Focus Week Primordial Black Holes 2023

New research directions in high-frequency gravitational waves

Keisuke Inomata, Kazunori Kohri, Takahiro Terada, arXiv:2306.17834 [astro-ph.CO]
Asuka Ito, Kazunori Kohri, Kazunori Nakayama, arXiv:2309.14765 [gr-qc]

Kazunori Kohri
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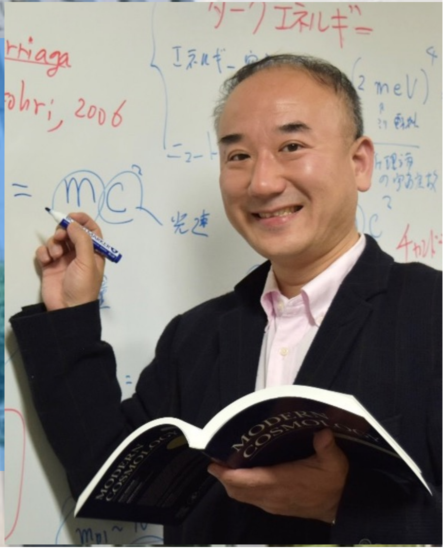


SOKENDAI QUP NAOJ iPNS KEK 理論センター THEORY CENTER KAVLI IPMU

Contents

- The search for high-frequency GWs is a new direction for investigating phenomena in the early Universe.
- The targets are so many:

  1. GWs from merging binary PBHs with subsolar mass
  2. Thermal/nonthermal graviton just after inflation,
  3. 1st-order phase transition at E >> weak scale
  4. …

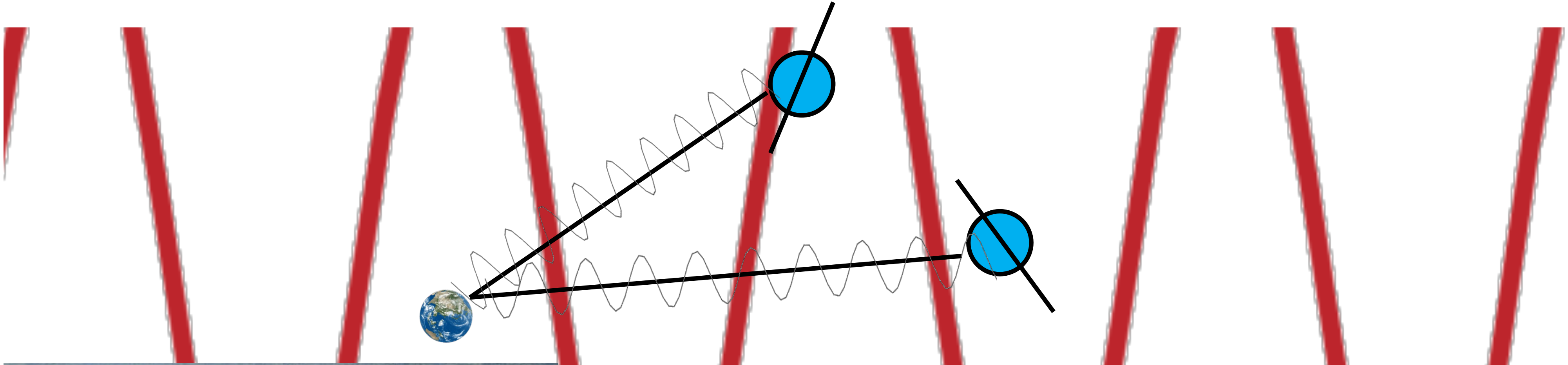
- We can test high-frequency GWs by observing the electromagnetic wave converted from the GWs.

Gabriella Agazie, et al, The NANOGrav15yr collaboration, arXiv:2306.16213 [astro-ph.HE]

NANOGrav 15yr

(North American Nanohertz Observatory for Gravitational Waves)

found stochastic GWs through pulsar timing

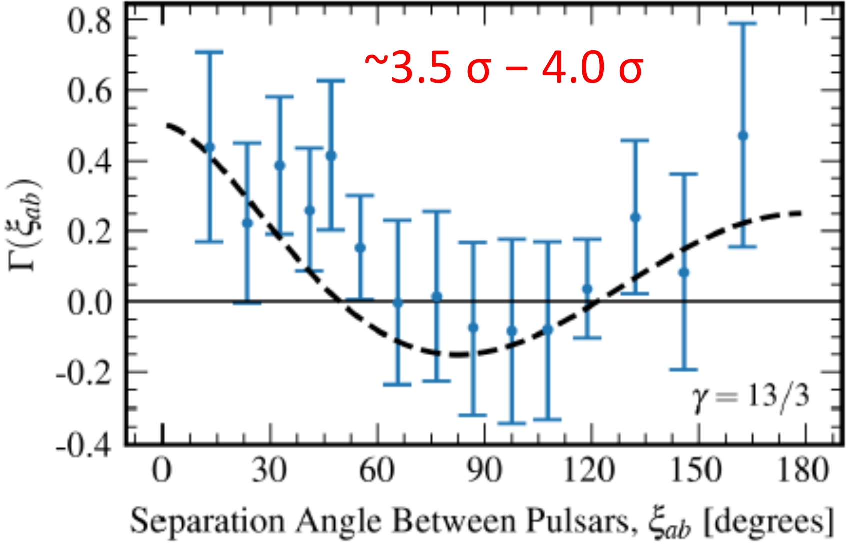


The 305-meter dish of the William E.
Gordon Telescope, The Arecibo Obs.

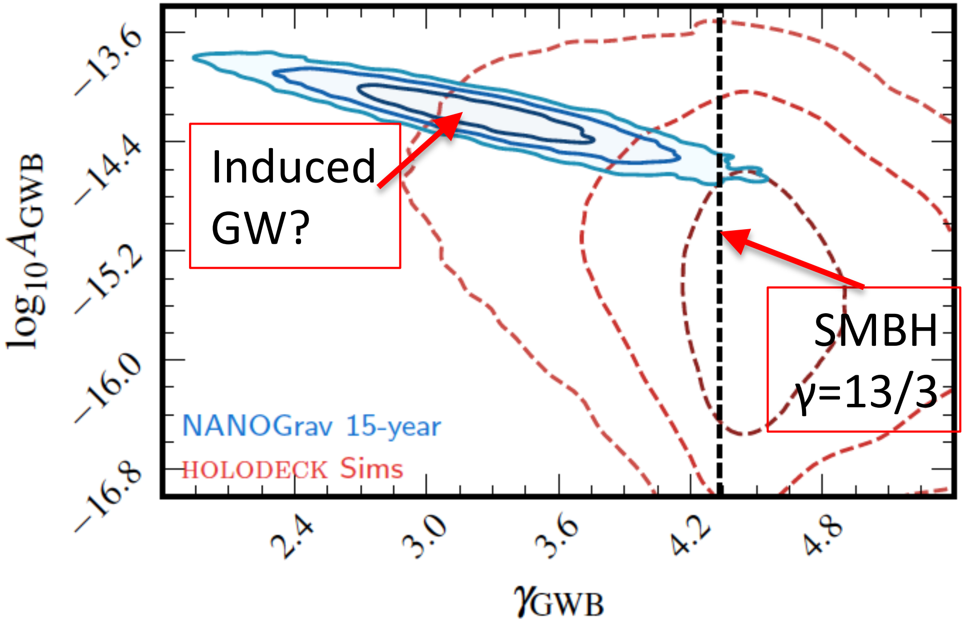
The 100-meter Green Bank Telescope

The NANOGrav 15-year Data Set: Evidence for a Gravitational-Wave Background

Gabriella Agazie, et al, The NANOGrav15yr collaboration, arXiv:2306.16213 [astro-ph.HE]


Hellings-Downs Curve

~3.5 σ – 4.0 σ


Induced GW?

SMBH
γ=13/3

$$h_c(f) = A_{\rm GWB}\left(\frac{f}{f_{\rm yr}}\right)^{\alpha}$$

$$S_{ab}(f) = \Gamma_{ab}\,\frac{A_{\rm GWB}{}^2}{12\pi^2}\left(\frac{f}{f_{\rm yr}}\right)^{-\gamma} f_{\rm yr}^{-3}$$

$$\Omega(f) = \frac{2\pi}{3H_0^2} f^2 h_c(f)^2 = \Omega_{yr}\left(\frac{f}{f_{\rm yr}}\right)^{\beta}$$

$\gamma = 3 - 2\alpha = 5 - \beta$

$\beta = 5 - \gamma\ (\sim 2)$

Implications of NANOGrav15yr for Inflation and/or Dark Matter

Keisuke Inomata, Kazunori Kohri, Takahiro Terada, arXiv:2306.17834 [astro-ph.CO]

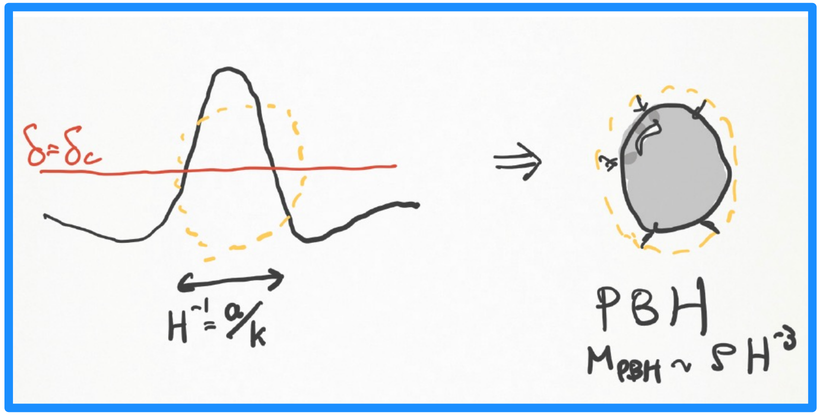
- Possibility of stochastic induced GW (iGW)

$$\Omega_{GW} \sim 10^{-8} \propto \delta^4 \text{ at } f \sim 10^{-8}\ \text{Hz}$$

- Suggests large density fluctuations $\langle\delta^2\rangle$ on small scales

$$\langle\delta^2\rangle \sim O(0.01) \gg 10^{-9} \text{ at } k \sim 10^{7}\ \text{Mpc}^{-1}$$

- The same fluctuations simultaneously create a PBH



$$M_{PBH} \sim O(10^{-5})\ M_\odot$$

$$f_{PBH} = \Omega_{PBH}/\Omega_{CDM} \sim O(0.01)$$

$1M_\odot = 2\times10^{33}$g

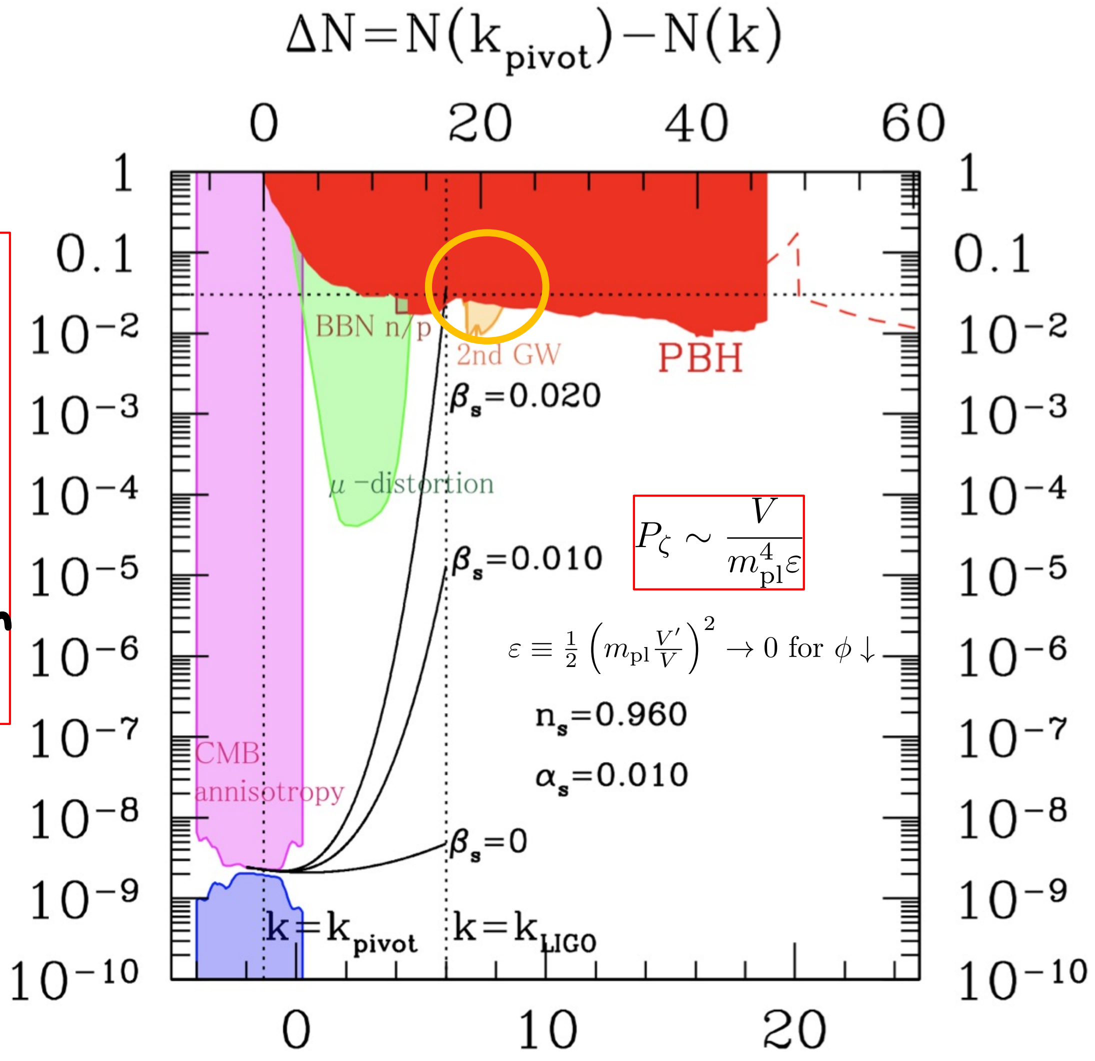
Curvature perturbation $P_\zeta(k)$

The δ-function like form

Kohri and T.Terada, 2018
Alabidi, Kohri, Sasaki, Sendouda, 2013

Amplitude of curvature perturbation

$P_\zeta \sim \langle \delta^2 \rangle$



$$P_\zeta \sim \frac{V}{m_{\rm pl}^4 \varepsilon}$$

Planck (2018)

