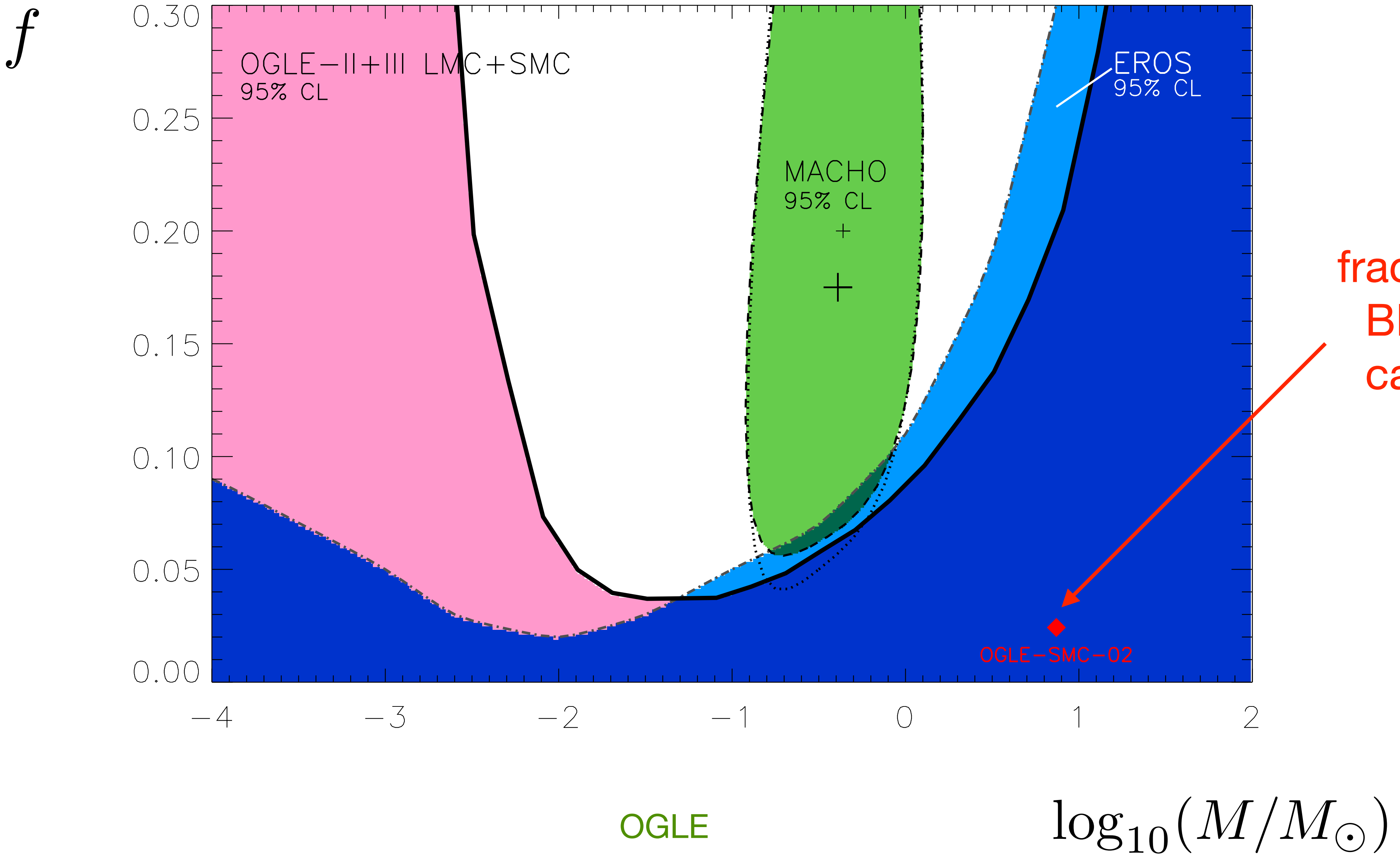


best-fit binary microlensing fit
also including parallax



Residuals

Constraints on fraction of halo in compact objects, f, (assuming a delta-function mass function):



