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Production of primordial black holes as a probe of high-scale inflation and supersymmetry (supergravity)

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with Takahiro Terada, Yermek Aldabergenov, Daniel Frolovsky, Costas Pallis, ...

arXiv:1406.0252, 1408.6524, 1607.05366, 2302.06153, 2304.12558, ...

Our Motivation and Strategy

- generalize single-field (quintessence) models of viable inflation, having (i) attractor-type inflationary solutions, (ii) good agreement with CMB, and (iii) compatibility with supergravity, toward PBH production with minimal number of extra parameters;

- use minimal no-scale supergravity as the unifying framework for inflation, PBH production, dark matter, dark energy and spontaneous SUSY breaking; connect it to MSSM and thus unify our models with high-energy particle physics beyond the SM via gravity mediation and renormalization.

Our Tools

- the alpha-attractor models of inflation (*Kallosh, Linde, 2013*) generalizing the basic *Starobinsky* model (1980);

- the PBH formation mechanism based on an ultra-slow-roll phase of inflation between two slow-roll phases, which are generated by a near-inflection point in the inflaton potential (*Ivanov, Naselsky, Novikov, 1994*);

- no-scale supergravity implied by heterotic string compactifications and modified supergravity (*Cremmer, Ferrara, Kounnas, Nanopoulos, 1983; Starobinsky and SVK, 2011*);

- High-Scale SUSY breaking with gravity mediation of SUSY breaking to the MSSM and the visible sector (*Giudice, Strumia, 2014; Addazi, Khlopov, SVK, 2017*)

Plan of talk

- Quintessence and Starobinsky model as the basic example
- Single-field extensions of the Starobinsky potential for viable inflation
- PBH production and induced GW in the generalized models
- Inflation with a near-inflection point in the minimal no-scale supergravity
- Spontaneous high-scale SUSY breaking, dark energy and MSSM
- Renormalization group equations ("*run and match*" the effective parameters), Higgs mass, EW vacuum stability and dark matter
- Conclusion

Modified gravity

- Modified gravity theories are generally-covariant non-perturbative extensions of Einstein-Hilbert gravity theory by the higher-order terms. These terms are irrelevant in the Solar system but are relevant in the high-curvature regimes (inflation, black holes) or for large cosmological distances (dark energy).
- A modified gravity action has the higher-derivatives and generically suffers from Ostrogradsky instability and ghosts. However, there are exceptions. For example, in the modified gravity Largrangian quadratic in the spacetime curvature, the only ghost-free term is given by R^2 with a positive coefficient. It leads to the Starobinsky model (1980) of modified gravity with the action

$$S_{\rm Star.} = \frac{M_{\rm Pl}^2}{2}\int {\rm d}^4x\sqrt{-g}\left(R+\frac{1}{6M^2}R^2\right) \equiv \frac{M_{\rm Pl}^2}{2}\int {\rm d}^4x\sqrt{-g}\,F(R)\ ,$$

having the only (mass) parameter M, where $M_{\rm Pl} = 1/\sqrt{8\pi G_{\rm N}} \approx 2.4\times 10^{18}$ GeV, the spacetime signature is $(-,+,+,+,)$.

Starobinsky model of inflation

- In the high-curvature regime, the EH term can be ignored and the pure R^2-action becomes scale-invariant.
- The Starobinsky gravity has the special (attractor) solution in the FLRW universe with the Hubble function

$$H(t) \approx \left(\frac{M}{6}\right)^2 (t_{\rm end} - t) ,$$

for $M(t_{\rm end} - t) \gg 0$. This solution spontaneously breaks the scale invariance of the R^2-gravity and, hence, implies the existence of the associated Nambu-Goldstone boson called scalaron.

- Scalaron is the physical (scalar) excitation of the higher-derivative gravity. It can be revealed by rewriting the Starobinsky action into the quintessence form by the field redefinition (Legendre-Weyl transform)

$$\varphi = \sqrt{\frac{3}{2}} M_{\rm Pl} \ln F'(\chi) \quad \text{and} \quad g_{\mu\nu} \to \frac{2}{M_{\rm Pl}^2} F'(\chi) g_{\mu\nu} \, , \quad \chi = R \, ,$$

which leads to

$$S[g_{\mu\nu}, \varphi] = \frac{M_{\rm Pl}^2}{2} \int {\rm d}^4x \sqrt{-g} R - \int {\rm d}^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + V(\varphi) \right] \, ,$$

with the potential $V(\varphi) = \frac{3}{4} M_{\rm Pl}^2 M^2 \left[1 - \exp\left(-\sqrt{\frac{2}{3}} \varphi / M_{\rm Pl} \right) \right]^2 \equiv V_0 [1-y]^2$.
This potential is suitable for describing slow-roll inflation with scalaron φ as the inflaton of mass m due to the infinite plateau of the positive height $\approx V_0$ for $y \ll 1$.

- The UV cutoff of the potential is $\Lambda_{\rm UV} = M_{\rm Pl}$. The higher-order curvature terms are supposed to be suppressed by $M_{\rm Pl} \gg M$. A string theory derivation of the Starobinsky inflation is still challenging (unknown).

Starobinsky model (1980) and CMB measurements (2020)

No phenomenological input was used so far. Nevertheless, the very simple Starobinsky model of inflation is still in excellent agreement with the current CMB measurements (Planck, BICEP/Keck).

A duration of inflation is usually measured by the e-foldings number

$$N = \int_{t_*}^{t_{\text{end}}} H(t)dt \approx \frac{1}{M_{\text{Pl}}^2} \int_{\varphi_{\text{end}}}^{\varphi_*} \frac{V}{V'} d\varphi \ .$$

The standard slow roll parameters are defined by

$$\varepsilon_{\text{sr}}(\varphi) = \frac{M_{\text{Pl}}^2}{2} \left(\frac{V'}{V}\right)^2 \quad \text{and} \quad \eta_{\text{sr}}(\varphi) = M_{\text{Pl}}^2 \left(\frac{V''}{V}\right) \ .$$

The amplitude of scalar (curvature) perturbations at the horizon crossing with the pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$ is determined by the WMAP normalization,

$$A_s = \frac{V_*^3}{12\pi^2 M_{\mathrm{Pl}}^6 (V_*{}')^2} = \frac{3M^2}{8\pi^2 M_{\mathrm{Pl}}^2} \sinh^4\left(\frac{\varphi_*}{\sqrt{6} M_{\mathrm{Pl}}}\right) \approx 1.96 \cdot 10^{-9}$$

that implies no free parameters in the Starobinsky model,

$$M \approx 3 \cdot 10^{13}\ \mathrm{GeV} \quad \text{or} \quad \frac{M}{M_{\mathrm{Pl}}} \approx 1.3 \cdot 10^{-5}\,, \quad \text{and} \quad H \approx \mathcal{O}(10^{14})\ \mathrm{GeV}\,.$$

The CMB measurements give the tilt of scalar perturbations $n_s \approx 1 + 2\eta_{\mathrm{sr}} - 6\varepsilon_{\mathrm{sr}} \approx 0.9649 \pm 0.0042\ (68\%\mathrm{CL})$ and restrict the tensor-to-scalar ratio as $r \approx 16\varepsilon_{\mathrm{sr}} < 0.032\ (95\%\mathrm{CL})$. The Starobinsky inflation gives $r \approx 12/N^2 \approx 0.003$ and $n_s \approx 1 - 2/N$, with the best fit at $N \approx 55$.

Single-field extensions of Starobinsky potential

The Starobinsky inflaton potential can be generalized to the α-attractors (Kallosh, Linde, 2013) either by modifying the exponential term as (called E-models)

$$y = \exp\left(-\sqrt{\frac{2}{3\alpha}}\frac{\varphi}{M_{\rm Pl}}\right)$$

with the parameter $\alpha > 0$, or/and by using another function (called T-models)

$$V(\varphi) = V_0 \tanh^2\left(\frac{\varphi/M_{\rm Pl}}{\sqrt{6\alpha}}\right) \equiv V_0 u^2 \, , \quad u = \tanh\frac{\varphi/M_{\rm Pl}}{\sqrt{6\alpha}} \, .$$

These extensions maintain the Mukhanov-Chibisov formula for the tilt of scalar perturbations, $n_s \approx 1 - \frac{2}{N}$ but modify the tensor-to-scalar ratio as $r_\alpha \approx \frac{12\alpha}{N^2}$, so that $r_\alpha \approx 3\alpha(1-n_s)^2$.