$$n_s = 0.9586 \pm 0.0056,$$
$$\alpha_s = 0.009 \pm 0.010,$$
$$\beta_s = 0.025 \pm 0.013.$$

at 68% C.L.

For inflation models with a big running, see Kohri, Lin, Lyth (2008)
See also, A. Green, arXiv:1805.05178

Wave number $\log_{10}(k/\mathrm{Mpc}^{-1})$

k = p x a

Secondary gravitational wave induced (IGW) from large curvature perturbation ($P_\zeta >> r$) at small scales

K. N. Ananda, C. Clarkson, and D. Wands, 2006
D.Baumann, P.J.Steinhardt, K.Takahashi and K.Ichiki,2007
R.Saito and J.Yokoyama, 2008
José Ramón Espinosa, Davide Racco, Antonio Riotto, 2018
Kohri and T.Terada, 2018
R.-G. Cai, S. Pi, and M. Sasaki, 2019

- Power spectrum of the tensor mode

$$\langle h^r_{\boldsymbol{k}}(\eta) h^s_{\boldsymbol{k}'}(\eta)\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_h(k,\eta)\delta(\boldsymbol{k}+\boldsymbol{k}')\delta^{rs}, \qquad h_{ij}(\boldsymbol{x},\eta) = \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\left[h^+_{\boldsymbol{k}}(\eta)\mathrm{e}^+_{ij}(\boldsymbol{k}) + h^\times_{\boldsymbol{k}}(\eta)\mathrm{e}^\times_{ij}(\boldsymbol{k})\right]$$

- Omega parameter well inside the horizon

$$\Omega_{\mathrm{GW}}(k,\eta) = \frac{1}{3}\left(\frac{k}{\mathcal{H}}\right)^2 \mathcal{P}_h(k,\eta).$$

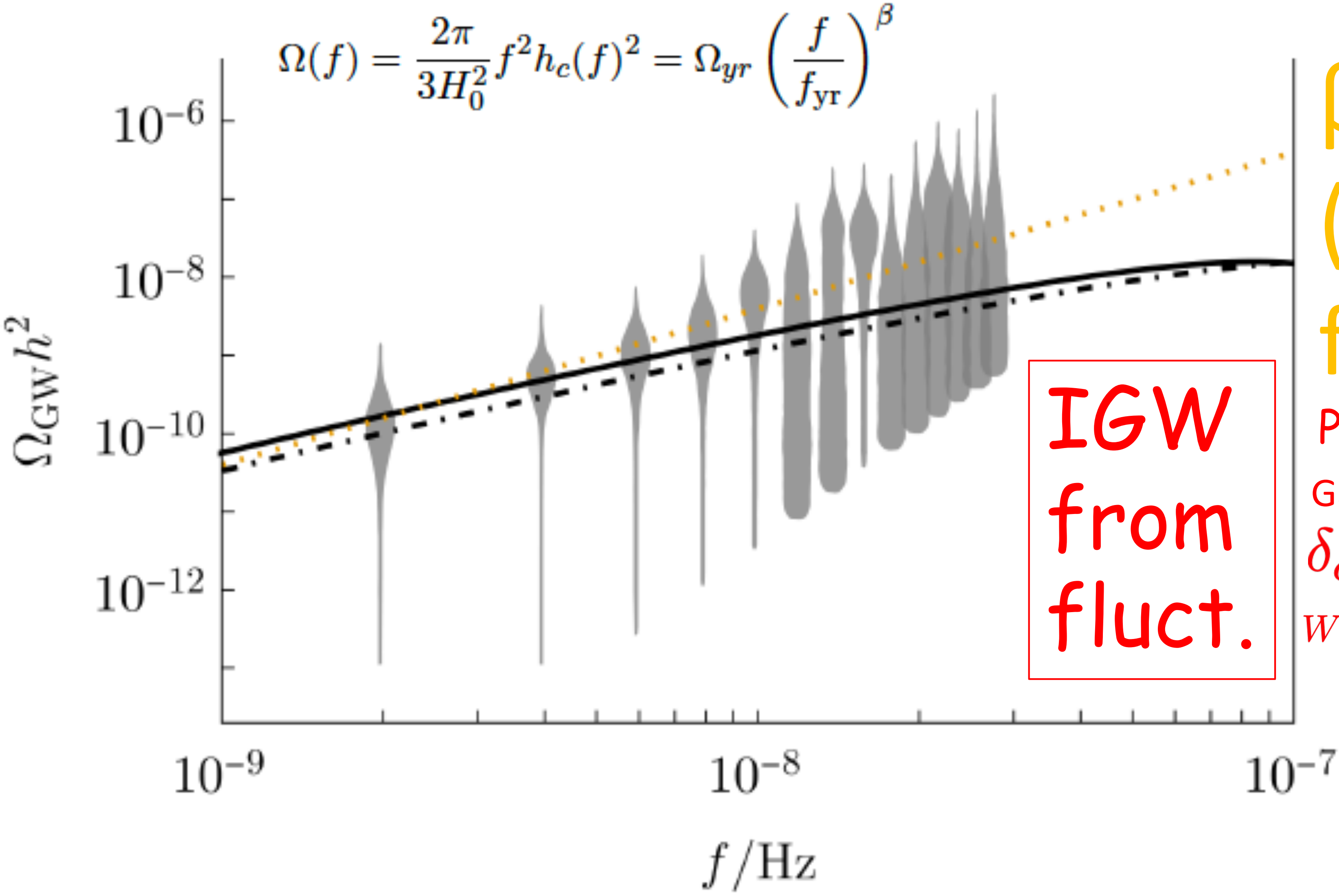
- Substituting the solution into this

$$\Omega_{\mathrm{GW},c}(f) = \frac{1}{12}\left(\frac{f}{2\pi a H}\right)^2 \int_0^\infty \mathrm{d}t \int_{-1}^{1} \mathrm{d}s \left[\frac{t(t+2)(s^2-1)}{(t+s+1)(t-s+1)}\right]^2 \times \overline{I^2(t,s,k\eta_c)}\,\mathcal{P}_\zeta\left(\frac{(t+s+1)f}{4\pi}\right)\mathcal{P}_\zeta\left(\frac{(t-s+1)f}{4\pi}\right)$$

$$\boxed{P_\zeta \sim \delta^2}$$

NANOGrav15yr by Induced GW and sub-solar PBHs

Keisuke Inomata, Kazunori Kohri, Takahiro Terada, arXiv:2306.17834 [astro-ph.CO]



$$\Omega(f) = \frac{2\pi}{3H_0^2} f^2 h_c(f)^2 = \Omega_{yr}\left(\frac{f}{f_{\rm yr}}\right)^{\beta}$$

IGW
from
fluct.

β=2
(γ=3)
for
P_ζ~δ-func.
Gaussian ζ

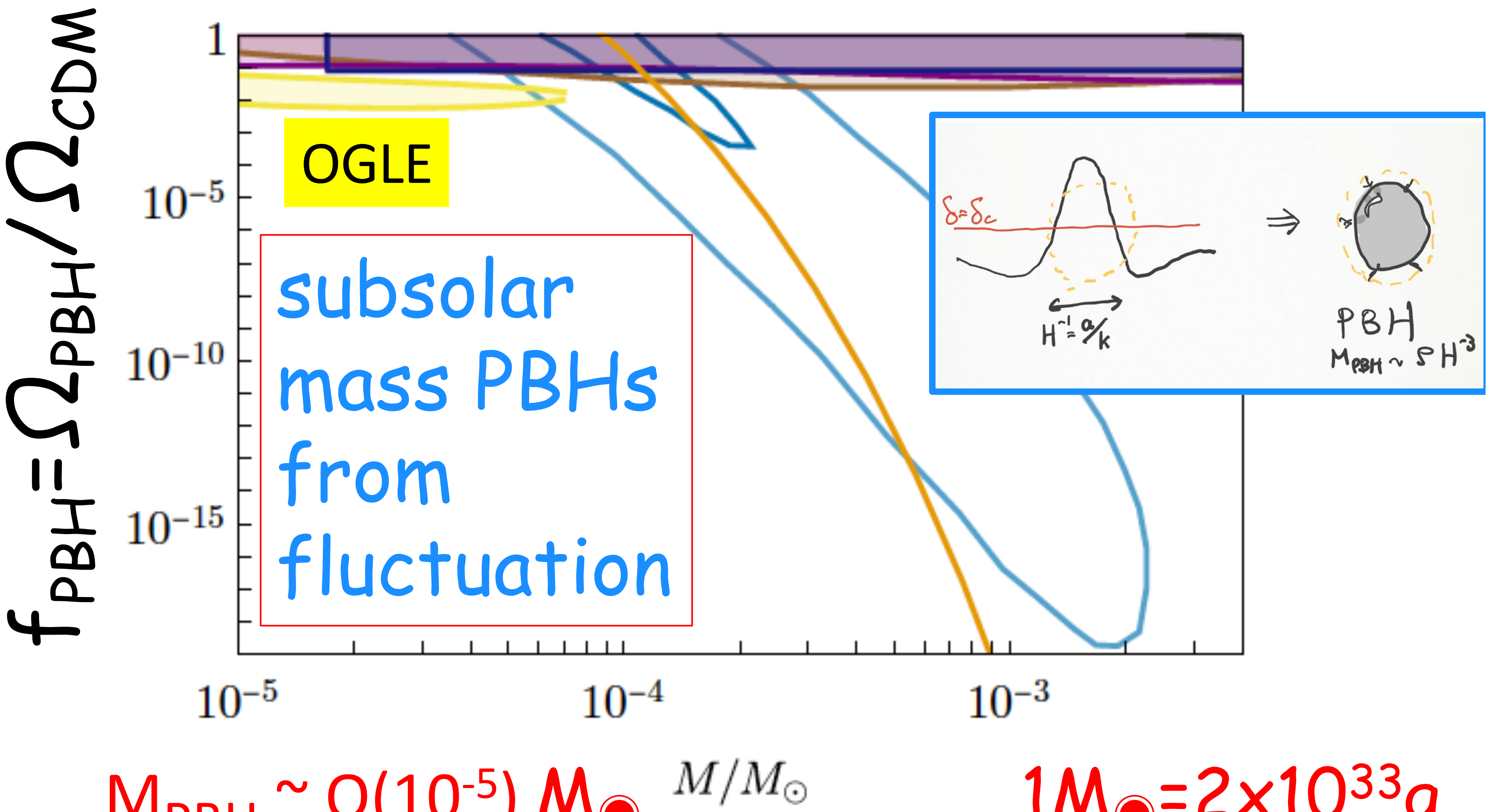
$$\delta_c = 0.45$$

$$W(x) = e^{-x^2/2}$$

NANOGrav15yr by Induced GW and sub-solar PBHs

Keisuke Inomata, Kazunori Kohri, Takahiro Terada, arXiv:2306.17834 [astro-ph.CO]

