Formation of trapped vacuum bubbles during inflation, and consequences for PBH scenarios

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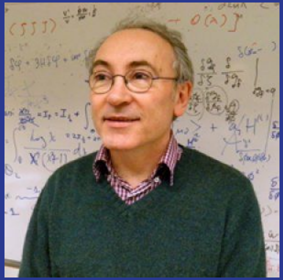
Focus week on PBHs at IPMU. 14/11/2023



Based on: A. Escrivà, V. Atal and J. Garriga. ArXiv:2306.09990 [JCAP10 (2023)035]

About PBHs…

Y.B. Zel'dovich and I.D. Novikov. Soviet Ast. 10 (1967) 602
S. Hawking. Mon. Not. Roy. Astron. Soc. 152 (1971) 75
B. Carr and S. Hawking. MNRAS 168 (1974) 399

- The most standard mechanism for PBH production is from the **collapse of sufficiently large (very rare event) adiabatic fluctuations generated during inflation**, which reenter the cosmological horizon during the radiation epoch.

- But indeed there are several mechanism for the production of PBHs: isocurvature fluctuations, cosmic strings, domains walls, Q-balls , quark confinement, etc

(see A. Escrivà, F. Kuhnel, Y. Tada. ArXiv:2211.05767 for a detailed list and a brief description)

What about baby Universes?

baby Universes: can be formed from false vacuum bubbles generated during inflation

Inside the bubble: an observer see an inflating Universe

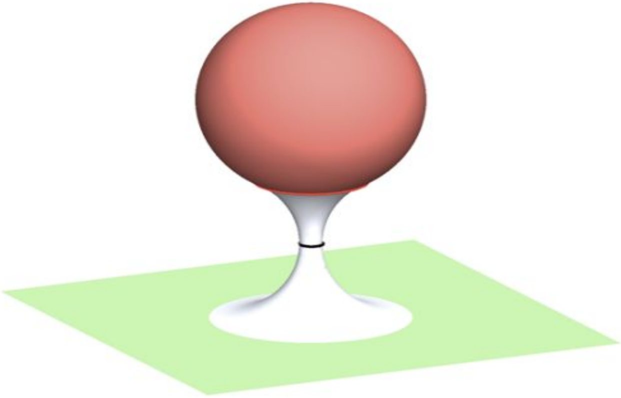
Outside the bubble: an observer will see a black hole (PBH)

J. Garriga, A. Vilenkin. Arxiv: 1210.7540
J. Garriga, A. Vilenkin, J. Zhang. Arxiv:1512.01819
H. Deng, A. Vilenkin. Arxiv:1710.02865
A. Kusenko, M. Sasaki, S. Sugiyama, M. Takada, V. Takhistov, E. Vitagliano. Arxiv:2001.09160

vacuum bubbles may be produced by quantum tunneling during inflation.

In this scenarios, tunneling is assumed to be a Poissonian process which can happen with nearly constant probability per unit time and volume

PBH mass function is rather broad.

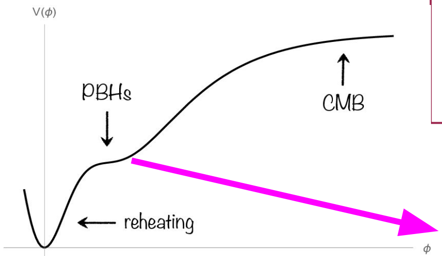


Another scenario:

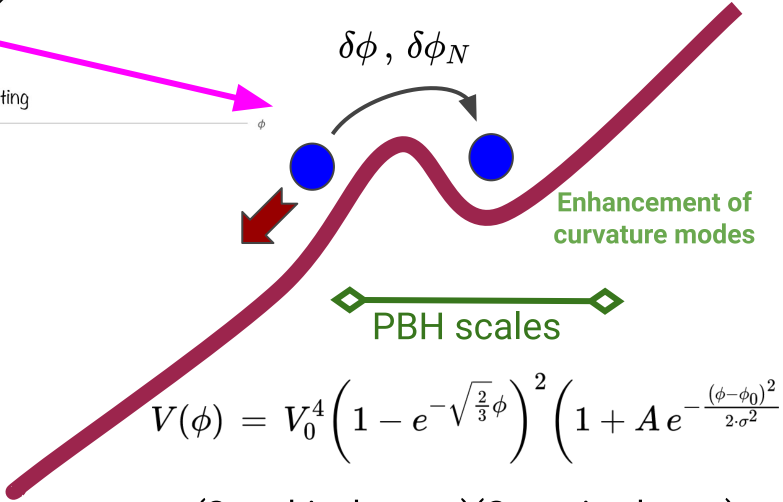
False vacuum bubbles can be formed from large backward fluctuations of the inflaton in single-field inflationary models containing a bump.

V. Atal, J. Garriga and A. M. Caballero. Arxiv:1905.13202
V. Atal, J. Cid, A. Escriva, J. Garriga. Arxiv:1908.11357

(See also R. Kawaguchi, T. Fujita, M. Sasaki. Arxiv:2305.18140)



Include a bump in the inflationary potential



$$V(\phi) = V_0^4\left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2\left(1 + A\, e^{-\frac{(\phi-\phi_0)^2}{2\cdot\sigma^2}}\right)$$

(Starobinsky type)(Gaussian bump)

The inflaton exits inflation, but sufficiently large backward fluctuations can prevent it from overshooting the barrier in horizon sized region

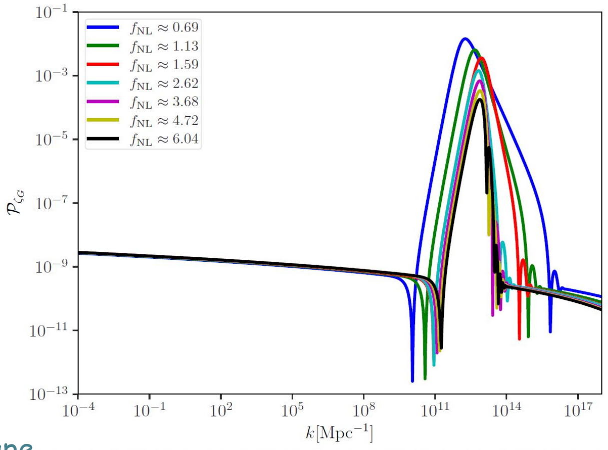
Formation of localized false vacuum bubbles!

