Primordial Black Holes

from R^2 gravity theory with a non-minimally coupled scalar field

Xinpeng Wang

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XW, Misao Sasaki & Ying-li Zhang, in preparation.

Contents

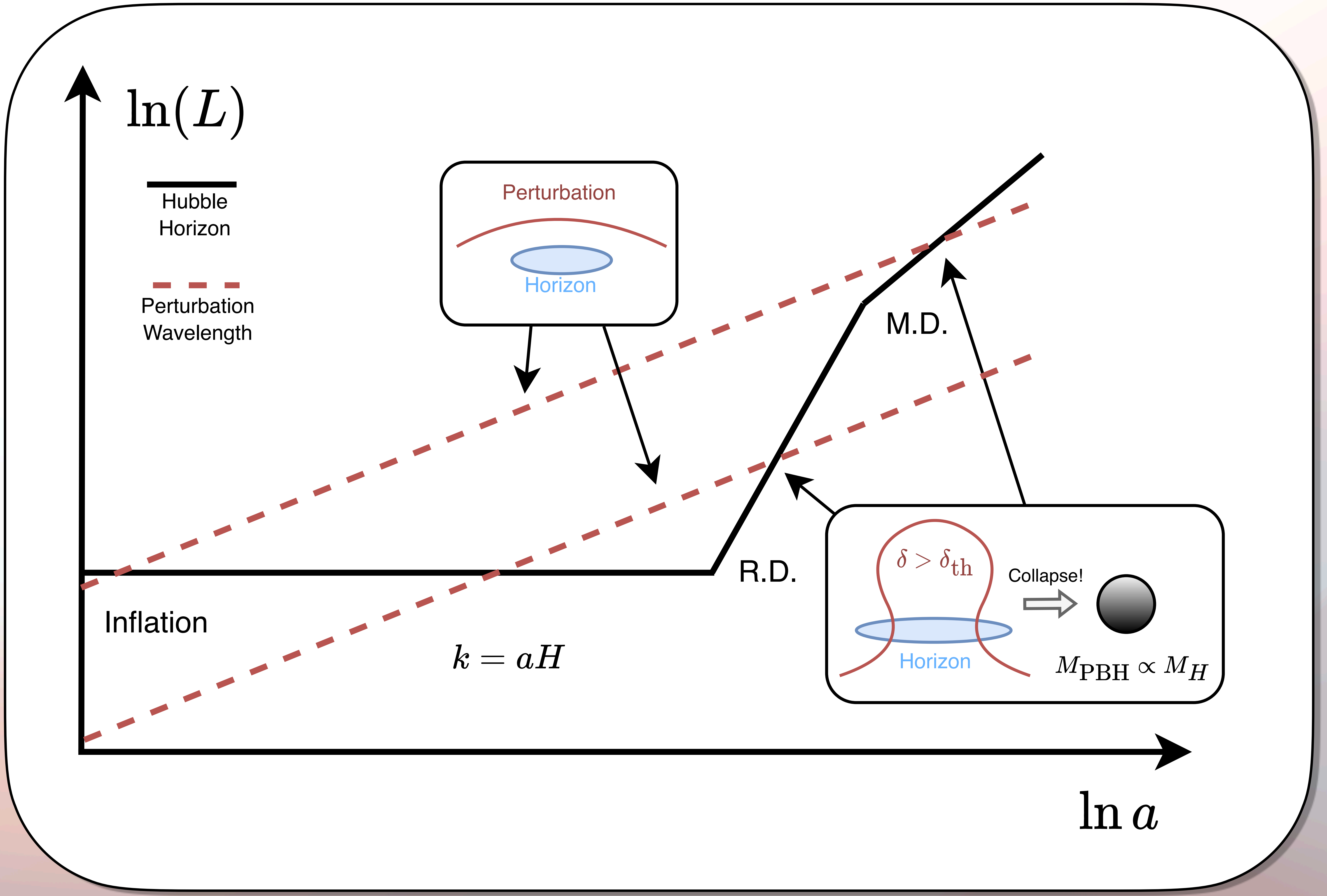
Inflation and Primordial Black Holes (PBHs)

Zel'dovich and Novikov, 1966; Hawking, 1971

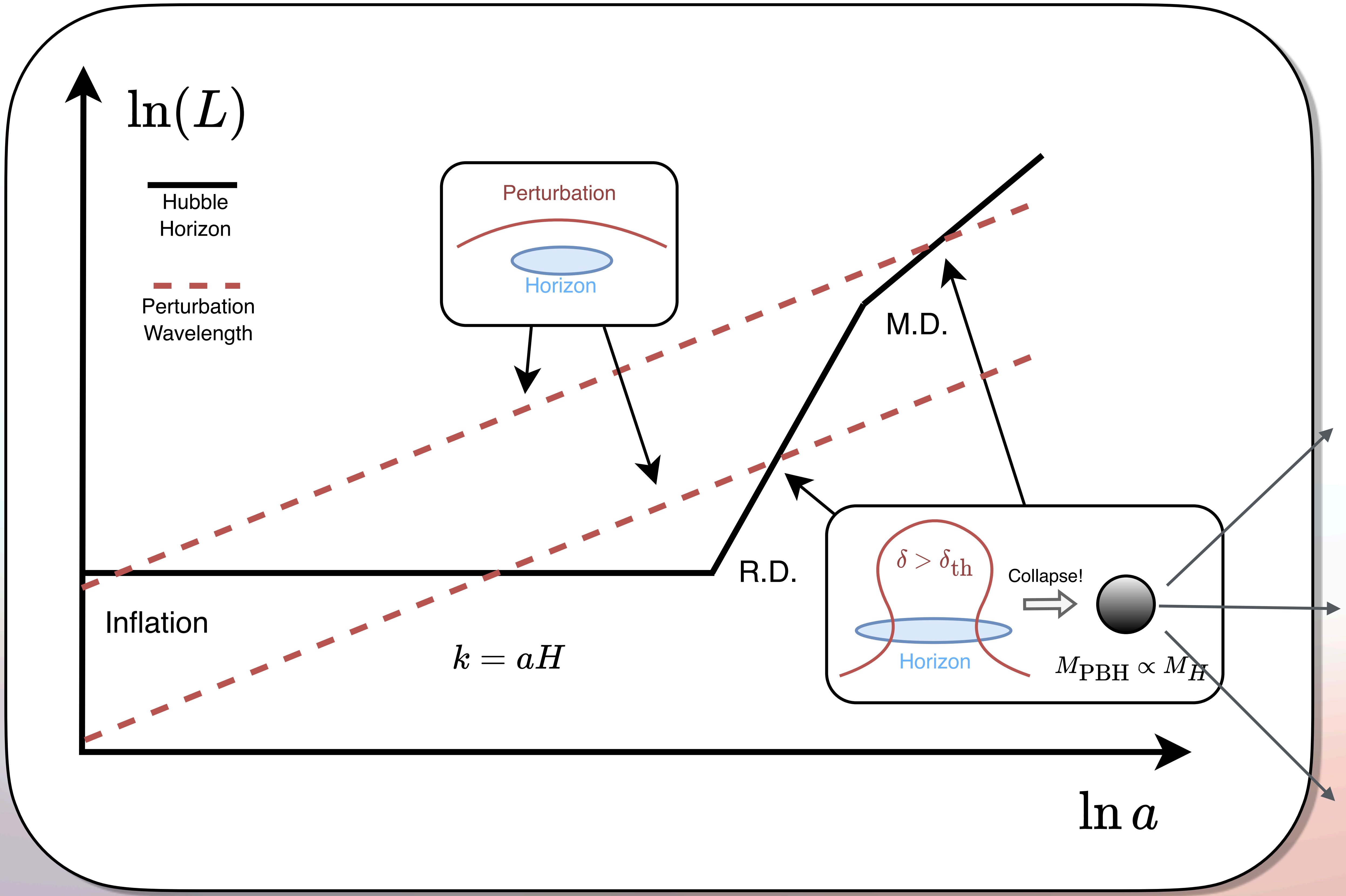
BHs form in the early universe!

1. **Large density perturbation**
2. **Domain Wall**
3. **Vacuum bubbles**
4. **Cosmic string loops**
5. **Q balls**

……



Inflation and Primordial Black Holes (PBHs)

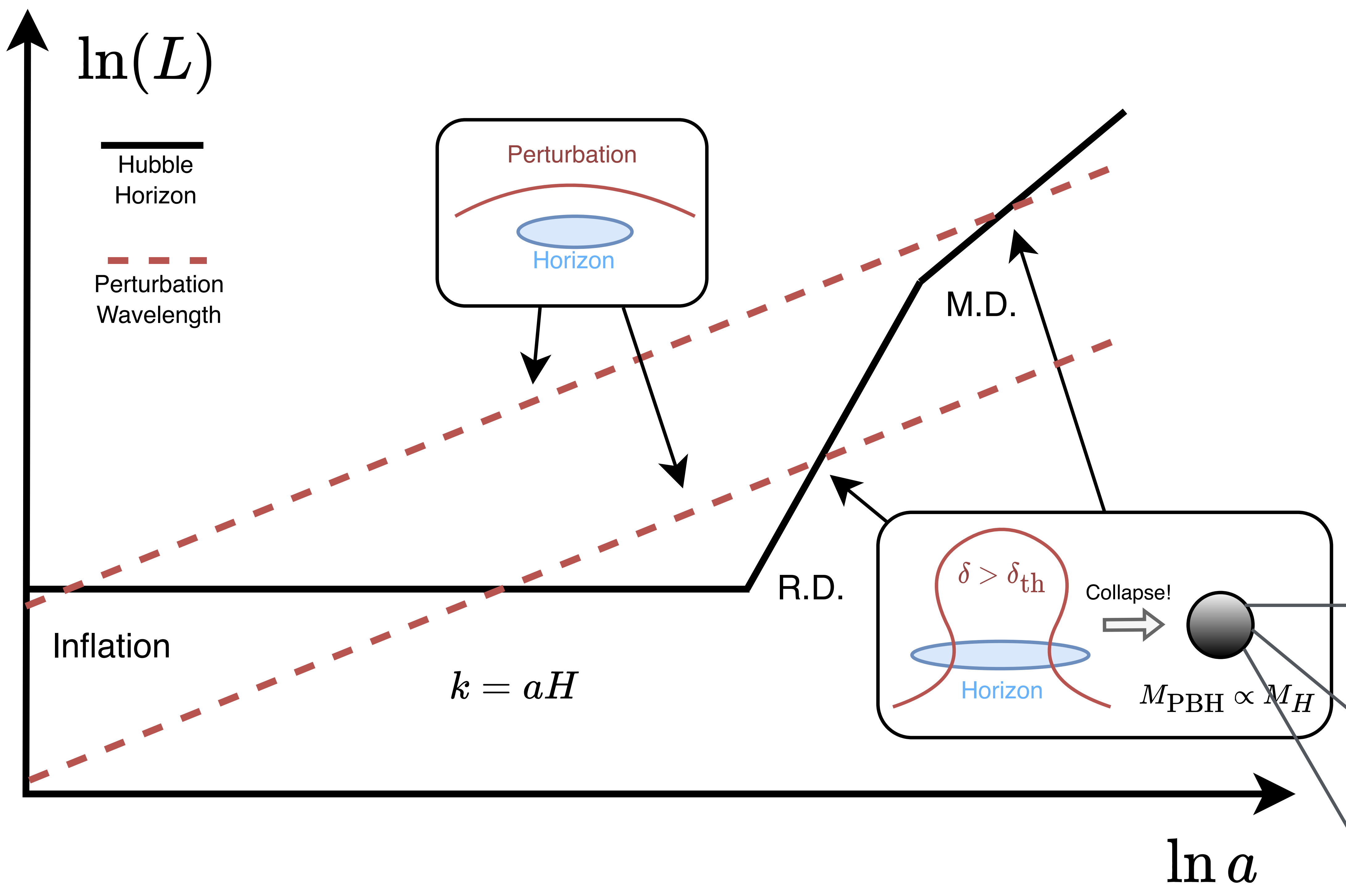


SGWB
- Large scalar perturbation
- PBH binary
- Initial PBH distribution...

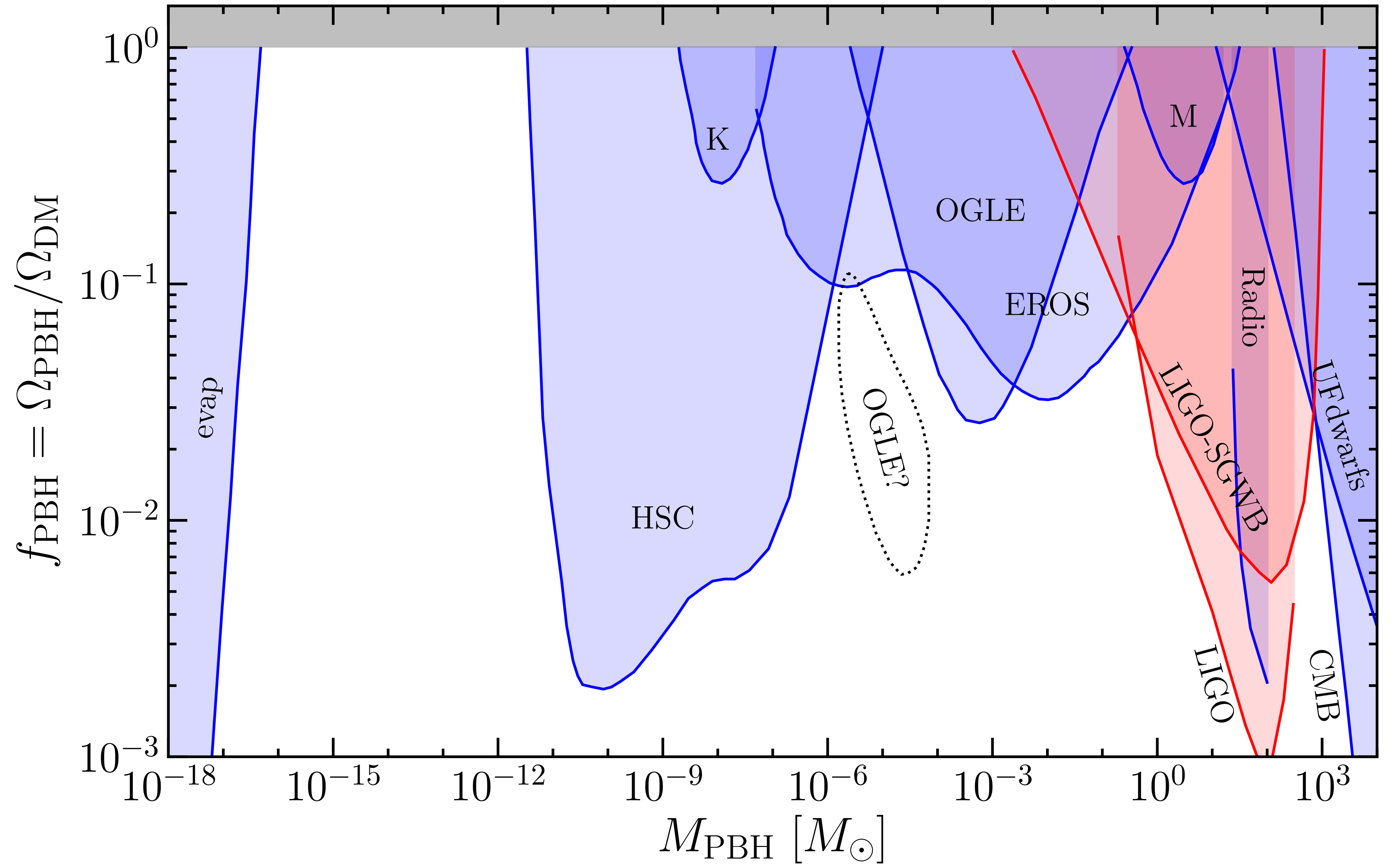
Dominate CDM
- Asteroid mass PBHs
- $M \sim 10^6 \sim 10^{22} g$

Candidate for SMBHs /Subsolar mass BHs
- Galatic Core?
- OGLE Observation?