$f_{PBH} = \Omega_{PBH}/\Omega_{CDM} \sim O(0.01)$ -- $O(0.1)$

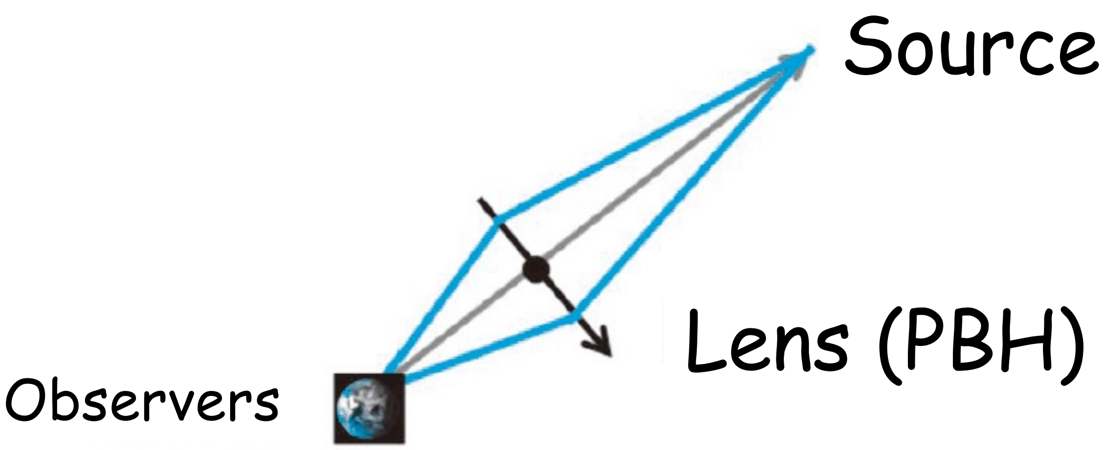


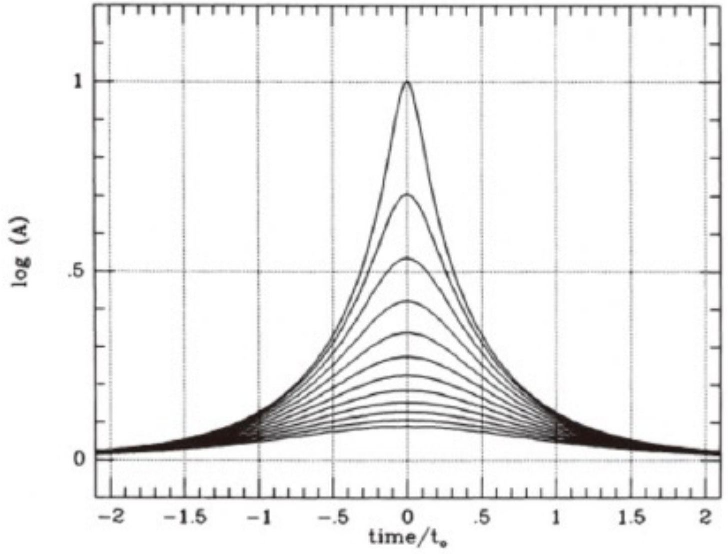
subsolar mass PBHs from fluctuation

$M_{PBH} \sim O(10^{-5})\, M_\odot$

$1M_\odot = 2\times10^{33}g$

Gravitational Lensing





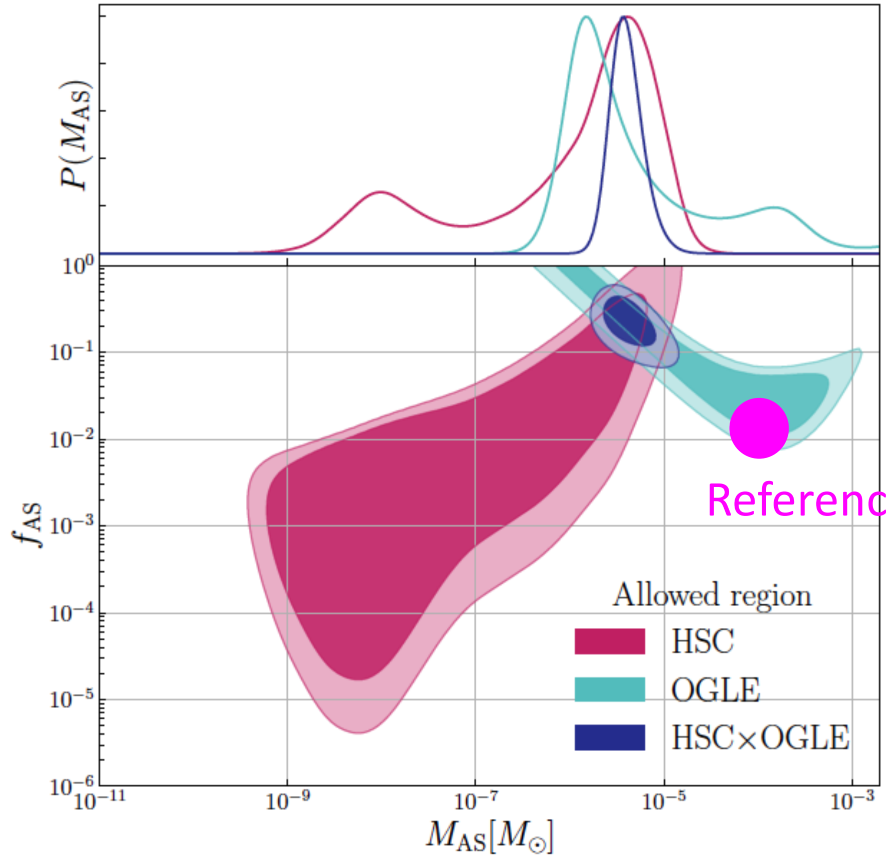
Hiroko Niikura, https://stg.asj.or.jp/jp/activities/geppou/item/113-1_6.pdf

HSC x OGLE events

Sunao Sugiyama, Masahiro Takada, Alexander Kusenko, arXiv:2108.03063 [hep-ph]

Hiroko Niikura, Masahiro Takada, Shuichiro Yokoyama, Takahiro Sumi, Shogo Masaki, arXiv:1901.07120 [astro-ph.CO]

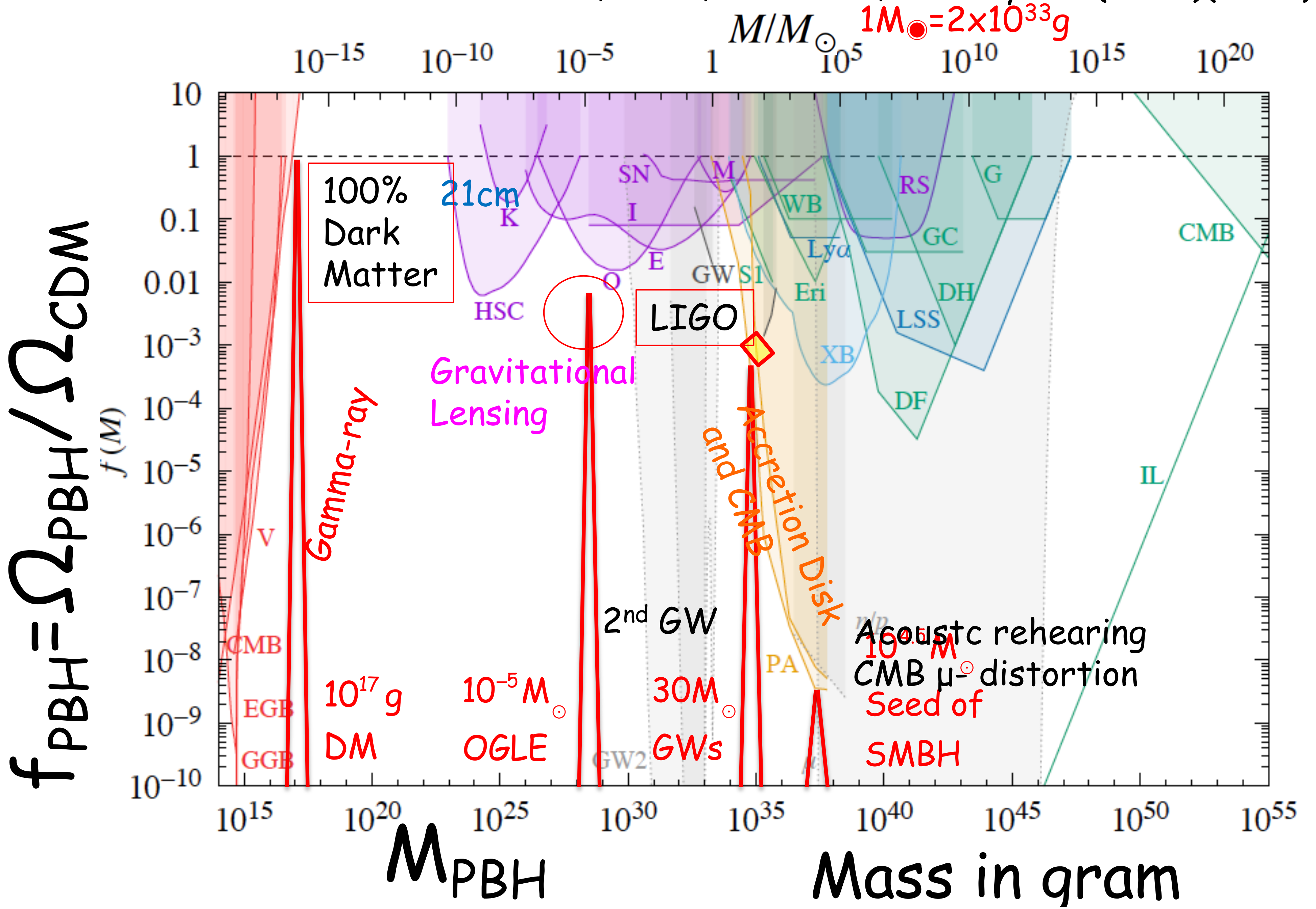
Masahiro Takada, Naoki Yasuda, Robert H. Lupton, Takahiro Sumi, Surhud More, Toshiki Kurita, Sunao Sugiyama, Anupreeta More, Masamune Oguri, Masashi Chiba, arXiv:1701.02151 [astro-ph.CO]



Reference point 1

Upper bounds on the fraction to CDM

Carr, Kohri, Sendouda, J.Yokoyama (2009)(2020)



NANOGrav15yr by Induced GW and sub-solar PBHs

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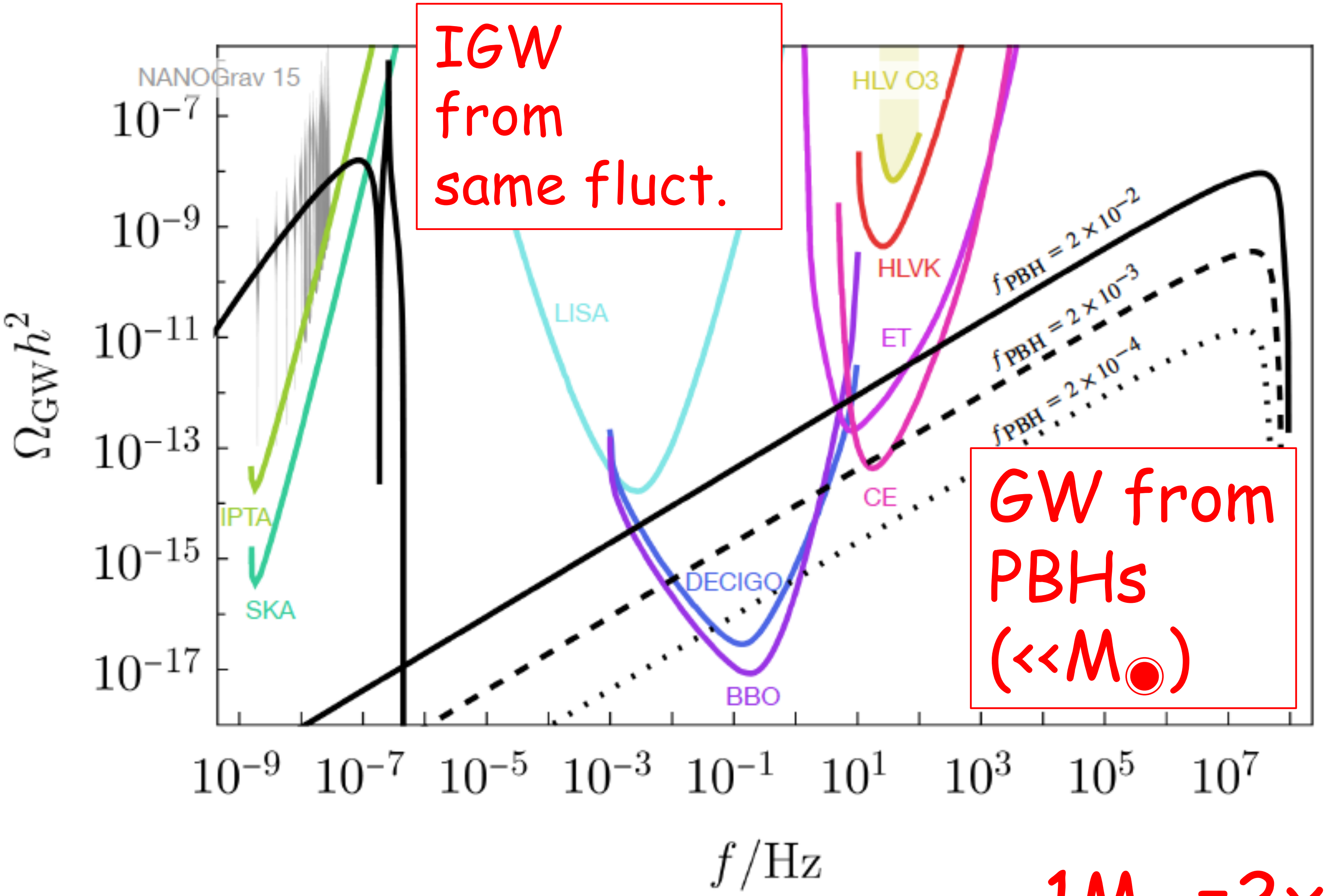


IGW from same fluct.