Two coexistent channels for the production of PBHs

- From large adiabatic fluctuations (the standard one)
- From false vacuum bubbles → Never explored numerically!

Let's start first with the adiabatic channel

Solving the MS equation numerically we obtain the power spectrum of the Gaussian curvature fluctuation.



Gaussian curvature fluctuation

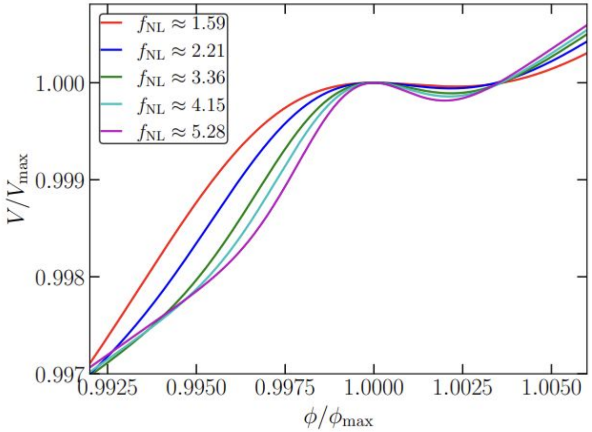
$$< (\Delta\zeta_G)^2 >= \sigma_0^2[1 - \Psi^2_{\zeta_G}(r)]$$

$$\zeta_G(r) = \mu_a \Psi_{\zeta_G}(r) \pm \Delta\zeta_G = \mu_a \frac{1}{\sigma_a^2}\int_{k_i}^{k_f} \mathcal{P}_{\zeta_G} \frac{\sin(kr)}{kr} d\ln k \pm \Delta\zeta_G$$

(considering very large peaks $\nu_a = \mu_a/\sigma_a \sim \mathcal{O}(10)$ -> spherically symmetric)

The dispersion in the shape from the mean profile is small

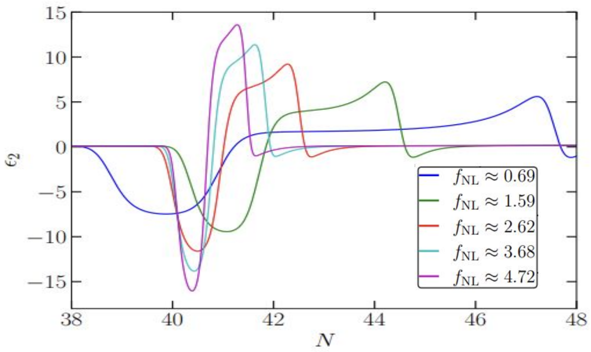
The small bump induces a non-gaussian contribution->invalidates Gaussian assumption $\zeta \neq \zeta_G$



$$f_{NL} = \frac{5}{12}\left[-3 + \sqrt{9 - 12\frac{V''(\phi_{\rm max})}{V(\phi_{max})}}\right]$$

$$\epsilon_1 = \dot{\phi}_b^2/2$$

$$\epsilon_2 = \dot{\phi}_b\ddot{\phi}_b/\epsilon_1$$



Full-non gaussian curvature fluctuation

V. Atal, J. Garriga and A. Marcos-Caballero. Arxiv:1905.13202

$$\zeta \neq \zeta_G$$

See also S.Pi and M. Sasaki. arXiv:2211.13932

$$\zeta = -\mu_\star \ln\left(1 - \frac{\zeta_G}{\mu_\star}\right)$$

$$\zeta_G = \mu(1 - e^{-\zeta/\mu_\star})$$

$$\mu_\star = \frac{5}{6\, f_{NL}}$$

From delta N formalism

$$\zeta \to \infty \Rightarrow \zeta_G \to \mu_\star$$

PDF for the NG curvature fluctuation

$$P[\zeta] = P_G[\zeta_G(\zeta)]\, \frac{d\zeta_G}{d\zeta}$$

"Exponential tail"

$$\frac{d\,\zeta_G}{d\,\zeta} = e^{-\zeta/\mu_\star} \qquad P[\zeta] \propto e^{-\zeta/\mu_\star}, \quad (\zeta \to \infty)$$

$\zeta_G = \mu_\star$ Singular point

$P_G[\zeta_G]$ normalized

$P[\zeta]$ No normalized!

$$\int P[\zeta]\, D\zeta = \int_{\zeta_G < \mu_\star} P_G[\zeta_G]\, D\zeta_G < 1$$

→ The singularity indicates the presence of alternative channels for PBH production, which restores unitarity.

We need the threshold and mass to estimate the PBH abundance…

$$ds^2 = -dt^2 + a^2(t)e^{2\zeta(r)}\left[dr^2 + r^2 d\Omega^2\right]$$

(initial condition at super-horizon scales)

We can follow two procedures to obtain the threshold:

Make the full numerical simulation (very difficult for some cases of large NGs…) A.Escrivà. ArXiv:1907.13065

Use the analytical estimate based on the compaction function

The compaction function has been shown to be useful in the context of PBH formation

M. Sasaki, M. Shibata. Arxiv:gr-qc/9905064
T. Harada, C.M. Yoo, T. Nakama, Y. Koga. Arxiv:1503.03934

$$\mathcal{C}(r,t) = 2\frac{M - M_b}{R}$$

twice the local excess-mass over the co-moving areal radius

(at super-horizon scales)

$$\mathcal{C}(r) = \frac{2}{3}\left[1 - \left(1 + r\,\zeta'(r)\right)^2\right]$$

We need the threshold and mass to estimate the PBH abundance…

To find the threshold: Find iteratively the value of the amplitude such that satisfy the relation:

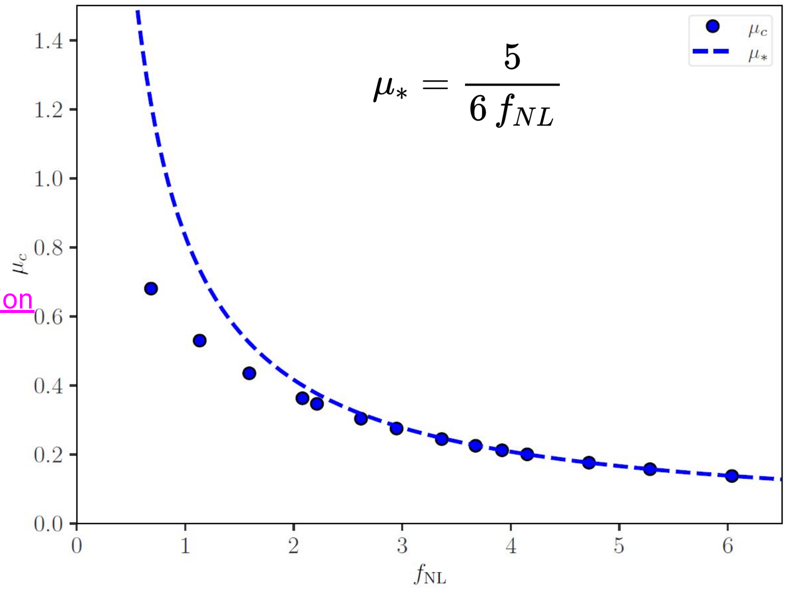
$$\mathcal{C} = \frac{2}{3}\left[1 - \left(1 + r\,\zeta'\right)^2\right]$$

A.Escrivà, C.Germani, R. K.Sheth. Arxiv:1907.13311
(see also A. Escrivà, C.M. Yoo. Arxiv:2310.16482)

$$\bar{\mathcal{C}}_c = 2/5$$

Universal value! Independent on the shape of the fluctuation within 2% deviation

$$\bar{\mathcal{C}}_c = \frac{3}{r_m^3(\mu_{a,c})e^{3\zeta(r_m(\mu_{a,c}))}} \int_0^{r_m(\mu_{a,c})} \mathcal{C}_c(r)(1 + r\zeta')e^{3\zeta(r)}r^2 dr.$$



What about the PBH mass near the critical regime?

N. Kitajima, Y. Tada, S.Yokoyama, C.M. Yoo. Arxiv:2109.00791

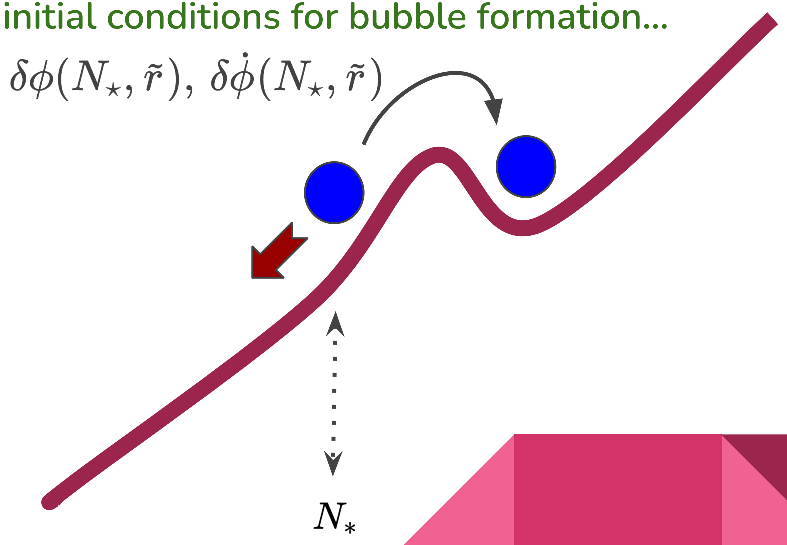
$$M_{\rm PBH}(\mu_a) = \mathcal{K}_a(\mu_{a,c})M_k(k)x_m^2(\mu_a)e^{2\zeta(r_m(\mu_a))}(\mu_a - \mu_{a,c})^{\gamma_a}$$

Let's move to the bubble channel->Numerical formation of bubbles

We need to solve the KG field equation taking into account a radial dependence

$$\ddot{\phi} + \dot{\phi}\left(3 - \frac{1}{2}\dot{\phi}_b^2\right) - \left(\frac{a_I H_I}{a(N)H(N)}\right)^2 \Delta\phi + \frac{1}{H^2}\frac{V_\phi(\phi)}{V(\phi)} = 0$$

But we have a problem!: we need to find the correct initial conditions for bubble formation...



In general, we can consider:

$$\phi(N_\star, \tilde{r}) = \phi_b(N_\star) + \delta\phi(N_\star, \tilde{r})$$
$$\dot{\phi}(N_\star, \tilde{r}) = \dot{\phi}_b(N_\star) + \delta\dot{\phi}(N_\star, \tilde{r})$$

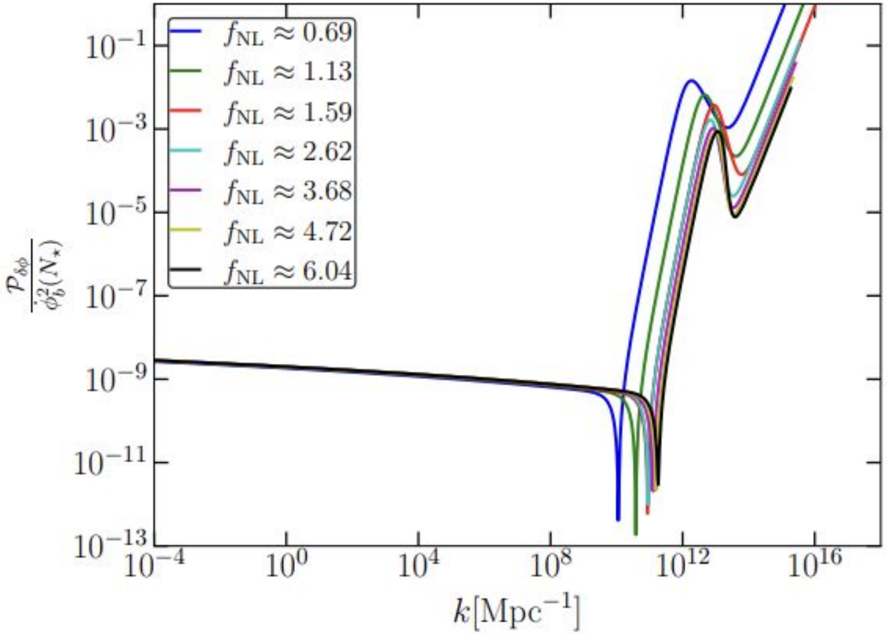
Initial conditions for bubble formation

$$\mathcal{P}_{\delta\phi}(N_\star, k) \gg \frac{k^2}{a^2(2\pi)^2}$$

Let's consider first the perturbation for the field space

$$\mathcal{P}_{\delta\phi}(N_\star, k) = \frac{k^3}{2\pi^2}\dot{\phi}_b^2(N_\star) \mid \zeta_G(N_\star, k) \mid^2$$

we evaluate the " k " $-$modes at N_*



$$\Psi_b(N_\star, \tilde{r}) = \frac{1}{\sigma_b^2}\int_{k_i}^{k_f} \mathcal{P}_{\delta\phi}(N_\star, k)\,\mathrm{sinc}(k\tilde{r})d\ln k$$

$$\delta\phi(N_\star, \tilde{r}) = \mu_b \Psi_b(N_\star, \tilde{r})$$

(like in the adiabatic channel)

Initial conditions for bubble formation

What about the perturbation for the velocity?