Inflation and Primordial Black Holes (PBHs)



https://github.com/bradkav/PBHbounds



Dominate CDM — Asteroid mass PBHs $M \sim 10^6 \sim 10^{22} g$

Candidate for SMBHs /Subsolar mass BHs — Galatic Core? OGLE Observation?

Main Target and Previous Works

Fit the CMB Observation.

Produce big Curvature Power Spectrum $\mathscr{P}_{\mathscr{R}}$ / POSITIVE Non-Gaussianity at $k > k_{\rm CMB}$.

End inflation in 50-60 e-folds.

Main Target and Previous Works

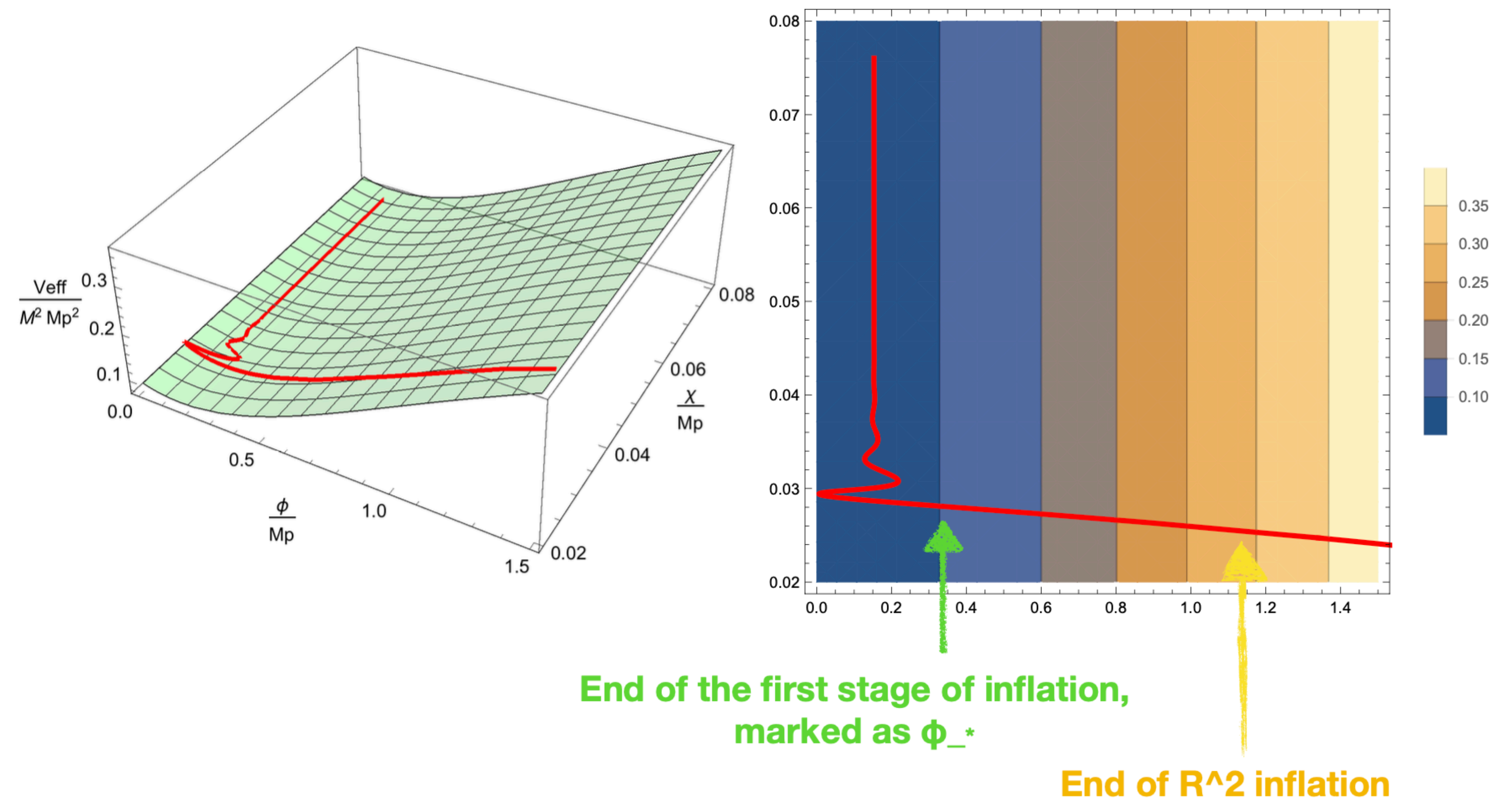
R^2 gravity and a non-minimally coupled scalar field ——— 2 stage slow rolls connected by a right angle turn of inflation

Shi Pi, Ying-li Zhang, Qing-Guo Huang and Misao Sasaki, 2017 (1712.09896)

$$S_J = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}f(R) - \frac{1}{2}g^{\mu\nu}\partial_\mu\partial_\nu\chi - V(\chi)\right],$$

$$f(R) = R + \frac{R^2}{6M^2} - \frac{\xi R}{M_{\rm pl}^2}\chi^2,$$

$$V(\chi) = V_0 - \frac{1}{2}m^2\chi^2\,.$$



Fit the CMB Observation.

Produce monochromatic PBH mass function.

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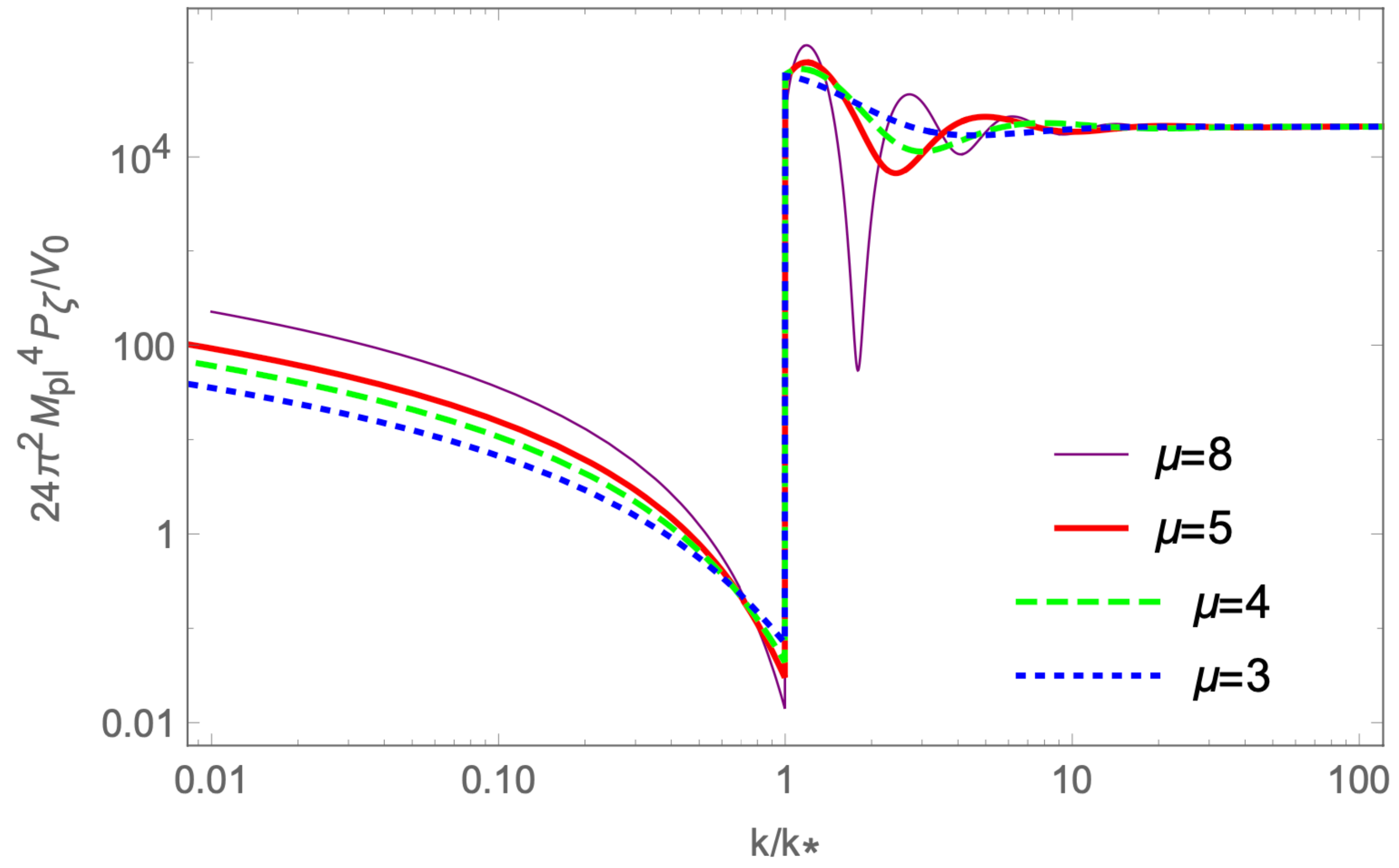