GW from PBHs ($\ll M_\odot$)

$1M_\odot = 2\times10^{33}$g

NANOGrav15yr and Inflation / Dark Matter

$1M_{\odot}=2\times10^{33}g$

- Possibility of cosmological nonlinear 2nd-order GW

$\Omega_{GW}\sim10^{-8} \propto \delta^4$ at $f \sim 10^{-8}$ Hz

- Suggesting big density fluctuation $<\delta^2>$ at small scale

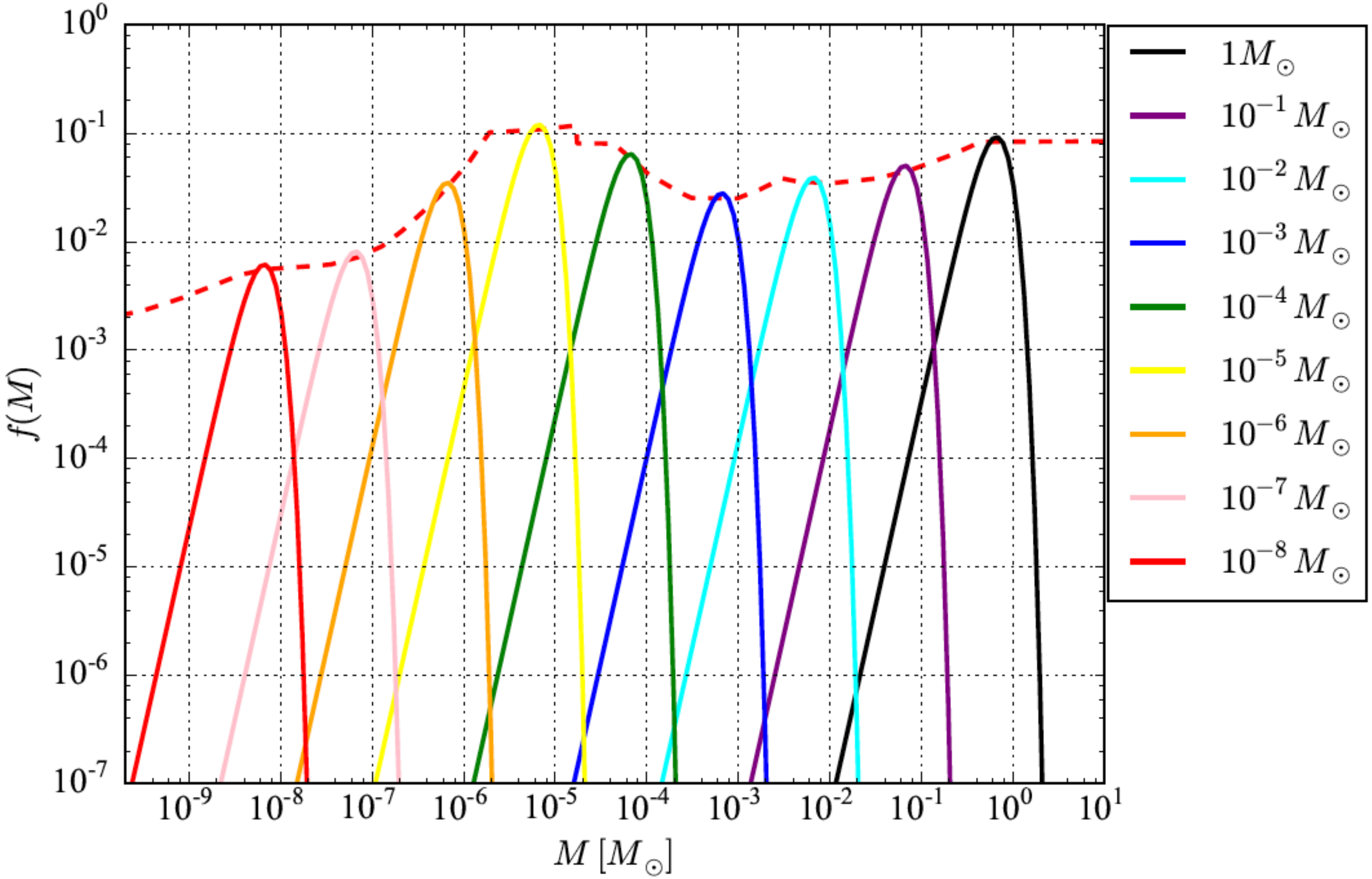
$\langle\delta^2\rangle \sim O(0.01) \gg 10^{-9}$ at $k \sim 10^7$ Mpc^{-1}

- The same fluctuation can simultaneously produce light primordial black holes much smaller than solar mass

$M_{BH} \sim O(10^{-5})\, M_{\odot}$, $f_{PBH} = \Omega_{PBH}/\Omega_{CDM} \sim O(0.01)$

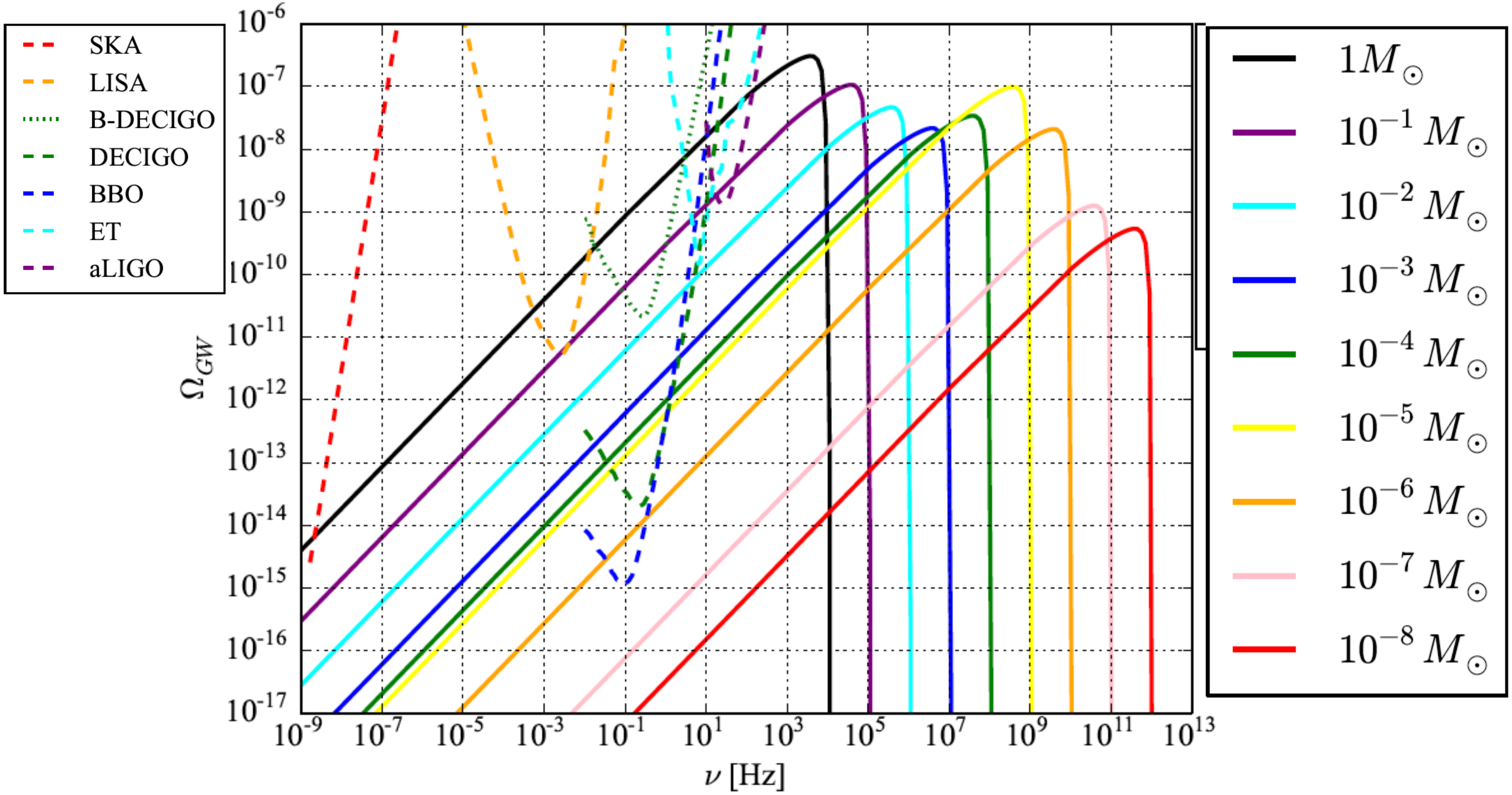
Subsolar-mass PBHs

S. Wang, K. Kohri, and T. Terada, arXiv:1903.05924v2 [astro-ph.CO]



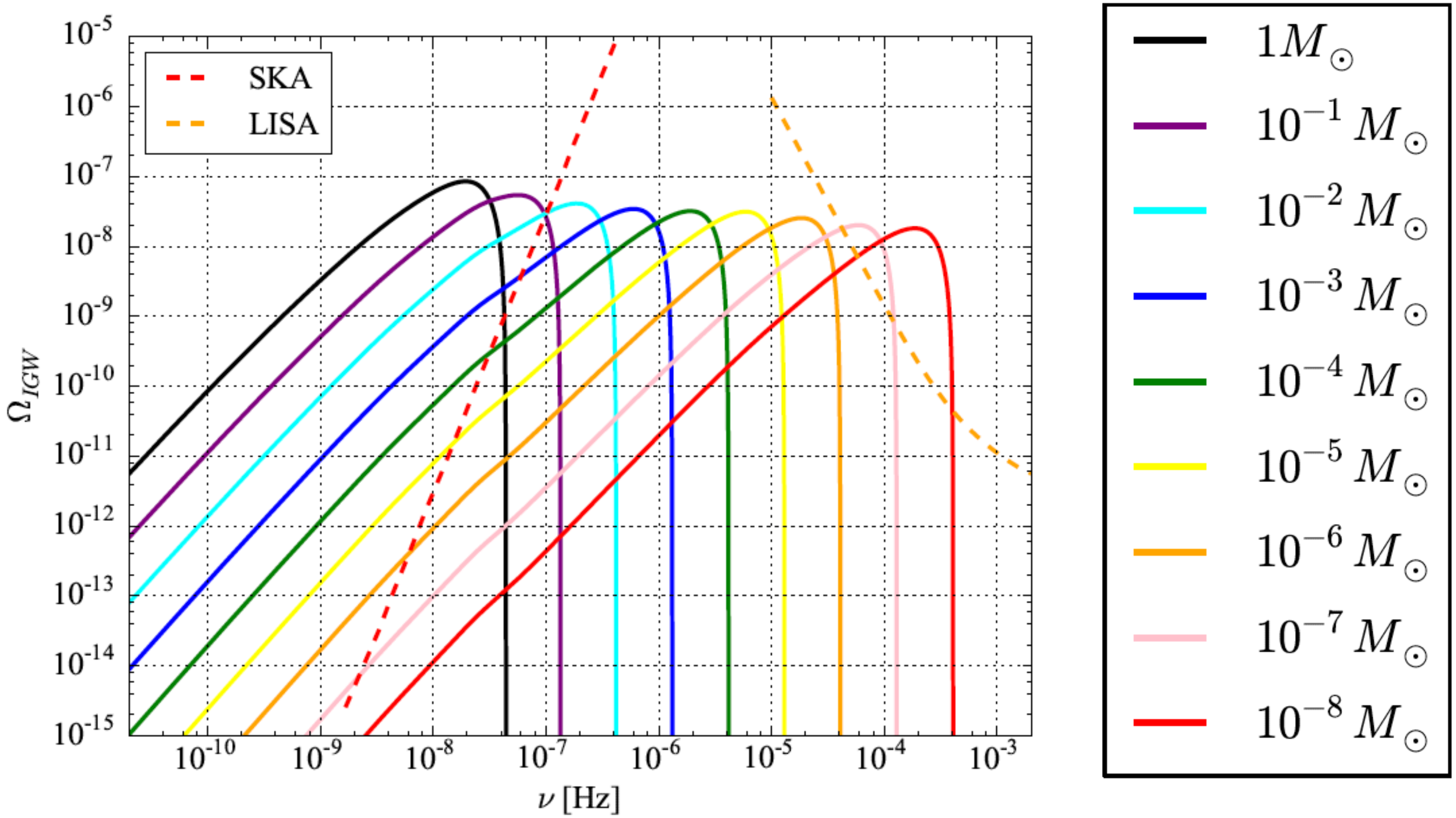
Merger signals from subsolar-mass binary PBHs