Further generalizations of T-models and E-models

It is possible to go further, while keeping agreement with CMB observations, by defining the generalized T-type α-attractors with the scalar potential (Kallosh, Linde, 2013)

$$V_{\mathrm{T-gen.}}(\varphi) = f^2\left(\tanh\frac{\varphi/M_{\mathrm{Pl}}}{\sqrt{6\alpha}}\right) \equiv f^2(u)\,,$$

and the generalized E-type α-attractors (Vernov, Pozdeeva, SVK, 2021) with the potential

$$V_{\mathrm{E-gen.}}(\varphi) = \frac{3}{4}M_{\mathrm{Pl}}^2M^2\left[1 - y + y^2\zeta(y)\right]^2\,,$$

with regular functions $f(u)$ and $\zeta(y)$ that do not significantly affect the CMB tilts. The idea: *use this functional freedom to produce PBH on the scales below the inflationary scale.* (See also Dalianis, Kehagias, Tringas, 2019). The Starobinsky model is reproduced with $\alpha = 1$, $\zeta(y) = 0$ and $f(u) = \sqrt{3}M_{\mathrm{Pl}}^2M^2u/(1+u)$.

Power spectrum of perturbations

Primordial scalar perturbations (ζ) and tensor perturbations g (primordial GW) are defined by a perturbed FLRW metric,

$$ds^2 = dt^2 - a^2(t)\left(\delta_{ij} + h_{ij}(\vec{r})\right) dx^i dx^j \quad , \qquad i,j = 1,2,3 \; ,$$

where

$$h_{ij}(\vec{r}) = 2\zeta(\vec{r})\delta_{ij} + \sum_{b=1,2} g^{(b)}(\vec{r}) e_{ij}^{(b)}(\vec{r}) \quad , \quad H = \frac{da/dt}{a} \; ,$$

in terms of a local basis $e^{(b)}$ with $e_i^{i(b)} = 0$, $g_{,j}^{(b)} e_i^{j(b)} = 0$, $e_{ij}^{(b)} e^{ij(b)} = 1$.
The primordial spectrum $P_\zeta(k)$ of scalar (density) perturbations is defined by the 2-point correlation function of scalar perturbations,

$$\left\langle \frac{\delta\zeta(x)}{\zeta}\frac{\delta\zeta(y)}{\zeta} \right\rangle = \int \frac{d^3k}{k^3} e^{ik\cdot(x-y)} \frac{P_\zeta(k)}{P_0} \quad .$$

For instance, the observed CMB power spectrum is described by the Harrison-Zeldovich fit,

$$P_{\zeta}^{\mathrm{HZ}}(k) \approx 2.21^{+0.07}_{-0.08} \times 10^{-9} \left(\frac{k}{k_*}\right)^{n_s-1}$$

with the pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$. In the slow-roll (SR) approximation, relevant for inflation, one finds

$$P_{\zeta} = \frac{H^2}{8M_{\mathrm{Pl}}^2\pi^2}\left(\frac{1}{\varepsilon_{\mathrm{sr}}}\right) \quad .$$

Therefore, it is possible to generate a large peak (enhancement) in the power spectrum by engineering $\epsilon_{\mathrm{sr}} \to 0$, called the ultra-slow-roll (USR) regime or the PBH production mechanism based a near-inflection point in the potential. This implies the double inflation scenario (SR $\to$ USR $\to$ SR) with two plateaus in the potential $V(\varphi)$ and in the Hubble function $H(t)$. Warning: USR is not SR !

Our generalized E-model

is defined by the potential with the dimensionless parameters $(\alpha, \beta, \gamma, \theta)$ as

$$V(\varphi) = \frac{3}{4} M_{\rm Pl}^2 M^2 \left[1 - y + \theta y^{-2} + y^2 (\beta - \gamma y) \right]^2 , \; y = \exp\left(-\sqrt{\frac{2}{3\alpha}} \frac{\varphi}{M_{\rm Pl}} \right) .$$

Let us replace (β, γ) with the new parameters (ϕ_i, ξ) having better meaning as

$$\beta = \frac{1}{1-\xi^2} \exp\left[\sqrt{\frac{2}{3\alpha}} \frac{\phi_i}{M_{\rm Pl}} \right] , \quad \gamma = \frac{1}{3(1-\xi^2)} \exp\left[2\sqrt{\frac{2}{3\alpha}} \frac{\phi_i}{M_{\rm Pl}} \right] .$$

When $\xi = 0$, the potential has an inflection point at $\phi = \phi_i$; when $0 < \xi \ll 1$, there is also a local minimum (dip) $y_{\rm ext}^-$ on the r.h.s. of ϕ_i and a local maximum (bump) $y_{\rm ext}^+$ on the l.h.s. of ϕ_i, while both extrema are equally separated from the inflection point, $y_{\rm ext}^{\pm} = y_i \, (1 \pm \xi)$, (see also Iacconi, Assadullahi, Fasiello, Wands, 2021, for using this parametrization).

Good features of our model

(i) the existence of an attractor inflationary solution in good agreement with CMB measurements of the scalar tilt $n_s \approx 0.965$ within 1σ and the tensor-to-scalar ratio $r < 0.032$,
(ii) the two extra terms with the fine-tuned coefficients (β, γ) are needed for engineering a near-inflection point in the scalar potential and a large enhancement (peak) in the power spectrum of scalar perturbations, with the factor of 10^7 against the CMB level,
(iii) adding another term with a negative power of y and a small negative coefficient θ removes the infinite (Starobinsky) plateau, thus restricting from above the total number of e-folds for inflation, while being also needed for better (within 1σ) agreement with the observed tilt n_s of CMB.

USR regime

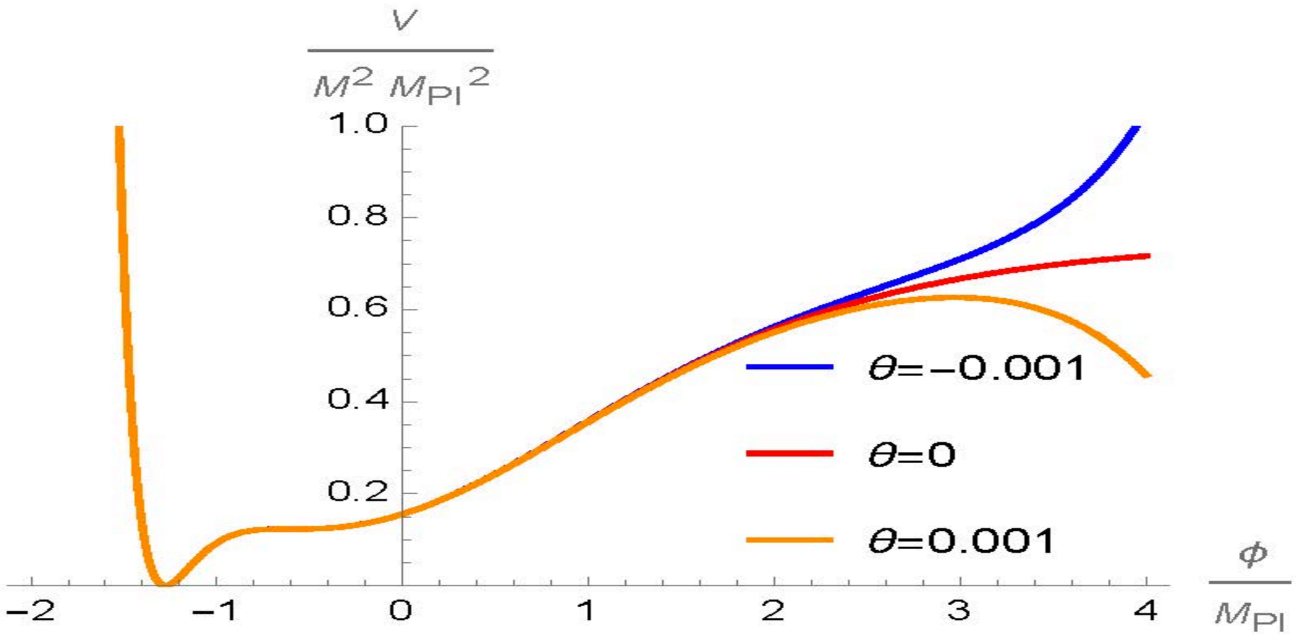
To study the USR regime, we introduce the Hubble flow functions

$$\epsilon(t) = -\frac{\dot{H}}{H^2}\,, \qquad \eta(t) = \frac{\dot{\epsilon}}{H\epsilon}\,.$$