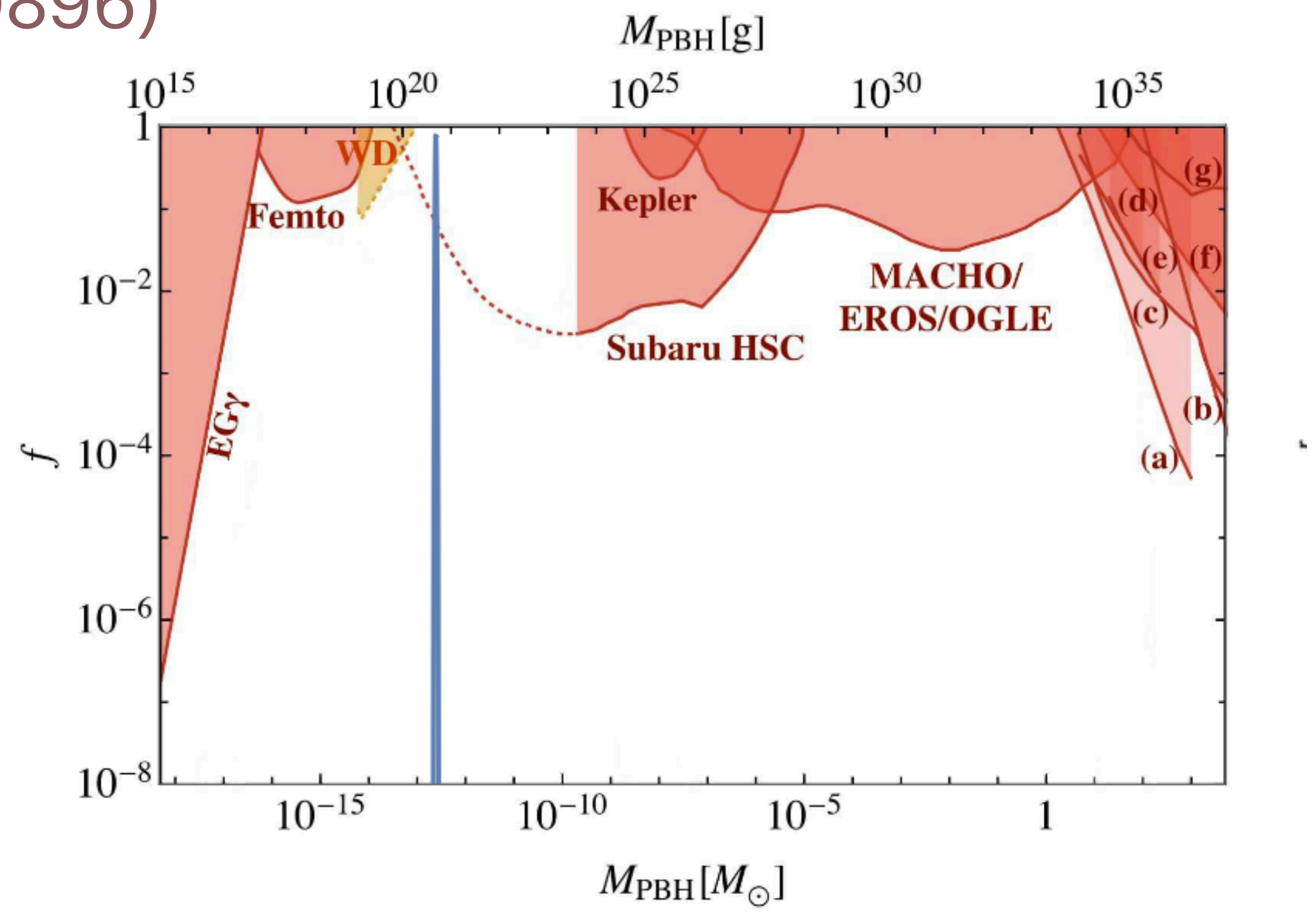

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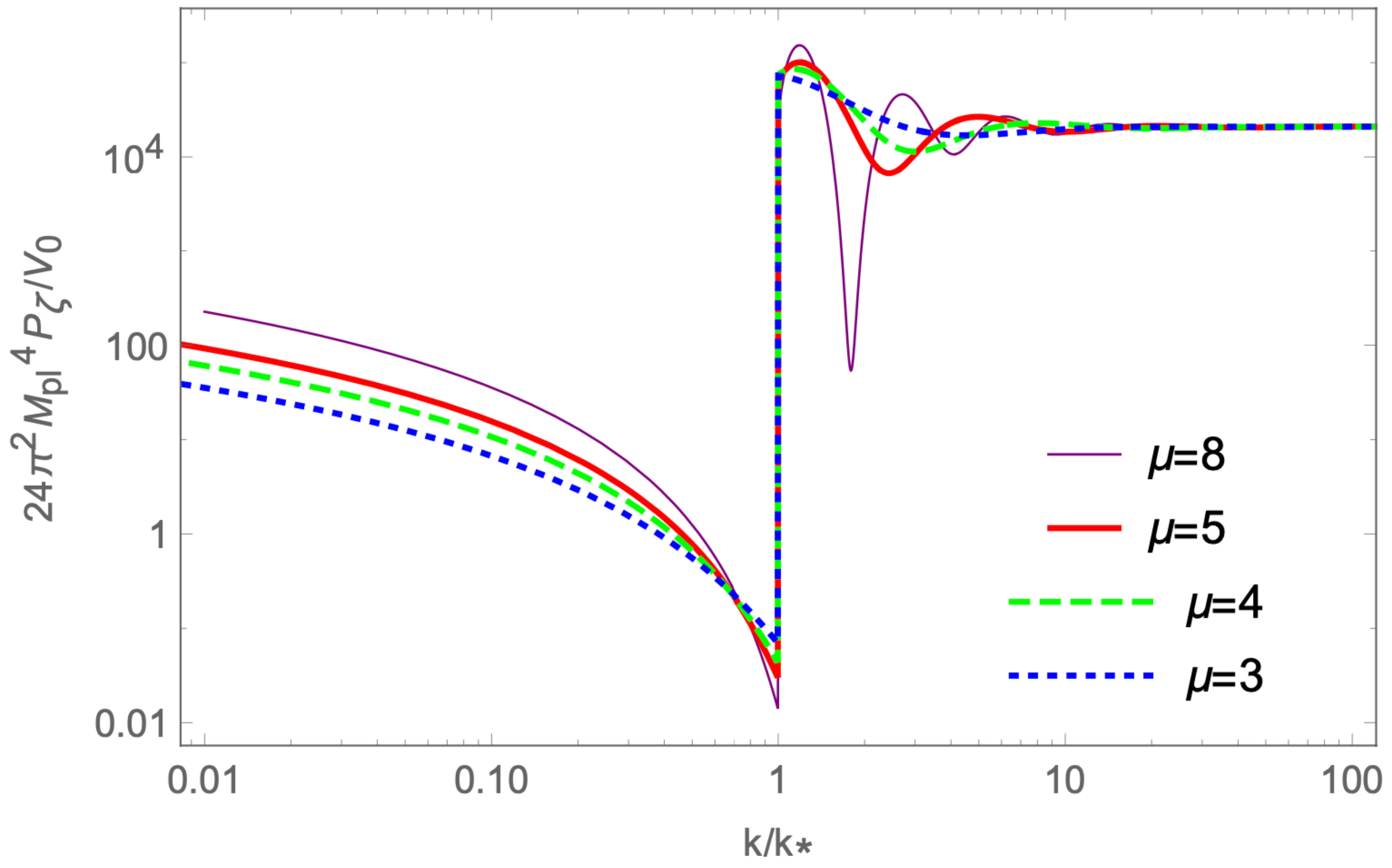
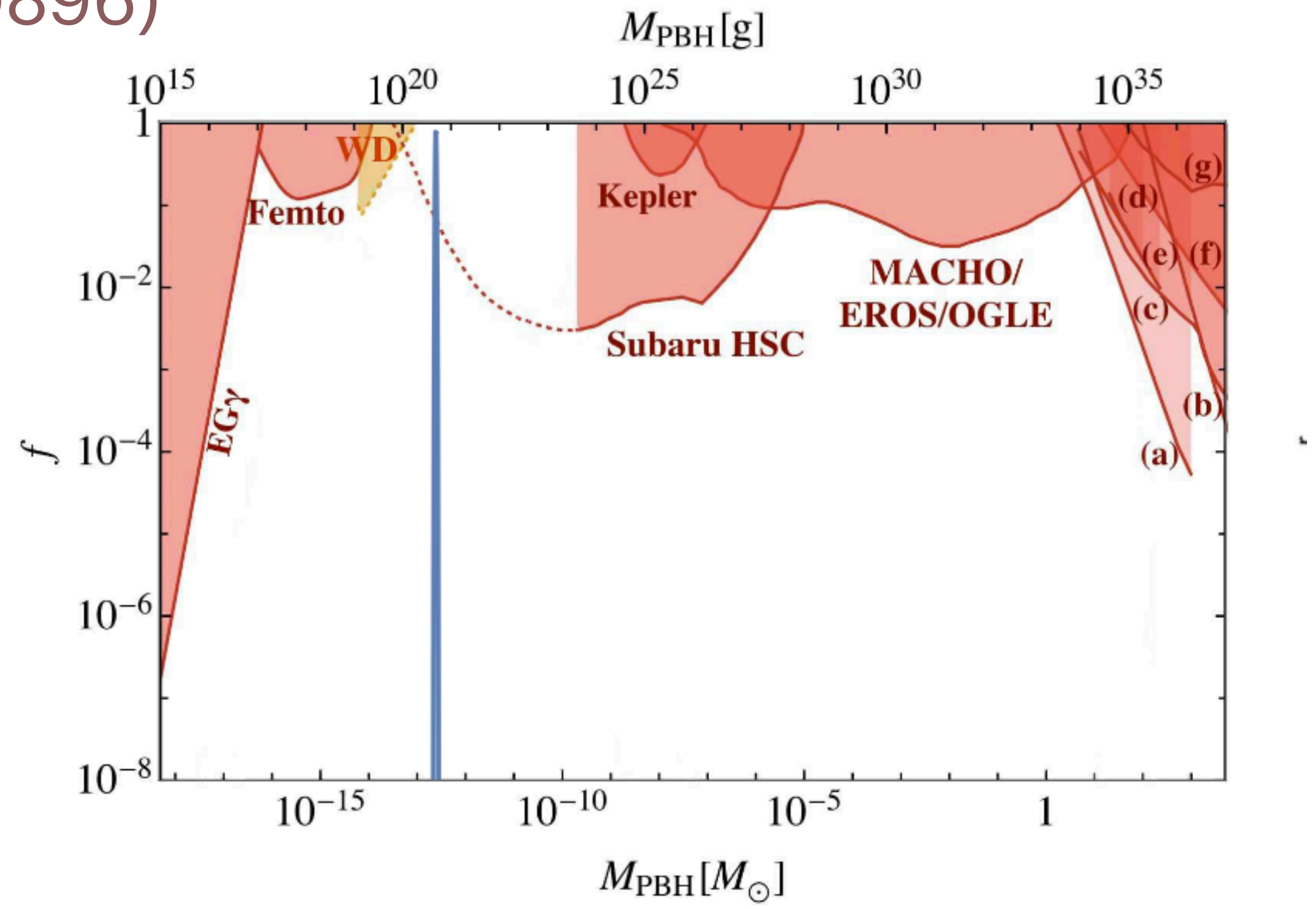




Perturbation evolution after exit the horizon? Iso-Curvature ?

End inflation in 50-60 e-folds ?

Non Gaussianity? SIGW?

Our model: The potential

Action in Jordan frame:

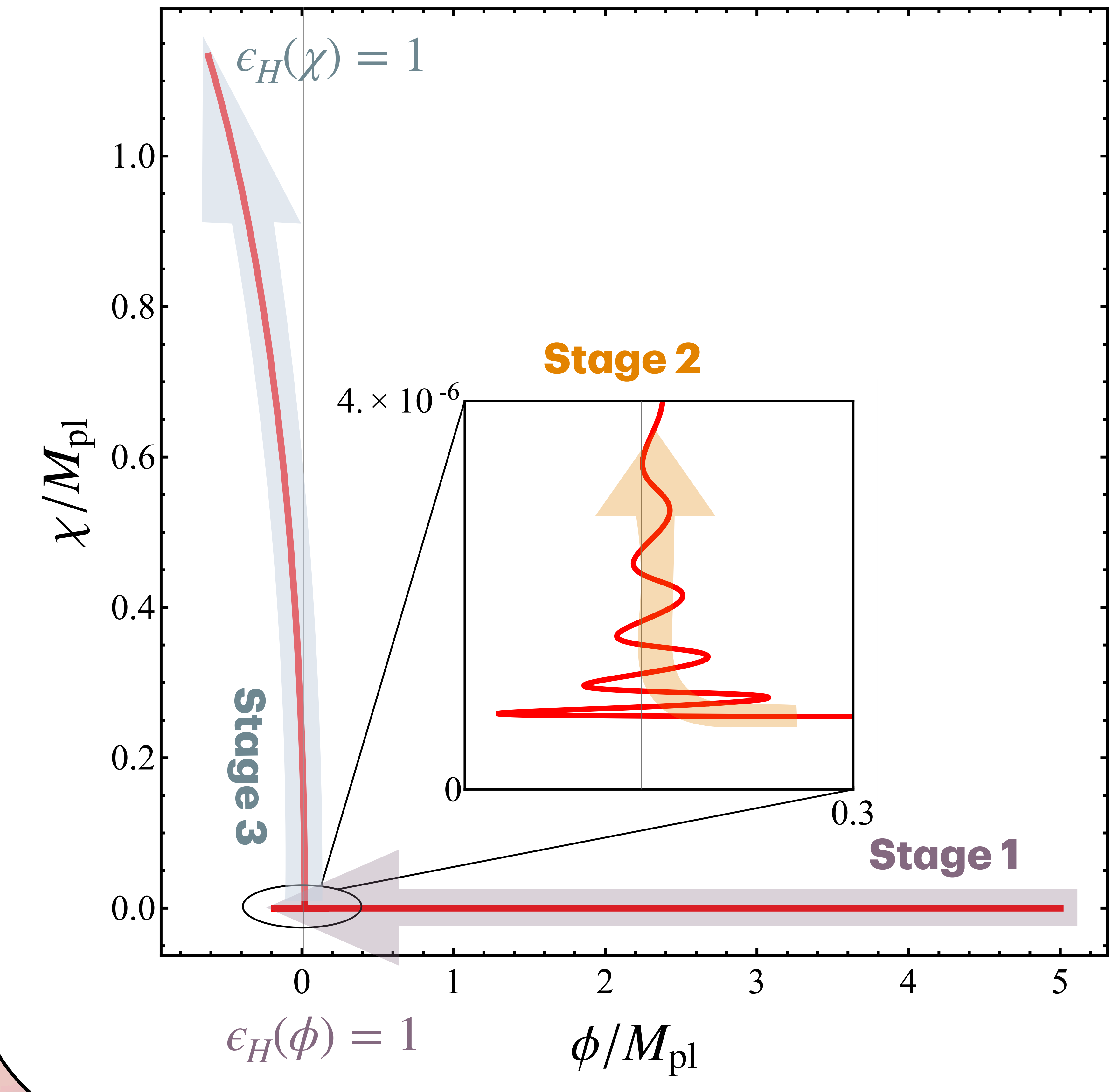
$$S_J = \int d^4x\sqrt{-g}\,\left[\frac{M_{\rm pl}^2}{2}f(R) - \frac{1}{2}g^{\mu\nu}\partial_\mu\partial_\nu\chi - V(\chi)\right],$$

$$f(R) = R + \frac{R^2}{6M^2} - \frac{\xi R}{M_{\rm pl}^2}(\chi-\chi_0)^2,$$

$$V(\chi) = V_0 - \frac{1}{2}m^2\chi^2 + \frac{1}{4}\lambda\chi^4 .$$

Conformal Transformation

$$F \equiv \partial f/\partial R = e^{\sqrt{2/3}\phi/M_{\rm pl}}$$



Two scalar fields ϕ, χ in Einstein frame

Our model: The potential

Action in Jordan frame:

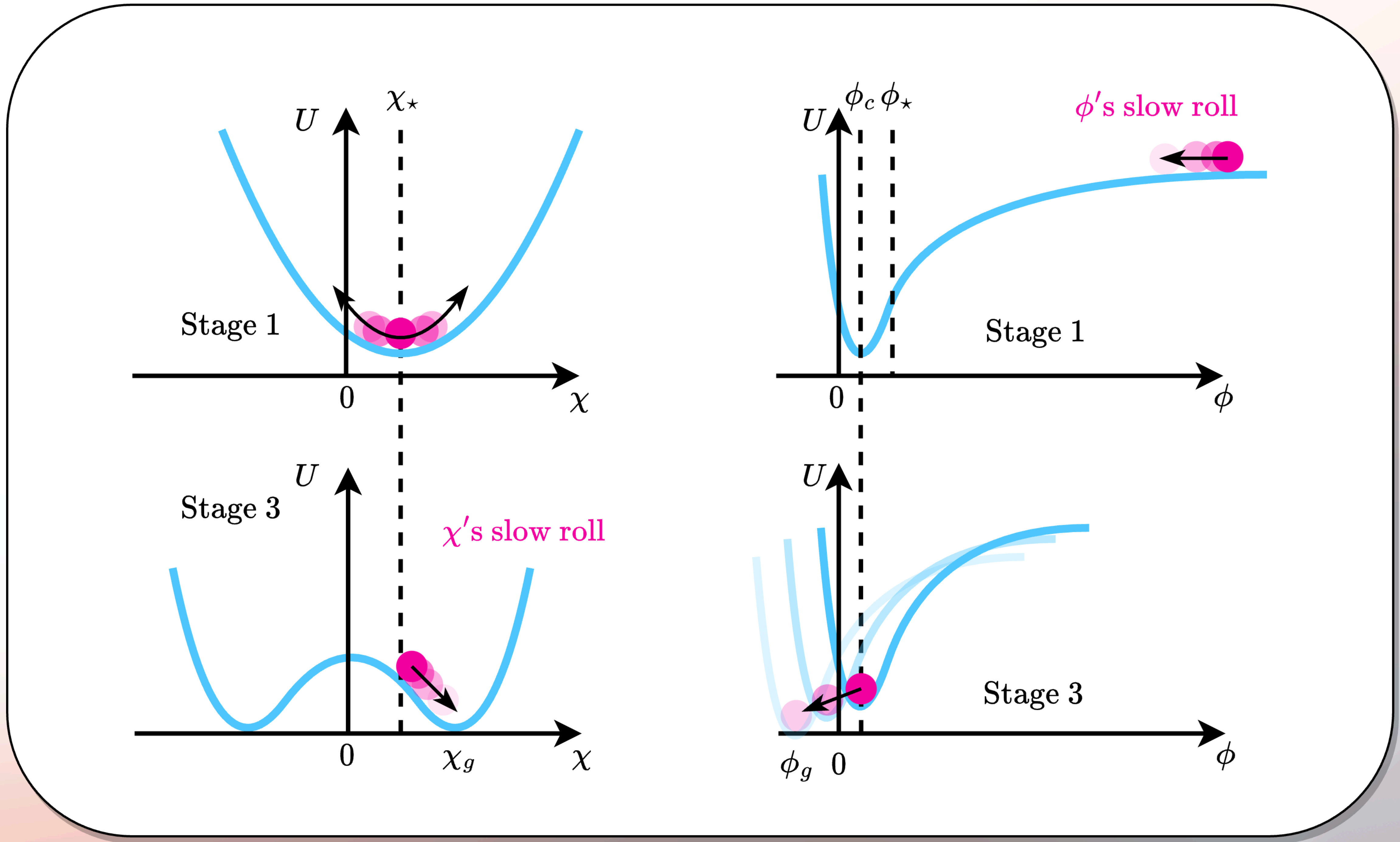
$$S_J = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}f(R) - \frac{1}{2}g^{\mu\nu}\partial_\mu\partial_\nu\chi - V(\chi)\right],$$