During the attractor regime:
clear relation between position and momentum (which also holds for the perturbations)

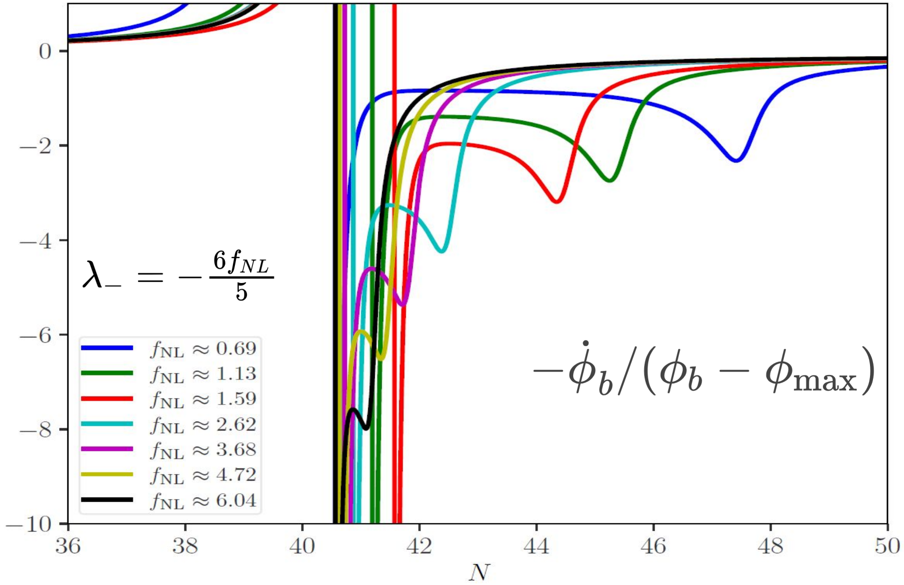
$$\dot{\phi}_b \approx -\lambda_-(\phi_b - \phi_{\max}) \Rightarrow \delta\dot{\phi} \approx -\lambda_- \delta\phi$$

⬇

Atractor regime

How do we choose?

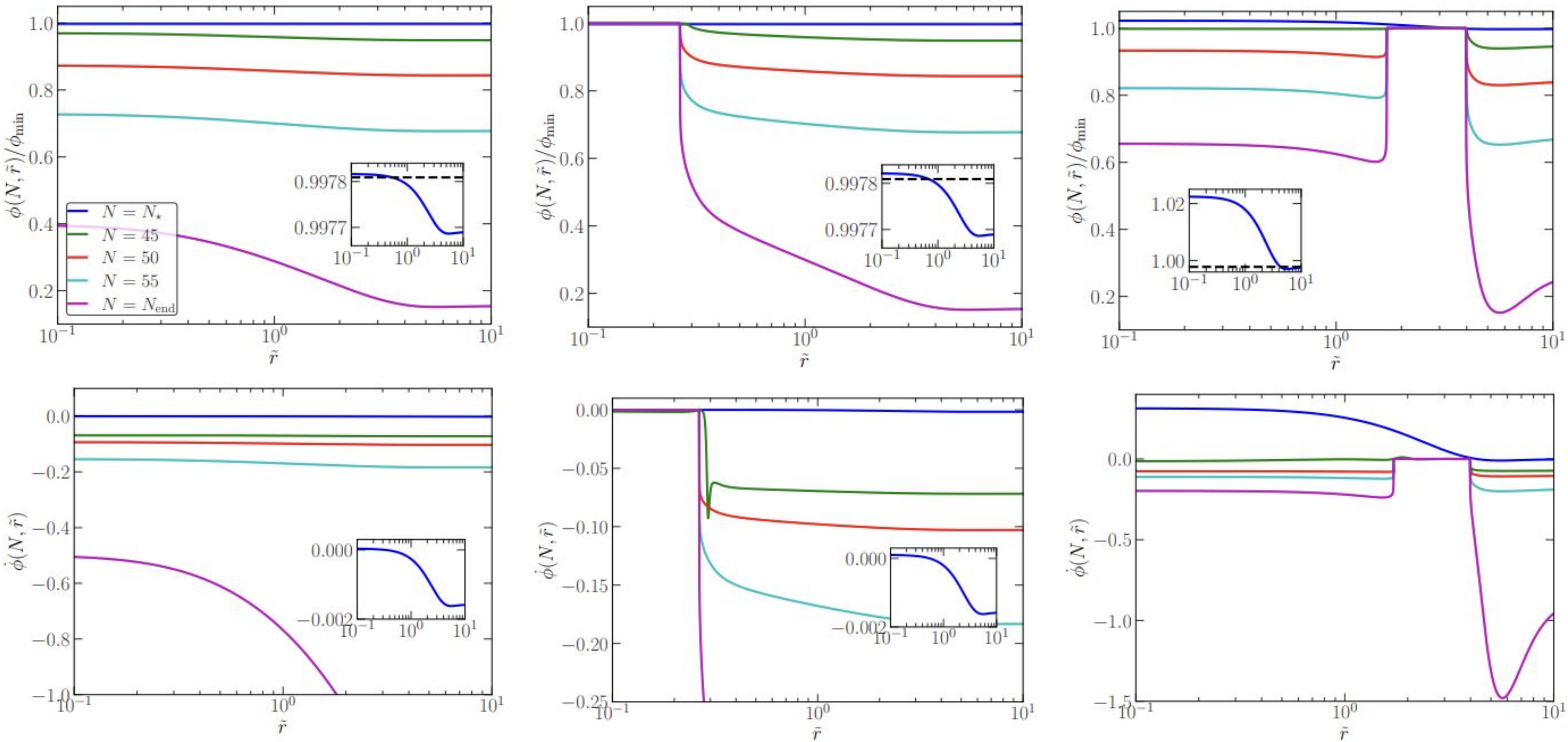
N_* ⟶ When the numerics is closer to the attractor regime