S. Wang, K. Kohri, and T. Terada, arXiv:1903.05924v2 [astro-ph.CO]



IGW from density perturbation to produce the same subsolar-mass PBHs

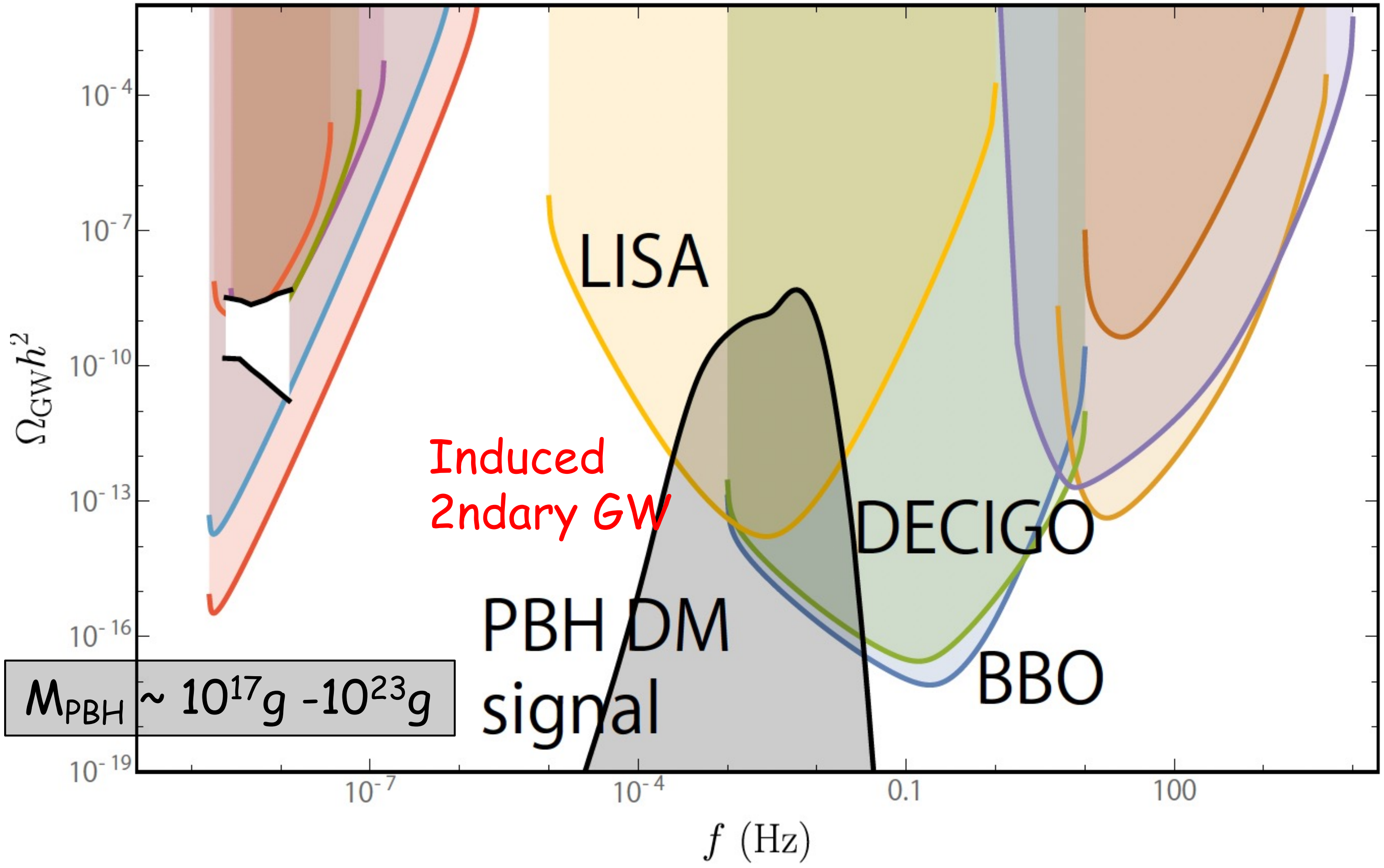
S. Wang, K. Kohri, and T. Terada, arXiv:1903.05924v2 [astro-ph.CO]



Primordial Black Holes and Second Order Gravitational Waves from Tachyonic Instability induced in Higgs-R^2 Inflation

Dhong Yeon Cheong, Kazunori Kohri, Seong Chan Park, arXiv:2205.14813 [hep-ph]

See also, K. Kohri and T. Terada, arXiv:2009.11853



Induced 2ndary GW

$M_{\rm PBH}$ ~ 10^{17}g - 10^{23}g

Gravitational wave search through electromagnetic telescopes

Asuka Ito, Kazunori Kohri, Kazunori Nakayama, arXiv:2309.14765 [gr-qc]

Gravitational wave search through electromagnetic telescopes

A. Ito, K. Kohri, K. Nakayama, arXiv:2309.14765 [gr-qc]
See also, M. E. Gertsenshtein, Sov. Phys. JETP 14 (1962) 84.
V. Domcke, C. Garcia-Cely, arXiv:2006.01161 [astro-ph.CO]
T. Fujita, K. Kamada, Y. Nakai, arXiv:2002.07548 [astro-ph.CO]

- Action of EM + gravity

$$S = \int d^4x \sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

$$\begin{cases} \mathcal{A}_\mu(x) = \bar{A}_\mu + A_\mu(x), \\ g_{\mu\nu}(x) = \eta_{\mu\nu} + \dfrac{2}{M_{\rm pl}}h_{\mu\nu}(x). \end{cases}$$

$h_{\mu\nu}$ has mass dimension one

$$S = \int d^4x \sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \boxed{+\frac{\alpha^2}{90m_e^4}\left((F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right)}\right].$$

Euler-Heisenberg term

Gravitational wave search through electromagnetic telescopes

Asuka Ito, Kazunori Kohri, Kazunori Nakayama, arXiv:2309.14765 [gr-qc]

- 2nd-order action

$$\delta S^{(2)} = \int d^4x \Big[-\frac{1}{2}(\partial_\mu h_{ij})^2 - \frac{1}{2}(\partial_\mu A_i)^2 + \frac{2}{M_{\rm pl}}\epsilon_{ijk}\bar{B}^k h^{jl}\partial_i A^l$$

h + B → γ

$$+\frac{\alpha^2}{90 m_e^4}\Big(16\bar{B}^i\bar{B}^j\Big(\delta_{ij}(\partial_k A_l)^2 - (\partial_k A_i)(\partial_k A_j) - (\partial_i A_k)(\partial_j A_k)\Big) + 28\Big((\partial_0 A_i)\bar{B}_i\Big)^2\Big)\Big]$$

$$\left[i\partial_z + \begin{pmatrix} -\frac{1}{2\omega}\frac{\omega^2\omega_{p,(i)}^2}{\omega^2-\omega_{c,(i)}^2} + \frac{1}{2\omega}\frac{16\alpha^2\bar{B}^2\omega^2}{45 m_e^4} & i\frac{B}{\sqrt{2}M_{\rm pl}} \\ -i\frac{B}{\sqrt{2}M_{\rm pl}} & 0 \end{pmatrix} \right] \begin{pmatrix} A^+(z) \\ h^+(z) \end{pmatrix} \simeq 0, \tag{13}$$

$$\left[i\partial_z + \begin{pmatrix} -\frac{\omega_{p,(i)}^2}{2\omega} + \frac{1}{2\omega}\frac{28\alpha^2\bar{B}^2\omega^2}{45 m_e^4} & i\frac{B}{\sqrt{2}M_{\rm pl}} \\ -i\frac{B}{\sqrt{2}M_{\rm pl}} & 0 \end{pmatrix} \right] \begin{pmatrix} A^\times(z) \\ h^\times(z) \end{pmatrix} \simeq 0. \tag{14}$$

Gravitational wave search through electromagnetic telescopes

Asuka Ito, Kazunori Kohri, Kazunori Nakayama, arXiv:2309.14765 [gr-qc]

- Oscillation probability

$$P^{(\times)}_{(i)}(A^\times \leftrightarrow h^\times) = \frac{\frac{8\bar{B}^2\omega^2}{M_{\rm pl}^2}}{\left(\omega_{p,(i)}^2 - \frac{28\alpha^2\bar{B}^2\omega^2}{45m_e^4}\right)^2 + \frac{8\bar{B}^2\omega^2}{M_{\rm pl}^2}} \times \sin^2\left(\frac{\sqrt{\left(\omega_{p,(i)}^2 - \frac{28\alpha^2\bar{B}^2\omega^2}{45m_e^4}\right)^2 + \frac{8\bar{B}^2\omega^2}{M_{\rm pl}^2}}}{4\omega}\Delta r\right)$$

The plus mode is small M. E. Gertsenshtein, Sov. Phys. JETP 14 (1962) 84.

$$P^{(+)}_{(i)}(A^+ \leftrightarrow h^+) = \frac{\frac{8\bar{B}^2\omega^2}{M_{\rm pl}^2}}{\left(\frac{\omega^2\omega_{p,(i)}^2}{\omega^2-\omega_{c,(i)}^2} - \frac{16\alpha^2\bar{B}^2\omega^2}{45m_e^4}\right)^2 + \frac{8\bar{B}^2\omega^2}{M_{\rm pl}^2}} \times \sin^2\left(\frac{\sqrt{\left(\frac{\omega^2\omega_{p,(i)}^2}{\omega^2-\omega_{c,(i)}^2} - \frac{16\alpha^2\bar{B}^2\omega^2}{45m_e^4}\right)^2 + \frac{8\bar{B}^2\omega^2}{M_{\rm pl}^2}}}{4\omega}\Delta r\right)$$

Plasma freq. $$\omega_p = \sqrt{\frac{4\pi\alpha n_e}{m_e}}.$$

Synchrotron freq. $$\omega_{c,(i)} = 1.8 \times 10^{19} \times \left(\frac{511\,\mathrm{keV}}{m_i}\right)\left(\frac{\bar{B}}{10^{12}\,\mathrm{G}}\right)\mathrm{Hz},$$

QED mass $$\omega_{\rm QED} \equiv \frac{\alpha\bar{B}\omega}{m_e^2} = 3.4 \times 10^{16} \times \left(\frac{\bar{B}}{10^{12}\,\mathrm{G}}\right)\left(\frac{\omega/2\pi}{10^{19}\,\mathrm{Hz}}\right)\mathrm{Hz},$$

Mixing $$\Omega \equiv \sqrt{\frac{8\bar{B}\omega}{M_{\rm pl}}} = 2.5 \times 10^{9} \times \left(\frac{\bar{B}}{10^{12}\,\mathrm{G}}\right)^{1/2}\left(\frac{\omega/2\pi}{10^{19}\,\mathrm{Hz}}\right)^{1/2}\mathrm{Hz},$$

Passing some segments with their magnetic field

Asuka Ito, Kazunori Kohri, Kazunori Nakayama, arXiv:2309.14765 [gr-qc]

- Probability of oscillation

$$P^{(\sigma)} = N_{\rm G} \frac{\frac{8\tilde{B}_{\rm G}^2\omega^2}{M_{\rm pl}^2}}{\left(\omega_p^2 - \omega_{{\rm QED},\sigma}^2 - \omega_{\rm CMB}^2\right)^2 + \frac{8\tilde{B}_{\rm G}^2\omega^2}{M_{\rm pl}^2}} \times \sin^2\left(\frac{l_{\rm G}}{l_{os}}\right)$$

$$\begin{aligned} \omega_p &= \sqrt{\frac{4\pi\alpha n_e}{m_e}}, \\ \omega_{{\rm QED},\sigma} &= \sqrt{\frac{8\lambda_\sigma\alpha^2\omega^2B^2}{45m_e^4}}, \\ \omega_{\rm CMB} &= \sqrt{\frac{88\pi^2\alpha^2\omega^2T^4}{2025m_e^4}} \end{aligned}$$