$$f(R) = R + \frac{R^2}{6M^2} - \frac{\xi R}{M_{\rm pl}^2}(\chi - \chi_0)^2,$$

Drive the 2nd SR

$$V(\chi) = V_0 - \frac{1}{2}m^2\chi^2 + \frac{1}{4}\lambda\chi^4.$$

End the Inflation

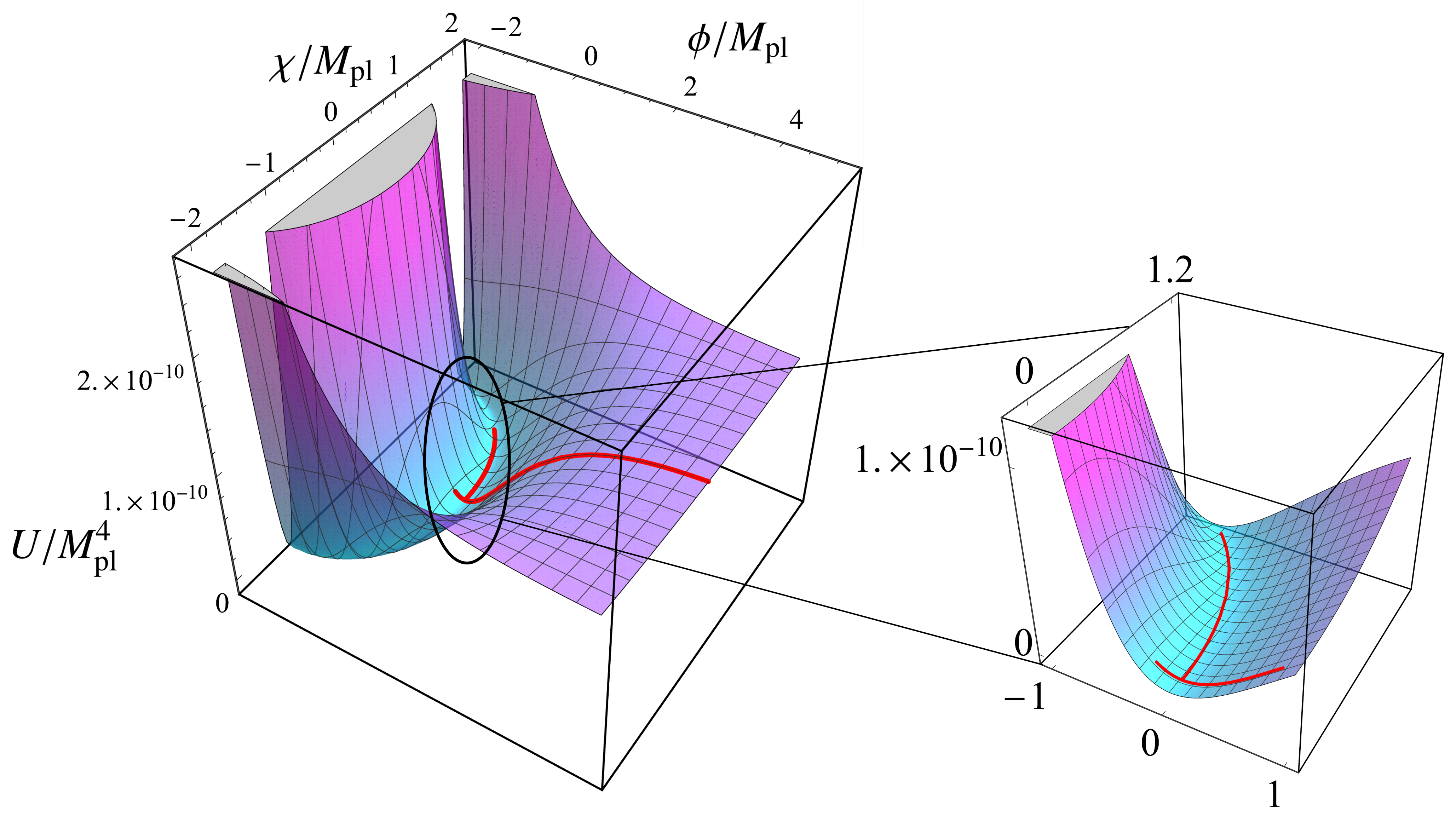


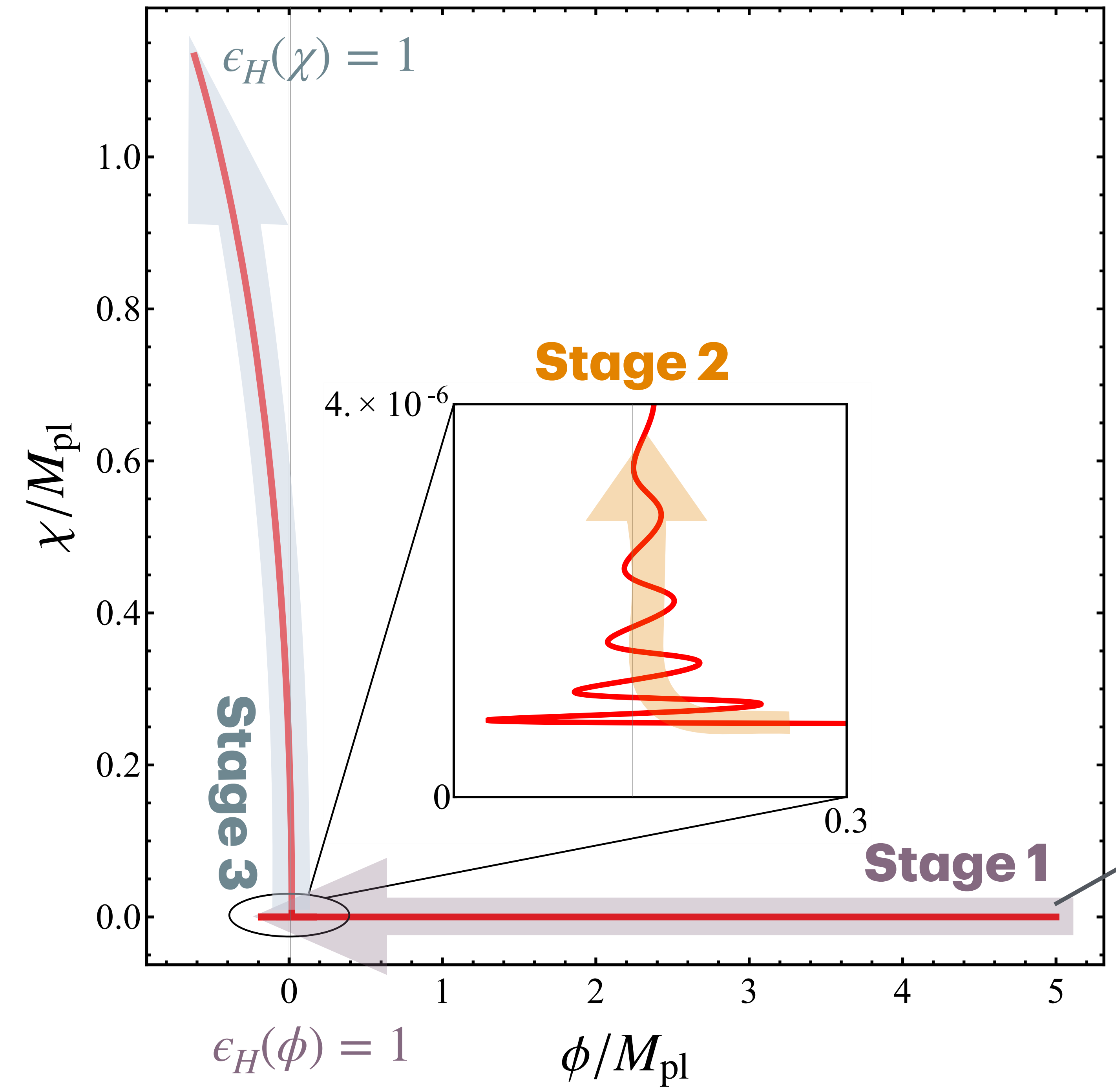
Conformal Transformation

$$\Rightarrow \quad F \equiv \partial f/\partial R = e^{\sqrt{2/3}\phi/M_{\rm pl}}$$

Our model: Background

$$\mathcal{N}_1 = \mathcal{N}_1(\phi) = \frac{3}{4}\left(F_{\rm ini} - F_{\star 1}\right)$$





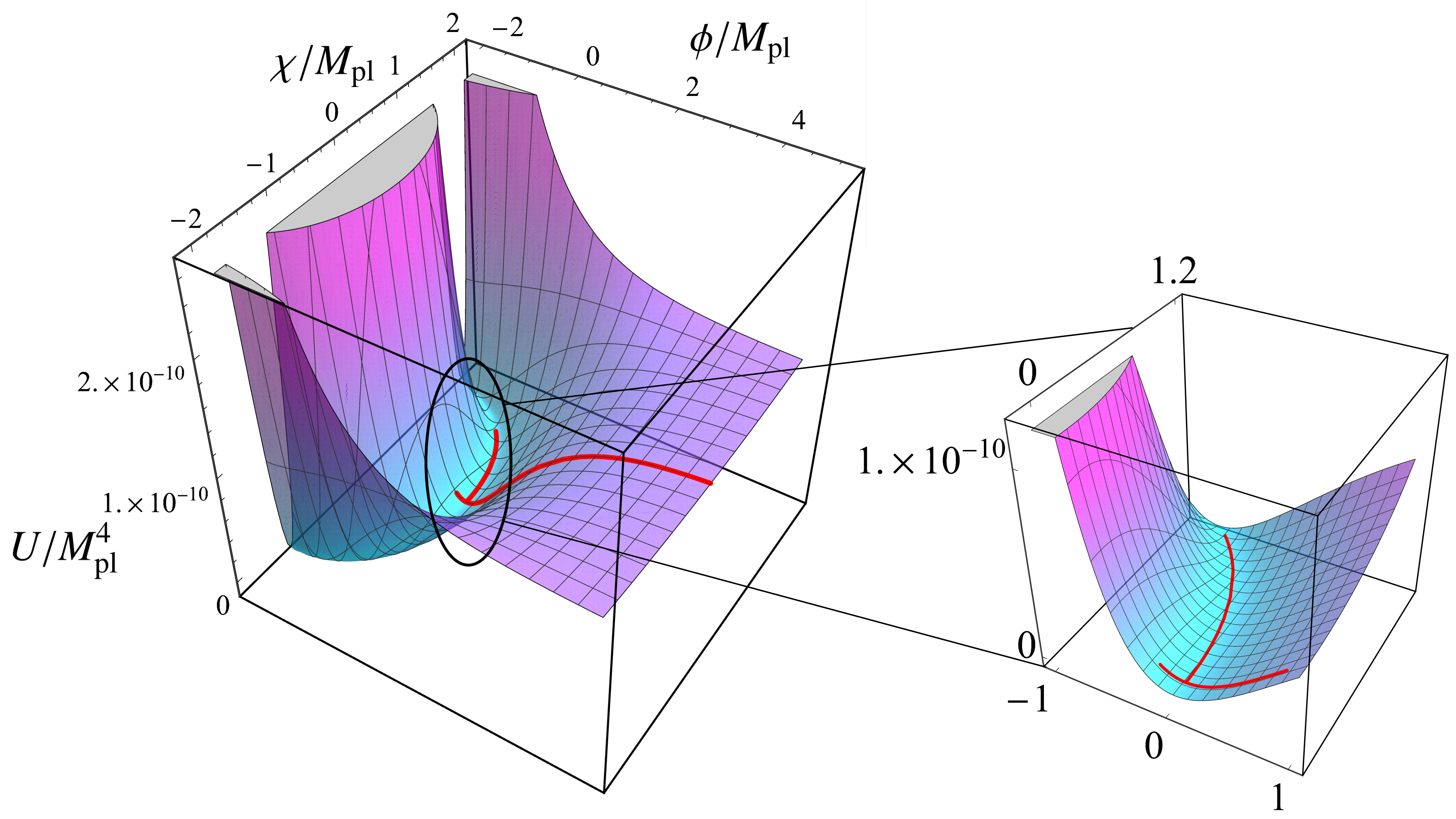
Inflaton rolls along ϕ direction (R^2 inflation), χ behaves like a damped* oscillator around χ_0.