Dynamics of bubble formation

$\mu_{b,c} \approx 5 \cdot 10^{-4}$

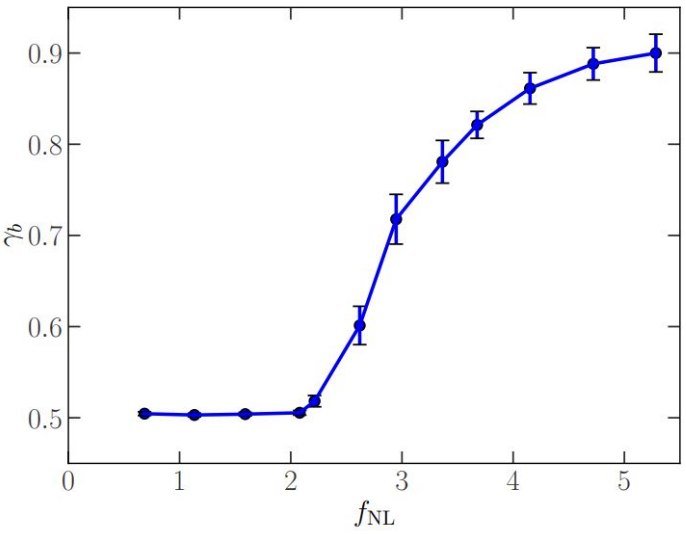
$\mu_b < \mu_{b,c}$ $\mu_b > \mu_{b,c}$ $\mu_b \gg \mu_{b,c}$



Comoving size of the bubbles

We find a critical regime for the bubble size!

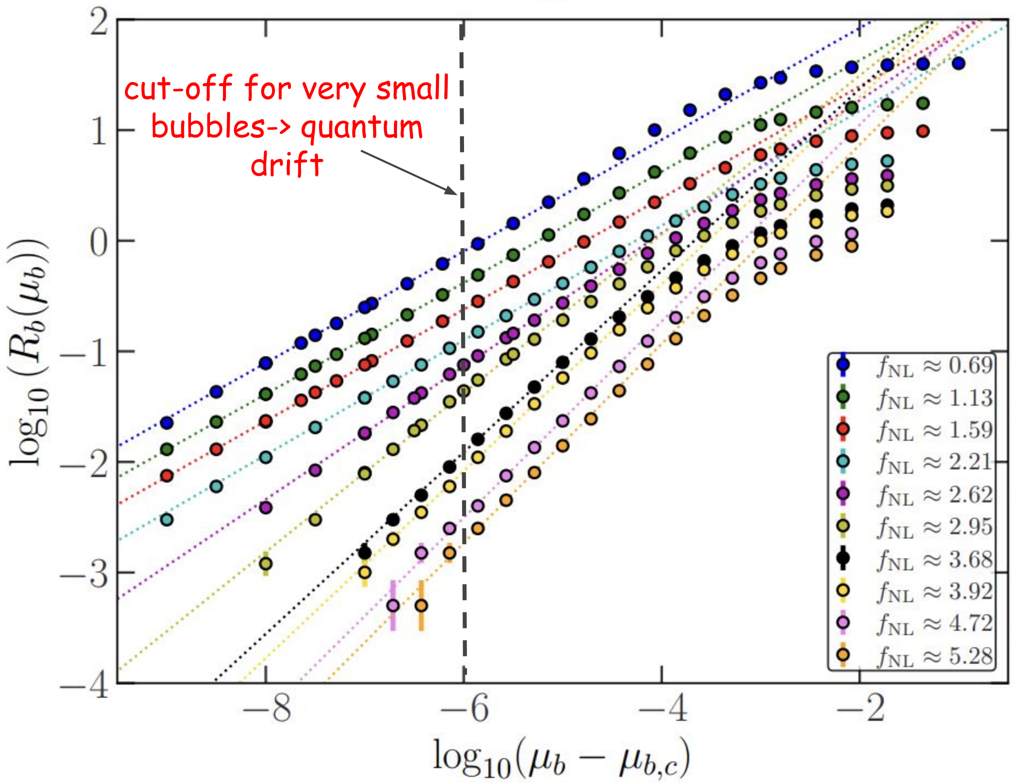
$$R_b(\mu_b) = \mathcal{K}_b(\mu_{b,c})(\mu_b - \mu_{b,c})^{\gamma_b(f_{\rm NL})}$$



$$\Delta\mu \sim \frac{H}{2\pi}(\Delta N)^{1/2} \gtrsim (\mu_b - \mu_{b,c})$$

$$\mu_{b,(\text{cut-off})} - \mu_{b,c} \approx \sqrt{-\ln(R_b(\mu_b))}\frac{H(N_\star)}{2\pi}$$

Reminiscent to the critical collapse in gravitational collapse phenomena, but gravity forces here does not play a role