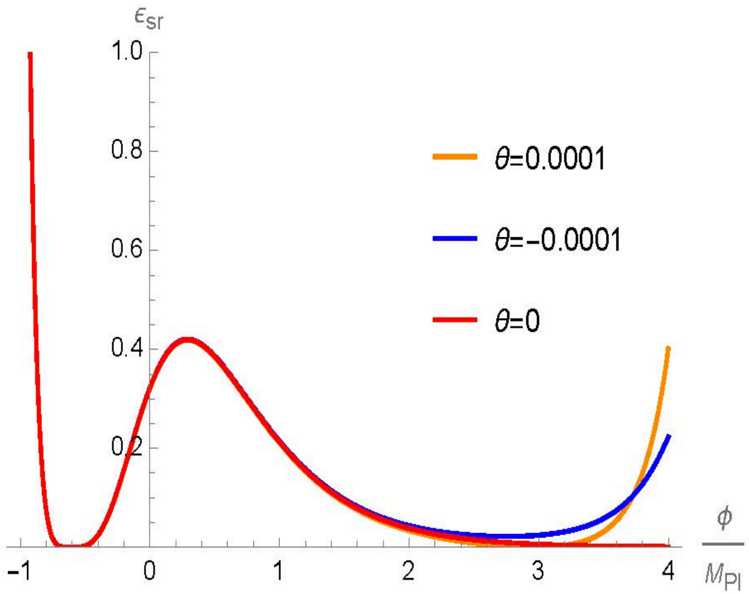
During the USR regime, the function $\epsilon(t)$ drops to very low values, whereas the function $\eta(t)$ goes from nearly zero to (-6) and back.

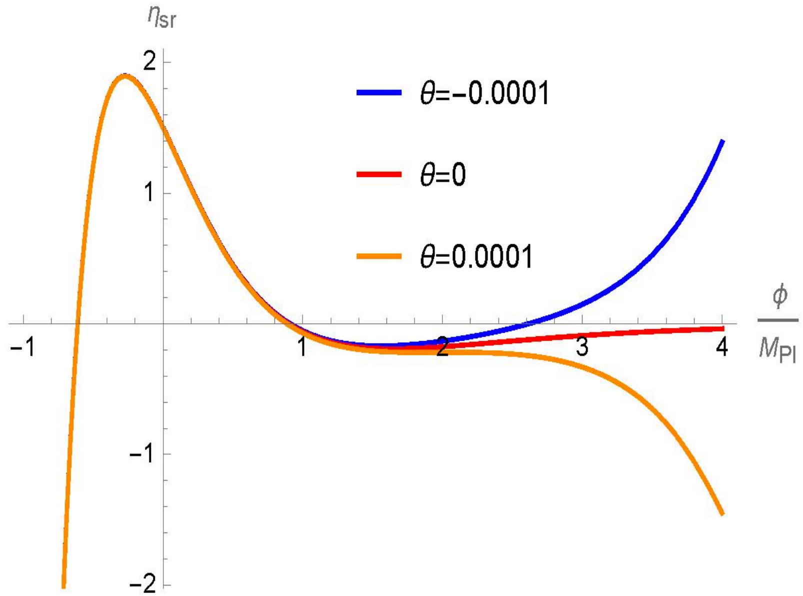
A standard procedure of (numerically) computing the power spectrum $P_R(k)$ of scalar (curvature) perturbations depending upon scale k is based on the Mukhanov-Sasaki (MS) equation. We used both approaches in our models and found that the difference between the results from numerically solving the MS equation and those derived from the SR formula is small.