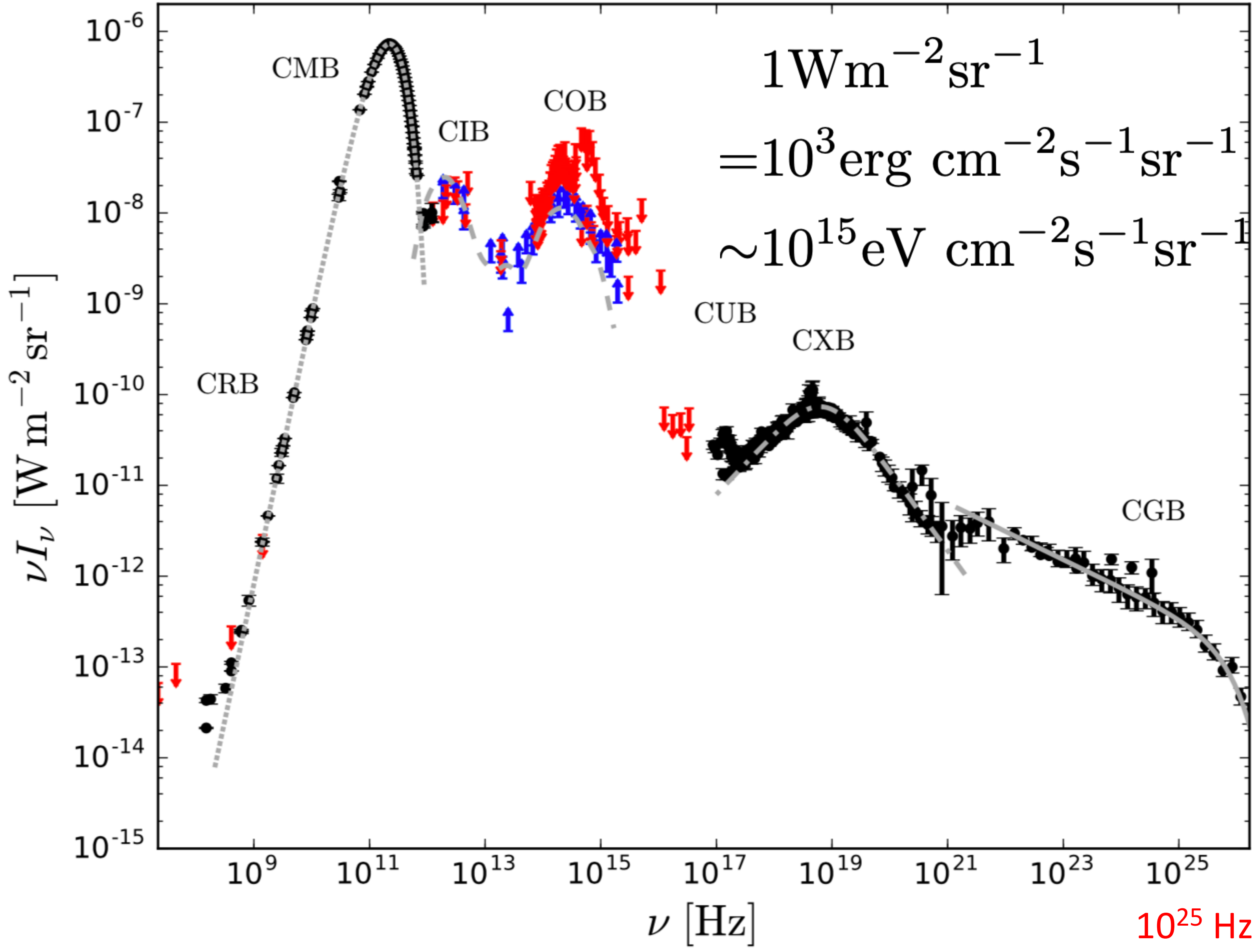
N_G : a number of patches for coherent B
l_G : The length of a patch for coherent B

- Oscillation length

$$l_{os} = \frac{4\omega}{\sqrt{\left(\omega_p^2 - \omega_{{\rm QED},\sigma}^2 - \omega_{\rm CMB}^2\right)^2 + \frac{8\tilde{B}_{\rm G}^2\omega^2}{M_{\rm pl}^2}}}$$

Cosmic photon background

Ryley Hill, Kiyoshi W. Masui, Douglas Scott, arXiv:1802.03694 [astro-ph.CO]

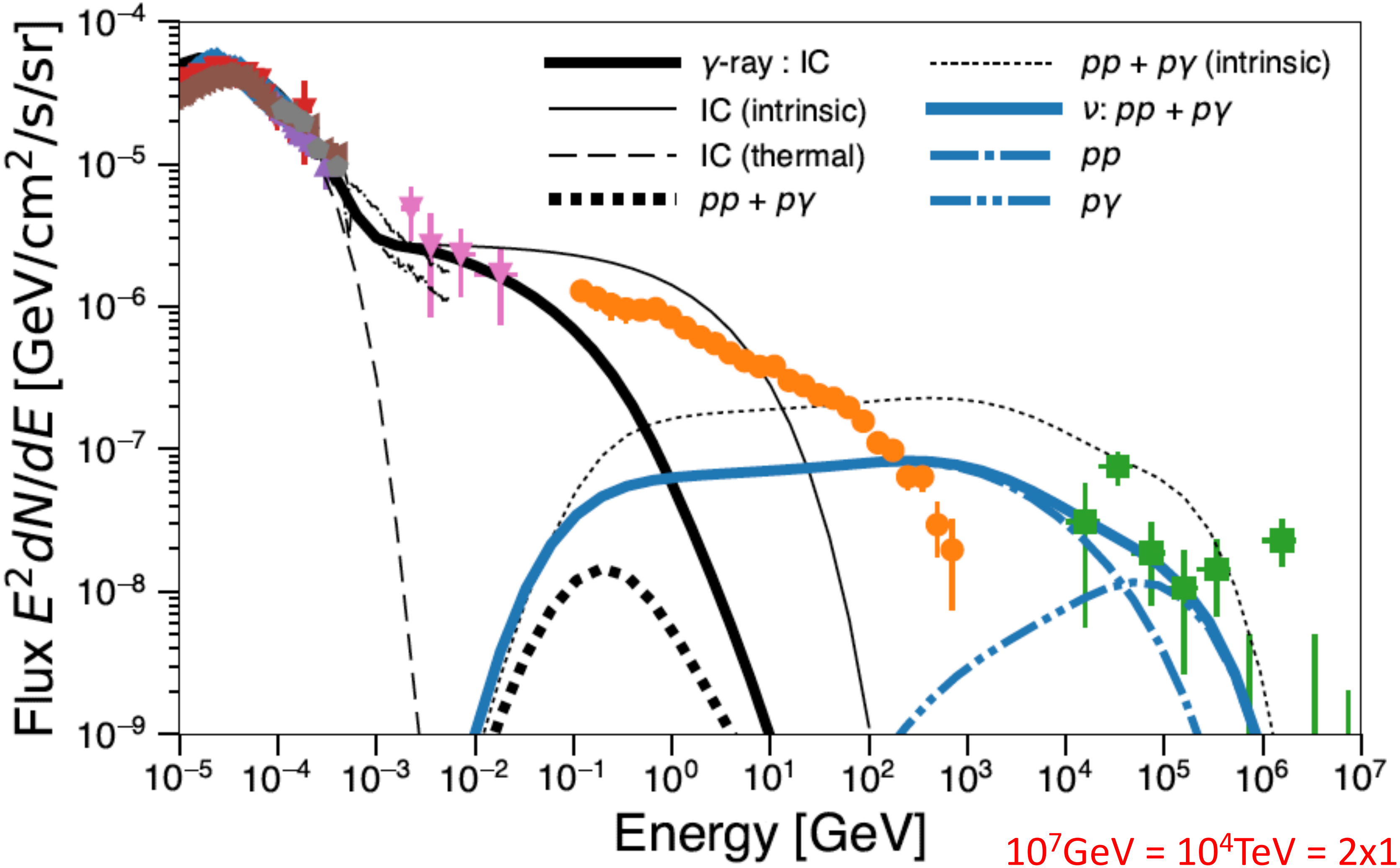


$$1\mathrm{W m^{-2} sr^{-1}}$$
$$= 10^{3}\mathrm{erg\ cm^{-2} s^{-1} sr^{-1}}$$
$$\sim 10^{15}\mathrm{eV\ cm^{-2} s^{-1} sr^{-1}}$$

10^{25} Hz = 5 GeV

Cosmic X-ray and gamma-ray background [and neutrino background]

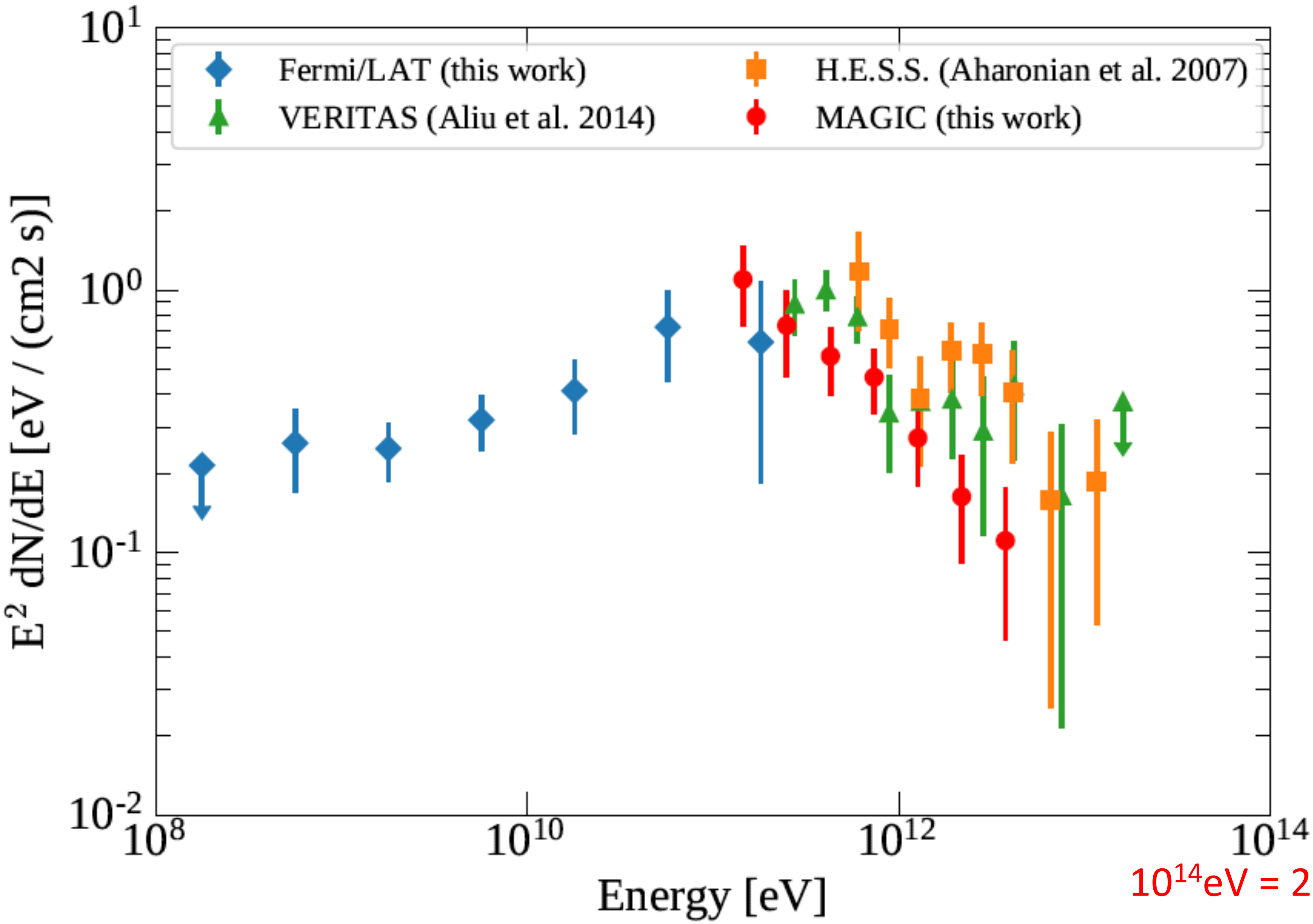
Yoshiyuki Inoue, Dmitry Khangulyan, Susumu Inoue, Akihiro Doi, arXiv:1904.00554 [astro-ph.HE]



10^7GeV = 10^4TeV = $2\text{x}10^{31}$ Hz

TeV gamma-rays from the extragalactic source (AGN (z=0.14), 1ES 0229+200)