Ratio of PBH production between the two channels

The prov. distribution of bubbles is Gaussian

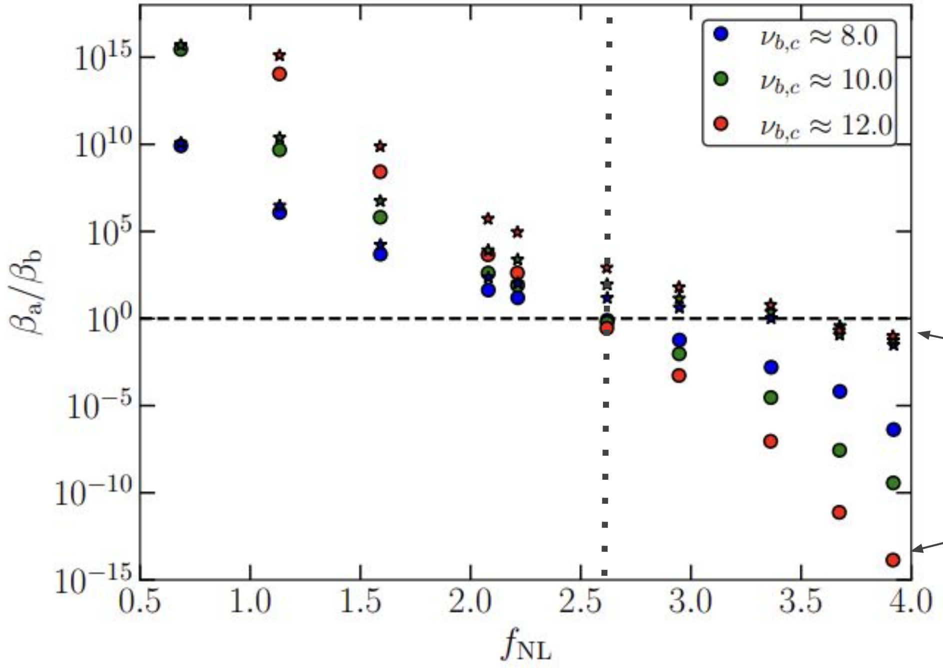
Apply peak theory in curvature peaks

C.M. Yoo, T. Harada, J. Garriga, K. Kohri. ArXiv:1805.03946

$$\mathcal{N}_{peak} = \left(\frac{\sigma_1}{\sqrt{3}\sigma_0}\right)^3 (\nu^3 - 3\nu) e^{-\nu^2/2}$$

$$\beta_a = \int_{\nu_{a,c}}^{\nu_*} \mathcal{N}_a(\nu) d\nu$$

$$\beta_b = \int_{\nu_{b,c}}^{\infty} \mathcal{N}_b(\nu) d\nu$$



$f_{NL} \approx 2.6$ → Both channels give same abundance of peaks

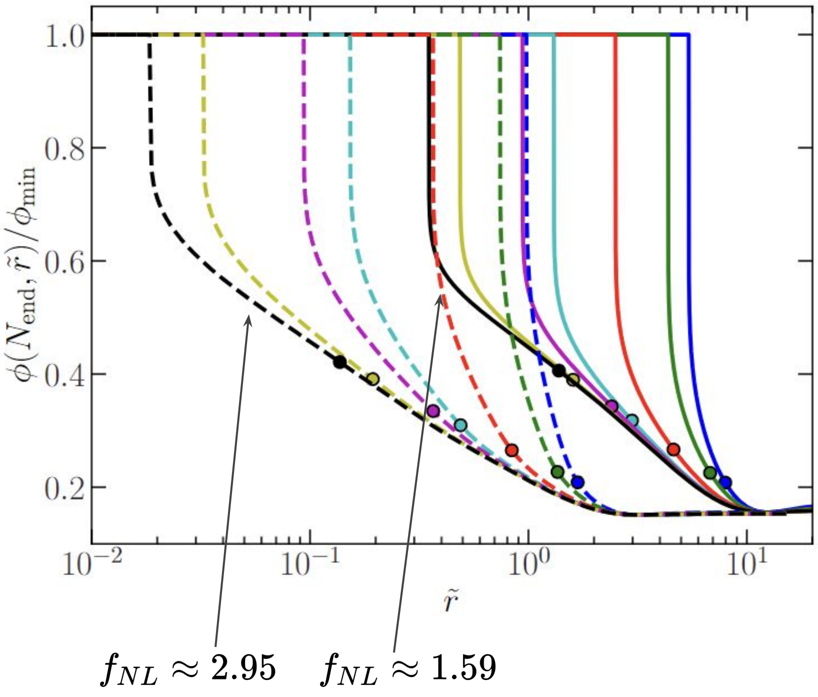
Using the analytical estimate $\mu_{b,c} = \mu_\star$

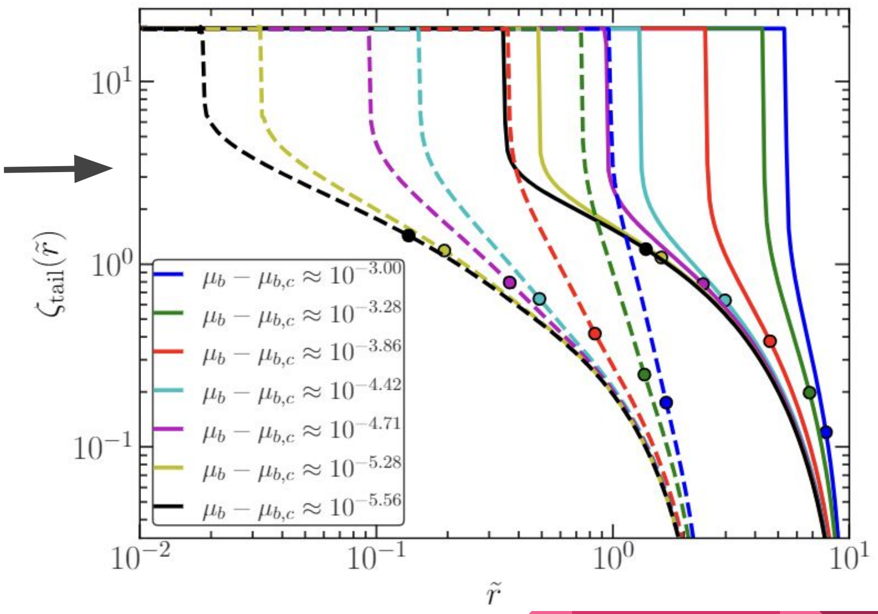
Using the numerical results

Vacuum bubbles are more easily formed than what we previously found!