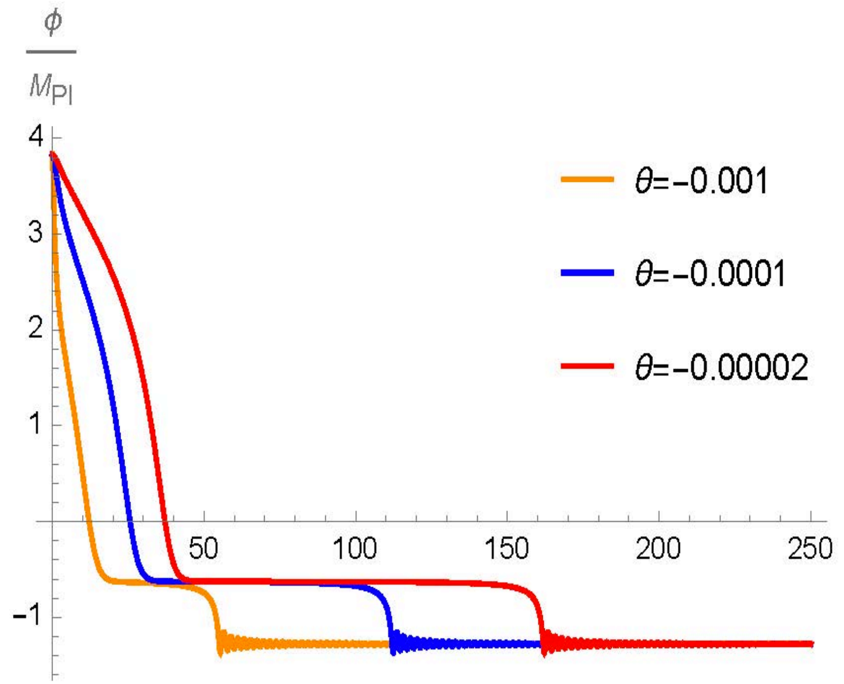
Numerical results

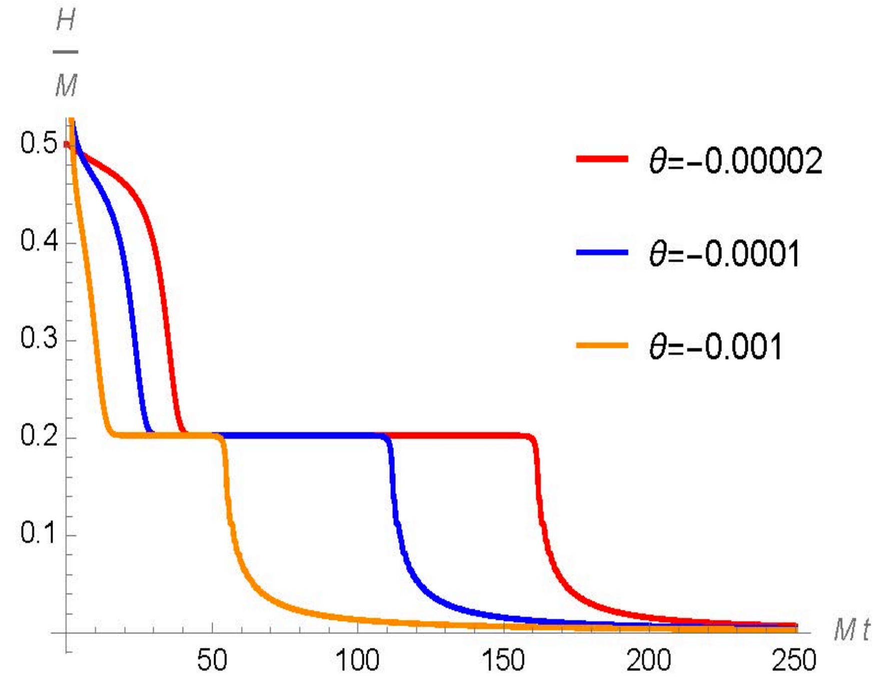


with α=0.743, ξ=0.012

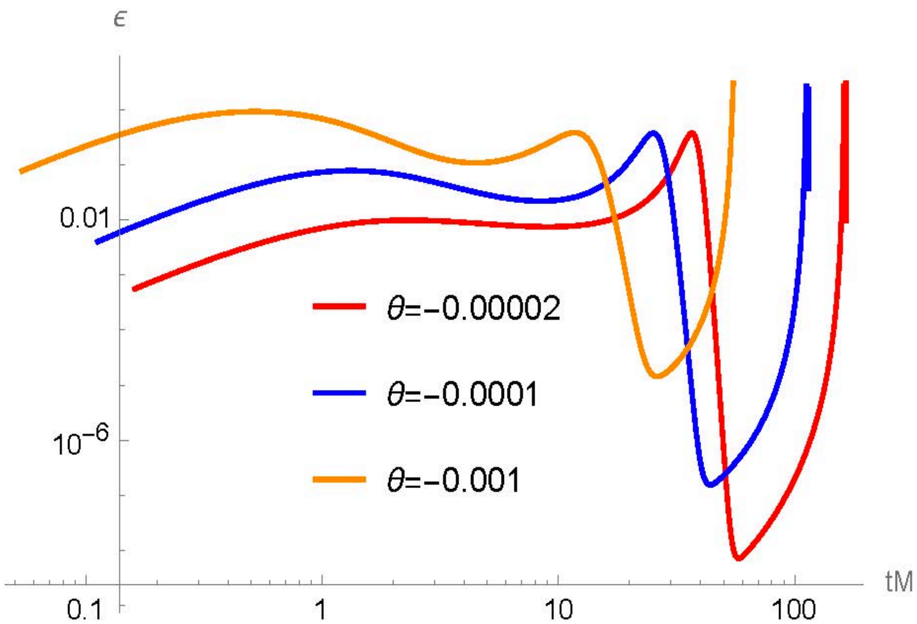


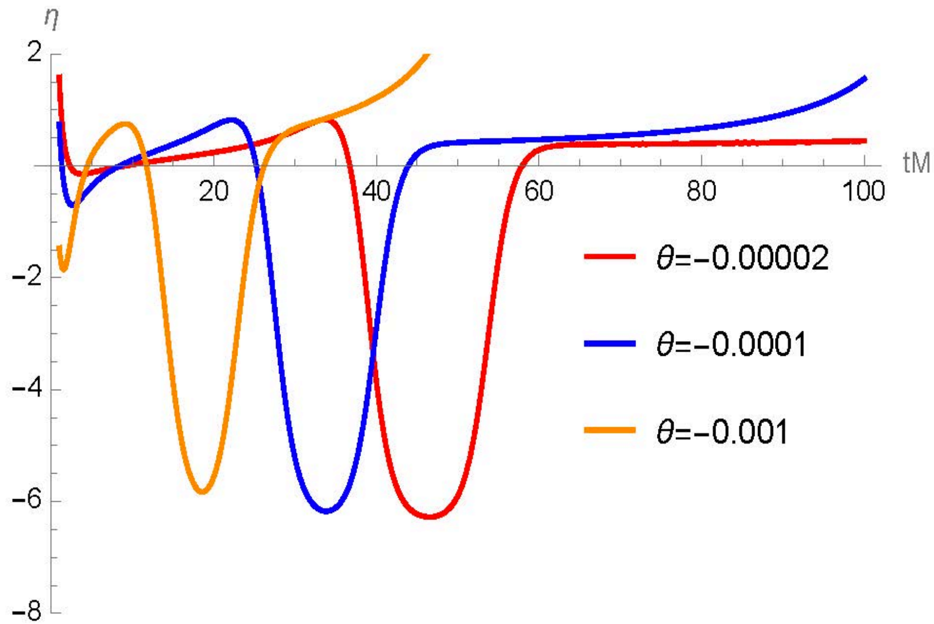




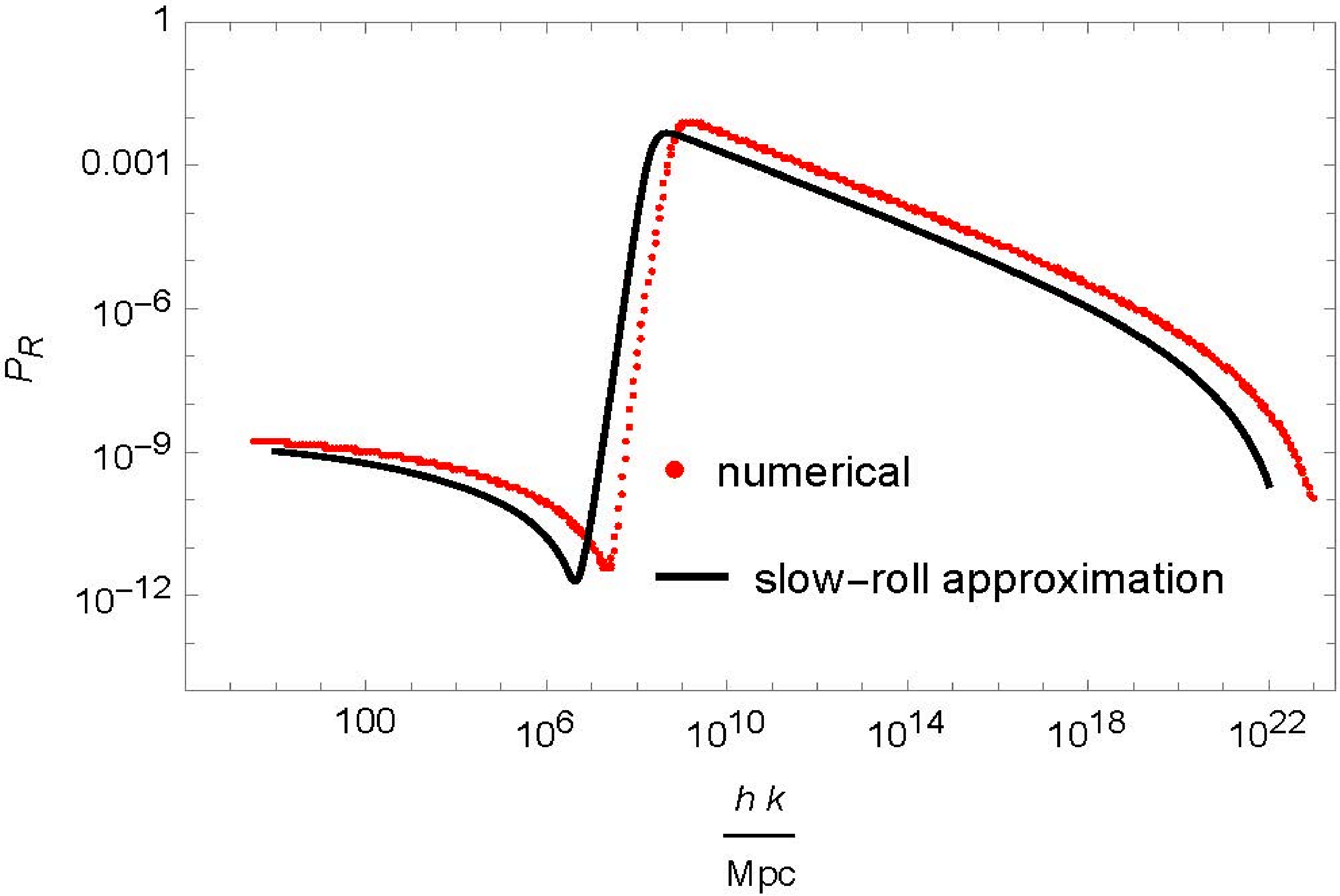


Numerical results





Comparison of our results from the Mukhanov-Sasaki equation for perturbations and from the slow-roll approximation formula



PBH masses

PBH may be formed by gravitational collapse of large density perturbations (Carr, Hawking, 1974). The masses of PBH can be estimated from given peaks (power spectrum enhancement) as follows (Pi, Sasaki, 2017):

$$M_{\rm PBH} \simeq \frac{M_{\rm Pl}^2}{H(t_{\rm peak})} \exp\left[2(N_{\rm total} - N_{\rm peak}) + \int_{t_{\rm peak}}^{t_{\rm total}} \varepsilon(t)H(t)dt\right]$$

that is very sensitive to the value of $\Delta N = N_{\rm total} - N_{\rm peak}$, while the integral gives a sub-leading correction. Increasing ΔN leads to decreasing the tilt n_s of CMB, which limits ΔN by 20 from above. On the other hand, ΔN cannot be too small when $M_{\rm PBH}$ have to exceed the Hawking (black hole) evaporation limit of 10^{15} g, which restricts ΔN from below (above 13).

After fine-tuning the parameters ξ and θ, we obtained the PBH masses in the asteroid-size range between 10^{17} g and 10^{21} g. Compare $M_{\odot} \approx 2 \cdot 10^{33}$ g.

Energy density of PBH induced GW

The present-day GW density function Ω_{GW} in the 2nd order with respect to perturbations is given by (Espinosa, Racco, Riotto, 2018)

$$\frac{\Omega_{\mathrm{GW}}(k)}{\Omega_r} = \frac{c_g}{72}\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \mathrm{d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} \mathrm{d}s \left[\frac{(s^2-\frac{1}{3})(d^2-\frac{1}{3})}{s^2+d^2}\right]^2 \times P_\zeta(kx)P_\zeta(ky)\left(I_c^2+I_s^2\right) ,$$

where the constant $c_g \approx 0.4$ in the SM, and $\Omega_r = 8.6\cdot 10^{-5}$ according to the present CMB temperature.