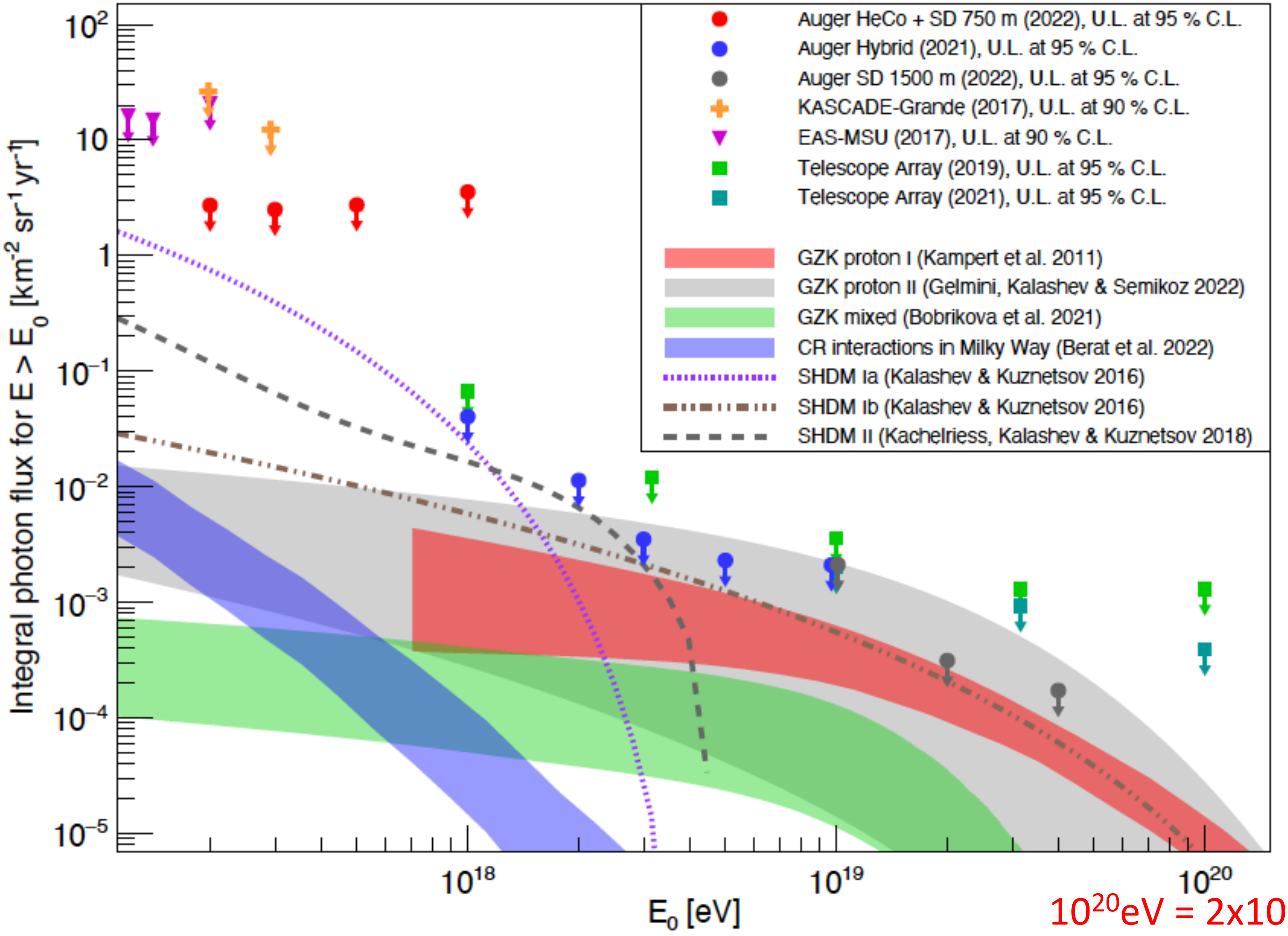
V. A. Acciari et al, the MAGIC Collaboration, arXiv:2210.03321 [astro-ph.HE]



10^{14}eV = 2x10^{29} Hz

Searches for Ultra-High-Energy Photons at the Pierre Auger Observatory

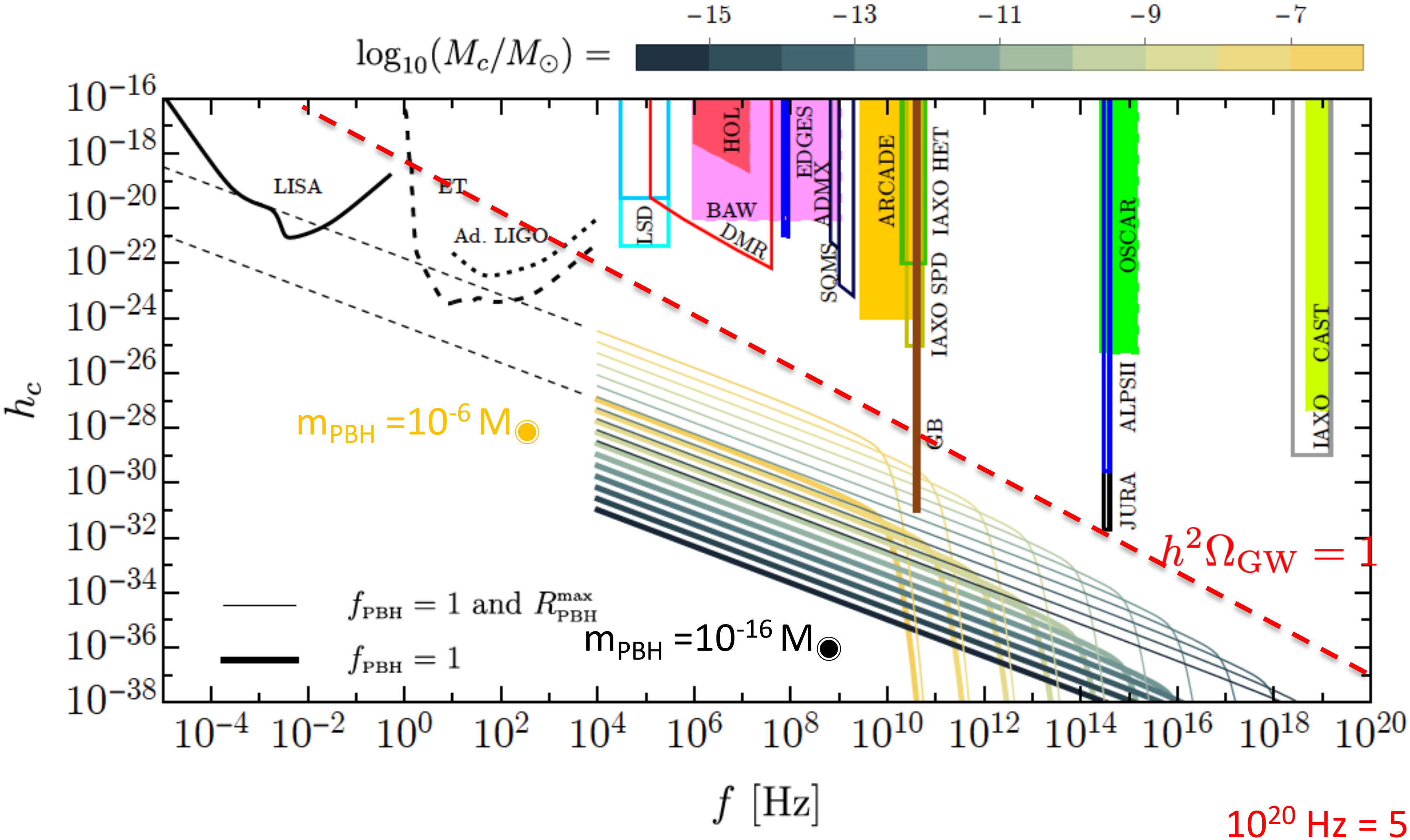
The Pierre Auger Collaboration: P. Abreu, et al., arXiv:2210.12959 [astro-ph.HE]



10^{20}eV = $2\mathrm{x}10^{35}$ Hz

Accumulated merger rates of binary PBHs with subsolar masses

Gabriele Franciolini, Anshuman Maharana, Francesco Muia, arXiv:2205.02153 [astro-ph.CO]



$m_{\rm PBH} = 10^{-6}\,M_\odot$

$m_{\rm PBH} = 10^{-16}\,M_\odot$

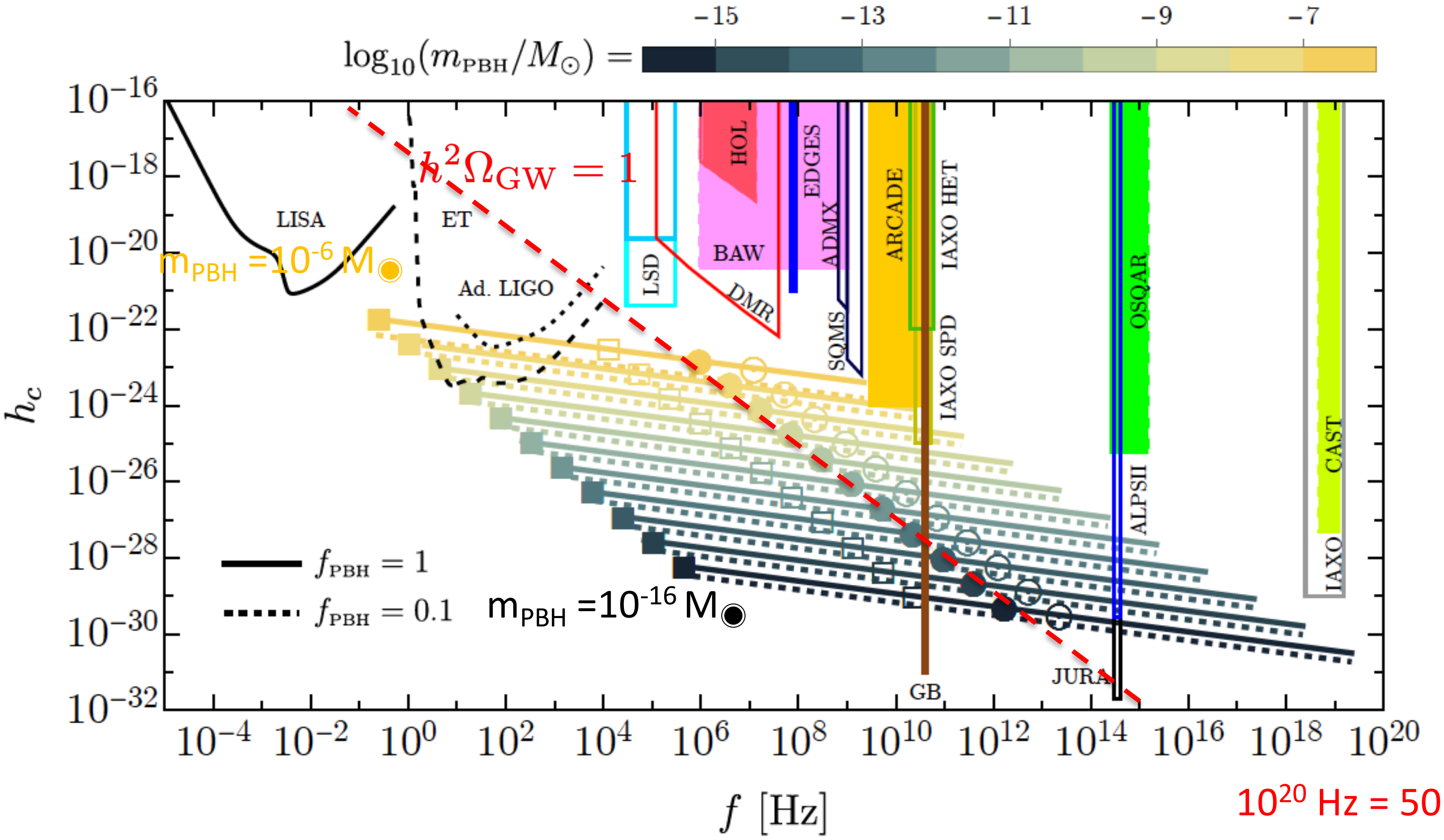
$h^2\Omega_{\rm GW} = 1$

10^{20} Hz = 50 keV

Rates for transient mergers of binary PBHs with subsolar masses

Gabriele Franciolini, Anshuman Maharana, Francesco Muia, arXiv:2205.02153 [astro-ph.CO]