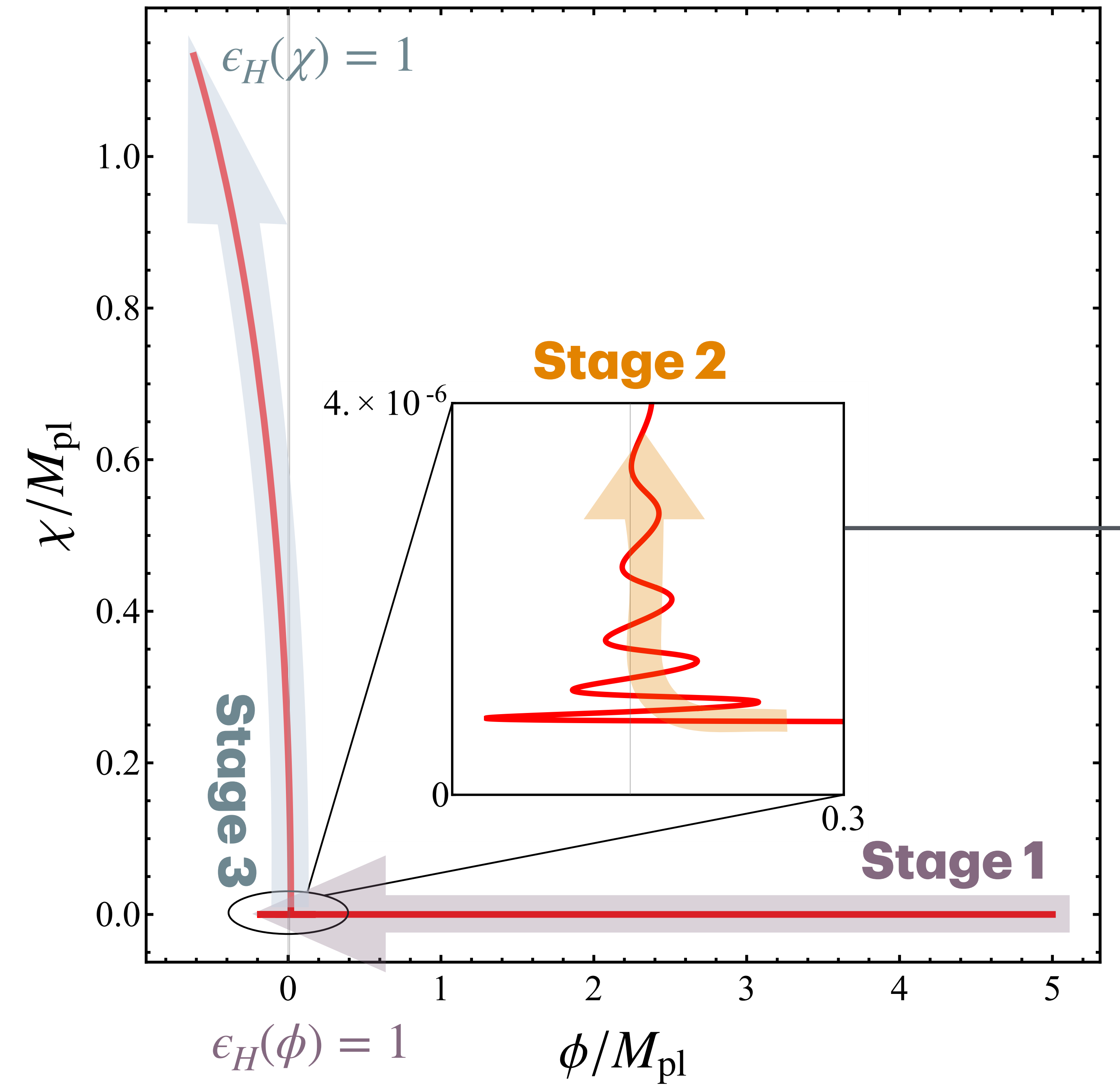
*$\xi > 3/16$ for under-damped χ

Our model: Background

$$\mathcal{N}_2 = \frac{1}{3}\ln\left[\frac{4}{3}\mu^2\left(\frac{\phi_\star}{M_{\rm pl}}\right)^2\right] = \text{const}\,.$$

$$\mu^2 \equiv H_1/H_2$$



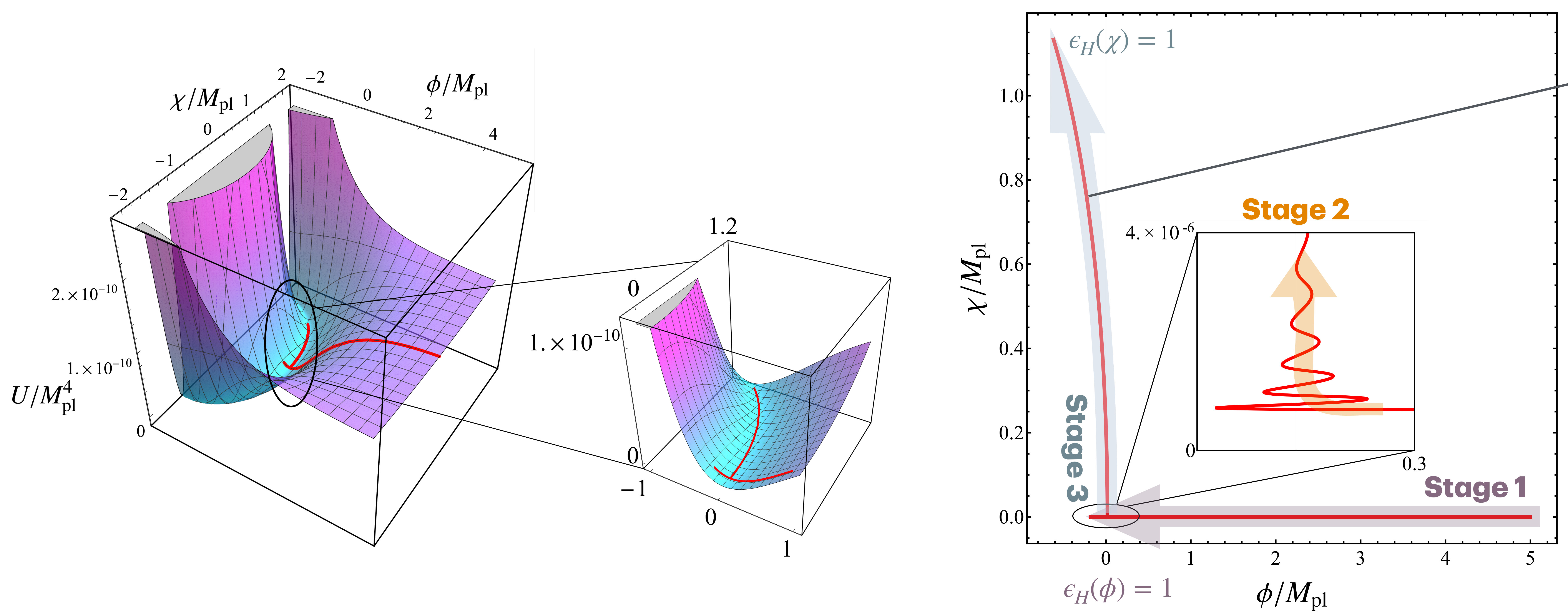


The slow roll is shortly violated by inflaton's oscillation along ϕ direction.

χ accelerates to attractor phase.

Our model: Background

$$\mathcal{N}_3 = \mathcal{N}_3(\chi) = \frac{1}{4\xi(A-1)} \ln\left[\frac{\sqrt{2}}{4}\left(\frac{\chi_g}{M_{\rm pl}}\right)\left(\frac{\chi_g}{\chi_0}\right)\right]$$



Inflation rolls along χ direction,
ϕ behaves like a under-damped oscillator around it's potential valley. The inflation ends when $\epsilon_H(\chi) = 1$.

Our model: Perturbations

Numerical results

Co-moving Curvature $$\mathscr{R} = H\frac{\dot{\phi}\delta\phi + F^{-1}\dot{\chi}\delta\chi}{\dot{\phi}^2 + F^{-1}\dot{\chi}^2},$$

Iso-Curvature $$\mathscr{S} = HF^{-1/2}\frac{\dot{\chi}\delta\phi - \dot{\phi}\delta\chi}{\dot{\phi}^2 + F^{-1}\dot{\chi}^2}.$$

A generalized description of our model

Field vector

$$\psi^a = (\phi, \chi)$$

Field Metric

$$h_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & F^{-1} \end{pmatrix}, \quad a, b = 1,2.$$

The Equations of motion:

$$\frac{D\dot{\psi}^a}{dt} + 3H\dot{\psi}^a + h^{ab}U_{,b} = 0,$$

$$\frac{D^2\delta\psi_k^a}{dt} + 3H\frac{D\delta\psi_k^a}{dt} + \frac{k^2}{a^2}\delta\psi_k^a + V^{;a}_{\ ;b}\delta\psi_k^b - R^a_{bcd}\dot{\psi}^b\dot{\psi}^c\delta\psi_k^d - \left[\frac{1}{a^3}\frac{d}{dt}\left(\frac{a^3}{H}\dot{\psi}^a\dot{\psi}^b\right)\right]h_{bc}\delta\psi_k^b = 0.$$

Our model: Perturbations

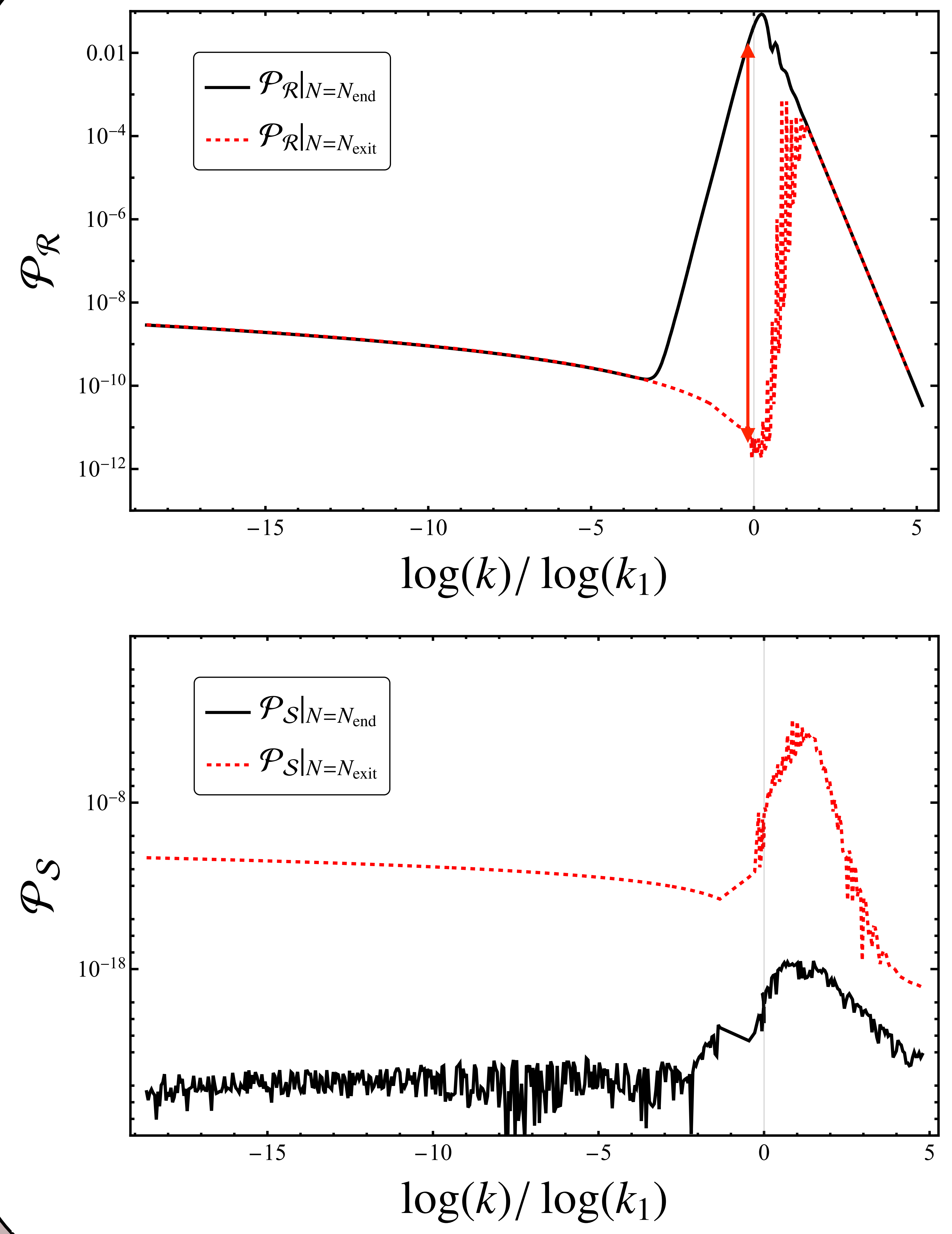
Numerical results

Co-moving Curvature $\mathscr{R} = H\frac{\dot{\phi}\delta\phi + F^{-1}\dot{\chi}\delta\chi}{\dot{\phi}^2 + F^{-1}\dot{\chi}^2},$

Iso-Curvature $\mathscr{S} = HF^{-1/2}\frac{\dot{\chi}\delta\phi - \dot{\phi}\delta\chi}{\dot{\phi}^2 + F^{-1}\dot{\chi}^2}.$

N_{end}———— **Time at the end of Inflation**

N_{exit}———— **Time at the Horizon exit stage**



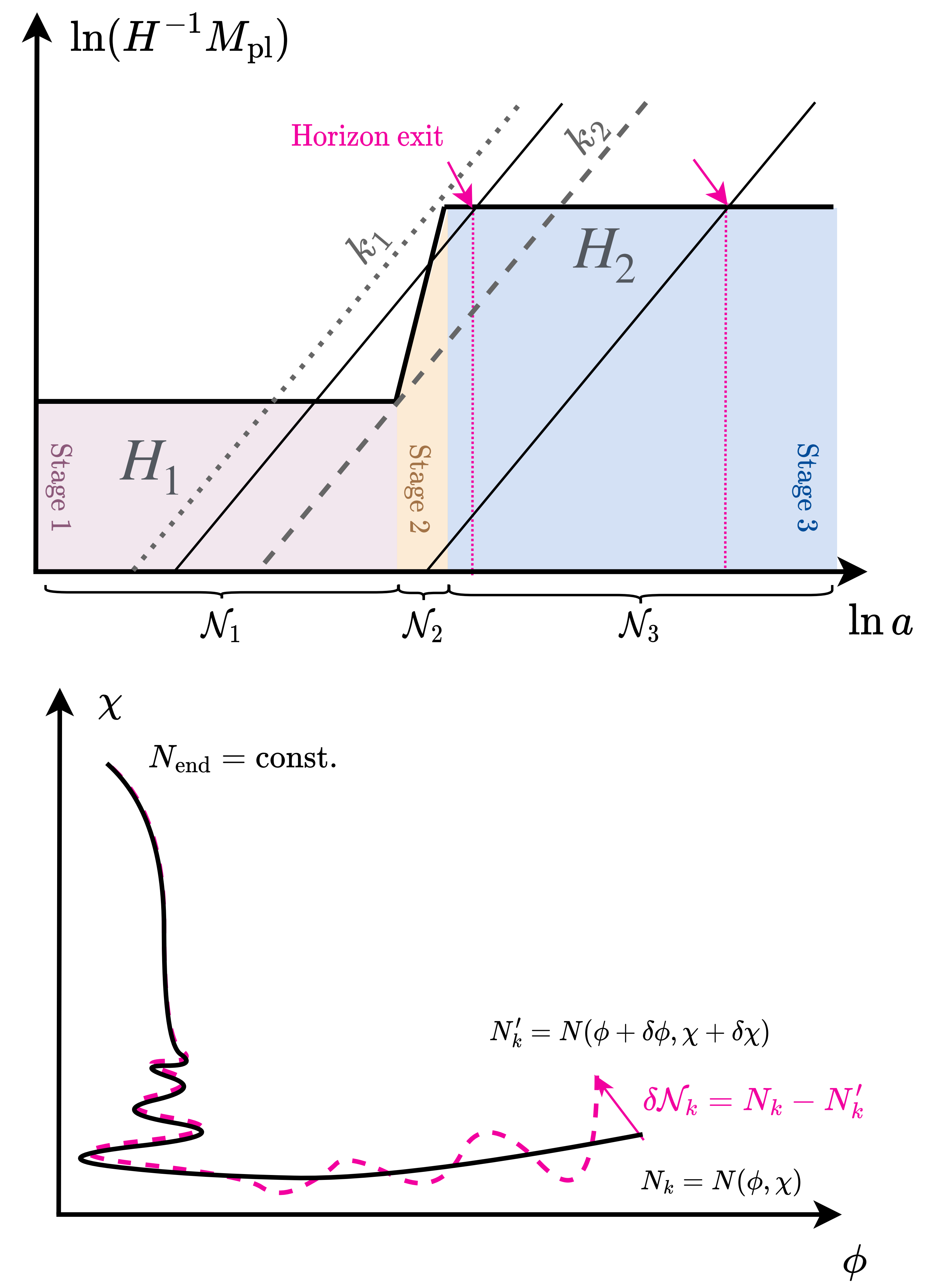
Our model: Perturbations

$\delta\mathcal{N}$ formalism

Kodama & Sasaki, 1984

Sasaki & Stewart, 1995 (astro-ph/9507001)

$$
\begin{aligned}
\delta\mathcal{N}(k) &= \sum_i \delta\mathcal{N}_i \\
&\approx \begin{cases} \delta\mathcal{N}_1(\phi) + \delta\mathcal{N}_3(\chi) & \text{for } k < k_1 \\ \delta\mathcal{N}_3(\chi) & \text{for } k \geq k_1 \end{cases} \\
&= \begin{cases} \left.\dfrac{\partial\mathcal{N}_1}{\partial\phi}\right|_{N=N_k} \delta\phi_k(N_k) + \left.\dfrac{\partial\mathcal{N}_3}{\partial\chi}\right|_{N=N_{\star 2}} \delta\chi_k(N_{\star 2}) & \text{for } k < k_1 \\ \left.\dfrac{\partial\mathcal{N}_3}{\partial\chi}\right|_{N=N_k} \delta\chi_k(N_k) & \text{for } k \geq k_1 \end{cases}
\end{aligned}
$$