halo
fraction from
BH binary
candidate
event

OGLE

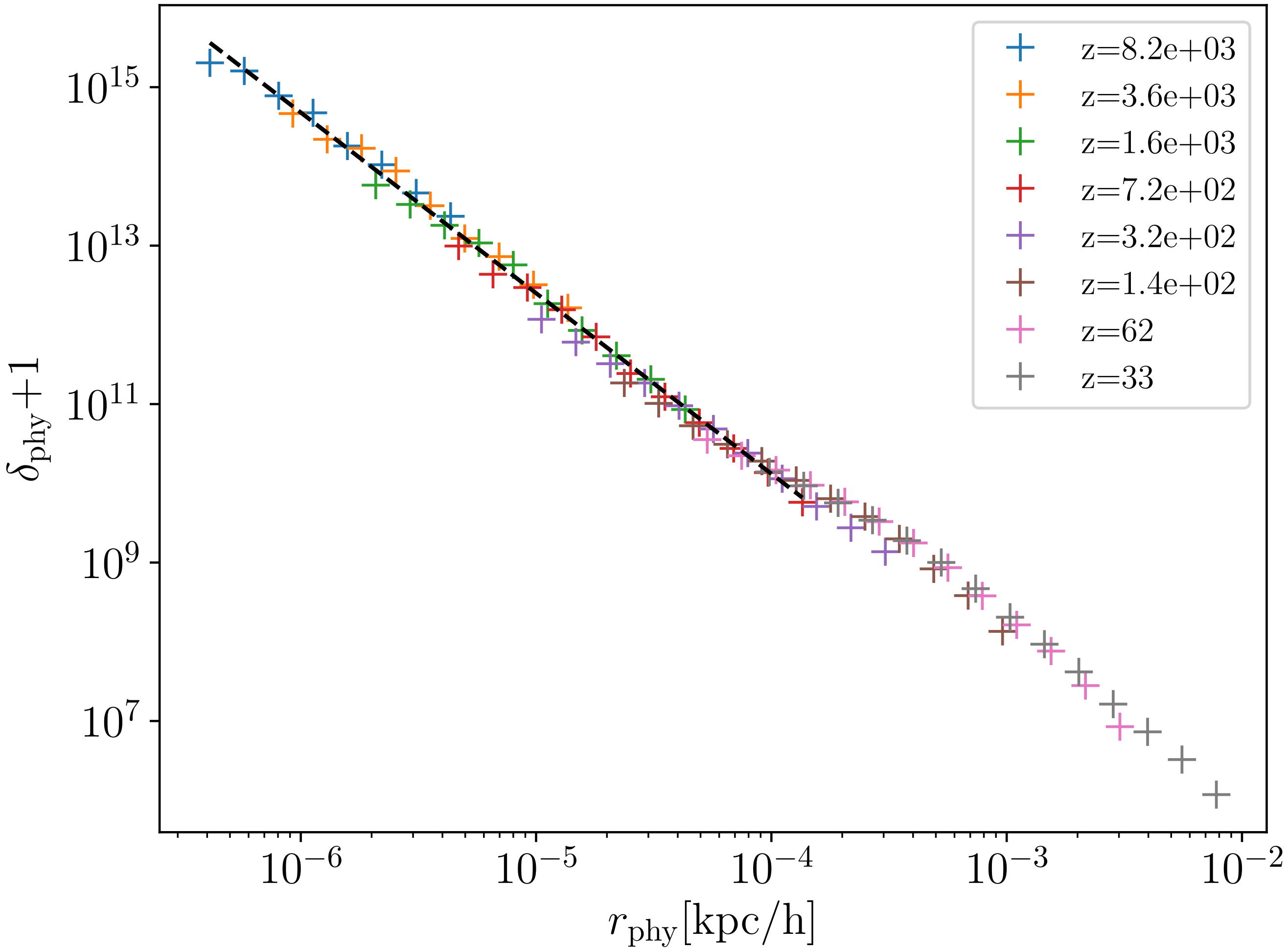
Mixed PBH + WIMP DM

mixed PBH-particle dark matter

If PBHs don’t make up all of the DM ($0 < f_{\rm PBH} < 1$) then isolated PBHs accrete a halo of particle DM with a steep density profile: $\rho(r) \propto r^{-9/4}$

Mack, Ostriker & Ricotti; Adamek et al.; Inman & Ali-Haïmoud

Density profile, in physical units, formed around a $30 M_{\odot}$ PBH



Adamek et al

If the DM were a mixture of PBHs and WIMPs would get large flux of gamma-rays (and neutrinos and positrons) from WIMP annihilation in halos around PBHs: all of the DM being a mixture of WIMPs and PBHs is excluded. Lacki & Beacom

If $f_{WIMP} \sim 1$ then $f_{PBH} \lesssim 10^{-9}$.
If $f_{PBH} \sim 10^{-3}$ (if LIGO-Virgo events are PBH binary mergers) then $f_{WIMP} \lesssim 10^{-6}$.
Adamek, Byrnes, Gosenca, Hotchkiss