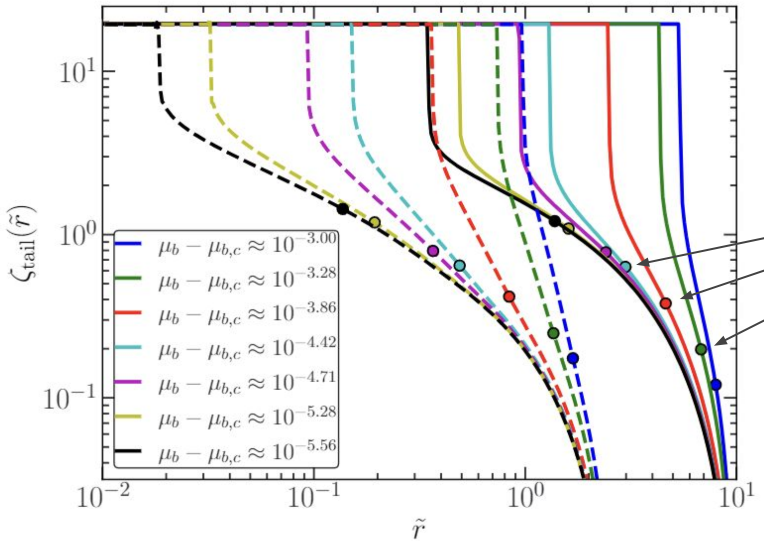
Inflaton field at the end of inflation

Corresponding curvature fluctuation with delta N formalism





Surprisingly: surrounding fluctuation of type II



Fluctuation of type II->Areal radius non-monotonic increasing function

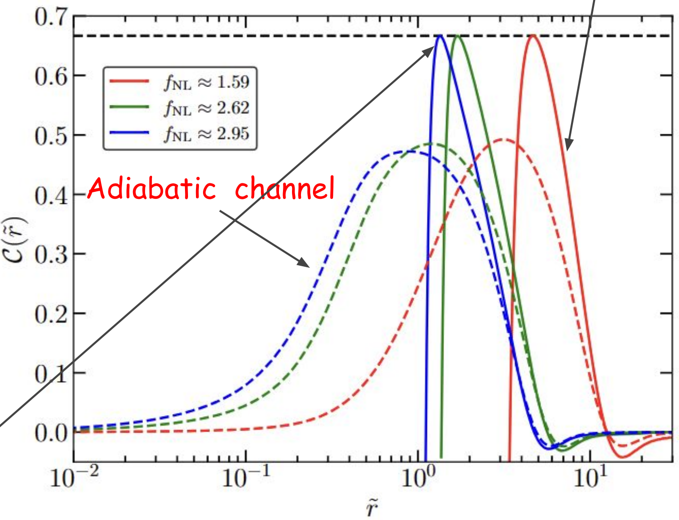
$$R = a\, r\, e^{\zeta_{tail}} \Rightarrow R' = a e^{\zeta_{tail}}\left(1 + r\, \zeta'_{tail}\right)$$

$$R' < 0 \Rightarrow 1 + r_{tail}\, \zeta'_{tail}(r_{tail}) = 0$$

$$\mathcal{C}(r_{tail}) = 2/3$$

Bubble channel

Adiabatic channel



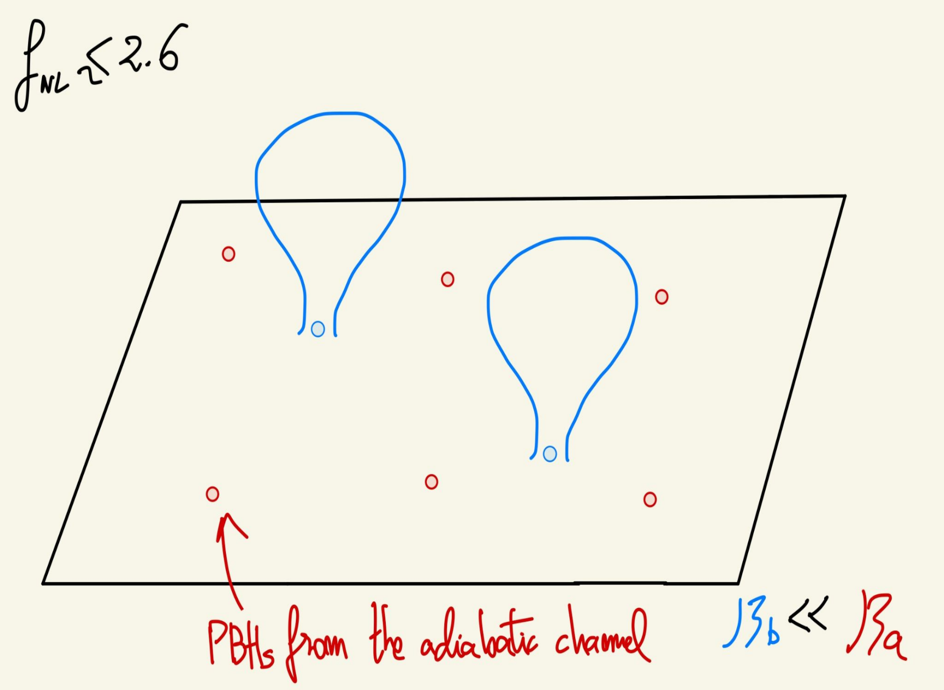
Fluctuations of type II corresponds to very large amplitudes far away from the threshold value->largely statistically suppressed in the adiabatic channel in our model

But in the bubble channel, actually gives the dominant contribution!

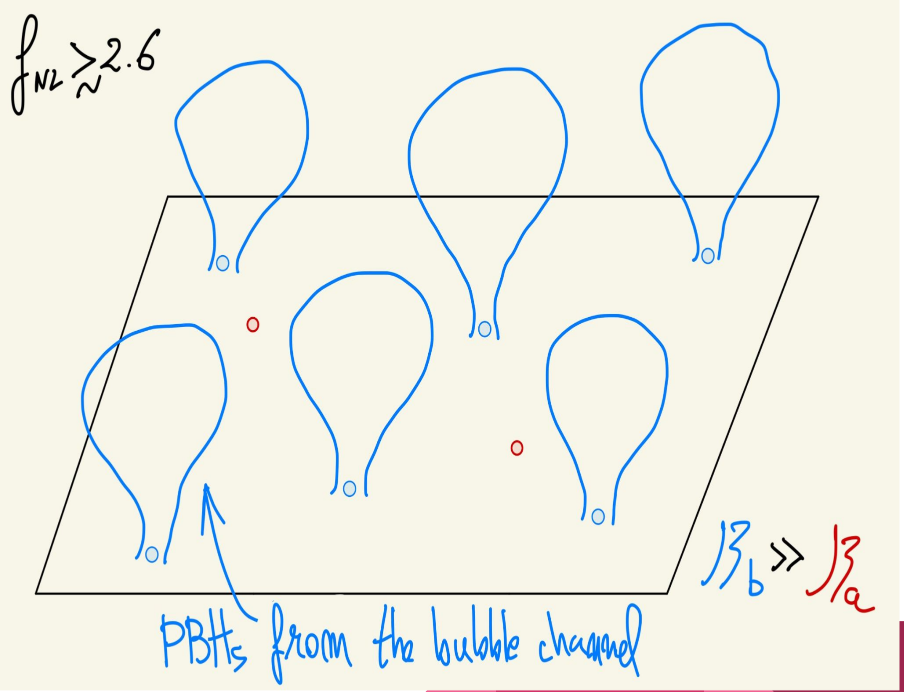
$$M_{bubble\ channel} \sim M_k(k_{tail}) \qquad k_{\rm tail}(\nu_b) = \frac{1}{r_{\rm tail}(\nu_b)\, e^{\zeta_{\rm tail}(r_{\rm tail}(\nu_b))}}$$