Our model: Perturbations

Analytical Approximation

$$\mathcal{P}_{\mathcal{R}}(k) \approx \frac{M^2}{4(2\pi)^2} \begin{cases} \left[\frac{2}{3}\left(\ln\frac{k_1}{k}+\frac{3}{4}F_\star\right)^2 + g_1^2 h^2 \chi_0^{-2}\left(\frac{k}{k_1}\right)^{\alpha}\right] \times \left[1+\left(\ln\frac{k_1}{k}+\frac{3}{4}F_\star\right)^{-1}\right] & \text{for } k < k_1 \\ g_2^2 h^2 \chi_0^{-2} \mu^{-2}\left(\frac{k}{k_1}\right)^{\beta} & \text{for } k \geq k_1 \,. \end{cases}$$

α is determined by

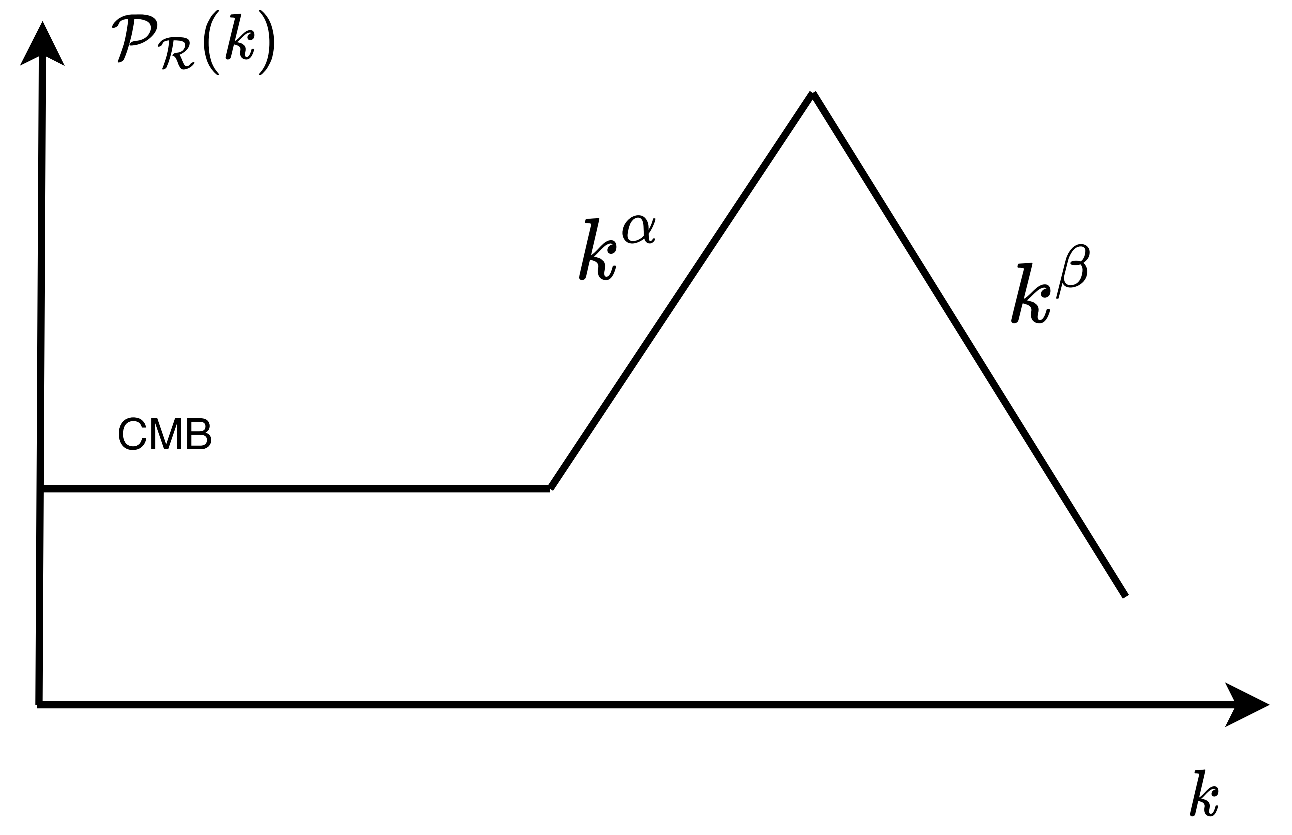
the oscillation behavior of

χ in the end first stage!

Broken power-law!

$$\alpha \equiv \mathrm{Re}\left(3 - 3\sqrt{1-\frac{16}{3}\xi}\right)$$

$$\beta \equiv 3 - \sqrt{3 + \frac{48\xi(A-1)}{1+1/\mu^2}},$$



Our model: Perturbations

Analytical Approximation

$$\mathcal{P}_{\mathcal{R}}^{\text{peak}} \approx \mathcal{P}_{\mathcal{R}}(k_1) = \frac{g_2^2 h^2}{4(2\pi)^2} \mu^{-2}(M/\chi_0)^2$$

$\mu^2 \gg 1$ is the ratio of H_1 to H_2

χ_0/M is the ratio of field perturbation to background

Broken power-law!

$$\alpha \equiv \text{Re}\left(3 - 3\sqrt{1 - \frac{16}{3}\xi}\right)$$

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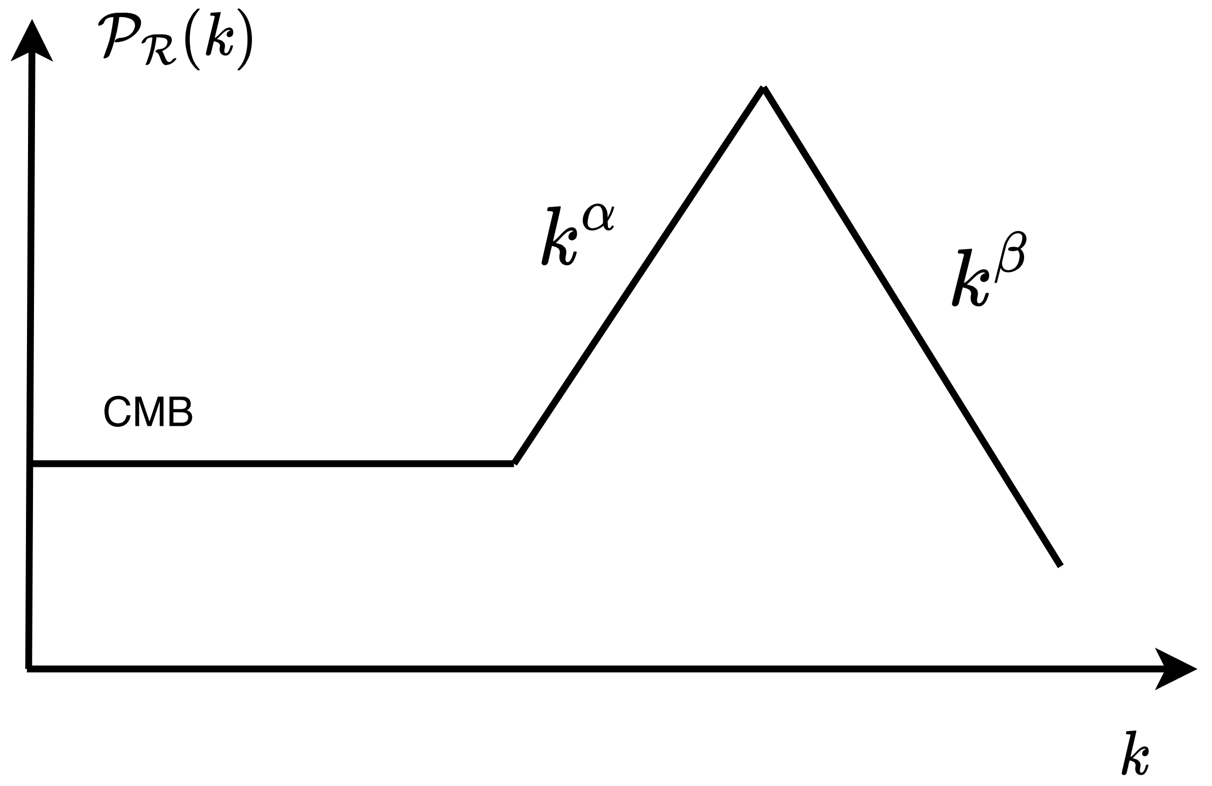
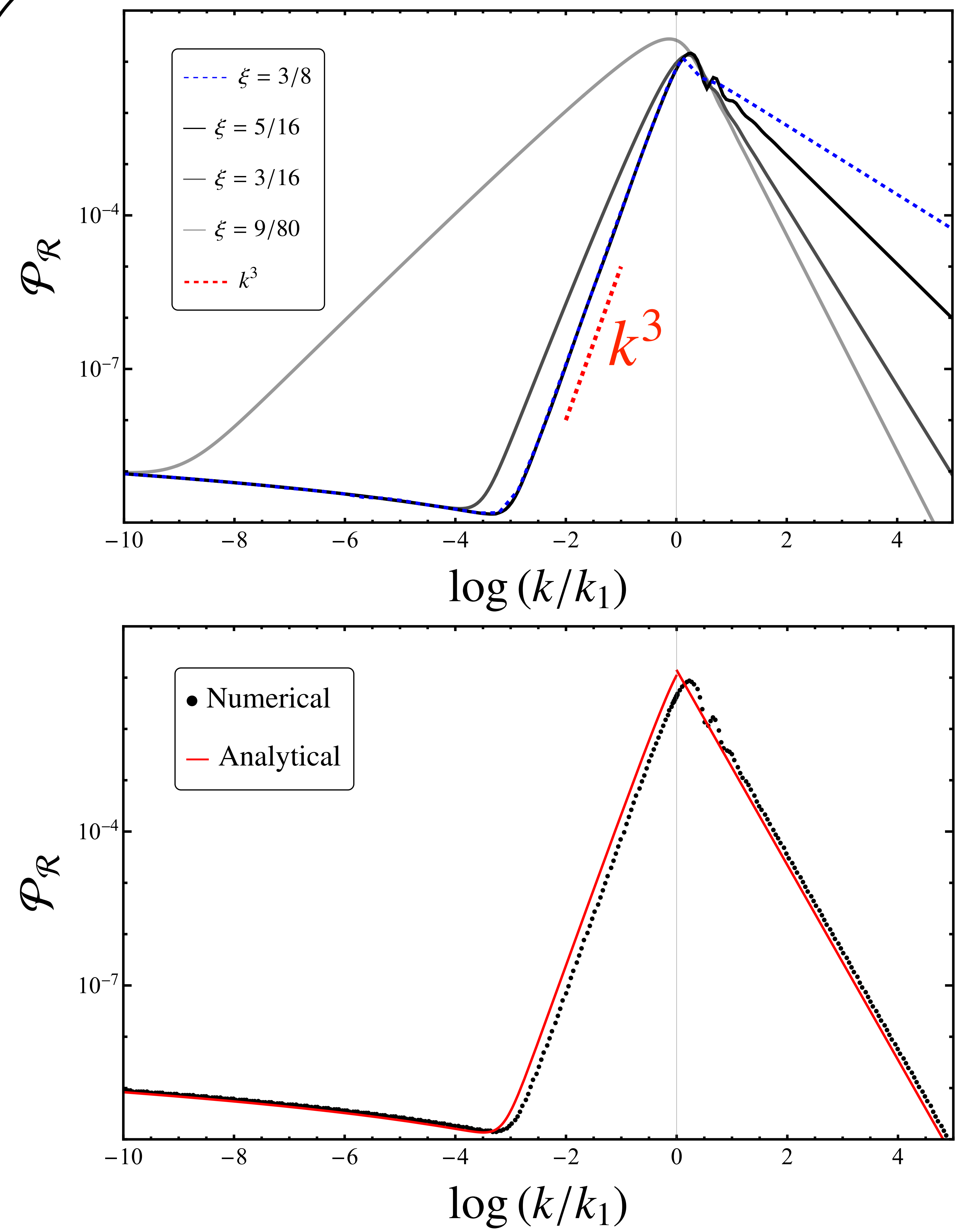
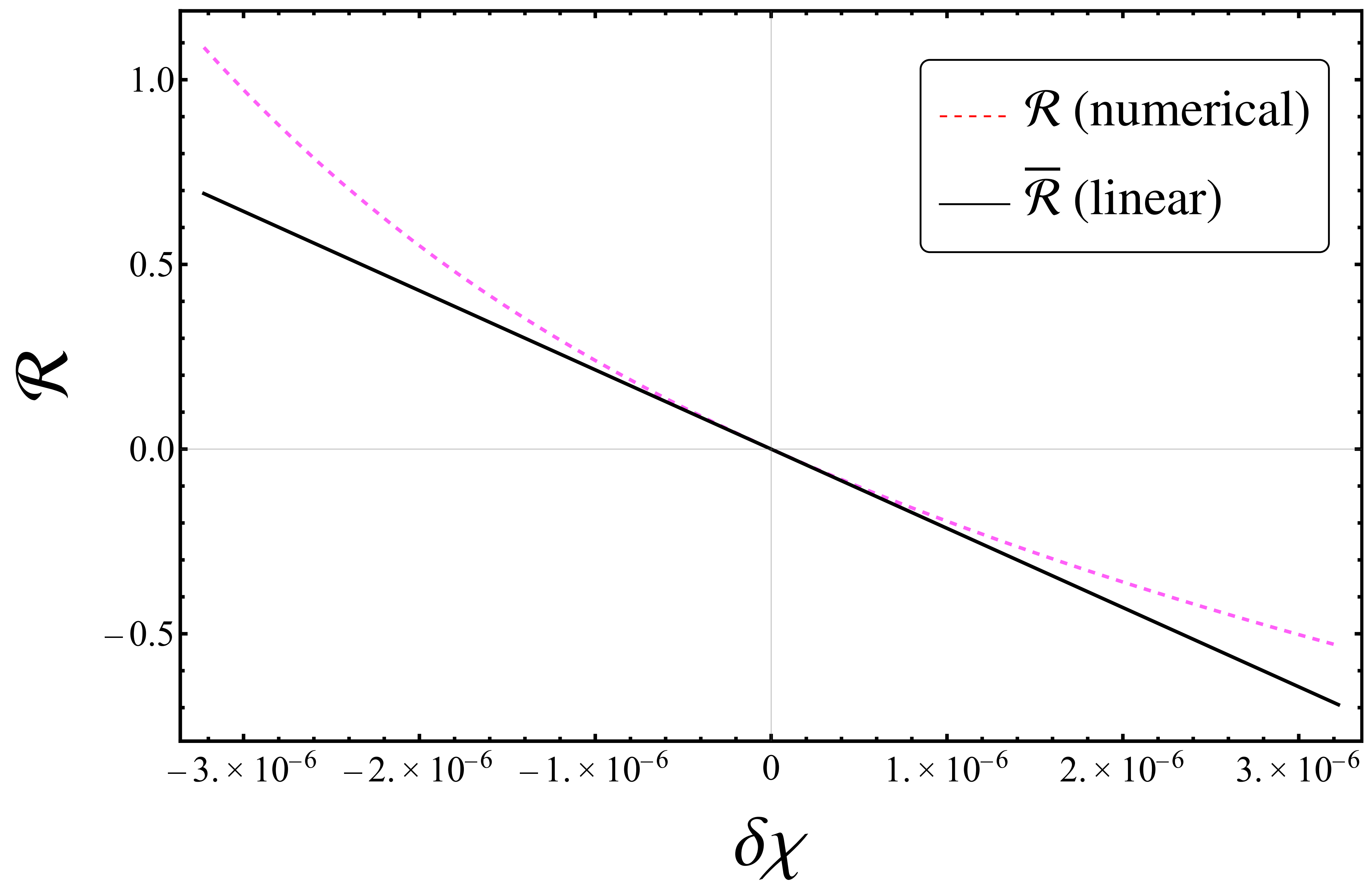