The variables (x,y) are related to the integration variables (s,d) as

$$x = \frac{\sqrt{3}}{2}(s+d)\ , \quad y = \frac{\sqrt{3}}{2}(s-d)\ .$$

The functions I_c and I_s of $x(s,d)$ and $y(s,d)$ are (Espinosa, Racco, Riotto, 2018)

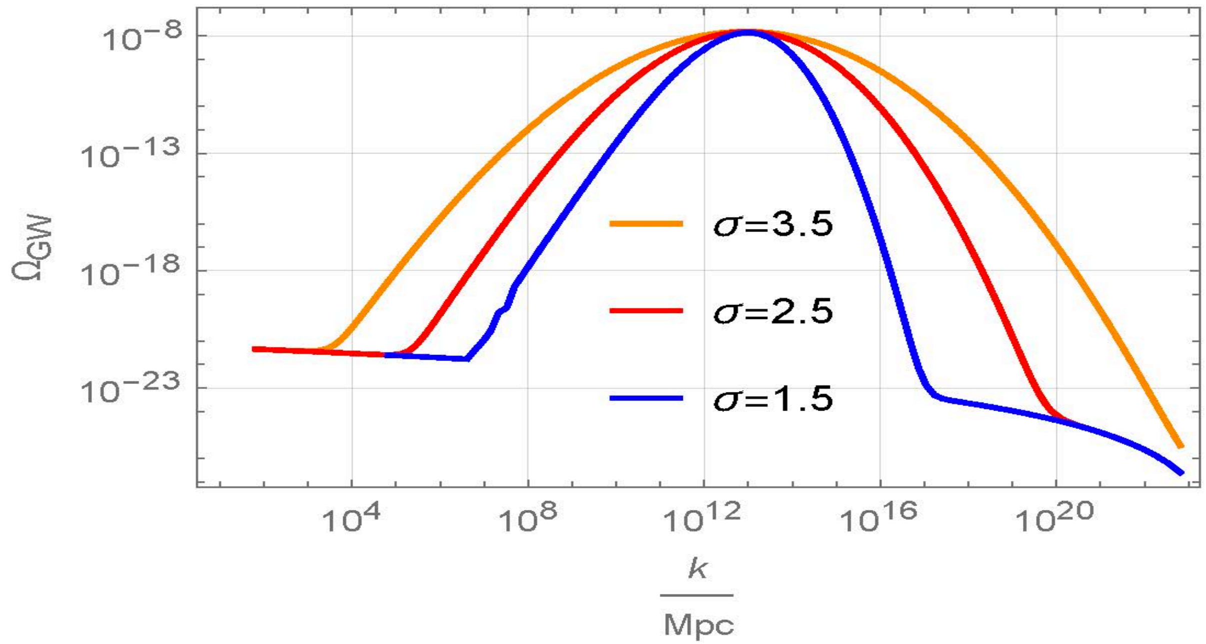
$$I_c = -36\pi \frac{(s^2+d^2-2)^2}{(s^2-d^2)^3}\theta(s-1)\ ,$$
$$I_s = -36 \frac{s^2+d^2-2}{(s^2-d^2)^2}\left[\frac{s^2+d^2-2}{s^2-d^2}\ln\left|\frac{d^2-1}{s^2-1}\right| + 2\right]\ .$$

In our models, $\Omega^{\rm GW}(k) \sim 10^{-6} P_R^2(k)$. Frequencies of PBH-induced GW are simply related to PBH masses as (De Luca, Franciolini, Riotto, 2020)

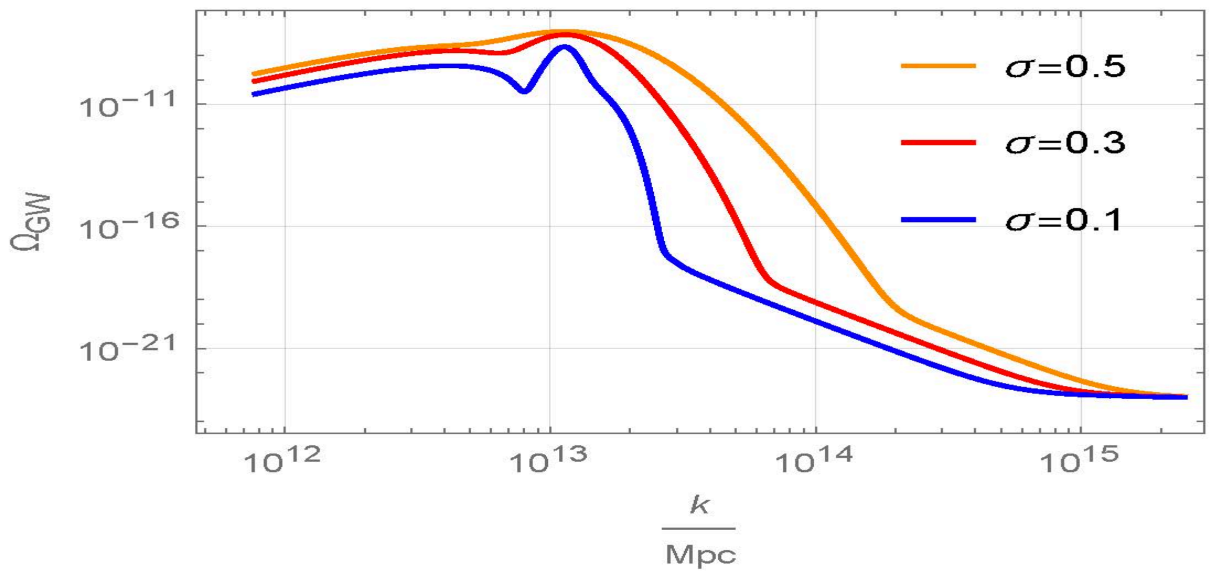
$$f \approx 5.7 \left(\frac{M_\odot}{M_{\rm PBH}}\right)^{1/2} 10^{-9}\ {\rm Hz}$$

that implies $\sim 10^{-3}$ Hz in our models, cf. NANOGrav GW frequences of 3 to 400 nHz.

Numerical results



with the peak width σ



PBH production in modified gravity after Starobinsky inflation

We propose the modified Appleby-Battye-Starobinsky (ABS) model (2010) of $F(R)$ gravity for that purpose, defined by the smooth F-function

$$F(R) = (1-g_1)R + gE_{AB}\ln\left[\frac{\cosh\left(\frac{R}{E_{AB}}-b\right)}{\cosh(b)}\right] + \frac{R^2}{6M^2} - \delta\frac{R^4}{48M^6} \ ,$$

where $g_1 = -g\tanh b$, $g \approx 2.25$ and $b \approx 2.89$, $0 < \delta < 4\cdot 10^{-6}$, and

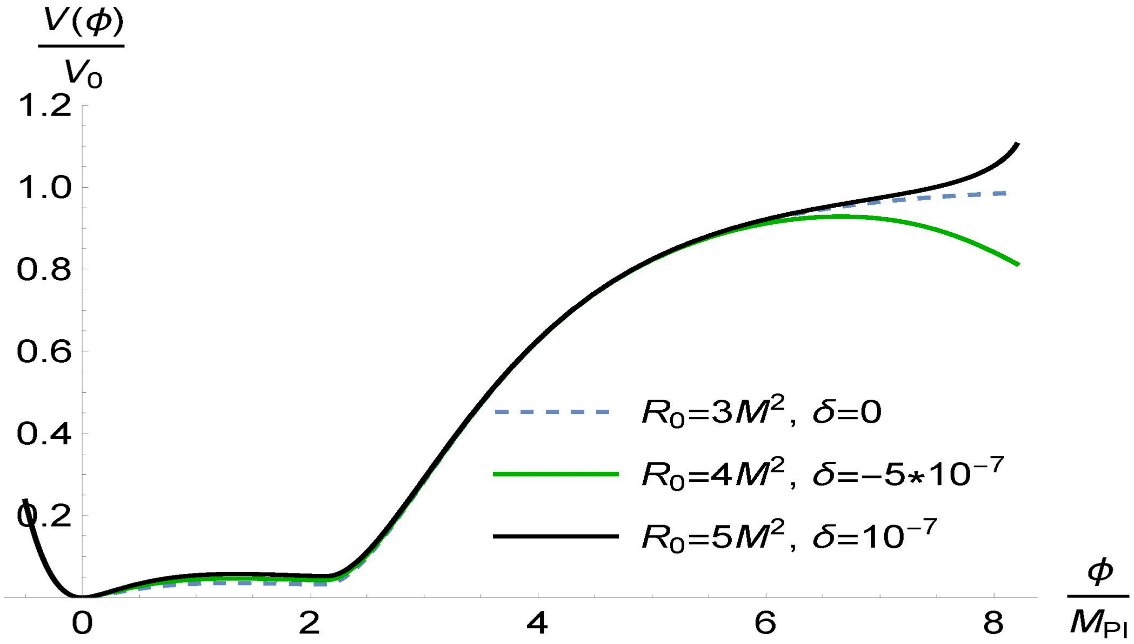
$$E_{AB} = \frac{R_0}{2g\ln(1+e^{2b})} \quad \text{with} \quad R_0 \approx 3M^2, \ \ M \sim 10^{-5}M_{\rm Pl}\ .$$