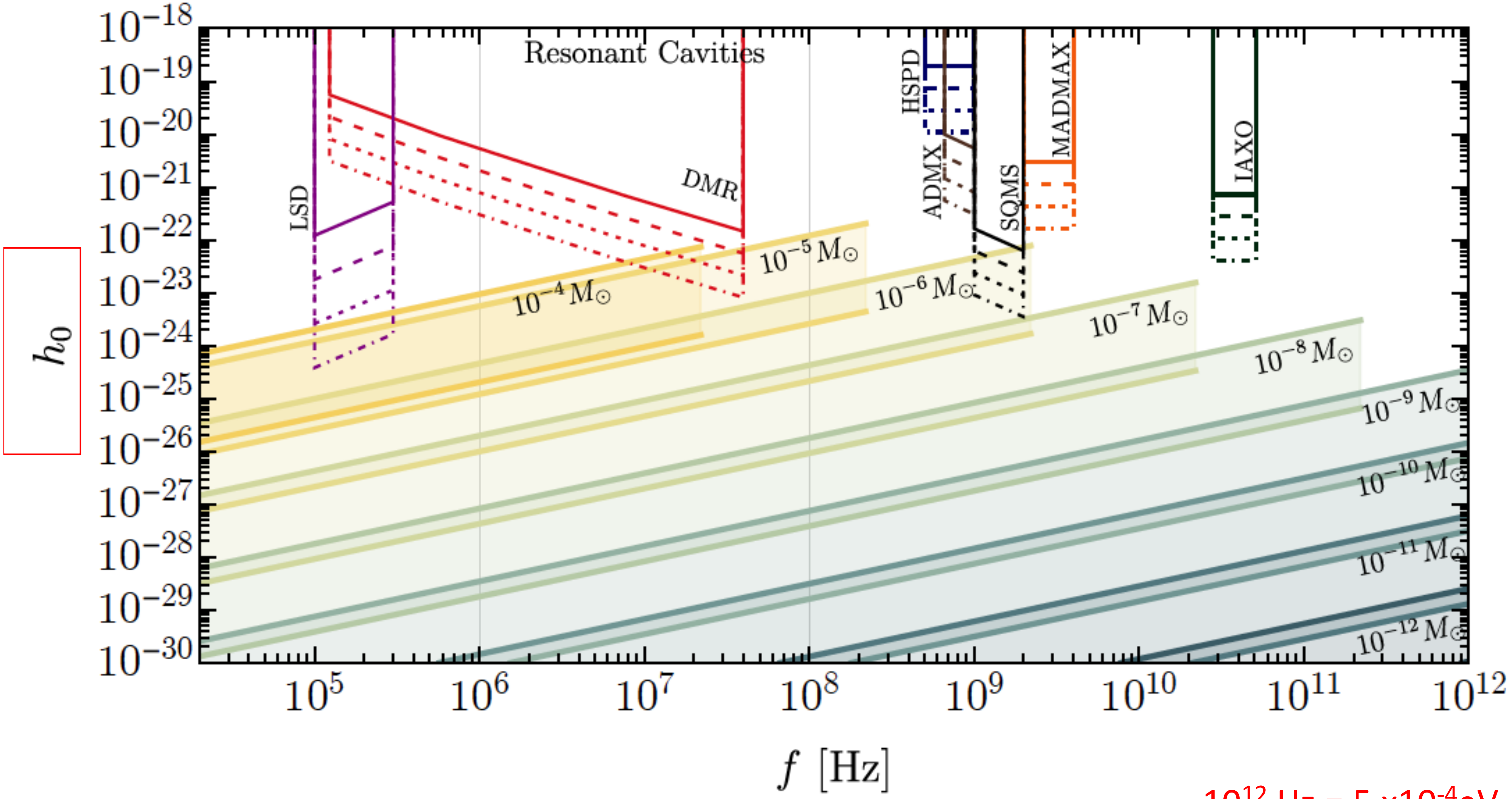
$$|h_c(f)| \simeq 4.54 \times 10^{-28} \left(\frac{m_{\rm PBH}}{10^{-12} M_\odot}\right)^{5/6} \left(\frac{d_{\rm L}}{\rm kpc}\right)^{-1} \left(\frac{f}{\rm GHz}\right)^{-1/6}$$



10^{20} Hz = 50 keV

Rates for transient mergers of binary PBHs with subsolar masses

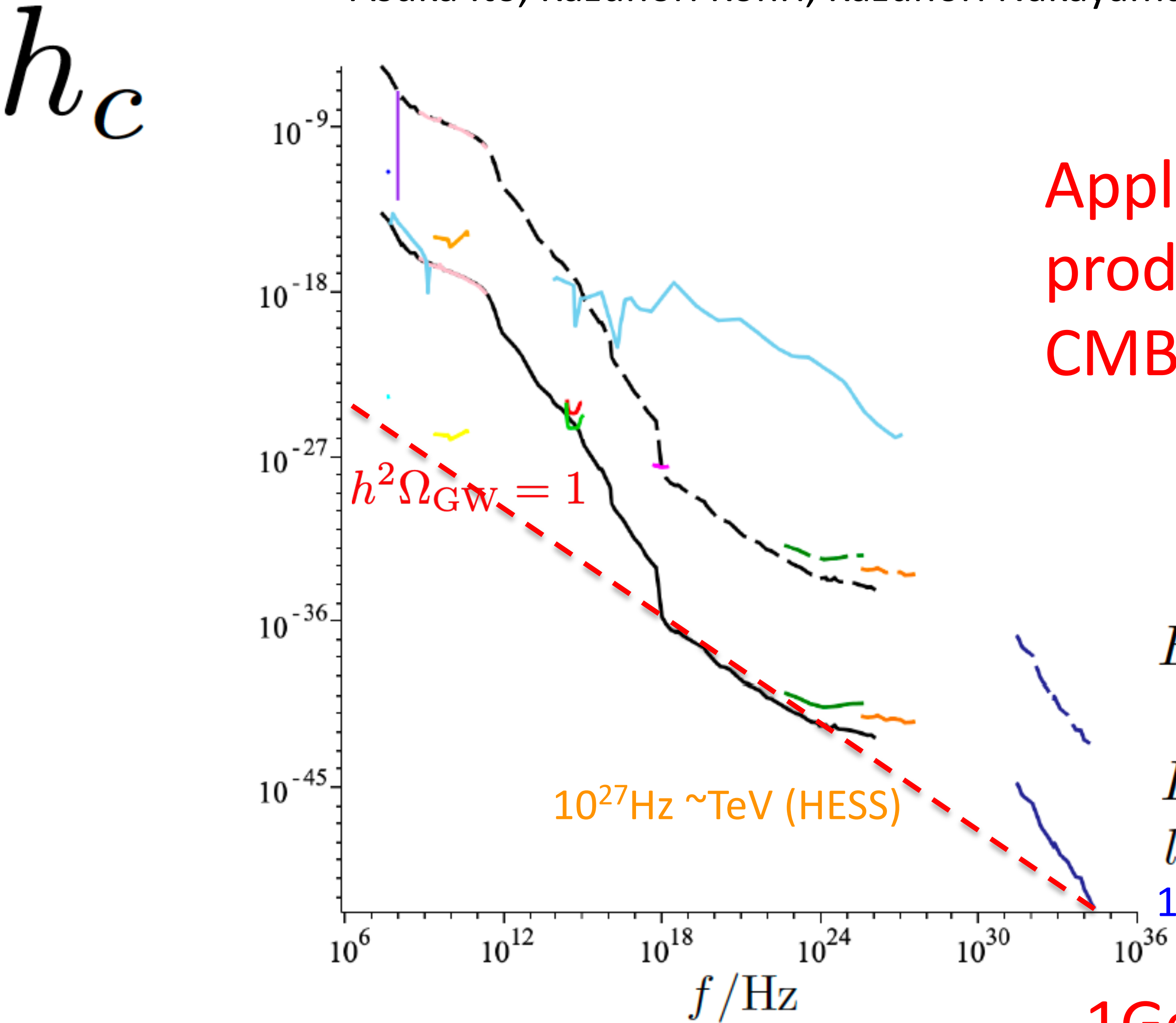
Gabriele Franciolini, Anshuman Maharana, Francesco Muia, arXiv:2205.02153 [astro-ph.CO]



10^{12} Hz = 5 x10^{-4}eV

Gravitational wave search through electromagnetic telescopes

Asuka Ito, Kazunori Kohri, Kazunori Nakayama, arXiv:2309.14765 [gr-qc]



Applicable for signals produced after CMB recombination

$B_{\mathrm{IG}} = 10^{-17}\mathrm{G}$

$B_{\mathrm{IG}} = 0.1\mathrm{nG}$

$l_{\mathrm{IG}} = 4000\mathrm{Mpc}$

10^{34}Hz ~10^{20}eV (Auger)

1GeV = 2x10^{24} Hz

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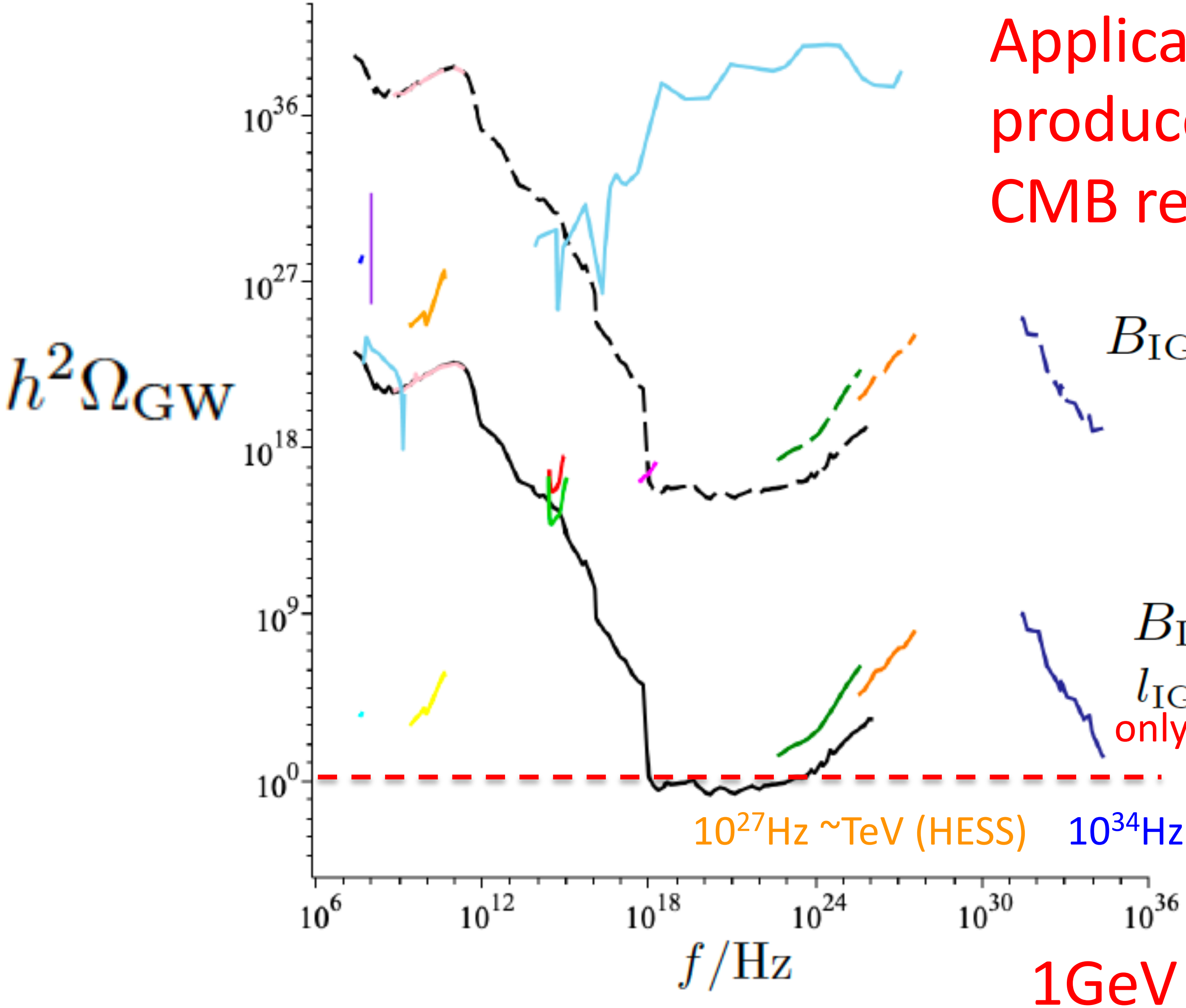

Applicable for signals produced after CMB recombination

$B_{\mathrm{IG}} = 10^{-17}\mathrm{G}$

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only within γ-ray's horizon

10^{27}Hz ~TeV (HESS) 10^{34}Hz ~10^{20}eV (Auger)

1GeV = 2x10^{24} Hz

Conclusion

- The search for high-frequency GWs is a new direction for investigating phenomena in the early Universe.
- The targets are so many:

1. GWs from merging binary PBHs with subsolar mass
2. Thermal/nonthermal graviton just after inflation,
3. 1st-order phase transition at E >> weak scale
4. …

- We can test high-frequency GWs by observing the electromagnetic wave converted from them.