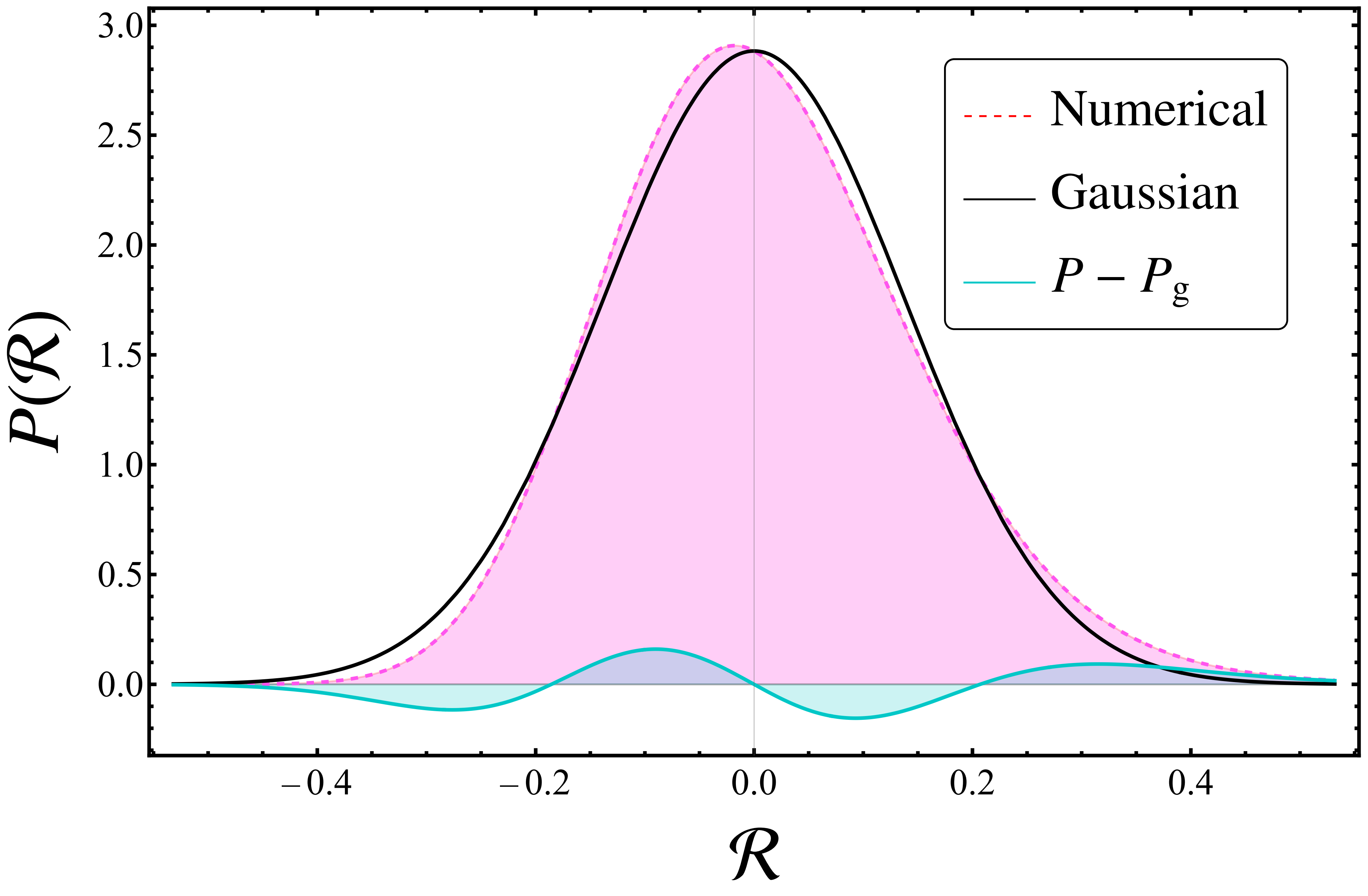


PBH formation — A brief discussion of NG (at the peak)

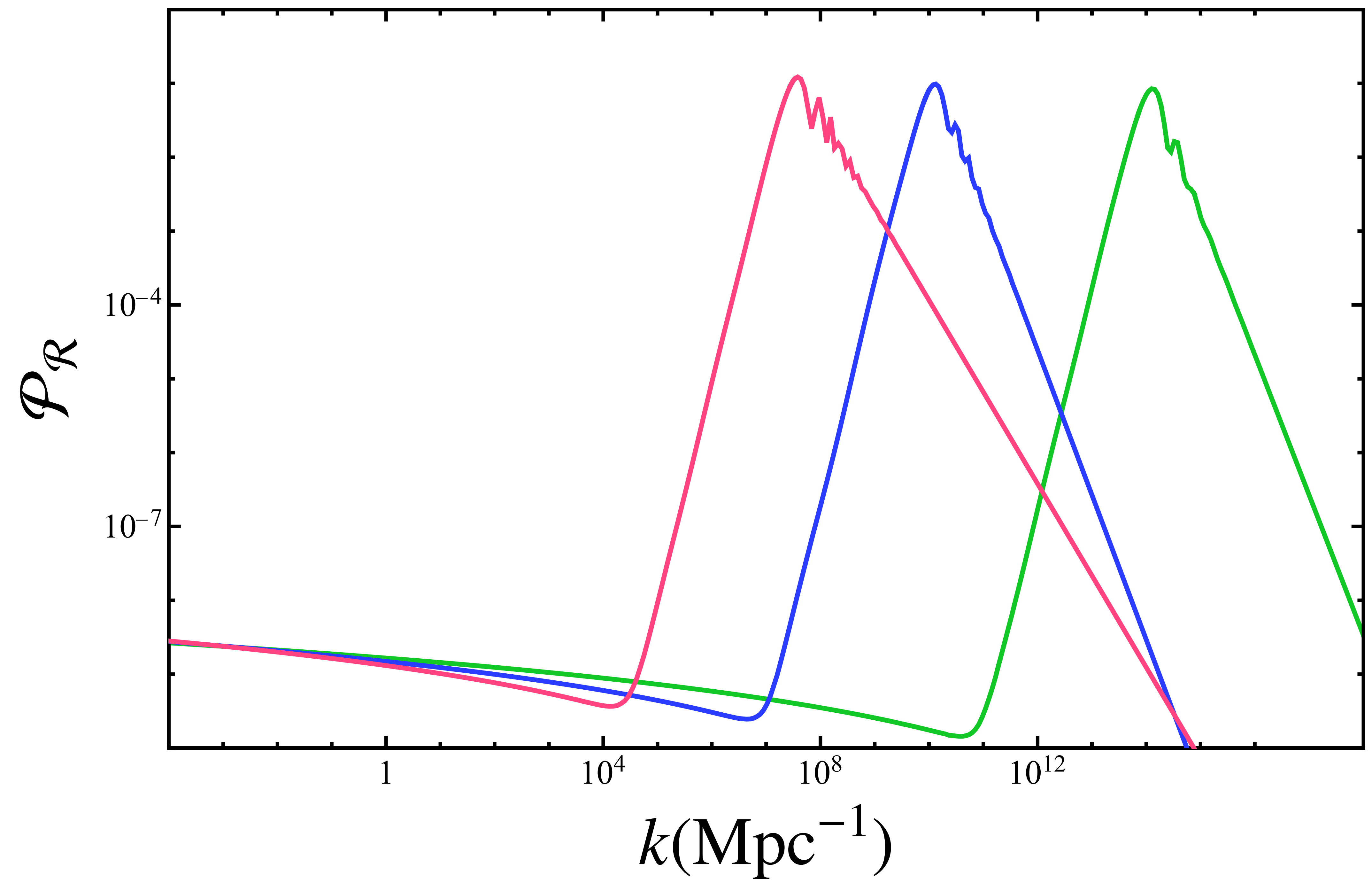
$$\mathcal{R} = \delta\mathcal{N}_3 = \frac{\rho_1}{M_{\rm pl}}\delta\chi + \frac{\rho_2}{M_{\rm pl}^2}\delta\chi^2 + \frac{\rho_3}{M_{\rm pl}^3}\delta\chi^3 + \cdots \Rightarrow f_{\rm NL}^{\rm local} \approx 2(A-1)\xi \ll 1$$