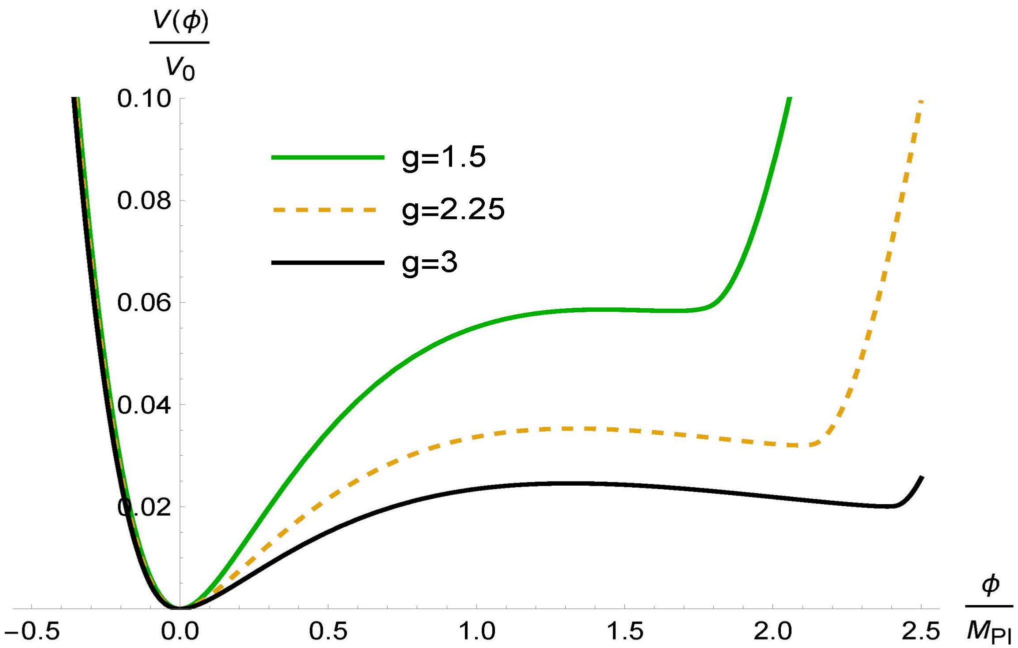
It is consistent with Starobinsky inflation and CMB measurements, has no ghosts ($F'(R) > 0$, $F''(R) > 0$), and the corresponding inflaton potential has two plateaus, leading to a large peak in the power spectrum. The last term can be interpreted as a quantum correction.

Consistency with CMB, and PBH masses

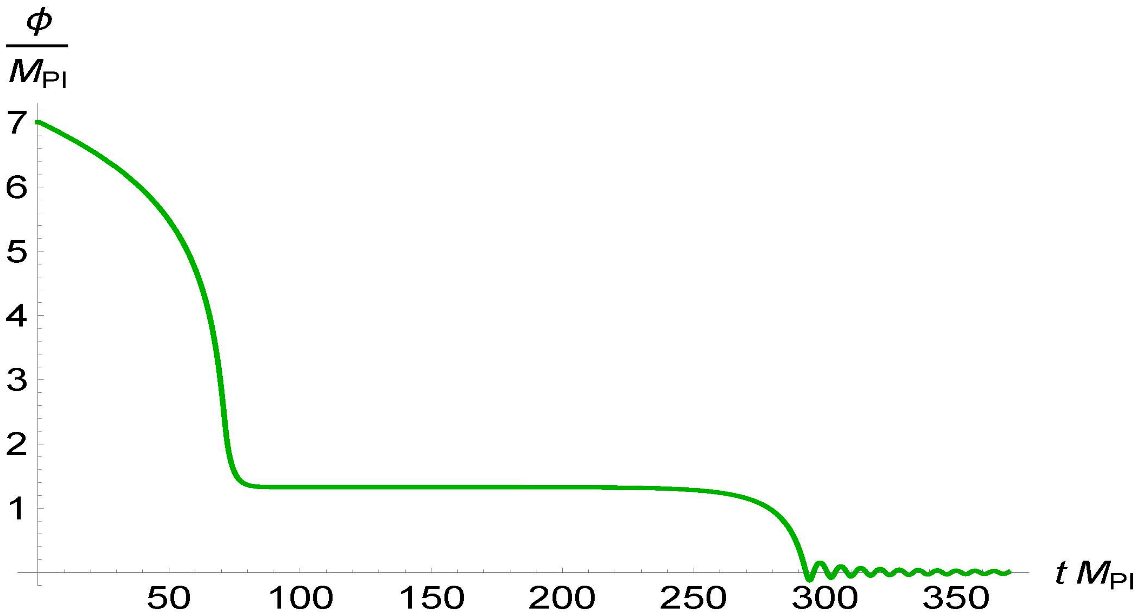
Demanding:
(i) a large enhancement (peak) in the power spectrum by the factor of 10^7 against the CMB level of 10^{-9},
(ii) consistency with the latest CMB measurements,
$n_s = 0.9649 \pm 0.0042$ (within 1σ) and $r < 0.032$, and
(iii) PBH masses beyond 10^{15} g,
we found ΔN must be restricted between 17 and 22 e-folds, while the total duration of inflation is between 54 and 66 e-folds.
The possible range of the parameter δ is between $1.02 \cdot 10^{-8}$ and $8.74 \cdot 10^{-8}$.
The PBH masses found are between 10^{16} g and 10^{20} g, i.e. of the asteroid-size again.