Qualitative picture

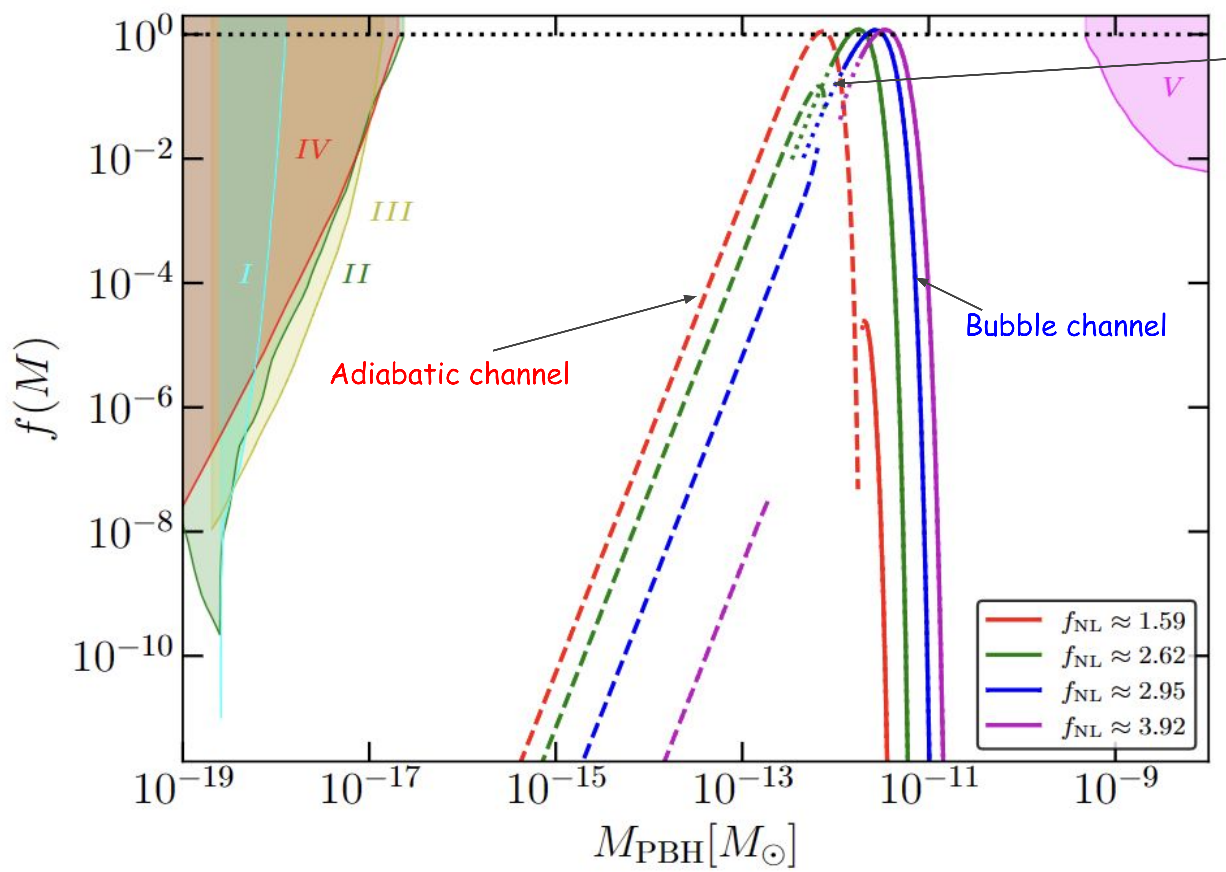
Fluctuations type I-> dominant contribution



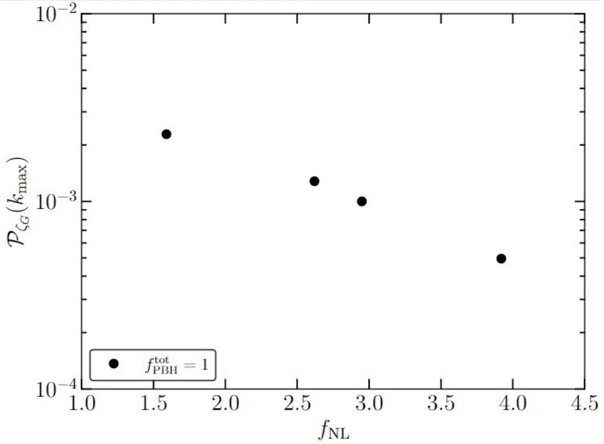
Fluctuations type II-> dominant contribution



Mass function from both channels



$$f_{\rm a}(M_a) = \frac{M_{\rm a}(\nu_a)\mathcal{N}_{\rm pk}(\nu_a(M_a))}{\rho_{\rm critical}\Omega_{\rm DM}}\left|\frac{d\ln M_a(\nu_a)}{d\nu_a}\right|^{-1}$$



Thanks for your attention!

Conclusions and messages to take home:

- The dynamics of vacuum bubble formation have been studied and clarified. We find a critical regime for the size of the bubbles.
- The log-relation for the full NG curvature fluctuation is successfully accurate in predicting the bubble channel of PBH production for small non-gaussianity (attractor regime condition).
- Bubbles are more easily formed than previously expected. The bubble channel is already dominant for fnl>2.6.
- A surrounding fluctuation of type II dominates the mass of PBHs generated from the bubble channel.
- The mass function for the bubble channel is highly monochromatic.
- The presence of alternative channels for PBH production in models with local-type non-Gaussianity can be easily inferred from unitarity considerations.