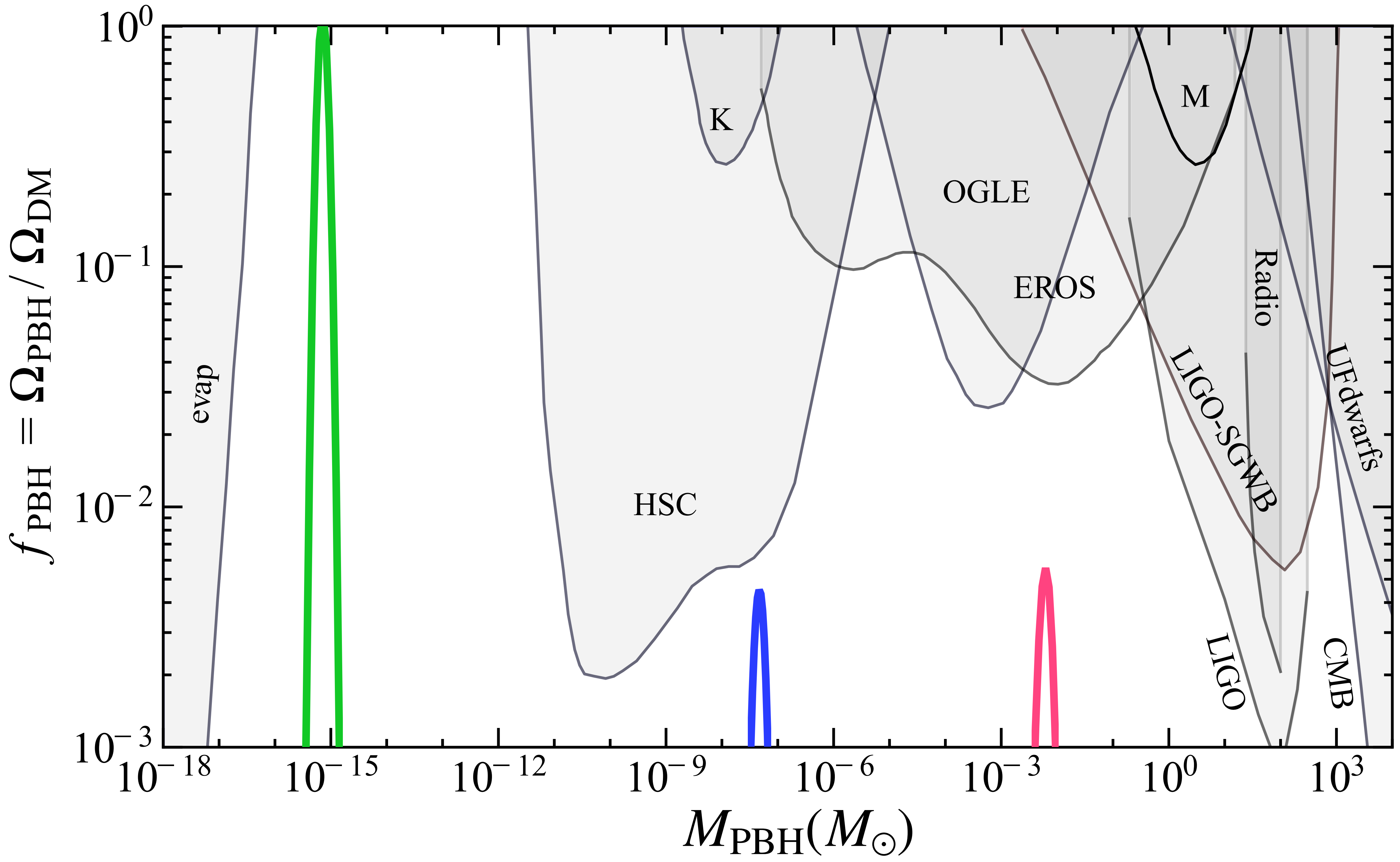
Small and Positive





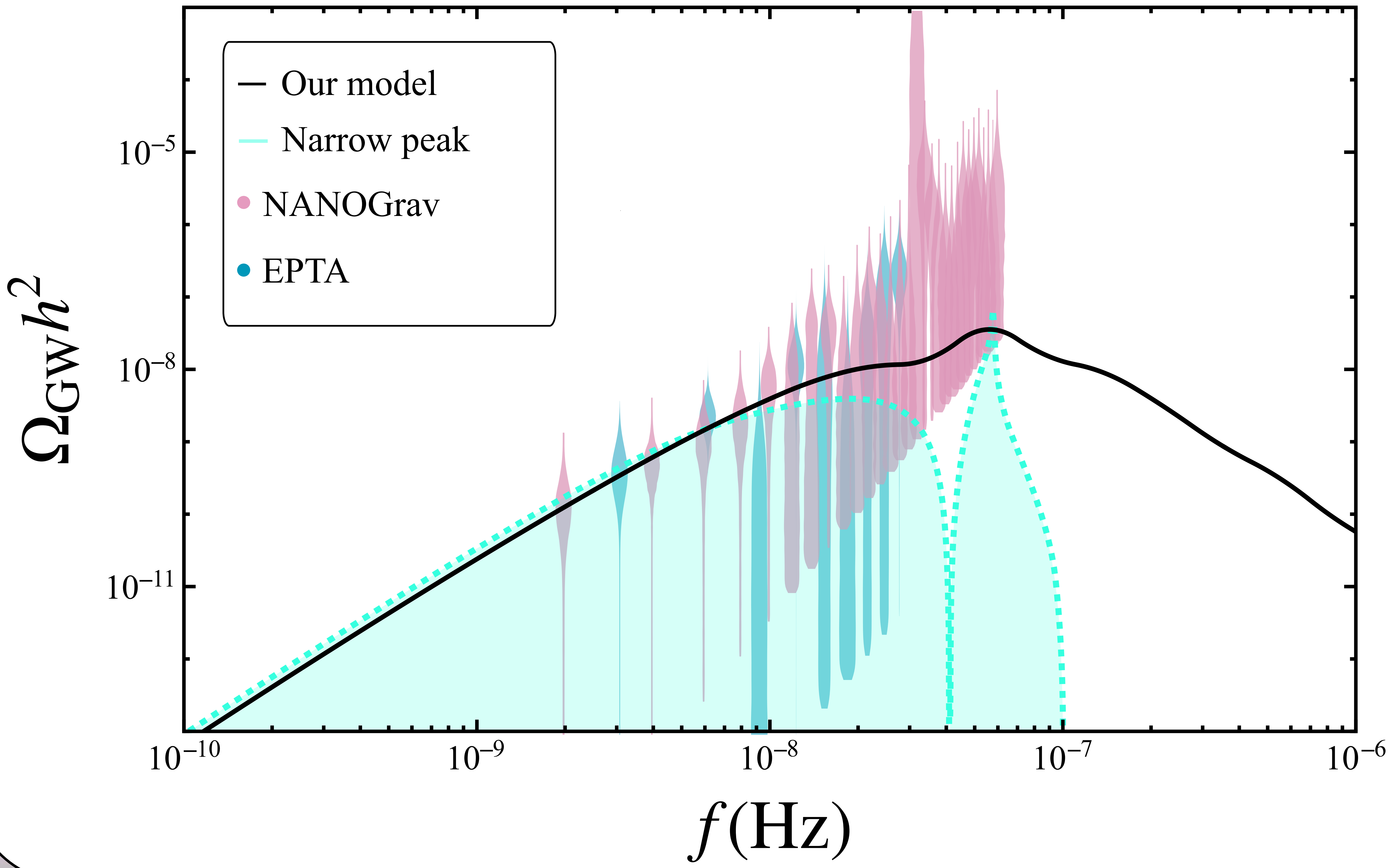
PBH formation





Press-Schechter formalism

Scalar Induced Gravitational Wave



Pi & Sasaki, 2020 (2005.12306) NANOGrav, 2023 (2306.16219) EPTA, 2023 (2306.16227)

Conclusion

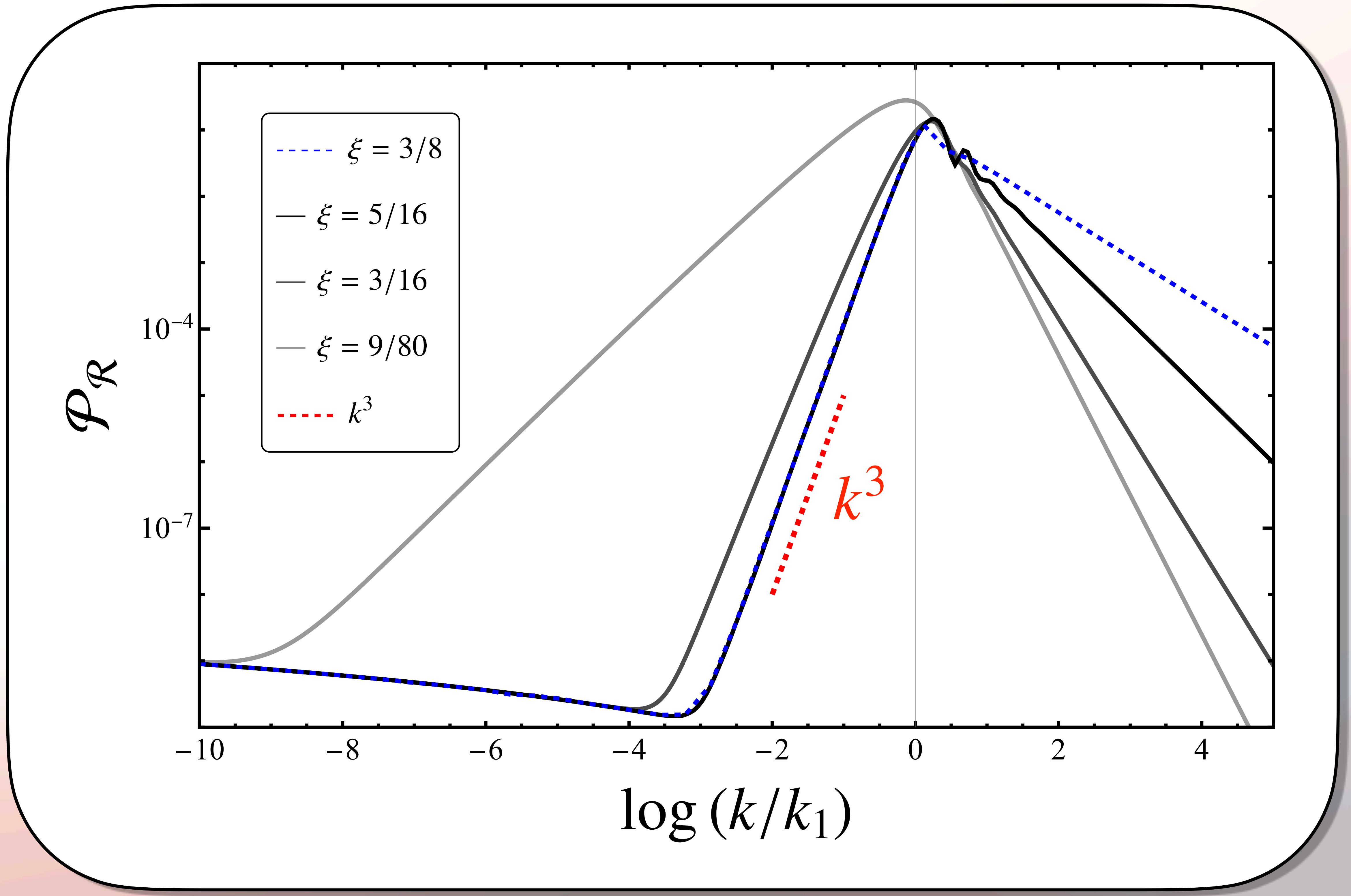
On the basis of fitting CMB observation & ending the inflation.......

1. We can **enhance the power spectrum by 2-stage inflation dominated by different fields connected by a sharp turn (Iso-curvature).**

2. The power spectrum can be easily recognized (or excluded) by observation of it's **broken power law shape especially the growing feature of k^3 .**

3. The non-Gaussianity is **positive** and **small**.

4. Our model produce a nearly **monochromatic PBH mass function**.



If you are interested, please see the details in our up coming paper :)

Look a step further

1. Our model probably can lead to an interesting reheating.

2. What happens for the **“I”-Stage “J”-field** inflation?
   🤔 **Muti-peak power spectrum? Muti-modal distribution of PBH?** Carr and Kuhnel, 2018 (1811.06532)
   🤔 **Will the growing behavior of power spectrum still be limited to k^3?**

3. What if there is a USR phase in between of the two stages?

To be continued...

Primordial Black Holes from R^2 gravity theory with a non-minimally coupled scalar field

Thanks For Listening!

ご視聴ありがとう！

感谢聆听！

Xinpeng Wang

Tongji University (Shanghai, China) & Kavli IPMU (Chiba, Japan)

XW, Misao Sasaki & Ying-li Zhang, in preparation.

CMB fitting

$n_s^{R^2}(k_{\rm CMB}) \simeq 0.947 < 0.965$

$(M = 2\times 10^{-5} M_{\rm pl}, \ln(k_1/k_{\rm CMB}) \approx 36)$

R^3 gravity

$$f(R) \equiv \left(R + \frac{R^2}{6M^2} + q\frac{R^3}{3M^4} \right) - \frac{1}{M_{\rm pl}^2}\xi R \chi^2$$

R^3 V.S. R^2

$$\frac{\mathcal{P}_{\mathcal{R}}^{R^3}(k)}{\mathcal{P}_{\mathcal{R}}^{R^2}(k)} \approx 1 + 6p_M q + \mathcal{O}(q^2)$$

$$n_s^{R^3}(k_{\rm CMB}) - n_s^{R^2}(k_{\rm CMB}) \approx -8\sqrt{p_M}q$$

$$p_M \equiv 9.7\times 10^{-7} M_{\rm pl}^2 M^{-2}$$

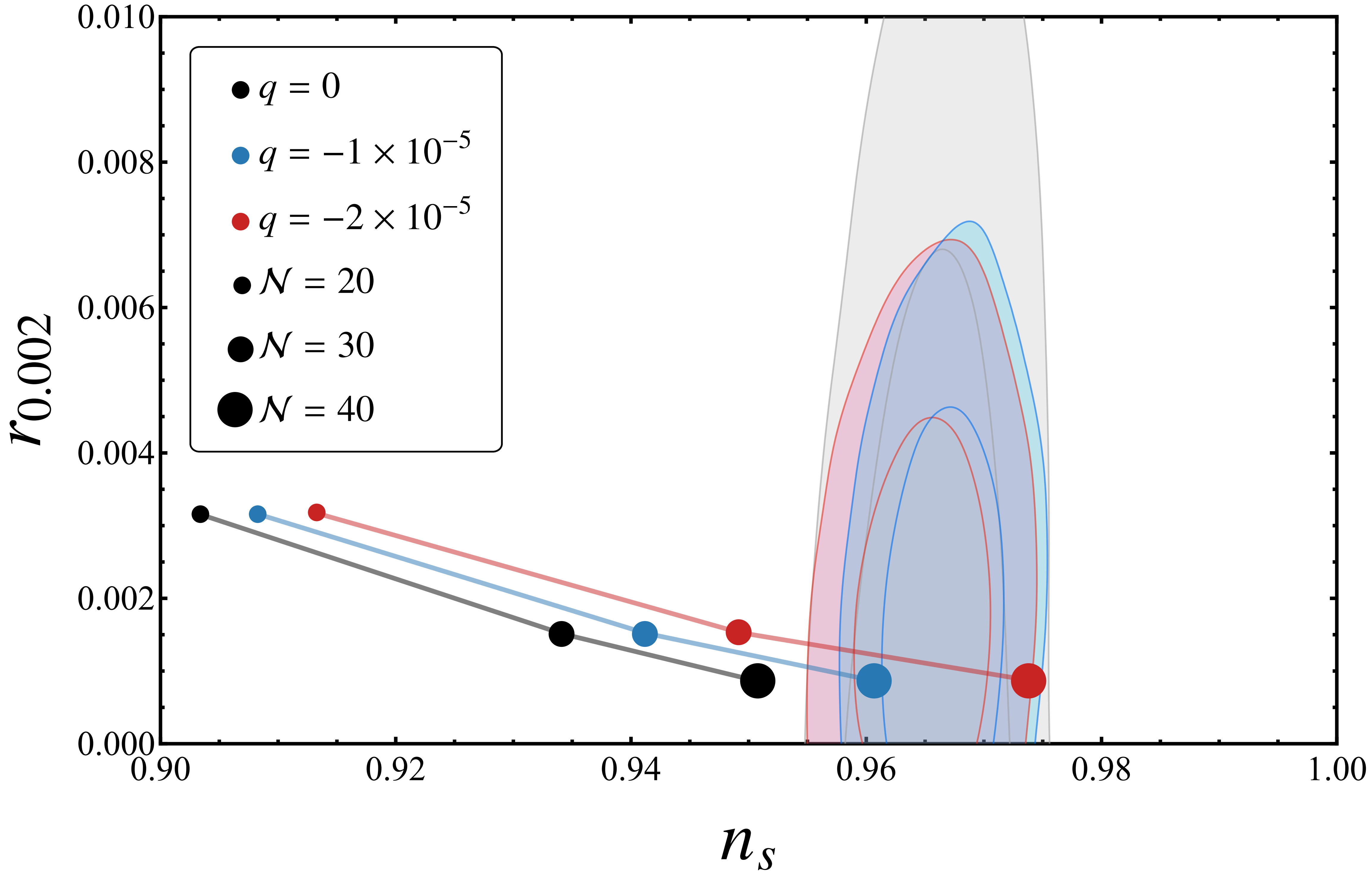
CMB fitting

R^3 V.S. R^2

$$\frac{\mathcal{P}_{\mathcal{R}}^{R^3}(k)}{\mathcal{P}_{\mathcal{R}}^{R^2}(k)} \approx 1 + 6p_M q + \mathcal{O}(q^2)$$

$$n_s^{R^3}(k_{\rm CMB}) - n_s^{R^2}(k_{\rm CMB}) \approx -8\sqrt{p_M}q$$

$$p_M \equiv 9.7 \times 10^{-7} M_{\rm pl}^2 M^{-2}$$



Parameters

Case	1	2	3	4
$M/M_{\rm pl}$	1.8×10^{-5}	1.59×10^{-5}	2.8×10^{-5}	1.8×10^{-5}
$m/M_{\rm pl}$	5.4×10^{-6}	4×10^{-6}	4×10^{-6}	5.4×10^{-6}
ξ	$5/16$	$4/16$	$5/16$	/
A	2	2.3	1.6	$15/32\xi^{-1}$
B	$0.193\sqrt{1/(2\pi^2)}$	$0.12\sqrt{1/(2\pi^2)}$	$0.1\sqrt{1/(2\pi^2)}$	$0.193\sqrt{1/(2\pi^2)}$
$\delta_{\rm th}$	0.45	0.45	0.45	/

Table 1: The parameters used for numerical calculation