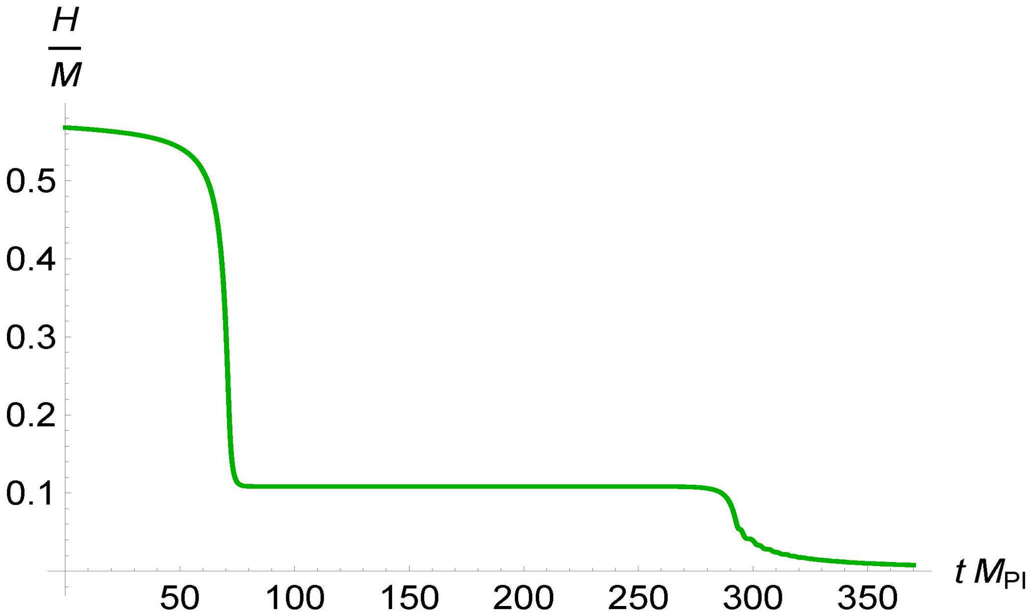
Numerical results



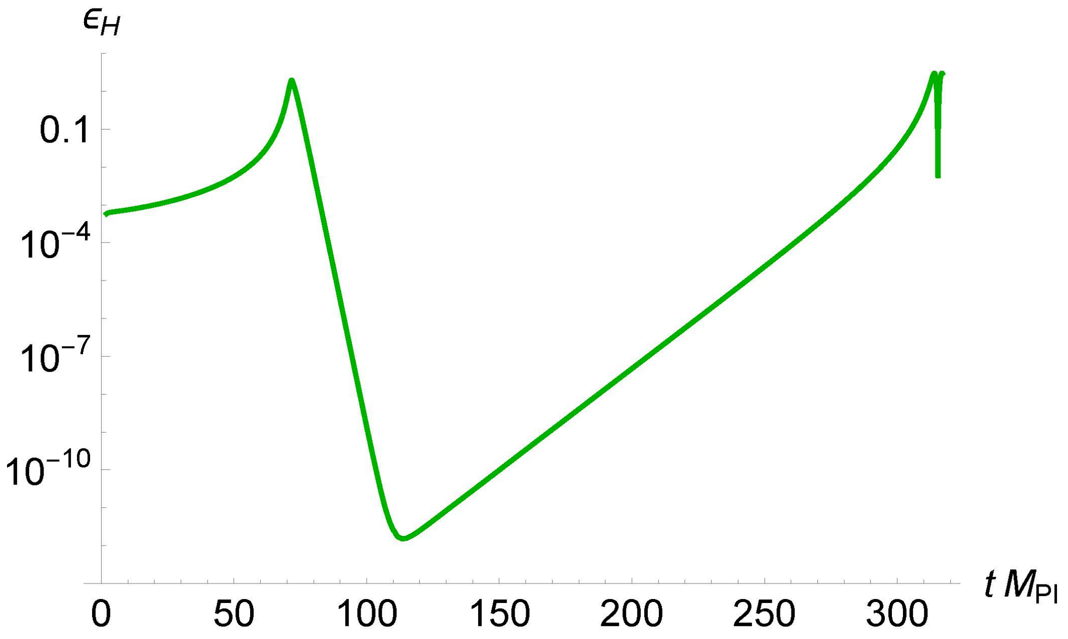


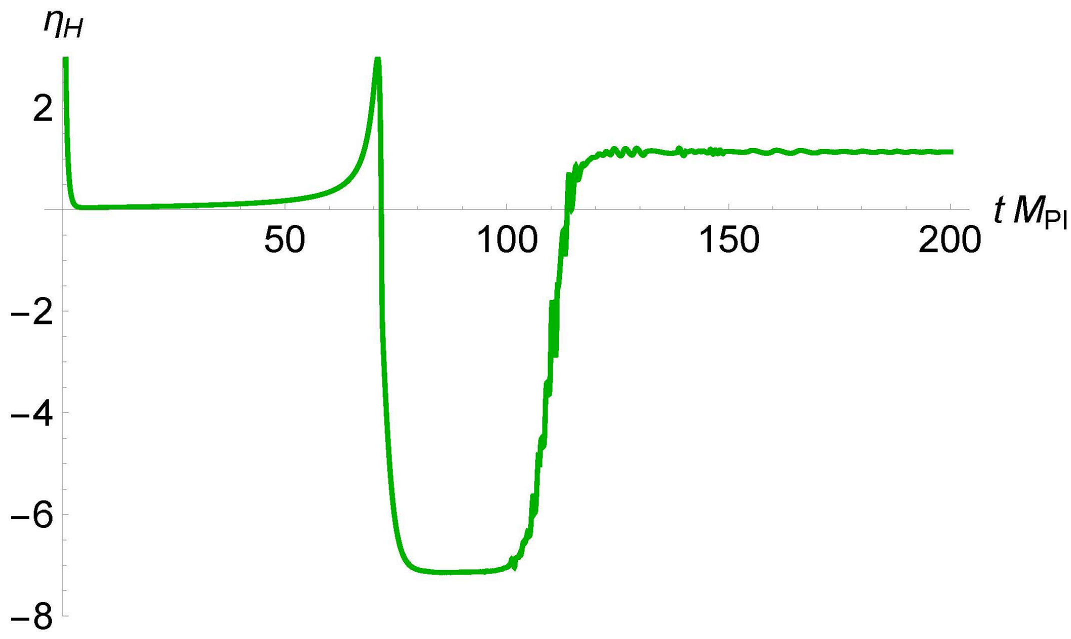
Potential and dynamics



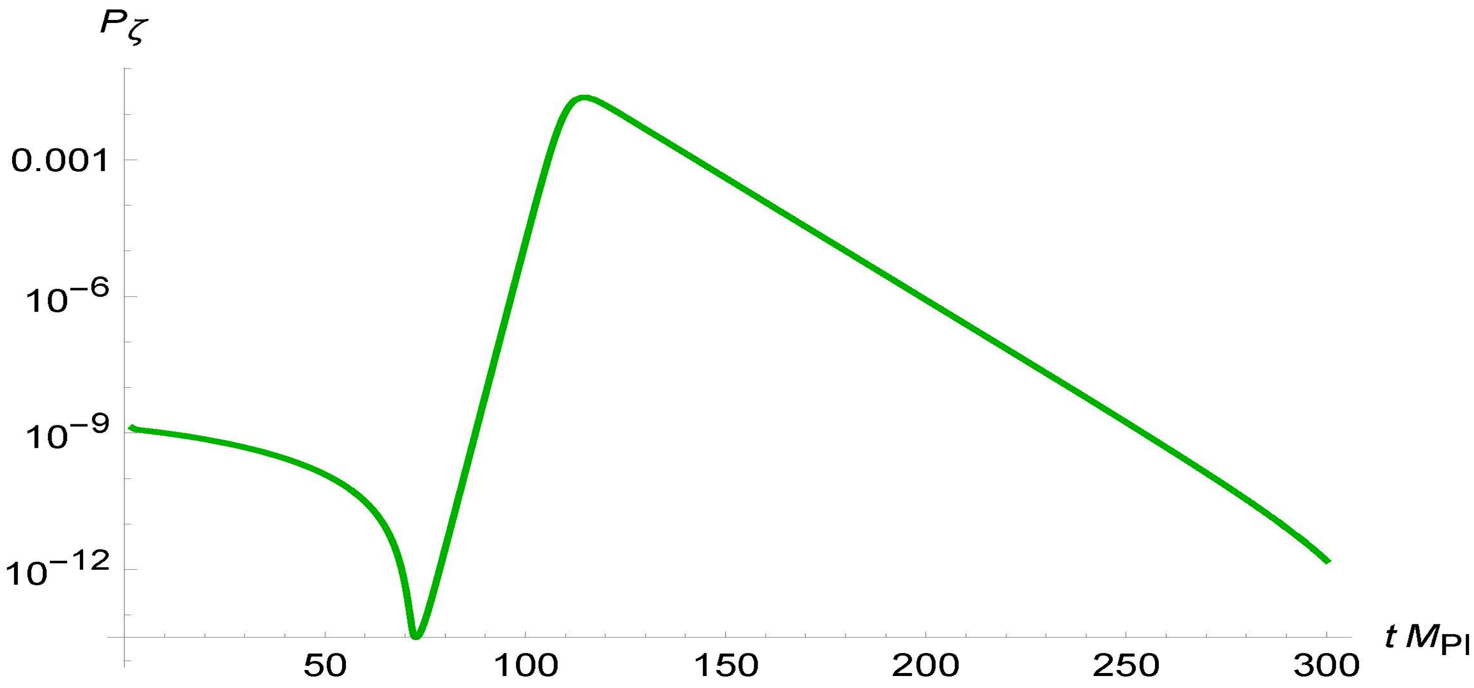


Numerical results





Hubble flow parameters and power spectrum



Modified supergravity

Modified supergravity is the (old-minimal) $N = 1$ local SUSY extension of the $(R + \alpha R^2)$ gravity. Manifest SUSY is achieved by using curved superspace. A generic action is given by a sum of D-type and F-type terms,

$$S = \int d^4x d^4\theta E^{-1} N(\mathcal{R}, \bar{\mathcal{R}}) + \left[\int d^4x d^2\Theta 2\mathcal{E} F(\mathcal{R}) + h.c \right] ,$$

where the covariantly chiral superfield $\mathcal{R}$ has the spacetime scalar curvature R among its field component. *See also Dalianis, Farakos, Kehagias, Riotto, Unge (2015)*.

The Starobinsky inflation scale $H \sim 10^{14}$ GeV (close to the GUT scale) is the scale where SUSY is expected to play a significant role.

The F-term can be included into the D-term (except a constant). We distinguish them by collecting the R-symmetry preserving terms in the N-potential, and the R-symmetry violating terms in the F-potential.

Superfield transfer to Einstein matter-coupled supergravity

After introducing the Lagrange multiplier superfield $\mathbf{T}$ as (Terada and SVK, 2013)

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left\{ -\tfrac{1}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R})N(\mathbf{S},\bar{\mathbf{S}}) + \mathcal{F}(\mathbf{S}) + 6\mathbf{T}(\mathbf{S}-\mathcal{R}) \right\} + \text{h.c.} \, ,$$

varying the Lagrangian w.r.t. the $\mathbf{T}$ gives back the original Lagrangian. On the other hand, the Lagrangian can be rewritten to the form

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left\{ \tfrac{3}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R}) \left[\mathbf{T} + \bar{\mathbf{T}} - \tfrac{1}{3}N(\mathbf{S},\bar{\mathbf{S}}) \right] + \mathcal{F}(\mathbf{S}) + 6\mathbf{T}\mathbf{S} \right\} + \text{h.c.}$$

that can be put into the standard form in supergravity,

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E} \left[\tfrac{3}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R})e^{-K/3} + W \right] + \text{h.c.} \, ,$$

where the Kähler potential K takes the no-scale supergravity form

$$K = -3\log(\mathbf{T} + \bar{\mathbf{T}} - \tilde{N}) \, , \quad \tilde{N} \equiv \mathbf{S}\bar{\mathbf{S}} - \tfrac{3}{2}\zeta(\mathbf{S}\bar{\mathbf{S}})^2 \, ,$$

but the modified supergravity origin of K and W becomes hidden.

See also Ellis, Nanopoulos and Olive (2013); first observed by Cecotti (1987).

Minimal No-scale Supergravity I

is obtained by identifying the inflaton superfield T with the goldstino superfield S (*Terada, SVK, 2014)*. The scalar potential in supergravity reads $(M_{\text{Pl}} = 1)$

$$V_{\text{SUGRA}} = e^{G}\left[G_{,T}\left(G_{,T\bar{T}}\right)^{-1}G_{,\bar{T}} - 3\right]$$

where $G = K + \ln|W|^2$. For example, when $K = -3\ln(T + \bar{T})$ and $W = W_0 T^3$, one gets $V = 0$, while SUSY can be broken along a flat direction. Spontaneous SUSY breaking occurs when $\langle F_T\rangle \neq 0$ with $m_{3/2} = \left\langle e^{G/2}\right\rangle$. Then goldstino is eaten up by gravitino (the well known super-Higgs mechanism). To realize inflation with $V > 0$, one can add a stabilizing term as (*Pallis, 2023*)

$$K = -p\ln\left[T + \bar{T} + \xi^2(T + \bar{T} - 2v)^4\right]$$

with the new parameters (p, ξ, v).

Minimal No-scale Supergravity II

The superpotential is fixed by demanding no-scale $(V = 0)$ in the absence of the stabilizing term $(\xi = 0)$. It yields

$$W = W_1 + W_2, \quad W_{1,2} = m_{1,2} T^{q_{1,2}}, \quad q_{1,2} = \frac{1}{2}\left(p \pm \sqrt{3p}\right) ,$$

with mass scales $m_{1,2}$. The m_1 is identified with the inflation scale $\sim 10^{13}$ GeV, and m_2 is identified with the dark energy (c.c.) scale $\sim 10^{-3}$ eV.
The stabilizing term breaks no-scale, leading to a positive potential, selects the vacuum with $\langle T \rangle = v$, and stabilizes the inflationary trajectory along $T = \bar{T}$ by giving a mass $\mathcal{O}(m_{\text{inf}})$ to sinflaton (phase of T). Good solutions (without branch cuts) arise for integer powers $q_{1,2}$. For instance, $q_{1,2} = (3, 9)$ for $p = 12$.

Minimal No-scale Supergravity, PBH and MSSM

The remaining parameters can be tuned as $\xi \approx 1.6$ and $v \approx 0.25$ to get a near-inflection point in the effective single-field inflaton potential (*Pallis, 2023*), which leads to the PBH production very similar to that obtained in our models without SUSY. However, there is much more!

Spontaneous high-scale SUSY breaking takes place with $m_{3/2} \sim 10^{11}$ GeV. The MSSM can be added to the minimal no-scale supergravity at the high scale by modifying the superpotential and the Kähler potential, $W \to W + W_{\text{MSSM}}$ and $K \to K + K_{\text{MSSM}}$, where

$$W_{\text{MSSM}} = h_{\alpha\beta\gamma}\Phi_\alpha\Phi_\beta\Phi_\gamma + \mu H_u H_d \ , \quad \Phi_\alpha = (Q, L, d^c, u^c, e^c, H_d, H_u) \ ,$$

and $K_{\text{MSSM}} = \sum_\alpha |\Phi_\alpha|^2$. The EFT is then obtained by the RGE renormalizing the parameters $(h_{\alpha\beta\gamma}, \mu)$ by the factor $\langle T \rangle^{-p/2}$ and leading to soft SUSY breaking terms after decoupling of supergravity in the limit $M_{\text{Pl}} \to \infty$. Consistency with the observed Higgs mass $M_H = (125.15 \pm 0.25)$ GeV is also achieved.

High-scale SUSY breaking, MSSM and Higgs mass

$$H_{\rm SM} = H_u \sin\beta + H_d^\dagger \cos\beta\ , \quad \lambda_{\rm SUSY} = \frac{1}{4}\left(g_1^2 + g_2^2\right)\cos^2 2\beta\ ,$$

$$m_{\rm H} = (125.15 \pm 0.25)\ {\rm GeV}\ , \quad m_{\rm t} = (173.134 \pm 0.76)\ {\rm GeV}\ ,$$

imply via the MSSM 2-loop RGE (Giudice, Strumia, 2014)

$$m_{3/2} \leq \mathcal{O}(10^{12})\ {\rm GeV} \quad {\rm and} \quad \tan\beta \sim \mathcal{O}(1)\ ,$$

as well as stability of the EW vacuum. However, SUSY cannot be responsible for the hierarchy $m_{\rm H}/M_{\rm Pl} \sim 10^{-16}$, cf. the cosmological constant fine-tuning of 10^{-120}. In the gravity decoupling limit, soft SUSY breaking terms ensure stability.

- Baryogenesis via non-thermal leptogenesis can be activated, cf. *Jeong, Kamada, Starobinsky, Yokoyama (2023)* for Starobinsky inflation+supermassive RH Majorana neutrinos.

Conclusion I

- Our approach is motivated by modified gravity and supergravity, leads to viable inflation, efficient PBH production and induced GW, and can be consistently connected to MSSM and SM.

- The PBH masses are possible in the window from 10^{17} g to 10^{21} g, in all our models. Those PBH may form (the whole or part of) current dark matter.

- The PBH-induced GW may be detectable by the future space-based gravitational interferometers (LISA, DECIGO, TianQin, Taiji) with mHz frequency.

- The near-inflection mechanism of PBH production can be employed in the minimal no-scale supergravity with the stabilizing term and viable single-large-field inflation. *This is different from multi-field inflation and PBH: SUSY2023 in UK.*

Conclusion II

- Unification of inflation and PBH production with high-scale SUSY breaking, MSSM and dark energy (tiny c.c.) is possible in the minimal no-scale supergravity in agreement with the known Higgs mass and the (meta)stable EW scale ($\lambda_H \geq 0$). There is no Polonyi problem and no overproduction problem but no SUSY explanation for the scale hierarchy (fine tuning).

- After inflation, inflaton decays into other particles (gravitinos, etc.), while heavy LSP gravitinos with the mass of $\mathcal{O}(10^{11})$ GeV is also a candidate for dark matter (Addazi, Khlopov, SVK, 2017). In our model, $T_{\rm reh} \sim 10^7$ GeV.

- PBH production during inflation in supergravity leads to the significant constraints on the parameters of high-energy particle physics and strong predictions: (i) high-scale SUSY breaking, (ii) PBH & gravitino DM, (iii) the MSSM mixing angle, $\tan\beta \approx 1$, etc.

Thank You for Your Attention!